MATH5004 Tutorial 4

Weak formulation of BVPs

Derive weak formulation of the following BVPs:

1. A steady two-point BVP:

$$u_{xx} = f(x), \qquad x \in (0,1)$$

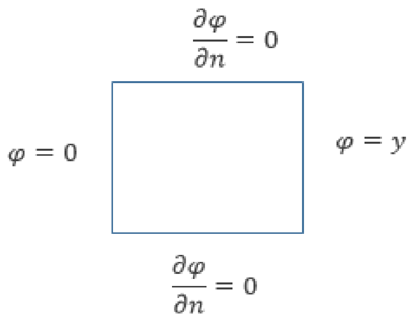
$$u(0) = 1, \qquad u(1) = -1,$$

where $f(x) = -\pi^2 \cos(\pi x)$.

2. The Laplace's equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = f(x,y)$$

on a 2×2 square with boundary condition as shown.



3. The Laplace's equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = f(x,y,z)$$

on a $2 \times 2 \times 2$ cube with boundary condition $\frac{\partial \varphi}{\partial n} = 0$.

Assignment II

Question 1. (LUT-WK4)

Derive weak formulation of the following BVPs:

a) The steady state heat conduction problem

$$k(x)\frac{\partial^2 u}{\partial x^2} = f(x), \quad 0 < x < 1$$

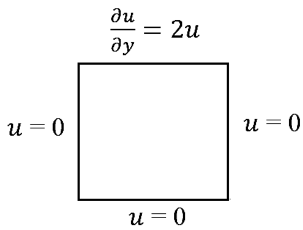
$$u(0) = 1, \qquad \frac{\partial}{\partial x}u(1) = 0,$$

where $f(x) = -\cos(\pi x), \ k = \pi x^2$.

b) The Poisson problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x - 2y$$

on a 2×2 square with boundary condition as shown.



c) The heat conduction problem

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$$\nabla \cdot (k \nabla T) = f(x, y, z)$$

on a $2 \times 2 \times 2$ cube with boundary condition

$$k\frac{\partial T}{\partial n} = h(T - T_\infty).$$

Note: Assignments II (25%): Assignment Questions will be given weekly.

In this week, Questions 1 (TUT-WK4) are a part of Assignment II, please submit a document file with MATLAB code via Blackboard by the due date of Assignment I on Friday 23 October 2020