

Lecture 4e: Choice over Time

Applications of Present Bias

EC 404: Behavioral Economics
Professor: Ben Bushong

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Application 1: Present Bias & Saving

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[Based on work by David Laibson and his collaborators.]

An Extended 3-Period Saving-Consumption Example

- ▶ You consume in 3 different periods — in the end, you choose a consumption bundle (c_1, c_2, c_3) .
 - ▶ Let (Y_1, Y_2, Y_3) denote your income flows.
 - ▶ Let r be the market interest rate, no liquidity constraints.
- ⇒ Your budget constraint is

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} \leq Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2} \equiv W$$

To keep things simple, let's use specific numerical values. In particular, let's use $r = 10\%$ and $W = \$1000$, and so the budget constraint is

$$c_1 + \frac{c_2}{1.1} + \frac{c_3}{(1.1)^2} \leq \$1000$$

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You have β, δ intertemporal preferences:

$$U^t(c_t, c_{t+1}, \dots, c_T) = 2(c_t)^{1/2} + \beta \sum_{x=1}^{T-t} \delta^x 2(c_{t+x})^{1/2}.$$

Note: Instantaneous utility is $u(c) = 2(c)^{1/2}$.

Again, to keep things simple, let's use specific numerical values. In particular, let's use $\beta = .8$ and $\delta = .9$.

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Long-Run Desired Behavior

Long-run desired behavior $(c_1^{**}, c_2^{**}, c_3^{**})$ maximizes

$$\begin{aligned}U^0(c_1, c_2, c_3) &= 2(c_1)^{1/2} + \delta 2(c_2)^{1/2} + \delta^2 2(c_3)^{1/2} \\&= 2(c_1)^{1/2} + (.9) 2(c_2)^{1/2} + (.9)^2 2(c_3)^{1/2}\end{aligned}$$

subject to

$$c_1 + \frac{c_2}{1.1} + \frac{c_3}{(1.1)^2} \leq \$1000.$$

Solution:

$$c_1^{**} = \$372.46 \quad c_2^{**} = \$365.05 \quad c_3^{**} = \$357.78$$

Note: This represents the person's ideal behavior when asked from a removed perspective — what she would follow if she were to commit prior to period 1.

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Propensity to Over-Consume

Recall: Long-run desired behavior is

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Actual Behavior for Naifs

Suppose you are naive.

In period 1, you start following your period-1 desired behavior, and so

$$c_1^N = c_1^* = \$481.16.$$

[Note: You plan $c_2 = c_2^* = \$301.81$ and $c_3 = c_3^* = \$295.81$.]

In period 2, you reassess:

- ▶ Given you've consumed \$481.16, period-2 wealth is

$$\begin{aligned} W_2^N &\equiv (W - c_1^N)(1 + r) \\ &= (\$1000 - \$481.16)(1.10) \\ &= \$570.72. \end{aligned}$$

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Conditional on having period-2 wealth $W_2^N = \$570.72$, you choose (c_2, c_3) to maximize

$$\begin{aligned}U^2(c_2, c_3) &= 2(c_2)^{1/2} + \beta\delta 2(c_3)^{1/2} \\ &= 2(c_2)^{1/2} + (.8)(.9) 2(c_3)^{1/2}\end{aligned}$$

subject to

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Solution:

$$c_2^N = \$363.46 \quad c_3^N = \$227.99$$

Hence, actual behavior for naifs is:

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Actual Behavior for Sophisticates

Suppose you are sophisticated.

Use backward induction:

Consider how you would behave in period 2 as a function of your chosen period-1 consumption:

- ▶ If you consumed c_1 in period 1, your period-2 wealth would be

$$\begin{aligned}W_2 &= (W - c_1)(1 + r) \\&= (\$1000 - c_1)(1.10).\end{aligned}$$

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$$c_2 + \frac{c_3}{1.1} \leq (\$1000 - c_1)(1.10).$$

Actual Behavior for Sophisticates

Solution for period-2 behavior as a function of c_1 :

$$c_2(c_1) = 0.70053 * (\$1000 - c_1)$$

$$c_3(c_1) = 0.43942 * (\$1000 - c_1)$$

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Knowing this, in period 1 you choose c_1 to maximize

$$\begin{aligned}
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Actual Behavior for Sophisticates

Solution for period-1 consumption:

$$c_1^S = \$484.17.$$

After choosing c_1^S , in period 2 you actually choose:

$$c_2^S = c_2(c_1^S) = 0.70053(\$1000 - c_1^S) = \$361.35$$

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Summary of the Basic Example

Long-run desired behavior is

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$c_1^* > c_1^{**}$ reflects that the present bias creates a propensity to over-consume (or under-save).

