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Parametric Classes of Generalized Conjunction and Disjunction Operations for Fuzzy Modeling

Ildar Batyrshin and Okyay Kaynak, *Senior Member, IEEE*

Abstract— It is argued that inference procedures of fuzzy models do not always require commutativity and associativity of the operations used. This raises the possibility of considering nonassociative and noncommutative conjunction and disjunction operations. Such operations are investigated in this paper and different methods for their generation are proposed. A number of new types of conjunction operations that are simpler than the known parametric classes of T-norms are given and, as an application example, the approximation of a function by a fuzzy inference system is considered.

Index Terms— Associativity, commutativity, conjunction, disjunction, fuzzy modeling, T-conorm, T-norm.

I. INTRODUCTION

AFTER the publication of the seminal paper of Zadeh [39], many different definitions of conjunction and disjunction operations have been introduced and studied [2], [6], [8]–[10], [12]–[15], [17], [21], [22], [27], [28], [34], [36]. In [6], the axiomatization of fuzzy operations is constructed by generalizing the properties of operations on usual sets and operations of binary logic. From these axioms it follows that min and max operations are the unique, possible conjunction and disjunction operations, respectively (see also [33]). As is shown in [2] and [34], this is because of the restrictions put on the possible forms of conjunction and disjunction operation by the distributivity property. The elimination of the latter from the set of axioms breaks the uniqueness of min and max operations and gives rise to the possibility of constructing a wide range of fuzzy connectives, based on the concepts of T-norm and T-conorm [2], [15], [19], [21], [27], [34]. The distributivity property is very important in fuzzy logic because it gives possibility to making equivalent transformations of logical forms from disjunctive to conjunctive forms and vice versa. In many practical applications of fuzzy models, however, such transformations are not needed and for this reason the distributivity property may be deleted from the system of axioms of fuzzy connectives.

The concepts of T-norm and T-conorm have come into the theory of fuzzy sets from the theories of functional equations and probabilistic metric spaces [1], [30]. The related axioms give possibility to build an infinite number of logical connectives. If the associativity property in the definition of T-norms and T-conorms is considered as a functional equation, then T-norms and T-conorms will be solutions of this equation and they may be generated by functions with one argument. A large amount of parametric classes of T-norms and T-conorms have been introduced in this way and they play an important role in fuzzy logic. But, from another point of view, the associativity property also appears as restrictive, because, in the construction of optimal fuzzy models, the parametric classes of T-norms and T-conorms constructed in such ways are generally far too complicated to render an easy hardware realization and optimization of the parameters [7], [37]. In this paper, it is proposed that as is done to the distributivity property, the commutativity and the associativity properties are also eliminated from the set of axioms of fuzzy connectives. This has the goal of reducing the limitations on the form of possible parametric classes of fuzzy connectives that may be used in fuzzy models. The properties of associativity and commutativity of fuzzy connectives are important, for example, in fuzzy models of multicriteria decision-making because one of the reasonable requirements laid on decision-making procedures is their nondependence on the order of consideration of criteria. But for fuzzy inference systems, these properties are not always necessary, especially when positions of variables in fuzzy rules are fixed.

The simplest fuzzy inference systems that have wide applications are based on the rules such as

R_i: IF x is A_i AND y is B_i THEN z is C_i,

R_i: IF x is A_i AND y is B_i THEN $z = f_i(x, y)$

where A_i, B_i, C_i are some fuzzy sets, and f is some function [16], [18], [19], [23]–[25], [29]. For given crisp values of x^* and y^* the firing value α of the rules is calculated as $\alpha = T_1(\mu_{Ai}(x^*), \mu_{Bi}(y^*))$, where T_1 is some T-norm representing connective AND and $\mu_{Ai}(x^*)$, $\mu_{Bi}(y^*)$ are the membership values. A conclusion of rules may then be calculated as $\mu_{C^*i}(z) = T_2(\alpha, \mu_{Ci}(z))$, and $z^* = T_2(\alpha, f_i(x^*, y^*))$, respectively, where T_2 is a T-norm perhaps different from T_1. For the aggregation of conclusions of rules some disjunction operation (often some T-conorm) may be used. The construction of optimal fuzzy models is traditionally based on a tuning of membership functions of fuzzy sets used in the rules. When these membership functions are given parametrically then such tuning may be based on the optimization of these parameters.

The influence of fuzzy logic operations on the behavior of fuzzy systems and optimization of parameters of operations of fuzzy models have been studied theoretically and experimentally in many papers [3], [7], [16], [20], [26], [32], [35], [38]. The tuning or optimization of parametric operations may be done in addition to or instead of tuning of membership functions. But this approach would turn out to be somewhat complicated because of the complexity of existing parametric classes of T-norms and T-conorms used as conjunction and disjunction operations. Also, hardware realization of such complicated operations is not easy [37]. From this point of view it appears that simpler parametric classes of conjunction and disjunction operators would have many advantages.

It is easy to see that the associativity of conjunction operation is not required if only two variables in premises of rules and different conjunction operations T_1 and T_2 are used. In a more general case, when positions of variables in premises of rules and the procedure of calculation of firing values are fixed neither the commutativity nor the associativity of conjunction operations are needed. In this case, the conjunction of several arguments may be calculated sequentially in correspondence with given order of variables. Moreover, noncommutativity and nonassociativity may be desirable in some cases. For example, if x and y denote "error" and "change in error" correspondingly as in fuzzy control systems, then noncommutativity of conjunction operation may be used for taking into account different influences of these variables on the control process. So, if the commutativity of conjunction implies equality of rights of both operands, then the noncommutativity of conjunction with fixed positions of operands gives the possibility to build context dependent operations. We may propose also that parametric operations T_2 and T_1 may be "rule dependent" that gives possibility of separate tuning of parameters of these operations for rules related to different parts of control process, for example near points with maximal or zero errors and so on [29].

In Section II, the nomenclature is established by a review of the concepts of T-norm and T-conorm and the most popular examples of the parametric classes of T-norms are considered. In the following sections, the definition of noncommutative and nonassociative conjunction and disjunction operations is given and a number of different ways for generating new types of fuzzy connectives are proposed. Various examples of parametric classes of conjunction operations are then considered that are simpler than the parametric classes of T-norms reported in the literature. Finally, a simple example of fuzzy modeling based on the optimization of a parametric class of new operation is discussed.

