Project 8

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Section 1

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1 Use the pumping lemma for context-free languages to show that the following language is not context-free: $\{0^n1^n0^n1^n | n \geq 0\}$.

Proof: Assume that L is context free. Then by the pumping lemma for CFL, there must be a pumping length p such that if s is a string in the language with a magnitude greater than p, then s satisfies the conditions of the pumping lemma.

Let $s = \{0^p1^p0^p1^p\}$. Clearly $|s| \geq p$ as required by the pumping lemma. According to the pumping lemma, $s = uvxyz$ with $|vxy| \geq p$. This means that there are three cases that describe vxy.

1. vxy is comprised of all os and s contained entirely within either the first or second string of 0s since $|vy| > 0$, then either v or y must contain at least one 0. Now consider uv^0xy^0z. This forces either the first or second string of 0s to have at least one fewer 0 than the other. Thus $uv^0xy^0z \notin L$ which is a contradiction of the pumping lemma.

2.vxy is comprised of all 1s and is contained entirely within either the first or second string of 1s. By the same reasoning in case 1, we can see a contradiction is derived.

3. vxy is comprised of a mix of 0s and 1s. This really describes two cases, where vxy is a string of 0s followed by a string of 1s or vxy is a string of 1s followed by a string of 0s. Taking the first case as an example, vxy either straddles the first $0-1$ division or the second $0-1$ division. Since $|vxy| \leq p$, it follows that pumping either up or down will only affect the substrings immediately adjacent to the division that is straddled. The other two substrings will be unaffected. Thus the length of the straddled substrings will be changed by pumping while the length of the other two will not be. This results in pumping a string that is not in the language and a contradiction is once again derived.

Since for every case, s cannot be pumped. We have a contradiction in the pumping lemma. This makes our original assumption false and we can conclude that L is not context free.

2 For languages A and B, the perfect shuffle of A and B is the language: $\{w|w = a_1b_1...a_mb_m$ where $S_1 = a_1...a_m \in A$ and $S_2 = b_1...b_m \in B,$ where each $a_ib_i \in \Sigma^*\}$

Notice that $|S_1| = |S_2|$.
Here are two languages: $A = \{0^k1^k|k \geq 0\}, B = \{a^jb^{3j}|j \geq 0\}$.
Describe the language resulting from their perfect shuffle using set notation.
Hint: To compute a perfect shuffle, set k and j to some reasonable value, giving two strings, S_1 and S_2. Write S_1 and S_2 on a sheet of paper, one above the other. Shuffle A and B as defined above. This is the language, C, the perfect shuffle of A and B.

If we set $k = 2$ and $j = 1$, we get:
$S_1 = 0^{(2)}1^{(2)} = 0011$
$S_2 = a^{(1)}b^{3*(1)} = abbb$
This satisfies the condition for a perfect shuffle that $|S_1| = |S_2|$, this value is m.
Now that we have a valid S_1 and S_2, we can shuffle them like this:
$C = \{a_i, b_i|0 \leq i \leq m$ where $a_i \in S_1$ and $b_i \in S_2\forall i\}$
$a_1 = 0, a_2 = 0, a_3 = 1, a_4 = 1$
$b_1 = a, b_2 = b, b_3 = b, b_4 = b$
In this case, for $m = 4$, we will end with: $\{a_1b_1, a_2b_2, a_3b_3, a_4b_4\}$
This results in the set: $C = \{0a, 0b, 1b, 1b\}$.
Because each pairing of $a_ib_i \in \Sigma^*$, we can now define new language is $0a, 0b, 1b$.
This language can be defined in set notation as $C = \{0a, 0b, 1b\}$

3 Using the pumping lemma for context free languages, show that the language resulting from the perfect shuffle in problem 2 is not context-free.