processes are independent (*22*). The sinusoidal oscillations of $N_{\pm}$ with the phase φ_a, indicating that the coherence between the two diamond modes has been preserved, are plotted in Fig. 2 for the Stokes detector D_s.

The fringe visibility is $V = (61 \pm 3)\%$ (for N_+), which, when compared to the theoretical maximum of ~75% (*18*) for a two-mode squeezed state, implies that we have two well-defined modes, with good mode overlap between the Raman-scattered modes interfered from the two diamonds. Causes of decorrelation include spontaneous emission of anti-Stokes photons (*18*) and decoherence of the phonon, as well as limited coupling efficiency due to scattering into higher-order spatial modes, indicating that our measurement represents a lower bound on the intrinsic correlation between the phonons.

To verify entanglement between the two phonon modes of the diamonds, we examine the entanglement between the Stokes and anti-Stokes photons (here we consider only N_+ from Fig. 2). Because the phonon-to-photon mapping is a local operation (acting separately on each diamond) that cannot increase entanglement (*16*), the photon entanglement is a strict lower bound to the phonon entanglement. Therefore, considering the photon modes, we must show that the probability of generating higher-order terms is inconsistent with any separable state (e.g., of the form $|\Psi_a\rangle = (1 + \varepsilon_a a_L^\dagger)(1 + \varepsilon_a a_R^\dagger)|\mathrm{vac}\rangle$). That is, the probability of twin anti-Stokes readout detection events $(a_L^\dagger a_R^\dagger|\ \mathrm{vac}\rangle)$ must be shown to be sufficiently small compared to the single anti-Stokes detection probability.

To formalize this argument, we evaluate the concurrence (*23*), which is a monotonic measure of two-qubit entanglement that is zero for any separable state and positive for all entangled states. Here the concurrence is defined over a subspace that consists of detecting zero or one anti-Stokes photon at the two detectors heralded on the detection of a Stokes photon. We assume that the density matrix ρ describing the joint state of the anti-Stokes modes has the form shown in Fig. 3. The indices [0,1] indicate the number of photons detected in the anti-Stokes mode generated from diamond [L,R], conditioned on detecting a Stokes photon with detector D_s. The off-diagonal coherence d between $a_L^\dagger|\mathrm{vac}\rangle$ and $a_R^\dagger|\mathrm{vac}\rangle$ is estimated to be $d = V(p_{01} + p_{10})/2$, and all other terms are set to zero. This makes our estimate of the amount of entanglement conservative, as nonzero elements can only increase the concurrence (*16*). Higher-order photon number contributions are neglected. With these assumptions, the concurrence provides a strict lower bound to the entanglement between the diamonds (*16*).

The concurrence of ρ is therefore (*16*, *23*)

$$C = 2\ \max(|d| - \sqrt{p_{00}p_{11}}, 0) \qquad (4)$$

We estimate the concurrence to be $(5.2 \pm 2.6) \times 10^{-6}$, which is on the order of the maximum value of the concurrence (for $V = 1$, and $p_{11} = 0$ we have $C_{max} = p_{01} + p_{10} = 2.3 \times 10^{-5}$). The maximum value is limited by coupling and detector efficiencies and the readout probability (the probability of converting a phonon into an anti-Stokes photon, conditioned on detecting a Stokes photon).

To determine our confidence in concluding that the system is entangled, we calculate the Poissonian confidence level (*24*) for positive concurrence when we detect X twin readout events

