Problem Set Party

[Due March 13th at 11:59 PM. Problems 1 and 2 are OPTIONAL; completing these problems can earn you a check+]

Question 1:

Consider a simple two-period model of intertemporal choice. Suppose that a person receives income \$48,326 in period 1 and additional income \$44,928 in period 2. The market interest rate at which the person can both borrow and save is 4%. Finally, the person's preferences are given by

$$U(c_1,c_2) = \frac{3}{2}(c_1)^{2/3} \ + \ \delta\frac{3}{2}(c_2)^{2/3}.$$

(a) Derive the budget constraint that the person faces.

(b) Solve for the optimal c_1 and c_2 as a function of δ.

(c) As δ increases, what happens to the optimal c_1 and c_2? Provide some intuition for your answer.

(d) For what values of δ will the person save, and for what values of δ will she borrow?

[**Note: Please report your answer to 5 decimal points.]**

(e) For what values of δ is the person's c_1 larger than her c_2? How does this compare to the condition that we discussed in class (for the case of log utility)?

(f) Consider the following alternative income streams:

Alternative A: Receive \$55,814 in period 1 and \$37,440 in period 2.

Alternative B: Receive \$44,026 in period 1 and \$49,400 in period 2.

Discuss how the person's optimal consumption path under these alternatives would compare to her optimal consumption path under the initial income stream — that is, discuss whether c_1 is larger, smaller, or the same, and discuss whether c_2 is larger, smaller, or the same. In addition, discuss how the person's period-1 saving under these alternatives would compare to her priod-1 saving under the initial income stream. **[Note: You can and should answer part (f) without re-solving for the person's behavior.]**

Question 2:

Consider a model exactly like that in Question 1 — where the person receives income \$48,326 in period 1 and additional income \$44,928 in period 2 — except let's now suppose that the person faces a liquidity constraint. Specifically, she can still save at an interest rate of 4%, but if she borrows, then she must pay an interest rate of 8%.

(a) If the person wants to save, the relevant interest rate is 4%. For what values of δ is it optimal to save? [Hint: You already know the answer from Question 1.]

(b) If the person wants to borrow, the relevant interest rate is 8%.

(i) Suppose the interest rate is 8%, and solve for the optimal c_1 and c_2 as a function of δ.

(ii) If the interest rate is 8%, for what values of δ is it optimal to borrow?

[**Note: Please report your answer to 5 decimal points.]**

(c) Given the liquidity constraint, for what values of δ is it optimal to neither borrow nor save? [Hint: Two conditions must hold: (i) δ must be such that the person does NOT want to save at an interest rate of 4%, and (ii) δ must be such that the person does NOT want to borrow at an interest rate of 8%.]

(d) Draw three pictures that illustrate the three cases — when it's optimal to save, when it's optimal to borrow, and when it's optimal to neither borrow nor save. Each picture should depict (i) the budget constraint, (ii) some indifference curves, and (iii) the optimal c_1 and c_2.

Question 3

Suppose that Mr. X has a tree growing in his yard that has become too large, and he must cut it down within the next four years. Cutting down the tree requires effort, and he would like to wait and cut down the tree later. However, each year that he delays, the tree gets bigger and requires even more effort to cut down. Specifically, suppose that the effort cost to cut down the tree in year $\bar{\tau}$ is $c(\bar{\tau})$, where $c(1) = 24$, $c(2) = 28$, $c(3) = 35$, and $c(4) = 46$. Suppose that Mr. X is an exponential discounter with discount factor δ, and that the only utility consequence of cutting down the tree is the disutility of effort experienced when the tree is cut.

(a) From a year-1 perspective:

(i) For what δ does Mr. X prefer cutting in year 1 rather than year 2?

(ii) For what δ does Mr. X prefer cutting in year 2 rather than year 3?

(iii) For what δ does Mr. X prefer cutting in year 3 rather than year 4?

(b) As a function of δ, when is Mr. X's preferred time to cut down the tree?

(c) Suppose Mr. X does not harvest the tree in year 1. From a year-2 perspective:

(i) For what δ does Mr. X prefer cutting in year 2 rather than year 3?

(ii) For what δ does Mr. X prefer cutting in year 3 rather than year 4?

(d) Is there any δ for which Mr. X would decide in year 2 to deviate from his initial year-1 plan?

Question 4

We can write the standard T-period saving-consumption model as:

Choose $(c_1, c_2, ..., c_T)$ to maximize

$$U(c_1, c_2, ..., c_T) = u(c_1) + \delta u(c_2) + ... + \delta^{T-1} u(c_T)$$

subject to

$$c_1 + \frac{c_2}{1+r} + ... + \frac{c_T}{(1+r)^{T-1}} \leq W.$$

In this model, there are three motivations that influence the allocation of consumption across periods: "impatience", "future consumption is cheaper", and "consumption smoothing".

(a) For each of these motivations, (i) describe in words how the motivation impacts the allocation of consumption across periods (i.e., how c_1 compares to c_2, c_3, and so forth), (ii) describe mathematically how it enters the problem above, and (iii) describe mathematically the special case of the problem above when the motivation is absent.

(b) Suppose that $T = 2$ and $u(c) = a + bc$. Derive the optimal consumption path as a function of δ and r.

Hint: Use the substitution method in part **(b)**.

Hint: Your answer to part **(b)** might provide insight that helps you to answer part **(a)**.