UNIVERSITY OF CAPE TOWN

MEC4110W

FINAL YEAR PROJECT

COMBINED MATHEMATICAL ANALYSIS

Analytical Model of BISRU two stage light gas gun

Author:	*Student Number:*
Joseph Goemans	GMNJOS001
Kayla Burger	BRGKAY001
Kimeel Govender	GVNKIM010
Scelo Fakude	FKDSCE001

June 8, 2020

1 Introduction

This document will demonstrate a basic model of the BISRU gas gun shown below.

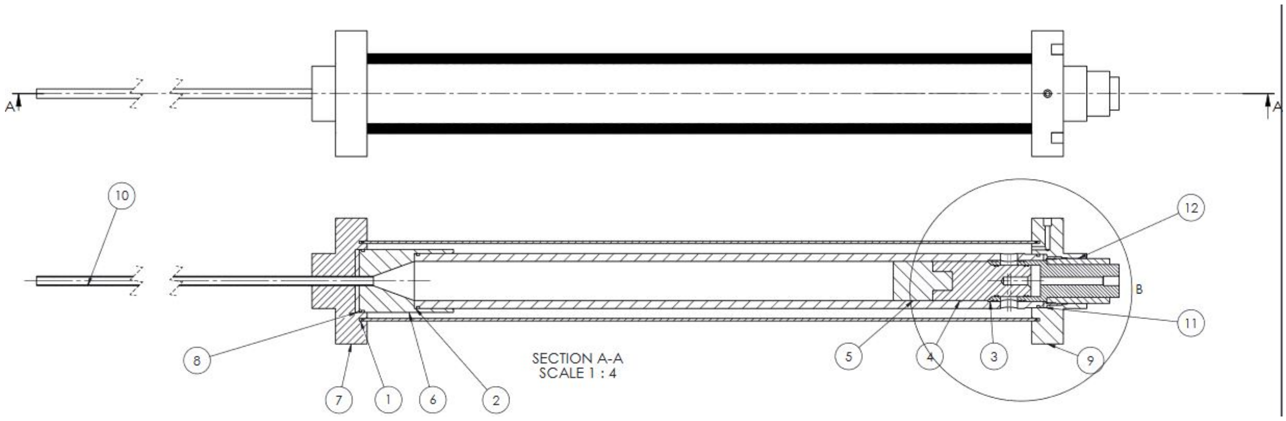

Figure 1: Overall view of the BISRU Two-Stage Gas Gun **?**

1.1 Approach

In order to determine the response of the gun, the basic geometry of the gun must be studied. The following steps will be followed.

1. First, the volume of the reservoir will be calculated.
2. Then, taking the pressure of the reservoir at this stage all the initial conditions are determined.
3. Due to the design, the piston needs to move past the slots so that the gas can expand behind it. The extra volume occupied by the gas in this no work expansion is then calculated.
4. From the new total volume. Ideal gas relations are used to determine the new initial pressure of the reservoir.
5. The expansion of the gas begins to do work on the piston which transfers the work at a rapid rate to the driver gas. To determine the volume through which the gas will expand, the volume of the space in front of the piston is calculated. A section of the transition cone will be utilised, so this is calculated with the dependent variable as the depth of deformation.
6. From the previous volume calculations, the amount of work done by the expanding gas is calculated using isentropic relations.
7. The work available from the expanding gas is then transferred into the driver gas. Assuming that the projectile only begins to move once the piston stops, the initial and final

volumes of this compression are calculated.

8. A balance is then performed until the deformation of the piston head into the cone is determined. With the known deformation in this ideal system, the second stage of the gas-gun begins.
9. The final pressure and temperature of the driver gas in the first stage are calculated and provide the initial conditions for the second stage.
10. Next, the final volume of driver gas behind the projectile in the barrel is calculated. This will be the final volume in the work equation for the projectile movement.
11. Using isentropic relations, the maximum work done by the driver gas on the projectile is then calculated.
12. Assuming that all energy is converted to kinetic, the maximum ideal velocity of the projectile is then determined.

