Create reduced binary decision tree

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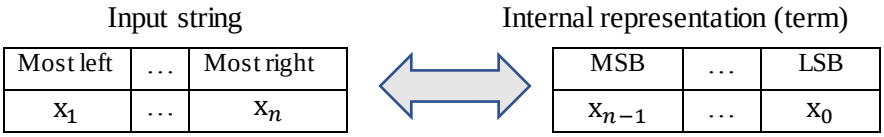
Introduction

The specification for this assignment is to produce a function (buildcompactbdt) that generate a reduced binary tree with minimum nodes, and a function (evalcompactbdt) will use this tree to evaluate input string to produce true "1", or false "0".

Description of Algorithm

The input for buildcompactbdt is a list of string, where each entry in string corresponds to a row in truth table which evaluate to 1. This input can be considered as an POS (Product of Sum) expression, where each string corresponds to a minterm, with the left most side as x1, which corresponds to LSB (Least significant bit) of the minterm, as illustrated by the diagram below.



To simplify the binary decision tree, instead of approach where seems most forward, to generate a tree then rotate and remove repetitive node.
I decide to per-process the string in term of simplify SOP expression, remove "don't cares" and get (essential) prime implicants, which is the simplest form possible, if done as binary decision graph, there will be using least node possible.
But the requirement of this assignment is binary decision tree, so I need to turn the graph into tree by creating duplicate node, with the least amount of duplication possible.

Buildcompactbdt

This function takes one argument, which is input strings (fvalues) and the return with root node of the decision tree. Below are the descriptions of the algorithm

1. Convert vector of string to vector of minterm

2. Using Quine–McCluskey[1,2] algorithm to find prime implicants.
   By using this method, up to this step, the output is optimal, there are no possible simplification to further reduce the term.

3. (optional)[3] Using chart(heuristic) or Petrick's method[4] to find essential prime implicants
   By using Petrick's method will ensure the number of term is optimal, but a much faster approached is to use heuristic method to approximate the essential prime implicants. However, as shown in later experimental section, this step does not produce a much different for the node of binary decision tree.

4. Using heuristic algorithm to generate the binary decision tree, with aim to make duplicated node appear as deeper node as possible.
   By using 3 conditions in order of propriety to choice the node, firstly choice node with most don't care, secondly choice node with most unbalanced 1 and 0, finally choice by numerical order from node 1 to node n.

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[1]Donald Krambeck (2016) *Everything About the Quine-McCluskey Method* https://www.allaboutcircuits.com/technical-articles/everything-about-the-quine-mccluskey-method/ [29th/04/2018]

[2] corporate author (2018) *Quine–McCluskey algorithm* https://en.wikipedia.org/wiki/Quine-McCluskey_algorithm [29th/04/2018]

[3] This step was considered at the start, but I had chosen not to implement due to the level of optimisation and time constraint. Please refer to 1st part of Experimental Evaluation for the experiment.

[4]Donald Krambeck (2016) *Prime Implicant Simplification Using Petrick's Method* https://www.allaboutcircuits.com/technical-articles/prime-implicant-simplification-using-petricks-method/ [29th/04/2018]

Below is an example of work through, for input fvalues of 0000(0), 1000(1), 0100(2), 1010(5), 0110(6), 1110(7), 0001(8), 1001(9), 0101(10) and 0111(14).

The website "allaboutcircuits"[5] showed how to simplify the fvalues above with Quine–McCluskey algorithm to obtain essential prime implicants so I don't repeat it here. At start of step 4, the table below shows the representation of prime implicants.

essential prime implicants	Internal representation in struct implicant		
$x_1x_2x_3x_4$[6]	$x_4x_3x_2x_1$	mask	Minterm
1-10	01-[7]1	0010	0101
-00-	-00-	1001	0000
01--	--10	1100	0010

In step4, 3 condition are used for this heuristics algorithm, listed below in the order of how decision is taken:

i. choice the x_n with least don't care, to make duplicated node appear as deeper node as possible,
ii. If there are multiple x_n with same number of don't care, additional screening is used to choose the node with most unbalanced 1 and 0. Such that for example with internal representation ($x_4x_3x_2x_1$) of 0001 and 0010, node x_3 then x_4 will be chosen as it is most unbalanced. Reference the full example from "Testing on case where can't be merged" in Evaluation section.
iii. If there are multiple x_n have same ranking from condition i and ii, then numerical order is choice.

Continue with the example, reference the table above, x_3,x_2,x_1 have the least don't care ("-" represented as 1 in mask), and same unbalance of 1 and 0 (all have 1 ones and 2 zeros). Which means numerical order will be used, which choice x_1 as root of the tree.

When x_1 is chosen as root node, the table above can be split into 2.

When x_1 is 0, left child			When x_1 is 1, right child		
$x_4x_3x_2x_1$	mask	Minterm	$x_4x_3x_2x_1$	mask	Minterm
-00(0)[8]	1000	0000	01-(1)	0010	0101
--1(0)	1100	0010	-00(1)	1000	0000

[5]Donald Krambeck (2016) *Everything About the Quine-McCluskey Method* https://www.allaboutcircuits.com/technical-articles/everything-about-the-quine-mccluskey-method/ [29th/04/2018]

[6]$x_1x_2x_3x_4$ is the order of the input string fvaules, but internally is stored as $x_4x_3x_2x_1$. Later on will always using internal representation unless when evaluate against fvalue(s).

[7] "-" here means don't care, 01-1 is same as $\overline{x_4}x_3x_1$

[8] Bracket means this bit (x_1 in this case) had been envaulted, the mask and minterm at this bit will be normalised to 0

Further consider node (bit) not been evaluated, using same heuristic strategy, x_2 will be chosen, as x_4, x_3, x_2have don't care 2,1,0 respectively, for the internal node for left child of root node x_1.By the same method, x_3 will be chosen as internal node for the right node. Which produces a table as shown below.

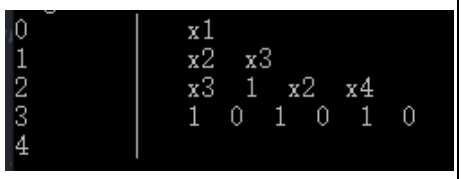
When x_1 is 0, left child		When x_1 is 1, right child	
When x_2 is 0	When x_2 is 1	When x_3 is 0	When x_3 is 1
$x_4x_3x_2x_1$	$x_4x_3x_2x_1$	$x_4x_3x_2x_1$	$x_4x_3x_2x_1$
−0(0)(0)	--(1)(0)	-(0)0(1)	0(1)-(1)

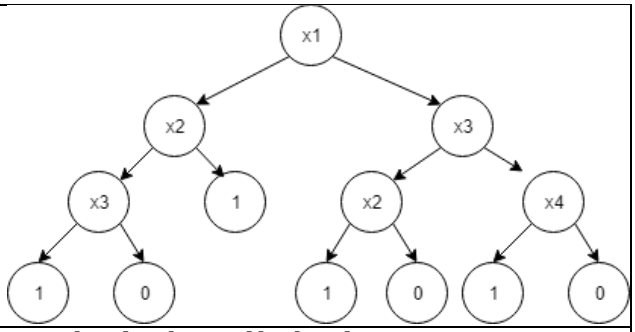
By repeat steps above, it will reach 2 kind of base case. First base case is when there is no implicant in the table, second base case is when there is *one* implicant with non-evaluated bit are all don't care ("-"). For example, when $x_1$is 1,x_3 is 0.

