

Homework 3

Due: Wednesday, October 20, 2021 before the start of class at 11am.

Turn-in via Gradescope.

Let me know if you're having difficulties and I'll give you help/hints.

Zoom office hours are Tuesdays/Thursdays at 8pm.

Problem 1:

Implement coupling from the past for the Glauber dynamics of the ferromagnetic Ising model on a $\sqrt{n} \times \sqrt{n}$ grid (with no boundary which is called free boundary or with the toroidal boundary which is called periodic boundary, your choice). Do it for large n , do many trials and look at the median coupling time (by coupling time I mean the coupling from the past convergence time M) or look at it as a function of n . How does the coupling time behave as you increase β ? The mixing time is $O(n \log n)$ for all $\beta < \beta_c$ but when does your coupling from the past convergence time start to blow up? Submit a figure to demonstrate your results, you decide what exactly is a useful figure to clearly explain your results. Be sure to let me know details of your experiment, e.g., what's the n you used, how many experiments did you do, what language did you use (any is fine), did you have to do any non-trivial implementation ideas to get it to run for a larger n ?

Part b: Try the same experiment with all + boundary condition. Does it slow down? Also try for very large β (i.e., small temperature). It's conjectured that the mixing time of the Glauber dynamics is polynomial in n for all β when there is the all + boundary condition.