

# Project 5

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**1 Prove the following lemma by PMI. If  $u, v \in \Sigma^*$ , then  $(uv)^R = v^R u^R$ .**

Basis:

If  $|v|$  and  $|u|$  are 0,  $v \in \Sigma^*$ ,  
then  $u = \lambda$   
s.t.

$$\begin{aligned}(uv)^R &= (\lambda v)^R \\ &= (v)^R \\ &= v^R(\lambda)^R \\ &= v^R \lambda^R \\ &= v^R u^R\end{aligned}$$

Inductive Step:

Let  $|u| = n > 0$   
assume  $u = aw$ , where  $w$  is a string,  $|w| = n - 1$  and  $a \in \Sigma$   
Then,

$$\begin{aligned}(uv)^R &= (awv)^R \\ &= (awv)^R \\ &= v^R(aw)^R \\ &= v^R w^R (a)^R \\ &= v^R w^R a^R \lambda^R \\ &= v^R w^R a^R \lambda v^R w^R a^R \\ &= v^R (wa)^R \\ &= v^R u^R\end{aligned}$$

We have now proven that  $(uv)^R = v^R u^R$  through mathematical induction.

**2 Using induction on the length of the string, and the lemma you proved in problem 1, prove that  $(w^R)^R = w$  for all strings  $w \in \Sigma^*$ .**

Let  $w = uv$

$$\begin{aligned}
 ((w)^R)^R &= ((uv)^R)^R \\
 &= (v^R u^R)^R \\
 &= u^R (v^R)^R \\
 &= u^R v^R \\
 &= (uv)^R \text{ [From Q1 Lemma]} \\
 &= vu \\
 &= w
 \end{aligned}$$

We have now proven that  $((w)^R)^R = w$  through mathematical induction.

**3 Let  $J$  be the set of palindromes over  $\{a, b\}$ .**

**3.1 Give a formal definition of  $J$  using set notation.**

$$J = \{w \mid w \in \{a \cup b\}^* \text{ s.t. } w^R = w\}$$

**3.2 Using the pumping lemma for regular languages, show that  $J$  is not regular.**

The pumping lemma can be used to determine if  $J$  is a regular language by determining if there is a number  $P$ , the pumping length, where if  $s$  is any string in  $L$  of length at least  $P$ , then  $s$  may be divided into 3 parts,  $S = xyz$  such that:

1. for each  $i \geq 0$ ,  $xy^i z \in J$
2.  $|y| > 0$
3.  $|xy| \leq P$

In this case, due to  $J$  being the set of palindromic strings formed from  $\{a, b\}$ , we can define  $S = a^p b a^p$ .

Through the first criteria for the modified version of  $s$  used in the pumping lemma,  $\forall i > 0, xy^i z \in J$ , we can deduce the following.

If  $S = a^h a^i a^{p-h-i} b^p a^p \in J$ , then  $S' = a^h a^i a^i a^{p-h-i} b^p a^p \in J$  which is equivalent to  $a^{i+p} b^p a^p \in J$ .

However, when defining this, we specified that  $i > 0$ , which is contradicted by this statement.

Therefore:

Since  $i \neq 0$ ,  $S'^R \neq S'$ , so  $S' \notin J$ .

This means that  $S$  is not a regular language.

4 Let  $L = \{ww \mid w \in \{a, b\}^*\}$ . Using the pumping lemma for regular languages, show that  $L$  is not regular.

**Lets assume that  $L$  is regular:**

Consider the constant  $k$  ( $k \geq 1$ )

Consider a string  $s = a^m b^m b^m a^m$  where  $m \geq 1$

Now consider decomposition's for  $s$ .

They will be of the form  $s = xy$ , where  $y = a^j$  where  $1 \leq j \leq k$ .

Now by pumping lemma,  $xv^i y$  belongs to  $L$  for all  $i \geq 0$ .

Consider  $i = 0$ , then we have a string in the form  $a^l b^m a^m b^m$  where  $l < m$ .

Since the number of  $a$ 's on the left side of the string is less than  $a$ 's on the right side of the string, they can never be represented as  $ww$ , where  $w \in ab^*$ .

This **violates** our assumption that  $L$  is regular.

So by **contradiction**, we have shown that  $L$  is not regular.