LTE Quick Reference - BeamForming   Home : www.sharetechnote.com

What is BeamForming ? It means just as it sounds. BeamForming is a techology to 'Form' a 'beam'.
Then what does it mean by 'beam' in this context ? I would say it means 'electromagnatic wave radiation pattern(propagation pattern) for a set of antenna system'. Simply put, BeamForming is a technic that constuct the antenna radation pattern as shown in < Case 2 > of the following illustration.

High level meaning is simple like this.. but real implementation would be very complicated and is out of my understanding. So I would just give you only a big picture of this technology.

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Motivation (Why we need BeamForming ?)

Why we need beamforming ?
It is simple. Let's look at the two illustrations as shown below. There are two antenna system and let's assume that the two antenna is transmitting the exactly same amount of total energy.
In case 1, the antenna system is radiating the energy in almost same amount in all direction. The three UEs around the antenna would receive almost same amount of the energy but a large portions of energy not directed to those UEs is wasted.
In case 2, the signal strength of the radiation pattern ('beam') is specially 'formed' in such a way that the radiated energy in direction to UEs are much stroger than the other parts which is not directed to UEs.

< Case 1 >   < Case 2 >
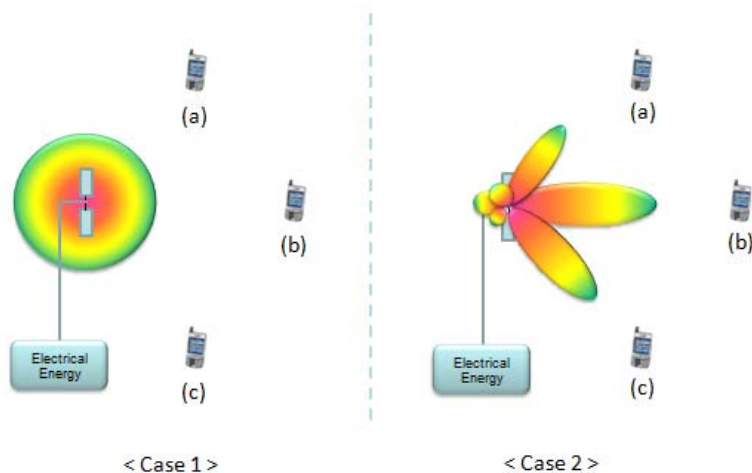
Technology for BeamForming

There are several different ways to implement the beamforming. Followings are couple of techniques most commonly used (As I mentioned, the details of implementation is out of my understanding).

For a little bit further details, see All Beamforming Solutions Are Not Equal. This is mainly for WLAN, but can be a good introduction.

Switched Array Antenna : This is the technique that change the beam pattern (radiation form) by switching on/off antenna selectively from the array of a antenna system.

DSP Based Phase Manipulation : This is the technique that change the beam pattern (radiation form) by changing the phase of the signal going through each antenna. Using DSP, you can change the signal phase for each antenna port differently to form a specific beam pattern that is best fit for one or multiple specific UEs.

Beamforming by Precoding : This is the technique that change the beam pattern (radiation form) by applying a specific precoding matrix. This is the technique used in LTE. In LTE, following transmission mode is implemeting 'BeamForming' implictely or explicitely.
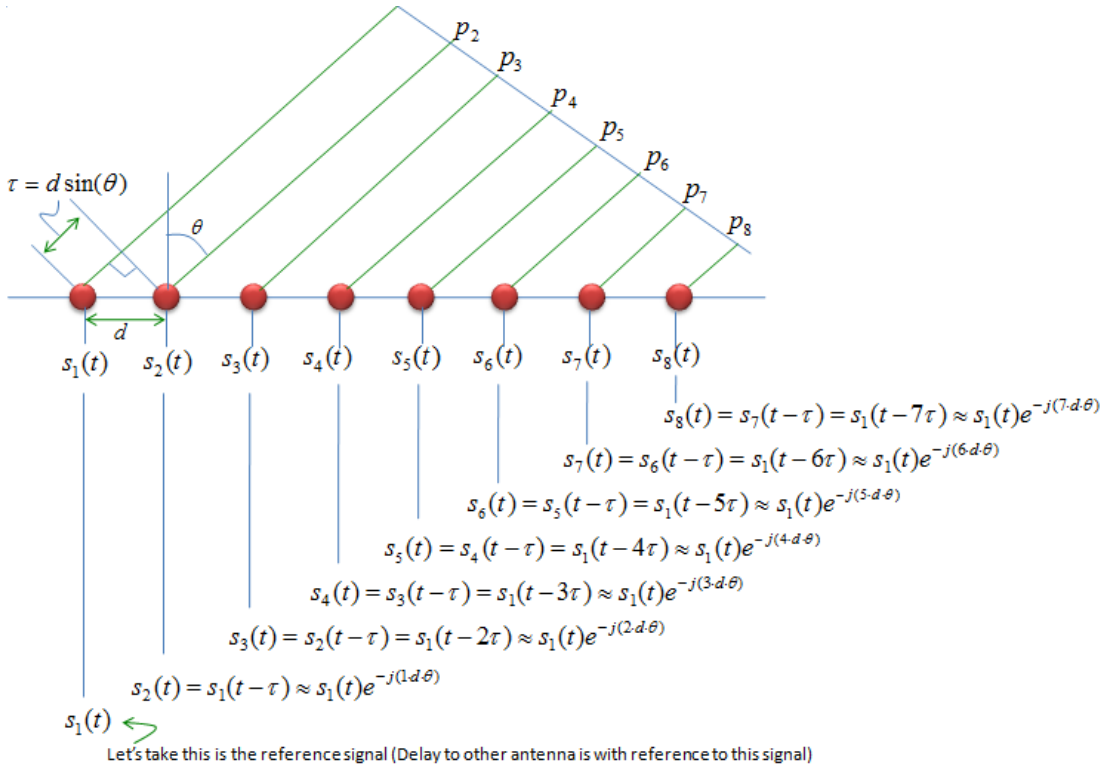
- TM 6 - Closed loop spatial multiplexing using a single transmission layer.
- TM 7 - Beamforming (Antenna port 5)
- TM 8 - Dual Layer Beamforming (Antenna ports 7 and 8)