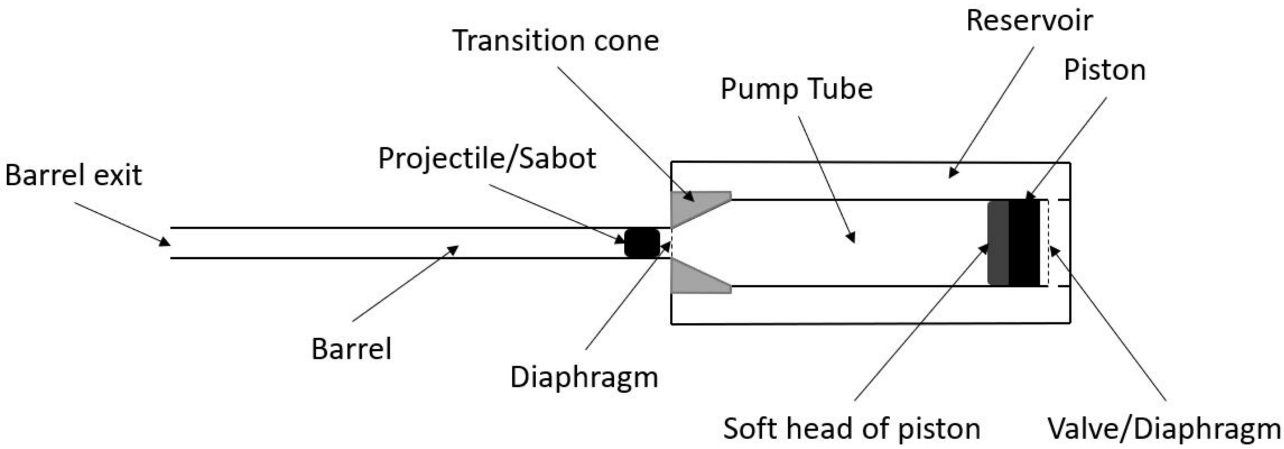

Figure 2: Simplified Schematic of the BISRU Gas Gun

2 Stage 1

2.1 Initial state of the Reservoir

The dimensions of the reservoir are given below:

$$d_{o.res} = 95.24mm$$

$$d_{i.res} = 70mm$$

The length of the reservoir is given by the length of the tube minus the length the tube is embedded into the flange on either end.

$$L_{res} = 870mm - 34mm = 836mm$$

As shown in figure 2.1, the transition cone occupies some of the space of the reservoir. So this volume must be calculated and subtracted from the overall volume.

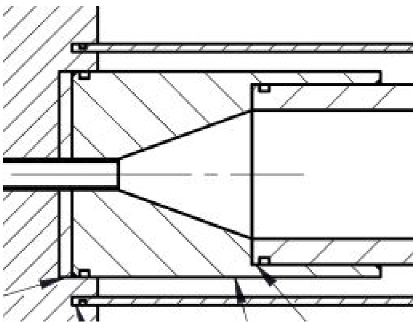

Figure 3: Section view of the conical section of the BISRU Two-Stage Gas Gun **?**.

$$d_{o.cone} = 80mm$$

$$L_{cone.inres} = 120mm$$

The final section of the reservoir to be considered is the volume of the space in the slots shown in the back end of the pump tube in figure 1.
The slots are $18x22mm^2$. They extend from the outer diameter of the pump tube which is the inner diameter of the reservoir, down to:

$$d_{i.slot} = 42mm$$

A simplifying assumption that the surface area of a flat slot and a slot in a cylinder are approximately equal is made to determine this volume.
The equation for the area of a slot is thus given by.

$$A_{slot} = \pi(\frac{l_{slot}}{2})(\frac{w_{slot}}{2}) \tag{1}$$

Where l_{slot} is the length of the slot. In this case, $l_{slot} = 22mm$. And w_{slot} is the width of the slot. In this case, $w_{slot} = 18mm$.

