Project 7

Jesse Adams, Dominic Orsi, Ben Puryear
Section 1

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1 Show that A is context free. You will need a definition and a construction to do this.

$A = \{a^m b^n c^n | m, n \geq 0\}$.

Definition of context free language:
A language is context free if and only if there is a PDA that recognizes it.
CFL's are generated by context free grammars.

Construction of context free grammar:
$S \Rightarrow XY$
$X \Rightarrow aX|\epsilon$
$Y \Rightarrow bYc|\epsilon$

Construction of a PDA that accepts A:
$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$
$\Sigma = \{a, b, c\}$
$\Gamma = \{b, \$\}$
$q_{\text{start}} = q_0$
$F = \{q_5\}$

$\delta =$

Input	Stack	Rule
ϵ	\$	$\epsilon, \epsilon \Rightarrow \$$
b	$b\$$	$b, \epsilon \Rightarrow 0$
a	\$	$a, \epsilon \Rightarrow \epsilon$
c, b	$b\$$	$c, b \Rightarrow \epsilon$
\$	ϵ	$\epsilon, \$ \Rightarrow \epsilon$

2 Show that B is context free. You will need a definition and a construction to do this.

$B = \{a^n b^n c^m | m, n \geq 0\}$.

Definition of context free language:
A language is context free if and only if there is a PDA that recognizes it.
CFL's are generated by context free grammars.

Construction of context free grammar:
$S \Rightarrow XY$
$X \Rightarrow aX|\epsilon$
$Y \Rightarrow bYc|\epsilon$

Construction of a PDA that accepts A:
$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$
$\Sigma = \{a, b, c\}$
$\Gamma = \{a, \$\}$
$q_{\text{start}} = q_0$
$F = \{q_5\}$

$\delta =$

Input	Stack	Rule
ϵ	\$	$\epsilon, \epsilon \Rightarrow \$$
c, ϵ	\$	$c, \epsilon \Rightarrow \epsilon$
a	\$	$a, \epsilon \Rightarrow \epsilon$
b, a	$\$a$	$b, a \Rightarrow \epsilon$
\$	ϵ	$\epsilon, \$ \Rightarrow \epsilon$

3

3.1 Let $C = A \cap B$. Using set notation, describe C.

$C = \{a^n b^n c^n | n \geq 0\}$.

3.2 Use the results from problems 1, 2, and examples 2.36 in Sipser, to prove that the class of context-free languages is not closed under intersection.

Assume C is context free. The pumping length, p, is n. Therefore $C = a^p b^p c^p$. However, this contradicts the first property of CFL (for each $i \geq 0, uv^2xy^2z \in A$) because it is impossible to describe an equal amount of a, b, and c.