II. T-NORMS AND T-CONORMS

T-norm and T-conorm are defined as functions T, S: $[0, 1]$ $\times [0, 1] \rightarrow [0, 1]$ satisfying the following properties:

(boundary condition)

$$T(x, 1) = x$$
$$S(x, 0) = x$$

(monotonicity)

$$T(x, y) \leq T(u, v) \quad \text{and} \quad S(x, y) \leq S(u, v)$$
$$\text{if } x \leq u, y \leq v$$

(commutativity)

$$T(x, y) = T(y, x)$$
$$S(x, y) = S(y, x)$$

(associativity)

$$T(T(x, y), z) = T(x, T(y, z))$$
$$S(S(x, y), z) = S(x, S(y, z)).$$

A *negation* is defined [31] as a function N: $[0, 1] \rightarrow [0, 1]$ satisfying the properties: $N(0) = 1$, $N(1) = 0$, and $N(x) \leq N(y)$ if $y \leq x$. A negation is called *an involution* if on $[0, 1]$ it is fulfilled involutivity property $N(N(x)) = x$. Parametric class of Sugeno involutive negations has the form: $N(x) = (1 - x)/(1 + \lambda x)$, $\lambda > -1$. When $\lambda = 0$ we obtain the negation of Zadeh [39]: $N(x) = 1 - x$. Noninvolutive negations are studied in [4], [5], and [11].

T-norms and T-conorms can be obtained one from another as follows:

$$S(x, y) = N(T(N(x), N(y)))$$
$$T(x, y) = N(S(N(x), N(y)))$$

where N is an involution. The simplest examples of T-norms and T-conorms mutually related by these relations for $N(x) = 1 - x$ are the following:

$$T_c(x, y) = \min\{x, y\}$$
$$S_c(x, y) = \max\{x, y\}$$
$$T_p(x, y) = xy$$
$$S_p(x, y) = x + y - xy$$
$$T_d(x, y) = \begin{cases} x, & \text{if } y = 1 \\ y, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}$$
$$S_d(x, y) = \begin{cases} x, & \text{if } y = 0 \\ y, & \text{if } x = 0 \\ 1, & \text{otherwise} \end{cases}$$
$$T_b(x, y) = \max\{0, (x + y - 1)\}$$
$$S_b(x, y) = \min\{1, (x + y)\}.$$

These simplest functions will later be used for the construction of parametric conjunction and disjunction operations. Generally, for any T-norm and T-conorm it follows that

$$T_d(x, y) \leq T(x, y) \leq T_c(x, y) \leq S_c(x, y)$$
$$\leq S(x, y) \leq S_d(x, y).$$

Hence, T-norms T_d and T_c are the minimal and the maximal boundaries for all T-norms (see Fig. 1). Similarly T-conorms S_c and S_d are the minimal and the maximal boundaries for all T-conorms (see Fig. 2). These inequalities are very important from a practical point of view because they establish the boundaries of the possible range of operations T and S.

T-norms and T-conorms, as functions satisfying the associativity property, can be generated by generators of several types [1], [2], [15], [19], [21], [30]. For example, T-norms may
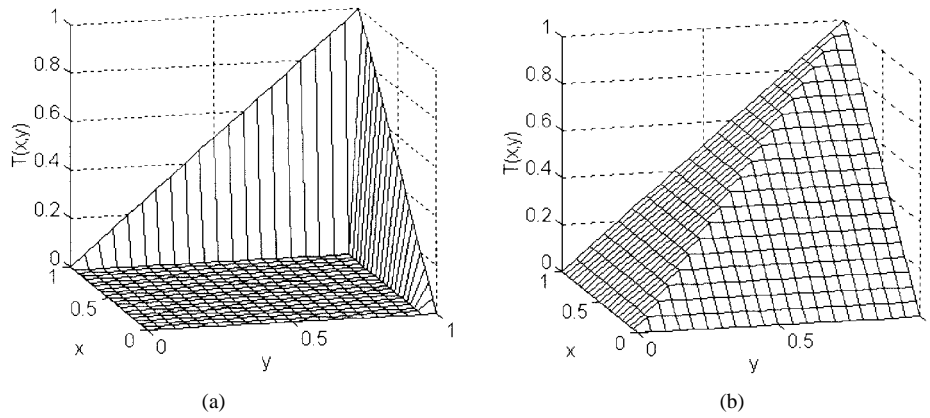
(a)   (b)

Fig. 1.   (a) T-norm T_d. (b) T-norm T_c. For all T-norms T it follows: $T_d(x, y) \leq T(x, y) \leq T_c(x, y)$.

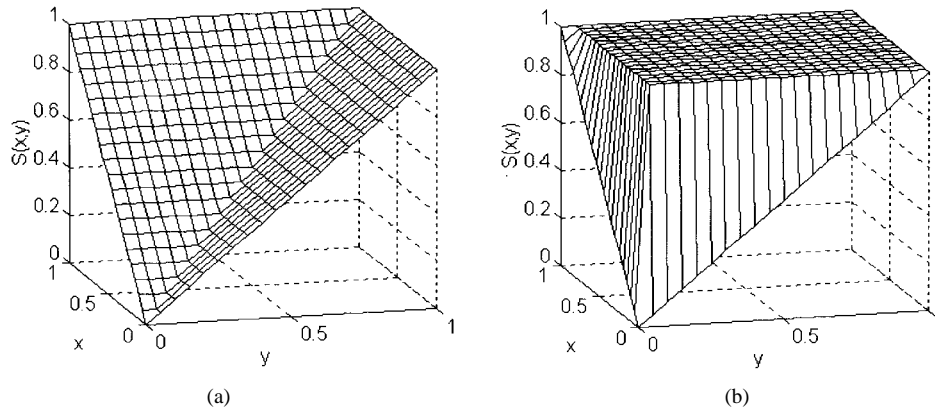(a)   (b)

Fig. 2.   (a) T-conorm S_c. (b) T-conorm S_d. For all T-conorms S it follows: $S_c(x, y) \leq S(x, y) \leq S_d(x, y)$.

be generated as follows: $T(x, y) = \varphi^{-1}(\varphi(x)\varphi(y))$, where φ is any increasing bijection (automorphism) $\varphi: [0, 1] \rightarrow [0, 1]$ with $\varphi(0) = 0$ and $\varphi(1) = 1$ and φ^{-1} is the inverse of φ. T-norm may be generated by means of another T-norm T_0 as follows: $T(x, y) = \varphi^{-1}(T_0(\varphi(x), \varphi(y)))$. Recent discussions of methods of generation of T-norms and T-conorms may be particularly found in [17] and [27]. Parametric classes of T-norms are generally complicated due to the necessity of using inverse functions for their construction. Below one can find examples of parametric classes of T-norms varying from T_d to T_c.

