MATH5004

ADVANCE NUMERICAL ANALYSIS

4.08.2015

OUTLINE

- Overview of BVPs
- Finite Difference Method
- General Finite Element Formulation
- Classes of Admissible Functions & Piecewise Polynomial Approximation
- Two-Point Boundary Value Problems
- Elliptic Boundary Value Problems
- Parabolic Boundary Value Problems
- Element Calculations
- Applications
- Applications:
  - Stokes Problems and Incompressible Flow
  - Multi-Phase Flows under EM Force
  - Blood Flows in Arteries with Stenosis
  - Black-Sholes Modell

ASSESSMENT

- 2 Assignments: 50% (each one 25%)

  Week: Week 6

  Day: Friday 11 September

  Time: 6pm (via Blackboard)

  Week: Week 12

  Day: Friday 23 October

  Time: 6pm (via Blackboard)

- Final Examination: 50%

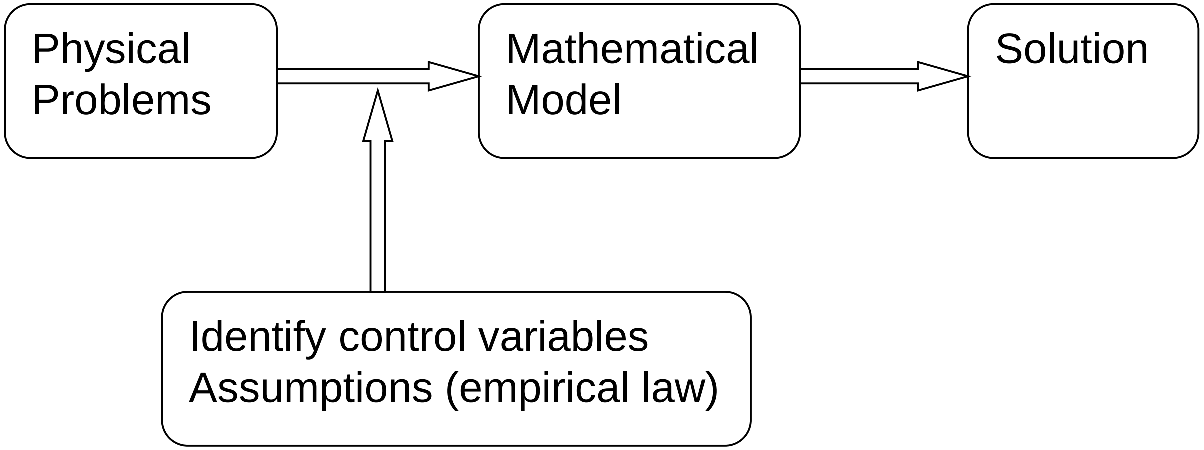
  Week: Exam period

  Day: TBA

  Time: TBA

Mathematical Model

(1) Modelling



(2) Types of Solution

	Exact Eq.	Approx. Eq.
Exact Sol.	◎	◎
Approx. Sol.	◎	◎

(3) Method of Solution

- Classical methods
- Numerical methods

  (I) Energy: Minimize an expression for the potential energy of the structure over the whole domain.

  (II) Boundary element: Approximates functions satisfying the governing differential equations not the boundary conditions.

  (III) Finite difference: Replaces governing differential equations and boundary conditions with algebraic finite difference equations.

  (IV) Finite element: Approximates the behavior of an irregular, continuous structure under general loadings and constraints with an assembly of discrete elements.

Methodology

FDM & FEM are numerical methods for solving a system of governing equations over *the domain of a continuous physical system*, which is discretized into simple geometric shapes.

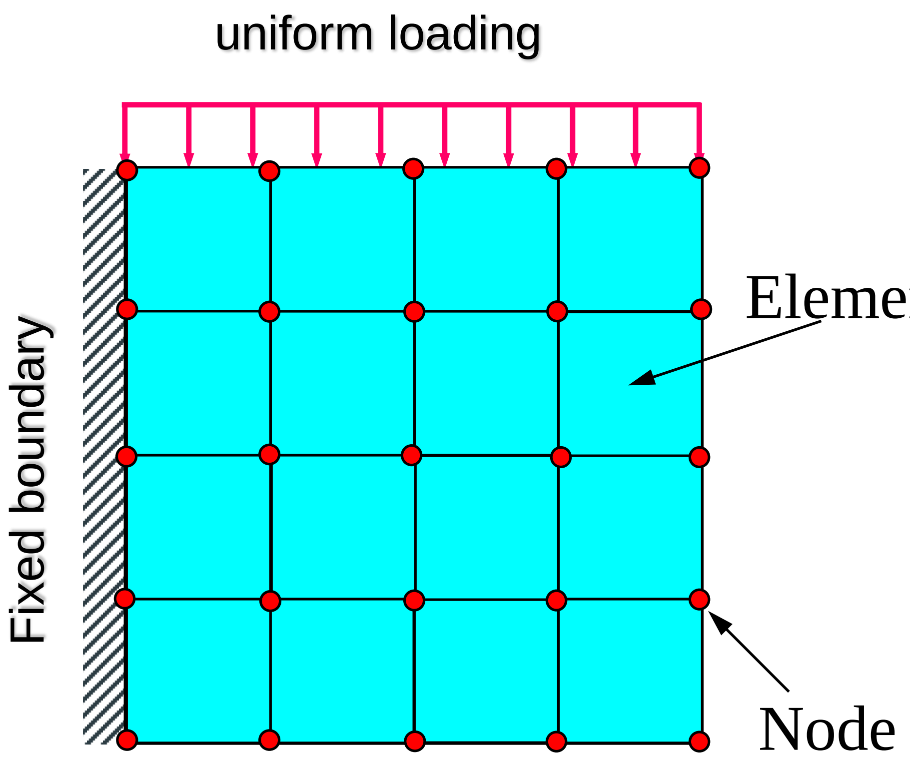
Continuous system
Time-independent PDE
Time-dependent PDE

→

Discrete system
Linear algebraic eq.
ODE

Discretisation

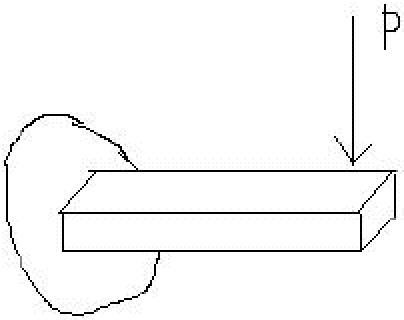
Modeling a body by dividing it into an equivalent system of finite elements interconnected at a finite number of points on each element called nodes.



