Stochastic Formulation

Minimizing Average Total Distance

Decision Variables:

First-stage:

$\mathbf{y}_{47x1}$: Binary, wether or not to open a point of dispense at each site

A_{Zx47} : Binary, whether or not to assign zip-code i to site j

Second stage:

$S_{47x10,000}$: Binary, wether or not to assign less than 10 medical professionals to a site

$M_{47x10,000}$: Binary, wether or not to assign between 10 and 20 medical professionals to a site

$L_{47x10,000}$: Binary, wether or not to assign more than 20 medical professionals to a site

$X_{47x100x10,000}$: Integer, the number of medical professionals for each point of dispense for every time period for every scenario

$I_{47x100x10,000}$: Integer, the amount of inventory to hold every day out of 100 days for each of 47 sites in 10,000 scenarios

Parameters:

T_{Zx47} : The time it takes to travel from zipcode i to site j

> Z is the number of zip codes

$\mathbf{p}_{Zx1}$: population of each zipcode

o : First stage fixed cost (opening cost)

v : Second stage variable cost

f : Second stage fixed cost

r : medical professional wage

h : Per unit daily holding cost

K : A constant serving as the budget constraint

$B_{Zx10,000}$: Binary, whether the population for a zipcode has an outbreak for each of the 10,000 scenarios

C : The number of available medical professionals

Helping Functions

Rate $= (25S_{jtk} + 50M_{jtk} + 100L_{jtk})X_{jtk}$

Total people served $= \sum_i B_{ik}A_{ij}p_i$

Average Total Distance $= \sum_{k=1}^{10,000}\sum_{i=1}^{Z}\sum_{j=1}^{47}\frac{1}{10,000}p_iB_{ik}T_{ij}A_{ij}$

> This equation sums the time each person has to travel from their zip code to their assigned point of dispense for each of 10,000 (equally likely) scenarios

Average Total Cost
$= \sum_{j=1}^{47} oy_j + \sum_{k=1}^{10,000}\sum_{j=1}^{47}\frac{1}{10,000}f(S_{jk} + M_{jk} + L_{jk}) + \sum_{k=1}^{10,000}\sum_{j=1}^{47}\sum_{i=1}^{Z}\frac{1}{10,000}vB_{ik}A_{ij}p_i + \sum_{k=1}^{10,000}\sum_{j=1}^{47}$
$\sum_{t=1}^{100}\frac{1}{10,000}(rX_{jtk} + hI_{jtk})$

> This adds the opening cost for each site with the cost of the medical professionals hired

Formulation

Minimize *Average Total Distance*

subject to

Average Total Cost $\leq K$

$\sum_{j=1}^{47} A_{ij} = 1, \forall i \in \{1, 2, \ldots, Z\}$

$A_{ij} \leq y_j, \forall i \in \{1, 2, \ldots, Z\}, \forall j \in \{1, 2, \ldots, 47\}$

$S_{jk} + M_{jk} + L_{jk} \leq 1$

$10M_{jk} + 20L_{jk} \leq X_{jtk} \leq 10S_{jk} + 20M_{jk} + 1,000L_{jk}$

$\sum_{t=1}^{100} 25X_{jtk} \geq \sum_{i=1}^{Z} B_{ik}A_{ij}p_i - (1 - S_{jk})1,000,000$

$\sum_{t=1}^{100} 50X_{jtk} \geq \sum_{i=1}^{Z} B_{ik}A_{ij}p_i - (1 - M_{jk})1,000,000$

$\sum_{t=1}^{100} 100X_{jtk} \geq \sum_{i=1}^{Z} B_{ik}A_{ij}p_i - (1 - L_{jk})1,000,000$

$\sum_{j=1}^{47} X_{jtk} \leq C$

$I_{jtk} + (1 - S_{jk})1,000,000 \geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100} 25X_{jtk}$

$I_{jtk} + (1 - M_{jk})1,000,000 \geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100} 50X_{jtk}$

$I_{jtk} + (1 - L_{jk})1,000,000 \geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100} 100X_{jtk}$

$X_{jtk} \leq \sum_i B_{ik}A_{ij}p_i - I_{jtk}$

$X_{jtk}, I_{jtk} \geq 0$, integer

$S_{jk}, M_{jk}, L_{jk}, A_{ij} \geq 0$, binary

Minimizing Average Max Distance

Decision Variables:

First-stage:

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Second stage:

$S_{47x10,000}$: Binary, wether or not to assign less than 10 medical professionals to a site

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Parameters:

T_{Zx47} : The time it takes to travel from zipcode i to site j

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$\mathbf{p}_{Zx1}$: population of each zipcode

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v : Second stage variable cost

f : Second stage fixed cost

r : medical professional wage

h : Per unit daily holding cost

K : A constant serving as the budget constraint

$B_{Zx10,000}$: Binary, whether the population for a zipcode has an outbreak for each of the 10,000 scenarios

C : The number of available medical professionals

Helping Functions

$\mathbf{t}_{10,000x1}$: Max travel time

Average Total Cost
$= \sum_{j=1}^{47} oy_j + \sum_{k=1}^{10,000}\sum_{j=1}^{47}\frac{1}{10,000}f(S_{jk} + M_{jk} + L_{jk}) + \sum_{k=1}^{10,000}\sum_{j=1}^{47}\sum_{i=1}^{Z}\frac{1}{10,000}vB_{ik}A_{ij}p_i + \sum_{k=1}^{10,000}\sum_{j=1}^{47}$
$\sum_{t=1}^{100}\frac{1}{10,000}(rX_{jtk} + hI_{jtk})$

Formulation

Minimize $\sum_{k=1}^{10,000}\frac{1}{10,000}t_k$

subject to

$t \geq B_{ik}T_{ij}A_{ij}$

Average Total Cost $\leq K$

$\sum_{j=1}^{47} A_{ij} = 1, \forall i \in \{1, 2, \ldots, Z\}$

$A_{ij} \leq y_j, \forall i \in \{1, 2, \ldots, Z\}, \forall j \in \{1, 2, \ldots, 47\}$

$S_{jk} + M_{jk} + L_{jk} \leq 1$

$10M_{jk} + 20L_{jk} \leq X_{jtk} \leq 10S_{jk} + 20M_{jk} + 1,000L_{jk}$

$\sum_{t=1}^{100} 25X_{jtk} \geq \sum_{i=1}^{Z} B_{ik}A_{ij}p_i - (1 - S_{jk})1,000,000$

$\sum_{t=1}^{100} 50X_{jtk} \geq \sum_{i=1}^{Z} B_{ik}A_{ij}p_i - (1 - M_{jk})1,000,000$

$\sum_{t=1}^{100} 100X_{jtk} \geq \sum_{i=1}^{Z} B_{ik}A_{ij}p_i - (1 - L_{jk})1,000,000$

$\sum_{j=1}^{47} X_{jtk} \leq C$

$I_{jtk} + (1 - S_{jk})1,000,000 \geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100} 25X_{jtk}$

$I_{jtk} + (1 - M_{jk})1,000,000 \geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100} 50X_{jtk}$

$I_{jtk} + (1 - L_{jk})1,000,000 \geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100} 100X_{jtk}$

$X_{jtk} \leq \sum_i B_{ik}A_{ij}p_i - I_{jtk}$

$X_{jtk}, I_{jtk} \geq 0$, integer

$S_{jk}, M_{jk}, L_{jk}, A_{ij} \geq 0$, binary