When x_1 is 1 and x_2 is 0					
When x_2 is 0			When x_2 is 1		
$x_4x_3x_2x_1$	mask	Minterm	$x_4x_3x_2x_1$	mask	Minterm
-(0)(0)(1)	1000	0000			

As shown on table above, on left side when x_2 is 0, the only term x_4 is not evaluated which is don't care. This node has label 1 and no child node, represented in Boolean is $\overline{x_3}\,\overline{x_2}\,x_1$ evaluate to 1.
On the other hand, on left side when x_2 is 1, there are no term as the table content. This node has label 0 and no child node, represented in Boolean is $x_3\,\overline{x_2}\,x_1$ evaluate to 0.

The binary decision tree produced by this method is shown below





*output from my print tree function | *graphical enhanced by hand

By using this method, only 13 nodes is required, compare to a full binary decision tree as did in assignment 1, requires 31 nodes.

Evalcompactbdt

This function takes two arguments, the first is the root of the tree, second is a string for input to evaluate.

In the binary decision tree, except for the leaf which will either be "1" or "0", the rest of node is in the form of "x*num*", where *num* is a index for a char at position (num-1). When evaluate the tree, if the string[num-1] is "0", it goes to left branch, otherwise, goes to right branch.

For example, continues example above, when evaluate string is "0100"($x_1x_2x_3x_4$), correspond to internal minterm 0010 ($x_4x_3x_2x_1$). Check it is not leaf node("1" or "0"), remove first char of string "x1", get 1, evaluate char of input string at index 0, which is 0, evaluate left node.
Check it is not leaf node, remove first char of string "x2", get 2, evaluate char of input string at index 1, which is 1, evaluate right node.
Check it is leaf node, output the content, returns "1".

Internal representation of minterm in C++

For flexibility of the code, all the algorithm and function implemented below uses same abstract data structure called “term”, which is treated as a vector consist of multiple word (integer) of constant length. All mask, minterm, implicant are implemented by using term.

The reason for use word (64 bit integer) is because of efficiency. Another way to implement such data structure is using std::vector<bool>, which although occupies same space (stores multiple bool in same word), but it requires mask when read and write and it does not support read and write multiple bits at a time, such as XOR of two 64-bit word.

Bitwise operator (~,|,^,&) had been overloaded, to allow readable and more maintainable code.

```
term operator~(const term& v1);
term operator&(const term& v1,const term& v2);
term operator|(const term& v1,const term& v2);
term operator^(const term& v1,const term& v2);
```

Below are how current implementation of term is defined. Each word is a 64 bit unsigned integer (unsigned long long), to form a vector of any length required by the fvalues.

```
#define wdlength 64
#define wdMax 0xFFFFFFFFFFFFFFFF
typedef uint64_t wd;
typedef std::vector<wd> term;
```

Impletion of Buildcompactbdt in C++

Generate minterm

This step is performed by function call genMinterm(fvalues,minterms) ,where fvalues is a vector of string of same length and minterms is vector of minterm, both pass by reference.

The algorithm can be represented by C++ like pseudo code below:

```
int numwd=fvalues[0].size() / wdlength;
int lastlen=fvalues[0].size() % wdlength;
For( str : fvalues){
        int p=fvalues[0].size()-1;
        term minterm(numwd+1,0); //init minterm to (numwd+1) word consist of 0

        //Most significant word of the term
        wd word=0
        Repeat lastlen times{
                if (str[p]==1){ word = word *2+1 } else { word = word *2};
                p--;
        }
        minterm[numwd]=word

        //All other less significant word of the term
        While(p>0){
                word=0;
                Repeat wdlength times{
                if (str[p]==1){ word = word *2+1 } else { word = word *2};
                p--;
                }
                minterm[(p+1) / wdlength]=word
        }
}
Minterms.push_back(minterm)
```

Find prime implicant[9]

Quine–McCluskey algorithm is used in this step, also by taking inspiration from (Jadhav Vitthal, 2012)[10], a structure is made to recursively compute this. The structure is called intermittent.

At the start the structure intermittent is populated by minterms with same mask, normalised by doing a bitwise AND with the mask and then separated into smaller group according to number of 1's to contain, the mask has same length as minterm. For example, $X_1\overline{X_4}$ and X_1X_4 will be populated to same instance of the structure, as they both have mask of 0110, but they will be speared to 2 group with minterm represented by 0001(one 1's) and 1001(two 1's) respectively.

The structure intermittent had a function called compareAll, which will compare neighbour groups of 1s iteratively by Quine–McCluskey algorithm, and create new instances of the structure by using "new " on HEAP and append it to a map with the common mask as key. For the part that did not use to create new instance, it is push backed to a vector of another structure implicant, as it is prime implicant.

Structure implicant only consist of two data field with no functions, first is the mask and the second is minterm, for example $X_3\overline{X_4}$ will have mask 0011 represent for "don't care" of term X_1X_2 and minterm 1000, where least significant 2 bit is forced to 0 due to the mask.

Pseudo code is not provided for this section, as it involved with multiple struct with their public data and function. A flowchart is used to given a overview of how these struct linked together to execute efficient Quine–McCluskey algorithm.

[9] ALL pseudo code from this sub-section onward, unless otherwise stated, treat term as a single integer, a simplification of how the real code is implemented.

[10] Corporate author (2012) *Modified Quine-McCluskey Method* https://pdfs.semanticscholar.org/650a/ae0c3e59bdd12e7b22158162166e4437949e.pdf [29th/04/2018]

The chart below shows how these structures linked together to find prime implicant.

