Resampling and CFO compensation

General parameters

$K=2048$: the total number of subcarriers.

$L=200$: the number of zero-padded symbols

$L+1$: the number of channel taps

$F_c=24kHz$: carrier frequency

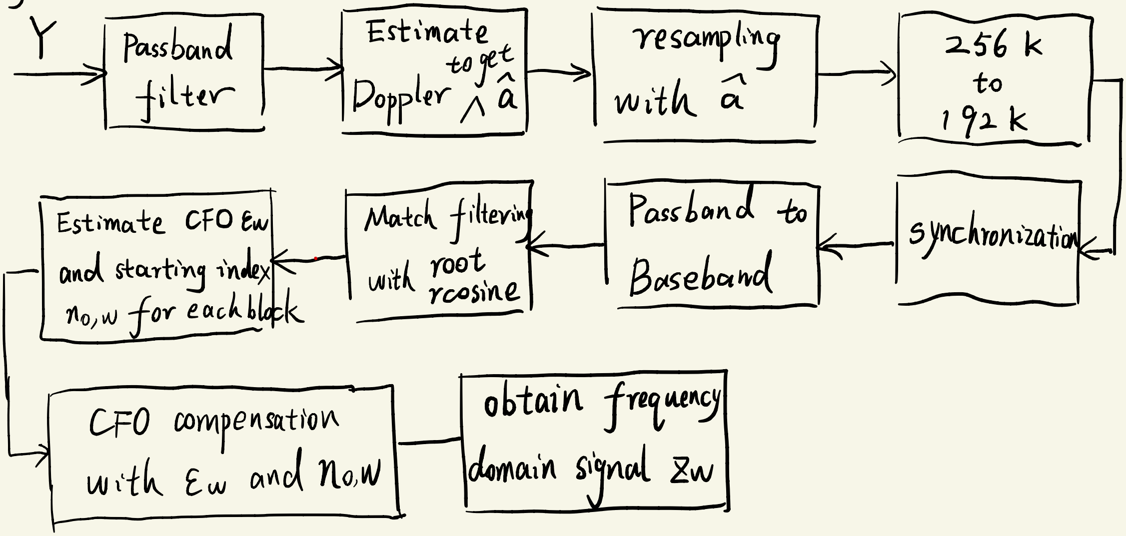
$B=8kHz$: Bandwidth. $[0, 8000]$ in baseband

sampling-rate : will change from 256kHz to 192kHz in the step 4.

$W=24$: the number of ZP-OFDM symols in one packet

$\lambda=24$: the oversampling factor

flowchart



1. use passband filter to remove the noise beyond the passband.

※ use matlab command "bandpass"

Y_{PB} = bandpass (Y, [$-1000+F_c$, $8000+F_c$], sampling_rate)

received data — upper and lower bound — sampling rate

$F_c = 24\,kHz$

sampling_rate = 256 kHz

When I generated the transmitted data, the baseband bandwidth was [0, 8000], not [-4000, 4000], you can also set [F_c, $8000+F_c$]. But in my processing, I set it as [$-1000+F_c$, $8000+F_c$]

2. Estimate the Doppler rate a

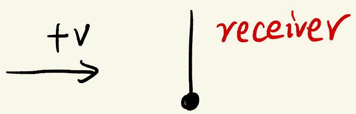
the Doppler rate a is also known as "Mach number"

the maximum $a = \frac{v}{c}$ $v \rightarrow$ moving speed, $c \rightarrow$ speed of sound