$c_2^N > c_2^*$ and $c_2^S > c_2^*$ reflects that the time inconsistency exacerbates the problem.

$c_1^S > c_1^N$ reflects that, in this example, sophistication exacerbates over-consumption in period 1. But unlike the above results, this result is not general — sophistication can exacerbate or mitigate period-1 over-consumption depending on the specific instantaneous utility function.

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Extended Example with an Illiquid Asset

Let's introduce an illiquid asset into our example:

- ▶ “Examples”: a CD account, a house, a retirement account.
- ▶ If in period 1 you invest z in this asset, then in period 3 you receive $z(1 + \hat{r})^2$ (but no access in period 2).

Let's initially suppose $\hat{r} = r$, and suppose further that $Y_1 = \$1000$ and $Y_2 = Y_3 = \$0$, and that you cannot borrow.

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Result: Sophisticates can now implement their period-1 desired behavior.

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With the illiquid asset, in period 1:

- ▶ consume \$481.16.
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BUT note that naifs are indifferent between using vs. not using the illiquid asset, because they (incorrectly) think that, even if they put their entire savings of \$518.84 in the bank, they would still consume \$301.81 in period 2 and \$295.81 in period 3.

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Illiquid Assets: Conclusions

- ▶ For people with self-control problems, tying up wealth in illiquid assets can be a useful commitment device to help counteract future over-consumption.
- ▶ Sophisticates are always on the lookout for such commitment opportunities, whereas naifs do not recognize the commitment value in illiquid assets.

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Illiquid Assets: A Few Comments

- ▶ The illiquid asset is not a perfect commitment technology, because you cannot prevent yourself from consuming current income. For instance, if $Y_1 = \$500$ and $Y_2 = \$550$, the illiquid asset would not help at all.
- ▶ An illiquid asset will not work as a commitment device if you can borrow against its future payoff. Hence, liquidity-enhancing instruments such as credit cards may in fact undermine the commitment value of illiquid assets.
- ▶ In the real world, illiquid assets usually have a larger return than liquid assets ($\hat{r} > r$).

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A Related Aside: Retirement Plans

Two features of retirement plans (IRA plans, 401(k) plans, etc):

- ▶ They are tax-exempt.
- ▶ They are illiquid (big penalty for early withdrawal).

Why have retirement plans?

- ▶ Goal: induce people to save for retirement.

To the extent that retirement plans are aimed at counteracting self-control problems:

- ▶ If people are sophisticated, the illiquidity feature of retirement plans is all that's needed to induce more retirement saving.
- ▶ If, in contrast, people are naive, then both features are crucial: the tax-exempt feature induces people to use retirement plans rather than some other form of saving, and then the illiquidity feature generates unexpected commitment benefits that “multiply” the effect.

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Angeletos *et al* (JEP 2001)

- ▶ They conduct a quantitative test of present bias (in the consumption-saving environment).

Basic idea:

- ▶ We observe people take on large credit-card debts at high interest rates, but also accumulate significant pre-retirement wealth.
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Applying These Ideas to Real Data

Angeletos *et al* (JEP 2001)

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They consider a rich, calibrated model:

- ▶ A “period” is one year.
- ▶ Households begin life at 20, retire at 63, and die at 90 (if not sooner).
- ▶ Demographics calibrated to the “U.S. population”.
- ▶ Labor income calibrated to the “U.S. population”.
- ▶ There is a liquid asset with real interest rate 3.75%.
- ▶ There is credit-card borrowing with real interest rate 11.75% (with a credit limit).
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Exponential simulation:

- ▶ Assume that the entire economy is populated by exponential discounters with discount factor δ_{exp} .
- ▶ Choose δ_{exp} so that the simulations generate a median wealth-to-income ratio for households aged 50-59 of 3.2.

Present bias simulation (sophisticates):

- ▶ Assume that the entire economy is populated by people with present bias with $\beta = .7$ and $\delta = \delta_{PB}$.
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Chosen discount factors:

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Angeletos et al: Performance of Simulated Models

Finally, they compare their simulated data to real-world data:

- ▶ households with liquid assets $>$ one-month's income:

Exponential simulation:	73%
Present Bias simulation:	40%
Data:	43%

- ▶ households with positive credit-card borrowing:

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Angeletos et al: Performance of Simulated Models

- mean credit-card borrowing (all households):

Exponential simulation:	\$900
Present Bias simulation:	\$3400
Data:	> \$5000

- consumption-income comovement:

Exponential simulation:	0.032
Present Bias simulation:	0.166
Data:	≈ 0.2