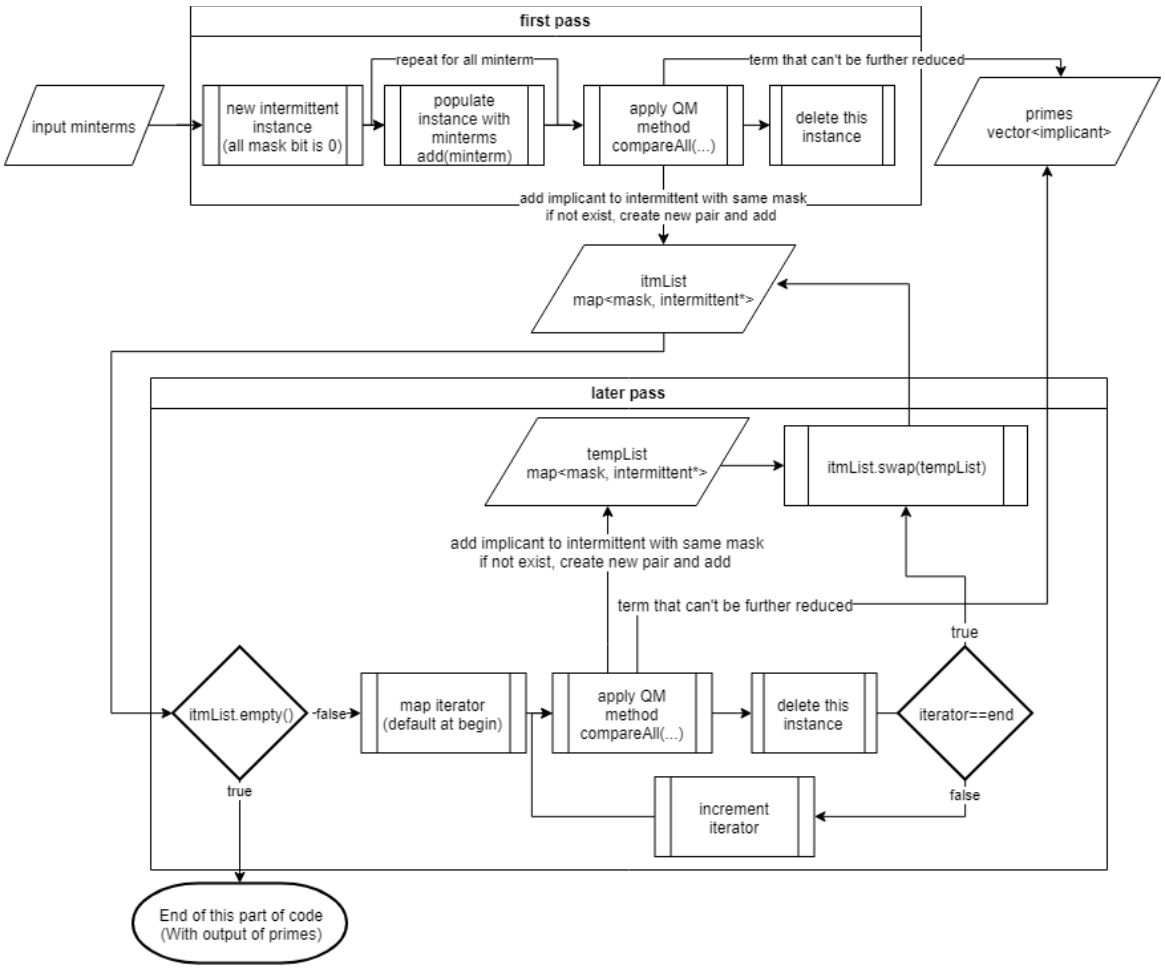


Figure 1

As shown from the chart, in the first pass, an intermittent is newed and constructed, then all the minterm produced by previous step is added to the struct, compareAll(...) function is called, produce an output of map itmList and primes for primes implicant. The current struct is than deleted to save HEAP space and avoid memory leak.

In all the later pass, itmList is firstly checked if it is empty. If true, means no more term need be further simplified and exist to next part, if false, iterate through the map for all the intermittent struct. For each intermittent struct, call compareAll(...) function but put output map to itmList, and delete the current instance and iterate to next struct in the itmList. After iteration finish, swap between tempList and itmList, ready for next pass. The pointer in itmList which swapped to tempList is no longer needed and delete automatedly as went out of scope.

The reason for the swap is feasible is due to implicant can only merge when mask is the same and its only different by one bit, but mask in tempList and itmList will never be, as tempList is like children of itmList, have 1 more bit of mask.

generate the binary decision tree

By using the algorithm introduced in Description of Buildcompactbdt from previous section. A recursive function can be used to implement this algorithm, with base cases of all don't cares and empty content.

Function recTreeConstructor(node, primes, nodeRemains) is used to implement the function. Where node is the root of the sub tree that is currently constructing, primes are the (essential) prime implicant produced by previous part, nodeRemains are the bit indicator of either bit had been used.
For example, for fvalue consist of 4 chars, if bit x_3, x_2 had been used, but x_4, x_1 haven't, the nodeRemains will be 1001 (by internal representation $x_4 x_3 x_2 x_1$).

Structure bitCount is used, is has function of construct(...), which takes vector of prime implicant (primes), count number of set for each bit of the don't care (primes.mask) and count number of ones for each bit of the minterm (primes.minterm).
It has a function called getMask (digit), which returns the sum of set bit for that digit. And function called getTermLeast(digit), which returns the minimum between sum of ones and sum of zeros for that digit, such that the smaller the number returns, the more unbalanced ones and zeros it has.

Below is C++ like pseudo code used to describe this function.

```
If(primes.size()==0){
        //2 base case
        Node.lable="0" //first base case
        return;
}else{
        For(prime :primes){
                maxMaskCount=popcount(nodeRemains)
                If( popcount(prime.mask)== maxMaskCount){
                        Node.lable="1" //second base case
                        return;
                }
        }
```

```
        // heuristic method to decide which root to choose
        bitCount cnt;
        cnt.constrct(primes, ...);
        unsigned int  mincount= (int)-1 //use type casting to set mincount to highes possible

        for(digit in nodeRemains){
                if(cnt.getMask(digit)< mincount){
                        // condition i
                        mincount= cnt. getMask (digit)
                        optimalDigit = digit
                }elseif(cnt.getMask(digit)==mincount && cnt.getTermLeast(digit)<mostUnbalanced){
                        // condition ii
                        mostUnbalanced= cnt.getTermLeast(digit)
                        optimalDigit = digit
                }
        }
        Node.labe=”x”+ optimalNode

        // prepare for the recursive call
        For(prime :primes){
                If(prime.minterm[optimalDigit]==0){
                        // as described in algorithm description, have 2 possible cases
                        If(prime.mask[digit]!=0){
                                tempPrime=prime
                                tempPrime.mask= prime.mask & (~optimalMask[11])
                                leftPrimes.push_back(prime)
                                rightPrimes.push_back(prime)

                        }else{
                                rightPrimes.push_back(prime)
                        }
                }else{
                        leftPrimes.push_back(primes[i]);
                }
        }
                // start recursive call
        recTreeConstructor(node->left,leftPrimes, nodeRemains& (~optimalMask))
        recTreeConstructor(node->right,rightPrimes, nodeRemains& (~optimalMask))

}
```

[11] optimalMask is a mask where only optimalDigit bit is set to 1, rest is 0. ~ means bitwise NOT, & means bitwise AND, it is used to normalise the optimalDigit bit on mask to 0

Impletion of Evalcompactbdt in C++

From algorithm introduced in Description of Evalcompactbdt, below is the simple implementation.

```
While(node is not 0 or 1){
        If(str[node.val]!=0){
                node=node.right
        }else{
                node=node.left
        }
}
Return node.val
```

Where node.val is the content in the node, str[i] is the i^{th} char in string str indexing from 0.

Experimental Evaluation

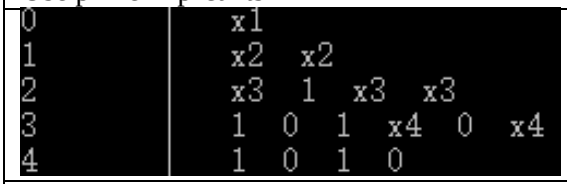
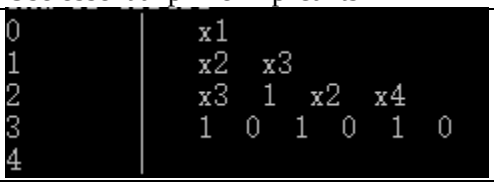
All the test is performed on Win10 Ubuntu, with average 13.5%±0.5% CPU (i7-7700K) with DDR3 RAM. The time shown is the time taken to execute buildcompactbdt() function.

In all of test below, unless stated otherwise, is done by taking integer set[12] as input, treated as minterm, generate fvalues and run buildcompactbdt(), and finally testing correctness by iterate all the possible combination and compare if "1" only get returned when it is a member of fvalues.

Is it worth to implement functions to find essential prime implicants?

I used my code without step 3 mentioned in Description of Buildcompactbdt algorithm to shown number of node produced by tree with only prime implicants as control group.
Used online resources[13] to generate the essential prime implicant, inject to step 4, to see the performance with essential prime implicants, as experimental group.