Two simplest parametric classes of T-norms of Schweizer and Sklar have the following form:

$$T(x, y) = \max\left(0, x^{-p} + y^{-p} - 1\right)^{-1/p},$$
$$T(x, y) = 1 - [(1 - x)^p + (1 - y)^p - (1 - x)^p(1 - y)^p]^{1/p}.$$

The behavior of a class of two-rule systems where the implication operator is formed by the first parametric class of T-norms is studied in [35]. Application of the second class of T-norms to optimal fuzzy modeling is discussed in [32].

Yager has introduced the following popular class of T-norms:

$$T(x, y) = 1 - \min(1, ((1 - x)^p + (1 - y)^p)^{1/p}), \qquad p > 0.$$

These operations are used in [20] for the construction of additive hybrid operators considered as nodes in network

structures for decision making and a training algorithm based on gradient descent method for this network is developed.

The class of the following T-norms is proposed by Dombi:

$$T(x, y) = 1/(1 + ((1/x - 1)^p + (1/y - 1)^p)^{1/p}), \qquad p > 0.$$

The tuning of these parametric operators in fuzzy modeling by means of gradient descent optimization method is considered in [7].

III. GENERALIZED CONJUNCTION AND DISJUNCTION OPERATIONS

Generalizations of the concepts of fuzzy conjunction and disjunction operators are considered in several papers. Here we consider a generalization of these operations with the aim of constructing parametric classes of these operations.

Definition 1: A conjunction operation and a disjunction operation are functions $T, S: [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following properties:

$$\begin{aligned} T(x, 1) &= T(1, x) = x \\ S(x, 0) &= S(0, x) = x \qquad\qquad (1) \\ T(x, y) &\leq T(u, v) \end{aligned}$$

and

$$S(x, y) \leq S(u, v) \qquad \text{if } x \leq u, y \leq v. \qquad (2)$$

Naturally any T-norm and T-conorm will be a conjunction and a disjunction with respect to this definition. Below T_c,

S_c, T_d, S_d, will denote the simplest T-norms and T-conorms considered above. It is easy to prove the following properties of fuzzy connectives.

Proposition 1: Conjunction and disjunction operations satisfy the following properties:

$$T(0, y) = T(x, 0) = 0$$
$$S(1, y) = S(x, 1) = 1 \tag{3}$$
$$T_d(x, y) \leq T(x, y) \leq T_c(x, y) \leq S_c(x, y)$$
$$\leq S(x, y) \leq S_d(x, y). \tag{4}$$

From (1)–(4) we obtain:

$$T(0, 0) = T(0, 1) = T(1, 0) = 0, \quad T(1, 1) = 1$$
$$S(0, 0) = 0, \quad S(0, 1) = S(1, 0) = S(1, 1) = 1. \tag{5}$$

We note that our definition of conjunction and disjunction operation coincides with the definition of T-seminorm and T-semiconorm on partially ordered sets in [8]. Conjunction operation belongs to the class of weak T-norms [12]. The properties (2) and (5) are considered in [22] as axioms of axiomatic skeleton for fuzzy sets intersection and union. Most general definition of fuzzy conjunction defined only by the properties (5) for T is considered in [13]. Nonstandard conjunctions are discussed also in [14].

It is easy to prove the following statement establishing the ways for generation disjunctions from conjunctions and vice versa.

Theorem 1: Suppose N is an involutive negation on [0, 1] and T, S are some conjunction and disjunction, then the following relations define correspondingly disjunction and conjunction functions:

$$S_T(x, y) = N(T(N(x), N(y)))$$
$$T_S(x, y) = N(S(N(x), N(y))). \tag{6}$$

It follows from (6) and from involutivity of N that for any T and $S = S_T$ and similarly for any S and $T = T_S$ the following De Morgan laws are fulfilled:

$$N(S(x, y)) = T(N(x), N(y))$$
$$N(T(x, y)) = S(N(x), N(y)).$$

IV. GENERATION OF CONJUNCTION AND DISJUNCTION OPERATIONS

Below we introduce two functions that will be used for generating conjunction and disjunction operations.

Definition 2: The functions t, s: [0, 1] × [0, 1] → [0, 1] satisfying the following properties:

$$t(0, y) = t(x, 0) = 0$$
$$s(1, y) = s(x, 1) = 1 \tag{7}$$
$$t(x, y) \leq t(u, v)$$

and

$$s(x, y) \leq s(u, v) \qquad \text{if } x \leq u, y \leq v \tag{8}$$

will be called a pseudoconjunction and a pseudodisjunction, respectively.

It is evident that any T-norm (T-conorm) will be a conjunction (disjunction) and any conjunction (disjunction) will be a pseudoconjunction (pseudodisjunction).

Theorem 2: Suppose T_1, T_2 are conjunctions, t is a pseudoconjunction, S_1 and S_2 are disjunctions and s is a pseudodisjunction, then the following functions:

$$T_3(x, y) = T_2(T_1(x, y), s(x, y))$$
$$S_3(x, y) = S_2(S_1(x, y), t(x, y)) \tag{9}$$
$$T_4(x, y) = T_2(s(x, y), T_1(x, y))$$
$$S_4(x, y) = S_2(t(x, y), S_1(x, y)) \tag{10}$$

will be conjunctions and disjunctions, respectively.

Proof:

$$T_3(x, 1) = T_2(T_1(x, 1), s(x, 1)) = T_2(x, 1) = x$$
$$T_3(1, y) = T_2(T_1(1, y), s(1, y)) = T_2(y, 1) = y.$$

A monotonicity of T_3 follows from the monotonicity of T_1, T_2, and s. Similarly we can show that T_4 is also a conjunction and S_3, S_4 are disjunctions.

Conjunctions (9) and (10) have the following properties.

Proposition 2:

$T(T_d, s) = T_d$ for all conjunctions T and pseudodisjunctions s;

$T_d(T, s) = T_d$ for all conjunctions T and pseudodisjunctions s such that $s(x, y) < 1$ if $x, y < 1$;

$T_c(T, S) = T$ for all conjunctions T and disjunctions S;

$T(T_c, S_c) = T$ for all commutative conjunctions T;

$T_b(T, S) = T_b$ for all pairs (T, S) of operators (T_c, S_c), (T_p, S_p), and (T_b, S_b).