Problem: Obtain the stresses/strains in the plate

- Approximate method
- Geometric model
- Node
- Element
- Mesh
- Discretization

Geometric Mode



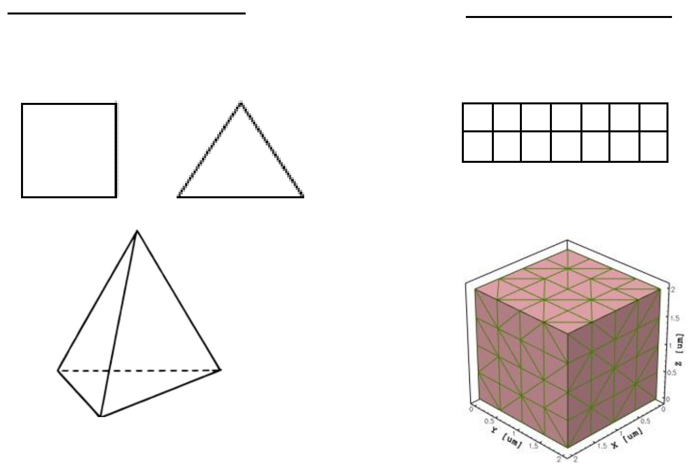
Build up geometric model

- 1D problem: Line
- 2D problem: surface
- 3D problem: solid

Discretization

(a) Element type

- 1D line element
- 2D element
- 3D tetrahedron element



(b) Total number of element

- 1D
- 2D
- 3D

Solve the System of Equations

A. Elimination method

- Gauss's method

B. Iteration method

- Gauss Seidel's method

C. Interpret the Results (Post Processing)

Role of simulation in design:

Boeing 777

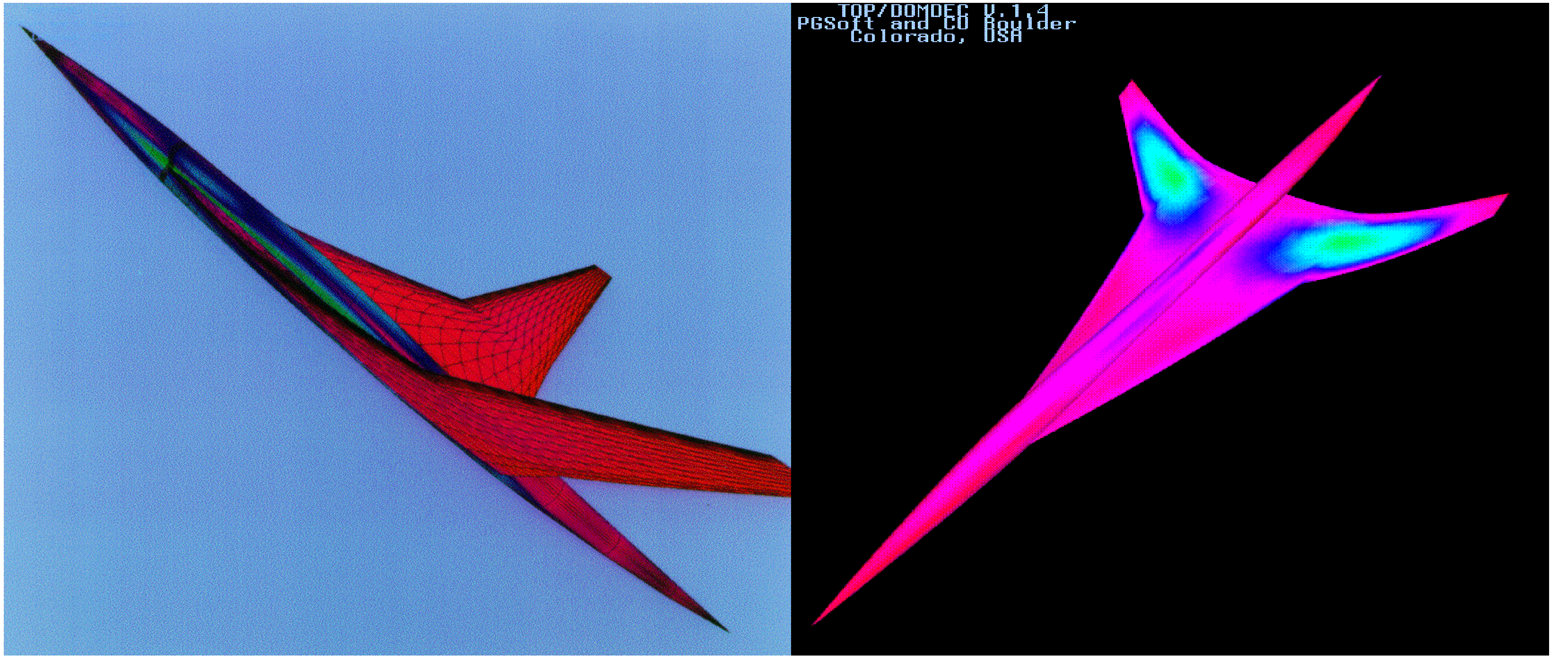
Source: Boeing Web site (http://www.boeing.com/companyoffices/gallery/images/commercial/).

Another success ..in failure:

Airbus A380

http://www.airbus.com/en/aircraftfamilies/a380/

Drag Force Analysis of Aircraft



- Question

    What is the drag force distribution on the aircraft?

