# Choice under Uncertainty (Lecture 1e)

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## Our Outline:

In the next (two? one and a half?) lectures we will "discover" some problems with vanilla Prospect Theory.

We'll also introduce a bunch of "new" value functions.

. . . except the value functions aren't new

. . . and the discoveries were lurking in the back of your mind this whole time.

And we'll do a million exercises. This material is tricky.

We'll follow the KR theory of reference-dependent utility with loss aversion. (Except it's really just the **right** way to do Prospect Theory.)

Their innovations address two major issues ("loopholes"):

1. What determines the reference point?
2. When do people experience loss aversion, and what is the magnitude of this experience?

They address these issues by incorporating two novel features:

A person's reference point is her recent beliefs or expectations about outcomes.

Gain-loss utility is directly tied to the intrinsic utility from consumption --- so that a person experiences more gain-loss utility for goods that involve more consumption utility.

## Model

Suppose there are $\mathbf{2}$ goods:

- Person chooses a vector $(x_A, x_B)$.
- Reference point is a vector $(r_A, r_B)$.

**Preferences:**

$$\begin{aligned} \text{Total Utility} \;\equiv\; & \left[\, m_A(x_A) \;+\; n_A(x_A|r_A) \,\right] \\ & + \left[\, m_B(x_B) \;+\; n_B(x_B|r_B) \,\right] \end{aligned}$$

- $m_A(x_A)$ is intrinsic utility for good $A$, and $m_B(x_B)$ is intrinsic utility for good $B$.

- $n_A(x_A|r_A)$ is gain-loss utility for good $A$, and $n_B(x_B|r_B)$ is gain-loss utility for good $B$.

How to formalize that gain-loss utility is directly tied to intrinsic utility:

Assume there exists a *universal gain-loss function* $\mu(z)$ such that the gain-loss utilities are:

$$v_A(x_A|r_A) = \mu\,(\,m_A(x_A) - m_A(r_A)\,)$$

$$v_B(x_B|r_B) = \mu\,(\,m_B(x_B) - m_B(r_B)\,)$$

In general, $\mu(z)$ takes form of the Kahneman-Tversky value function. But we'll focus on the "easy" case:

$$\mu(z) = \{ \quad \eta * z \quad \text{if} \quad z \geq 0$$
$$\eta * \lambda * z \quad \text{if} \quad z \leq 0$$

Example: Two goods, shoes ( $c$ ) and money ( $w$ ), with intrinsic utilities:

$$m(c) = \theta * c$$

$$m(w) = w$$

As with mugs, we can represent shoe utility in a 2x2 grid (see board)

Consider the following choice problem:

- Suppose Bogi starts with 0 shoes and wealth $w$, and has the option to purchase a shoe for price $p$. How does Bogi behave as a function of expectations?

**Case 1:** Suppose you expect to buy a pair of shoes $\Longrightarrow$ reference point is $(r_c = 1, r_m = w - p)$:

$$\text{Utility}(\text{Buy}) = [\theta + \eta 0] + [(w - p) + \eta 0]$$

$$\text{Utility}(\text{Not}) = [0 - \eta\lambda\theta] + [w + \eta p]$$

$$\text{Buy when Utility}(\text{Buy}) \geq \ \text{Utility}(\text{Not}) \Longleftrightarrow p \leq \frac{1 + \eta\lambda}{1 + \eta}\theta$$

Consider the following choice problem:

- Suppose Bogi starts with 0 shoes and wealth $w$, and has the option to purchase a shoe for price $p$. How does Bogi behave as a function of expectations?

**Case 2:** Suppose you expect not to buy any shoes $\Longrightarrow$ reference point is $(r_c = 0, r_m = w)$:

$$\text{Utility}(\text{Buy}) = [\theta + \eta\theta] + [(w - p) - \eta\lambda p]$$

$$\text{Utility}(\text{Not}) = [0 + \eta 0] + [w + \eta 0]$$

$$\text{Buy when Utility}(\text{Buy}) \geq \text{ Utility}(\text{Not}) \Longleftrightarrow p \leq \frac{1+\eta}{1+\eta\lambda}\theta$$

.