Basic Concept : Array Antenna

Since the BeamForming is mostly based on Array Antenna, let's take a look at the basic principles of Array Antenna. Let's suppose we have a antenna array in which each of the antenna is placed apart from each other with a certain distance (d). And then, suppose multiple rays of beam transmitted from a single source is received by each of the antenna in the array. If all the rays from the single source is coming directly (meaning theta in the following illustration is 0) into the antenna, there would be no differences in terms of distance along which each of the rays travelled from the source to each of the reciever antenna. So if you sum up the energy recieved by each antenna, it will be same as each of the wave sum up constructively. However, if the direction of the ray and the axis of antenna array is not in right angle (meaning the theta is not 0), the travel distance of each ray (p1,p2,..,p8) gets different and the difference can be expressed as d sin(theta) as shown in the following illustration.
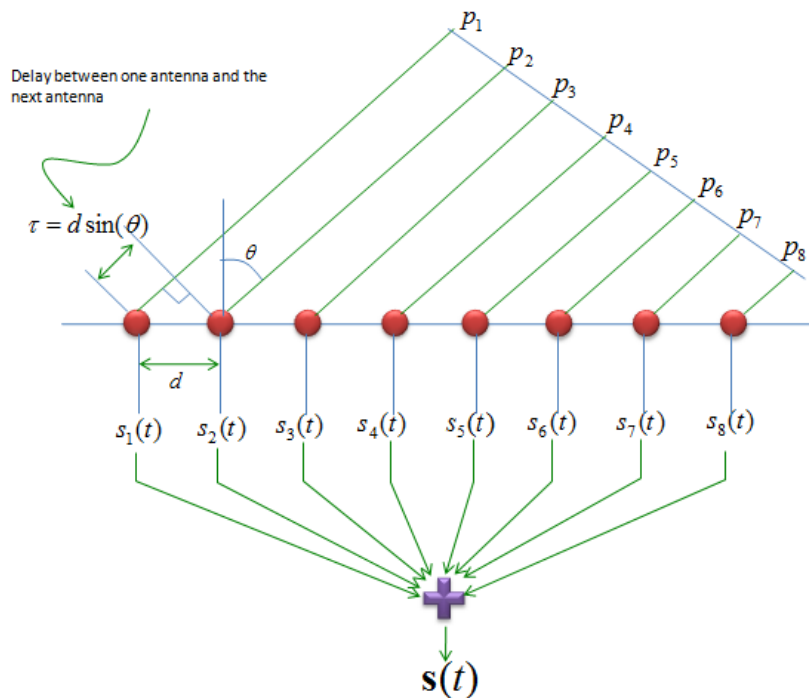
Now let's describe what I explained above into mathematical expression. If I take the received signal at the first antenna (the leftmost antenna) as the reference antenna and the signal received at each of the antenna can be represented as follows. (This would the point where many people gave up further reading/study because of the math. But in most of engineering and science, there are many cases where you can never understand completely without a certain degree of math. So, don't give up just because of the math... and try to understand the meaning of each mathematica terms. When I am explaining about this array antenna, I first talked about the travel path difference and angle between antenna array axis and the beam ray hitting the antenna. In the mathematical expression, you still see the angle term 'theta', but you don't see any direct term that look like 'travel path'. Actually we don't know the exact travel difference of each beam because we don't know the exact location of the beam source. The only thing we know is the travel path difference between each beam hitting on each antenna. That travel path difference can be expressed as e^(-j * n * d * theta) where n is the index of the antenna being numbered 0 from the leftmost antenna. Keeping this in mind, take close loot at each mathematical expression and it would make sense to you. If you are not familiar with interpreting the meaining of e^(-theta), you may refer to complex number page).

$\tau = d\sin(\theta)$

$s_8(t) = s_7(t-\tau) = s_1(t-7\tau) \approx s_1(t)e^{-j(7 \cdot d \cdot \theta)}$

$s_7(t) = s_6(t-\tau) = s_1(t-6\tau) \approx s_1(t)e^{-j(6 \cdot d \cdot \theta)}$

$s_6(t) = s_5(t-\tau) = s_1(t-5\tau) \approx s_1(t)e^{-j(5 \cdot d \cdot \theta)}$

$s_5(t) = s_4(t-\tau) = s_1(t-4\tau) \approx s_1(t)e^{-j(4 \cdot d \cdot \theta)}$

$s_4(t) = s_3(t-\tau) = s_1(t-3\tau) \approx s_1(t)e^{-j(3 \cdot d \cdot \theta)}$

$s_3(t) = s_2(t-\tau) = s_1(t-2\tau) \approx s_1(t)e^{-j(2 \cdot d \cdot \theta)}$

$s_2(t) = s_1(t-\tau) \approx s_1(t)e^{-j(1 \cdot d \cdot \theta)}$

$s_1(t)$

Let's take this is the reference signal (Delay to other antenna is with reference to this signal)

