Lecture 7b: Non-Bayesian Information Processing

EC 404: Behavioral Economics
Professor: Ben Bushong

December 9, 2021

An Alternative Model: The Law of Small Numbers
Based on Rabin (*QJE* 2002)

Assumes one particular misconception of randomness: People act as if they believe recent draws of one outcome increase the odds of the next draw being a different outcome.

Specifically, an *N-Freddy* acts as if he believes that probabilistic events are drawn from an "urn" with N balls without replacement — specifically, he believes that the ball just drawn is not in the "urn" for the current draw.

An Alternative Model: The Law of Small Numbers

An Alternative Model: The Law of Small Numbers
Based on Rabin (*QJE* 2002)

Assumes one particular misconception of randomness: People act as if they believe recent draws of one outcome increase the odds of the next draw being a different outcome.

Specifically, an *N-Freddy* acts as if he believes that probabilistic events are drawn from an "urn" with N balls without replacement — specifically, he believes that the ball just drawn is not in the "urn" for the current draw.

An Alternative Model: The Law of Small Numbers

An Alternative Model: The Law of Small Numbers
Based on Rabin (*QJE* 2002)

Assumes one particular misconception of randomness: People act as if they believe recent draws of one outcome increase the odds of the next draw being a different outcome.

Specifically, an *N-Freddy* acts as if he believes that probabilistic events are drawn from an "urn" with N balls without replacement — specifically, he believes that the ball just drawn is not in the "urn" for the current draw.

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) =$?
- If previous roll is M, then $\Pr(H) =$?

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) =$?
- If previous roll is M, then $\Pr(H) =$?

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) =$?
- If previous roll is M, then $\Pr(H) =$?

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) =$?
- If previous roll is M, then $\Pr(H) =$?

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) = \quad ?$
- If previous roll is M, then $\Pr(H) = \quad ?$

An Example

Suppose ($N = 12$)-Freddy faces a die with 4 H's and 2 M's.

On the first roll: $\Pr(H) = 2/3$

On the second roll:

- If first roll is H, then $\Pr(H) = 7/11 < 2/3$
- If first roll is M, then $\Pr(H) = 8/11 > 2/3$

More generally:

- If previous roll is H, then $\Pr(H) =$?
- If previous roll is M, then $\Pr(H) =$?

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias
(and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias (and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias
(and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias (and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias (and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias
(and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias
(and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias
(and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias (and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Formal Assumptions

Suppose the objective probability of outcome x is p.

On the first draw:

- N-Freddy believes $\Pr(x) = p$.

On the second draw:

- If first draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If first draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

A nice feature: The larger is N, the smaller is the bias (and as $N \to \infty$ an N-Freddy becomes a Bayesian).

Implication 1

Implication 1: An N-Freddy will exhibit the gambler's fallacy.

- If there is a known objective probability, then an N-Freddy will act as if he believes a recent draw of one outcome increases the odds of the next draw being a different outcome (nearly tautological).

Implication 1

Implication 1: An N-Freddy will exhibit the gambler's fallacy.

- If there is a known objective probability, then an N-Freddy will act as if he believes a recent draw of one outcome increases the odds of the next draw being a different outcome (nearly tautological).

Implication 2

Implication 2: An N-Freddy will exhibit over-inference from small samples.

Recall Example C: Suppose there are two dice, a red one and a blue one. On the red die, there are 4 H's and 2 M's, whereas on the blue die, there are 2 H's and 4 M's. I flip a fair coin, and if it comes up heads I use the red die, and if it comes up tails I use the blue die (and you cannot observe the coin flip). I then roll that die twice. If the die comes up HH, what is the likelihood that I'm rolling the red die?

For a Bayesian: $q_0(\text{red}) = q_0(\text{blue}) = 1/2$.

$\gamma(HH|\text{red}) = 4/9$ and $\gamma(HH|\text{blue}) = 1/9$.

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = 4/5.$$

Implication 2

Implication 2: An N-Freddy will exhibit over-inference from small samples.

Recall Example C: Suppose there are two dice, a red one and a blue one. On the red die, there are 4 H's and 2 M's, whereas on the blue die, there are 2 H's and 4 M's. I flip a fair coin, and if it comes up heads I use the red die, and if it comes up tails I use the blue die (and you cannot observe the coin flip). I then roll that die twice. If the die comes up HH, what is the likelihood that I'm rolling the red die?

For a Bayesian: $q_0(\text{red}) = q_0(\text{blue}) = 1/2.$

$\gamma(HH|\text{red}) = 4/9$ and $\gamma(HH|\text{blue}) = 1/9.$

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = 4/5.$$

Implication 2

Implication 2: An N-Freddy will exhibit over-inference from small samples.

Recall Example C: Suppose there are two dice, a red one and a blue one. On the red die, there are 4 H's and 2 M's, whereas on the blue die, there are 2 H's and 4 M's. I flip a fair coin, and if it comes up heads I use the red die, and if it comes up tails I use the blue die (and you cannot observe the coin flip). I then roll that die twice. If the die comes up HH, what is the likelihood that I'm rolling the red die?