- Solve
    - Navier-Stokes Partial Differential Equations.
- Recent Developments
    - Multigrid Methods for Unstructured Grids

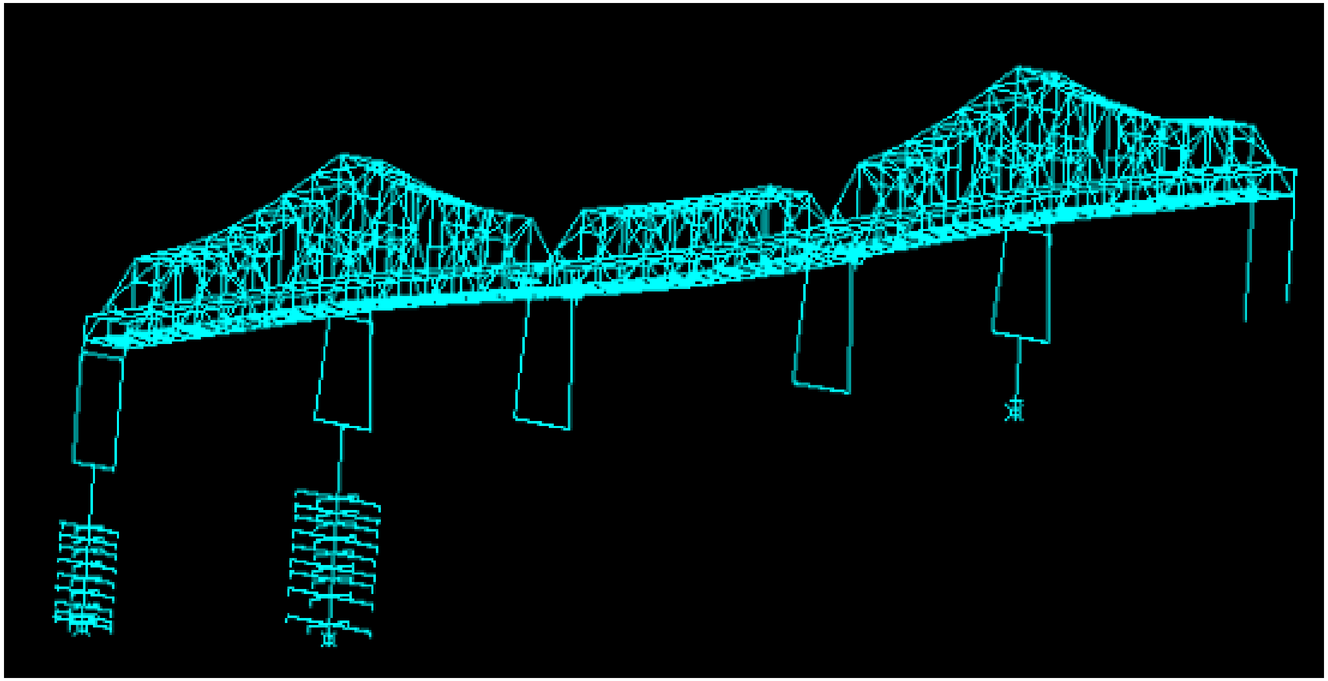
San Francisco Oakland Bay Bridge

Before the 1989 Loma Prieta earthquake

San Francisco Oakland Bay Bridge

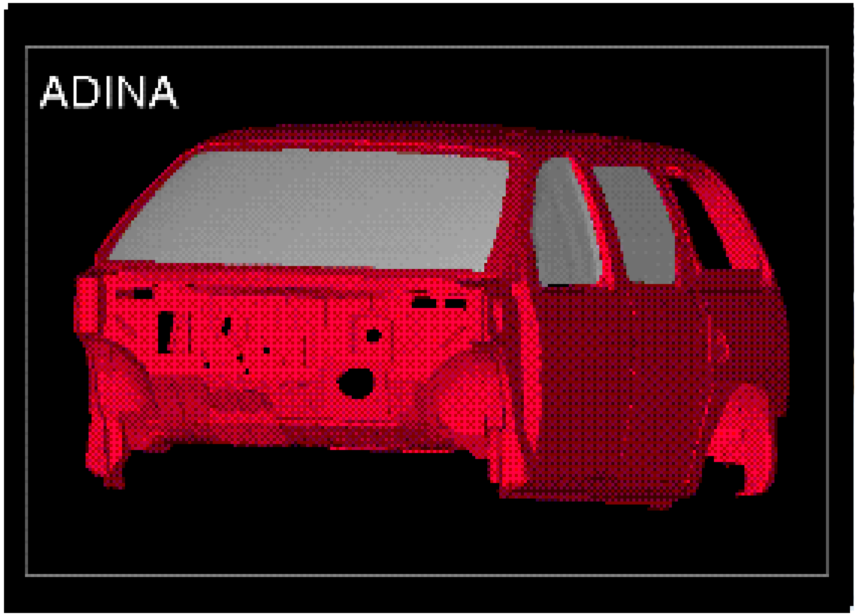
After the earthquake

San Francisco Oakland Bay Bridge



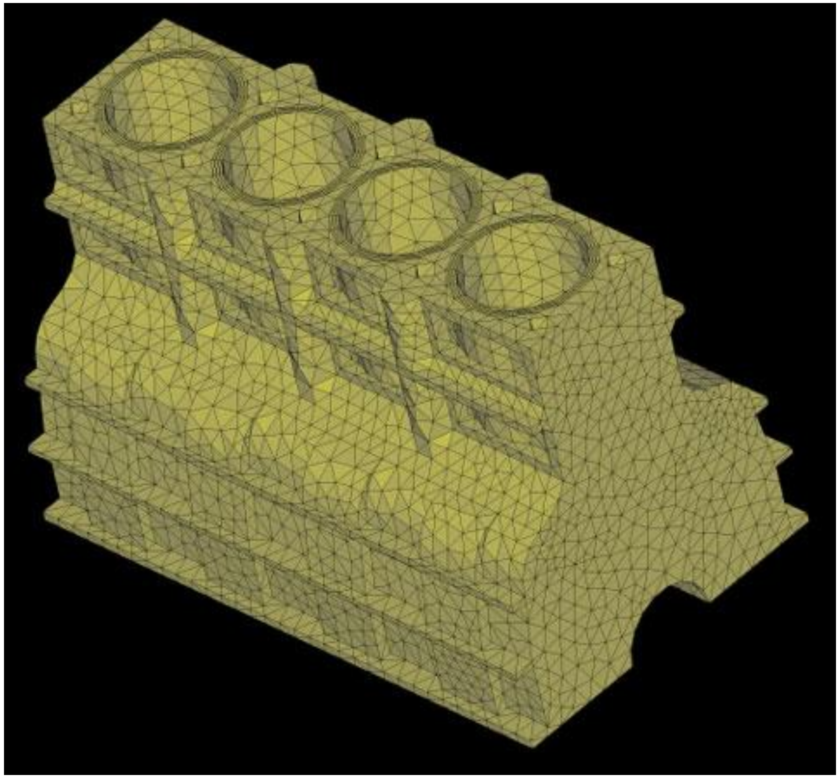
A finite element model to analyze the bridge under seismic loads

