Math 29 Writing Assignment

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1 Theorem 1. The Fish Problem

Is it possible to exactly fill an order for n pounds of fish with for integer $n \geq 32$ given an unlimited number of 5-pound and 9-pound fish?

Proof. We prove by induction.
Base case, $n = 32$: Observe

$$5(1) + 9(3) = 32 -> 5 + 27 = 32$$
$$32 = 32$$

Inductive Step:
Let x represent the number of 9-pound fish and y represent the number of 5-pound fish.

Assuming 5x + 9y $n \geq 32$, we will show $P(n)$ is true for $n + 1$.

If $x \geq 1$, then one 9-pound fish can be substituted with two 5-pound fish, which is equal to 10 pounds. This increases the total amount of pounds by one, satisfying the $n + 1$ inductive step.

Similarly, if $y \geq 7$, then we can substitute seven 5-pound (35 pounds) fish with four 9-pound fish (36 pounds), which also leads to filling up an order by $n + 1$.

Therefore, we have proven by induction that it is possible to exactly fill an order for n pounds of fish with for integer $n \geq 32$ given an unlimited number of 5-pound and 9-pound fish.

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