Choice under Uncertainty (Lecture 1d)

EC404; Fall 2021

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Last updated September 21, 2021

Our Outline:

(Coming soon when all slides are covered)

Guiding Principles

Here are the two we've discussed so far:

#1: Carefully Define Your Environment

#2: Use Extreme Cases to Clarify Your Thinking

Have you memorized the bold definitions?

Have you tried thinking about extremes?

Have you come to office hours? (Answer: no).

Prospect Theory: A One-Slide Reminder

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A person evaluates a prospect $(x_1, p_1; \ldots; x_n, p_n)$ according to:

$$V(x_1, p_1; \ldots; x_n, p_n) \quad = \sum_{i=1}^{N} \pi(p_i)\, v(x_i)\,.$$

Notable features:

- Value comes from **changes** in wealth, not absolute wealth
- $\pi(p) \neq p$ implies people "mess up" probabilities
- Otherwise similar to expected utility. **Not** radical!

A commonly assumed form of $v(\cdot)$ is

$$v(x) = \{x^{\alpha} \quad \text{if} \quad x \geq 0$$

$$\lambda(x)^{\beta} \quad \text{if} \quad x \leq 0$$

and we'll often just drop the exponents.

For many years, expected utility has been used by economists to capture risk preferences. Indeed, it is still used in almost all applications.

But economists are starting to recognize that some behaviors are hard to interpret in terms of expected utility; and for many such behaviors, prospect theory provides a natural interpretation.

To illustrate, we'll consider nine examples.

- I suspect much more work in this area in coming years.
- Only one example addresses probability weighting -- but I suspect it is running around in tons of seemingly-strange behaviors.

Example courtesy of Samuelson (1963)

Consider the following bet:

$$\textbf{win \$200 with prob } \mathbf{1/2}$$

$$\textbf{lose \$100 with prob } \mathbf{1/2}$$

Samuelson's colleague turned down this bet, but announced that he would accept 100 plays of the same bet.

Samuelson proved that his colleague was "irrational" --- by proving that it is inconsistent with expected-utility theory to turn down the single bet but accept 100 such bets.

But **was** his colleague "irrational"?

Class discussion: Suppose that a person turns down the bet **at some wealth levels.** Does EU imply that the person must turn down 100 such bets?

Continued: Suppose that a person turns down the bet **at all wealth levels**. Does EU imply that the person must turn down 100 such bets?

What is the basic intuition?

Intuition: Consider an individual who has said that he is unwilling to take one bet but is willing to play 100 such bets. Suppose this person has played 99 bets.

- If asked whether he would like to stop at this point he will say yes. By assumption, he dislikes one bet at any relevant wealth level.
- However, this means that if asked after 98 bets whether he would like to play number 99 he must also decline.
- He should realize (by backward induction) that he would reject bet 100, implying that bet 99 is a single play.

The same reasoning applies to the first bet.

Thinking about economics grad school? Prove this claim formally.

Consider an alternative "model":

- Suppose that a person evaluates bets according to the value function

$$v(x) = x \quad \text{if} \quad x \geq 0$$

$$v(x) = 2.5x \quad \text{if} \quad x \leq 0$$

Consider the single bet $y = [200, .5; -100, .5]$.

Consider taking two such bets. This means you face aggregate gamble $z = [400, .25; 100, .5; -200, .25]$.

Point: Unlike EU, loss aversion can lead a person to reject one play of the bet but to accept multiple plays of the bet.

Note: The Previous Explanation Was Underspecified

Mental accounting: the process a person uses to interpret a choice situation.

- **Any** application of prospect theory requires a mental-accounting assumption.
- Typically, this requires an assumption about how people decide what are the objects for evaluation.
- E.g., Kahneman & Tversky interpret the isolation effect as people ignoring seemingly extraneous parts of the problem.
- E.g., to explain the behavior of Samuelson's colleague, we assumed that the person collapses the aggregate bet into a single lottery and decides whether to accept that lottery.
- Sometimes, we must make an assumption about when and how people code outcomes as gains and losses. (We'll do this in later applications)

People tend to dislike risky prospects even when they involve an expected gain.

Rabin & Thaler's point, which should feel **very** repetitive by now:

- Calibrationwise, this explanation doesn't work, because according to EU, "anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes."
- Now we'll show that *loss aversion* is a useful alternative.

Suppose you have wealth \$20,000, and you turn down a 50-50 bet to win \$110 vs. lose \$100.

We showed that rejecting the bet implies that $\rho > 18.17026$.

