Exercise 1

After performing trial and error to determine the order of the model, an appropriate model structure with na = 1, nb = 9, and nk = 5 was chosen. As shown in Figure 1, the ARX model achieves a high fit of 87.41%, indicating that most of the measured output is accurately predicted by the model. Also, residual analysis, as shown in Figure 2, reveals that the autocorrelation of the residuals and the cross-corelation between residuals and inputs are well within the confidence interval of 99% for all samples values shown in the x axis. This suggests that the residuals are independent from each other in time and independent from the input, meaning they can be considered as white noise. Thus, it can be concluded that all useful information has been extracted and incorporated into the model.

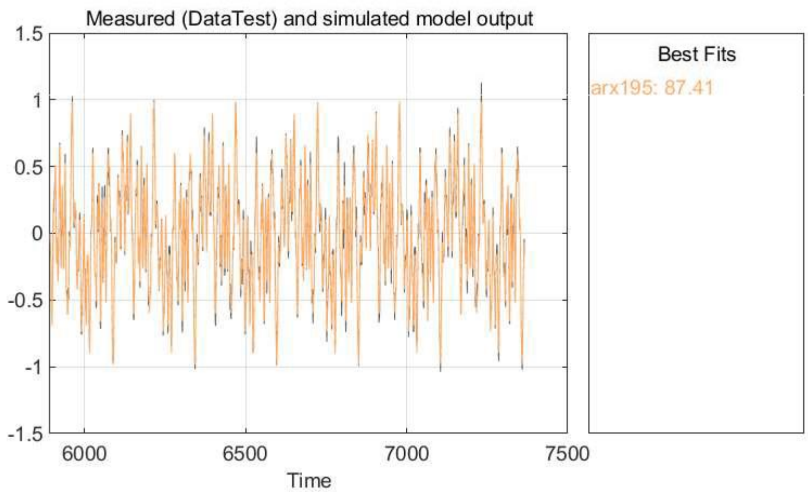

Figure 1 Comparison of model and measured output using ARX model

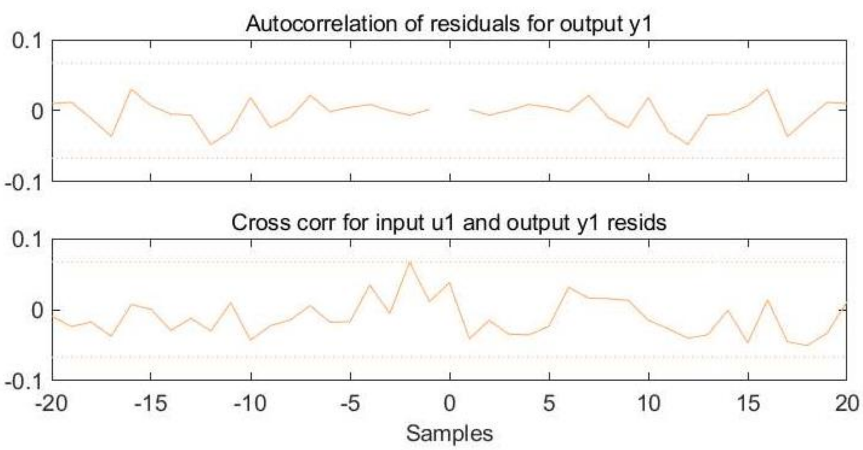

Figure 2 Residual analysis using ARX model

Exercise 2

Three different OE models, labeled as model oe125, oe235, and oe185, are identified, and their respective coefficients are shown in Table 1. Model oe125 is the simplest model, with the lowest order terms (nb = 1 and nf = 2), making it the most computationally efficient. However, as seen in Figure 3, it has the lowest fit percentage (79.26%) compared to the other models. This is further supported by residual analysis in Figure 4, where the autocorrelation and cross-corelation of the residuals occasionally fall outside of the confidence interval. Thus, indicating that underfitting is present in the model.

