Stochastic Formulation

Minimizing Average Total Distance

Decision Variables:

First-stage:

$\textbf{y}_{47x1}:$ Binary, wether or not to open a point of dispense at each site

$A_{Zx47}:$ Binary, whether or not to assign zip-code i to site j

Second stage:

$S_{47x10,000}:$ Binary, wether or not to assign less than 10 medical professionals to a site

$M_{47x10,000}:$ Binary, wether or not to assign between 10 and 20 medical professionals to a site

$L_{47x10,000}:$ Binary, wether or not to assign more than 20 medical professionals to a site

$X_{47x100x10,000}:$ Integer, the number of medical professionals for each point of dispense for every time period for every scenario

$I_{47x100x10,000}:$ Integer, the amount of inventory to hold every day out of 100 days for each of 47 sites in 10,000 scenarios

Parameters:

$T_{Zx47}:$ The time it takes to travel from zipcode i to site j

> Z is the number of zip codes

$\textbf{p}_{Zx1}:$ population of each zipcode

$o:$ First stage fixed cost (opening cost)

$v:$ Second stage variable cost

$f:$ Second stage fixed cost

$r:$ medical professional wage

$h:$ Per unit daily holding cost

$K:$ A constant serving as the budget constraint

$B_{Zx10,000}:$ Binary, whether the population for a zipcode has an outbreak for each of the 10,000 scenarios

$C:$ The number of available medical professionals

Helping Functions

Rate $= (25S_{jtk}+50M_{jtk}+100L_{jtk})X_{jtk}$

Total people served $= \sum_i B_{ik}A_{ij}p_i$

Average Total Distance $= \sum_{k=1}^{10,000}\sum_{i=1}^Z \sum_{j=1}^{47} \frac{1}{10,000} p_i B_{ik} T_{ij}A_{ij}$

> This equation sums the time each person has to travel from their zip code to their assigned point of dispense for each of 10,000 (equally likely) scenarios

Average Total Cost $=\sum_{j=1}^{47} o y_j + \sum_{k=1}^{10,000}\sum_{j=1}^{47} \frac{1}{10,000} f(S_{jk} + M_{jk} + L_{jk})+ \sum_{k=1}^{10,000}\sum_{j=1}^{47} \sum_{i=1}^Z \frac{1}{10,000} vB_{ik}A_{ij}p_i + \sum_{k=1}^{10,000}\sum_{j=1}^{47} \sum_{t=1}^{100} \frac{1}{10,000}(r X_{jtk} + h I_{jtk})$

> This adds the opening cost for each site with the cost of the medical professionals hired

Formulation

Minimize *Average Total Distance*

subject to

Average Total Cost $\leq K$

$\sum_{j=1}^{47} A_{ij} = 1$, $\forall i \in \{1, 2, ..., Z\}$

$A_{ij} \leq y_j$, $\forall i \in \{1, 2, ..., Z\}, \forall j \in \{1, 2, ..., 47\}$

$S_{jk} + M_{jk} + L_{jk} \leq 1$

$10 M_{jk} + 20 L_{jk} \leq X_{jtk} \leq 10 S_{jk} + 20 M_{jk} + 1,000 L_{jk}$

$\sum_{t=1}^{100}25 X_{jtk} \geq \sum_{i=1}^Z B_{ik}A_{ij} p_i - (1-S_{jk})1,000,000$

$\sum_{t=1}^{100}50 X_{jtk} \geq \sum_{i=1}^Z B_{ik}A_{ij} p_i - (1-M_{jk})1,000,000$

$\sum_{t=1}^{100}100 X_{jtk} \geq \sum_{i=1}^Z B_{ik}A_{ij} p_i - (1-L_{jk})1,000,000$

$\sum_{j=1}^{47} X_{jtk} \leq C$

$I_{jtk} + (1-S_{jk})1,000,000\geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100}25X_{jtk}$

$I_{jtk} + (1-M_{jk})1,000,000\geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100}50X_{jtk}$

$I_{jtk} + (1-L_{jk})1,000,000\geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100}100X_{jtk}$

