Lecture 2e: Choice over Time
Applications of Present Bias

EC 404: Behavioral Economics
Professor: Ben Bushong

March 17, 2022

Application 1: Present Bias & Saving

[Based on work by David Laibson and his collaborators.]

An Extended 3-Period Saving-Consumption Example

- You consume in 3 different periods — in the end, you choose a consumption bundle (c_1, c_2, c_3).
- Let (Y_1, Y_2, Y_3) denote your income flows.
- Let r be the market interest rate, no liquidity constraints.

$\Longrightarrow$ Your budget constraint is

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} \leq Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2} = W$$

To keep things simple, let's use specific numerical values. In particular, let's use $r = 10\%$ and $W = \$1000$, and so the budget constraint is

$$c_1 + \frac{c_2}{1.1} + \frac{c_3}{(1.1)^2} \leq \$1000$$

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Preferences

You have β, δ intertemporal preferences:

$$U^t(c_t, c_{t+1}, ..., c_T) = 2(c_t)^{1/2} + \beta \sum_{x=1}^{T-t} \delta^x \; 2(c_{t+x})^{1/2}.$$

Note: Instantaneous utility is $u(c) = 2(c)^{1/2}$.

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Long-Run Desired Behavior

Long-run desired behavior $(c_1^{**}, c_2^{**}, c_3^{**})$ maximizes

$$U^0(c_1, c_2, c_3) = 2(c_1)^{1/2} + \delta\ 2(c_2)^{1/2} + \delta^2\ 2(c_3)^{1/2}$$

$$= 2(c_1)^{1/2} + (.9)\ 2(c_2)^{1/2} + (.9)^2\ 2(c_3)^{1/2}$$

subject to

$$c_1 + \frac{c_2}{1.1} + \frac{c_3}{(1.1)^2} \leq \$1000.$$

Solution:

$$c_1^{**} = \$372.46 \qquad c_2^{**} = \$365.05 \qquad c_3^{**} = \$357.78$$

Note: This represents the person's ideal behavior when asked from a removed perspective — what she would follow if she were to commit prior to period 1.

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Propensity to Over-Consume

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Actual Behavior for Naifs

Suppose you are naive.

In period 1, you start following your period-1 desired behavior, and so

$$c_1^N = c_1^* = \$481.16.$$

[Note: You plan $c_2 = c_2^* = \$301.81$ and $c_3 = c_3^* = \$295.81$.]

In period 2, you reassess:

- Given you've consumed \$481.16, period-2 wealth is

$$\begin{aligned} W_2^N &\equiv (W - c_1^N)(1+r) \\ &= (\$1000 - \$481.16)(1.10) \\ &= \$570.72. \end{aligned}$$

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Conditional on having period-2 wealth $W_2^N = \$570.72$, you choose (c_2, c_3) to maximize

$$\begin{aligned} U^2(c_2, c_3) &= 2(c_2)^{1/2} + \beta\delta\; 2(c_3)^{1/2} \\ &= 2(c_2)^{1/2} + (.8)(.9)\; 2(c_3)^{1/2} \end{aligned}$$

subject to

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Solution:

$$c_2^N = \$363.46 \qquad c_3^N = \$227.99$$

Hence, actual behavior for naifs is:

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Actual Behavior for Sophisticates

Suppose you are sophisticated.

Use backward induction:

Consider how you would behave in period 2 as a function of your chosen period-1 consumption:

- If you consumed c_1 in period 1, your period-2 wealth would be

$$\begin{aligned} W_2 &= (W - c_1)(1 + r) \\ &= (\$1000 - c_1)(1.10). \end{aligned}$$

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$$c_2 + \frac{c_3}{1.1} \leq (\$1000 - c_1)(1.10).$$

Actual Behavior for Sophisticates

Solution for period-2 behavior as a function of c_1:

$$c_2(c_1) = 0.70053 * (\$1000 - c_1)$$

$$c_3(c_1) = 0.43942 * (\$1000 - c_1)$$

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Knowing this, in period 1 you choose c_1 to maximize

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Solution for period-1 consumption:

$$c_1^S = \$484.17.$$

After choosing c_1^S, in period 2 you actually choose:

$$\begin{aligned} c_2^S &= c_2(c_1^S) = 0.70053(\$1000 - c_1^S) = \$361.35 \\ c_3^S &= c_3(c_1^S) = 0.43942(\$1000 - c_1^S) = \$226.67 \end{aligned}$$

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Summary of the Basic Example

Long-run desired behavior is

$$c_1^{**} = \$372.46 \qquad c_2^{**} = \$365.05 \qquad c_3^{**} = \$357.78$$

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$c_1^* > c_1^{**}$ reflects that the present bias creates a propensity to over-consume (or under-save).

$c_2^N > c_2^*$ and $c_2^S > c_2^*$ reflects that the time inconsistency exacerbates the problem.

$c_1^S > c_1^N$ reflects that, in this example, sophistication exacerbates over-consumption in period 1. But unlike the above results, this result is not general — sophistication can exacerbate or mitigate period-1 over-consumption depending on the specific instantaneous utility function.