Crush Analysis of Ford Windstar



- Question
  - What is the load-deformation relation?
- Solve
  - Partial Differential Equations of Continuum Mechanics
- Recent Developments
  - Meshless Methods, Iterative methods, Automatic Error Control

Engine Thermal Analysis

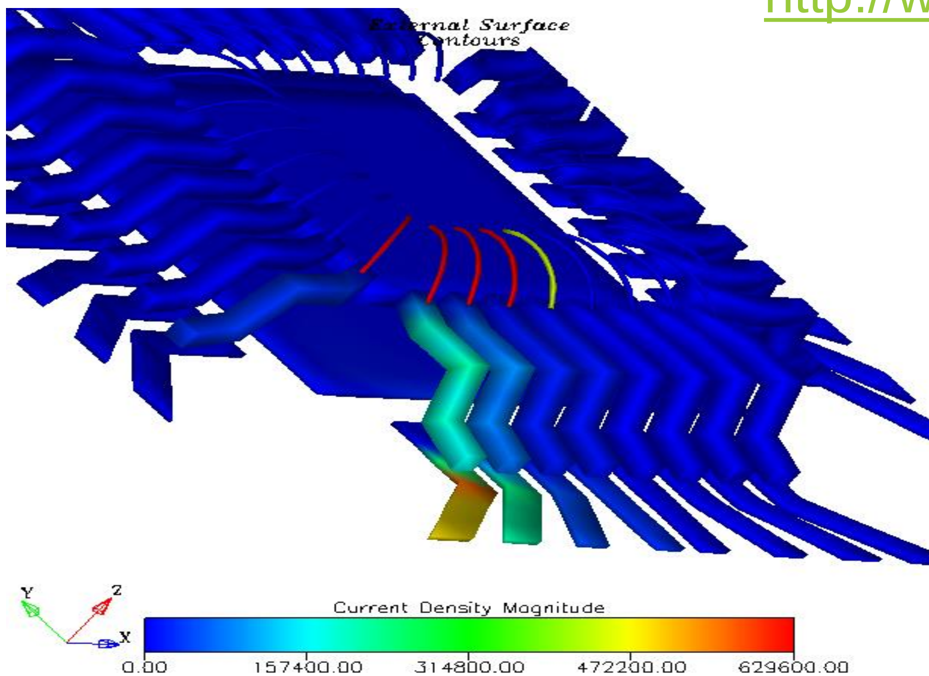


Picture from http://www.adina.com

- Question
  - What is the temperature distribution in the engine block?
- Solve
  - Poisson Partial Differential Equation.

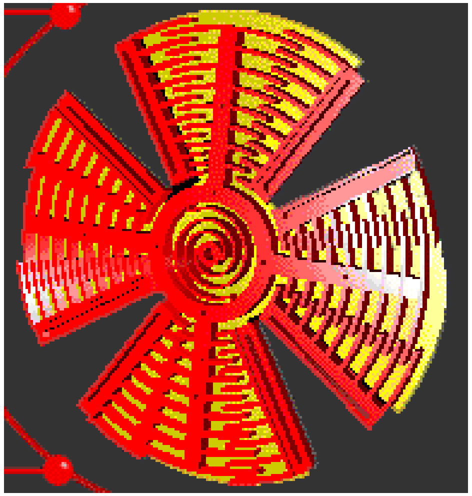
Electromagnetic Analysis of Packages

http://www.coventor.com



- Solve
  - Maxwell's Partial Differential Equations
- Recent Developments
  - Fast Solvers for Integral Formulations

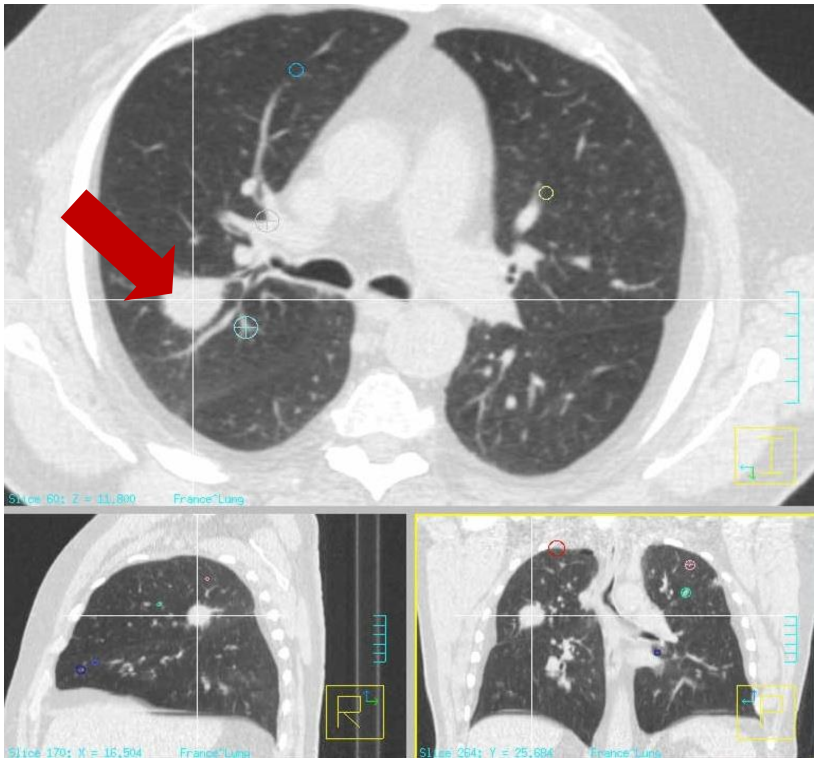
Micromachine Device Performance Analysis

