Choice over Time: Evidence that Contradicts the Standard Model

EC404; Spring 2024

Prof. Ben Bushong

Last updated February 22, 2024

General Version of Discounted Utility

Recall the general version of the discounted-utility model:

$$U^t = \sum_{x=0}^{T-t} D(x)\, u_{t+x}.$$

- U^t is intertemporal utility from perspective of period t.
- u_τ is instantaneous utility in period τ ("well-being" in period t).
    - x is the delay before receiving some utility.
    - $D(x)$ is a discount function that specifies the amount of discounting associated with delay x.

Again, in principle, we could have any discount function.

Exponential discounting **assumes** $D(x) = \delta^x$.

Typical procedure elicits indifference points of the form:

$(A \text{ at date } x) \sim (B \text{ at date } x + y)$

To interpret, typically make four assumptions (aka **The Usual Assumptions**)

1. People obey discounted utility model.
2. People treat these amounts as "bursts" of consumption.
3. Utility is linear in the amount.
4. Normalize $D(0) = 1$.

An implication of the usual assumptions:

$$(A \textbf{ now}) \;\sim\; (B \textbf{ at date } x)$$

$$\iff D(0)u(A) = D(x)u(B)$$

$$A = D(x)B$$

$$D(x) = \frac{A}{B}$$

More generally, the usual assumptions imply:

$$(A \text{ at date } x) \sim (B \text{ at date } x+y)$$

$$\iff \qquad D(x)u(A) = D(x+y)u(B)$$

$$D(x)A = D(x+y)B$$

$$\frac{D(x+y)}{D(x)} = \frac{A}{B}$$

An alternative procedure elicits WTP now for something to be received later.

E.g., if WTP up to $\boldsymbol{A}$ now to obtain $\boldsymbol{B}$ at date $\boldsymbol{x}$, this implies:

$(-\boldsymbol{A}$ now & $+\boldsymbol{B}$ at date $\boldsymbol{x}) \sim$ (no changes)

Applying the usual assumptions here yields:

$$D(0)u(-A) + D(x)u(B) = 0$$

$$D(x)B = A$$

$$D(x) = \frac{A}{B}$$

Using a (hypothetical) matching technique, Thaler found that people were indifferent between:

Option	Comparison
a.	(\$15 now) ~ (\$30 in 3 months)
b.	(\$15 now) ~ (\$60 in 1 year)
c.	(\$15 now) ~ (\$100 in 3 years)

Implications under the usual assumptions:

a. $D(\mathbf{3}\text{ months })= \frac{\mathbf{15}}{\mathbf{30}} = \mathbf{0.50}$.

b. $D(\mathbf{1}\text{ year })= \frac{\mathbf{15}}{\mathbf{60}} = \mathbf{0.25}$.

c. $D(\mathbf{3}\text{ years })= \frac{\mathbf{15}}{\mathbf{100}} = \mathbf{0.15}$.

We can convert each $D(x)$ into an implicit yearly discount rate:

Definition: The *average yearly discount rate* applied to delay x (where x is specified in years) is the ρ such that

$$e^{-\rho x} = D(x) \qquad \text{or} \qquad \rho = \frac{1}{x}(-\ln D(x)).$$

Applying this definition:

a. $D(3 \text{ months}) = 0.50 \Longrightarrow$ 277% yearly discounting.

b. $D(1 \text{ year}) = 0.25 \Longrightarrow$ 139% yearly discounting.

c. $D(3 \text{ years}) = 0.15 \Longrightarrow$ 63% yearly discounting.

