YANSHAN UNIVERSITY

EXAMINATION

End of Semester 2, 2020

ATACM Advanced Topics in Applied and Computational Mathematics

This is an OPEN BOOK examination

For questions in Part IV, to obtain full marks for a question you must **clearly** show appropriate working.

TIME ALLOWED: 150 minutes

TOTAL MARKS: 100

MATERIALS: Not permitted to use the internet other than getting access to Blackboard, MS EXCEL, R, MS WORD, MATLAB and PYTHON.

INSTRUCTIONS TO STUDENTS:

1. Attempt as many questions or part questions as possible.
2. You are required to submit your answer sheets in a single file in WORD format or PDF format. Photo submission or any other format will not be marked.
3. Remember to write your name, student number and version of your exam paper in the Answer Sheet and name your submission/solution file as *Your Name_Your ID* (i.e. *EmmaSmith_20145327*.pdf).
4. Late submissions will not be marked.

Part I. Multiple choice questions (10 marks)

Part II. True or False questions (20 marks)

Part III. Matching questions. Draw the line for matching (15 Marks)

Part IV. Long-answer questions. (55 marks)

Question 4.1 1D Galerkin's method (20 marks)

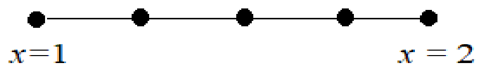
Consider the boundary value problem

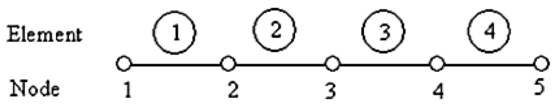
$$PDE,$$

$$BC$$

A. Derive the variational statement of the problem. (5 points)

B. For the mesh of four linear elements with the same length, write down two linear shape functions $\phi_1^{(4)}(x)$ and $\phi_2^{(4)}(x)$ on the element Ω_4. (5 points)





C. Using the Galerkin method and the shape functions in **B**, write out the expressions for local element matrix $K^{(4)} = \left(k_{ij}^{(4)}\right)_{2\times 2}$ and the local element vector $F^{(4)} = \left(f_i^{(4)}\right)_{2\times 1}$ (You do not need to calculate the integrals). (5 points)

D. Use the linear shape functions $N_1(\xi)$ and $N_2(\xi)$ on the master element to write out the expressions for the entries $k_{ij}^{(4)}$ and $f_i^{(4)}$ $(i,j = 1,2)$ (You do not need to calculate the integrals). Is the local element matrix symmetric? (5 points)

Question 4.2 2D Galerkin's method (35 marks)

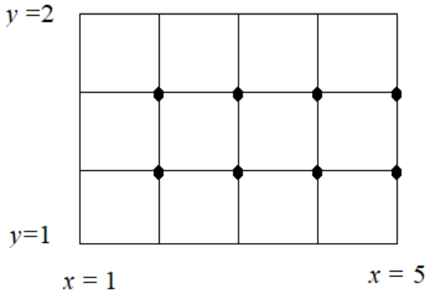
Consider the boundary value problem

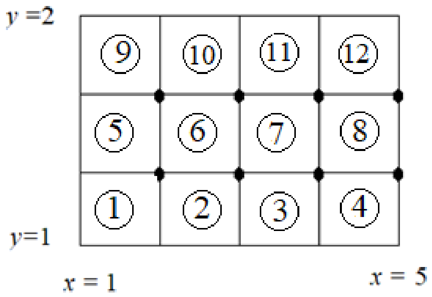
$$PDE$$

$$BC$$

A. Derive the variational statement of the problem. (9 points)

B. For the mesh of twelve bi-linear rectangular elements, write out the expressions for **four** bi-linear shape functions $\phi_1^{(5)}(x)$, $\phi_2^{(5)}(x)$, $\phi_3^{(5)}(x)$ and $\phi_4^{(5)}(x)$ of the element Ω_e (8 points)





C. For the master element, the **four** bi-linear shape functions are

$$N_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)\ , i = 1,\ldots,4.$$

Determine the element transformation

$$T_e\colon \begin{array}{l} x = x(\xi,\eta) \\ y = y(\xi,\eta) \end{array}$$

and find the Jacobian matrix of transformation of the element Ω_4.

(8 points)

D. Given $K^{(e)} = \left(k_{ij}^{(e)}\right)_{2\times 2}$ where $k_{ij}^{(e)} = \int_{\Omega_e} \frac{\partial\phi_i^{(e)}}{\partial x}\frac{\partial\phi_j^{(e)}}{\partial x} + \frac{\partial\phi_i^{(e)}}{\partial y}\frac{\partial\phi_j^{(e)}}{\partial y} - \phi_i^{(e)}\phi_j^{(e)}\, d\Omega,$ and $F^{(e)} = \left(f_i^{(e)}\right)_{2\times 1}$ where $f_i^{(e)} = \int_{\partial\Omega_{x=5}} \phi_i^{(e)} y\, dy + \int_{\Omega_e} \phi_i^{(e)}\, d\Omega,$ use the bi-linear shape functions $N_1(\xi,\eta)$, $N_2(\xi,\eta)$, $N_3(\xi,\eta)$ and $N_4(\xi,\eta)$ to write out the expression for the entries $k_{ij}^{(4)}$ and $f_i^{(4)}$ $(i,j = 1,2,3,4)$ of the element Ω_4 (You do not need to calculate the integrals).

Is the local element matrix symmetric? (10 points)

END OF EXAMINATION PAPER