

# Lecture 4b: Non-Bayesian Information Processing

EC 404: Behavioral Economics  
Professor: Ben Bushong

April 21, 2022

# An Alternative Model: The Law of Small Numbers

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Based on Rabin (*QJE* 2002)

Assumes one particular misconception of randomness: People act as if they believe recent draws of one outcome increase the odds of the next draw being a different outcome.

Specifically, an *N-Freddy* acts as if he believes that probabilistic events are drawn from an “urn” with  $N$  balls without replacement — specifically, he believes that the ball just drawn is not in the “urn” for the current draw.

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# An Example

Suppose ( $N = 12$ )-Freddy faces a die with 4  $H$ 's and 2  $M$ 's.

On the first roll:  $\Pr(H) = 2/3$

On the second roll:

- ▶ If first roll is  $H$ , then  $\Pr(H) = 7/11 < 2/3$
- ▶ If first roll is  $M$ , then  $\Pr(H) = 8/11 > 2/3$

More generally:

- ▶ If previous roll is  $H$ , then  $\Pr(H) = ?$
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Suppose the objective probability of outcome  $x$  is  $p$ .

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A nice feature: The larger is  $N$ , the smaller is the bias  
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# Implication 1

Implication 1: An  $N$ -Freddy will exhibit the gambler's fallacy.

- ▶ If there is a known objective probability, then an  $N$ -Freddy will act as if he believes a recent draw of one outcome increases the odds of the next draw being a different outcome (nearly tautological).

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## Implication 2

Implication 2: An  $N$ -Freddy will exhibit over-inference from small samples.

Recall Example C: Suppose there are two dice, a red one and a blue one. On the red die, there are 4  $H$ 's and 2  $M$ 's, whereas on the blue die, there are 2  $H$ 's and 4  $M$ 's. I flip a fair coin, and if it comes up heads I use the red die, and if it comes up tails I use the blue die (and you cannot observe the coin flip). I then roll that die twice. If the die comes up  $HH$ , what is the likelihood that I'm rolling the red die?

For a Bayesian:  $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ .

$\gamma(HH|\text{red}) = 4/9$  and  $\gamma(HH|\text{blue}) = 1/9$ .

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = 4/5.$$

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For an ( $N = 12$ )-Freddy:

- ▶  $q_0(\text{red}) = q_0(\text{blue}) = 1/2$  (no change).
- ▶  $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- ▶  $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
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Intuition: The law of small numbers makes Freddy think that  $HH$  is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll  $HH$  (i.e., for the blue die), Freddy will update too much toward the die more likely to roll  $HH$  (i.e., toward the red die).

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## Implication 3

Implication 3: An  $N$ -Freddy will exhibit the hot-hand fallacy.

# Implication 3

## A Simplified Example:

- ▶ Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ( $N = 8$ )-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- ▶ With probability  $q$ , the player's hit rate is 75%.
- ▶ With probability  $q$ , the player's hit rate is 25%.
- ▶ With probability  $1 - 2q$ , the player's hit rate is 50%.

Initially, Freddy doesn't know  $q$ , but Freddy attempts to infer  $q$  from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

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Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

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## A Simplified Example:

- ▶ Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ( $N = 8$ )-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- ▶ With probability  $q$ , the player's hit rate is 75%.
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What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a “large” sample, Freddy will see pairs in the following proportions:

- ▶  $(H, H) = 1/4$
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Even though the player does NOT have a hot hand, Freddy will come to believe that the player is “hot” in 25% of games, “cold” in 25% of games, and “average” in 50% of games.

Intuition: The law of small numbers makes Freddy think that a 50% shooter is less likely to have *HH* or *MM* games than the player actually is. Hence, when Freddy sees a lot of *HH* and *MM* games, Freddy is forced to conclude that the player must be having hot and cold games.

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I don't care if you struggled with the calculus in this course or if you found the algebra tedious. (So did I, when I was coming up with all this stuff)

I want you to take these lessons:

- ▶ People are dumb (duh)
- ▶ People are systematically dumb
- ▶ Formal models can help us understand the implications of their dumbness.
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