$X_{jtk} \leq \sum_i B_{ik}A_{ij}p_i - I_{jtk}$

$X_{jtk}, I_{jtk} \geq 0$, integer

$S_{jk}, M_{jk}, L_{jk}, A_{ij} \geq 0$, binary

Minimizing Average Max Distance

Decision Variables:

First-stage:

$\textbf{y}_{47x1}:$ Binary, wether or not to open a point of dispense at each site

$A_{Zx47}:$ Binary, whether or not to assign zip-code i to site j

Second stage:

$S_{47x10,000}:$ Binary, wether or not to assign less than 10 medical professionals to a site

$M_{47x10,000}:$ Binary, wether or not to assign between 10 and 20 medical professionals to a site

$L_{47x10,000}:$ Binary, wether or not to assign more than 20 medical professionals to a site

$X_{47x100x10,000}:$ Integer, the number of medical professionals for each point of dispense for every time period for every scenario

$I_{47x100x10,000}:$ Integer, the amount of inventory to hold every day out of 100 days for each of 47 sites in 10,000 scenarios

Parameters:

$T_{Zx47}:$ The time it takes to travel from zipcode i to site j

> Z is the number of zip codes

$\textbf{p}_{Zx1}:$ population of each zipcode

$o:$ First stage fixed cost (opening cost)

$v:$ Second stage variable cost

$f:$ Second stage fixed cost

$r:$ medical professional wage

$h:$ Per unit daily holding cost

$K:$ A constant serving as the budget constraint

$B_{Zx10,000}:$ Binary, whether the population for a zipcode has an outbreak for each of the 10,000 scenarios

$C:$ The number of available medical professionals

Helping Functions

$\textbf{t}_{10,000x1}:$ Max travel time

Average Total Cost $=\sum_{j=1}^{47} o y_j + \sum_{k=1}^{10,000}\sum_{j=1}^{47} \frac{1}{10,000} f(S_{jk} + M_{jk} + L_{jk})+ \sum_{k=1}^{10,000}\sum_{j=1}^{47} \sum_{i=1}^Z \frac{1}{10,000} vB_{ik}A_{ij}p_i + \sum_{k=1}^{10,000}\sum_{j=1}^{47} \sum_{t=1}^{100} \frac{1}{10,000}(r X_{jtk} + h I_{jtk})$

Formulation

Minimize $\sum_{k=1}^{10,000} \frac{1}{10,000}t_k$

subject to

$t \geq B_{ik}T_{ij} A_{ij}$

Average Total Cost $\leq K$

$\sum_{j=1}^{47} A_{ij} = 1$, $\forall i \in \{1, 2, ..., Z\}$

$A_{ij} \leq y_j$, $\forall i \in \{1, 2, ..., Z\}, \forall j \in \{1, 2, ..., 47\}$

$S_{jk} + M_{jk} + L_{jk} \leq 1$

$10 M_{jk} + 20 L_{jk} \leq X_{jtk} \leq 10 S_{jk} + 20 M_{jk} + 1,000 L_{jk}$

$\sum_{t=1}^{100}25 X_{jtk} \geq \sum_{i=1}^Z B_{ik}A_{ij} p_i - (1-S_{jk})1,000,000$

$\sum_{t=1}^{100}50 X_{jtk} \geq \sum_{i=1}^Z B_{ik}A_{ij} p_i - (1-M_{jk})1,000,000$

$\sum_{t=1}^{100}100 X_{jtk} \geq \sum_{i=1}^Z B_{ik}A_{ij} p_i - (1-L_{jk})1,000,000$

$\sum_{j=1}^{47} X_{jtk} \leq C$

$I_{jtk} + (1-S_{jk})1,000,000\geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100}25X_{jtk}$

$I_{jtk} + (1-M_{jk})1,000,000\geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100}50X_{jtk}$

$I_{jtk} + (1-L_{jk})1,000,000\geq \sum_i B_{ik}A_{ij}p_i - \sum_t^{100}100X_{jtk}$

$X_{jtk} \leq \sum_i B_{ik}A_{ij}p_i - I_{jtk}$

$X_{jtk}, I_{jtk} \geq 0$, integer

$S_{jk}, M_{jk}, L_{jk}, A_{ij} \geq 0$, binary