Because $\lambda > 1$ implies $\frac{1+\eta\lambda}{1+\eta} > \frac{1+\eta}{1+\eta\lambda}$, there are three cases:

1. If $p > \frac{1+\eta\lambda}{1+\eta}\theta$, don't buy no matter your beliefs.
2. If $p < \frac{1+\eta}{1+\eta\lambda}\theta$, buy no matter your beliefs.
3. If $\frac{1+\eta}{1+\eta\lambda}\theta < p < \frac{1+\eta\lambda}{1+\eta}\theta$, buy if you expect to buy, and don't buy if you expect not to buy.

**Point:** If the reference point depends on expectations, then, even in the same situation, a person might exhibit different outcomes depending on which set of self-fulfilling expectations he happens to have.

# An Exercise

Suppose there are two goods, candy bars ($c$) and money ($m$). Paige has initial income $I$, and she is deciding whether to buy 0, 1, or 2 candy bars at a price of $p$ per candy bar. Paige's total utility is the sum of her candy-bar utility and her money utility, and her intrinsic utilities for the two goods are:

$$w_c(c) \equiv \begin{cases} 0 & \text{if} \quad c = 0 \\ \theta_1 & \text{if} \quad c = 1 \\ \theta_1 + \theta_2 & \text{if} \quad c = 2 \end{cases}$$

Where $\theta_1 > \theta_2$ and $w_m(m) \equiv m$ .

# An Exercise

**(a)** If Paige were a standard agent who only cares about her intrinsic utilities, how would she behave as a function of the price $p$? In other words, for what prices would she buy zero candy bars, for what prices would she buy one candy bar, and for what prices would she buy two candy bars?

**(b)** Now suppose that Paige behaves according to the Koszegi-Rabin model. In other words, in addition to intrinsic utilities, she also cares about gain-loss utility, where the gain-loss utility for each good is derived from the universal gain-loss function described above.

If Paige expects to buy no candy bars, how would she behave as a function of the price $p$? In other words, for what prices would she buy zero candy bars, for what prices would she buy one candy bar, and for what prices would she buy two candy bars?

In a second paper, Köszegi and Rabin investigate the implications of their approach for basic risk preferences.

Assume one good, money ($x$), with intrinsic utility $w(x) = x$.

- Note: $w(x) = x$ implies there is no intrinsic risk aversion --- all risk aversion will derive from gain-loss utility!

Applying their approach, if consume money $x$ given reference point $r$, then total utility is

$$u(x|r) = \{x + \eta * (x - r) \quad \text{if} \quad x > r$$

$$x + \eta * \lambda * (x - r) \quad \text{if} \quad x \leq r$$

How to incorporate uncertainty:

- If consume lottery $X \equiv (x_1, p_1; \ldots; x_N, p_N)$ given reference point $r$, then "expected" total utility is

$$U(X|r) \;=\; \sum_{i=1}^{N} p_i \, u(x_i|r).$$

**Example:** If $X = (200, \frac{1}{4}; 0, \frac{3}{4})$ and $r = 100$, then

$$U(X|r) = \frac{1}{4}u(200|100) + \frac{3}{4}u(0|100).$$

But might expect a lottery, in which case **the reference point would be a lottery**.

If consume money $x$ given reference point $R \equiv (r_1, q_1; \ldots; r_M, q_M)$, then "expected" total utility is

$$U(x|R) \ = \ \sum_{j=1}^{M} q_j \, u(x|r_j).$$

- If consume lottery $X \equiv (x_1, p_1; \ldots; x_N, p_N)$ given reference point $R \equiv (r_1, q_1; \ldots; r_M, q_M)$, then total utility is

$$\begin{aligned} U(X|R) &= \sum_{i=1}^{N} p_i \, U(x_i|R) \\ &= \sum_{j=1}^{M} q_j \, U(X|r_j) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{M} p_i \, q_j \, u(x_i|r_j). \end{aligned}$$

**Example:** If $X = (200, \frac{1}{4}; 0, \frac{3}{4})$ and $R = (150, \frac{1}{3}; 50, \frac{2}{3})$, then

$$U(X|R) = \frac{1}{4}\left[\frac{1}{3}u(200|150) + \frac{2}{3}u(200|50)\right] + \frac{3}{4}\left[\frac{1}{3}u(0|150) + \frac{2}{3}u(0|50)\right]$$

or

$$U(X|R) = \frac{1}{3}\left[\frac{1}{4}u(200|150) + \frac{3}{4}u(0|150)\right] + \frac{2}{3}\left[\frac{1}{4}u(200|50) + \frac{3}{4}u(0|50)\right]$$

or

$$U(X|R) = \frac{1}{12}u(200|150) + \frac{1}{6}u(200|50) + \frac{1}{4}u(0|150) + \frac{1}{2}u(0|50)$$

.