Proof: From (7) and Theorem 2 it follows that it is sufficient to consider the cases when $x, y < 1$. Then we have $T(T_d(x, y), s(x, y)) = T(0, s(x, y)) = 0 = T_d(x, y)$. $T_d(T(x, y), s(x, y)) = 0 = T_d(x, y)$ since $T(x, y) \leq \min(x, y) < 1$ and $s(x, y) < 1$ for $x, y < 1$. From (4) we have $T_c(T(x, y), S(x, y)) = \min(T(x, y), S(x, y)) = T(x, y)$. From commutativity of T it follows that $T(T_c(x, y), S_c(x, y)) = T(\min(x, y), \max(x, y)) = T(x, y)$. Let us show that $T_b(T(x, y), S(x, y)) = \max(0, T(x, y) + S(x, y) - 1) = T_b(x, y)$. It is sufficient to show that $T(x, y) + S(x, y) = x + y$ for all considered pairs of T, S operators

$$T_c(x, y) + S_c(x, y)$$
$$= \min(x, y) + \max(x, y) = x + y$$
$$T_p(x, y) + S_p(x, y)$$
$$= xy + x + y - xy = x + y$$
$$T_b(x, y) + S_b(x, y)$$
$$= \max(0, x + y - 1) + \min(1, x + y) = x + y$$

for both possibilities $x + y < 1$ and $x + y \geq 1$.

Similarly, it can be shown that disjunctions (9) and (10) have the following properties.

Proposition 3:

$S(t, S_d) = S_d$ for all pseudoconjunctions t and disjunctions S;

$S_d(t, S) = S_d$ for all disjunctions S and all pseudocon-junctions t such that $t(x, y) > 0$ if $x, y > 0$;

$S_c(T, S) = S$ for all conjunctions T and disjunctions S;

$S(T_c, S_c) = S$ for all commutative disjunctions S;

$S_b(T, S) = S_b$ for all pairs (T, S) of T-operators (T_c, S_c), (T_p, S_p), and (T_b, S_b).

As follows from Theorem 2, we can build conjunction and disjunction operations from well-known T-norms and T-conorms using them as (pseudo) conjunction and (pseudo) disjunction operations. But for obtaining new operators we must take into account Propositions 2 and 3. For example, starting from T_c, T_p, S_c, S_p we can get the following commutative conjunction and disjunction operations related to each other by De Morgan laws [with $N(x) = 1 - x$]:

$$T(x, y) = (x + y - xy) \min(x, y)$$
$$S(x, y) = \max(x, y) + xy - \max(x, y)xy$$
$$T(x, y) = \max(x, y)xy$$
$$S(x, y) = \min(x, y) + x + y - xy$$
$$- \min(x, y)(x + y - xy)$$
$$T(x, y) = xy(x + y - xy)$$
$$S(x, y) = x + y - xy(x + y - xy).$$

For obtaining more interesting parametric classes of conjunction and disjunction operations we may use in (9) and (10) pseudoconnectives different from T-norms and T-conorms.

Proposition 4: Suppose N is any negation on [0, 1] and t, s are some pseudoconjunction and pseudodisjunction then the following relations define correspondingly pseudodisjunction and pseudoconjunction functions:

$$s_t(x, y) = N(t(N(x), N(x)))$$
$$t_s(x, y) = N(s(N(x), N(x))). \tag{11}$$

Proof: $s_t(x, 1) = N(t(N(x), N(1))) = N(t(N(x), 0)) = N(0) = 1$. Similarly, we obtain $s_t(1, x) = 1$. Since t is monotonically increasing on both arguments and N is monotonically decreasing function we get the monotonicity property of s_t. Proof for t_s is similar.

We note that if in (11) the negation N is an involution then pseudoconnectives t and $s = s_t$, s and $t = t_s$ will be mutually related by De Morgan laws. We will consider the following pairs of pseudoconnectives mutually related with negation $N(x) = 1 - x$:

$$t_B(x, y) = 0, \qquad \text{for all } x, y \in [0, 1]$$
$$s_B(x, y) = 1, \qquad \text{for all } x, y \in [0, 1]$$

$$t_D(x, y) = \begin{cases} 0, & \text{if } x = 0 \\ 0, & \text{if } y = 0 \\ 1, & \text{otherwise,} \end{cases} \qquad s_D(x, y) = \begin{cases} 1, & \text{if } x = 1 \\ 1, & \text{if } y = 1 \\ 0, & \text{otherwise} \end{cases}$$

$$t_x(x, y) = \begin{cases} 0, & \text{if } y = 0 \\ x, & \text{otherwise,} \end{cases} \qquad s_x(x, y) = \begin{cases} 1, & \text{if } y = 1 \\ x, & \text{otherwise} \end{cases}$$

$$t_y(x, y) = \begin{cases} 0, & \text{if } x = 0 \\ y, & \text{otherwise,} \end{cases} \qquad s_y(x, y) = \begin{cases} 1, & \text{if } x = 1 \\ y, & \text{otherwise.} \end{cases}$$

It is easy to see that for all pseudoconjunctions t and pseu-dodisjunctions s the following inequalities are fulfilled:

$$t_B(x, y) \le t(x, y) \le t_D(x, y)$$
$$s_D(x, y) \le s(x, y) \le s_B(x, y).$$

Any pseudoconjunction t differs from any pseudodisjunction s at least in two points (0, 1) and (1, 0) because of the following:

$$t(0, 1) = t(1, 0) = 0, \qquad s(0, 1) = s(1, 0) = 1.$$

Theorem 3: Suppose s is some parametric pseudo-disjunction varying from s_D to s_B and T_1 is an arbitrary conjunction, then by means of any conjunction T_2 applying (9) and (10) we can generate conjunctions varying from T_d to T_1.

Proof: From (9) we have $T_2(T_1(x, y), s_B(x, y)) = T_2(T_1(x, y), 1) = T_1(x, y)$. Denote $T(x, y) = T_2(T_1(x, y), s_D(x, y))$. If $x = 1$ then we have $T(1, y) = T_2(T_1(1, y), s_D(1, y)) = T_2(y, 1) = y$. If $y = 1$, similarly, we have $T(x, 1) = x$. If $x \ne 1$ and $y \ne 1$ we have $T(x, y) = T_2(T_1(x, y), s_D(x, y)) = T_2(T_1(x, y), 0) = 0$. Hence, $T = T_d$, and we obtain by (9) T_1 and T_d when $s = s_B$ and $s = s_D$, respectively. Suppose by varying parameter in s one can build pseudoconjunctions s_a and s_b such that $s_a \le s_b$. Denote conjunctions obtained by (9) on the base of s_a and s_b as T_a and T_b, respectively. Then we have $s_D \le s_a \le s_b \le s_B$ and from monotonicity of all functions in (9) we obtain $T_d \le T_a \le T_b \le T_1$.