The first set I tested is with minterm used in previous work through 0,1,2,5,6,7,8,9,10,14 which forms a fvalues with size 10 and each fvalue string is made by 4 chars.
The result from control group, without reduce term to find essential prime implicants is 17 nodes. There is only one form of essential prime implicant, which is $\overline{x_2}\,\overline{x_3} + \overline{x_1}x_2 + \overline{x_4}x_3x_1$, by inject to step 4, creating a tree with 13 node. Both tree are drawn in the table below:

Use prime implicants	Use essential prime implicants
	
17 nodes	13 nodes

The second set I tested is with minterm[14] 40,51,37,38,27,50,14,5,42,53,61,20,21,16,15,22, 63,30,25,47,6,19,17,23,13,56,26,9,52,45,24,12,1,7,62,41,28,32,58,8,0,29,39,35,43,60,57,59,1 1,31,3,44,54,46,36,4,48,2,34,18, which forms a fvalues with size 59 and each fvalue string is made by 6 chars.
The result from control group, without reduce term to find essential prime implicants is 29 nodes, the result from experimental group with all 12 different form of simplification measured, it had max of 35 nodes, minimum of 31 nodes, median of 35 nodes.
Interestingly for this case, by find essential prime implicants increases the number of node required.

The last set I tested is with minterm0,2,4,8,9,10,11,12,16,21,22,23,24,25,26,27,28,29,30, 31,32,39,40,44,46,49,51,52,61,63which forms a fvalues with size 29 and each fvalue string is made by 6 chars.
The result from control group, without reduce term to find essential prime implicants is 61 nodes, the result from experimental group with all 2 different form of simplification is 57 nodes and 63 nodes.

As shown, from 3 set of data above, step 3 does not always lead to reduced to node and due to time constraint, it is not implemented.

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[12] All testing data are generated randomly from random.org by using integer set generator.
[13] Frédéric Carpon (2012) *quine-mccluskey-frederic-carpon-implementation* http://quine-mccluskey-frederic-carpon-implementation.e-geii.eu/ [29th/04/2018]
[14] Randomly generated from random.org https://www.random.org/integer-sets/

Memory and time evualtion and optimisation

Before I optimise the step 2 of Buildcompactbdt (Quine–McCluskey), the runtime of step 3 is heavily depends on the size of the input fvaules, it will take significant part of HEAP after fvalues size reaches 64 and above, also the runtime is often greater than 1 minutes. Hence, I decide to debug the find out how can I improve it.

Here is the performance before optimise, with input of minterms from 0 to 31, generate a fvaules of size 32 and each fvalue string have 5 chars.
Before the optimisation, the Buildcompactbdt function will take 15.34 seconds to run, uses 34.4MB on HEAP maximum, but there is no memory leak, as all struct created by new are deleted before pointer go out of scope.
After the optimisation, with the same input, Buildcompactbdt functions only takes 104.1 ms to construct the tree, with 101kB on HEAP at maximum, with 99.3% reduction on time and 99.7% reduction on HEAP.
See second row of table below for original output, first 5 lines are my debug code in main(), after that are information provided by valgrind.

I start this evaluation by identity the part where takes most of time and memory happened at "later pass" described in flowchart Figure 1. By using "cout" to show intermediate process in this section, I found the problem is due to duplicate entry to each of intermittent struct tempList.
To solve this problem, I changed the code in add function of struct intermittent to perform additional check of duplicate, shown on third row of table below.

Before optimsed	After optimesed
the tree has 1 nodes compare to tree in ass1 require 63 node reduced by 98% the buildcompactbdt function takes 15360.3ms ==414== ==414== HEAP SUMMARY: ==414== in use at exit: 72,704 bytes in 1 blocks ==414== total heap usage: 933 allocs, 932 frees, 34,356,724 bytes allocated ==414== ==414== LEAK SUMMARY: ==414== definitely lost: 0 bytes in 0 blocks ==414== indirectly lost: 0 bytes in 0 blocks ==414== possibly lost: 0 bytes in 0 blocks ==414== still reachable: 72,704 bytes in 1 blocks ==414== suppressed: 0 bytes in 0 blocks ==414== Rerun with --leak-check=full to see details of leaked memory ==414== ==414== For counts of detected and suppressed errors, rerun with: -v ==414== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)	the tree has 1 nodes compare to tree in ass1 require 63 node reduced by 98% the buildcompactbdt function takes 103.015ms ==420== ==420== HEAP SUMMARY: ==420== in use at exit: 72,704 bytes in 1 blocks ==420== total heap usage: 597 allocs, 596 frees, 100,756 bytes allocated ==420== ==420== LEAK SUMMARY: ==420== definitely lost: 0 bytes in 0 blocks ==420== indirectly lost: 0 bytes in 0 blocks ==420== possibly lost: 0 bytes in 0 blocks ==420== still reachable: 72,704 bytes in 1 blocks ==420== suppressed: 0 bytes in 0 blocks ==420== Rerun with --leak-check=full to see details of leaked memory ==420== ==420== For counts of detected and suppressed errors, rerun with: -v ==420== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0) ben@DESKTOP-6IA49Pb: /ass/algds/2$

minterm=minterm&baseNot;
int pcount=popcount(minterm);
 	
minterms[pcount].push_back(minterm);
used[pcount].push_back(false);
	minterm=minterm&baseNot;
	int pcount=popcount(minterm);
	
	if(notRepeated[15](minterms[pcount],minterm)){
	used[pcount].push_back(false);
	}

[15] notRepeated(A,B) function will return Boolean whether B is already in A, and push back B to A if not exist

Testing on case where can't be merged

One special case that I have experimented is when there are not term to merge to produce don't care, such that the only factor is going to make a difference is the order of the node chosen.

Below is the example of how node will be chosen for 00001, 00010 and 10000, 01000.

Term $x_1 x_2 x_3 x_4 x_5$	with condition i and ii[16]	All 3 condition i,ii,iii
00001 00010	0 x1 1 x2 0 2 x3 0 3 x4 0 4 x5 x5 5 0 1 1 0 13 nodes in total	0 x1 1 x2 0 2 x3 0 3 x4 0 4 x5 x5 5 0 1 1 0 13 nodes in total
10000 01000	0 x1 1 x2 x2 2 0 x3 x3 0 3 x4 0 x4 0 4 x5 0 x5 0 5 1 0 1 0 19 nodes in total	0 x3 1 x4 0 2 x5 0 3 x1 0 4 x2 x2 5 0 1 1 0 13 nodes in total

As shown from the table above, with condition ii, choice the node with most unbalanced 1 and 0 applies, it create minimum node consistently. Whereas if after condition i just choice numerically by condition iii, the number of nodes is heavily depend on the order of the input, leading to 46% increasement on number of nodes in this case from 13 nodes to 19 nodes.

Testing on large fvalues

Below is the result for 10,000[17] terms of fvalue. From in the range of 000000000000000 (minterm 0) to 1111111111111 (minterm 2^{15}-1).

Minterm Set reference number[18]	Number of nodes with condition i ii	Number of nodes with condition i ii iii	Number of nodes without simplification
Set 1	16861	16857	65535
Set 2	16659	16639	65535
Set 3	16815	16875	65535
Set 4	16991	16915	65535
Set 5	16817	16787	65535
Average Nodes	16828.6	16814.6	65535

As shown, there is a average of 74.3% of reduction by using my method compare to the node without simplification. By using condition ii, before choosing node numerically will reduce number of node further by 14 on average, although there are times where by using only condition i ii performs, but it is just luck.