Now we formulated the signal coming into each antenna. Now let's formulate the signal that combined all the signal coming into each antenna. It is simple, you only have sum up all the signals comining into each antenna and it can be presented as follows.

$$|\mathbf{s}(t)| = |s_1(t) + s_2(t) + s_3(t) + s_4(t) + s_5(t) + s_6(t) + s_7(t) + s_8(t)|$$

$$= |(1 + e^{-j(1 \cdot d \cdot \theta)} + e^{-j(2 \cdot d \cdot \theta)} + e^{-j(3 \cdot d \cdot \theta)} + e^{-j(4 \cdot d \cdot \theta)} + e^{-j(5 \cdot d \cdot \theta)} + e^{-j(6 \cdot d \cdot \theta)} + e^{-j(7 \cdot d \cdot \theta)})s_1(t)|$$

You can represent this equation into a vector notation as follows. (Many people even including me get intimidated when seeing this kind of matrix/vector expression. Don't get scared. We have already learned basic operations from high school or freshmen university math course. We just didn't have enough chance for enough practice. So if you get somehow uneasy feeling about this, just expand the following equation by hand and you will get more familiar expression as above. Doing this kind of practice whenever you see this kind of vector/matrix notation, gradually you will get more and more familiar and such an uneasy feeling would disappear. Nobody else can do this kind of practice for you)

$$|\mathbf{s}(t)| = \left| \begin{bmatrix} 1 \\ e^{-j(1 \cdot d \cdot \theta)} \\ e^{-j(2 \cdot d \cdot \theta)} \\ e^{-j(3 \cdot d \cdot \theta)} \\ e^{-j(4 \cdot d \cdot \theta)} \\ e^{-j(5 \cdot d \cdot \theta)} \\ e^{-j(6 \cdot d \cdot \theta)} \\ e^{-j(7 \cdot d \cdot \theta)} \end{bmatrix} s_1(t) \right|$$

Norm = sum of all elements

Vector (Representing the vector into $\mathbf{a}(\theta)$ for simplicity)

Scalar (removed the subscription just for simplicity)

$$= |\mathbf{a}(\theta)s(t)|$$

Give a single number

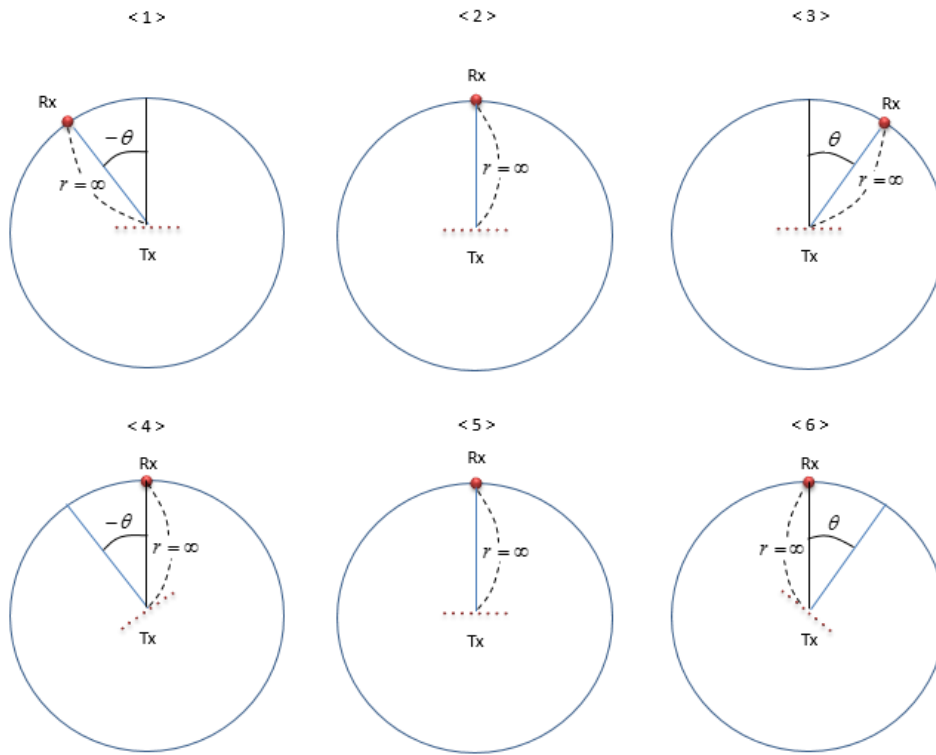## Charecterization of Array Antenna - Beam Pattern Plot

When you characterize (represent the performance of an Antenna array), you might have seen the types of plots as shown below. In some documents, you would see the plot like the one on the left and in some other document you would see the one on the right. Actually these two represents the exact the same thing. They just plot the same thing in two different coordinate system. The one on the left is the representation of (trancemitted or received power) vs angle in Cartesian coordinate and the right one is the representation of the same data in polar coordinate.