The proof of the theorem for (10) is similar.

As follows from this theorem, if we can build parametric class of pseudodisjunctions s, varying from s_D to s_B, then applying s and $T_1 = T_c$ in (9) or (10) we can vary conjunctions in all possible ranges between T_d and T_c. Of course the types of conjunctions that will be generated between T_d and T_c depend on the form of s and T_2.

Dually we can formulate the following theorem.

Theorem 4: Suppose t is some parametric pseudoconjunc-tion varying from t_B to t_D, and S_1 is an arbitrary disjunction, then by means of any disjunction S_2 applying (9) and (10) we can generate disjunctions varying from S_1 to S_d.

From Theorems 3 and 4 it follows that for generation para-metric classes of conjunctions and disjunctions it is sufficient to generate suitable parametric classes of pseudoconnectives. This question is considered in the following section.

V. GENERATION OF PSEUDOCONJUNCTIONS AND PSEUDODISJUNCTIONS

Proposition 5: Suppose t_1 and t_2 are pseudoconjunctions, s_1, s_2 are pseudodisjunctions, f_1, f_2, g_1, g_2, h: [0, 1] $\rightarrow$ [0, 1] are nondecreasing functions such that $f_1(0) = f_2(0) = 0$, $g_1(1) = g_2(1) = 1$, then the following functions:

$$t_3(x, y) = t_1(f_1(x), f_2(y))$$
$$s_3(x, y) = s_1(g_1(x), g_2(y)) \tag{12}$$
$$t_4(x, y) = f_1(t_1(x, y))$$
$$s_4(x, y) = g_1(s_1(x, y)) \tag{13}$$
$$t_5(x, y) = t_2(t_1(x, y), h(y))$$
$$s_5(x, y) = s_2(s_1(x, y), h(y)) \tag{14}$$
$$t_6(x, y) = t_2(h(x), t_1(x, y))$$
$$s_6(x, y) = s_2(h(x), s_1(x, y)) \tag{15}$$

will be pseudoconjunctions and pseudodisjunctions, respectively.

Proof: From $f_1(0) = f_2(0) = 0$ and from fulfillment of (7) for t_1 and t_2, we obtain fulfillment of (7) for functions t in (12)–(15). The monotonicity of functions t follows from the monotonicity of t_1, t_2, f_1, f_2, and h. The proof for pseudodisjunctions is similar.

We note that due to possible noncommutativity of functions f_1, f_2, g_1, and g_2 the functions (14) and (15) may be different.

Multiple recursive application of (12)–(15) gives possibility to build various pseudo-conjunctions and pseudodisjunctions and then, by means of Theorem 2 and Proposition 4, various conjunctions and disjunctions may be constructed.

Functions f and g defined in Proposition 5 will be called f-generators and g-generators, respectively. It is easy to see that by means of any negation N we can obtain from f-generator some g-generator and vice versa

$$g(x) = N(f(N(x))), \qquad f(x) = N(g(N(x))).$$

For example, by using (9) and (12) we may obtain the following conjunction:

$$T(x, y) = T_2(T_1(x, y), s(g_1(x, p_1), g_2(y, p_2)))$$

where T_2, T_1 some T-norms, s is a T-conorm and $g_1(x, p_1)$, $g_2(y, p_2)$ some generators dependent on parameters p_1, p_2. For obtaining more or less simple parametric classes of conjunctions we may choose T_2, T_1 between T_c, T_p, T_d, T_b, choose s between S_c, S_p, S_d, S_b, and use simple functions g_1 and g_2.

In what follows, mainly conjunction operations are considered. Corresponding disjunction operations can be obtained dually or from conjunction operations by means of negation operations.

Let us consider the following generators:

$$f_B(x) = 0, \qquad \text{for all } x \in [0, 1]$$
$$g_B(x) = 1, \qquad \text{for all } x \in [0, 1]$$
$$f_D(x) = \begin{cases} 0, & \text{if } x = 0 \\ 1, & \text{otherwise} \end{cases}$$
$$g_D(x) = \begin{cases} 1, & \text{if } x = 1 \\ 0, & \text{otherwise.} \end{cases}$$

For any f- and g-generators we have

$$f_B(x) \le f(x) \le f_D(x), \qquad g_D(x) \le g(x) \le g_B(x).$$

Taking into account that the following functions

$$f_x(x) = x, \qquad g_x(x) = x$$

are also generators, we can change in (12)–(15) generators and functions h by their arguments.

It is easy to see that by substituting $t_1 = T$ and $s_1 = S$ in (12) we obtain for arbitrary conjunction T and disjunction S

and for any generators f and g the following relations:

$$T(f_B(x), f(y)) = T(f(x), f_B(y)) = t_B(x, y)$$
$$S(g_B(x), g(y)) = S(g(x), g_B(y)) = s_B(x, y)$$
$$T(f_D(x), f_D(y)) = t_D(x, y)$$
$$S(g_D(x), g_D(y)) = s_D(x, y)$$
$$T(f_D(x), y) = t_y(x, y)$$
$$S(g_D(x), y) = s_y(x, y)$$
$$T(x, f_D(y)) = t_x(x, y)$$
$$S(x, g_D(y)) = s_x(x, y).$$

Taking these relations into account we obtain the following result from Theorem 3.

Theorem 5: Suppose T_1 and T_2 are arbitrary conjunctions, S is an arbitrary disjunction, g_1 and g_2 are some parametric classes of g-generators such that one of them varies from g_D to g_B and another from g_D to some g^*, then by means of relation

$$T(x, y) = T_2(T_1(x, y), S(g_1(x), g_2(y))) \qquad (16)$$

we can generate conjunctions varying from T_d to T_1.