In our underwater field test, $v = 2\,\text{knots} = 1.03\ m/s$

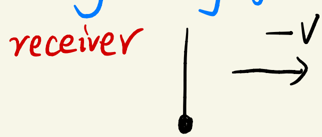
$c = 1500\ m/s$, so the maximum $a \approx 6.87e^{-4}$

moving towards receiver,



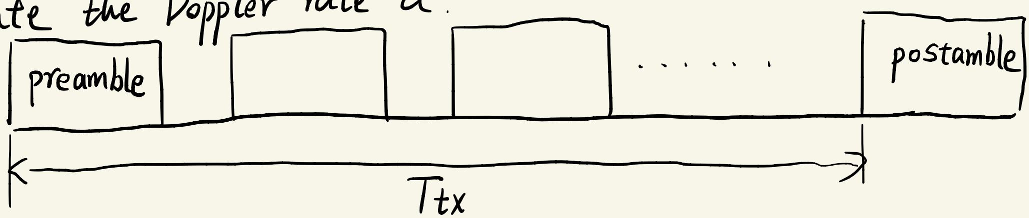
the signal will get compressed.

moving away from receiver,



the signal will get stretched out.

According to the compression (or stretching out) ratio, we can estimate the Doppler rate a.



$$\hat{a} = \frac{T_{tx}}{\hat{T}_{rx}} - 1$$

, where $\hat{T}_{rx}$ is the estimated received signal duration

T_{tx} is the transmitted signal duration

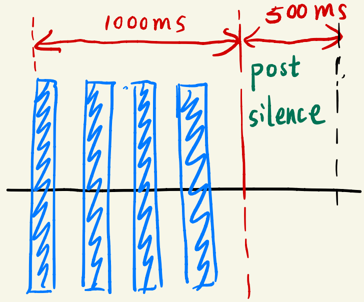
if $\hat{T}_{rx} > T_{tx}$, $\hat{a} < 0 \Rightarrow$ moving away from receiver

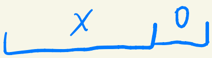
$\hat{T}_{rx} < T_{tx}$, $\hat{a} > 0 \Rightarrow$ moving towards receiver

How to calculate T_{tx} :

I have given the value at the bottom. You don't need to calculate it.

The preamble has the duration of 1500 ms. In my preamble design, I have the post silence.





2) W=24 ZP-OFDM block: $24 \times (2048+200) = 53952$ samples

3) one zero-padding in front of 24 blocks : $53952 + 200 = 54152$ samples

4) oversampling : $54152 \times 24 = 1299648$ samples

5) filtering with the square root rcosine : I used "filter" function, the input length and the output length are the same. so the total number of samples is still 1299648

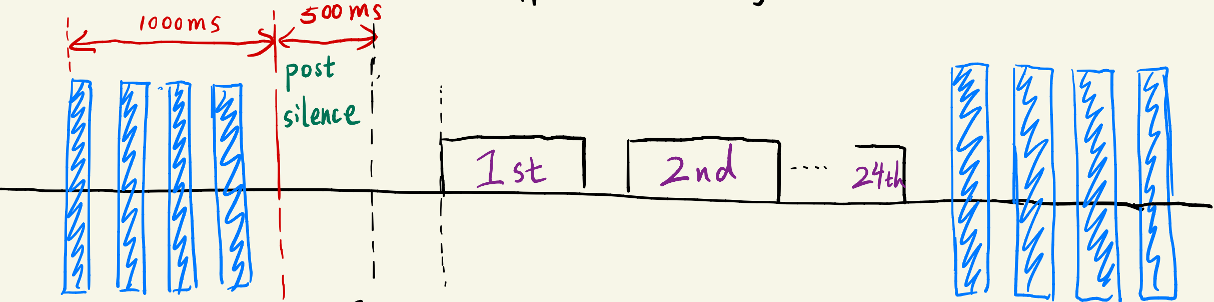
6) bassband to passband : the # of samples is still 1299648.

7) Predistortion (filtering with a pre-defined vector), the vector size is 101 : I used "conv" function, so the output length is $1299648 + 101 - 1 = 1299748$

8) data part duration : $1299748 / 192000 = 6769.52$ ms

9) the total is $T_{tx} = 6769.52 + 1500 = 8269.52$ ms ≈ 8.2695 s

How to estimate the Doppler rate a of our underwater data ?



find the duration $\hat{T}_{rx}$ between two chirps from the preamble and postamble.

