Supplementary Material for "On Weighted Optimality of Excitation Trajectory for Inertial Parameter Identification with Hierarchical Importance"

This document presents additional results for the experiment in the main paper.

A. Inertial Parameter Estimation Error

We have evaluated the identifiability of each parameter in the transformed coordinates $\hat{\Phi}_s = U^{-1}\hat{\Phi}$. The results indicate that the leading parameters achieve higher identifiability, owing to the larger weights assigned to them. To further support this finding, we also present the estimation errors of Φ_s, which are consistent with the identifiability analysis.

To obtain the inertial parameter error, the ground truth of the inertial parameter is required. Although the gripper we used has nominal CAD parameters, the installation of an additional flange between the robot and the gripper changed its inertial properties. As a result, the original CAD parameters no longer accurately represent the system and are unsuitable as ground truth. Instead, we utilize the average results of four methods in the case of 12 seconds as the ground truth, in which the excitation trajectory is sufficiently rich. The error is defined as

$$\varepsilon = \|\hat{\Phi}_s - \hat{\Phi}^*_{s,12s}\| \qquad \varepsilon^i : i\text{-th element in } \varepsilon \tag{1}$$

where $\hat{\Phi}^*_{s,12s}$ the is average results of 12 seconds excitation.

The estimation errors are reported in Table I. Under both weighting scenarios, the leading parameters generally achieve high estimation accuracy, with the first parameter in $\hat{\Phi}_s$ being the most accurate across all methods. This improvement arises because the proposed method assigns the largest weight to the first elements. Although the estimation errors inevitably contain noise and unmodeled effects, the overall results remain consistent with the identifiability analysis.

B. Conditional Number of Excitation Trajectories

The proposed weighted method assigns weights to the covariance of the inertial parameters. As discussed in the paper, this weighting tends to align the regression matrix with the weighting matrix. Consequently, an unsuitable weighting matrix may, in principle, lead to an ill-conditioned regression matrix. However, our findings indicate this effect is limited. Following [1], we adopt the *condition number* as a measure of matrix conditioning, with results summarized in Table II. The results show that, compared to other methods, the proposed weighting has only a minor impact on the conditioning, even when the weighting matrix is somewhat ill-conditioned. Therefore, we think that as long as the weighting matrix is not extremely ill-conditioned (e.g., nearly singular), the proposed criterion can still provide a reliable excitation trajectory.

TABLE II: Condition Number of Regression Matrix

	cond$(A^T\Sigma^{-1}A)$
With *known* planning trajectory (8s)	
No-weighting-Coor.1	55.54
No-weighting-Coor.2	113.34
Proposed	70.5934
Pullback	148.4176
Weighting matrix M_d	3076.68
With *unknown* planning trajectory (8s)	
No-weighting-Coor.1	55.54
No-weighting-Coor.2	113.34
Proposed	130.24
Pullback	148.42
Weighting matrix M_d	28.15

Note: cond(M) = cond(M^{-1})

References

[1] Maxime Gautier and Wisama Khalil. Exciting trajectories for the identification of base inertial parameters of robots. *The International journal of robotics research*, 11(4):362–375, 1992.

TABLE I: Inertial parameter estimation errors (lower is better).

	Estimation error of each inertial parameter $\varepsilon = \|\hat{\Phi}_s - \hat{\Phi}^*_{s,12s}\|$ ε^i : i-th element in ε									
	ε^1	ε^2	ε^3	ε^4	ε^5	ε^6	ε^7	ε^8	ε^9	ε^{10}
Weighting matrix $M_d = A_d^T\Sigma^{-1}A_d$ With ***known*** planning trajectory (8s trajectory)										
No-weighting-Coor.1	0.2712	0.0615	**0.0535**	0.3592	**0.0782**	0.5211	0.7096	0.0847	0.8120	0.8565
No-weighting-Coor.2	0.1200	**0.0058**	0.0970	0.2606	0.4117	**0.1218**	0.4233	0.0856	**0.1475**	**0.1799**
Proposed	**0.0709**	0.0333	0.0583	**0.0007**	0.2863	0.3530	**0.3190**	**0.0281**	0.2032	0.3510
Pullback	0.1981	0.1214	0.2165	0.1710	0.3601	0.3735	0.7837	0.0581	0.9018	0.2746
Weighting matrix $M_d = A_r^T\Sigma^{-1}A_r$**:** With ***unknown*** planning trajectory (8s trajectory)										
No-weighting-Coor.1	0.0521	0.1047	0.1231	0.0465	0.2974	0.0827	0.1195	0.0193	0.2821	0.4255
No-weighting-Coor.2	0.0813	0.4168	0.1641	0.3107	**0.0613**	0.3913	**0.0829**	0.0947	**0.0138**	**0.0200**
Proposed	**0.0003**	**0.0013**	0.2265	**0.0351**	0.2100	0.1972	0.2118	**0.0020**	0.3311	0.0623
Pullback	0.3446	0.1086	**0.0953**	0.1792	0.4620	**0.0336**	0.3187	0.3311	0.0520	0.9400