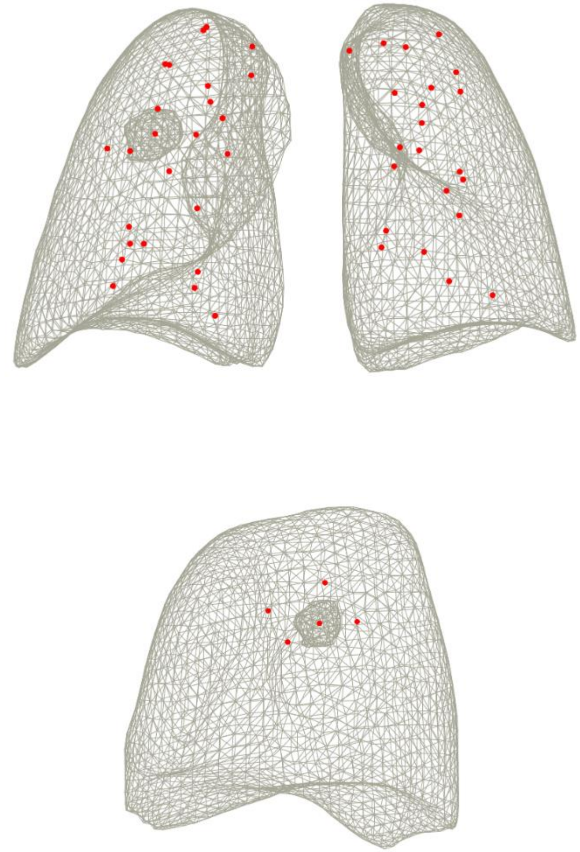


www.memscap.com

- Equations
  - Elastomechanics, Electrostatics, Stokes Flow.
- Recent Developments
  - Fast Integral Equation Solvers, Matrix-Implicit Multi-level Newton Methods for coupled domain problems.

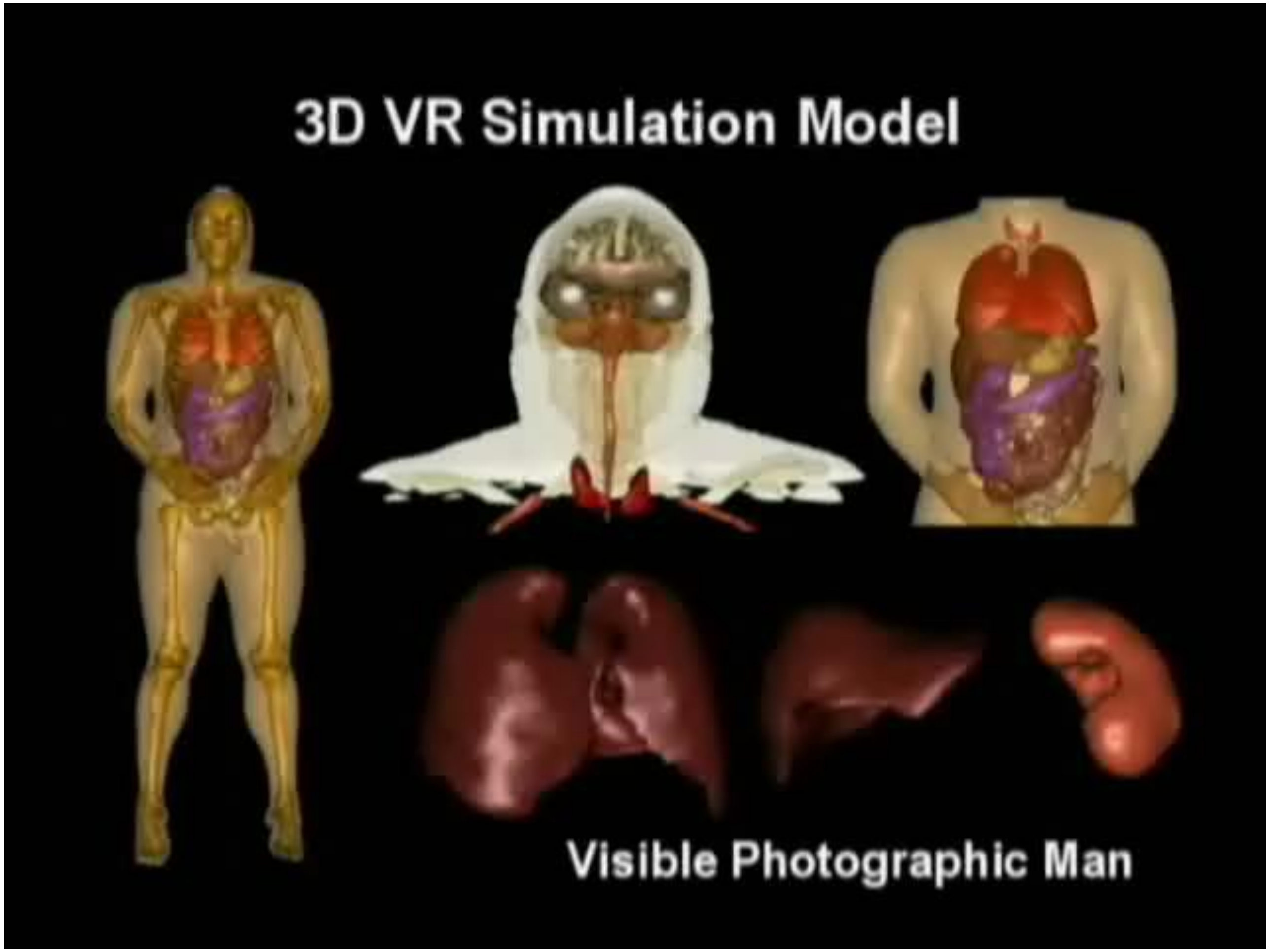
Radiation Therapy of Lung Cancer





http://www.simulia.com/academics/research_lung.html

Virtual Surgery



Engineering design

General scenario..

Physical Problem

Question regarding the problem
...how large are the deformations?
...how much is the heat transfer?

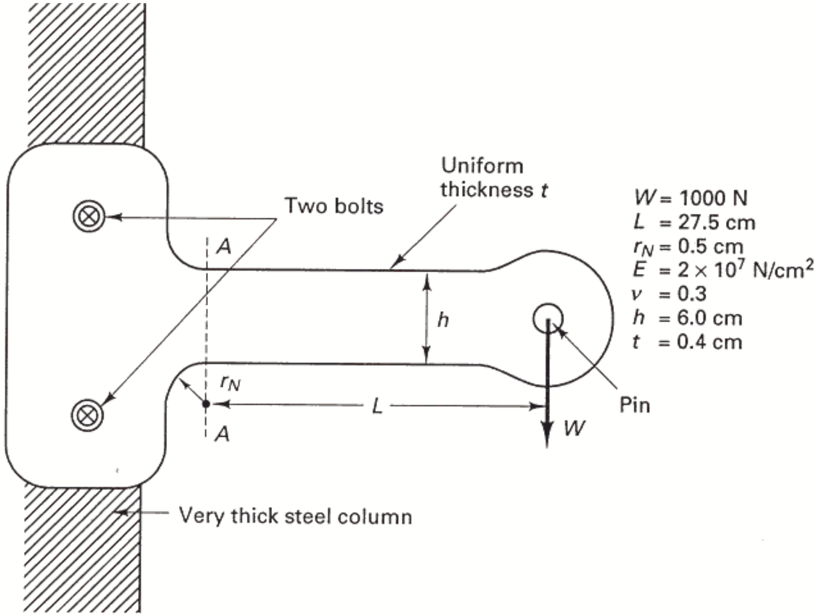
Mathematical model

Governed by differential equations

Assumptions regarding
Geometry
Kinematics
Material law
Loading
Boundary conditions
Etc.