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Beam Pattern
- Cartesian Plot -

Beam Pattern
- Polar Plot -
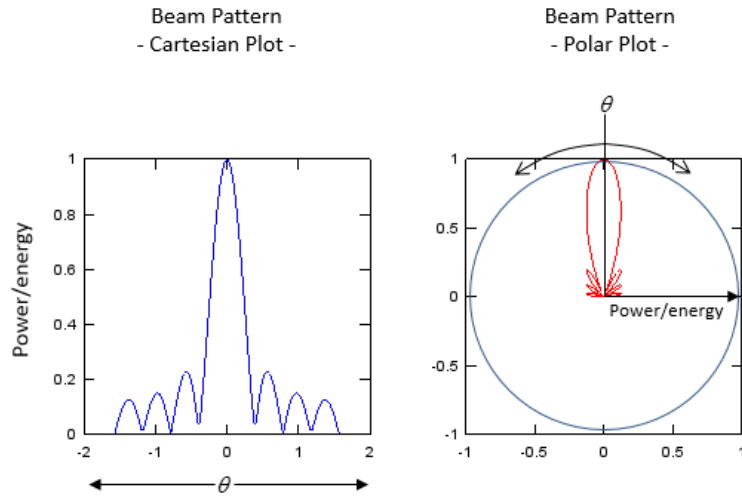
Power and Energy in the plot would be familiar to you. Then, what does the theta (angle) represents ? It is the angle between the direction of ray and the axis of antenna array. If reciever is the array antenna and the transmitter is a single transmitter far enough from the reciever antenna, the angle can be represented as follows. In this illustration, I set the distance (r) to be infinite to assume that the all the rays reaches the reciever antenna in paralelle. It reality, the distance cannot be infinite. it is just far enough to make the assumption meaningful. You can set the angle (theta) by placing the transmitter antenna to different location on the circumference (as in the top track) or by rotating the antenna array (as in the bottom track). In this case, you may think that the plots shown above reprents total power recieved by all the antenna on the array.
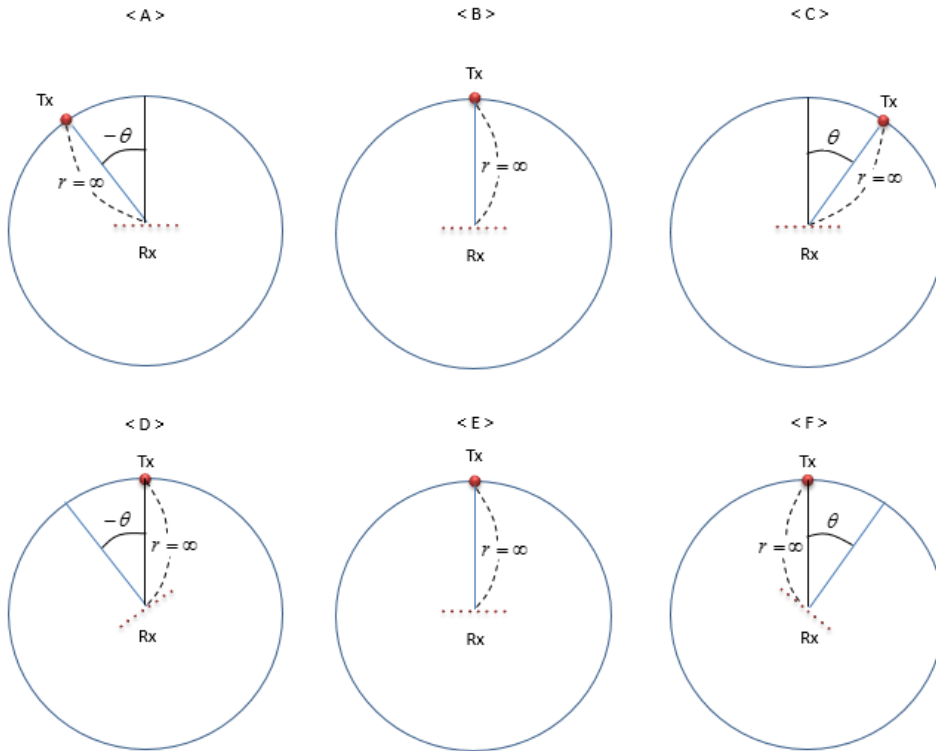

The same logic applies when the transmitter antenna is the array antenna and the reciever antenna is a point far away from the transmitter. The angle is the angle between the direction of ray and the axis of antenna array. The angle can be represented as follows. In this illustration, I set the distance (r) to be infinite to assume that the all the rays reaches the reciever antenna in paralelle. It reality, the distance cannot be infinite. it is just far enough to make the assumption meaningful. You can set the angle (theta) by placing the recieving antenna to different location on the circumference (as in the top track) or by rotating the antenna array (as in the bottom track). In this case, you may think that the plots shown above represents total power recieved by the reciever antenna.

<1> <2> <3>

Rx
Rx
Rx

$-\theta$

$r = \infty$

θ

$r = \infty$

$r = \infty$

Tx   Tx   Tx

<4> <5> <6>

Rx   Rx   Rx

$-\theta$

$r = \infty$

$r = \infty$

θ

$r = \infty$

Tx   Tx   Tx

In above illustration, you have seen four different cases of representing the angle. Even though the function of the antenna array (being reciever or transmitter) is different and the way making the angle (by moving reciever/transmitter antenna by rotating the antenna array), the overall shape of the plot is same (at least in theory). The only difference would be the scale of Engergy/Power depending on whether it represents the reciever power or transmitter power. So if we ignoare the scale of the engery/power axis, you can interpret the graph (Power/energy vs theta) as shown below.

As you see here, the energy/power gets maximum when the angle (theta) is 0. To get the max energy when the angle is zero, the distance between one antenna in the array and another antenna next to it should be a half of wave length.