Question: What about with loss aversion?

Two plausible features of preferences consistent with loss aversion:

1. How you feel about absolute gambles is largely insensitive to your wealth --- e.g., you might reject $(101, .5; -100, .5)$ for all w.
2. At the same time, scaling outcomes proportionally need not change your preferences much --- e.g., you might have

$$(12, .5; -10, .5) \sim (0, 1)$$

$$(120, .5; -100, .5) \sim (0, 1)$$

$$(1200, .5; -1000, .5) \sim (0, 1)$$

Equity-Premium Puzzle (Mehra & Prescott, 1985)

Equity premium: The difference between the returns on stocks and the returns on fixed-income securities.

- The (historical) equity premium is quite large. For instance, since 1926, the real return on stocks has been about 7%, and the real return on T-Bills has been about 1%.

The puzzle: The equity premium is "too large" --- Mehra and Prescott estimate that investors would need to have absurd levels of risk aversion to explain the historical equity premium.

Standard Economics/Finance View of Financial Decisions

You have wealth w, and you use this wealth for your lifetime consumption profile $(c_1, c_2, \ldots, c_T)$.

Your lifetime consumption profile yields lifetime utility

$$u(c_1) + \delta u(c_2) + \delta^2 u(c_3) + \ldots + \delta^{T-1} u(c_T)$$

Wealth that's targeted for future consumption is invested in financial assets (stocks and bonds).

- Hence, any risk in your financial portfolio gets translated into risk in future consumption.
- And therefore any risk aversion that have with regard to future consumption gets translated into risk aversion with regard to your financial portfolio.

Assume a CRRA utility function (over consumption):

$$u(c) = \frac{(c)^{1-\rho}}{1-\rho}.$$

- *Note:* The larger is ρ, the **less** risk one takes on in one's financial portfolio (fewer stocks, more bonds).
- Mehra & Prescott show that to explain the observed equity premium, we need to assume that people have $\rho > 30$.
- But empirical estimates and theoretical arguments suggest $\rho \approx 1$ (log utility) and definitely not more than 5. (Remember the exercise we did?)

Two Interpretations for Mehra and Prescott Result

- Given the historical equity premium, under EU (and CRRA utility) people's observed willingness to hold a mix of stocks and bonds can be explained only by a $\rho > 30$, which is clearly absurd (i.e., it would imply absurd behavior in other domains).
- Given EU and reasonable levels of risk aversion ($\rho =1$ or perhaps even $\rho = 5$), under the historical equity premium, we should observe people investing exclusively in stocks.

Benartzi & Thaler's explanation: "Myopic Loss Aversion"

- Two components: loss aversion and a specific mental-accounting assumption.

Basic foundation: From time to time, a person evaluates her portfolio and experiences joy/pain from watching it grow/shrink.

Objects for Evaluation

Suppose a person evaluates her portfolio at dates

$$t, t + \triangle, t + 2\triangle, \ldots$$

Let Y_τ be the value of her portfolio at date τ.

Let $x_{\tau+\Delta} \equiv Y_{\tau+\Delta} - Y_\tau$.

At date τ, person chooses between lotteries over $x_{\tau+\Delta}$.

Key idea: The person's portfolio allocation chosen at date τ generates a lottery over $x_{\tau+\Delta}$ --- that is, a lottery over how her portfolio will change in value between now (τ) and the next evaluation period ($\tau + \Delta$).

A (Much) Simplified Example:

Suppose there are two assets, stocks and bonds, and that between τ and $\tau + \Delta$ the returns are:

- For bonds: $(+1\%, 1)$
- For stocks: $(+10\%, \frac{1}{2}; -5\%, \frac{1}{2})$

Suppose further that the person must choose a proportion α of her wealth to invest in stocks, with the remainder invested in bonds. As a function of α, the resulting lottery over $x_{\tau+\Delta}$ is

$$\textbf{Good Outcome:} \qquad \alpha w(.10) + (1-\alpha)w(.01)\ ,\ \frac{1}{2}$$

$$\textbf{Bad Outcome:} \qquad \alpha w(-.05) + (1-\alpha)w(.01)\ ,\ \frac{1}{2}$$

Again, at date τ, person chooses between lotteries over $x_{\tau+\Delta}$.

> Note: Major deviation from the standard approach!