Engineering design

Example: A bracket



Physical problem

Questions:

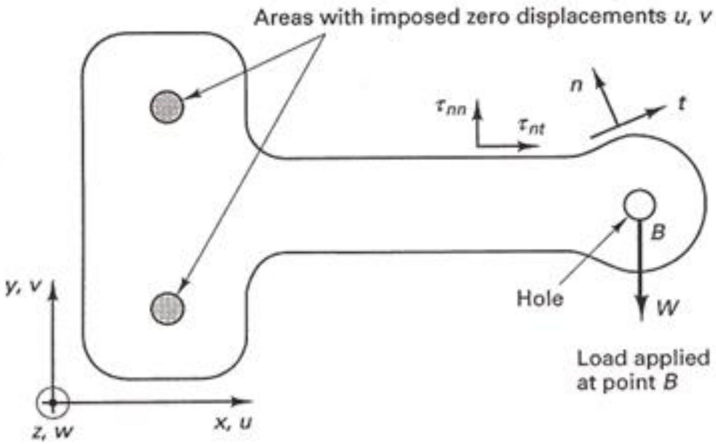
1. What is the bending moment at section AA?
2. What is the deflection at the pin?

Engineering design

Example: A bracket

Mathematical model 2: plane stress

Difficult to solve by hand!



Stress-strain relation

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$$

E = Young's modulus, ν = Poisson's ratio

Strain-displacement relations

$$\epsilon_{xx} = \frac{\partial u}{\partial x}; \qquad \epsilon_{yy} = \frac{\partial v}{\partial y}; \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

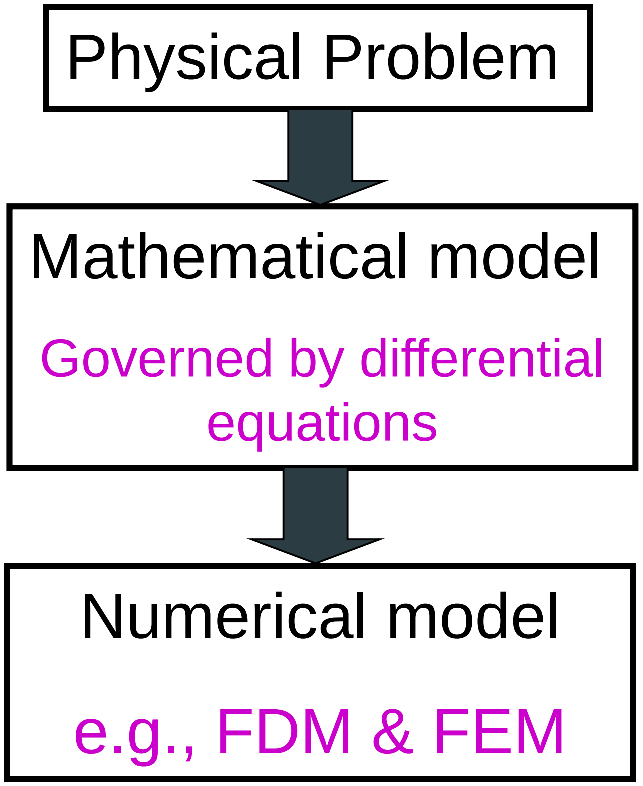
Equilibrium equations

$$\left.\begin{aligned} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} &= 0 \end{aligned}\right\} \text{ in domain of bracket}$$

$\tau_{nn} = 0$, $\tau_{nt} = 0$ on surfaces except at point B and at imposed zero displacements

Engineering design

..General scenario..



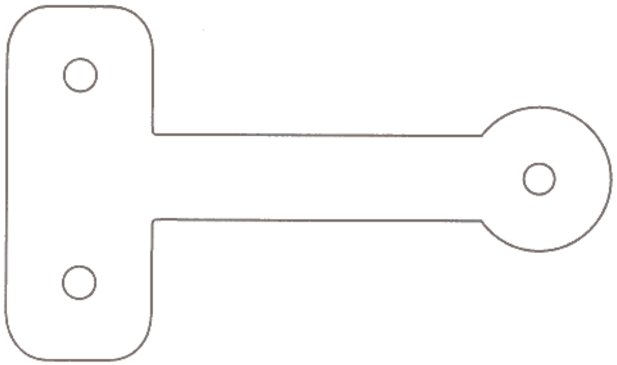
Engineering design

..General scenario..