<A>,<D>   <B>,<E>   <C>,<F>
<1>,<4>   <2>,<5>   <3>,<6>
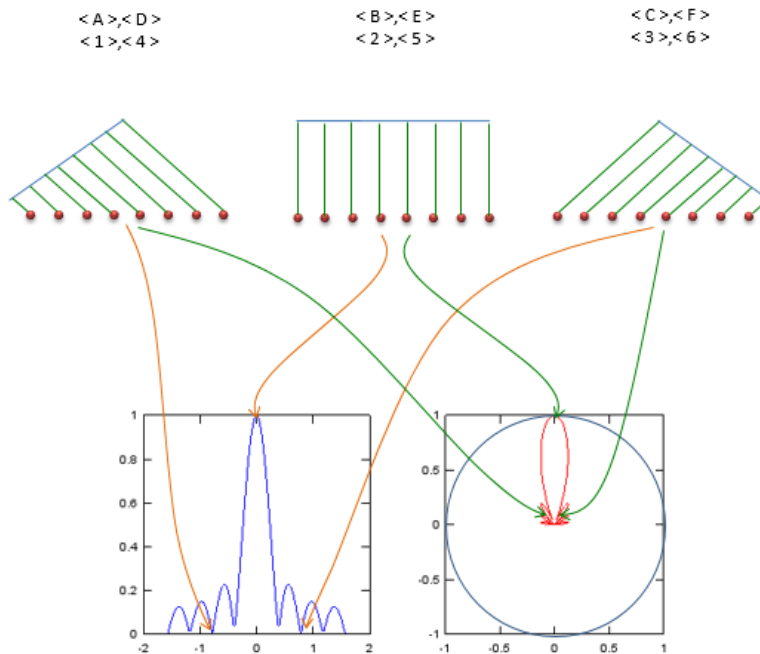
If you want to play with this graph, try running the follow matlab toy code.

```
theta = -0.5*pi:pi/100:0.5*pi;
d = 2.0;
a_theta = [exp(-j .* 0 .* d .* theta) ;
        exp(-j .* 1 .* d .* theta);
        exp(-j .* 2 .* d .* theta);
        exp(-j .* 3 .* d .* theta);
        exp(-j .* 4 .* d .* theta);
        exp(-j .* 5 .* d .* theta);
        exp(-j .* 6 .* d .* theta);
        exp(-j .* 7 .* d .* theta)];
a_theta_sum = sum(a_theta);
a_theta_sum_abs = abs(a_theta_sum);
a_theta_sum_abs = a_theta_sum_abs ./ max(a_theta_sum_abs);
```

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```
a_theta_sum_abs_dB = 10 .* log(a_theta_sum_abs);

for i = 1:length(a_theta_sum_abs_dB)
   if a_theta_sum_abs_dB(i) <= -30
      a_theta_sum_abs_dB(i) = -30;
   end;
end;

a_theta_sum_abs_dB = a_theta_sum_abs_dB - min(a_theta_sum_abs_dB);
a_theta_sum_abs_dB = a_theta_sum_abs_dB/max(a_theta_sum_abs_dB);

subplot(2,2,1);plot(theta,a_theta_sum_abs);
xlim([-pi/2 pi/2]);
set(gca,'xtick',[-pi/2 -pi/4 0 pi/4 pi/2]);
set(gca,'xticklabel',{'-pi/2' '-pi/4' '0' 'pi/4' 'pi/2'});

subplot(2,2,2);polar(theta + 0.5*pi,a_theta_sum_abs,'-r');
t = findall(gcf,'type','text');
delete(t);

subplot(2,2,3);plot(theta,10 .* log(a_theta_sum_abs));ylim([-30 1]);
xlim([-pi/2 pi/2]);
set(gca,'xtick',[-pi/2 -pi/4 0 pi/4 pi/2]);
set(gca,'xticklabel',{'-pi/2' '-pi/4' '0' 'pi/4' 'pi/2'});

subplot(2,2,4);polar(theta + 0.5*pi,a_theta_sum_abs_dB,'-r');
t = findall(gcf,'type','text');
delete(t);
```
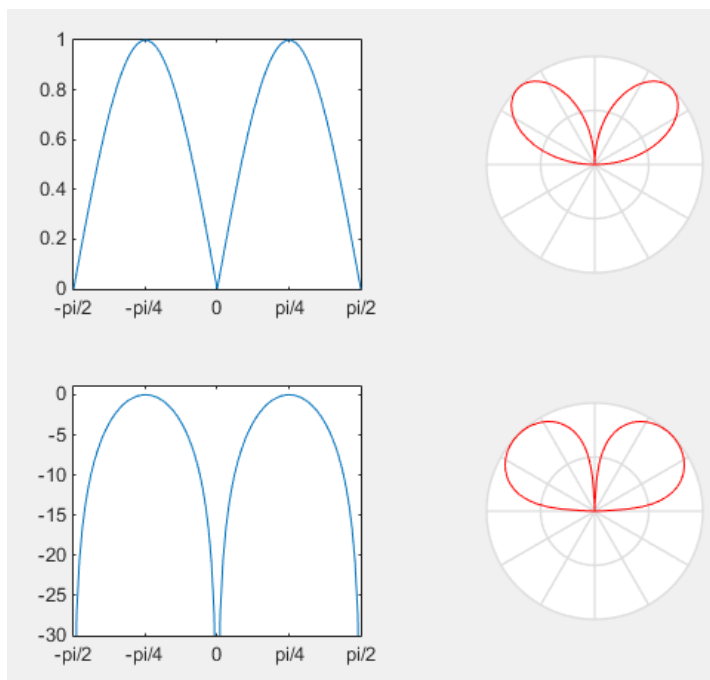
By tweaking the phase of each antenna, you can create a beam pointing to multiple direction as shown the following example. (Upper track is the power/energy in linear scale and Lower track is the power/energy in dB scale)

Following is the matlab code for the plot shown above. Play with this by changing theta_shift or any other values in a_theta array as you like until you get the intuitive understanding of each parameters.