The Volume of the slot is then given by:

$$V_{slot} = (A_{slot})(d_{i.res} - d_{i.slot}) \tag{2}$$

Which gives:

$$V_{slot} = 8.708 \times 10^{-3} L = 8.708 \times 10^{-6} m^3$$

With all the different components of the reservoir, the initial volume can be calculated by the following:

$$V_{res} = \pi((\frac{d_{o.res}^2 - d_{i.res}^2}{4})L_{res} - (\frac{d_{o.cone}^2 - d_{i.res}}{4})L_{cone.inres}) + 6V_{slot} \tag{3}$$

Which produces the initial volume of:

$$V_{res} = 2.649L = 2.649 \times 10^{-3} m^3$$

From the parameters of the gun, the initial Pressure is $10bar$, and the initial temperature will be room temperature. So these will be the three initial conditions

$$V_0 = V_{res}$$

$$P_0 = 10bar$$

$$T_0 = 296K$$

2.2 Final conditions before work is done on the driver gas

Before work can be done on the piston, it must slide forward such that the air can flow behind the piston. Figure 4, 5 and 6 show where this point will be.

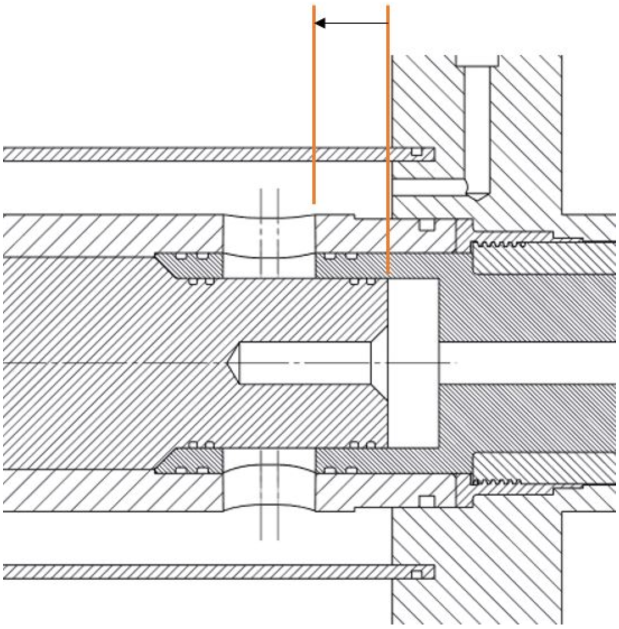

Figure 4: Section view of the rear end of the Gas Gun demonstrating the movement required before work **?**.

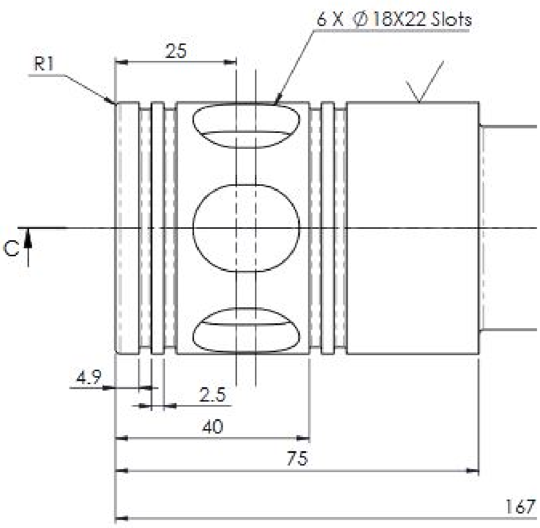

Figure 5: Dimensions of the piston insert relating to the required movement of the piston before work **?**.

The space that the gas needs to fill before work is done can then be calculated from the volume of the area between the start of the slots and the back of the inside of the pump insert shown in

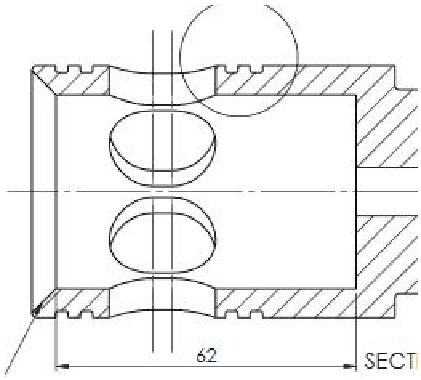

Figure 6: Section view showing the internal dimensions of the piston insert ?.