※ Since the SNR in this filed test is good, you don't need to use "correlation" to determine the starting time point for each chirp. You can just zoom in the figure to observe the starting time point.

For (8K, 2048) $T_{tx} = 8.2675$ seconds

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3. resampling with $\hat{a}$

※ use matlab command "resample"

$$Y_{PB\_re} = \text{resample}(\,Y_{PB},\ \text{round}((1+\hat{a})\cdot 1e^5),\ 1e^5)$$

$\hat{a}$ is the estimated doppler rate.

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4. resampling from 256 kHz to 192 kHz

Our transmitter uses 192 kHz as sampling rate, but the receiver takes 256 kHz as sampling rate. We need to resample the signal to have the sampling rate of 192 kHz

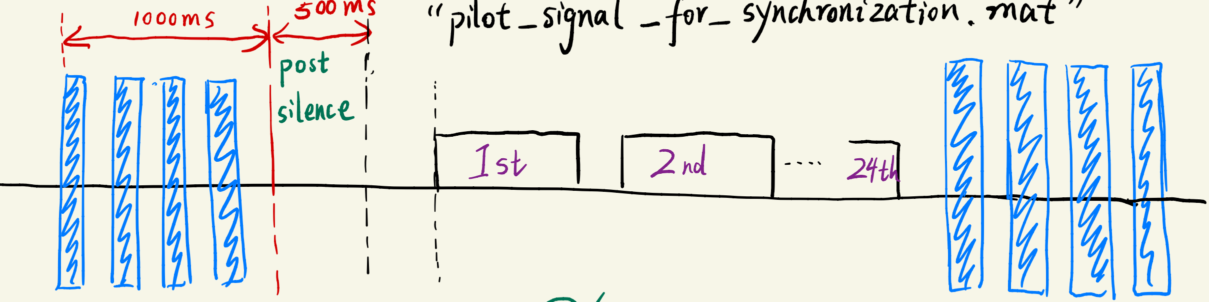
※ use command "upfirdn"

$L_s = 192$; $M_s = 256$; $L_p = 24$; $N = L_p \cdot L_s - 1$;

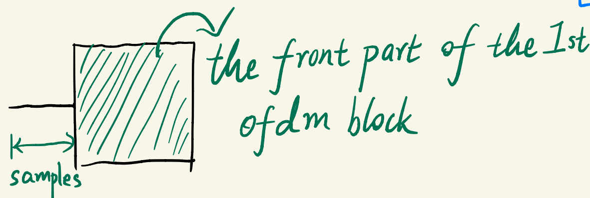
$h = L_s \cdot \text{fir1}(N, 1/M_s, \text{kaiser}(N+1, 7.8562))$

$$\tilde{Y}_{PB\_re} = \text{upfirdn}(\,Y_{PB\_re},\ h,\ L_s,\ M_s)$$

5. synchronization : the pilot signal is stored in "pilot_signal_for_synchronization.mat"