Evaluating Lotteries

At date t, person chooses her portfolio to maximize her "prospective utility"

$$\sum_{x_{t+\Delta}} \pi(x_{t+\triangle})v(x_{t+\triangle}).$$

Let's use the value function

$$v(x) = x^{\alpha} \qquad \text{if} \quad x \geq 0$$

$$v(x) = -\lambda(-x)^{\beta} \qquad \text{if} \quad x \leq 0$$

The authors assume $\alpha = \beta = .88$ and $\lambda = 2.25$ (Tversky & Kahneman, 1992).

$\pi(x_{t+\triangle})$ reflects probability weighting. The authors use the cumulative form --- including the suggested parameter values --- from Tversky & Kahneman, 1992.

General (Simulation) Approach

1. Draw samples from historical (1926-1990) monthly returns on stocks, 5-yr bonds, and T-Bills.

- E.g., if 10 observations of actual monthly returns on an asset were

$$-2\%, 1\%, 0\%, 1\%, -1\%, 1\%, 2\%, 0\%, 1\%, 0\%$$

... then for that asset they'd set $\Pr(1\%) = 0.4$, $\Pr(0\%) = 0.3$, etc.

1. Then, consider n-month evaluation periods, for $n = 1, 2, 3, \ldots$, where the distribution of returns for an n-month evaluation period is constructed from n IID draws from the distribution of monthly returns.

- E.g., if the monthly return distribution is $(20\%, 1/2; 0\%, 1/2)$, then the 2-month return distribution is $(44\%, 1/4; 20\%, 1/2; 0\%, 1/4)$.

Results

First question: What evaluation period n would make investors *indifferent* between holding all stocks vs. holding all bonds?

Answer: The historical data are consistent with their model applied as if people evaluated their portfolios about once a year.

Second question: Assuming yearly evaluations, what is the **optimal** mix of stocks and bonds?

Answer: The optimal holdings, given the historical data, are to hold roughly equal amounts in stocks and bonds (as we observe in the world).

Snowberg and Wolfers (2010) explore non-linearity in probabilities "in the wild" by investigating horse-race bets.

If Horse A is 2:1 odds this should mean both:

1. That the implied probability of winning is $\frac{1}{3}$. Losing is twice as likely as winning and either the horse wins or loses.

2. If you put $1 on horse A, you either receive $3 ($2 winnings + $1 stake) or zero, since $\frac{1}{3} \times 3 + \frac{2}{3} \times 0 = 1$.

That is: Betting and not betting should yield the same expected return.

(Of course, there is a track profit such that the expected return is a bit negative. But we'll ignore this margin for now.)

Point: If all odds were appropriate, every horse would have equal expected value and thus equal expected returns.

Finding: The "Favorite-Longshot Bias"

- Longshots have low expected return, given how rarely they win...
- and bettors value favorites too little given how **often** they win.
- Concretely, betting on a horse with 100/1 odds yields returns of about -61%.
- Betting **randomly** yields average returns of -23%.
- Betting on a horse with 1/3 odds yields returns of only -5.5%

Application #4: Non-Linear Probabilities in the Wild

"Disposition Effect": When investors sell their stocks, they are more prone to sell their winners than their losers.

- A stock is a "winner" if its current price is above its purchase price, and it is a "loser" if its current price is below its purchase price (as in Shefrin & Statman 1985).

Odean (1998) provides a nice empirical test, and assesses several potential explanations.

Odean (1998) has a dataset of individual traders at a small brokerage house and observes each individual's stock portfolio and all trades made each day.

For every individual-day on which he observes trades, he calculates:

1. "Proportion of Gains Realized":

$$PGR \equiv \frac{\text{\# of winners sold}}{\text{\# of winners in portfolio}}$$

1. "Proportion of Losses Realized":

$$PLR \equiv \frac{\text{\# of losers sold}}{\text{\# of losers in portfolio}}$$

"Disposition Effect": $PGR > PLR$.

Big Question: What's the explanation?

Rational Explanation #1: Sell winners to rebalance your portfolio.

- If the disposition effect is driven by rebalancing, then if we restrict attention to trades in which (i) only entire holdings of a stock are sold or (ii) no new purchases are made, we should no longer observe a disposition effect.

- When Odean does this, the effect *does not go away*.

Rational Explanation #2: Sell winners because losers are better.

- Odean finds that the winners people sell *outperform* the losers they keep (over various horizons --- 1/3 year, 1 year, 2 years).

Odean's Explanations

- Loss aversion with a mental-accounting assumption that you experience gain-loss utility for a particular stock when you sell that stock.

- Or an irrational belief in mean reversion.