[16] As condition described in step 4 of section Buildcompactbdt
[17] Max number of integers allowed to generate by random.org
[18] The Minterm Set used in the test bench can be find from end of appendix

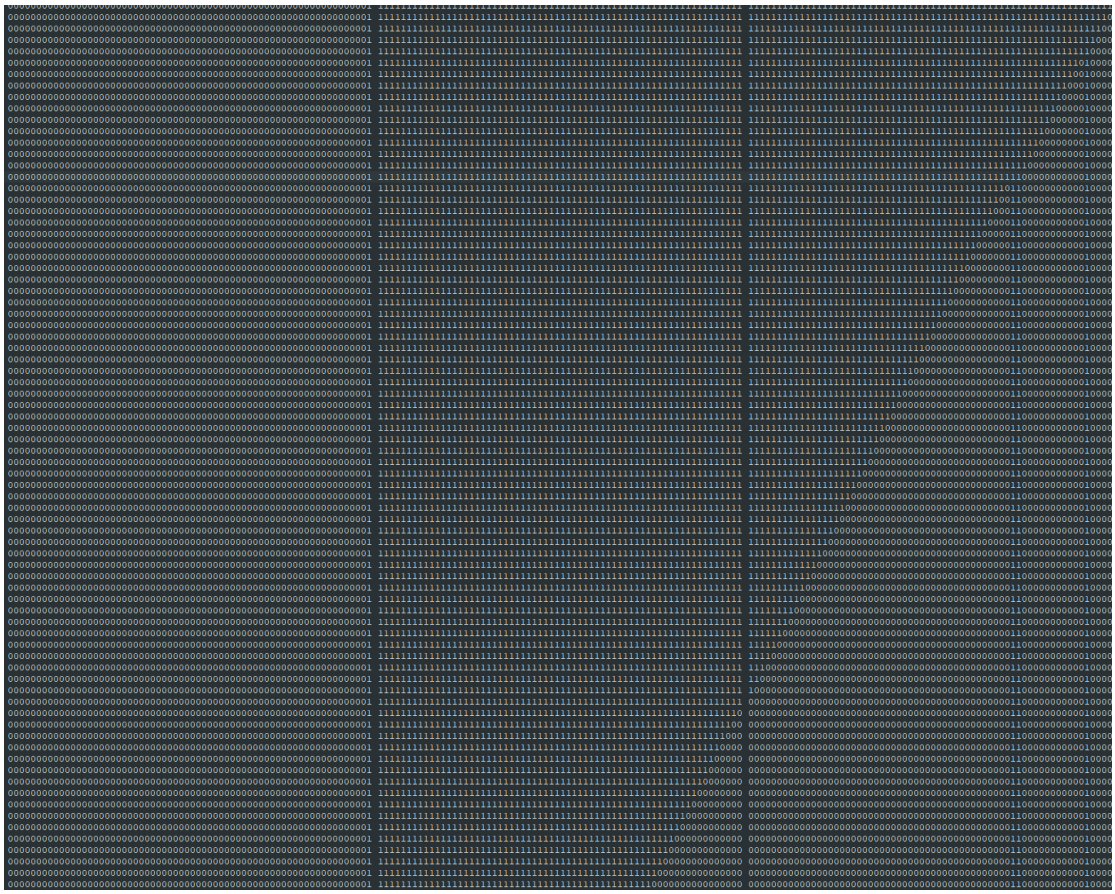
Manual test on fvalue with length greater than 64

Due to longest integer supported by c++11 is 64bit, for fvalue longer than 64 bit, automated test can't be easily performed. Hence manual test is done to confirm that it still functioning correctly.

The test contain 5 input fvalues each consist of 129 chars as shown below

```
10001010101000000001111111111100000000000000000011111111111000000000000000000111111111100000000000000001000000000000
10000010101000000001111111111100000000000000000011111111111000000000000000000111111111100000000000000001000000000000
10001010101000000100111111111100000000000000000011111111111000000000000000000111111111100000000000000001000000000000
10001010101000001000111111111100000000000000000011111111111000000000000000000111111111100000000000000001000000000000
10001010101000001001111111111100000000000000000011111111111000000000000000000111111111100000000000000001000000000000
```

To check if the tree is correct, all 5 string in fvalues are evaluated and confirmed to return 1. To check all other string will return 0, tree is printed on screen and verified the tree is the same as I expected.

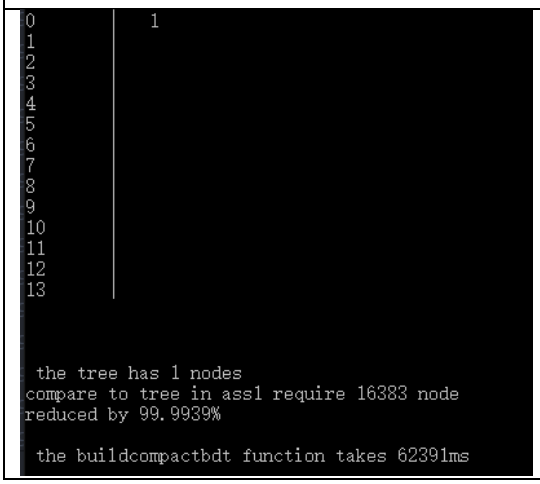
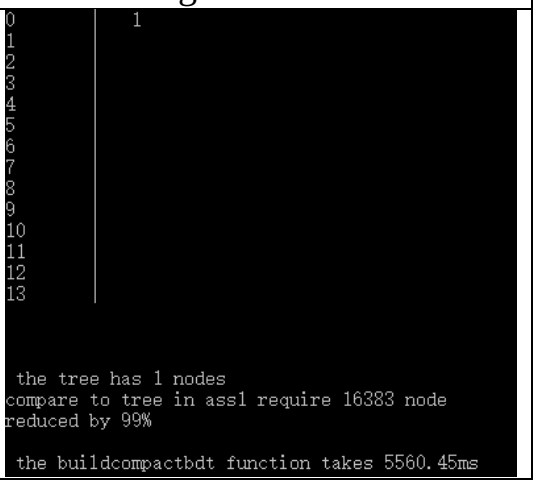


The picture shown above is the terminal output of how node is considered, just as expected, in general node is picked from left to right (from x_0), where bit turn from 1 to 0 meaning it was in consideration. It's interesting to note that bit 5 and 17,18 had stayed 1, it is due to condition ii, due to x_5 and x_{17}, x_{18} can be merged while calculating the implicant.

Effect of speed on how Internal representation of minterm is implemented

For the cause of general purpose, the minterm is internal represented as vector in the end, to allow simplification of trees with each of fvalue string longer than 64 bit, which can not otherwise performed with a single word.

However, it creates overhead for strings that is shorter or equal to 64 bit, which leading to longer run time. To understanding the effect, a test is done, by generating fvalues string from 00000000000 (minterm 0) all to 1111111111111 (minterm 2^{13}-1), total of 2^{13} fvaules. With the result shown in the table below, is about 10 times slower, but consider the number of term access during the MQ algorithm growth exponentially (NP-hard) its affect will become more significant if number of term increase, such as 2^{32} terms.

Term as vector	Term as single 64-bit word
	

Appendix

Binary trick

As some of the binary comparison will be performed many times, and the time complexity of the problem trying to solve is NP-hard. It is essential to have an efficient binary operation, as it will reduce the base of the exponential time function.