figure 2.2. The inner diameter, $d_{i.pumpinsert}$, is not shown in the figures but in Mr. Govender's report it is recorded as:

$$d_{i.pumpinsert} = 42mm$$

The length of this volume is given by:

$$L_{no.work} = (62mm + 4.9mm) - (25mm + 22mm - 9mm) = 28.9mm$$

So the volume through which no work can be done is given as:

$$V_{no.work} = \pi(\frac{d_{i.pimpinsert}}{2})^2 L_{no.work} \tag{4}$$
$$V_{no.work} = 0.04L = 4 \times 10^{-5} m^3$$

From this, we can find the final volume before work begins. This is given by:

$$V_{res.final} = V_{res} + V_{no.work} = 2.689L = 2.689 \times 10^{-3} m^3$$

Using the ideal gas law given in equation 5, we find the final conditions before work is done on the piston. Due to the small volume change, we can assume that the temperature remains constant.

$$PV = mRT = Const \tag{5}$$
$$P_2 = \frac{P_1 V_1}{V_2}$$

So the final reservoir gas pressure before work is then:

$$P_{res.0.5} = \frac{P_0 V_0}{V_{res.final}} = 9.851 bar$$

2.3 Work done by expansion of the reservoir gas

The volume that the piston displaces can be calculated by the volume ahead of the piston plus a section of the transition cone that is dependent on the depth of deformation. Figure 7 demonstrates the empty volume to be calculated as the space between the piston and the blue line on the left.

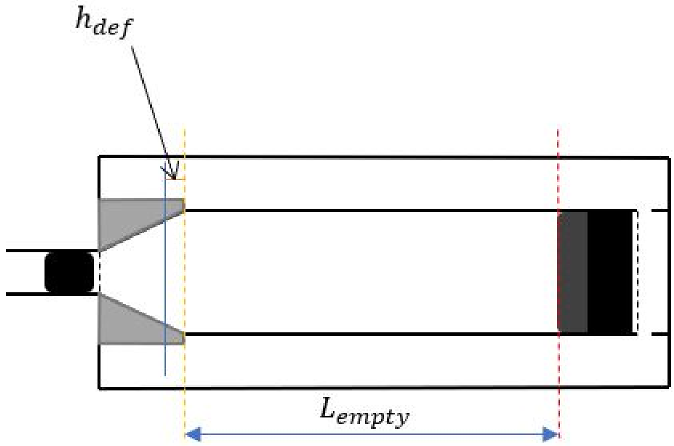

Figure 7: Pump tube before expansion of the reservoir gas.

The volume through which the reservoir gas expands is then given by the following.
The piston starts the compression cycle moved a distance into the pump tube. This is given by the depth into the pump tube to the back of the piston, $L_{start.depth}$, plus the length of the piston, L_{piston} and is calculated to be:

$$L_{start} = L_{start.depth} + L_{piston} \tag{6}$$

Where:

$$L_{start.depth} = 4.1mm + L_{no.work} = 33mm$$
$$L_{piston} = 175mm$$

The diameter of the pump tube is given by:

$$d_{ptube} = 50mm$$

The volume of a truncated cone is given by:

$$V_{trunc} = \frac{1}{3}\pi(r_1^2 + r_1 r_2 + r_2^2)h \tag{7}$$

The angle of the cone is given as:

$$\theta_{cone} = 20\deg$$

And the diameter of the cone is equal to the diameter of the pump tube, so:

$$r_1 = 25mm$$

r_2 for the volume of the deformation is dependent on the depth of the deformation, h_{def}, and is given by the following expression.

$$r_2 = r_1 - h_{def} \tan(\theta_{cone}) \tag{8}$$

The depth of deformation, h_{def}, is dependent on the amount of energy available. In order to obtain an accurate result for this, an iterative process was used. The result of which was:

$$h_{def} = 2.776mm$$

Plugging the values into equation 7, the volume of the deformation, V_{def}, is obtained.

$$V_{def} = 5.234 \times 10^{-3} L = 5.234 \times 10^{-6} m^3$$

The total volume of the space displaced by the piston in the expansion process is then given by:

$$V_{p.disp} = \pi \left(\frac{d_{ptube}}{2} \right)^2 L_{empty} + V_{def}$$

$$V_{p.disp} = 1.168L = 1.168 \times 10^{-3} m^3$$

From this, the final volume of the expansion process is able to be determined.

