Topic 4A: Warm-Up: Interest Rates, Compounding, PDV,....

EC 404: Behavioral Economics
Professor: Ben Bushong

October 12, 2021

Overview of Topic 4

Topic 4: Intertemporal Choice — Making Choices Over Time

Many interesting questions in economics involve choice over time:

- How do people allocate their wealth between current consumption and future consumption?
- How do people decide when to work on tasks?
- For goods that yield short-term consumption utility but generate negative consequences in the long-term — e.g., alcohol, cigarettes, potato chips — how do people trade off the short-term benefits vs. the long-term costs?

The standard model — "exponential discounting" — assumes:

- people treat time in a relatively even-handed manner.
- people carry out their plans.
- people know what they'll like in the future.

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Let's understand the standard model and (quickly) do some example problems.

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Interest Rates and Compounding

Example A: Suppose you put \$1000 into a bank account that pays 10% interest per year.

- After 1 year, you'll have $(\$1000) * (1.10) = \$1100$.
- After 2 years, you'll have $(\$1100) * (1.10) = \$1210$.
- After 3 years, you'll have $(\$1210) * (1.10) = \$1331$.

More generally:

- If you put $\$P$ into a bank account that pays interest rate r per year, its future value in T years will be $(\$P) * (1 + r)^T$.

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Some Definitions:

Compound interest is interest paid on past interest earned.

Compounding is earning interest on past interest earned.

The *frequency of compounding* is the frequency at which interest is credited to your account (after which it's starts earning compound interest).

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Interest Rates and Compounding

Example B: Suppose you put \$1000 into a bank account that pays a 10% annual interest rate that is compounded every six months.

Because a 10% annual interest rate implies a 5% semi-annual interest rate:

- After 6 months, you'll have $(\$1000) * (1.05) = \$1050$.
- After 1 year, you'll have $(\$1050) * (1.05) = \$1102.50$.

Example C: Suppose you put \$1000 into a bank account that pays a 10% annual interest rate that is compounded every month.

Because a 10% annual interest rate implies a $0.8\bar{3}\%$ monthly interest rate:

- After 1 year, you'll have $(\$1000) * (1.008\bar{3})^{12} = \$1104.71$.

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Interest Rates and Compounding

More generally, if you put $\$P$ into a bank account that pays an annual interest rate of r that is compounded n times per year:

- Its future value after 1 year will be $(\$P) * (1 + r/n)^n$.
- Its future value after T years will be $(\$P) * [(1 + r/n)^n]^T$.

Note: For continuous compounding, $\lim_{n\to\infty}(1 + r/n)^n = e^r$, and $\lim_{n\to\infty}[(1 + r/n)^n]^T = e^{rT}$.

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Discrete-Time Models

Suppose there is some set of periods $0, 1, 2, ..., T$ (perhaps $T = \infty$).

- Note: The length of a period might be one year, one month, one day, or whatever is most appropriate for the particular application.

Suppose there is a per-period interest rate r, and interest is compounded every period.

If P_t is the principal in your bank account in period t, then:

- $P_1 = (1 + r) * P_0$
- $P_2 = (1 + r)^2 * P_0$
- $P_t = (1 + r)^t * P_0$
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Present Discounted Value (PDV)

Suppose that you will be paid \$1100 one year from today. If the market interest rate is 10% (and yearly compounding), how much is this future payment be worth to you now?

We can answer this question by asking how much you could borrow now such that you would have to pay back exactly \$1100 in one year.

- Answer: \$1000 — because $(1.10) * (\$1000) = \$1100$.

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Definition: Given per-period interest rate r, the *present discounted value* (or sometimes just *present value*) of $\$P$ to be paid T periods in the future is

$$\frac{P}{(1+r)^T}$$

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Some PDV's for $P = \$1000$ and yearly compounding:

r	1 Year	2 Years	3 Years	10 Years	20 Years
3%	\$971	\$943	\$915	\$744	\$554
4%	\$962	\$925	\$889	\$676	\$456
5%	\$952	\$907	\$864	\$614	\$377
6%	\$943	\$890	\$840	\$558	\$312
7%	\$935	\$873	\$816	\$508	\$258

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PDV of a Stream of Payoffs

Suppose that you will be paid \$1100 one year from today, another \$1100 two years from today, and yet another \$1100 three years from today.

If the market interest rate is 10% (and yearly compounding), how much is this stream of payoffs worth to you now?

- Answer: Add up the individual *PDV*'s:

$$PDV = \frac{\$1100}{(1.10)} + \frac{\$1100}{(1.10)^2} + \frac{\$1100}{(1.10)^3} = \$2735.54.$$

More generally: Given per-period interest rate r, a stream of future revenues $(R_1, R_2, ..., R_N)$ — where revenue R_n is received in period n — has a present discounted value of

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