Euler Method

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1 Derivation

We want to find the solution $u(t)$ in the time domain $[a, b]$ to the first order ODE given by

$$u' = f(u, t), \quad u(t_0) = u_0, \quad t_0 = a. \tag{1}$$

In order to numerically approximate the solution we begin by discretising the domain into $N + 1$ equidistant points in time given by $t_{n+1} = t_n + h$ where h is the constant time step $h = \frac{b-a}{N}$.

The derivation of the Euler method and its algorithm is given below.

$$u_{n+1} = u(t_{n+1}) = u(t_n + h). \tag{2}$$

Then after taylor expansion about the point t_n we get

$$u(t_n + h) = u(t_n) + hu'(t_n) + O(h^2). \tag{3}$$

Disregarding the h^2 terms and higher we obtain

$$u_{n+1} = u_n + hu'(t_n). \tag{4}$$

Then using equation (1) we acquire the desired result

$$u_{n+1} = u_n + hf(u_n, t_n). \tag{5}$$

Algorithm 1 Euler Method

Require: $a, b, N, u_0 = u(a)$

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h ← (b−a)/N
t_0 ← a
for n in 0, 1, 2, ..., N do
    u_{n+1} ← u_n + h f(u_n, t_n)
    t_{n+1} ← t_n + h
end for
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