Choice under Uncertainty (Lecture 1e)

EC404; Fall 2021

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Last updated September 28, 2021

Our Outline:

This space will be filled by week's end.

"Participation" Questions

1. Suzy is risk loving. She faces a lottery (100, .4; -100, .6). She is offered the opportunity to avoid this lottery by paying 15 dollars. Will she do it?

(Yes / No / We need more information)

1. Alexandre has preferences according to prospect theory (with the three properties of the value function we previously discussed). He faces the same lottery. What will he do?

(Pay $15 / Play the lottery / We need more information)

1. Roxanne is confused by the content in this course. However, Roxanne has not come to office hours. Is Roxanne making wise life decisions?

(Yes / No / We need more information)

In the next (two? one and a half?) lectures we will "discover" some problems with vanilla Prospect Theory.

We'll also introduce a bunch of "new" value functions.

. . . except the value functions aren't new

. . . and the discoveries were lurking in the back of your mind this whole time.

And we'll do a million exercises. This material is tricky.

We'll follow the KR theory of reference-dependent utility with loss aversion.
(Except it's really just the **right** way to do Prospect Theory.)

Their innovations address two major issues ("loopholes"):

1. What determines the reference point?
2. When do people experience loss aversion, and what is the magnitude of this experience?

They address these issues by incorporating two novel features:

A person's reference point is her recent beliefs or expectations about outcomes.

Gain-loss utility is directly tied to the intrinsic utility from consumption --- so that a person experiences more gain-loss utility for goods that involve more consumption utility.

Model

Suppose there are 2 goods:

- Person chooses a vector (x_A, x_B).
- Reference point is a vector (r_A, r_B).

Preferences:

$$\begin{aligned} \text{Total Utility} \;\equiv\; & \left[\, m_A(x_A) \;+\; n_A(x_A|r_A) \,\right] \\ & + \left[\, m_B(x_B) \;+\; n_B(x_B|r_B) \,\right] \end{aligned}$$

- $m_A(x_A)$ is intrinsic utility for good A, and $m_B(x_B)$ is intrinsic utility for good B.
- $n_A(x_A|r_A)$ is gain-loss utility for good A, and $n_B(x_B|r_B)$ is gain-loss utility for good B.

How to formalize that gain-loss utility is directly tied to intrinsic utility:

Assume there exists a *universal gain-loss function* $\mu(z)$ such that the gain-loss utilities are:

$$v_A(x_A|r_A) = \mu\,(\; m_A(x_A) - m_A(r_A)\;)$$

$$v_B(x_B|r_B) = \mu\,(\; m_B(x_B) - m_B(r_B)\;)$$

In general, $\mu(z)$ takes form of the Kahneman-Tversky value function. But we'll focus on the "easy" case:

$$\mu(z) = \{ \quad \eta * z \quad \text{if} \quad z \geq 0$$

$$\eta * \lambda * z \quad \text{if} \quad z \leq 0$$

Example: Two goods, shoes (c) and money (w), with intrinsic utilities:

$$m(c) = \theta * c$$

$$m(w) = w$$

As with mugs, we can represent shoe utility in a 2x2 grid (see board)

Consider the following choice problem:

- Suppose Bogi starts with 0 shoes and wealth w, and has the option to purchase a shoe for price p. How does Bogi behave as a function of expectations?

Case 1: Suppose you expect to buy a pair of shoes $\Longrightarrow$ reference point is $(r_c = 1, r_m = w - p)$:

$$\text{Utility(Buy)} = [\theta + \eta 0] + [(w - p) + \eta 0]$$

$$\text{Utility(Not)} = [0 - \eta\lambda\theta] + [w + \eta p]$$

$$\text{Buy when Utility(Buy)} \geq \text{ Utility(Not)} \Longleftrightarrow p \leq \frac{1 + \eta\lambda}{1 + \eta}\theta$$

Consider the following choice problem:

- Suppose Bogi starts with 0 shoes and wealth w, and has the option to purchase a shoe for price p. How does Bogi behave as a function of expectations?