Model oe185 achieves the highest fit percentage of 87.44%, as shown in Figure 3, and has good residual analysis results, with all autocorrelation and cross-correlation values within the confidence interval along all sample values (Figure 4). However, it is also noted that model oe185 has the highest order terms, with nf = 8 (Table 1). This increases its complexity and could result in a heavier computational load.

As for model oe235, it has a high fit percentage (86.98%) as shown in Figure 3 while also having good autocorrelation and cross-corelation results where all the values are inside the confidence interval as shown in Figure 4. Model oe235 achieves a balance between model accuracy and complexity by having less order terms than model oe185. A downside model oe235 might have is that it may miss finer details of the system dynamics compared to model oe185, as indicated by its slightly lower fit percentage.

Table 1 Variation of OE models

Model	nb	nf	nk	Model equation
oe125	1	2	5	B(z) = 0.1238 z^-5 F(z) = 1 - 1.468 z^-1 + 0.5962 z^-2
oe235	2	3	5	B(z) = 0.07956 z^-5 + 0.05349 z^-6 F(z) = 1 - 1.595 z^-1 + 0.9072 z^-2 - 0.1799 z^-3
oe185	1	8	5	B(z) = 0.07982 z^-5 F(z) = 1 - 2.254 z^-1 + 2.376 z^-2 - 1.69 z^-3 + 1.014 z^-4 - 0.5333 z^-5 + 0.2129 z^-6 - 0.04877 z^-7 + 0.002556 z^-8