For a Bayesian: $q_0(\text{red}) = q_0(\text{blue}) = 1/2$.
$\gamma(HH|\text{red}) = 4/9$ and $\gamma(HH|\text{blue}) = 1/9$.

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = 4/5.$$

Implication 2

Implication 2: An N-Freddy will exhibit over-inference from small samples.

Recall Example C: Suppose there are two dice, a red one and a blue one. On the red die, there are 4 H's and 2 M's, whereas on the blue die, there are 2 H's and 4 M's. I flip a fair coin, and if it comes up heads I use the red die, and if it comes up tails I use the blue die (and you cannot observe the coin flip). I then roll that die twice. If the die comes up HH, what is the likelihood that I'm rolling the red die?

For a Bayesian: $q_0(\text{red}) = q_0(\text{blue}) = 1/2$.
$\gamma(HH|\text{red}) = 4/9$ and $\gamma(HH|\text{blue}) = 1/9$.

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = 4/5.$$

Implication 2

For an ($N = 12$)-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \quad 14/17$$

Intuition: The law of small numbers makes Freddy think that HH is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll HH (i.e., for the blue die), Freddy will update too much toward the die more likely to roll HH (i.e., toward the red die).

Implication 2

For an $(N = 12)$-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \qquad 14/17$$

Intuition: The law of small numbers makes Freddy think that HH is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll HH (i.e., for the blue die), Freddy will update too much toward the die more likely to roll HH (i.e., toward the red die).

Implication 2

For an ($N = 12$)-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \qquad 14/17$$

Intuition: The law of small numbers makes Freddy think that HH is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll HH (i.e., for the blue die), Freddy will update too much toward the die more likely to roll HH (i.e., toward the red die).

Implication 2

For an $(N = 12)$-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \quad 14/17$$

Intuition: The law of small numbers makes Freddy think that *HH* is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll *HH* (i.e., for the blue die), Freddy will update too much toward the die more likely to roll *HH* (i.e., toward the red die).

Implication 2

For an ($N = 12$)-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \qquad 14/17$$

Intuition: The law of small numbers makes Freddy think that *HH* is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll *HH* (i.e., for the blue die), Freddy will update too much toward the die more likely to roll *HH* (i.e., toward the red die).

Implication 2

For an $(N = 12)$-Freddy:

- $q_0(\text{red}) = q_0(\text{blue}) = 1/2$ (no change).
- $\gamma(HH|\text{red}) = 2/3 * 7/11 = 14/33$
- $\gamma(HH|\text{blue}) = 1/3 * 3/11 = 1/11$
- Now apply Bayes' Rule:

$$q_1(\text{red}|HH) = \frac{\gamma(HH|\text{red})q_0(\text{red})}{\gamma(HH|\text{red})q_0(\text{red}) + \gamma(HH|\text{blue})q_0(\text{blue})} = \qquad 14/17$$

Intuition: The law of small numbers makes Freddy think that *HH* is less likely to occur than it actually is; but since this reduction is larger for the die less likely to roll *HH* (i.e., for the blue die), Freddy will update too much toward the die more likely to roll *HH* (i.e., toward the red die).

Implication 3

Implication 3: An N-Freddy will exhibit the hot-hand fallacy.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ($N = 8$)-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ($N = 8$)-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ($N = 8$)-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an $(N = 8)$-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an $(N = 8)$-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an $(N = 8)$-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

A Simplified Example:

- Suppose that in each game a player takes two shots. In reality, the player's hit rate is always 50%.

Consider an ($N = 8$)-Freddy who entertains the notion that the player might have hot and cold games. Specifically, for any given game, he believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Initially, Freddy doesn't know q, but Freddy attempts to infer q from watching the player's performance across a large number of games. What will Freddy infer?

Note: Let's assume that Freddy's processing reflects that there is a new urn at the start of every game.

Implication 3

What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a "large" sample, Freddy will see pairs in the following proportions:

- $(H, H) = 1/4$
- $(M, M) = 1/4$
- (H, M) or $(M, H) = 1/2$

Implication 3

What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a "large" sample, Freddy will see pairs in the following proportions:

- $(H, H) = 1/4$
- $(M, M) = 1/4$
- (H, M) or $(M, H) = 1/2$

Implication 3

What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a "large" sample, Freddy will see pairs in the following proportions:

- $(H, H) = 1/4$
- $(M, M) = 1/4$
- (H, M) or $(M, H) = 1/2$

Implication 3

What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a "large" sample, Freddy will see pairs in the following proportions:

- $(H, H) = 1/4$
- $(M, M) = 1/4$
- (H, M) or $(M, H) = 1/2$

Implication 3

What does Freddy actually see?

Given that the true hit rate is always 50%, after observing a “large” sample, Freddy will see pairs in the following proportions:

- $(H, H) = 1/4$
- $(M, M) = 1/4$
- (H, M) or $(M, H) = 1/2$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our ($N = 8$)-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our ($N = 8$)-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our $(N = 8)$-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our $(N = 8)$-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our $(N = 8)$-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our $(N = 8)$-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our $(N = 8)$-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our ($N = 8$)-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1-2q$, the player's hit rate is 50%.