**Point 1**: Risk aversion when no possible "losses".

Consider choice

$$A \equiv (y, 1) \text{ with } y \leq 100 \qquad \text{vs.} \qquad B \equiv \left( 200, \frac{1}{2} \; ; \; 0, \frac{1}{2} \right)$$

**Case 1:** Suppose expect $A \implies$ reference point is $r = y$:

$$U(A|r) = y + \eta * (0) = y$$

$$U(B|r) = 100 + \left[ \frac{1}{2}\eta(200 - y) + \frac{1}{2}\eta\lambda(0 - y) \right]$$

This implies that you choose $A$ if $y \geq \frac{1+\eta}{1+\frac{1}{2}\eta+\frac{1}{2}\eta\lambda} 100 \equiv \bar{y}_1$

- **Note:** $\lambda > 1$ implies $\bar{y}_1 < 100$ --- risk averse!

**Case 2:** Suppose expect lottery $B \Longrightarrow$; reference point is $R = (200, \frac{1}{2}; 0, \frac{1}{2}$

$$U(A|R) = y + \left[\frac{1}{2}\eta(y-0) + \frac{1}{2}\eta\lambda(y-200)\right]$$

$$U(B|r) = 100 + \frac{1}{2}\left[\frac{1}{2}\eta(200-0) + \frac{1}{2}\eta(200-200)\right] +$$

$$\frac{1}{2}\left[\frac{1}{2}\eta(0-0) + \frac{1}{2}\eta\lambda(0-200)\right]$$

**Result:** Choose $A$ if $y \geq 100 \equiv \bar{y}_2$.

- Note: $\bar{y}_2 > \bar{y}_1$; that is, *expecting risk* makes you less risk averse!
- *Intuition:* When expecting risk, even certain outcomes involve gains and losses, and thus they lose part of their advantage relative to risky outcomes.

**Point 2:** Above feature helps explain demand for insurance at actuarially unfair prices.

Suppose you have wealth \$1000, but there is a 10% chance that you will suffer a loss of \$250.

Full insurance is available at price $\pi > 25$ .

- If insure, face lottery $(1000 - \pi, 1) \equiv A$.
- If don't, face lottery $(1000, .9; 750, .1) \equiv B$.

**Note:** If reference point is $r = 1000$, don't insure! (Prove this.)

Could it be that you expect to be insured, and still prefer to be insured?

In other words, given reference point $r = 1000 - \pi$, do you prefer lottery $A \equiv (1000 - \pi, 1)$ over $B \equiv (1000, .9; 750, .1)$?

In other words, given reference point $r = 1000 - \pi$, do you prefer lottery $A \equiv (1000 - \pi, 1)$ over $B \equiv (1000, .9; 750, .1)$?

$$U(A|r) = [1000 - \pi] + [0]$$

$$U(B|r) = 975 + [.9\eta(\pi) + .1\eta\lambda(\pi - 250)]$$

- **Result:** Insure if $\pi \leq \frac{1+\eta\lambda}{1+\eta\lambda-.9\eta(\lambda-1)}25 \equiv \bar{\pi}$
- Note: $\lambda > 1$ implies $\bar{\pi} > 25$ --- indeed willing to insure at actuarially unfair prices.
- *Intuition:* Because expect to pay premium, it's not felt as a loss.

## Camerer, Babcock, Loewenstein, & Thaler (1997)

For many jobs, people choose how to allocate their labor from day-to-day, or from week-to-week, or from month-to-month.

**Benchmark:** The standard life-cycle model of labor supply says that, if your wage varies over time, you should work more when the wage is high than you do when the wage is low.

- Simple intuition: efficiently allocate your work effort.
- Authors test this prediction on NYC cab drivers.

**First finding:** Their data permits them to calculate an average hourly wage for cab drivers, and they conclude that wages are highly correlated within a day, but not correlated across days.

Hence, they take their unit of observation to be a day --- in particular, they estimate a daily wage equation:

$$\ln H_t = \gamma \ln W_t + \beta X_t + \varepsilon_t$$

- $H_t \equiv$ hours worked on day $t$
- $W_t \equiv$ average wage on day $t$

Standard model predicts $\gamma > 0$, but they find $\gamma < 0$.