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Let's introduce an illiquid asset into our example:

- "Examples": a CD account, a house, a retirement account.
- If in period 1 you invest z in this asset, then in period 3 you receive $z(1+\hat{r})^2$ (but no access in period 2).

Let's initially suppose $\hat{r} = r$, and suppose further that $Y_1 = \$1000$ and $Y_2 = Y_3 = \$0$, and that you cannot borrow.

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Sophisticates and the Illiquid Asset

Result: Sophisticates can now implement their period-1 desired behavior.

Recall: Period-1 desired behavior is

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With the illiquid asset, in period 1:

- consume \$481.16.
- save \$274.37 in the bank (which yields \$301.81 in period 2).
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Sophisticates strictly prefer to use the illiquid asset in this way — indeed, they choose to do so even for $\hat{r}$ smaller than r.

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With the illiquid asset, in period 1 naifs COULD:

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BUT note that naifs are indifferent between using vs. not using the illiquid asset, because they (incorrectly) think that, even if they put their entire savings of \$518.84 in the bank, they would still consume \$301.81 in period 2 and \$295.81 in period 3.

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Illiquid Assets: Conclusions

- For people with self-control problems, tying up wealth in illiquid assets can be a useful commitment device to help counteract future over-consumption.
- Sophisticates are always on the lookout for such commitment opportunities, whereas naifs do not recognize the commitment value in illiquid assets.

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Illiquid Assets: A Few Comments

- The illiquid asset is not a perfect commitment technology, because you cannot prevent yourself from consuming current income. For instance, if $Y_1 = \$500$ and $Y_2 = \$550$, the illiquid asset would not help at all.

- An illiquid asset will not work as a commitment device if you can borrow against its future payoff. Hence, liquidity-enhancing instruments such as credit cards may in fact undermine the commitment value of illiquid assets.

- In the real world, illiquid assets usually have a **larger** return than liquid assets ($\hat{r} > r$).

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A Related Aside: Retirement Plans

Two features of retirement plans (IRA plans, 401(k) plans, etc):

- They are tax-exempt.
- They are illiquid (big penalty for early withdrawal).

Why have retirement plans?

- Goal: induce people to save for retirement.

To the extent that retirement plans are aimed at counteracting self-control problems:

- If people are sophisticated, the illiquidity feature of retirement plans is all that's needed to induce more retirement saving.
- If, in contrast, people are naive, then both features are crucial: the tax-exempt feature induces people to use retirement plans rather than some other form of saving, and then the illiquidity feature generates unexpected commitment benefits that "multiply" the effect.

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Applying These Ideas to Real Data

Angeletos *et al* (*JEP* 2001)

- They conduct a quantitative test of present bias (in the consumption-saving environment).

Basic idea:

- We observe people take on large credit-card debts at high interest rates, but also accumulate significant pre-retirement wealth.
- Under exponential discounting, it is very difficult to accommodate both.
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- We observe people take on large credit-card debts at high interest rates, but also accumulate significant pre-retirement wealth.
- Under exponential discounting, it is very difficult to accommodate both.
- Under present bias, this combination can be (roughly) understood as credit-card debt being driven by short-term impatience (β) and accumulation of pre-retirement wealth being driven by long-term patience (δ).

Applying These Ideas to Real Data

Angeletos *et al* (*JEP* 2001)

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Angeletos et al: The Model

They consider a rich, calibrated model:

- A "period" is one year.
- Households begin life at 20, retire at 63, and die at 90 (if not sooner).
- Demographics calibrated to the "U.S. population".
- Labor income calibrated to the "U.S. population".
- There is a liquid asset with real interest rate 3.75%.
- There is credit-card borrowing with real interest rate 11.75% (with a credit limit).
- There is an illiquid asset that generates annual consumption flow equal to 5% of its value (can be sold only with a transaction cost).
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Angeletos et al: Simulations

Exponential simulation:

- Assume that the entire economy is populated by exponential discounters with discount factor δ_{exp}.
- Choose δ_{exp} so that the simulations generate a median wealth-to-income ratio for households aged 50-59 of 3.2.

Present bias simulation (sophisticates):

- Assume that the entire economy is populated by people with present bias with $\beta = .7$ and $\delta = \delta_{PB}$.
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Chosen discount factors:

- $\delta_{\text{exp}} = .944$
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Angeletos et al: Performance of Simulated Models

Finally, they compare their simulated data to real-world data:

- households with liquid assets $>$ one-month's income:

Exponential simulation:	73%
Present Bias simulation:	40%
Data:	43%

- households with positive credit-card borrowing:

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Angeletos et al: Performance of Simulated Models

- mean credit-card borrowing (all households):

Exponential simulation:	\$900
Present Bias simulation:	\$3400
Data:	$>$ \$5000

- consumption-income comovement:

Exponential simulation:	0.032
Present Bias simulation:	0.166
Data:	≈ 0.2