More Evidence from Thaler (1981)

Thaler (1981) also found that people were indifferent between:

Option	Comparison
e.	(\$250 now) ~ (\$300 in 3 months)
f.	(\$250 now) ~ (\$350 in 1 year)
g.	(\$250 now) ~ (\$500 in 3 years)

Implications under the usual assumptions:

e. $D(\mathbf{3}$ months $)= \frac{\mathbf{250}}{\mathbf{300}} = \mathbf{0.83} \Longrightarrow$ 73% yearly discounting.

f. $D(\mathbf{1}$ year $)= \frac{\mathbf{250}}{\mathbf{350}} = \mathbf{0.71} \Longrightarrow$ 34% yearly discounting.

g. $D(\mathbf{3}$ years $)= \frac{\mathbf{250}}{\mathbf{500}} = \mathbf{0.50} \Longrightarrow$ 23% yearly discounting.

Conclusions from Thaler (1981)

Conclusion #1: The amount matters --- there is more discounting for smaller amounts ("magnitude effect").

But the **key** conclusion is:

Conclusion #2: For either amount, discount rates are higher in the short run than in the long run (sometimes referred to as "declining discount rates").

This finding is inconsistent with exponential discounting!

Consider the following (hypothetical) choice scenarios:

Choice 1:

$$[\text{10 M\&Ms now}] \qquad \text{vs.} \qquad [\text{15 M\&Ms tomorrow}]$$

Choice 2:

$$[\text{10 M\&Ms in 7 days}] \qquad \text{vs.} \qquad [\text{15 M\&M in 8 days}]$$

A plausible pattern:

$$(\text{10 M\&Ms now }) \succ (\text{15 M\&Ms tomorrow})$$

$$(\text{15 M\&Ms in eight days}) \succ (\text{10 M\&Ms in seven days})$$

Implication: Under the usual assumptions, this pattern implies

$$D(0)10 > D(1)15 \qquad \Longleftrightarrow \qquad \frac{D(0)}{D(1)} > 1.5$$

$$D(7)10 < D(8)15 \qquad \Longleftrightarrow \qquad \frac{D(7)}{D(8)} < 1.5$$

$$\textbf{Hence:}\ \frac{D(0)}{D(1)} > \frac{D(7)}{D(8)}$$

Conclusion: If you exhibit this pattern, then you are more impatient toward the now-vs.-near-future tradeoff than you are toward the near-future-vs.-further-future tradeoff.

In fact, Kirby & Herrnstein (1995) show that, for 23 of their 24 subjects, they can make the subject exhibit this type of "preference reversal" .

For instance, for each of these 23 subjects, they find an $x > 0$ and a $y > 0$ such that the subject's preferences are

$$(\textbf{45 now }) \succ (\textbf{52 in } y \textbf{ days})$$

$$(\textbf{ 45 in } x \textbf{ days}) \succ (\textbf{ 52 in } y \textbf{ days})$$

Much as above, under the usual assumptions, this implies

$$\frac{D(0)}{D(y)} > \frac{D(x)}{D(x+y)}$$

Again, more impatient toward the now-vs.-near-future tradeoff than toward the near-future-vs.-further-future tradeoff.

Another approach to the same type of data is to directly compare two functional forms:

1. Exponential Discounting: $D(x) = e^{-kx}$
2. Hyperbolic Discounting: $D(x) = \frac{1}{1+kx}$

In these comparisons, the answer is that hyperbolic discounting is virtually always a better fit (occasionally, they're equally good).

\frametitle{Evidence of " Hyperbolic Discounting" }

For instance, Kirby (1997) elicited WTP's for \$20 to be received in x days, where each subject answered for every odd x between 1 and 29.

He then tested for each subject whether their discount function was better fit by the exponential functional form or the hyperbolic functional form.

Results: Hyperbolic was a better fit for 59 of 67 subjects, exponential was a better fit for 6 subjects, and for 2 subjects the functions were equally good.

In the experimental data, there seems to be a key feature that virtually always holds:

- Discount rates are higher in the short run than in the long run (sometimes referred to as "declining discount rates").

In terms of our notation, the evidence seems to suggest:

$$\frac{D(0)}{D(1)} > \frac{D(1)}{D(2)} > \frac{D(2)}{D(3)} > \ldots$$

- When thinking about some future period, you'd like to behave relatively patiently; but when the time comes to actually choose your behavior, you want to behave relatively impatiently.