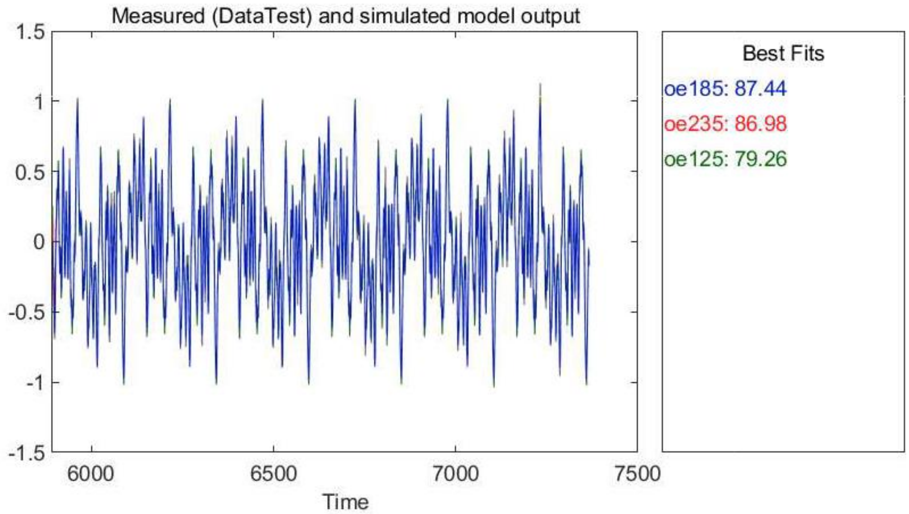

Figure 3 Comparison of model and measured output using OE model

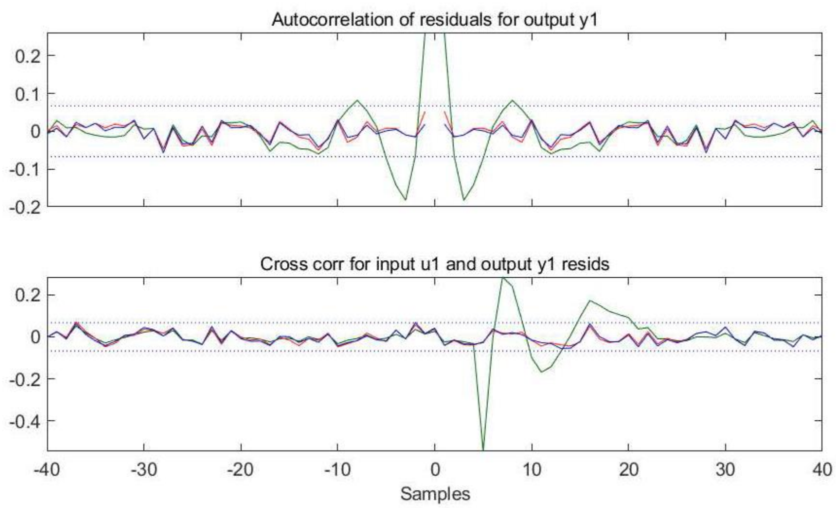

Figure 4 Residual analysis using OE model

Exercise 3

Based on the results of Exercise 2, it can be concluded that model oe235 is the most appropriate choice. This is because it does not underfit as model oe125 and also does not use as many terms as model oe185. By observing Figure 4, model oe235 and oe185 have similar autocorrelation and cross-correlation results, with all values lying within the confidence interval, while model oe235 achieves this with fewer terms. Also, by examining the model equations in Table 1, as the order of terms increases in model oe185, its coefficients tend to approach zero, indicating that the model's order is higher than necessary, and the extra parameters may be modelling measurement noise. Thus, model oe235 is the appropiate choice, offering better balance between model complexity and accuracy.

Exercise 4

Based on the outcomes of Exercise 1 and Exercise 3, it was concluded that model arx195 and oe235 were the most appropriate models. Both models were compared by observing their fit percentage, as shown in Figure 5. The ARX model performs slightly better, with a 0.43% higher fit percentage. However, it achieves this with twice the number of parameters than the OE model, with 10 total parameters for the ARX model and 5 for the OE model. This higher parameter count makes the ARX model more complex and computationally heavier.

Further comparison between the models was conducted through residual analysis, as shown in Figure 6. In Figure 6, both models display autocorrelation and cross-correlation values that remain within the confidence interval across all sample values, indicating a good fit. Additionally, the models were compared by examinining a histogram plot of their residuals, which are shown in Figure 7. From this figure, it is observed that the ARX model as well as OE model are both normally distributed.

Upon closer inspection of the normal distribution shown in Figure 7, it reveals that OE model has a tighter spread and greater density around the zero point. Also, the peak of both distributions is very close, with the OE model just slightly having a higher peak. This indicates that the OE model has more residuals close to zero, implying that it may be more accurate in minimizing residual values. Thus, it can be concluded that the OE model is the more appropriate choice, as it achieves good autocorrelation and cross-correlation results, a sufficient fit, and a higher concentration of residuals around zero, all with fewer model parameters.

