

# Gurobi Optimization MiniMax

```
In [1]: import gurobipy as gp
from gurobipy import *
import numpy as np
import csv
import os
import matplotlib.pyplot as plt
import warnings
import math
import pandas as pd
import random

warnings.filterwarnings("ignore") # To ignore warnings produced

In [2]: zips_df = pd.read_csv('Zipcode.csv')
# zips_df
zips_df = zips_df.loc[(zips_df['COUNTYNAME'] == 'ALLEGHENY') & (zips_df['population'] > 0)]
zips_df

Out[2]:
```

	OBJECTID	ZIP	NAME	ZIPTYPE	STATE	STATEFIPS	COUNTYFIPS	COUNTYNAME	S3DZIP	LAT	...	MFUD	SFC
0	4	15057	BAKERSTOWN	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	150.0	40.361610	...	0.0	101
1	4	15206	BAKERSTOWN	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	150.0	40.467710	...	0.0	101
2	4	15214	BAKERSTOWN	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	150.0	40.483220	...	0.0	101
3	4	15229	BAKERSTOWN	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	150.0	40.517330	...	0.0	101
4	4	15228	BAKERSTOWN	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	150.0	40.508810	...	0.0	101
...	...	...	...	...	...	...	...	...	...	...	...	...	...
118	65824	15047	GREENOCK	PO BOX	PA	42.0	42003.0	ALLEGHENY	150.0	40.609477	...	0.0	...
121	65828	15032	CURTISVILLE	PO BOX	PA	42.0	42003.0	ALLEGHENY	150.0	40.382620	...	346.0	4134
123	65837	15221	PITTSBURGH	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	152.0	40.467750	...	3486.0	12256
124	65839	15205	PITTSBURGH	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	152.0	40.483220	...	1718.0	8991
125	65841	15202	PITTSBURGH	NON-UNIQUE	PA	42.0	42003.0	ALLEGHENY	152.0	40.370729	...	2933.0	10066

96 rows x 23 columns

```
In [3]: zips = zips_df['ZIP'].to_list()
# zips
all_n = np.array(zips)
# # zips
# np.in1d(POD_sites, all_n)
# 0,1,5,9,10,13,15,16,21,22,24,27,29,36
# POD_zips[0,1,5,9,10,13,15,16,21,22,24,27,29,36]

In [4]: zipcodes_path = 'pittsburgh-alleggheny-county.csv'
data = np.genfromtxt(zipcodes_path, dtype=str, delimiter=',', encoding='utf-8-sig')
# neighborhoods = data.astype(np.int)
neighborhoods = all_n.astype(np.int)

POD_sites_path = 'POD Sites.xlsx'
POD_df = pd.read_excel(POD_sites_path)
POD_df = POD_df[['SCHOOL/FACILITY NAME', 'STRIIP MAP']]
POD_df['ZIPCODE'] = POD_df['STRIIP MAP'].apply(Lambda x: str(x)[-5:])
POD_df = POD_df[['ZIP']]
POD_df

POD_zips = np.array(pd.to_numeric(POD_df.ZIPCODE).values)
print(POD_zips)
print(neighborhoods)
```

