MATH5004 Tutorial 5

1D Finite Element formulation

Exercise 1. Find variational statement of the following boundary value problems:

a) $u'' + c^2\lambda^2 u = 0, \quad u(a) = f(x),\ u(b) = g(x).$

b) $u'' + u' + u = 0, \quad u'(a) = f(x),\ u(b) = g(x).$

c) $u'' - u' - c^2\lambda^2 u = 0, \quad u'(a) = f(x),\ u'(b) = g(x).$

Exercise 2. Consider the following BVP:

$$-u_{xx} = 1 + x \qquad \text{in } \Omega = (0,1),$$
$$u(0) = 1, \qquad u'(1) + u(1) = 1.$$

a) Develop a variational statement of the problem.
b) For the mesh of four linear elements of the same length, write down two linear shape functions N_1^2 and N_2^2 on the second element.
c) Sketch the linear finite element basis function $\emptyset_3(x)$ and $\emptyset_4(x)$ at the nodes x_3 and x_4.
d) By Galerkin approximation, derive Finite Element Formulation (the stiffness matrix and load vector) of the problem.

Assignment II

Question 1. (LUT-WK5)

For the mesh of four linear elements of the same length, derive Finite Element Formulation of the BVP:

$$k(x)\frac{d^2u}{dx^2} = f(x), \quad 0 < x < 1$$

where $f(x) = -\cos(\pi x)$, $k = \pi x^2$ and the following boundary conditions:

(a) $u(0) = 1,\ u(1) = 0,$
(b) $\frac{d}{dx}u(0) = \frac{d}{dx}u(1) = 1.$

Note: Assignments II (25%): Assignment Questions will be given weekly.

In this week, Questions 1 (TUT-WK5) is a part of Assignment II, please submit a document file with MATLAB code via Blackboard by the due date of Assignment I on Friday 23 October 2020