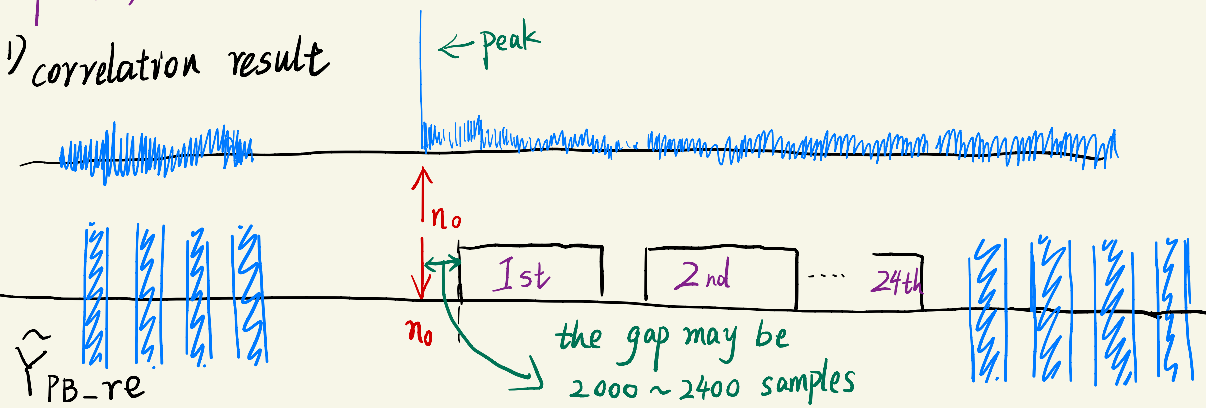


the pilot passband signal

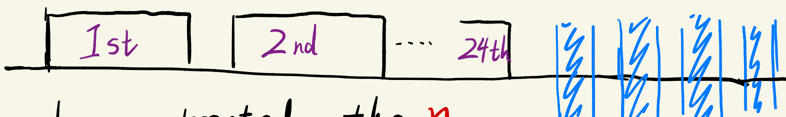


correlate the $\tilde{Y}_{PB\_re}$ with the pilot passband signal, you will find a peak, whose index can be used as the starting index n_0

1) correlation result



2) then extract the data part starting from n_0 : $\tilde{Y}_{PB\_re} = \tilde{Y}_{PB\_re}(n_0 : end)$



3) since the data part has been extracted, the n_0 becomes unuseful.

6. Match filtering: first stage

In retrospect, as I analyze the field test where I generated the transmitted signal, I now realize that there was one step that, in today's perspective, appears to be unnessary.

This step was "convolving the passband signal with a pre-defined vector". This vector is stored in "itc_1007_compfilter".

In my original understanding, this vector was used for predistortion. But I believe this is an unnessary step after analyzing my data.

before doing the conversion from passband to baseband, you need to convolve your signal with this vector.

Alert: if you plot this vector, you will find that it has the span of 101 samples. Henceforth, it will cause 50 samples deley. Please remember to remove this delay as you have done in the part I.

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7. Passband to Baseband

Now, $f_s = 192\,kHz$

$t_s = 1/f_s$

$$Y_{BB}[n]\_I = \tilde{Y}_{PB\_re}[n] \cdot 2\cos(2\pi f_c n t_s)$$

$$Y_{BB}[n]\_Q = -\tilde{Y}_{PB\_re}[n] \cdot 2\sin(2\pi f_c n t_s)$$

$$Y_{BB}[n] = Y_{BB}[n]\_I + 1i\, Y_{BB}[n]\_Q$$

8. Match filtering: second stage

$$\bar{Y}_{BB} = Y_{BB} * R$$

$*$ ← convolution

Here, R is the square root raised cosine filter we defined in the part I.

Alert: remember to remove the delay caused by convolving with R

9. find the starting index $n_{0,1}$ of the 1st OFDM block

find the carrier frequency offset (CFO) ε_1 of the 1st OFDM block

$L=200$
$k=2048$
$\lambda=24$

$f_s=192\,kHz$
$t_s=1/f_s$
$L=200$

Grid search

For $\hat{n}_{0,1} = [2200:1:2400]$

For $\varepsilon_1 = [-2:0.1:2]$

1) CFO compensation:

$$\hat{Y}_{BB\_1}[n] = \bar{Y}_{BB}[\hat{n}_{0,1}+n]\cdot e^{-j2\pi\varepsilon_1(n_{0,1}+n)t_s}, \quad n=0,1,2,\cdots,(k+L)\lambda-1$$

2) Down-sampling: $\check{Y}_{BB\_1}[i] = \hat{Y}_{BB\_1}[k\lambda]$, $i=0,\cdots,k+L-1$