[15237 15236 15102 15216 15227 15106 15210 15220 15025 15108 15024 15110 15137 15037 15038 15146 15101 15065 15216 15132 15136 15108 15228 15090 15229 15202 15235 15214 15044 15206 15221 15239 15056 15139 15209 15133 15057 15129 15144 15120 15136 15025 15241 15126 15122 15221 15221] [15057 15206 15214 15229 15228 15108 15101 15146 15037 15024 15106 15236 15237 15007 15014 15015 15018 10202 15034 15030 15035 15046 15049 15064 15069 15082 15110 15112 15129 15131 15132 15133 15135 15137 15144 15148 15223 15211 15213 15224 15225 15232 15239 15275 15006 15051 15233 15227 16229 15282 15083 15088 15116 15122 15204 15216 15102 15234 15120 15017 15215 15139 15028 15145 15104 15086 15142 15241 15226 15207 15075 15076 15200 15243 15136 15046 15056 15045 15090 15084 15147 15235 15238 15220 15217 15233 15143 15210 15289 15044 15126 15047 15032 15221 15205 15202]

```
In [5]: num_neighborhoods = len(neighborhoods)
num_sites = len(POD_zips)

random.seed(20)

zipcodes_df = pd.read_csv('Zipcode.csv')
# zipcodes_df = zipcodes_df.loc[zipcodes_df['type'].isin(['STANDARD', 'UNIQUE'])]
zipcodes_df = zipcodes_df.loc[zipcodes_df['population'] > 0]
zipcodes_df_filtered = zipcodes_df[['ZIP', 'NAME', 'LAT', 'LON', 'population']]

from math import sin, cos, sqrt, atan2, radians

def calcDistBetweenTwoPoints(pt1, pt2):
    # approximate radius of earth in km
    R = 6373.0
    lat1 = radians(pt1[0])
    lon1 = radians(pt1[1])
    lat2 = radians(pt2[0])
    lon2 = radians(pt2[1])
    dlon = lon2 - lon1
    dlat = lat2 - lat1
    a = sin(dlat / 2)**2 + cos(lat1) * cos(lat2) * sin(dlon / 2)**2
    c = 2 * atan2(sqrt(a), sqrt(1 - a))
    distance = R * c
    return distance

def getLatLongFromZip(zipcode, df):
    lat = df.loc[df['ZIP'] == zipcode]['LAT'].values[0]
    long = df.loc[df['ZIP'] == zipcode]['LON'].values[0]
    return (lat, long)

def getLatLongFromZip(POD_zips[j], zips_df):
    lat = df.loc[df['ZIP'] == zipcodes['LAT']].values[0]
    long = df.loc[df['ZIP'] == zipcodes['LON']].values[0]
    return (lat, long)

problematic_n = []
problematic_s = []
distances = []
# nomi.query_postal_code('latitude')
for i in range(num_neighborhoods):
    temp = []
    for j in range(num_sites):
        try:
            print(POD_zips[j])
            pt1 = getLatLongFromZip(neighborhoods[i], zips_df)
            pt2 = (nomi.query_postal_code(str(neighborhoods[i]))['latitude'], nomi.query_postal_code(str(neighborhoods[i]))['longitude'])
            # print(pt1)
            pt2 = getLatLongFromZip(POD_zips[j], zips_df)
            dist = calcDistBetweenTwoPoints(pt1, pt2)
            temp.append(dist)
        except:
            problematic_n.append(neighborhoods[i])
            problematic_s.append(POD_zips[j])
    # print(neighborhoods[i], POD_zips[j])
    print("-"*50)
    temp.append(100000)
distances.append(temp)
distances = np.array(distances)
# distances = np.delete(distances, (12), axis=0)
```