Case 2: Suppose you expect not to buy any shoes $\implies$ reference point is $(r_c = 0, r_m = w)$:

$$\text{Utility(Buy)} = [\theta + \eta\theta] + [(w - p) - \eta\lambda p]$$

$$\text{Utility(Not)} = [0 + \eta 0] + [w + \eta 0]$$

$$\text{Buy when Utility(Buy)} \geq \text{ Utility(Not)} \iff p \leq \frac{1+\eta}{1+\eta\lambda}\theta$$

.

Because $\lambda > 1$ implies $\frac{1+\eta\lambda}{1+\eta} > \frac{1+\eta}{1+\eta\lambda}$, there are three cases:

1. If $p > \frac{1+\eta\lambda}{1+\eta}\theta$, don't buy no matter your beliefs.
2. If $p<\frac{1+\eta }{1+\eta \lambda }\theta $, buy no matter your beliefs.
3. If $\frac{1+\eta}{1+\eta\lambda}\theta < p < \frac{1+\eta\lambda}{1+\eta}\theta$, buy if you expect to buy, and don't buy if you expect not to buy.

Point: If the reference point depends on expectations, then, even in the same situation, a person might exhibit different outcomes depending on which set of self-fulfilling expectations he happens to have.

Köszegi & Rabin (2006) Example

Suppose there are two goods, candy bars (c) and money (m). Paige has initial income I, and she is deciding whether to buy 0, 1, or 2 candy bars at a price of p per candy bar. Paige's total utility is the sum of her candy-bar utility and her money utility, and her intrinsic utilities for the two goods are:

w_{c}(c) \equiv &\left\{ 0 \text{if }c=0

$$\theta_1 \text{if } c = 1$$

\qquad \theta _{1}+\theta _{2} & \text{if }c=2

\qquad \qquad \text{where }\theta _{1}>\theta _{2} \text{and } w_{m}(m) &\equiv

class

(a) If Paige were a standard agent who only cares about her intrinsic utilities, how would she behave as a function of the price p? In other words, for what prices would she buy zero candy bars, for what prices would she buy one candy bar, and for what prices would she buy two candy bars?

(b) Now suppose that Paige behaves according to the Koszegi-Rabin model. In other words, in addition to intrinsic utilities, she also cares about gain-loss utility, where the gain-loss utility for each good is derived from the universal gain-loss function described above.

If Paige expects to buy no candy bars, how would she behave as a function of the price p? In other words, for what prices would she buy zero candy bars, for what prices would she buy one candy bar, and for what prices would she buy two candy bars?

Köszegi & Rabin (2007)

In a second paper, Köszegi and Rabin investigate the implications of their approach for basic risk preferences.

Assume one good, money (\$x\$), with intrinsic utility $w(x) = x$.

- Note: $w(x) = x$ implies there is no intrinsic risk aversion --- all risk aversion will derive from gain-loss utility!

Applying their approach, if consume money x given reference point r, then total utility is

u(x|r)=\left\{x+\eta (x-r) & \text{if }x\geq r

\quad \quad x+\eta \lambda (x-r) & \text{if }x\leq r

How to incorporate uncertainty:

- If consume lottery $X \equiv (x_1, p_1; \ldots; x_N, p_N)$ given reference point r, then "expected" total utility is

$$U(X|r) \;=\; \sum_{i=1}^{N} p_i \, u(x_i|r).$$

Example: If $X = (200, \frac{1}{4}; 0, \frac{3}{4})$ and $r = 100$, then

$$U(X|r) = \frac{1}{4}u(200|100) + \frac{3}{4}u(0|100).$$

But might expect a lottery, in which case **the reference point would be a lottery**.

If consume money x given reference point $R \equiv (r_1, q_1; \ldots; r_M, q_M)$, then "expected" total utility is

$$U(x|R) \;=\; \sum_{j=1}^{M} q_j \, u(x|r_j).$$

- If consume lottery $X \equiv (x_1, p_1; \ldots; x_N, p_N)$ given reference point $R \equiv (r_1, q_1; \ldots; r_M, q_M)$, then total utility is

$$\begin{aligned} U(X|R) &= \sum_{i=1}^{N} p_i \, U(x_i|R) \\ &= \sum_{j=1}^{M} q_j \, U(X|r_j) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{M} p_i \, q_j \, u(x_i|r_j). \end{aligned}$$

Köszegi & Rabin (2007)