Hence, as a function of q, our ($N=8$)-Freddy expects to see pairs in the following proportions:

- $(H,H) = 3/14 + q/7$
- $(M,M) = 3/14 + q/7$
- (H,M) or $(M,H) = 8/14 - 2q/7$

Implication 3

What does Freddy expect to see?

Recall that Freddy believes:

- With probability q, the player's hit rate is 75%.
- With probability q, the player's hit rate is 25%.
- With probability $1 - 2q$, the player's hit rate is 50%.

Hence, as a function of q, our ($N = 8$)-Freddy expects to see pairs in the following proportions:

- $(H, H) = 3/14 + q/7$
- $(M, M) = 3/14 + q/7$
- (H, M) or $(M, H) = 8/14 - 2q/7$

Implication 3

What does Freddy infer about q?

Recall: Freddy actually sees:

- (H, H) — $1/4$
- (M, M) — $1/4$
- (H, M) or (M, H) — $1/2$

Recall: Freddy expects to see:

- (H, H) — $3/14 + q/7$
- (M, M) — $3/14 + q/7$
- (H, M) or (M, H) — $8/14 - 2q/7$

Hence, an $N = 8$ Freddy's estimate q will be the q such that ...

Implication 3

What does Freddy infer about q?

Recall: Freddy actually sees:

- (H, H) — $1/4$
- (M, M) — $1/4$
- (H, M) or (M, H) — $1/2$

Recall: Freddy expects to see:

- (H, H) — $3/14 + q/7$
- (M, M) — $3/14 + q/7$
- (H, M) or (M, H) — $8/14 - 2q/7$

Hence, an $N = 8$ Freddy's estimate q will be the q such that ...

Implication 3

What does Freddy infer about q?

Recall: Freddy actually sees:

- (H, H) — $1/4$
- (M, M) — $1/4$
- (H, M) or (M, H) — $1/2$

Recall: Freddy expects to see:

- (H, H) — $3/14 + q/7$
- (M, M) — $3/14 + q/7$
- (H, M) or (M, H) — $8/14 - 2q/7$

Hence, an $N = 8$ Freddy's estimate q will be the q such that ...

Implication 3

What does Freddy infer about q?

Recall: Freddy actually sees:

- (H, H) — $1/4$
- (M, M) — $1/4$
- (H, M) or (M, H) — $1/2$

Recall: Freddy expects to see:

- (H, H) — $3/14 + q/7$
- (M, M) — $3/14 + q/7$
- (H, M) or (M, H) — $8/14 - 2q/7$

Hence, an $N = 8$ Freddy's estimate q will be the q such that ...

Implication 3

Even though the player does NOT have a hot hand, Freddy will come to believe that the player is “hot” in 25% of games, “cold” in 25% of games, and “average” in 50% of games.

Intuition: The law of small numbers makes Freddy think that a 50% shooter is less likely to have *HH* or *MM* games than the player actually is. Hence, when Freddy sees a lot of *HH* and *MM* games, Freddy is forced to conclude that the player must be having hot and cold games.

Implication 3

Even though the player does NOT have a hot hand, Freddy will come to believe that the player is "hot" in 25% of games, "cold" in 25% of games, and "average" in 50% of games.

Intuition: The law of small numbers makes Freddy think that a 50% shooter is less likely to have *HH* or *MM* games than the player actually is. Hence, when Freddy sees a lot of *HH* and *MM* games, Freddy is forced to conclude that the player must be having hot and cold games.

Class Conclusion Slide

I don't care if you struggled with the calculus in this course or if you found the algebra tedious. (So did I, when I was coming up with all this stuff)

I want you to take these lessons:

- People are dumb (duh)
- People are systematically dumb
- Formal models can help us understand the implications of their dumbness.
- Formal thinking can help us understand what we don't understand.

Class Conclusion Slide

I don't care if you struggled with the calculus in this course or if you found the algebra tedious. (So did I, when I was coming up with all this stuff)

I want you to take these lessons:

- People are dumb (duh)
- People are systematically dumb
- Formal models can help us understand the implications of their dumbness.
- Formal thinking can help us understand what we don't understand.

Class Conclusion Slide

I don't care if you struggled with the calculus in this course or if you found the algebra tedious. (So did I, when I was coming up with all this stuff)

I want you to take these lessons:

- People are dumb (duh)
- People are systematically dumb
- Formal models can help us understand the implications of their dumbness.
- Formal thinking can help us understand what we don't understand.

Class Conclusion Slide

I don't care if you struggled with the calculus in this course or if you found the algebra tedious. (So did I, when I was coming up with all this stuff)

I want you to take these lessons:

- People are dumb (duh)
- People are systematically dumb
- Formal models can help us understand the implications of their dumbness.
- Formal thinking can help us understand what we don't understand.

Class Conclusion Slide

I don't care if you struggled with the calculus in this course or if you found the algebra tedious. (So did I, when I was coming up with all this stuff)

I want you to take these lessons:

- People are dumb (duh)
- People are systematically dumb
- Formal models can help us understand the implications of their dumbness.
- Formal thinking can help us understand what we don't understand.