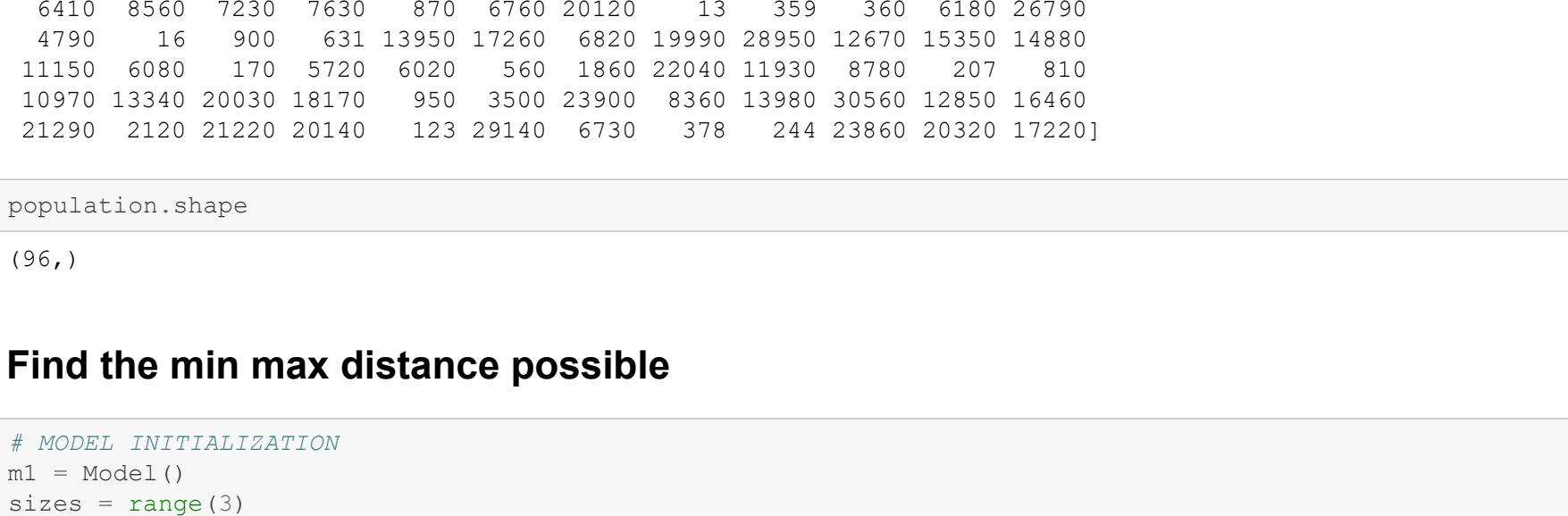
```
In [6]: POD_lats = []
POD_longs = []

for z in POD_zips:
    lat, long = getLatLongFromZip_graph(z, zips_df)
    POD_lats.append(lat)
    POD_longs.append(long)

neighborhood_lats = []
neighborhood_longs = []

for z in neighborhoods:
    lat, long = getLatLongFromZip_graph(z, zips_df)
    neighborhood_lats.append(lat)
    neighborhood_longs.append(long)

plt.scatter(neighborhood_longs, neighborhood_lats, marker='o', c = 'blue')
plt.scatter(POD_longs, POD_lats, marker='x', c = 'red')
plt.xlabel('Longitude')
plt.ylabel('Latitude')
plt.title('Neighborhoods (blue) vs possible POD sites (red)')
```



```
In [7]: population = []
for i in range(num_neighborhoods):
    try:
        population.append(zipcodes_df_filtered.loc[zipcodes_df_filtered['ZIP'] == neighborhoods[i]]['population'].values[0])
    except:
        pass

population = np.array(population)
pop_mean = np.mean(population)

#for i in range(len(population)):
#    if population[i] == 0:
#        population[i] = pop_mean
print(population)

[ 4739 22090 12010 13410 17180 37850 24110 25680 9730 7970 16810 29410
 42230 360 2650 1290 750 15150 1350 850 1770 2360 860 310
 9890 386 3950 2650 10260 7280 14700 5460 4510 8540 3580 1860
 6560 8730 7630 870 6760 20120 13 359 360 6180 26790
 4790 16 900 631 13950 17260 6820 19990 28950 12670 15350 14890
11150 6080 170 5720 6020 560 1860 22040 11930 8780 207 810
10970 13340 20030 18170 950 3500 23900 8360 13980 30560 12850 16460
21290 2120 15220 20140 123 20140 4730 378 244 23860 20320 17220]
```

```
In [8]: population.shape
```

```
Out[8]: (96,)
```

