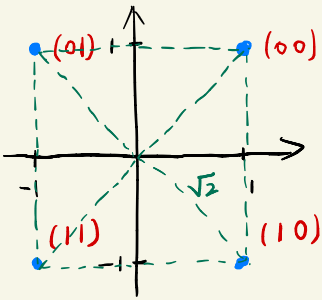
ZP-OFDM Transmitter Design and Validation

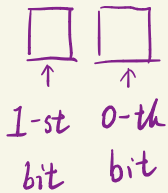
Flowchart overview

Baseband ZP-OFDM signal → insert zero → filtered rcosine → Bassband to passband → LFM concatenation →

---

1. generate QPSK symbol sequence S





$00 \to \frac{1}{\sqrt{2}}[1+1j]$

$01 \to \frac{1}{\sqrt{2}}[-1+1j]$

$10 \to \frac{1}{\sqrt{2}}[-1-1j]$

$11 \to \frac{1}{\sqrt{2}}[1-1j]$

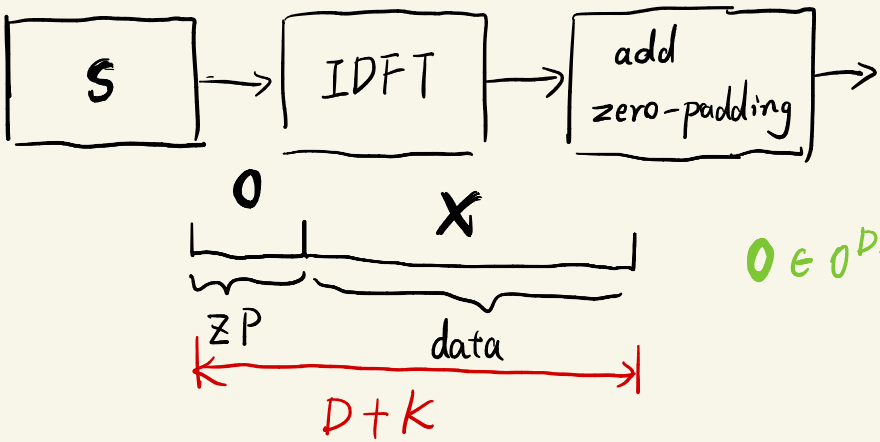
---

2. implementation for one ofdm symbol

$$x[n] = \sum_{k=0}^{K-1} s[k] e^{j2\pi \frac{kn}{K}}, \quad n = [0, 1, \cdots, K-1]$$

subcarrier data : $\mathbf{S} = [s[0], s[1], \cdots, s[K-1]]^T \in \mathbb{C}^{K\times 1}$

time-domain data $\mathbf{X} = [x[0], x[1], \cdots, x[K-1]]^T \in \mathbb{C}^{K\times 1}$



$\mathbf{0} \in 0^{D\times 1}$

3. Concatenation

W is the number of OFDM symbols.

$$\bar{X} = [\mathbf{0}\ \mathbf{x}_1\ \mathbf{0}\ \mathbf{x}_2\ \mathbf{0}\ \mathbf{x}_3\ \mathbf{0}, \cdots, \mathbf{0}, \mathbf{x}_W, \mathbf{0}]^T$$

$$\bar{X} \in \mathbb{C}^{(W(D+k)+D)\times 1}$$, length is $W(D+k)+D$

4. Insert zero

λ: oversampling factor $\lambda = 24$

$$\bar{X}_{ovF}[n] = \begin{cases} \bar{X}[n/\lambda], & \text{if } \mathrm{mod}(n,\lambda)=0 \\ 0, & \text{else} \end{cases}$$

$$n = [0, 1, \cdots, (W(D+k)+D)\lambda - 1]$$

5. filter with rcosine function $R[n]$

$$\tilde{X}_{BB}[n] = \bar{X}_{ovF}[n] * R[n]$$

✳ Hint: MATLAB command: rcosine(1, Fs/Fd, 'sqrt', β, delay)

key parameters: 1) roll-off factor $\beta = 0.125$

2) oversampling factor $\lambda = \frac{F_s}{F_d} = 24$

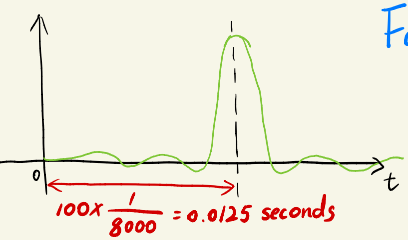
F_s: filter sampling frequency $F_s = 192$ kHz

F_d (or R_s): data/transfer rate $F_d = 8$ kHz

3) Type: "sqrt" square root raised cosine

4) delay: 100 samples before oversampling

delay = 100



$R[n]$ is a discrete filter with the length of $2L+1$,

where $L = 100 \times 24 = 2400$, so the total length is 4801.

