Complexity

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November 13, 2024

You do not have to type this homework (though I would appreciate it if your handwriting is not ideal). You can hand-write it and scan into a pdf (there are lots of scanner apps out there).

1 DNF is easy

Show that the problem of determining the satisfiability of boolean formulas in disjunctive normal form (DNF) is polytime solvable. (As opposed to CNF, which is NP-complete.)

2 2-CNF is easy

2-CNF is the problem of determining satisfiability of a formula in CNF but with exactly two terms per clause. (For example $(x_1 \vee x_3) \wedge (\neg x_2 \vee x_4) \ldots$). Show that this can be solved in polytime by describing an algorithm to solve the problem.

3 Clique

This is to help you understand the proof that Clique is NPC. Consider the 3-CNF problem: Is there an assignment to

$$\phi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_1 \vee x_2 \vee x_3))$$

that satisfies ϕ? Convert this problem into a graph problem of the form : "Here is a graph. Does it have a clique of size 3?" I want to see the graph. Draw it as cleanly as possible.

4 Vertex Cover

This is to help you understand the proof that Vertex Cover is NPC. Consider the Clique problem: Is there a clique of size 4 in the following graph

$G = (\{0,1,2,3,4,5\}, \{(0,1),(0,2),(0,3),(0,4),(1,2),(1,3),(1,5),(2,3),(2,4),(4,5)\})$

Convert this problem to a Vertex Cover problem. Give me the graph and the exact question.

5 Subset sum

Construct the subset sum matrix corresponding to the Vertex Cover of the preceeding question.