But to improve readability of the code, most of the binary tricks are warped inside structure/functions with self-explanatory names.
Please refer to the start of "Find prime implicant" for the explanation of the most important and complicated structure "intermittent" .
Structure "bitcount" is explained in the next page, to explain the concept of this optimisation.
If have further question about how these function are implemented, please refer to the source file which have comment explains the why such a binary bitwise operation is performed.

Tricks for bitcount

Tricks for bit count is quite complicated, so here given a overview of how does it count all the digit efficiently.

Counting each digit

At start assume term are single integer. For example to count the following term vector: 001111 000001 001000 001111 000000. As shown, it the vector has size 5 (0b101), so the maximum count for any digit will max by 3 bit. Hence, leave space 3 for each count.

Start by creating the mask, which will be 100100 in this case. Do shift and mask and save to a vector by using method show below, for the 001111.

Term been counting	Mask and shift	Value stored in vector	Vector index
001111	>>0 & 100100[19]	001001	0
001111	>>1 & 100100	000001	1
001111	>>2 & 100100	000001	2

Do the same for next term 000001, get {001010, 000001, 000001} and repeat for all term, finally get {011011, 000010, 000010}

Reading each digit

To read count for digit x, simply to let column be the quotient and row be the remainder of x/3. As table below, for example for x=2, it have count of 2. (bolded)

Vector index	Higher 3 bit		Lower 3 bit	
	Count digit	value	Count digit	value
0	Digit 3	011 (3)	Digit 0	011 (3)
1	Digit 4	000 (0)	**Digit 1**	**010 (2)**
2	Digit 5	000 (0)	Digit 2	010 (2)

For term as vector

The same method can be applied for vector of integers.

By splitting vector to word of integers called zones and count them separately. For example, with terms being $\{\{a_1, a_0\},\{b_1, b_0\},\{c_1, c_0\}\}$, the zone 0 will be count for a_0, b_0, c_0 and zone 1 will count for a_1, b_1, c_1.

And store the count vector as 1D, by setting the start index for each zone being the zone number multiplied by space, for start index zone 1 in this case is 1*2=2.

[19] >>0 & 100100 means means right shift 001111 by 0, bitwise AND with 100100

List of testing case for large fvalues:

All the term in the list below are in form of minterm representation of fvaules, all from range 0 to 2^{15}-1, generated by integer set from random.org.

Set 1

[illegible]

[illegible]

Set 2

[illegible]

[illegible]

Set 3

[illegible]

[illegible]

Set 4

[illegible]