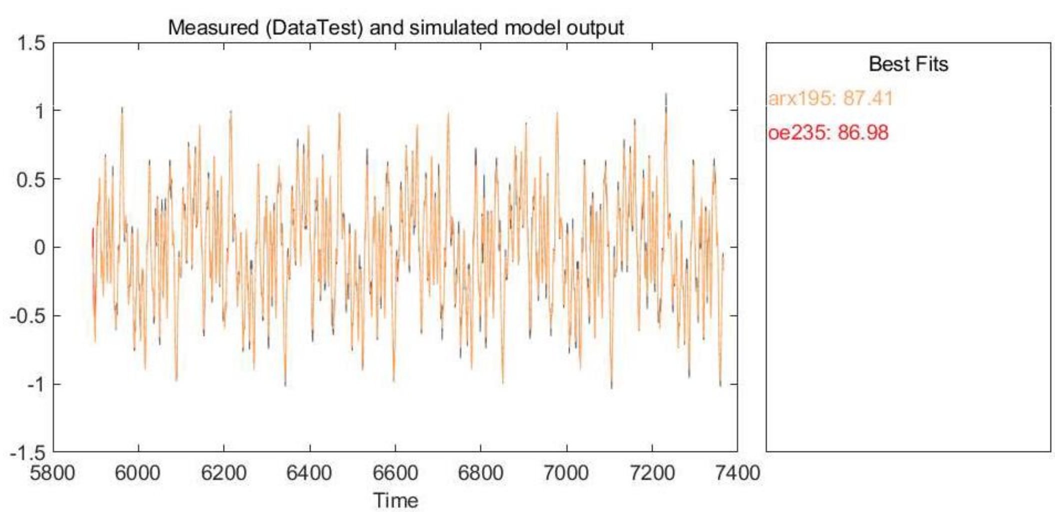

Figure 5 Comparison of model and measured output using ARX and OE model

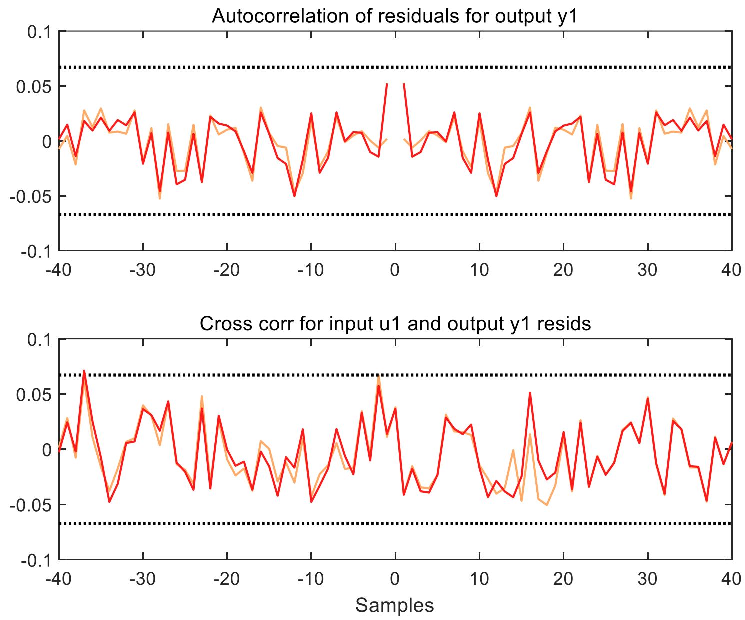

Figure 6 Residual analysis between ARX and OE model

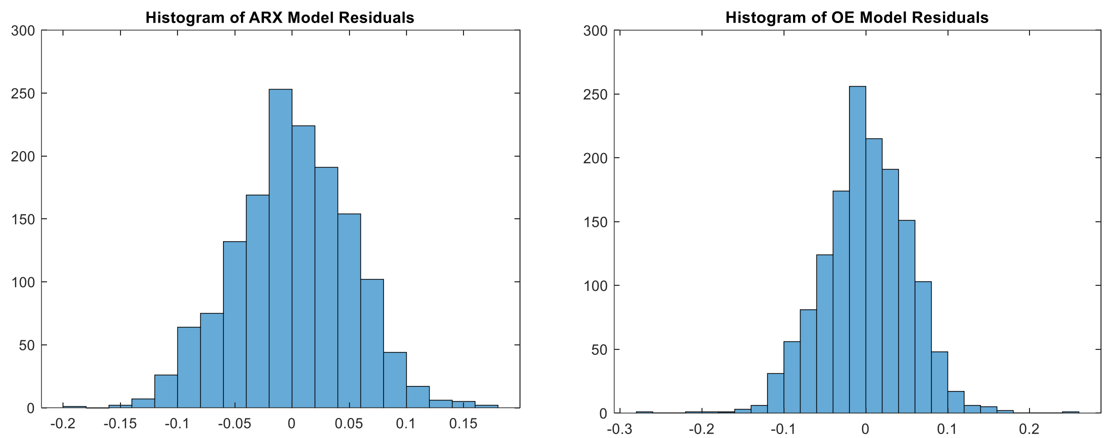

Figure 7 Histogram comparison of ARX and OE models