In words, the standard model predicts positive wage elasticities, but they find **negative** wage elasticities.

Their explanation is income targeting driven by loss aversion:

- Drivers have one-day time horizon for decision making.
- Their reference point is a daily income target.
- They feel losses relative to the target loom larger than gains.

## Farber (2005)

Provides several critiques of Camerer et al (1997):

- There is a "division bias": wages are calculated as earnings divided by hours, but hours are endogenous.
- $\Rightarrow$ Negative bias in wage elasticity estimates.
- After cutting the data in a different way, Farber finds that it is not so clear there is more inter-day variation in the wage than intra-day variation in the wage.

**Main point:** There is a better approach that gets around these problems: instead of estimating usual wage regressions, estimate a probit optimal-stopping model.

> [And now for a little econometrics.]

## Probit optimal-stopping model:

- Stop when $R(\tau) \geq 0$, where $R(\tau) = \gamma_1 h_\tau + \gamma_2 y_\tau + \beta X_\tau + \varepsilon_\tau$.
- $h_\tau \equiv$ hours worked today after trip $\tau$ .
- $y_\tau \equiv$ earnings today after trip $\tau$ .

**Note:** Standard model predicts $\gamma_1 > 0$ and $\gamma_2 = 0$.

Farber indeed finds evidence consistent with $\gamma_1 > 0$ and $\gamma_2 = 0$, as in the standard model.

- BUT it's not clear whether this result is inconsistent with income targeting, since income targeting does not imply $\gamma_1 = 0$.

## Crawford & Meng (AER 2011)

They apply the Köszegi-Rabin perspective to this debate:

- There should be gain-loss utility over each dimension of consumption. Here, this means over income (as usual) but also over hours worked.
- Take the reference point to be people's expectations about outcomes; in particular, take them to be people's average experienced outcomes.

$H_t \equiv$ hours worked on day $t$ $Y_t \equiv$ income on day $t$ $W_t \equiv Y_t/H_t$

$H^e \equiv$ average of $H_t \setminus Y^e \equiv$ average of $Y_t \setminus W^e \equiv Y^e/H^e$

Their Hypothesis: Reference point is $(H^e, Y^e)$.

- Working fewer than $H^e$ hours generates gain utility, and working more than $H^e$ hours generates loss utility.
- Earning more than income $Y^e$ generates gain utility, and earning less than income $Y^e$ generates loss utility.

Key Idea:

- On high-wage days ( $W_t > W^e$ ), hit $Y^e$ first and $H^e$ second.
- On low-wage days ( $W_t < W^e$ ), hit $H^e$ first and $Y^e$ second.

This suggests splitting the sample into high-wage days vs. low-wage days, because this model predicts that we should see different patterns of behavior.