## Find the min max distance possible

```
In [9]: # MODEL INITIALIZATION
m1 = Model()
sizes = range(3)
zipcodes = range(distances.shape[0])
sites = range(distances.shape[1])
days = range(25)

# CONSTANTS
D = distances
p = population
total_pop = population.sum()
e = np.array([172, 85, 100])
o = 20000
v = 50
f = 8000
r = 206.76
h = 12.5
c = 1370

# DECISION VARIABLES
A = m1.addVars(zipcodes, sites, vtype = GRB.BINARY)
S = m1.addVars(sites, vtype = GRB.BINARY)
M = m1.addVars(sites, vtype = GRB.BINARY)
L = m1.addVars(sites, vtype = GRB.BINARY)
U = m1.addVars(sites, days, vtype = GRB.BINARY)
X = m1.addVars(sites, days)#, vtype = GRB.INTEGER)
I = m1.addVars(sites, days)#, vtype = GRB.INTEGER)

# OBJECTIVE
MAX = m1.addVar(lb = 0.0)
m1.setObjective(MAX)
m1.modelSense = GRB.MINIMIZE

# CONSTRAINTS
for i in zipcodes:
    m1.addConstr(sum(A[i,j] for j in sites) == 1)

    for j in sites:
        m1.addConstr(sum(A[i,j] * p[i] for i in zipcodes)
            m1.addConstr(sum(e[0]*X[i,t] + M[j] + L[j] <= 1)
            m1.addConstr(sum(e[0]*X[i,t] for t in days) >= assigned_pop - (1-S[j])*total_pop)
            m1.addConstr(sum(e[1]*X[i,t] for t in days) >= assigned_pop - (1-M[j])*total_pop)
            m1.addConstr(sum(e[2]*X[i,t] for t in days) >= assigned_pop)
            m1.addConstr(S[j] >= 0)
            m1.addConstr(M[j] >= 0)
            m1.addConstr(L[j] >= 0)

    for t in days:
        administered[0] += e[0]*X[i,t]
        administered[1] += e[1]*X[i,t]
        administered[2] += e[2]*X[i,t]
        m1.addConstr(X[i,t] + 10*M[j] + 20*L[j] - 20*(1-U[j,t]))
        m1.addConstr(X[i,t] <= C*U[j,t])
        m1.addConstr(U[j,t] <= S[j] + M[j] + L[j])
        m1.addConstr(I[j,t] + (1-S[j])*total_pop >= assigned_pop - administered[0])
        m1.addConstr(I[j,t] + (1-M[j])*total_pop >= assigned_pop - administered[1])
        m1.addConstr(I[j,t] + (1-L[j])*total_pop >= assigned_pop - administered[2])
        m1.addConstr(X[i,t] >= 0)
        m1.addConstr(U[j,t] >= 0)

for t in days:
    m1.addConstr(sum(X[j,t] for j in sites) <= c)
```

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```
In [10]: #m1.Params.MIPGap = 0.1
m1.Params.TimeLimit = 20*60
m1.optimize()
```

Changed value of parameter TimeLimit to 1200.0  
Prev: inf Min: 0.0 Max: inf Default: inf  
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64)  
Thread count: 6 physical cores, 12 logical processors, using up to 12 threads  
Optimize a model with 25736 rows, 8179 columns and 465901 nonzeros  
Model fingerprint: 0x52a4fd94  
Variable types: 2351 continuous, 5828 integer (5828 binary)  
Coefficient statistics:  
Matrix range [2e+01, 1e+06]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+00, 1e+00]  
RHS range [1e+00, 1e+06]  
Presolve removed 11853 rows and 1175 columns  
Presolve time: 0.30s  
Presolved: 13883 rows, 7004 columns, 66250 nonzeros  
Variable types: 1376 continuous, 5828 integer (5828 binary)  
Found heuristic solution: objective 47.6903713  
Found heuristic solution: objective 35.9605456  
Found heuristic solution: objective 32.0191374

Root relaxation: Current 2.849850e+00, 1865 iterations, 0.12 seconds

Nodes	Obj	Depth	IntInf	I	Incumbent	BestBd	Gap	It/Node	Time
0	0	2.84985	0	28	32.01914	2.84985	91.1%	-	0s
H	0	0	0	31	3664648	2.84985	90.9%	-	0s
H	0	0	0	228	7704195	2.84985	90.1%	-	0s
H	0	0	0	17.5052689	2.84985	83.7%	-	0s	0s
H	0	0	0	14.6579122	2.84985	80.6%	-	0s	0s

Cutting planes:  
MIR: 6  
Flow cover: 6  
GUB cover: 1  
RLT: 32  
Relax-and-lift: 24

Explored 1 nodes (1710 simplex iterations) in 1.04 seconds  
Thread count was 12 (of 12 available processors)

Solution count 7: 14.6579 17.5053 28.7704 ... 47.6904

Optimal solution found (tolerance 1.00e-04)  
Best objective 1.465791215778e+01, best bound 1.465791215778e+01, gap 0.0000%

