Project 6

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Section 1

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1 Let $L = \{a^n b^m c^{2n+m} | n, m \geq 0\}$. Construct a context-free grammar, G, that generates L.

Our context free grammar, G, that generates L, starts with string S:

$$
\begin{aligned}
&S \Rightarrow AB \\
&A \Rightarrow aABCC|aACC|\lambda \\
&B \Rightarrow bBC|\lambda \\
&C \Rightarrow cC\lambda
\end{aligned}
$$

2

2.1 State the rules for Chomsky Normal Form (CNF).

A CFG is in CNF if every rule is of the form:

$A \Rightarrow BC$

$A \Rightarrow a$

$S \Rightarrow \lambda$

Where:

$a \in \Sigma$

$A, B, C \in V$

S is the start variable

2.2 Convert the following grammar to CNF.

$$
\begin{aligned}
S &\Rightarrow aA|ABA \\
A &\Rightarrow AA|a \\
B &\Rightarrow AbA|bb
\end{aligned}
$$

2.2.1 Guarantee Start Variable is only on LHS.

No action needed

2.2.2 Eliminate all rules of $A \Rightarrow \lambda$ where $A \neq S$.

No action needed

2.2.3 Eliminate all unit rules.

No action needed

2.2.4 Fix remaining rules.

Let $U_1 \Rightarrow a$ and $U_2 \Rightarrow b$

$$
\begin{aligned}
S &\Rightarrow U_1A|ABA \\
A &\Rightarrow AA|a \\
B &\Rightarrow AU_2A|bb \\
U_1 &\Rightarrow a \\
U_2 &\Rightarrow b
\end{aligned}
$$

Let $B_1 \Rightarrow BA$ and $B_2 \Rightarrow U_2A$

$$\begin{aligned}
S &\Rightarrow U_1A|AB_1 \\
A &\Rightarrow AA|a \\
B &\Rightarrow AB_2|U_2U_2 \\
B_1 &\Rightarrow BA \\
B_2 &\Rightarrow U_2A \\
U_1 &\Rightarrow a \\
U_2 &\Rightarrow b
\end{aligned}$$

This grammar is now in Chomsky Normal Form.

3 Using Example 2.14, show that the machine, M1 in Figure 2.15 recognizes 0011.

Input String	Stack Content	Rule	Description
ϵ	\$	$\epsilon, \epsilon \Rightarrow \$$	Input ϵ, pop ϵ from stack, push \$.
0	0\$	$0, \epsilon \Rightarrow 0$	Input 0, pop ϵ from stack, push 0.
0	00\$	$0, \epsilon \Rightarrow 0$	Input 0, pop ϵ from stack, push 0.
1	0\$	$1, 0 \Rightarrow \epsilon$	Input 1, pop 0 from stack, push ϵ.
1	\$	$1, 0 \Rightarrow \epsilon$	Input 1, pop 0 from stack, push ϵ.
ϵ		$\epsilon, \$ \Rightarrow \epsilon$	Input 1, pop \$ from stack, push ϵ.