VI. EXAMPLES OF PARAMETRIC CLASSES OF CONJUNCTIONS

Example 1: We can propose the following parametric classes of threshold dependent generators:

$$f(x, p) = \begin{cases} 0, & \text{if } x \le p \\ 1, & \text{otherwise} \end{cases}$$
$$g(x, p) = \begin{cases} 1, & \text{if } x \ge p \\ 0, & \text{otherwise,} \end{cases} \qquad p \in [0, 1]$$

with the following properties:

$$f_B(x) = f(x, 1) \le f(x, p) \le f(x, 0) = f_D(x)$$
$$g_D(x) = g(x, 1) \le g(x, p) \le g(x, 0) = g_B(x).$$

For any T and S we have

$$T(f(x, p), f(y, q)) = \begin{cases} 0, & \text{if } x \le p \text{ or } y \le q \\ 1, & \text{otherwise} \end{cases}$$
$$S(g(x, p), g(y, q)) = \begin{cases} 1, & \text{if } p \le x \text{ or } q \le y \\ 0, & \text{otherwise.} \end{cases}$$

Applying in (16) $T_1 = T_c$ and generators $g(x, p)$ and $g(y, q)$ for arbitrary T_2 and S, we obtain the following conjunction operation:

$$T(x, y) = \begin{cases} \min(x, y), & \text{if } p \le x \text{ or } q \le y \\ 0, & \text{otherwise} \end{cases}$$

and particularly $T = T_d$ when $p = 1$, $q = 1$, and $T = T_c$ when $p = 0$ or $q = 0$. The graph of this conjunction for $p = 0.4$, $q = 0.8$ is shown on Fig. 3.
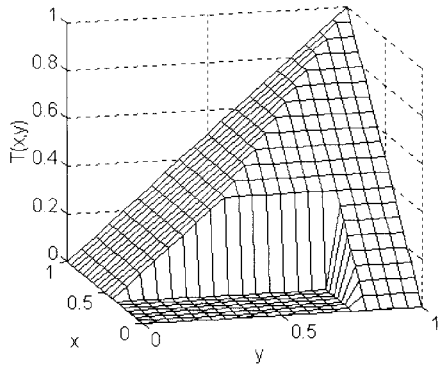
Fig. 3. Conjunction for $p = 0.4$, $q = 0.8$ in Example 1.

Example 2: Let us consider the parametric classes of linear generators:

$$f(x, p) = \min(px, 1)$$
$$g(x, p) = \max(1 - p(1 - x), 0), \qquad p \geq 0.$$

We have for these generators

$$f_B(x) = f(x, 0) \leq f(x, p) \leq f(x, \infty) = f_D(x),$$
$$g_D(x) = g(x, \infty) \leq g(x, p) \leq g(x, 0) = g_B(x)$$

where $f(x, \infty) = \lim f(x, p)$, $p \to \infty$, and $g(x, \infty) = \lim g(x, p)$, $p \to \infty$.

Applying in (16) $T_1 = T_c$, $T_2 = T_p$, and $S = S_c$ we obtain conjunction

$$T(x, y) = \min(x, y) \max\{1 - p(1 - x), 1 - q(1 - y), 0\}$$

with $T = T_c$ when $p = 0$ and $T \to T_d$ when $p, q \to \infty$. The graphs of this conjunction for $p = 1.2$, $q = 4$ and for $p = 2$ and $q = 4$ are shown on Fig. 4.

Example 3: Let us consider the parametric classes of power generators

$$f(x, p) = x^p$$
$$g(x, p) = x^p, \qquad p \geq 0$$

where we suppose that $0^{-p} = 0$ for all $p > 0$, $f(0, 0) = 0$, $f(1, \infty) = 0$, but $g(0, 0) = 1$, $g(1, \infty) = 1$. Then we have

$$f_B(x) = f(x, \infty) \leq f(x, p) \leq f(x, 0) = f_D(x)$$
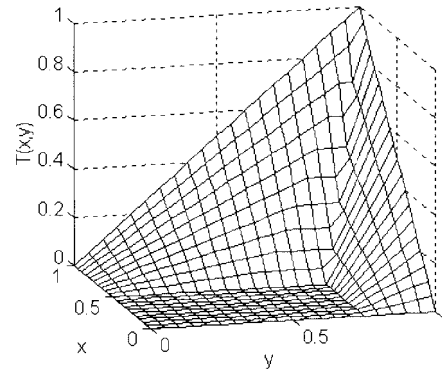$$g_D(x) = g(x, \infty) \leq g(x, p) \leq g(x, 0) = g_B(x).$$

By means of these generators we can build many conjunctions with attractive features. For example from Theorem 5 it follows that applying these generators in (16) with $T_1 = T_c$ we will obtain parametric classes of conjunctions varying from T_c (when $p, q \to 0$) to T_d (when $p, q \to \infty$).

Example 3.1: For $T_2 = T_c$ and $S = S_c$ we will obtain the following conjunction:

$$T(x, y) = \min\{\min(x, y), \max(x^p, y^q)\}.$$
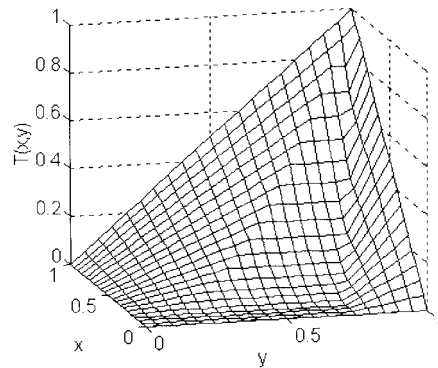
(a)

(b)

Fig. 4. Conjunctions in Example 2. (a) For $p = 1.2$, $q = 4$. (b) For $p = 2$, $q = 4$.
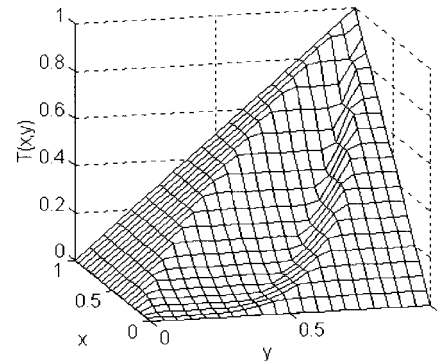

Fig. 5. Conjunction for $p = 2$, $q = 4$ in Example 3.1.

