Problem Set 1

[Due in class on Tuesday, January 23rd.]

Question 1:

Consider the following choice:

$$\mathbf{x} \equiv (\$1000, 0.7; -\$1000, 0.3)$$
 vs. $\mathbf{y} \equiv \left(\$600, \frac{1}{3}; \$300, \frac{1}{2}; \$0, \frac{1}{6}\right)$

- (a) Ting is an EV maximizer with initial wealth \$10,000. Which does she choose?
- **(b)** Grace and Sunflower are EU maximizers with initial wealth \$10,000.
 - (i) If Grace has a CRRA utility function with $\rho = 1/2$, which does she choose?
 - (ii) If Sunflower has a CRRA utility function with $\rho = 2$, which does she choose?

Question 2:

Suppose that Sonya has initial wealth \$20,000 that she plans to invest for a year and then consume it. There are two assets in which she can invest. First, there is a risk-free asset that yields a certain return of 3%. Second, there is a risky asset that yields a return of 45% with probability 1/3 and a return of -15% with probability 2/3.

If Sonya is an expected-utility maximizer with utility function $u(x) = \ln x$, how much would she invest in each asset? (Note: She must invest all of her initial wealth in one asset or the other.)

Question 3:

Consider the following three choice situations:

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Choice (i): ($2500, 1/2; $0, 1/2) vs. ($1300, 1)

Choice (ii): (-$6000, 1/4; -$1000, 3/4) vs. (-$2300, 1)

Choice (iii): ($600, 1/4; $400, 1/4; $200, 1/4; $0, 1/4) vs. ($400, 1/2; $200.1/2)
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- (a) Suppose Josh is a risk-averse expected utility maximizer. For each choice, can we determine which option Josh will choose, or do we need more information?
- **(b)** For each choice, as Josh becomes more risk averse, does he become more prone to choose the first or second option?
- (c) Now suppose Josh has a CRRA utility function. For each choice, as Josh's wealth becomes larger, does he become more prone to choose the first or second option?

(Note: You should be able to intuit an answer to part (c) — no math is needed.)

Question 4: (Demand for Insurance)

Suppose you are a risk-averse expected utility maximizer with utility function u(x). You have initial wealth \$20,000, but before you consume it, you are subject to the following health risk (these events are mutually exclusive):

Required Medical Payment	Probability
\$100	20%
\$500	8%
\$1500	2%
\$10,000	1%

Now suppose that an insurance agent offers to sell you health insurance with 10% coinsurance — that is, for any medical payment that you require, you must pay for 10% of the payment and the insurance company pays the rest. The price of this insurance is *p*.

- (a) If you do not buy the insurance, what lottery do you face? If you buy the insurance, what lottery do you face?
- (b) Let p^* denote your willingness to pay for the insurance so that you prefer to buy the insurance if $p < p^*$ and you prefer not to buy the insurance if $p > p^*$. Provide an equation from which you could derive p^* .
- (c) If you are risk-neutral, what can we say about your p^* ? If you are risk-averse, what can we say about your p^* ? As you become more risk averse, what happens to your p^* ? Briefly explain your answers.

(d) If the coinsurance rate goes up to 15%, what happens to your p^* ? Briefly explain your answer.

Question 5:

Consider the following choice (where $p \in [0,1]$, $q \in [0,1]$, and $p+q \le 1$):

$$\mathbf{x} \equiv (\$600, p ; \$400, q ; \$0, 1 - p - q)$$
 vs. $\mathbf{y} \equiv \left(\$500, \frac{1}{3} ; \$300, \frac{1}{2} ; \$0, \frac{1}{6}\right)$

- (a) For what values of p and q does lottery \mathbf{x} dominate lottery \mathbf{y} ?
- (b) For what values of p and q does lottery y dominate lottery x?

Question 6: [optional. Nothing to gain except knowledge here.]

Short-Answer Questions: For each question below, please provide a short, concise answer — your answer only needs to be a few sentences.

- (a) Suppose Karen is a risk-averse expected utility maximizer. As Karen's wealth goes up, will she act more or less risk-averse? Briefly explain your answer.
- (b) Suppose Julian is a risk-averse expected utility maximizer. If he were extremely risk-averse, could he prefer the lottery (1000, 1) over the lottery $(1000, \frac{1}{2}; 1100, \frac{1}{2})$? Briefly explain your answer.
- (c) Does the St. Petersburg Paradox suggest that expected utility is a bad model of human behavior? Briefly explain your answer.