Set 5

447, 5835, 4060, 7303, 14098, 12494, 12972, 9366, 11176, 8201, 5889, 11408, 10523, 6027, 4788, 11642, 14058, 10773, 9721, 6114, 15607, 10835, 755, 2836, 327, 8267, 1851, 15293, 3906, 11630, 3729, 4650, 15013, 8723, 13965, 5981, 5324, 1201, 12304, 8918, 10795, 14680, 14773, 9037, 10982, 2, 511, 1154, 15823, 14096, 11444, 15224, 6453, 13687, 4277, 3308, 9776, 6085, 1575, 11129, 11586, 11057, 2981, 3563, 4578, 15111, 6763, 15187, 3036, 3156, 9604, 4990, 8763, 3387, 13005, 4020, 9141, 48, 10758, 2863, 13572, 14861, 15291, 10414, 15379, 5421, 1918, 12088, 15411, 8551, 5400, 15891, 4052, 4927, 10149, 6780, 9509, 3473, 10711, 12022, 15534, 1078, 14375, 10616, 42, 10340, 3215, 1319, 5633, 3487, 6279, 304, 5688, 14292, 16136, 14186, 4590, 8906, 330, 1722, 8276, 7162, 10135, 10644, 6015, 15550, 8535, 9389, 13124, 4402, 2936, 7092, 13161, 2751, 15561, 10858, 3713, 10718, 12242, 1028, 4469, 13767, 2607, 10573, 2496, 15499, 2741, 14532, 631, 3675, 6016, 10222, 6404, 11103, 8319, 11981, 5121, 1860, 1864, 7503, 12910, 7572, 8658, 364, 13149, 15603, 2449, 9981, 14147, 3725, 16000, 3815, 6886, 9664, 6290, 654, 16061, 1293, 1945, 5245, 10518, 11517, 6826, 1070, 14484, 3153, 88, 16027, 9665, 399, 6034, 2548, 422, 11975, 15376, 10596, 2131, 5197, 7952, 14953, 5256, 8575, 488, 11278, 13809, 1566, 6534, 531, 16226, 4951, 1533, 3861, 3665, 968, 7312, 7960, 1624, 6084, 1180, 11465, 15256, 5745, 5997, 8993, 11386, 6823, 4586, 320, 10059, 16260, 13403, 781, 11248, 1437, 549, 4698, 12446, 1559, 7717, 59, 11919, 2640, 6006, 2493, 16272, 1663, 3619, 795, 3967, 13142, 6179, 6496, 10292, 4791, 16249, 11992, 9629, 12128, 15104, 4430, 1774, 7136, 5678, 10196, 3455, 15367, 7864, 13853, 4207, 3712, 6501, 3220, 2979, 7267, 15874, 12089, 7816, 10231, 1616, 10242, 15297, 3956, 12576, 1060, 11125, 11674, 6817, 9606, 13953, 166, 12648, 10409, 15209, 4855, 12166, 11836, 5463, 4358, 10679, 13252, 9100, 2507, 3644, 4682, 3848, 13952, 10390, 7838, 4739, 3654, 15633, 2523, 14316, 15095, 8197, 162, 2620, 6039, 6157, 11582, 13431, 15062, 5500, 1189, 8381, 9015, 9944, 3660, 3013, 357, 9393, 12032, 8982, 16207, 6679, 611, 13821, 6253, 8527, 11677, 4044, 15011, 3826, 14135, 13352, 12103, 14574, 15635, 4675, 10087, 5864, 5924, 4193, 10429, 7962, 14819, 14107, 10841, 5494, 13465, 13316, 8663, 8495, 8189, 6405, 852, 13964, 1233, 10887, 5449, 3642, 6742, 4156, 6134, 11271, 4598, 13698, 1423, 174, 15686, 4811, 1122, 1691, 7823, 804, 8795, 6761, 12403, 559, 15032, 3017, 4187, 5711, 1714, 13781, 5524, 5707, 2622, 2586, 7790, 2994, 7402, 9056, 4333, 10802, 13577, 10568, 15259, 6288, 6968, 8139, 287, 122, 15139, 12484, 199, 7744, 9994, 4595, 572, 13334, 1799, 387, 1401, 4029, 16280, 9594, 5969, 8294, 8165, 5793, 3846, 14719, 4118, 1038, 13601, 13482, 12015, 8512, 12779, 5747, 3453, 5584, 6721, 425, 831, 3142, 15558, 15910, 5803, 4943, 12796, 12135, 5203, 15584, 2417, 6361, 1801, 7848, 2553, 10410, 14185, 9685, 11097, 1229, 16107, 1146, 1542, 7077, 11003, 1678 2651, 12536, 7867, 10622, 14016, 4627, 1866, 16140, 15682, 13547, 12184, 771, 15798, 9838, 7666, 6928, 15972, 6379, 5168, 10510, 7761, 10926, 2070, 9783, 953, 2103, 15324, 6428, 4234, 8429, 11284, 15846, 10079, 5562, 1384, 13823, 2848, 8621, 15605, 9802, 10166, 2314, 4849, 4067, 5264, 9205, 7813, 3000, 4476, 6229, 8671, 8570, 1174, 5685, 341, 5021, 7101, 12867, 1754, 16360, 3218, 12344, 5295, 10105, 128, 16029, 8155, 13712, 16020, 15727, 12474, 787, 4267, 6400, 10236, 8476, 9759, 9813, 13468, 5387, 3805, 2559, 14919, 219, 933, 11939, 6479, 10501, 1271, 93, 4366, 13430, 14891, 7074, 7910, 2054, 2872, 10040, 3357, 1544, 4668, 5354, 12252, 2292, 14559, 1882, 8289, 4611, 2785, 15231, 4711, 3377, 10167, 7514, 3730, 9459, 14980, 11029, 276, 4749, 12005, 12041, 15835, 13169, 6299, 11290, 4385, 5926, 4343, 1628, 15577, 5579, 6218, 10266, 7210, 540, 12669, 16016, 3990, 14200, 11588, 12653, 14973, 3224, 318, 5861, 5158, 15006, 6095, 2304, 3461, 2334, 39, 7299, 12397, 13575, 6605, 13278, 9302, 3981, 12271, 5734, 16046, 11208, 14181, 10996, 11186, 6752, 5828, 4325, 13498, 2249, 7277, 5772, 3748, 7727, 455, 5952, 3669, 14955, 7263, 4955, 13283, 1387, 11337, 13418, 11150, 5876, 3930, 2603, 230, 15178, 3875, 15353, 4688, 6063, 10729, 9761, 5379, 14275, 10385, 11295, 10422, 14658, 8113, 8734, 11512, 11023, 6294, 1168, 11884, 10432, 6556, 4872, 7567, 12751, 13112, 10023, 13968, 3097, 14059, 12528, 4328, 12414, 4563, 14551, 12196, 6923, 6494, 6590, 3688, 2053, 14944, 10641, 5111, 2101, 5251, 1539, 15486, 10940, 1771, 9043, 15205, 12511, 7668, 8681, 3249, 8749, 183, 13263, 8166, 16198, 10498, 7242, 14284, 3592, 10259, 3809, 11505, 714, 11963, 12625, 14790, 15871, 8911, 3594, 1786, 6999, 14065, 9906, 10456, 4799, 16243, 13902, 16235, 10090, 11297, 15171, 15107, 7778, 12325, 3261, 1605, 11782, 516, 6408, 4227, 1302, 1757, 2902, 10285, 9596, 740, 4805, 8589, 6571, 14243, 464, 761, 11606, 397, 8443, 653, 9583, 11680, 7428, 10076, 7570, 12784, 14741, 1277, 312, 9997, 13036, 13862, 9868, 10665, 7746, 1273, 9154, 1968, 3135, 16072, 7098, 3413, 4225, 3349, 12864, 5353, 5504, 14459, 11275, 11559, 10889, 139, 11174, 13388, 8026, 4039, 8554, 7222, 8540, 2252, 4987, 3532, 1321, 514, 9177, 16262, 5566, 10574, 4170, 11928, 1279, 1791, 1987, 6695, 6855, 11541, 4112, 7563, 12153, 6141, 2546, 5887, 6663, 1272 10127, 9858, 58, 3517, 1668, 2997, 2368, 8583, 11958, 916, 13342, 3539, 8203, 11834, 966, 2621, 11116, 10742, 10094, 15191, 8348, 2031, 12126, 12676, 4889, 1115, 12065, 9187, 3814, 7582, 10663, 13972, 6137, 10611, 7883, 9151, 8778, 12481, 4316, 14472, 11791, 3503, 10267, 11530, 4547, 1434, 6655, 137, 13692, 4604, 604, 460, 2022 6650, 3657, 3247, 4132, 9589, 15580, 7413, 9207, 5044, 10780, 11830, 11263, 12534, 1956, 14628, 8687, 4540, 1931, 7425, 16187, 4387, 5445, 5542, 15026, 2298, 10419, 1462, 9625, 2477, 16228, 5746, 3626, 10230, 14970, 10960, 15777, 10848, 8373, 12906, 3222, 2756, 13055, 11335, 8364, 15879, 15373, 5117, 14268, 10832, 4741, 12656 6219, 354, 13432, 11104, 4757, 11185, 7041, 11083, 14692, 6505, 461, 7470, 2813, 1734, 14929, 1428, 2357, 9574, 6925, 7295, 5642, 7917, 4144, 3047, 12217, 1675, 12830, 15716, 405, 14534, 9608, 1037, 13964, 3808, 697, 10207, 1475, 13961, 3411, 12823, 6502, 3062, 5426, 4918, 6067, 4821, 13021, 8474, 6866, 196, 14913, 9331, 226, 10801, 12050, 8382, 2355, 2611, 9788, 3655, 7157, 12392, 1161, 11086, 10496, 1323, 4609, 7687, 1378, 1041, 6967, 5886, 12911, 2278, 10467, 5079, 16293, 1473, 9901, 11888, 11837, 4998, 8690, 12160, 6440, 5943, 11523, 13132, 5443, 958, 4273, 2509, 10516, 501, 11634, 10219, 2436, 12033, 