Problem Set Henry

Due in class on Thursday, February 6.

You must indicate the date that you completed each problem next to the start of your answer to that problem.

Question 1:

Suppose you are an insurance agent, and you come across an individual who faces a potential loss L with probability p. You offer this individual an insurance plan wherein they can pay you αq to insure against proportion α of this loss for any $\alpha \in [0,1]$ they choose.

(a) Suppose the individual is risk-averse.

(i) What can we conclude about the α the individual might choose if the insurance you offer is actuarially underfair ($q > pL$)?

(ii) What can we conclude about the α the individual might choose if the insurance is actuarially overfair ($q < pL$)?

(b) Suppose the individual is risk-loving.

(i) What can we conclude about the α the individual might choose if the insurance you offer is actuarially underfair ($q > pL$)?

(ii) What can we conclude about the α the individual might choose if the insurance is actuarially overfair ($q < pL$)?

Note: Question 1 should be answered using only intuition/logic — no calculus is needed.

Question 2:

Suppose a person chooses Lottery A over Lottery B, and also chooses Lottery C over Lottery D, where:

Lottery A: $(50, 1)$	Lottery C: $(50, .5; 0, .5)$
Lottery B: $(100, .5; 0, .5)$	Lottery D: $(100, .1; 50, .3; 0, .6)$

(a) Does this person's behavior violate expected utility (without any restrictions on u)?

(b) Does this person's behavior violate expected utility with more is better?

(c) Does this person's behavior violate expected utility with risk aversion?

Explain your answers.

Question 3:

Suppose that David evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function

$$v(x) = \begin{cases} x^{0.75} & \text{if } x \geq 0 \\ 2*(-x)^{0.75} & \text{if } x \leq 0. \end{cases}$$

David owns an asset that yields a lottery $(\$500, \frac{3}{8}; \$10, \frac{1}{2}; -\$500, \frac{1}{8})$. If you offer to purchase this asset from David for an amount $\$z$, how large would $\$z$ need to be for David to accept your offer?

Question 4:

Suppose Dwight is a risk-averse expected utility maximizer. In contrast, Michael evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

Consider the following four choice situations:

Choice (i): $(-\$70, \frac{1}{3} ; -\$35, \frac{2}{3})$ vs. $(-\$45, 1)$

Choice (ii): $(\$500, \frac{1}{5} ; \$200, \frac{4}{5})$ vs. $(\$275, 1)$

Choice (iii): $(-\$1200, \frac{1}{2} ; \$0, \frac{1}{2})$ vs. $(-\$625, 1)$

Choice (iv): $(-\$250, \frac{3}{4} ; \$750, \frac{1}{4})$ vs. $(\$0, 1)$

For each choice, describe for both Dwight and Michael whether we can determine which option they will choose or whether we need more information.

Question 5:

Suppose that Mari evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

(a) If Mari faces a 5% chance of incurring a loss of \$10,000, would she be willing to purchase full insurance at a premium of \$500? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person would behave? If so, what is it? If not, why not?

(b) If Mari faces a 55% chance of incurring a loss of \$2,500, would she be willing to purchase full insurance at a premium of \$1,375? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

(c) If Mari chooses lottery $(\$500, .8; \$0, .2)$ over lottery $(\$900, .5; \$0, .5)$, could she also choose lottery $(\$900, .15; \$0, .85)$ over lottery $(\$500, .24; \$0, .76)$? Could prospect theory with $\pi(p) \neq p$ explain this pattern of choices? If so, how? If not, why not?

Question 6:

Note: Report your answers to 3 decimal points.

Consider the bet ($\$50, \frac{3}{4}\ ;\ -\$Y, \frac{1}{4}$).

(a) Suppose that April is an expected utility maximizer with $u(x) = \ln x$, and her initial wealth is \$1,000.

(i) For what values of Y does April reject a single play of the bet?

(ii) For what values of Y does April accept two independent plays of the bet?

(iii) Is it possible for April to reject the single bet but accept the aggregate bet?

(b) Suppose that Andy also has initial wealth \$1,000, but he evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.25x & \text{if } x \leq 0. \end{cases}$$

(i) For what values of Y does Andy reject a single play of the bet?

(ii) For what values of Y does Andy accept two independent plays of the bet?

(iii) Is it possible for Andy to reject the single bet but accept the aggregate bet?

Question 7

(a) Suppose that Harry is an expected utility maximizer with $u(x) = -e^{-0.002x}$, and his initial wealth is \$50,000. Derive how Harry feels about the following bets:

(i) Harry will accept $(-50, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Harry will accept $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Harry will accept $(-250, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iv) Harry will accept $(-800, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(b) Like Harry, Ron has initial wealth \$50,000. But unlike Harry, Ron evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x \leq 0. \end{cases}$$

If we observe that Ron accepts $(-50, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > 110$, what can we conclude about Ron's λ?

