Problem Set Intelligence

[Due by 11:59 PM on Tuesday, January 25.]

Question 1:

Consider the following choice:

$$\mathbf{x} \equiv (\$100, 0.8; -\$100, 0.2) \quad \text{vs.} \quad \mathbf{y} \equiv \left(\$70, \frac{5}{9}; \$40, \frac{1}{3}; \$0, \frac{1}{9}\right)$$

(a) Rachel is an EV maximizer with initial wealth \$1,000. Which does she choose?

(b) Stanley and Yusuf are EU maximizers with initial wealth \$10,000.

(i) If Stanley has a CRRA utility function with $\rho = 1/4$, which does he choose?

(ii) If Yusuf has a CRRA utility function with $\rho = 4$, which does he choose?

Question 2:

Suppose that Sarah has initial wealth \$50,000 that she plans to invest for a year and then consume it. There are two assets in which she can invest. First, there is a risk-free asset that yields a certain return of 4%. Second, there is a risky asset that yields a return of 20% with probability 2/3 and a return of -25% with probability 1/3. Assume these are the only two assets in which Sarah can invest, and she can split her wealth between them.

If Sarah is an expected-utility maximizer with utility function $u(x) = \ln x$, how much would she invest in each asset?

Question 3:

Consider the following three choice situations:

Choice (i): ($\$700, \frac{1}{2}$; $\$0, \frac{1}{2}$) vs. ($\$300, 1$)

Choice (ii): ($-\$300, \frac{1}{3}$; $-\$500, \frac{2}{3}$) vs. ($-\$400, 1$)

Choice (iii): ($\$700, \frac{1}{4}$; $\$500, \frac{1}{4}$; $\$300, \frac{1}{4}$; $\$100, \frac{1}{4}$) vs. ($\$450, \frac{1}{2}$; $\$350, \frac{1}{2}$)

(a) Suppose Gemma is a risk-averse expected utility maximizer. For each choice, can we determine which option Gemma will choose, or do we need more information?

(b) For each choice, as Gemma becomes more risk averse, does she become more prone to choose the first or second option?

(c) Now suppose Gemma has a CRRA utility function. For each choice, as Gemma's wealth becomes larger, does she become more prone to choose the first or second option?

(Note: You should be able to intuit an answer to part **(c)** — no math is needed.)

Question 4: (Demand for Insurance)

Suppose you are a risk-averse expected utility maximizer with utility function $u(x)$. You have initial wealth \$30,000, but before you consume it, you are subject to the following health risk (these events are mutually exclusive):

Required Medical Payment	Probability
\$100	15%
\$400	10%
\$800	4%
\$2,000	2%
\$15,000	1%

Now suppose that an insurance agent offers to sell you health insurance with 20% coinsurance — that is, for any medical payment that you require, you must pay for 20% of the payment and the insurance company pays the rest. The price of this insurance is p.

(a) If you do not buy the insurance, what lottery do you face? If you buy the insurance, what lottery do you face?

(b) Let p^* denote your willingness to pay for the insurance — so that you prefer to buy the insurance if $p < p^*$ and you prefer not to buy the insurance if $p > p^*$. Provide an equation from which you could derive p^*.

(c) If you are risk-neutral, what can we say about your p^*? If you are risk-averse, what can we

say about your p^*? As you become more risk averse, what happens to your p^*? Briefly explain your answers.

(d) If the coinsurance rate goes down to 15%, what happens to your p^*? Briefly explain your answer.

Question 5:

Consider the following choice (where $p \in [0,1]$, $q \in [0,1]$, and $p+q \leq 1$):

$$\mathbf{x} \equiv (\ \$40, p\ ;\ \$10, q\ ;\ \$0, 1-p-q\ ) \quad \text{vs.} \quad \mathbf{y} \equiv \left(\ \$30, \frac{1}{2}\ ;\ \$20, \frac{1}{6}\ ;\ \$0, \frac{1}{3}\ \right)$$

(a) For what values of p and q does lottery **x** dominate lottery **y**?

(b) For what values of p and q does lottery **y** dominate lottery **x**?

Question 6: **[optional, no extra credit, but worth trying]**

Short-Answer Questions: For each question below, please provide a short, concise answer — your answer only needs to be a few sentences.

(a) Suppose Karen is a risk-averse expected utility maximizer. As Karen's wealth goes up, will she act more or less risk-averse? Briefly explain your answer.

(b) Suppose Julian is a risk-averse expected utility maximizer. If he were extremely risk-averse, could he prefer the lottery $(1000, 1)$ over the lottery $(1000, \frac{1}{2}; 1100, \frac{1}{2})$? Briefly explain your answer.

(c) Does the St. Petersburg Paradox suggest that expected utility is a bad model of human behavior? Briefly explain your answer.