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STAT 2122 HW#6

1a. $P(x_1=1, x_2=1) = 0.15$

1b. $P(x_1/x_1=0) + P(x_2/x_2=1) + P(x_2/x_2=2) + P(x_1/x_1=3)$

$0.08 + 0.15 + 0.10 + 0.07 = 0.40$

1c. $A = (x_1, x_2) = (0,2), (0,3), (1,3), (2,0), (3,0), (4,0), (3,1), (4,1), (4,2)$

$0.04 + 0.00 + 0.04 + 0.05 + 0.00 + 0.00 + 0.03 + 0.01 + 0.05 = 0.22$

1d. Exactly 4 $\rightarrow P(x_1+x_2=4) = (1,3), (2,2), (3,1), (4,0) \rightarrow 0.04 + 0.1 + 0.03 + 0 = 0.17$

At least 4 $\rightarrow P(x_1+x_2 \geq 4) = (1,3), (2,2), (3,1), (4,0), (2,3), (3,2), (4,1), (3,3), (4,2), (4,3)$

$0.04 + 0.01 + 0.03 + 0 + 0.06 + 0.04 + 0.01 + 0.07 + 0.05 + 0.06 = 0.46$

2a. $\int_{20}^{30} \int_{20}^{30} K(x^2+y^2) dx dy = 1$

$K \int_{20}^{30} (x^2+y^2) dx dy = \int_{20}^{30} x^2 dx + \int_{20}^{30} y^2 dx = \left[\frac{x^3}{3} \right]_{20}^{30} + y^2 \left[dx \right]_{20}^{30}$

$\frac{(30)^3}{3} - \frac{(20)^3}{3} = \frac{19000}{3} + y^2(30-20) = 10y^2 + \frac{19000}{3}$

$\int_{20}^{30} (10y^2 + \frac{19000}{3}) dy \rightarrow \left[\frac{10(y^3)}{3} + \frac{19000}{3} y \right]_{20}^{30} = \frac{19000}{3} \cdot 10 + \frac{19000}{3} \cdot 10 = \frac{380000}{3}$

$\frac{380000}{3} K = 1 \rightarrow K = \frac{3}{380000}$

2b. $\int_{20}^{26} \int_{20}^{26} K(x^2+y^2) dx dy \rightarrow 3192 + 6y^2 \rightarrow K \int_{20}^{26} (6y^2 + 3192) dy = 38304$

$K(38304) = \frac{3}{380000} (38304) = \frac{114912}{380000} = \frac{189}{625} \approx 0.3024$

2c. $\int_{20}^{30} K(x^2+y^2) dy \rightarrow K \int_{20}^{30} (x^2+y^2) dy = K \left[10x^2 + \frac{19000}{3} \right]$

$\frac{3}{380000} \left(10x^2 + \frac{19000}{3} \right) = \frac{30x^2}{380000} + \frac{19000}{380000} \rightarrow f_X(x) = \frac{30x^2 + 19000}{380000}$

2d. $\left(\frac{3x^2 + 19000}{380000} \right) \cdot \left(\frac{3y^2 + 19000}{380000} \right) = \frac{(3x^2 + 19000)(3y^2 + 19000)}{(380000)^2} = \frac{3}{380000} (x^2 + y^2)$

X and Y are NOT independent r.v's

$$3. f_{x,y}(x,y) = f_x(x) \cdot f_y(y) = 6x^2y \quad \begin{matrix} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{matrix}$$

$$E[|X-Y|] = \int_0^1 \int_0^x (x-y)(6x^2y) dy dx$$

$$\int_0^x (x-y)(6x^2y) dy = x \left(\frac{x^2}{2} \right) - \frac{x^3}{3} = \frac{x^3}{2} - \frac{x^3}{3} = \frac{3x^3 - 2x^3}{6} = \frac{x^3}{6}$$

$$\downarrow$$

$$6x^2 \cdot \frac{x^3}{6} = x^5$$

$$\int_0^1 x^5 dx = \frac{1}{6} \rightarrow E[|X-Y|] = \frac{8}{12} + \frac{2}{12} - \frac{9}{12} = \frac{1}{12} \text{ or } 5 \text{ mins}$$