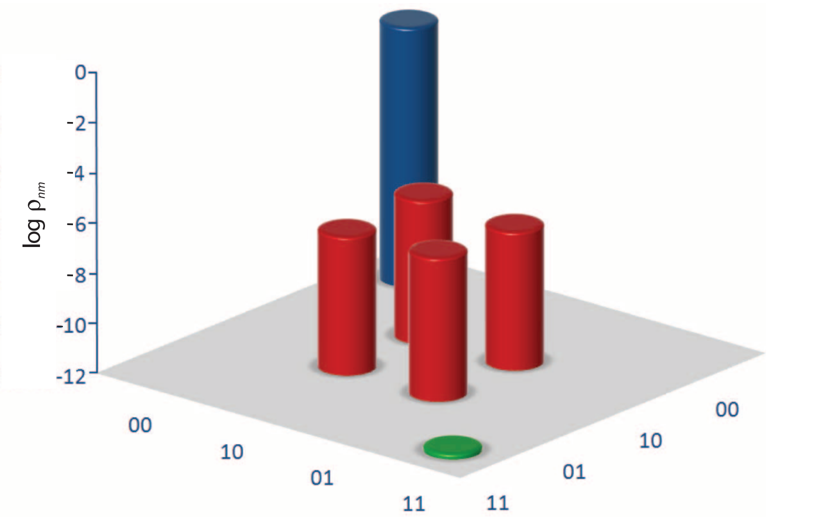

Fig. 3. Density matrix of the heralded anti-Stokes modes. The density matrix elements are $p_{00} = 1 - 2.3 \times 10^{-5}$, $p_{01} = 1.2 \times 10^{-5}$, $p_{10} = 1.1 \times 10^{-5}$, $d = 7.0 \pm 0.3 \times 10^{-6}$, $p_{11} = 2.0 \pm 1.1 \times 10^{-11}$. The diagonal element probabilities are maximum likelihood estimates, measured with no interference between the anti-Stokes modes of the two diamonds. No corrections for background counts, accidental coincidences, or system inefficiencies were made in these measurements.. The higher-order term is inherent to the process of spontaneous emission, and the vacuum component is related to the anti-Stokes readout, collection, and detector efficiencies.

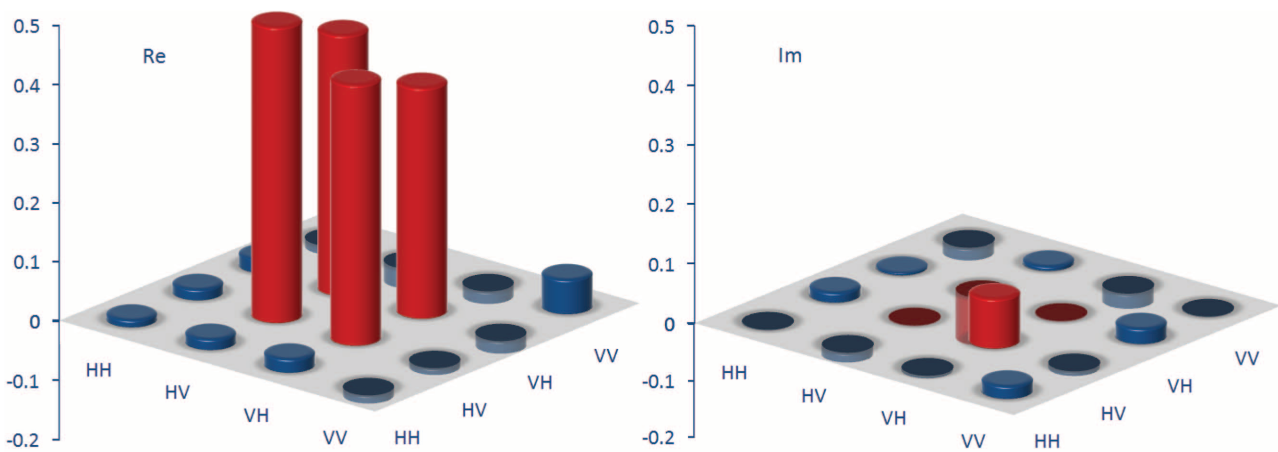

Fig. 4. Reconstructed real (Re) and imaginary (Im) components of the joint polarization state of the Stokes and anti-Stokes modes, projected into the subspace containing one photon in each mode. HV, for example, indicates a horizontally polarized Stokes photon and vertically polarized anti-Stokes photon, where polarization is used to encode the time ordering of pulses [see SOM (*18*)] The state appears to be highly entangled in polarization after postselection of this subspace, which demonstrates strong coherence between the diamonds, suggestive of near-maximal entanglement. This complements the evidence for genuine entanglement provided by Fig. 3.