$$V_{drive.1} = V_{res.final} + V_{p.disp} = 3.857L = 3.857 \times 10^{-3} m^3$$

The equation for isentropic work is dependent on pressure and volume. Equation 9 is the solved integral.

$$W = \frac{P_0 V_0}{\gamma - 1} \left[1 - \left(\frac{V_1}{V_0} \right)^{1-\gamma} \right] \tag{9}$$

The work done on the piston is therefore given by:

$$W_1 = \frac{P_{res.0.5} V_{res.final}}{\gamma_{air} - 1} \left[1 - \left(\frac{V_{drive.1}}{V_{res.final}} \right)^{1-\gamma_{air}} \right] = 889.6J$$

Where $\gamma_{air} = 1.4$.

2.4 Compression of the driver gas

Continuing with the no-loss assumption, this energy will then be equal to the work required to compress the driver gas to it's pre-expansion state. By finding the initial volume of the driver gas, the final volume and hence final state of the driver gas before expansion is obtained.

The initial volume of the driver gas is the sum of the volume in the entire transition cone plus the volume of the space in front of the piston in the pump tube. r_1 remains the same as above, but r_2 becomes the radius of the cone where the barrel screws in.

$$r_2 = 6mm$$

The volume of the transition cone is then given by:

$$V_{cone} = 0.044L = 4.4 \times 10^{-5} m^3$$

And the initial volume of the driver gas is given by:

$$V_{d.0} = V_{cone} + \pi \left(\frac{d_{ptube}}{2} \right)^2 L_{empty} = 1.207L = 1.207 \times 10^{-3} m^3$$

Rearranging equation 9, the final volume of the driver gas after compression is obtained.

$$V_{d.1} = V_{d.0} \sqrt[1-\gamma_{air}]{1 + \frac{W_1(\gamma_{air} - 1)}{P_{d.0} V_{d.0}}} = 0.039L = 3.9 \times 10^{-5} m^3$$

Where $P_{d.0}$ is atmospheric pressure.
This volume was then compared to the remaining volume in the transition cone beyond the deformation of the piston. An iterative loop was then used to determine an accurate value for the depth of deformation, h_{def}, this is the recorded value on page 8.

Using the isentropic relation between pressure and volume, the compressed pressure of the driver gas is obtained. And using the relation between pressure and temperature, the temperature is found.

$$P_{d.1} = \frac{P_{d.0}}{\left(\frac{V_{d.1}}{V_{d.0}} \right)^{\gamma_{air}}} = 122.403bar = 12.24MPa$$

$$\frac{P_{d.0}}{P_{d.1}} = \left(\frac{T_{d.0}}{T_{d.1}} \right)^{\frac{\gamma_{air}}{\gamma_{air}-1}}$$

Which gives:

$$T_{d.1} = 1169K$$

If all this energy is transferred to kinetic energy in the projectile of mass $m_{proj} = 2.48g$, the maximum obtainable velocity is.

$$v_{max} = \sqrt{\frac{2W_1}{m_{proj}}} = 847m/s$$

However, even in this idealized system there will be further energy losses in the second stage. Another relationship to be considered is the relationship between gas temperature and the maximum speed that can be obtained by a projectile through the gas.

$$u_{max} = \frac{2\sqrt{\gamma_{air} R_{air} T_{d.1}}}{\gamma_{air} - 1} = 3427m/s$$

3 Stage 2

The second stage of the gas gun begins with the expansion of the driver gas. The final conditions of the first stage become the initial conditions of the second stage. So, these are the initial conditions of stage two.

$$P_{0.2} = 122.403bar$$

$$V_{0.2} = 0.039L$$

$$T_{0.2} = 1169K$$

Figures 8 and 9 demonstrate the state of the second stage at start and finish.

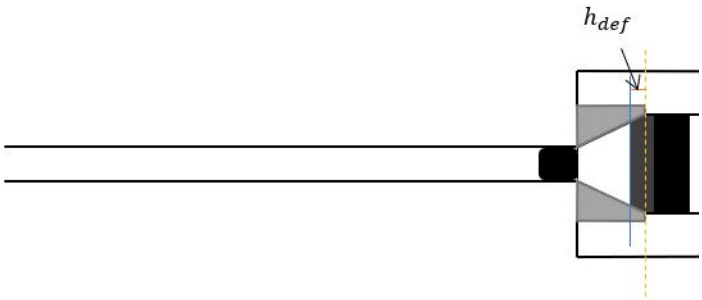

Figure 8: State 1 of the second stage of the BISRU two stage gas gun.