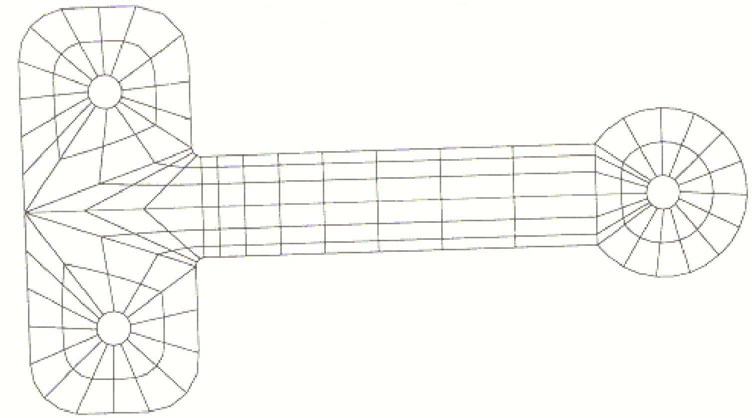
PREPROCESSING

1. Create a geometric model

2. Develop the finite element model



Solid model

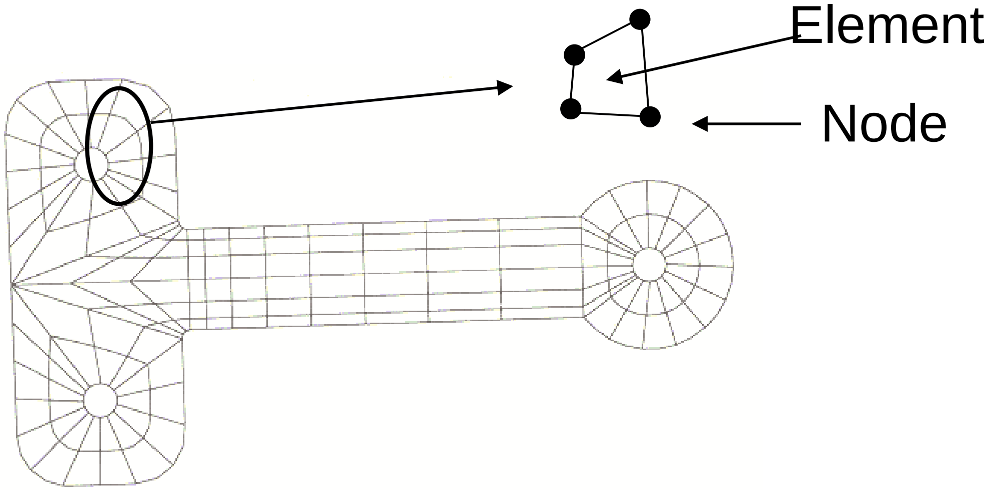


FD & FE models

Engineering design

<u>Analysis scheme</u>

Step 1: Divide the problem domain into non overlapping regions ("**elements**") connected to each other through special points ("**nodes**")



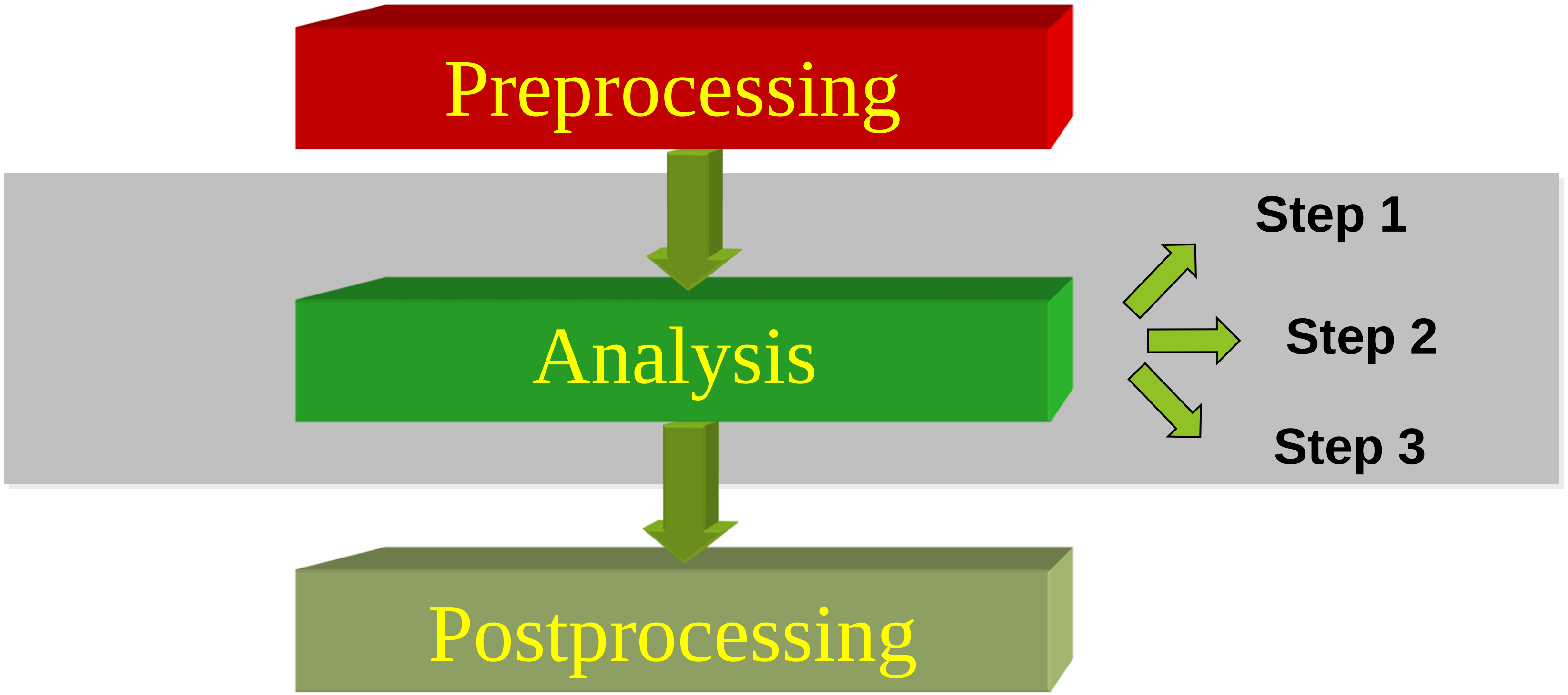
Engineering design

<u>FEM analysis scheme</u>

Step 2: Describe the behavior of each element

Step 3: Describe the behavior of the entire body by putting together the behavior of each of the elements (this is a process known as “**assembly**”)