```
In [11]: m1.objval
```

```
Out[11]: 14.657912157781494
```

The minimum max distance possible is 14.66

```
In [12]: num_sites = 0
for i in sites:
    if (S[i].x + M[j].x + L[j].x) > 0:
        num_sites += 1
print(num_sites)
```

```
Out[12]: 46
```

## Minimizing distance while keeping cost at a minimum

```
In [13]: distances.max(0).min()
```

```
Out[13]: 30.520364383667598
```

```
In [14]: np.argmax(distances.max(0))
```

```
Out[14]: 29
```

Use only one site, site 29

## Minimizing Cost while holding max distance at minimum



[illegible]

```

Site: 45      -0.0
Site: 46      -0.0
Site: 47      -0.0
Site: 47      -0.0

```

In [19]:

```

max_ts = []
for j in sites:
    max_t = 0
    for t in days:
        if np.allclose(l[j,t].x,0):
            max_t = t + 1
            max_ts.append(max_t)
    break

```

In [20]:

```

plt.hist(max_ts)
plt.title("The Distribution of the Number of days to administer vaccine ")
plt.show()

```

The Distribution of the Number of days to administer vaccine

In [21]:

```

days_not_serving = []
days_serving = []
for j in sites:
    num_days = 0
    serve = 0
    for t in days:
        if np.allclose(K[j,t].x, 0) and np.allclose(l[j,t].x, 0) is False:
            num_days += 1
        if np.allclose(K[j,t].x, 0) is False:
            serve += 1
    days_not_serving.append(num_days)
    days_serving.append(serve)

```

In [22]:

```

plt.hist(days_not_serving)
plt.title("Distribution of days POD is Idle")
plt.show()

```

Distribution of days POD is idle

In [23]:

```

plt.hist(days_serving)
plt.title("Distribution of days actually vaccinating")
plt.show()

```

Distribution of days actually vaccinating

In [24]:

```

utilized = []
for j in sites:
    if (S[j].x + M[j].x + L[j].x) > 0:
        utilized.append(j)

```

In [25]:

```

print(num_sites)
plt.scatter(neighborhood longs, neighborhood_lats, marker='o', c = 'blue')
plt.scatter(np.take(POD longs, utilized), np.take(POD_lats, utilized), marker='x', c = 'red')
plt.xlabel('Longitude')
plt.ylabel('Latitude')
plt.title('Neighborhoods (blue) vs optimal POD sites (red)')

```

7

Out[25]: Text(0.5, 1.0, 'Neighborhoods (blue) vs optimal POD sites (red)')

Neighborhoods (blue) vs optimal POD sites (red)

In [26]:

```

vaccinations = []
cum_vaccinations = []
cum_vaccinated = 0
for t in days:
    vaccinated = sum(K[j,t].x * e[0]*S[j].x + e[1]*M[j].x + e[2]*L[j].x)
    cum_vaccinated += vaccinated
    vaccinations.append(vaccinated)
    cum_vaccinations.append(cum_vaccinated)

```