When $p, q \leq 1$ we have $T = T_c$. The graph of this conjunction for $p = 2$ and $q = 4$ is shown on Fig. 5. When $p = q$ this conjunction has the following form:

$$T(x, y) = \begin{cases} \min(x^p, y) & \text{if } x > y \\ \min(x, y^p) & \text{if } x \leq y. \end{cases}$$

Example 3.2: For $T_2 = T_p$ and $S = S_c$ we will obtain another conjunction

$$T(x, y) = \min(x, y) \max(x^p, y^q).$$

The graphs of this conjunction for $p = 1.2$, $q = 4$, and

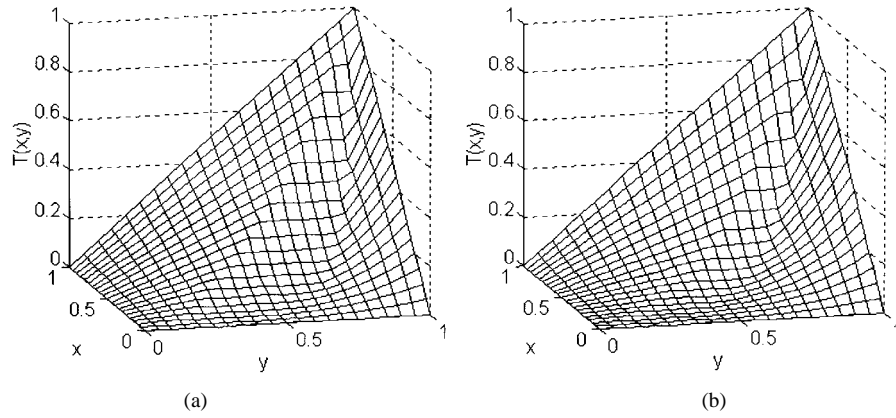(a)   (b)

Fig. 6.   Conjunction in Example 3.2. (a) For $p = 1.2$, $q = 4$. (b) For $p = 2$, $q = 4$.
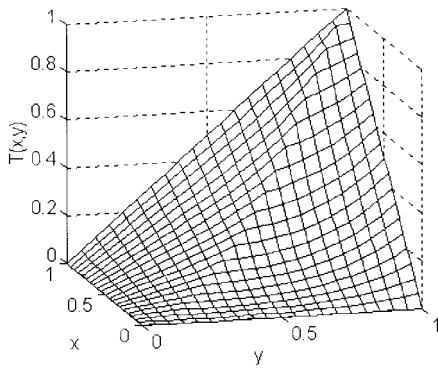

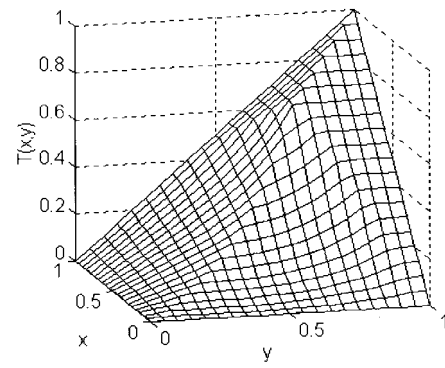Fig. 7.   Conjunction for $p = 1.2$, $q = 4$ in Example 3.3.

Fig. 8.   Conjunction for $p = 0.8$, $q = 4$ in Example 3.4.

for $p = 2$, $q = 4$ are shown on Fig. 6. When $p = q$ this conjunction has the following form:

$$T(x, y) = \begin{cases} yx^p, & \text{if } x > y \\ xy^p, & \text{if } x \leq y. \end{cases}$$

When $p = q = 1$ we have $T = T_p$.

Example 3.3: For $T_2 = T_p$ and $S = S_p$ we will obtain a new conjunction

$$T(x, y) = \min(x, y)(x^p + y^q - x^p y^q).$$

The graph of this conjunction for $p = 1.2$, $q = 4$ is shown on Fig. 7.

Example 3.4: For $T_2 = T_p$ and $S = S_b$ we will obtain the following conjunction:

$$T(x, y) = \min(x, y) \min(1, x^p + y^q).$$

The graph of this conjunction for $p = 0.8$, $q = 4$ is shown on Fig. 8.

Example 3.5: For $T_2 = T_c$, $S = S_c$ and two generators $g_D(x)$ and y^q we will obtain a following conjunction:

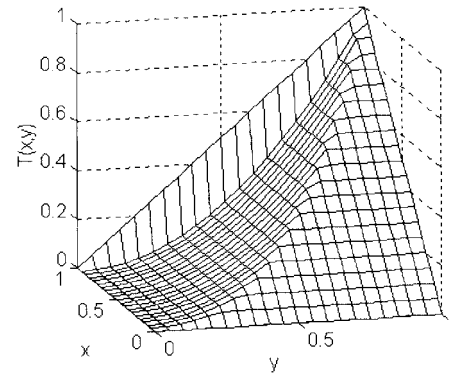$$T(x, y) = \min\{\min(x, y), \max(g_D(x), y^q)\}.$$

Fig. 9.   Conjunction for $q = 2$ in Example 3.5.

We have $T(x, y) = T_c$ for $q \leq 1$ and $T(x, y) \rightarrow T_d$ when $q \rightarrow \infty$. For $q \geq 1$ this conjunction can be represented as

$$T(x, y) = \begin{cases} y, & \text{if } x = 1 \\ \min(x, y^q), & \text{otherwise.} \end{cases}$$

The graph of this conjunction for $q = 2$ is shown on Fig. 9. An interchange of the variables x and y will result in another conjunction.

Example 4: We have considered conjunctions based on relations (9) and (12). Other types of conjunctions may be based on relations (9) and (13). For example by means of
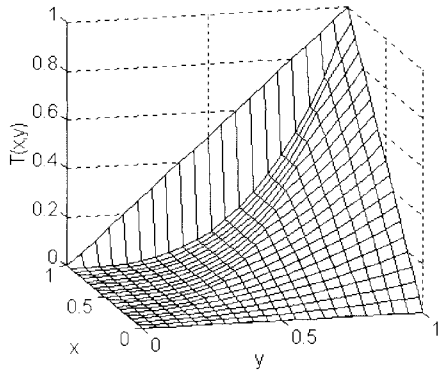
Fig. 10.   Conjunction for $q = 2$ in Example 5.

Fig. 11.   Surface of initial model.

Fig. 12.   Surface of final model.

generator $g(x, p) = x^p$ and T-norms $T_2 = T_p$ and $T_1 = T_c$ we will obtain the following conjunction:

$$T(x, y) = \min(x, y)(x + y - xy)^p$$

varying from T_c (when $p = 0$) to T_d (when $p \to \infty$).