✳ You can observe these signals in Frequency domain.

Hint: MATLAB command: fftshift(fft(*))

---

6. Bandwidth: $B = 8000$ Hz ; # of subcarriers: $K = 2048$

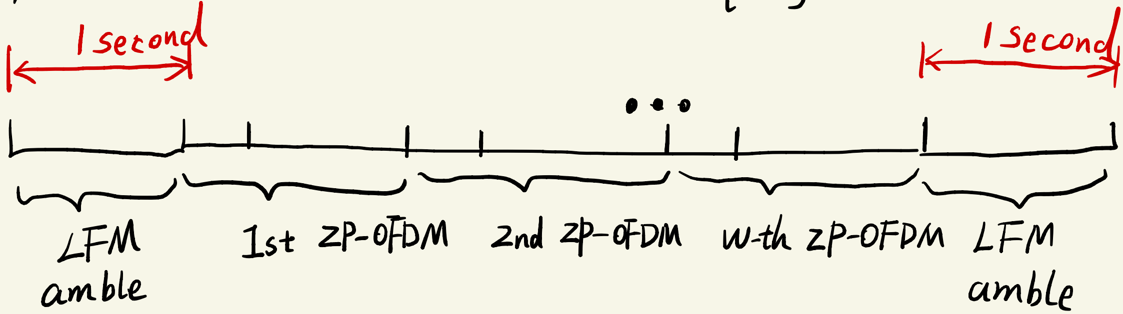
Subcarrier spacing: $\Delta f = \frac{B}{K} \approx 3.91$ Hz

$T_s = \frac{1}{B}$, $T = \frac{1}{\Delta f} = 0.256$ secs, $T_g = 0.025$ secs

Baseband to passband: f_c: the carrier frequency

$$X_{PB}[n] = \mathrm{Re}\left\{ \tilde{X}_{BB}[n] \, e^{j2\pi f_c n t_s} \right\}, \quad t_s = \frac{T_s}{\lambda} = \frac{1}{B} \cdot \frac{1}{24}$$

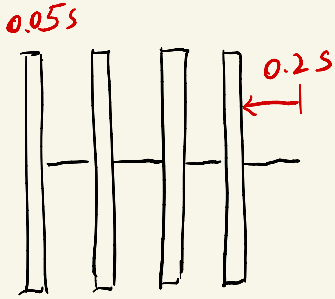
7. Concatenate LFM (Linear Frequency Modulation)



In LFM amble, there are four chirps. Each chirp is sampled at 192 kHz for 0.05 s. The instantaneous frequency is $f_c - 4000$ at $t=0$ and crosses $f_c + 4000$ at $t = 0.05$ second. Each chirp is tailed with zero signal of 0.2 seconds.

chirp: $\cos(2\pi f(t))$, $0 \le t < t_1$, $t_1 = 0.05$ secs

$f(t) = f_0 + \beta t$, $0 \le t < t_1$



$$f_0 = f_c - 4000, \quad f_1 = f_c + 4000$$

$$\beta = \frac{f_1 - f_0}{t_1} = \frac{8000}{0.05} = 160000$$

※ You can observe these signals in spectrum.

Hint: MATLAB command: spectrogram(   )

8. Validation

1) passband to baseband:

$$\tilde{X}_{BB}[n]\_I = X_{PB}[n]\cos(2\pi f_c n t_s)$$

$$\tilde{X}_{BB}[n]\_Q = -X_{PB}[n]\sin(2\pi f_c n t_s)$$

$$\tilde{X}_{BB}[n] = \tilde{X}_{BB}[n]\_I + 1i\,\tilde{X}_{BB}[n]\_Q$$

2) match filtering: $\bar{X}_{OVF}[n] = \tilde{X}_{BB}[n] * R[n]$

(is a low-pass filtering)

and down-sampling: $\bar{X}[n] = \bar{X}_{OVF}[n\lambda + \underset{\substack{\uparrow\\ \text{starting index}}}{n_0}]$

$n = 0, 1, 2, \cdots$

During the down-sampling, you should be very careful of the starting index n_0. Since there exists the delay shift when you convolute the signal with $R[n]$, you need to calculate the delay shift and then find the correct starting index.

3) extract the signal part corresponding to each ZP-OFDM, and then use DFT to get the frequency domain data

$$y[k] = \sum_{n=0}^{K+D-1} x[n]\, e^{-j2\pi\frac{nk}{K}}, \quad k = 0, \cdots, K+D-1$$