Project 9

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Section 1

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1 Argue that a DFA and a regular expression are equivalent.

Theorem 1.39:
Every nondeterministic finite automation has an equivalent deterministic finite automation.
Lemma 1.55:
If a language is described by a regular language, then it is regular.
Theorem 4.5:
EQ_{DFA} is a decidable language.
Arguing that a DFA and a regular expression are equivalent.

1.1 Express this problem as a language.

Define the language as:
$M = \{(D, R) | D$ is a DFA and R is a regular expression with $L(D) = L(R)\}$

Recall that the proof of theorem 4.5 defines a Turing machine F that decides the language $EQ_{\text{DFA}} = \{(A, B)|A$ and B are DFA's and $L(A) = L(B)\}$. Then the following Turing machine T decides M.

1.2 Show that it is decidable.

$T =$ on input (D, R), where D is a DFA and R is a regular expression:

- Convert R into a DFA C_R using the algorithm in Lemma 1.55.
- Run Turing machine decider F from Theorem 4.5 on input (D, C_R).
- If F accepts, accept. If F rejects, reject.

2 Suppose S is the set of all infinite sequences of 0s and 1s. 001010101... is a member of S, for example. Show that S is uncountable.

Assume S is countable $\Rightarrow \exists f : \mathbb{N} \to S$ (A bijection)

$n \to S_n$

$f(1) = 10011\ldots$ (Pick 1 at index 1, flip to be 0)
$f(2) = 01110\ldots$ (Pick 1 at index 2, flip to be 0)
$f(3) = 11001\ldots$ (Pick 0 at index 3, flip to be 1)
$\ldots$
$f(n)$
$\ldots$

Special sequence A, it takes the first digit in the first line, second digit in the second line, third digit in third line and so on, but flips each digit.

$A = 001\ldots$

A being a sequence of 0s and 1s is in S, but it is never hit by the bijection. In other words:

$A \in S$ but $\forall n \in \mathbb{N}, f \neq A$

3 Example 3.9 describes a Turing machine, M, that decides the language w#w, where w is any string over $\{0, 1\}^*$. Give the sequence of configurations that M produces on input 1#1.

$q_1 1\#1 \to q_6 1X1$
$Xq_3\#1 \to q_6 X1X1$
$X1q_5 1 \to q_6 \sqcup X1X1$
$Xq_6 X1 \to q_{\text{reject}} \sqcup X1X1$

4 Write a Python program that simulates the Turing Machine M_2.

See GitHub Classroom submission Q4.py submitted by Ben10164 (Ben Puryear) in the project-9-Ben10164 repository.

5 We know that if $A \leqq_M B$ and A is undecidable, then B is undecidable. We use this principle to conclude that an important problem in computational theory is undecidable. What's the problem?

The halting problem.

6 State the problem formally for the previous question.

$\mathrm{HALT}_{\mathrm{TM}} = \{(M, W) | M$ is a TM and M halts on input $W\}$.

7 What theorem did we call the "gloomiest theorem of them all?"

Rice's theorem.