

## Problem Set 6

[Due in class on Thursday, December 6.]

### Question 1:

This question asks you to reconsider the model of optimal sin taxes that we studied in class with a different distribution of types. Assume that everyone has  $\rho = 60$  and  $\gamma = 30$ . Assume further that proportion  $\alpha$  of the population has  $\beta = 0.95$  while proportion  $1 - \alpha$  has  $\beta = 1$  (both types have  $\delta = 1$ ).

- (a) As a function of  $\alpha$  and  $t$ , what is the uniform lump-sum transfer?
- (b) As a function of  $\alpha$  and  $t$ , derive an expression for social welfare.
- (c) Solve for the optimal tax.
- (d) How does the optimal tax depend on  $\alpha$ ? Provide some intuition for this answer.

### Question 2:

This question asks you to reconsider the model of optimal sin taxes that we studied in class when there is heterogeneity in people's susceptibility to health consequences from potato-chip consumption (in addition to heterogeneity in self-control problems). Suppose that everyone has  $\rho = 60$  (everyone has the same tastes for potato-chip consumption). Suppose that  $1/2$  of the population has  $\beta = 1$  while  $1/2$  of the population has  $\beta = 0.95$ . Suppose further that  $1/4$  of the population has  $\gamma = 50$  and the other  $3/4$  of the population has  $\gamma = 25$ , where the distributions of  $\beta$  and  $\gamma$  are independent.

Note that there are four types: (i) people with  $\beta = 1$  and  $\gamma = 50$ ; (ii) people with  $\beta = 1$  and  $\gamma = 25$ ; (iii) people with  $\beta = 0.95$  and  $\gamma = 50$ ; and (iv) people with  $\beta = 0.95$  and  $\gamma = 25$ .

- (a) As a function of  $t$ , how many potato chips will each type consume?
- (b) As a function of  $t$ , what is the uniform lump-sum transfer?
- (c) For each type, compare people's utility for  $t = 0\%$  vs.  $t = 10\%$ .
- (d) Are all types better off when  $t = 10\%$ ? Provide some intuition for this answer.
- (e) Are the two types with  $\beta = 1$  on average better off? Are the two types with  $\beta = 0.95$  on average better off? Provide some intuition for this answer.

### Question 3:

Suppose that Ogre and Donkey both have “social-welfare preferences” of the form introduced by Charness & Rabin (that we discussed in class). They differ, however, in that Ogre takes a utilitarian view of social welfare (he has  $\delta = 0$ ) while Donkey takes a maximin view of social welfare (he has  $\delta = 1$ ). Note: For each game, you should specify how their behavior depends on their  $\lambda$ .

(a) Solve for Ogre and Donkey’s behavior in the Prisoners’ Dilemma for the case when they believe that their opponent is playing  $D$  (use the version of the Prisoners’ Dilemma from class).

(b) Solve for Ogre and Donkey’s behavior in the Dictator Game.

(c) Solve for Ogre and Donkey’s behavior in the role of Player 2 in the Ultimatum Game when they are offered a share  $s \leq 1/3$ .

(d) To what extent can social-welfare preferences explain experimental results in the Prisoners’ Dilemma, the Dictator Game, and the Ultimatum Game?

### Question 4:

Suppose Ogre and Donkey have social-welfare preferences as in Question 3. In contrast, Fiona has “inequity aversion” of the form introduced by Fehr & Schmidt (that we discussed in class). Note: For each game, you should specify how Ogre and Donkey’s behavior depends on their  $\lambda$ , and how Fiona’s behavior depends on her  $\alpha$  and  $\beta$ . Also, if you like, you may assume that Player 1 can choose non-integer divisions — e.g., Player 1 might keep 25.9 tokens and give 24.1 tokens.

(a) Consider the following modified dictator game: Player 1 divides 50 tokens between Player 1 and Player 2. Each token is worth \$2 to Player 1, and each token is worth \$6 to Player 2. How would Ogre, Donkey, and Fiona behave in this game?

(b) Consider the following modified dictator game: Player 1 divides 40 BLUE tokens and 20 RED tokens between Player 1 and Player 2. Each BLUE token is worth \$2 to Player 1 and \$1 to Player 2. Each RED token is worth \$3 to Player 1 and \$5 to Player 2. How would Ogre, Donkey, and Fiona behave in this game?

Question 5:

Consider a simple dictator game in which Player 1 has 4 options from which to choose:

- (A) (\$10, \$10)      (B) (\$15, \$75)      (C) (\$25, \$40)      (D) (\$75, \$0)

How would Ogre, Donkey, and Fiona behave in this game? Provide some intuition for your answers. Note: You should specify how Ogre and Donkey's behavior depends on their  $\lambda$ , and how Fiona's behavior depends on her  $\alpha$  and  $\beta$ .

Question 6 (NOT TO BE TURNED IN):

Marge has inequity aversion, but with the following non-linear form:

$$u^1(x_1, x_2) = \begin{cases} 2(x_1)^{1/2} - \alpha[x_2 - x_1] & \text{if } x_1 \leq x_2 \\ 2(x_1)^{1/2} - \beta[x_1 - x_2] & \text{if } x_1 \geq x_2 \end{cases}$$

(a) Suppose Marge plays a dictator game in which she must divide \$10 between herself and another person. As a function of her  $\alpha$  and  $\beta$ , how will she behave?

Note: Rather than solve for the *share* that Marge offers (as we did in class), it is perhaps easier to solve for the *amount* that Marge offers — i.e., if she offers amount \$z, then she will keep \$(10 - z) for herself.

(b) In class, we discussed how the linear version of inequity aversion does not explain well the quantitative results in experimental dictator games. Does this non-linear version work better?

Question 7 (NOT TO BE TURNED IN):

Homer has simple altruism, but with the following non-linear form:

$$u^1(x_1, x_2) = \ln(x_1 + 1) + \phi [\ln(x_2 + 1)]$$

(a) Suppose Homer plays a dictator game in which he must divide \$10 between himself and another person. As a function of his  $\phi$ , how will he behave?

(b) In class, we discussed how the linear version of simple altruism does not explain well the quantitative results in experimental dictator games. Does this non-linear version work better?