```
theta = -0.5*pi:pi/100:0.5*pi;
d = 2.0;
theta_shift = (0.25*pi);

a_theta = [exp(-j .* 0 .* d .* (theta-theta_shift));
          exp(-j .* 1 .* d .* (theta-theta_shift));
          exp(-j .* 2 .* d .* (theta-theta_shift));
          exp(-j .* 3 .* d .* (theta-theta_shift));
          exp(-j .* 0 .* d .* (theta+theta_shift));
          exp(-j .* 1 .* d .* (theta+theta_shift));
          exp(-j .* 2 .* d .* (theta+theta_shift));
          exp(-j .* 3 .* d .* (theta+theta_shift))];

a_theta_sum = sum(a_theta);
a_theta_sum_abs = abs(a_theta_sum);
a_theta_sum_abs = a_theta_sum_abs ./ max(a_theta_sum_abs);
a_theta_sum_abs_dB = 10 .* log(a_theta_sum_abs);
for i = 1:length(a_theta_sum_abs_dB)
   if a_theta_sum_abs_dB(i) <= -30
```

```
        a_theta_sum_abs_dB(i) = -30;
    end;
end;

a_theta_sum_abs_dB = a_theta_sum_abs_dB - min(a_theta_sum_abs_dB);
a_theta_sum_abs_dB = a_theta_sum_abs_dB/max(a_theta_sum_abs_dB);

subplot(2,2,1);plot(theta,a_theta_sum_abs);
xlim([-pi/2 pi/2]);
set(gca,'xtick',[-pi/2 -pi/4 0 pi/4 pi/2]);
set(gca,'xticklabel',{'-pi/2' '-pi/4' '0' 'pi/4' 'pi/2'});

subplot(2,2,2);polar(theta + 0.5*pi,a_theta_sum_abs,'-r');
t = findall(gcf,'type','text');
delete(t);

subplot(2,2,3);plot(theta,10 .* log(a_theta_sum_abs));ylim([-30 1]);
xlim([-pi/2 pi/2]);
set(gca,'xtick',[-pi/2 -pi/4 0 pi/4 pi/2]);
set(gca,'xticklabel',{'-pi/2' '-pi/4' '0' 'pi/4' 'pi/2'});

