Project 1

Jesse Adams, Dominic Orsi, Ben Puryear
Section 1

September 7th, 2023

1 Write a short, informal English description for this set: {1,3,5,7, . . . }.

The set {1,3,5,7,...,} contains all positive odd integers.

2 Write a formal description of this set: The set containing all integers that are greater than 5.

$$\{x \in \mathbb{Z} | x \geq 5\}$$

3 Let $A = \{x, y, z\}$ and $B = \{x, y\}$

3.1 What is A x B, where x is the Cartesian product operator?

	x	y
x	(x, x)	(x, y)
y	(y, x)	(y, y)
z	(z, x)	(z, y)

4 Given sets A and B from problem 3, what is the power set of B?

$B = \{x, y\}$

The power set of B is $\{0, \{x\}, \{y\}, \{x, y\}\}$

5 Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$

Here at two functions, g and f, defined in the following tables, over X and Y:

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

n	f(n)
1	6
2	7
3	6
4	7
5	6

5.1 What is the value of f(2)?

7

5.2 What is the range and domain of f?

Domain: [1, 5]
Range: [6, 7]

5.3 What is the value of g(2,10)?

6

5.4 What is the range and domain of g?

Domain: [1, 10]
Range: [6, 10]

5.5 What is the value of g(4,f(4))?

8

6 Prove $2 = 1$

Basis: Consider the equation $a = b$

1. Multiply both sides by a to obtain $a^2 = ab$
2. Subtract b^2 from both sides to obtain $a^2 - b^2 = ab - b^2$
3. Factor each side, to obtain $(a - b)(a + b) = b(a - b)$
4. Divide by $(a - b)$ to get $(a + b) = b$
5. Now, let $a = b = 1$.

6. Therefore, $2 = 1$, which is what we set out to prove.

There seems to be an error here. What is it?

The error is occurs in step 4. This step involves dividing both sides of the equation by (a-b). However, this is impossible because the basis for this proof states that a=b, making dividing by (a-b) be dividing by zero. Because of its undefined properties, it is not mathematically possible to divide by zero.

7 Let $(n) = 1 + 2 + ... + n$ be the sum of the first n positive integers. Using PMI, prove that: $S(n) = \frac{1}{2}n(n+1)$

Prove:

$$S(n) = \frac{1}{2}n(n+1)$$

$$\frac{1}{2}n(n+1) = \frac{n(n+1)}{2}$$

Base Case:

$$S(1) = \frac{1(1+1)}{2}$$

$$\frac{1(1+1)}{2} = 1$$

$$S(1) = 1$$

Induction Step:

Assume true for k:

$$S(k) = \frac{k(k+1)}{2}$$

Define $S(k+1)$

$$S(k+1) = 1 + 2 + 3 + ... + k + (k+1)$$

Add $(k+1)$ to the defined version of $S(k)$

$$\frac{k(k+1)}{2} + (k+1)$$

Multiply $(k+1)$ by $\frac{2}{2}$ to equalize the base

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

Evaluate the addition

$$\frac{k(k+1) + 2(k+1)}{2}$$

Factor the numerator

$$\frac{(k+1)(k+2)}{2}$$

Expand the addition in $(k+2)$

$$\frac{(k+1)((k+1)+1)}{2}$$

Now the inducted equation matches the defined $S(k+1)$

$$\frac{(k+1)((k+1)+1)}{2} = S(k+1)$$

8 Let $C(n) = 13 + 23 + ... + n^3$ be the sum of the cubes of the first n positive integers. Using PMI, prove that: $C(n)$ has this closed form equivalent:$\frac{1}{4}(n^4 + 2n^3 + n^2)$

Prove:

$$C(n) = \frac{1}{4}(n^4 + 2n^3 + n^2)$$

$$\frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{n^4 + 2n^3 + n^2}{4}$$

Base Case:

$$C(1) = \frac{1^4 + 2(1)^3 + 1^2}{4}$$

$$\frac{1^4 + 2(1)^3 + 1^2}{4} = 1$$

$$C(1) = 1$$

Induction Step:

Assume true for some k

$$C(k) = \frac{k^4 + 2k^3 + k^2}{4}$$

Add $(k+1)^3$ for the next step in the equation

$$\frac{k^4 + 2k^3 + k^2}{4} + (k+1)^3$$

Expand $(k+1)^3$

$$\frac{k^4 + 2k^3 + k^2}{4} + ((k+1)(k+1)^2)$$

$$\frac{k^4+2k^3+k^2}{4}+((k+1)(k^2+2k+1))$$

$$\frac{k^4+2k^3+k^2}{4}+((k^3+2k^2+k)+(k^2+2k+1))$$

$$\frac{k^4+2k^3+k^2}{4}+(k^3+3k^2+k+1)$$

Multiply the RHS by $\frac{4}{4}$ to equalize the base

$$\frac{k^4+2k^3+k^2}{4}+\frac{4(k^3+3k^2+k+1)}{4}$$

$$\frac{k^4+2k^3+k^2}{4}+\frac{4k^3+12k^2+4k+4}{4}$$

Evaluate the addition

$$\frac{(k^4+2k^3+k^2)+(4k^3+12k^2+4k+4)}{4}$$

$$\frac{k^4+6k^3+13k^2+4k+4}{4}$$

Set the inducted equation equal to the assumed true equation for $C(k+1)$

$$\frac{k^4+6k^3+13k^2+4k+4}{4}=\frac{(k+1)^4+2(k+1)^3+(k+1)^2}{4}$$

Expand the RHS of the equation

[...]

$$\frac{k^4+6k^3+13k^2+4k+4}{4}=\frac{k^4+6k^3+13k^2+4k+4}{4}$$

We have now proved that the inducted equation works for C(1), C(k), and C(k+1), therefore C(n) has been proven a true equation through mathematical induction