3) obtain the frequency domain data: Z_1

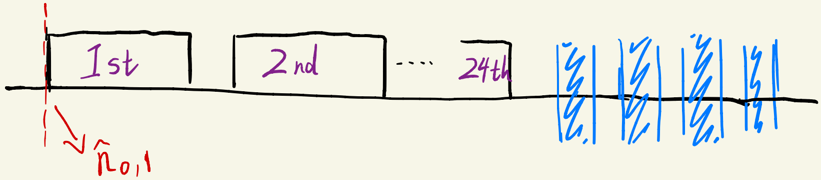
$$Z_1 = \begin{bmatrix} Z_{0,1} \\ Z_{1,1} \\ \vdots \\ Z_{k-1,1} \end{bmatrix} \in C^{k\times 1}, \quad Z_{m,1} = \sum_{i=0}^{k+L-1} \check{Y}_{BB\_1}[i]\, e^{-j\frac{2\pi mi}{k}}$$

4) calculate the power over null subcarriers.

$$P_{Null}(\hat{n}_{0,1}, \varepsilon_1) = \sum_{m\in D_{Null}} \| Z_{m,1} \|^2$$

end

end



Find the minimal P_{NULL}, and its corresponding $n_{0,1}$

1) set $n_{0,1} = [2200 : 1 : 2400]$, since the gap after the synchronization may be 2200 ~ 2400 samples.
   ↳ step

2) set $\varepsilon_1 = [-2 : 0.1 : 2]$, since the CFO should be small after resampling if the moving speed is relatively steady.
   ↳ step

"λ": the over-sampling factor ; "K": total number of subcarriers

"L": is # of zero-paddings ; "$L+1$" is # of channel taps

"t_s": the sampling duration $t_s = \frac{1}{192000}$

"P_{null}": the total power of null subcarriers

"D_{null}": the index set of null subcarriers. The null subcarrier index is labeled as "0" in "ofdm_map"

10. After knowing the $\hat{n}_{0,1}$, we need to obtain the frequency domain data Z_w of each ZP-OFDM symbol.

Fine tuning: find the starting index $\hat{n}_{0,w}$ and ε_w of each ZP-OFDM, and record the frequency domain data Z_w

$$\boxed{\text{initial: } \hat{n}_{0,0} = \hat{n}_{0,1} - (K+L)\lambda}$$

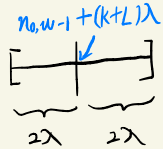
$\lambda = 24$	$K = 2048$

Ideally, when there is no channel, the $\hat{n}_{0,w}$ of the w-th ZP-OFDM should be $\hat{n}_{0,w-1}+(K+L)\lambda$. In reality, let's fine tune $\hat{n}_{0,w}$ as

ZP-OFDM index

For $w = [1:21]$

For $\hat{n}_{0,w} = \hat{n}_{0,w-1} + (K+L)\lambda + [-2\lambda : 1 : 2\lambda]$



For $\varepsilon_w = [-2 : 0.1 : 2]$

1) CFO compensation:

$$\hat{Y}_{BB\_w}[n] = \bar{Y}_{BB}[\hat{n}_{0,w}+n] \cdot e^{-j2\pi\varepsilon_w(\hat{n}_{0,w}+n)t_s}$$

$n = 0, 1, 2, \cdots, (K+L)\lambda - 1$

2) Down-sampling: $\check{Y}_{BB\_w}[i] = \hat{Y}_{BB\_w}[k\lambda]$, $i = 0, \cdots, K+L-1$

3) obtain the frequency domain data: Z_w

$$Z_w = \begin{bmatrix} z_{0,w} \\ z_{1,w} \\ \vdots \\ z_{K-1,w} \end{bmatrix} \in \mathbb{C}^{K\times 1}, \quad z_{m,w} = \sum_{i=0}^{K+L-1} \check{Y}_{BB\_w}[i]\, e^{-j\frac{2\pi m i}{K}}$$

4) calculate the power over null subcarriers.

$$P_{Null}(\hat{n}_{0,w}, \varepsilon_w) = \sum_{m \in D_{Null}} \| z_{m,w} \|^2$$

end

end

find the minimal $P_{Null}(\hat{n}_{0,w}, \varepsilon_w)$, and record the corresponding $\hat{n}_{0,w}$ and ε_w. And also record the frequency domain data Z_w

end