Problem Set Kenny

[Due Tuesday, November 9 at beginning of class.]

Question 1

This question asks you to explore when it is possible for a sophisticate to use an illiquid asset to implement her period-1 desired consumption path. Consider a 3-period saving-consumption example. First consider the case in which the person can borrow and save at an interest rate of $r = 0\%$, and suppose further that her lifetime wealth is $W = \$13,000$. For this case, the following facts are true:

The person's period-1 desired behavior is

$$c_1^* = \$5000 \qquad c_2^* = \$4000 \qquad c_3^* = \$4000.$$

The person's actual behavior is

$$c_1^S = \$4990 \qquad c_2^S = \$4450 \qquad c_3^S = \$3560.$$

Note: I have NOT given you enough information to solve for these values, because I have not given you an intertemporal utility function. But these values are all that's needed to answer the questions below.

Now consider the following case. The person cannot borrow, and she has two options for saving. First, she can put her money in the bank and earn an interest rate $r = 0\%$, in which case she will have full access to the money next period. Second, she can invest in an illiquid asset and earn an interest rate of $\hat{r} = 0\%$. If she invests in the illiquid asset in period 1, then she cannot get any access to these funds until period 3.

For each of the income flows below, is it possible to use the illiquid asset to implement period-1 desired behavior? If so, describe the full details of period-1 behavior (how much do you consume, how much do you put in the bank, and how much do you invest in the illiquid asset). If not, discuss what the person's actual consumption path is likely to look like.

(a) $Y_1 = \$6000$, $Y_2 = \$3500$, and $Y_3 = \$3500$.

(b) $Y_1 = \$6000$, $Y_2 = \$5000$, and $Y_3 = \$2000$.

(c) $Y_1 = \$6000$, $Y_2 = \$4400$, and $Y_3 = \$2600$.

(d) $Y_1 = \$6000$, $Y_2 = \$2800$, and $Y_3 = \$4200$.

Note: For each part, your answer should clearly state the person's actual consumption path — that is, her c_1, her c_2, and her c_3. For some of these, it might be that you cannot provide an exact answer; in that case, your answer should be a range in which the object would lie — e.g., $x \in (50, 100)$.

Question 2:

This question asks you to explore further the relationship between present bias and health-club usage. Suppose there are 30 days in a month, and that on each of these days you consider going to the health club.

Each visit to the health club generates a future benefit of 35. *Each* visit to the health club also carries an immediate cost (because exercise requires effort). However, because your motivation varies from day to day, this immediate cost will vary from day to day. Specifically, assume that for m days each month you will have a low cost of 10, for n days each month you will have a medium cost of 20, and for $(30 - m - n)$ days each month you will have a high cost of 30.

The health club offers two contracts:

A: No monthly fee, but you pay \$10 per visit.

B: Monthly fee of $\$X$, but then you pay nothing per visit.

You must choose your contract in advance (prior to the first day of the month).

Finally, suppose you treat any money spent as a future cost (linear in the amount of money spent). For instance, under Contract A, if you visit the club on a low-cost day, then you incur an immediate cost of 10 (effort), you incur a future cost of 10 (price paid), and you receive a future benefit of 35 (health benefits).

(a) Suppose you are a standard exponential discounter with $\delta = 1$.

(i) If you choose Contract A, on which days (low-cost, medium-cost, high-cost) will you visit the gym? From a prior perspective, what would be your total utility for the month?

(ii) If you choose Contract B, on which days (low-cost, medium-cost, high-cost) will you visit the gym? From a prior perspective, what would be your total utility for the month as a function of X?

(iii) For what values of X would you choose Contract B?

(iv) If you chose Contract B, what would end up being your average price per visit as a function of X? Can we say how this compares to $10?

(b) Repeat part **(a)**, except suppose you have sophisticated present bias with $\beta = .7$ and $\delta = 1$.

(c) Repeat part **(a)**, except suppose you have naive present bias with $\beta = .7$ and $\delta = 1$. Note: For parts (i)-(iii), you should be deriving what naifs predict for their own future behavior as they think about this problem from a prior perspective—which is when they decide between Contract A and Contract B.

THE FOLLOWING QUESTIONS ARE OPTIONAL AND WILL BE COVERED ON THURSDAY, NOV 5 and TUESDAY, NOV 10

The material from these questions will be covered on the exam.

Question 3:

This question explores utility from anticipation and your willingness to pay to avoid a shock.

When you experience it, the shock generates consumption utility $v(\text{shock}) = -\bar{v}$.

But your instantaneous utility in any given period depends on both current consumption utility and utility from anticipating future consumption utility. Specifically, suppose your instantaneous utility in period t is

$$u_t = v(c_t) + \phi\left[v(c_{t+1}) + v(c_{t+2})\right].$$

Suppose your period-t intertemporal utility is

$$U^t = u_t + \delta u_{t+1} + \delta^2 u_{t+2} + \ldots.$$

Finally, suppose that your utility from money is linear, so that if your period-1 intertemporal utility from receiving the shock is $U^1 < 0$, then your willingness to pay to avoid the shock is just $\$(-U^1)$.

(a) Solve for your willingness to pay to avoid the shock as a function of whether you receive the shock in period 1, 2, 3, or 4.

(b) For Loewenstein's subjects, the willingness to pay to avoid the shock was monotonically increasing in the delay until the shock. Assuming $\delta < 1$, does this result hold in the example here? Why or why not?

Question 4:

This question asks you to extend the model of projection bias and the endowment effect that we studied in class.

(a) Suppose we ask an unendowed person to predict what her selling price would be if she were endowed with a mug. Solve for her predicted selling price.

(b) Suppose we ask an endowed person to predict what her buying price would be if she were not endowed with a mug. Solve for her predicted buying price.

(c) How do these predictions compare to the actual selling and buying prices that we derived in class? How does your answer depend on α?

(d) How do these predictions compare to each other? How does your answer depend on α?

Question 5:

This question asks you to explore in more detail how projection bias influences order and return decisions for a winter coat.

Suppose that, if you buy it, you will use a coat on 5 cold days and 5 warm days. The actual utility of the coat is 20 on a cold day and 10 on a warm day — that is, $u(\text{coat,cold}) = 20$ and $u(\text{coat,warm}) = 10$.

Suppose further that you are an exponential discounter with $\delta = 1$, but you have simple projection bias with magnitude α.

Finally, suppose that your utility from money is linear, so that if the coat yields total utility of $\bar{u}$, then your willingness to pay for the coat is just $\$\bar{u}$.

(a) If you did not have projection bias, what would be your willingness to pay for the coat?

(b) If you decide whether to order on a cold day, what is your willingness to pay for the coat (as a function of α)? If you decide whether to order on a warm day, what is your willingness to pay for the coat (as a function of α)?

(c) Suppose that when you receive the coat you immediately decide whether to keep it or return it (and you stick with that decision). If you receive the coat on a cold day, what is your willingness to pay to keep the coat (as a function of α)? If you receive the coat on a warm day, what is your willingness to pay to keep the coat (as a function of α)?

(d) How does your propensity to return the coat depend on whether you order on a cold day vs. on a warm day?