(c) Suppose that Ron has the λ that you found in part **(b)**, and derive how he feels about the following bets:

(i) Ron will accept $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Ron will accept $(-250, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Ron will accept $(-800, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

Question 8

Suppose that you have \$10,000 to invest, and you can invest it in stocks or bonds. Each month, bonds yield a certain return of 1.1%. Each month, stocks yield a risky return of 2% with probability 0.8 and -1.2% with probability 0.2. Assume returns are independent across months.

You choose your portfolio as suggested by Benartzi & Thaler. Let x be the change in your portfolio's value between now and the next time you evaluate your portfolio. Of course x will be risky — that is, your choice will generate a lottery over possible outcomes for x. For any lottery $(x_1, p_1; ...; x_N, p_N)$, you evaluate this lottery according to prospective utility

$$\sum_{i=1}^{N} p_i \, v(x_i)$$

where

$$v(x_i) = \begin{cases} x_i & \text{if } x_i \geq 0 \\ 2.2x_i & \text{if } x_i \leq 0. \end{cases}$$

(a) Suppose you plan to evaluate your portfolio after one month. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(b) Suppose you plan to evaluate your portfolio after two months. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(c) How does your preference for stocks vs. bonds depend on your evaluation horizon? Discuss the significance of your answer for the equity premium puzzle.

(d) Suppose you plan to evaluate your portfolio after two months, and suppose further that you consider splitting your \$10,000 evenly between stocks and bonds. How do you feel about this allocation relative to all bonds or all stocks?

Question 9

In class, we developed a simple model with mug utility and money utility, and we derived implications for the reservation values of buyers, sellers, and choosers in endowment-effect experiments. This question asks you to think through several variants of that model. For all parts, assume that the person has wealth $w = \$10,000$, and that (Total Utility) = (Mug Utility) + (Money Utility).

(a) Let's begin with the case studied in class: Suppose that money utility is $u_m(m) = m$, and that mug utility is $u(c,r) = \mu c + v(c-r)$ where

$$v(x) = \begin{cases} \eta_{\text{mug}} * x & \text{if } x \geq 0 \\ \lambda_{\text{mug}} * \eta_{\text{mug}} * x & \text{if } x \leq 0. \end{cases}$$

(i) Derive the reservation values for buyers, sellers, and choosers as a function of μ, η_{mug}, and λ_{mug}.

(ii) Discuss how and why the three types differ. If it helps, consider the special case when $\mu = 2$, $\eta_{\text{mug}} = 0.75$ and $\lambda_{\text{mug}} = 4$.

(b) Now, let's introduce loss aversion over money: Suppose that mug utility is as in part **(a)**, but now suppose that money utility is $u_m(m, r_m) = m + v_m(m - r_m)$ where $r_m = w$ and

$$v_m(x) = \begin{cases} \eta_{\text{money}} * x & \text{if } x \geq 0 \\ \lambda_{\text{money}} * \eta_{\text{money}} * x & \text{if } x \leq 0. \end{cases}$$

(i) Derive the reservation values for buyers, sellers, and choosers as a function of μ, η_{mug}, λ_{mug}, η_{money}, and λ_{money}.

(ii) Discuss how and why the three types differ. If it helps, consider the special case when $\mu = 2$, $\eta_{\text{mug}} = 0.75$, $\lambda_{\text{mug}} = 4$, $\eta_{\text{money}} = 0.5$, and $\lambda_{\text{money}} = 3.5$.

(c) Next, instead of assuming loss aversion over money, let's assume diminishing marginal utility from money: Suppose again that mug utility is as in part **(a)**, but now suppose that money utility is $u_m(m) = 10{,}000 * \ln m$. To simplify things, assume that $\mu = 2$, $\eta_{\text{mug}} = 0.75$, $\lambda_{\text{mug}} = 4$.

(i) Derive the reservation values for buyers, sellers, and choosers.

(ii) Discuss how and why the three types differ.

(d) Finally, let's keep diminishing marginal utility from money, but now eliminate loss aversion over mugs: Suppose that mug utility is $u_c(c) = 3c$, and that money utility is $u_m(m) = 10{,}000 * \ln m$.

(i) Derive the reservation values for buyers, sellers, and choosers.

(ii) Discuss how and why the three types differ.

Note: For parts (c) & (d), report your answers to 4 decimal points.

Question 10: **[optional, no extra credit, but worth trying]**

Short-Answer Questions: For each question below, please provide a short, concise answer — your answer only needs to be a few sentences.

(a) Does the reflection effect violate expected utility? Briefly explain your answer.

(b) Briefly explain the difference between diminishing sensitivity and loss aversion.