Engineering design

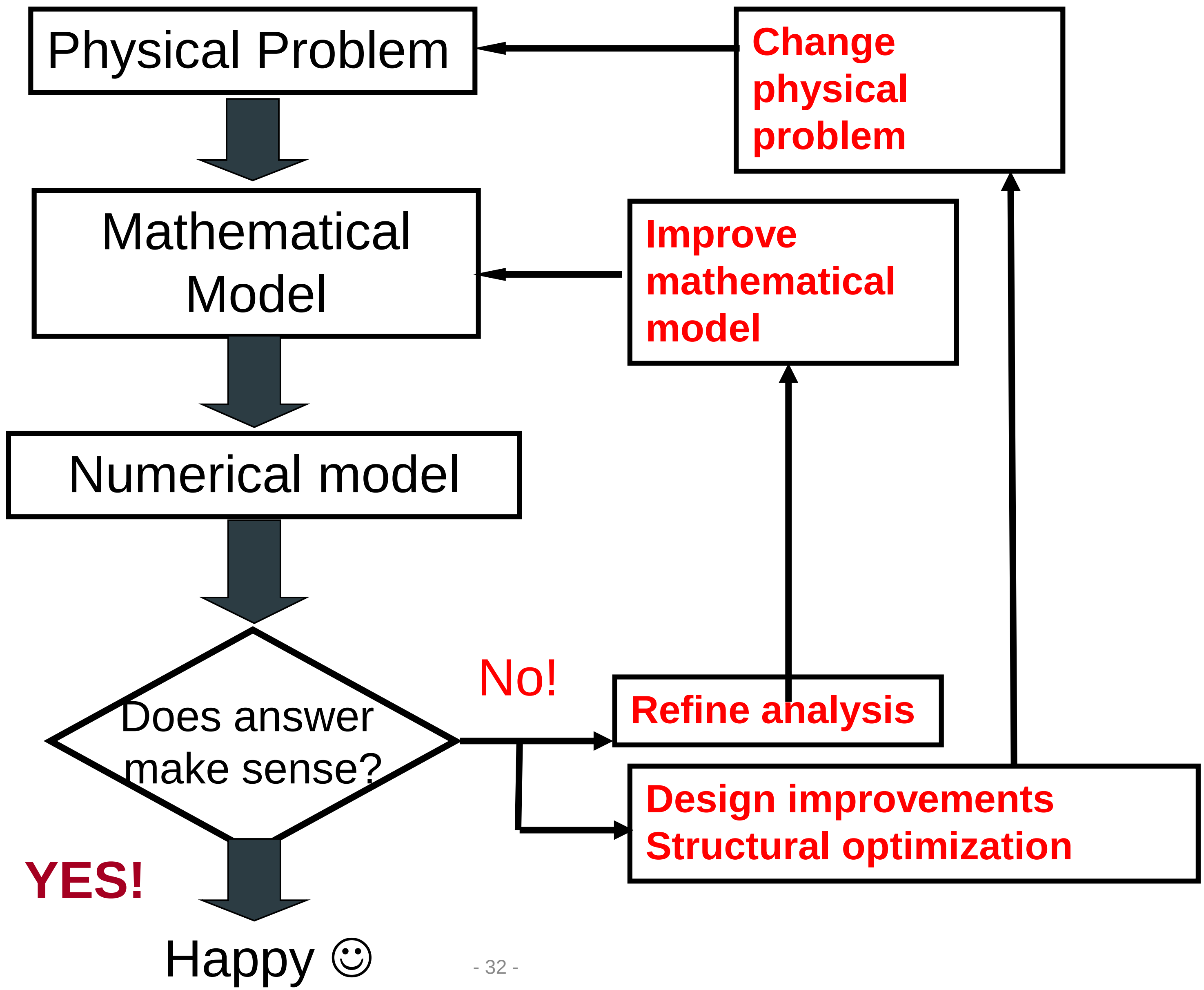


Engineering design

Example: A bracket

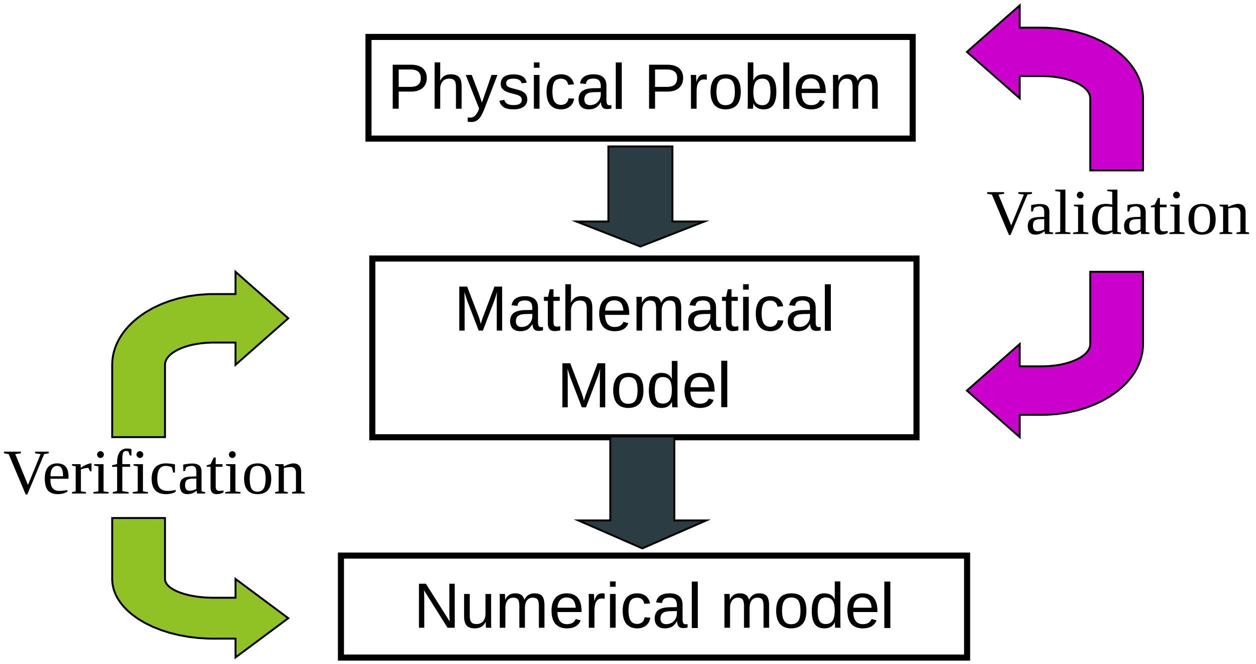
1. The **selection** of the mathematical model depends on the response to be predicted.
2. The **<u>most effective</u>** mathematical model is the one that delivers the answers to the questions in reliable manner with least effort.
3. **<u>The numerical solution is only as accurate as the mathematical model.</u>**

Modeling a physical problem



Modeling a physical problem

Verification and validation



MATLAB

- Matlab (**Mat**rix **lab**oratory) is an interactive software system for numerical computations and graphics.
- Matlab is especially designed for matrix computations:
  - solving systems of linear equations,
  - computing eigenvalues and eigenvectors,
  - factoring matrices, and so forth.
- Many such programs come with the system; a number of these extend Matlab's capabilities to nonlinear problems, such as the solution of initial value problems for ordinary differential equations.