Problem Set 4

[Due in class on Thursday, March 27.]

Question 1

Suppose we observe a person making the following pattern of choices:

$$\begin{aligned} \$71 \text{ Now} &\prec \$75 \text{ in 1 Week} \\ \$72 \text{ Now} &\prec \$75 \text{ in 1 Week} \\ \$73 \text{ Now} &\succ \$75 \text{ in 1 Week} \\ \$74 \text{ Now} &\succ \$75 \text{ in 1 Week} \end{aligned}$$

Under the "usual assumptions" (see Topic 3c), what can we infer about the person's $D(7 \text{ days})$? What can we infer about the person's average yearly discount rate applied to a 7-day delay?

Question 2

Suppose we observe that a person has the following indifference points:

$$\begin{aligned} \$140 \text{ Now} &\sim \$150 \text{ in 1 Year} \\ \$147 \text{ in 1 Year} &\sim \$150 \text{ in 2 Years} \\ \$149 \text{ in 2 Years} &\sim \$150 \text{ in 3 Years} \end{aligned}$$

(a) Under the "usual assumptions" (see Topic 3c), what can we infer about the person's $D(1$ Year$)$, $D(2$ Years$)$, $D(3$ Years$)$? Discuss whether, under the "usual assumptions", this data consistent with exponential discounting, hyperbolic discounting, and present bias.

(b) Instead of assuming that "utility" is linear in the amount, let's assume that the person evaluates gains according to a value function $v(x) = x^{0.5}$. Under this alternative, what can we infer about the person's $D(1$ Year$)$, $D(2$ Years$)$, $D(3$ Years$)$? Discuss whether, under this alternative, this data consistent with exponential discounting, hyperbolic discounting, and present bias.

(c) If a person actually values money according to a concave value function, but we incorrectly assume that her utility is linear in the amount, will we over-estimate or under-estimate how patient the person is?

Question 3:

This question asks you to reconsider the "doing-it-once" environment that we studied in class (Example 1 and Example 2 from Topic 3d), except that we now consider costs and rewards that do not rise monotonically. For both parts, suppose that you value rewards and costs linearly, and that you have β, δ preferences with $\beta = 1/2$ and $\delta = 1$.

(a) Suppose there is an onerous task that you must complete on one of the next 6 days. If you complete the task on day t, the cost is c_t, where $(c_1, c_2, c_3, c_4, c_5, c_6) = (10, 24, 18, 38, 34, 70)$. There is no reward.

(i) When is the best time to complete the task (given long-run preferences)?

(ii) When do naifs complete the task? When do sophisticates complete the task?

(iii) Discuss how the outcomes here compare to the outcomes in Example 1.

(b) Suppose there is a pleasurable task that you get to complete on one of the next 6 days. If you complete the task on day t, the reward is v_t, where $(v_1, v_2, v_3, v_4, v_5, v_6) = (6, 10, 8, 18, 34, 30)$. There is no cost.

(i) When is the best time to complete the task (given long-run preferences)?

(ii) When do naifs complete the task? When do sophisticates complete the task?

(iii) Discuss how the outcomes here compare to the outcomes in Example 2.

Question 4

This question also asks you to reconsider the "doing-it-once" environment, except that we now consider an activity that generates BOTH rewards and costs. Throughout, suppose that you value rewards and costs linearly, and that you have β, δ preferences with $\beta = 0.7$ and $\delta = 1$.

Suppose there is an activity that you will complete on one of the next 5 days. As a function of when you do the activity, your reward will be v_t and your cost will be c_t, where $(c_1, c_2, c_3, c_4, c_5) = (45, 45, 45, 62, 87)$ and $(v_1, v_2, v_3, v_4, v_5) = (35, 45, 60, 60, 60)$. However, there are two possible cases for **when** you receive these payoffs:

Immediate costs: you incur the cost when you do it, and receive the reward in the future.

Immediate rewards: you receive the reward when you do it, and incur the cost in the future.

For instance, if you do the activity in period 4, your cost is $c_4 = 62$ and your reward is $v_4 = 60$. For immediate costs, the cost $c_4 = 62$ is incurred in period 4 while the reward $v_4 = 60$ is received sometime later. For immediate rewards, the reward $v_4 = 60$ is received in period 4 while the cost $c_4 = 62$ is incurred sometime later. Since we are assuming $\delta = 1$, exactly when later is irrelevant.

(a) When is the best time to complete the task (given long-run preferences)? How does your answer depend on the timing of rewards and costs?

(b) Consider the case of immediate costs. When do naifs complete the task, and when do sophisticates complete the task?

(c) Consider the case of immediate rewards. When do naifs complete the task, and when do sophisticates complete the task?

Question 5:

This question asks you to explore further the relationship between present bias and health-club usage. Suppose there are 30 days in a month, and that on each of these days you consider going to the health club.

Each visit to the health club generates a future benefit of 35. *Each* visit to the health club also carries an immediate cost (because exercise requires effort). However, because your motivation varies from day to day, this immediate cost will vary from day to day. Specifically, assume that for m days each month you will have a low cost of 10, for n days each month you will have a medium cost of 20, and for $(30-m-n)$ days each month you will have a high cost of 30.

The health club offers two contracts:

A: No monthly fee, but you pay \$10 per visit.

B: Monthly fee of \$$X$, but then you pay nothing per visit.

You must choose your contract in advance (prior to the first day of the month).

Finally, suppose you treat any money spent as a future cost (linear in the amount of money spent). For instance, under Contract A, if you visit the club on a low-cost day, then you incur an immediate cost of 10 (effort), you incur a future cost of 10 (price paid), and you receive a future benefit of 35 (health benefits).

(a) Suppose you are a standard exponential discounter with $\delta = 1$.

(i) If you choose Contract A, on which days (low-cost, medium-cost, high-cost) will you visit the gym? From a prior perspective, what would be your total utility for the month?

(ii) If you choose Contract B, on which days (low-cost, medium-cost, high-cost) will you visit the gym? From a prior perspective, what would be your total utility for the month as a function of X?

(iii) For what values of X would you choose Contract B?

(iv) If you chose Contract B, what would end up being your average price per visit as a function of X? Can we say how this compares to \$10?

(b) Repeat part **(a)**, except suppose you have sophisticated present bias with $\beta = .7$ and $\delta = 1$.

(c) Repeat part **(a)**, except suppose you have naive present bias with $\beta = .7$ and $\delta = 1$. Note: For parts (i)-(iii), you should be deriving what naifs predict for their own future behavior as they think about this problem from a prior perspective—which is when they decide between Contract A and Contract B.