3829, 8405, 5201, 2428, 12487, 920, 1048, 13505, 10550, 8288, 14820, 8277, 14452, 6389, 6667, 11216, 12931, 13597, 10370, 5063, 14371, 10556, 443, 9553, 9195, 6794, 14756, 15339, 12266, 12448, 5336, 3110, 11012, 3059, 11748, 12114, 13848, 1980, 6489, 3540, 6552, 1365, 3934, 15844, 3397, 7105, 16051, 15804, 3385, 2633, 9915, 6622, 5626, 9412, 11382, 4519, 11394, 621, 3364, 1356, 1641, 12251, 7767, 8065, 11705, 16248, 15088, 11262, 2141, 7806, 832, 2127, 3454, 14009, 8094, 7676, 11317, 3892, 10444, 12383, 5410, 13978, 11637, 1112, 12281, 8043, 1346, 16327, 8454, 5883, 14067, 3451, 1960, 12692, 8962, 4561, 7232, 6707, 2837, 15461, 15282, 8002, 11253, 486, 9188, 175, 3176, 5950, 12876, 15064, 8529, 6478, 11330, 1845, 16363, 3315, 13549, 16247, 12067, 16087, 5786, 2701, 14584, 662, 14727, 13153, 8386, 12881, 2792, 1699, 15741, 12930, 10607, 9475, 15155, 15864, 2362, 7677, 5146, 7114, 15688, 7022, 4566, 15562, 7028, 1503, 229, 10121, 2599, 709, 9808, 16013, 8500, 6586, 4986, 6258, 4579, 8127, 8058, 5125, 16279, 11030 10760, 3721, 14057, 4400, 8159, 14389, 2642, 2489, 13802, 14011, 14465, 3668, 5959, 10853, 4373, 12439, 1495, 12794, 3128, 15452, 6874, 2843, 1160, 5059, 1849, 2158, 10260, 3065, 7548, 8110, 4672, 15697, 9274, 10014, 4529, 12349, 3608, 7541, 383, 7359, 14686, 6757, 965, 2095, 15978, 14577, 9605, 7626, 7765, 8669, 14422, 12565, 15150, 11376, 7789, 13795, 13642, 7515, 594, 15511, 9176, 11220, 4877, 1366, 1944, 11794, 12510, 4551, 10998, 1159, 13870, 11970, 9545, 4549, 15003, 726, 13064, 15290, 6120, 14398, 11179, 5001, 5071, 9472, 6683, 12158, 9754, 14743, 6191, 6277, 6813, 5663, 8157, 10185, 5841, 9656, 12555, 1392, 53, 5651, 1719, 4169, 9018, 12654, 3305, 14511, 8622, 2881, 288, 3674, 8733, 3505, 15189, 3355, 3162, 9161, 6187, 3410, 13890, 3682, 5515, 6368, 10654, 9082, 7200, 3304, 107, 603, 11912, 6573, 11449, 7018, 259, 9215, 5547, 9463, 4331, 12222, 5704, 15642, 7519, 14261, 10444, 14543, 13878, 12027, 13610, 7233, 8216, 1665, 7710, 10933, 12317, 12264, 11092, 11047, 13415, 4308, 12759, 11146, 12670, 12496, 14446, 2922, 9258, 14996, 8657, 5532, 10816, 10031, 16039, 11736, 13582, 13269, 14211, 8920, 3168, 4036, 7701, 10055, 4292, 4865, 11377, 10335, 15963, 587, 9693, 14037, 5821, 5695, 7808, 8078, 9088, 12839, 16053, 8756, 8890, 15774, 309, 9638, 14583, 7134, 754, 6099, 6387, 9218, 343, 12060, 13977, 3678, 9347, 6127, 13366, 12888, 8445, 1684, 8860, 8241, 586, 6596, 1130, 12631, 10735, 13589, 2962, 4219, 12097, 11662, 9383, 8576, 6367, 1870, 2832, 7903, 14073, 9493, 7545, 10515, 10012, 4433, 15542, 1985, 12760, 6160, 8767, 2454, 2805, 15641, 14425, 13044, 1391, 10272, 12731, 14835, 834, 14234, 3381, 14757, 6827, 3006, 3649, 8378, 11166, 6074, 2265, 1013, 3183, 4147, 10560, 8346, 11111, 272, 7260, 5497, 8086, 9835, 7809, 6858, 14245, 942, 3351, 979, 14883, 4968, 1763, 3933, 3004, 15481, 8436, 1249, 7637, 15983, 2348, 1085, 13059, 8935, 628, 10971, 9777, 4923, 16074, 3514, 13004, 1006, 13822, 2072, 213, 5967, 6892, 4703, 14262, 1308 1833, 13607, 10839, 16116, 2524, 111, 1612, 8806, 4131, 10386, 15839, 8017, 12316, 2783, 4166, 7949, 11134, 5977, 12443, 6352, 5530, 11726, 7446, 6681, 2171, 15997, 7549, 483, 872, 467, 919, 11976, 11831, 12960, 856, 7613, 1738, 9756, 13550, 13580, 8359, 13663, 14344, 8273, 1562, 12962, 13906, 8898, 10652, 3320, 708, 8816, 2580, 15285, 9344, 3085, 12092, 11235, 13540, 3122, 16312, 5233, 15581, 5037, 7163, 450, 688, 12270, 7802, 4448, 12261, 6212, 13134, 14210, 10362, 5124, 4222, 3916, 14985, 2373, 14766, 3811, 1597, 15593, 3182, 10762, 5843, 13025, 10311, 10020, 7733, 11845, 11596, 4306, 5008, 10310, 9769, 5956, 1429, 10985, 3879, 872 7556, 12941, 13756, 2186, 10606, 11747, 12538, 5791, 7383, 13387, 3088, 6953, 10698, 8452, 3780, 15687, 7032, 6398, 12190, 6729, 12350, 6072, 15498, 3188, 10997, 7216, 1762, 5849, 1881, 362, 3783, 4411, 6540, 2059, 14548, 1199, 13608, 10364, 4350, 2299, 12164, 701, 6748, 12717, 12926, 5632, 4298, 6407, 15229, 8605, 6861, 8424, 4424, 481, 5701, 9254, 13198, 14568, 9828, 15218, 11548, 3149, 13844, 9573, 13649, 11633, 12849, 1220, 159, 2304, 10782, 6283, 11554, 1055, 15298, 5143, 15349, 13406, 6985, 9253, 13638, 11202, 3728, 15733, 4917, 9303, 9738, 13928, 4565, 14614, 6255, 1583, 7741, 3623, 9004, 3755, 5047, 4370, 905, 7719, 11788, 11025, 5559, 2877, 16333, 6375, 11316, 3583, 10949, 16086, 13966, 4353, 5609, 14158, 7274, 14026, 13102, 10081, 4813, 14307, 6395, 15819, 6668, 15395, 4235, 4960, 7096, 12697, 10791, 12882, 13534, 5073, 14159, 4898, 4607, 6045, 11152, 13568, 9900, 7089, 1584, 15690, 3958, 9416, 5307, 2645, 98, 1596, 1535, 14433, 12809, 12735, 12647, 9071, 6915, 2723, 11078, 652, 12430, 11303, 16085, 3698, 7253, 12567, 15629, 1375, 15770, 10189, 16195, 67, 5738, 4125, 5062, 15192, 8146, 8056, 4948, 11920, 4154, 3329, 8347, 6504, 6044, 9506, 9569, 1525, 5611, 3460, 912, 10436, 15357, 8007, 10561, 1420, 5352, 3044, 2379, 15815, 8150, 3223, 14432, 5855, 198, 526, 14533, 7671, 4155, 799, 11342, 9227, 10017, 273, 12591, 13276, 9894, 2291, 13255, 16322, 13285, 914, 3332, 3209, 15147, 14469, 4628, 11785, 3297, 15505, 2411, 682, 11860, 8447, 10399, 14501, 3356, 12598, 11946, 13531, 14443, 9939, 6750, 10770, 9498, 12586, 5860, 9265, 6052, 13631, 11679, 14393, 8472, 2234, 4694, 3596, 14613, 11827, 3272, 8286, 4895, 6920, 7625, 11687, 12967, 7332, 15036, 1082, 377, 989, 4637, 4199, 2438, 10298, 5709, 9872, 944, 11697, 4264, 15488, 13851, 14366, 10058, 15156, 14317, 4701, 12771, 6441, 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384, 13713, 10107, 10108, 16341, 3790, 7037, 7368, 1204, 1983, 14483, 7702, 10064, 14961, 11752, 2629, 4870, 12228, 8255, 10338, 4559, 3882, 11589, 6831, 16070, 14689, 8830, 3366, 15487, 1541, 10796, 76, 10898, 3335, 5682, 15277, 6485, 11922, 10824, 7112, 7155, 8430, 8863, 7525, 6325, 4680, 4152, 11820, 12177, 11670, 7388, 2330, 6951, 2571, 5867, 997, 4911, 3309, 5472, 2253, 7640, 12480, 10256, 2274, 12924, 16342, 11415, 5218, 584, 5342, 9415, 7760, 7599, 12572, 5953, 3152, 10812, 13695, 14426, 9500, 14708, 6465, 12944, 3198, 15473, 2110, 15586, 6338, 14829, 6527, 13762, 5374, 14845, 11604, 3605, 10594, 1582, 4489, 6357, 1053, 9796, 8143, 7409, 9862, 3438, 7289, 13257, 14066, 12417, 480 5533, 1510, 12564, 16274, 6472, 15144, 1877, 8773, 5583, 16022, 2504, 4570, 4395, 4197, 10642, 15996, 5881, 14305, 15336, 10967, 5405, 15366, 12355, 15720, 6301, 2822, 15840, 14608, 6635, 4669, 1554, 13248, 745, 12602, 8682, 5459, 15447, 7926, 8811, 643, 6168, 10731, 10845, 9944, 14088, 6535, 14759, 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