1. DNF is easy

DNF meaning Disjunctive normal form. Described as an OR of ANDs, a sum of products.

Ex)
- $(A \wedge \bar{B} \wedge \bar{C}) \vee (\bar{D} \wedge E \wedge F \wedge D \wedge F)$
- $(A \wedge B) \vee (C)$
- $(A \wedge B)$
- A

Problem)

show that the satisfiability of boolean formulas in DNF is polytime solvable

Algorithm)

$O(m)$

For each clause:

$O(n)$

if clause contains x_i and $\overline{x_i}$:

unsatisfiable

else:

satisfiable

runtime)

n = number of variables

m = number of clauses

Pseudo polynomial

$m \leq s \leq n$

For each clause $O(m)$ search inside clause and find x_i and $\overline{x_i}$, $O(n)$,

So final runtime is $O(m \cdot n)$

2 2-CNF is easy

Show that the satisfiability of a formula in CNF but with exactly 2 terms per clause

Ex) $(x_1 \vee x_3) \wedge (\overline{x_2} \vee x_4) \wedge \dots$

Algorithm)

$O(n)$ For every variable x_i:

$O(n-1)$ For every variable x_j where x_i is not compared to itself (i.e. $x_i != x_j$):

if (x_i and x_j aren't in the same clause) and ($x_i == \overline{x_j}$)

unsatisfiable

else:

satisfiable

runtime)

$n =$ number of variables

for every variable x_i, $O(n)$, for every variable $x_j \neq x_i$, $O(n-1)$

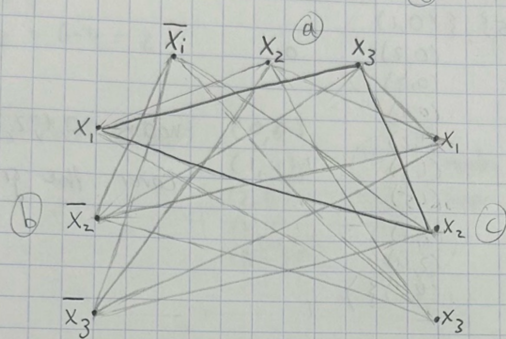
Final runtime: $O(n \cdot n-1) = O(n^2 - n) = O(n^2)$

3 Clique

Is there an assignment to

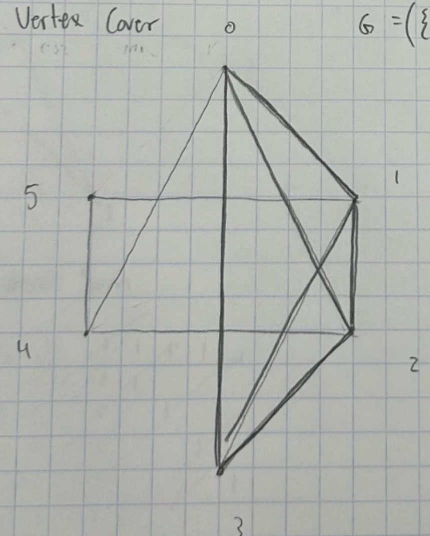
$$\phi = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee x_3)$$

a b c



x_1 = True
x_2 = True
x_3 = True

4 Vertex Cover



G = ({0,1,2,3,4,5}, {(0,1)
(0,2)
(0,3)
(0,4)
(1,2)
(1,3)
(1,5)
(2,3)
(2,4)
(4,5)})

is there a clique of size 4?

subG = ({0,1,2,3}, {(0,1)
(0,2)
(0,3)
(1,2)
(1,3)
(2,3)})

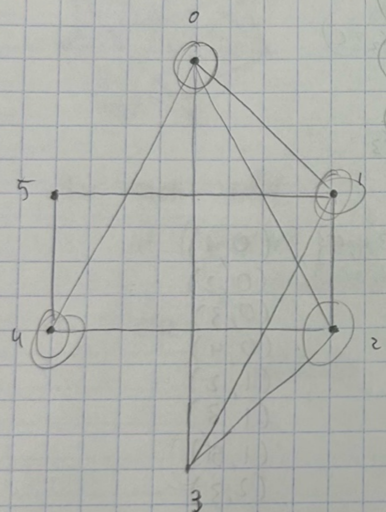
4 Vertex Cover

is there a Vertex Cover of Size 4?

G = ({0,1,2,3,4,5}, {(0,1)
(0,2)
(0,3)
(0,4)
(1,2)
(1,3)
(1,5)
(2,3)
(2,4)
(4,5)})

0
1
2
4

nodes 0,1,2,4
cover the graph



4 Vertex Cover

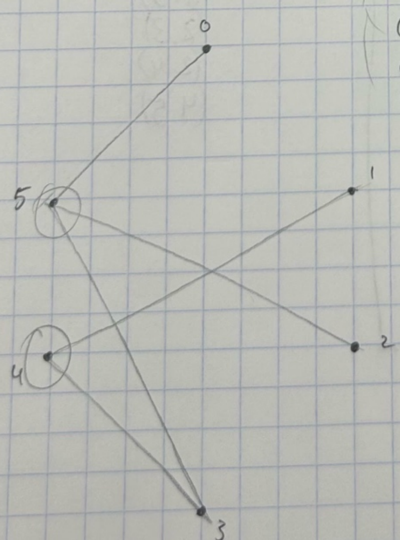
G has a clique of size 4 if and only if $\overline{G}$ has a vertex cover of size $|V| - 4$

$|V| = 6$

size $= 6 - 4 = 2$

$\overline{G} = (\{0,1,2,3,4,5\}, \{(0,5), (1,4), (2,5), (3,4), (3,5)\})$

nodes 4,5 have a Vertex Cover of size 2 on $\overline{G}$



~~5 Subset Sum~~

X_0 = 1 1 1 1 1

⑤ Subset sum

$$
\begin{array}{cc}
\begin{array}{cccccccccccccc}
 & & & 01 & 02 & 03 & 04 & 12 & 13 & 15 & 23 & 24 & 45 & &
\end{array} & \\
\left[\begin{array}{cccccccccccccc}
x_0 & = & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & = & 1984 \\
x_1 & = & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & = & 1592 \\
x_2 & = & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & = & 1318 \\
x_3 & = & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & = & 1172 \\
x_4 & = & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & = & 1091 \\
x_5 & = & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & = & 1033 \\
 & & & 1 & & & & & & & & & & = & 512 \\
 & & & & 1 & & & & & & & & & = & 256 \\
 & & & & & 1 & & & & & & & & = & 128 \\
 & & & & & & 1 & & & & & & & = & 64 \\
 & & & & & & & 1 & & & & & & = & 32 \\
 & & & & & & & & 1 & & & & & = & 16 \\
 & & & & & & & & & 1 & & & & = & 8 \\
 & & & & & & & & & & 1 & & & = & 4 \\
 & & & & & & & & & & & 1 & & = & 2 \\
 & & & & & & & & & & & & 1 & = & 1 \\
T & = & & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & = &
\end{array}\right] &
\begin{array}{c}
(0\ 1) \\ (0\ 2) \\ (0\ 3) \\ (0\ 4) \\ (1\ 2) \\ (1\ 3) \\ (1\ 5) \\ (2\ 3) \\ (2\ 4) \\ (4\ 5)
\end{array}
\end{array}
$$