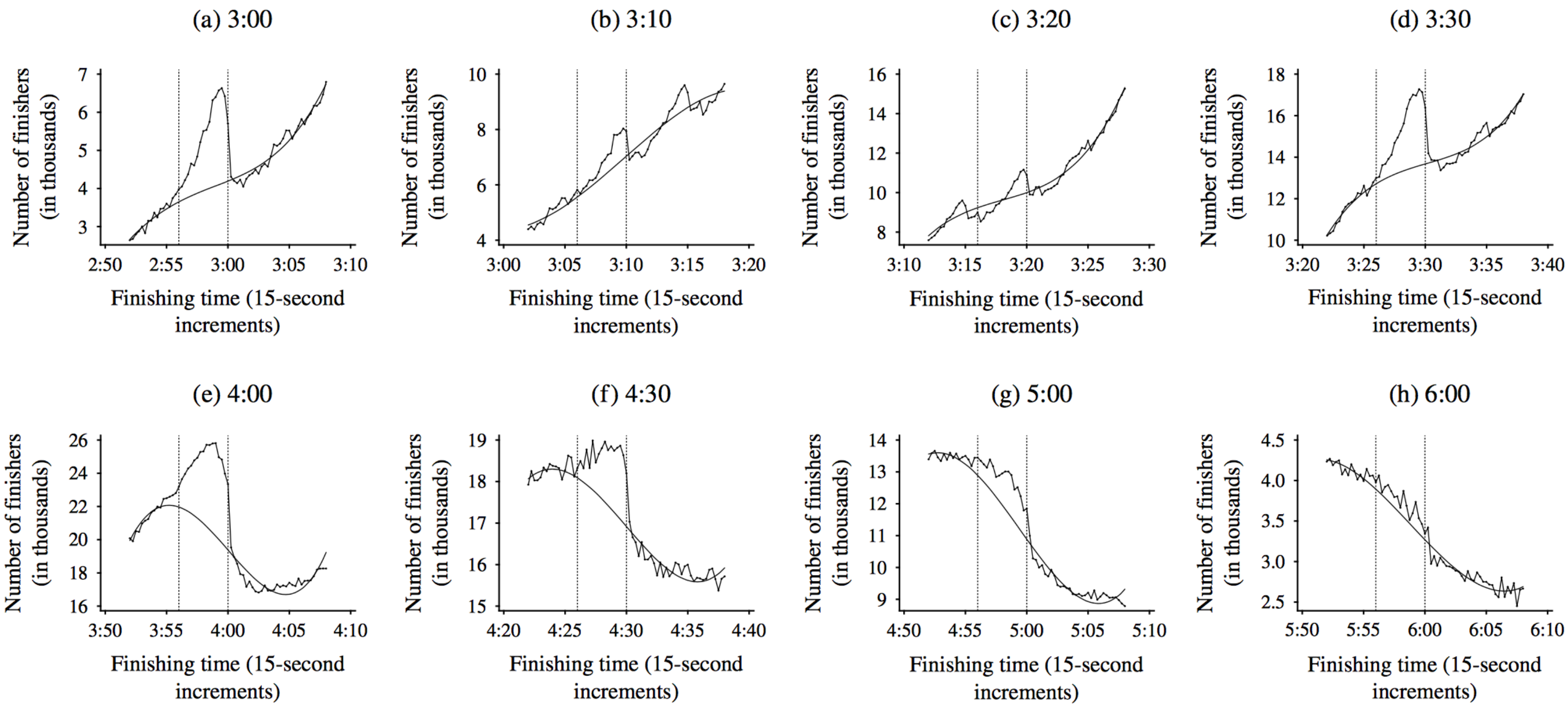
**And** when they do so, they find evidence consistent with their model and strongly reject Farber's analysis.

Moreover, they show that targets in hours loom larger than targets in wages, which is consistent with the theory.

Crawford and Meng (2011) and its predecessors have been influential because of the domain: labor supply. However, this domain can make the analysis more complex than it needs to be.

An alternative approach (innovated by Saez 2010; and Chetty et. al 2011): search for excess "bunching".

- Observed distribution of data exceeds a modeled counterfactual distribution or a normative distribution.
- Chetty et al. (2011) application: taxes and kinks in the tax schedule.
- Allen et al. (2017) looks for this in marathon runners.


(a) 3:00
(b) 3:10
(c) 3:20
(d) 3:30
(e) 4:00
(f) 4:30
(g) 5:00
(h) 6:00
Number of finishers (in thousands)
Finishing time (15-second increments)

## Are people worse off for having made loss-averse decisions?

- Samuelson showed us that they are worse of mathematically.
- Ultimately, answer to this question depends on modeler's beliefs about whether loss aversion is something that people really *feel*, or merely an artifact of some choice bias or mistake.

Two camps: (1) Loss aversion is an affective forecasting error; (2) Loss aversion is a real manifestation of preferences.

Surprisingly: Kahneman waffles between two; see e.g. Schkade and Kahneman (1998).

**Me:** (2.5) Loss aversion is a little bit an affective forecasting error and a little bit "real".

# Epilogue



What have we learned in $\approx 500$ years of studying risk preferences?

**Expected values** matter, but don't wholly determine choice.

- . . . except they probably should. [Begin rant.]
- . . . most of the time. [End rant.]

**Diminishing marginal utility** definitely does not explain most choices over risk.

- . . . and I'm suspicious of **all** evidence on diminishing marginal utility of wealth.
- Evidence conflated with reference-point effects (e.g. hedonic treadmill).

# Epilogue



**Prospect Theory** matters, but you need to apply it correctly.

- Misleading conclusions when you fail to account for beliefs.
- . . . but when you apply the "correct model", your intuitions are preserved.

We've also (sneakily) introduced a new category of model: *belief-based utility*.

We will return to some other models in this space.