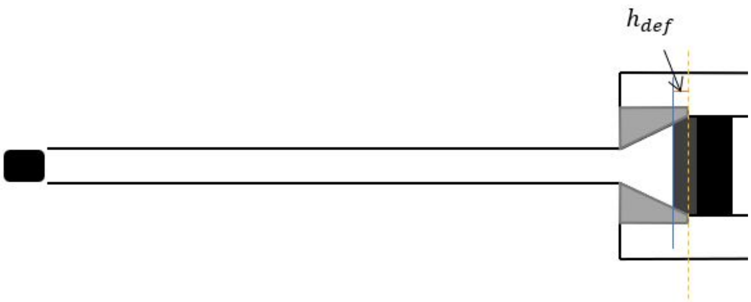

Figure 9: Final state of the second stage of the BISRU two stage gas gun.

The final volume to be considered is the sum of the volume of the entire barrel and the initial volume of stage 2. This is shown in figure 9. The dimensions of the barrel are:

$$L_{barrel} = 2.9m$$

$$r_{barrel} = 5mm$$

Therefore;

$$V_{barrel} = \pi \left(r_{barrel}\right)^2 L_{barrel} = 0.228L = 2.28 \times 10^{-4} m^3$$

Which means that the final volume of the second stage expansion will be:

$$V_{1.2} = 0.267L = 2.67 \times 10^{-4} m^3$$

To perform a basic simplification of the resistance experienced by the projectile due to the atmospheric air in the barrel, it is assumed that this resistance will result in a constant resistive force of $F_{resist} = P_{atm}A_{barrel}$ through the full length of the barrel. Considering the equation, $W = Fx$, the energy imparted onto the projectile through the second stage is given as the work done by the driver gas through expansion minus the work done on the projectile by the resistive force.

$$W_2 = \frac{P_{0.2}V_{0.2}}{\gamma_{air} - 1}\left[1 - \left(\frac{V_{1.2}}{V_{0.2}}\right)^{1-\gamma_{air}}\right] = 639.543J$$

$$W_{resist} = P_{atm}A_{barrel}L_{barrel} = 23.073J$$

The available energy to convert into kinetic energy is thus:

$$KE = W_2 - W_{resist} = 616.47J$$

So the maximum velocity obtainable in this ideal system is:

$$v_{max} = \sqrt{\frac{2KE}{m_{proj}}} = 705.091m/s$$

Using the weight of the hollow projectile specified in Ms. Edwards report from 2018.

$$m_{proj} = 2.48g$$

4 Discussion

The speed of sound in room temperature air is given by:

$$a_0 = \sqrt{\gamma RT} = 344.9m/s \tag{10}$$

With this information, the Mach number of the velocity obtained is

$$M = 2.044$$

Considering the extent of the idealised system presented, this number is unachievable in real life. After losses are included the results obtained by Ms. Edwards in 2018 **?** look more reasonable than previous models had simulated.
To complete the energy analysis, an efficiency can be calculated.
By integrating the equation:

$$W = \int_V PdV \tag{11}$$

The following is obtained:

$$W = \frac{P_0V_0}{\gamma - 1}\left[1 - \left(\frac{V_1}{V_0}\right)^{1-\gamma}\right]$$

As the limit of this equation as V_1 tends to infinity, the maximum work available from the reservoir is found to be:

$$W = \frac{P_0V_0}{1 - \gamma}$$

Putting in the values from the current gun, the maximum work available is shown to be:

$$W_{max} = 6.623kJ$$

So, considering that only $616.47J$ of work is actually transmitted to the projectile, this gives an efficiency of:

$$\%_{Eff} = 9.656\%$$

While the efficiency of energy transmission between the two stages is:

$$\%_{stageEff} = 71.9\% \tag{12}$$

The maximum work will never be obtained, but the efficiency shown between the two stages demonstrates the effectiveness of the energy transmission done by the two-stage gas gun.