subplot(2,2,4);polar(theta + 0.5*pi,a_theta_sum_abs_dB,'-r');
t = findall(gcf,'type','text');
delete(t);
```

Basic Concept : Beam Forming - Phased Array

In previous sections on basic Antenna array mode, we could see that just by arraning multiple antenna in an array we could create a beam with the directivity to a certain direction and by physically rotating the antenna array we can change the direction of the beam. However, there will be a lot of restrictions in terms of the shape of the beam and controlling the direction in simply placing multiple antenna in an array.

There are even smarter idea as invented as shown below. You can do everything explained above and do even more by placing the devices that can control phase and amplitude of a wave to each antenna as illustrated below.
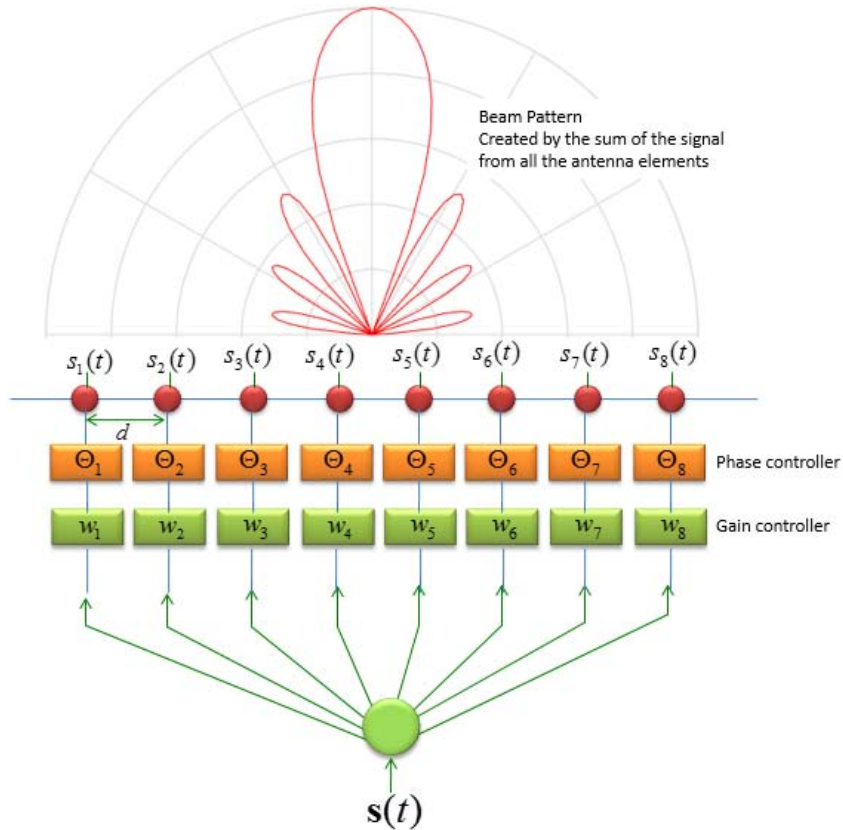
The total energy pattern (beam pattern) created by summing up all the transmitted energy coming out of each antenna can be described in mathematical form as shown below.

$$\left|\mathbf{s}(t)\right| = \left|s_1(t) + s_2(t) + s_3(t) + s_4(t) + s_5(t) + s_6(t) + s_7(t) + s_8(t)\right|$$

$$= \left|\left(\begin{array}{c} w_1 \cdot e^{-j(1 \cdot d \cdot \Theta_1)} + w_2 \cdot e^{-j(2 \cdot d \cdot \Theta_1)} + w_3 \cdot e^{-j(3 \cdot d \cdot \Theta_1)} + w_4 \cdot e^{-j(4 \cdot d \cdot \Theta_1)} \\ + w_5 \cdot e^{-j(5 \cdot d \cdot \Theta_1)} + w_6 \cdot e^{-j(6 \cdot d \cdot \Theta_1)} + w_7 \cdot e^{-j(7 \cdot d \cdot \Theta_1)} + w_8 \cdot e^{-j(8 \cdot d \cdot \Theta_1)} \end{array}\right) s(t)\right|$$

Again, you can simplify this long equation into a simple vector notation as shown below. Theoretically, you can create a beam with any shape and any direction by changing the gain and phase in the vector.

$$|\mathbf{s}(t)| = \left| \begin{bmatrix} w_1 \cdot e^{-j(1 \cdot d \cdot \Theta_1)} \\ w_2 \cdot e^{-j(2 \cdot d \cdot \Theta_2)} \\ w_3 \cdot e^{-j(3 \cdot d \cdot \Theta_3)} \\ w_4 \cdot e^{-j(4 \cdot d \cdot \Theta_4)} \\ w_5 \cdot e^{-j(5 \cdot d \cdot \Theta_5)} \\ w_6 \cdot e^{-j(6 \cdot d \cdot \Theta_6)} \\ w_7 \cdot e^{-j(7 \cdot d \cdot \Theta_7)} \\ w_8 \cdot e^{-j(8 \cdot d \cdot \Theta_8)} \end{bmatrix} s(t) \right|$$

Modeling of Beam Forming

As I mentioned above, theorectically you can create a beam with any shape and any angle by chaning angle (phase) and gain connected to each antenna. Then the question is how to find proper angle (phase) and gain value to each antenna to achieve the desired beam pattern.

Long story.. but the conclusion is not that complicated. Let's asume that we have the simplest array antenna made up of just two antenna as shown below. The antenna array on the left is transmitter antenna and the single antenna on the right is the reciever antenna. The blocks labeled p1 and p2 is a complex number that representing phase and amplitude controller (As you know, a complex number can represent both phase and amplitude). h1 and h2 is channel coefficient between transmitter and reciever antenna.
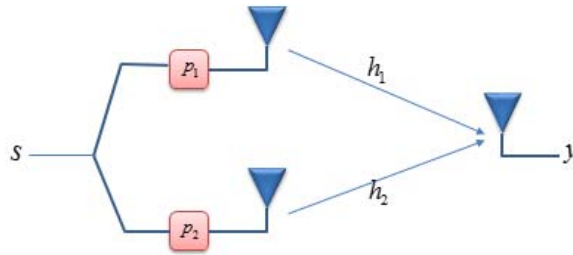
Now the question is how to represent the received signal y using these parameters. It can be presented as shown below (If you are not familiar with this kind of channel representation, refer to Channel Model page)

$$y = s \cdot p_1 \cdot h_1 + s \cdot p_2 \cdot h_2 = s \cdot (p_1 \cdot h_1 + p_2 \cdot h_2)$$

$$= \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} s$$