Example: If $X = (200, \frac{1}{4}; 0, \frac{3}{4})$ and $R = (150, \frac{1}{3}; 50, \frac{2}{3})$, then

$$U(X|R) = \frac{1}{4}\left[\frac{1}{3}u(200|150) + \frac{2}{3}u(200|50)\right] + \frac{3}{4}\left[\frac{1}{3}u(0|150) + \frac{2}{3}u(0|50)\right]$$

or

$$U(X|R) = \frac{1}{3}\left[\frac{1}{4}u(200|150) + \frac{3}{4}u(0|150)\right] + \frac{2}{3}\left[\frac{1}{4}u(200|50) + \frac{3}{4}u(0|50)\right]$$

or

U(X|R)=\frac{1}{12}u(200|150)+\frac{1}{6}u(200|50)+\frac{1}{4}u(0|150)+\frac{1}
{2}u(0|50)$$.

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# Köszegi & Rabin (2007)}

\textbf{Point 1:} Risk aversion when no possible "losses".



Consider choice%
\[
A\equiv (y,1)\ \text{with }y\leq 100\qquad \text{vs.\qquad }B\equiv (\text{ }%
200,\frac{1}{2}\text{ };\text{ }0,\frac{1}{2}\text{ })
\]

 %

Case 1: Suppose expect lottery $A\Longrightarrow $ reference point is `\(r=y\)`:

\begin{center}
    \begin{tabular}{llclc}
        &  &  &  &  \\
        `\(U(A|r)\)` & `\(=\)` & `\(y\)` & `\(+\)` & $\left[ 0\right] $ \\
        &  &  &  &  \\
        `\(U(B|r)\)` & `\(=\)` & `\(100\)` & `\(+\)` & $\left[ \frac{1}{2}\eta (200-y)+\frac{1
        \eta \lambda (0-y)\right] $ \\
        &  &  &  &
    \end{tabular}

    \[
    \text{Choose }A\text{ if }y\geq \frac{1+\eta }{1+\frac{1}{2}\eta +\frac{1}{2}%
        \eta \lambda }100\equiv \bar{y}_{1}\text{.}
```

Allen et al. 2017}

```
\begin{center}
\begin{figure}
  \includegraphics[scale=.55]{graphics/allenetal2017_1}
\end{figure}
\end{center}
```

Allen et al. 2017}


```
\begin{center}
\begin{figure}
\includegraphics[scale=.33]{graphics/allenetal2017_2}
\end{figure}
\end{center}
```

Reference-Dependent Welfare}

\dots not \textbf{that} kind of welfare. The economics kind.

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Put more concretely: are people worse off for having made risk averse decisions?

\begin{itemize} \item Samuelson showed us that they are worse of mathematically.

\vspace*{0.1in}

\item Ultimately, answer to this question depends on modeler's beliefs about whether loss aversion is something that people really \textit{feel}, or merely an artifact of some choice bias or mistake.

\item Two camps: (1) Loss aversion is an affective forecasting error; (2) Loss aversion is a real manifestation of preferences. \end{itemize}

Surprisingly: Kahneman waffles between two; see e.g. Schkade and Kahneman (1998).

Epilogue}

What have we learned in $\approx \mathbf{500}$ years of studying risk preferences?

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\textbf{Expected values} matter, but don't wholly determine choice.

\begin{itemize} \item \dots except they probably \textit{should}. [Begin rant.]

\vspace*{0.1in}

\item \dots most of the time. [End rant.] \end{itemize}

\vspace*{0.1in}

\textbf{Diminishing marginal utility} matters, but definitely does not explain most choices over risk.

\begin{itemize} \item \dots and I'm suspicious of \textit{all} evidence on diminishing marginal utility of wealth.

Epilogue}

\textbf{Prospect Theory} matters, but you need to apply it correctly.

\begin{itemize} \item Misleading conclusions when you fail to account for beliefs.

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\item \dots but when you apply the "correct model", your intuitions are preserved. \end{itemize}

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BUT: there are a lot of open questions about Köszegi and Rabin's theory.

\indent (Wanna go to grad school?)

Epilogue}

We've also (sneakily) introduced a new category of utility function:\textbf{ belief-based utility}.

We will return to some other models in this space.

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\begin{itemize}
\item \dots and in doing so we can address the lingering question: what \textit{is
\end{itemize}
```

%EndExpansion

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