Example 5: Let us propose another parametric class of conjunctions based on representations (9) and (14) with $T_1 = T_c$, $T_2 = T_p$, $s_1 = s_D$, $h(y) = y^p$, $s_2 = S_c$

$$T(x, y) = \min(x, y) \max(s_D(x, y), y^p)$$
$$= \begin{cases} \min(x, y), & \text{if } \max(x, y) = 1 \\ y^p \min(x, y), & \text{otherwise.} \end{cases}$$

We have $T = T_c$ when $p = 0$ and $T = T_d$ when $p \to \infty$. The graph of this conjunction for $q = 2$ is shown on Fig. 10.

Below one can find the conjunctions based on $T_1 = T_p$

$$T(x, y) = (xy) \max(x^p, y^q)$$
$$T(x, y) = xy(x^p + y^q - x^p y^q)$$
$$T(x, y) = (xy) \min(1, x^p + y^q).$$

These conjunctions vary from T_p to T_d.

The following conjunction is based on representations (9) and (14) with $T_1 = T_c$, $T_2 = T_p$, $s_1 = s_2 = S_c$, $h(y) = p$:

$$T(x, y) = \min(x, y) \max(x, y, p)$$
$$= \begin{cases} p \min(x, y), & \text{if } x, y \leq p \\ xy, & \text{otherwise,} \end{cases} \quad p \in [0, 1]$$

and varies from $T = T_c$ when $p = 1$ to $T = T_p$ when $p = 0$.

VII. EXAMPLE OF MODELING BY NEW OPERATIONS

In this section, the approximation of a given function $f(x, y)$ by a fuzzy inference system (FIS) is considered. Let the function $f(x, y)$ be the surface described by a first order Sugeno FIS with two inputs and one output. Each input variable has two terms: S (SMALL) and L (LARGE) defined by trapezoidal membership functions on $[0, 1]$ and FIS consists
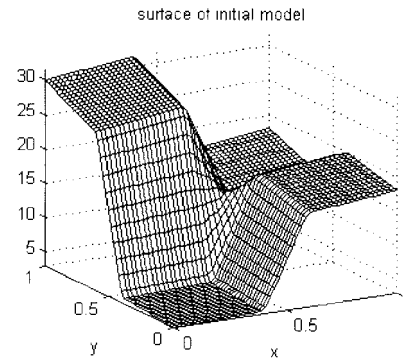
of the following four rules:

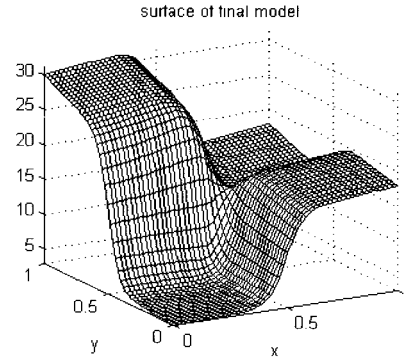IF x is S AND y is S THEN $z = x + 2y + 3$
IF x is S AND y is L THEN $z = 4x + 10y + 20$
IF x is L AND y is S THEN $z = 3x + 5y + 15$
IF x is L AND y is L THEN $z = 4x + 8y + 6.$

In this FIS, let us first use the min operation as the conjunction operation representing the connective AND. The surface of the function defined by this FIS is shown on Fig. 11.

Let us now approximate this initial FIS by the same FIS where trapezoidal membership functions S and L are replaced by triangular membership functions and min operation is replaced by the parametric operation $T(x, y) = \min(x, y)(x^p + y^q - x^p y^q)$. Four parameters p_S, p_L, q_S, and q_L are used in this operation for processing the membership value of x in S and L, y in S and L, respectively. For example, the firing value of the second rule is calculated as

$$\alpha = T(\mu_S(x^*), \mu_L(y^*))$$
$$= \min(\mu_S(x^*), \mu_L(y^*))(\mu_S(x^*)^{p_S} + \mu_L(y^*)^{q_L}$$
$$- \mu_S(x^*)^{p_S} \mu_L(y^*)^{q_L}).$$

The values of the parameters p and q are obtained as a result of the minimization of the mean-squared error between the surface of the initial FIS and the surface of the approximating FIS. Fifty points grid are used in each scale and, as result, 2500 points from initial surface is used for approximation. The optimal values of parameters are the following: $p_S = 6.45$, $p_L = 6.45$, $q_S = 5.89$, $q_L = 5.89$. The surface of the optimal
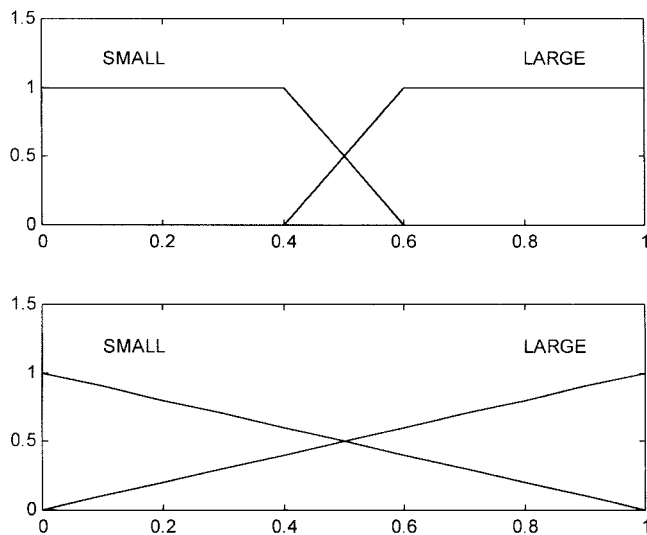
Fig. 13. Membership functions of initial and final inference systems.

approximating FIS is shown on Fig. 12. The membership functions of fuzzy sets used in these models are presented on Fig. 13.

VIII. CONCLUSIONS

In the construction of optimal fuzzy models, the optimization of the parameters of the fuzzy operations may be used instead of or in addition to the optimization of the parameters of the fuzzy sets. However, the known parametric classes of T-norms and T-conorms are, in general, far too complicated for optimization procedures and hardware realization. It is therefore desirable to have simpler parametric classes of conjunction and disjunction operations. This is the motivation behind the work reported in this paper. A novel approach to the construction of parametric classes of conjunction and disjunction operations is proposed, based on the consideration that the inference procedures of fuzzy models do not always require commutativity and associativity of the operations used. Several such parametric classes of conjunction operations are derived that are simpler than the known parametric classes of T-norms. The efficacy of the proposed operation is tested on the approximation of a given function by a fuzzy inference system in which the proposed parametric class of conjunctions is used. It is seen that the parameters of the conjunction operation can easily be tuned and the resulting FIS is a good approximation of the given function.

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