Now the question is how to find p1 and p2 value to make the beam properly point to the reciever antenna. If you rewrite the question a little bit, it can be 'what is the p1 and p2 value that maximize the received energy at the reciever antenna ?'. In real situation, h1 and h2 act as some factors distorting/deteriorating the signal between the transmitter and reciever antenna. So, if we can set p1 and p2 propery to compensate (undo) the effect of h1 and h2, that would be the p value to make the received energy maximum. h1 and h2 are complex numbers, meaning h1 and h2 has gain component and phase component. Therefore, the question becomes 'how to undo an effect caused by a complex number ?'.
Undoing a complex number is simple. Just multiplying a complex conjugate to the given complex number. (If you multiply a complex number with its conjugate, the phase of the resulting complex number become always '0', meaning that imaginary part become always 0. So in this kind of situation, we can say 'multiplying a complex number with its conjugate' is same as 'undoing the phase shift caused by the complex number'.
Therefore, if you put complex conjugate of h1 and h2 into p1 and p2 as shown below, this would undo the phase shifting effect of h1 and h2.
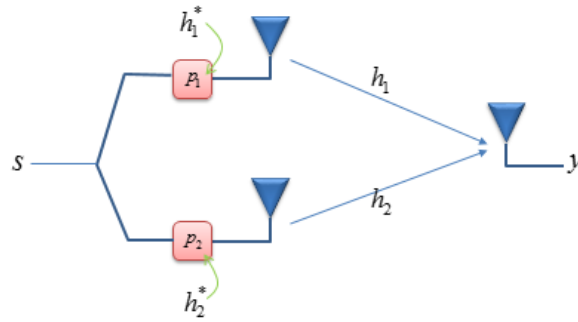
This looks very handy and simple solution. However, there is a problem with this kind of undo. By multiplying the channel coefficient with its conjudate, we can easily remove the phase part. However, the real value of the resulting number gets larger. To prevent this, you only have to normalize those conjugate numbers with the norm of h vector (a vetor made of h1 and h2) as shown below.

Now with this, we found the p1 and p2 to make the beam in the best direction to the reciever antenna.
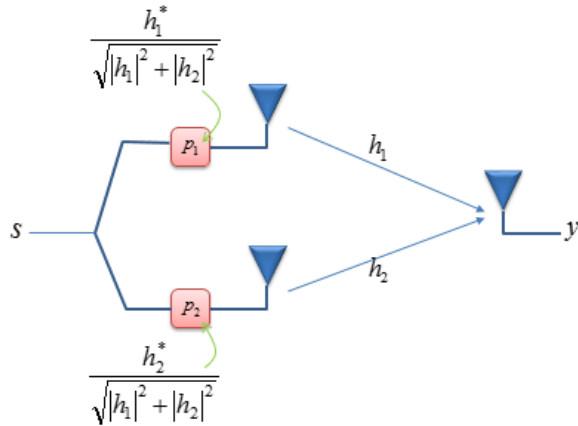
If you implement the p1 and p2 in DSP or FPGA, the illustration above is good enough. Because you can easily implement a complex number as it is. However, if we implement p1 and p2 as analog component, probably following illustration would be a better representation.
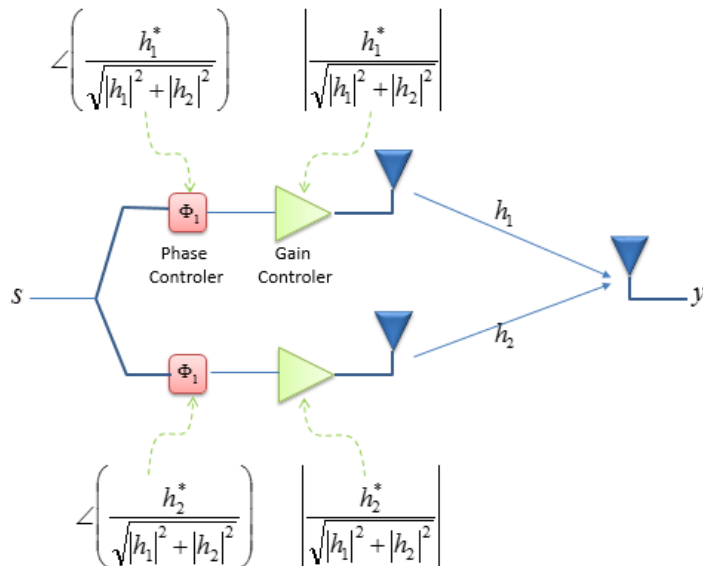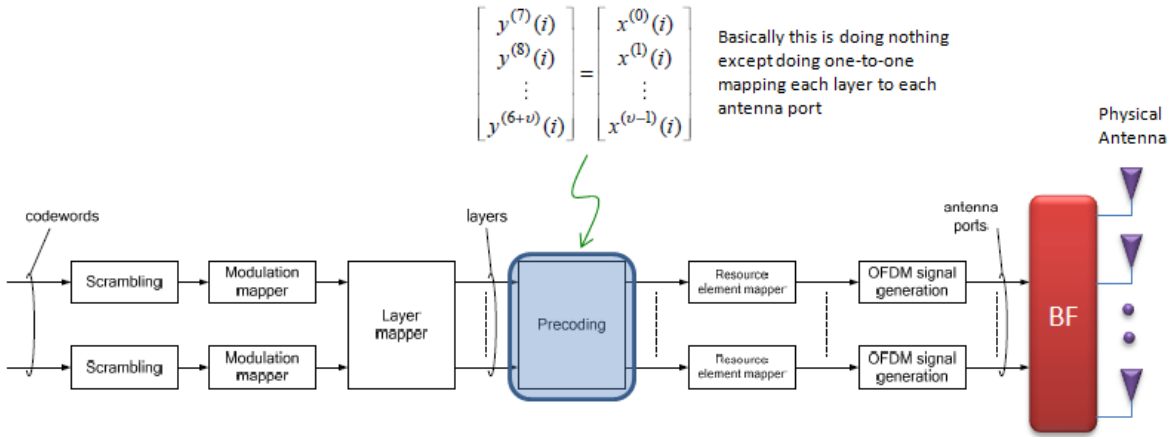

Beamforming in LTE

Overall procedure for LTE beamforming in the context of physical layer processing goes as shown below.

< Precoding for BeamForming >

In case of BeamForming, Precoding is doing almost nothing and in stead something similar to Precoding happens at the last stage labeled in BF(Beam Forming) stage.

36.211 - 6.3.4.4

$$\begin{bmatrix} y^{(7)}(i) \\ y^{(8)}(i) \\ \vdots \\ y^{(6+\upsilon)}(i) \end{bmatrix} = \begin{bmatrix} x^{(0)}(i) \\ x^{(1)}(i) \\ \vdots \\ x^{(\upsilon-1)}(i) \end{bmatrix}$$

Basically this is doing nothing except doing one-to-one mapping each layer to each antenna port

Basic Beamforming Model is described in 36.101 (B.4.1, B.4.2, B.4.3) and it propose three different categories as summarized below.

< Single-layer random beamforming (Antenna port 5, 7, or 8) : 36.101 B.4.1>

This type has following two cases. TM8, DCI Format 2B single layer would fall into this type.

i) without a simultaneous transmission on the other antenna port

36.211 Table 6.3.4.2.3-1:

Codebook index	Number of layers υ	
	1	2
0	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
1	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$
3	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$	-

Random Selection

$$\begin{bmatrix} y_{bf}(i) \\ \tilde{y}_{bf}(i) \end{bmatrix} = W(i)y^{(p)}(i)$$

$$p \in \{5,7,8\}$$

ii) with a simultaneous transmission on the other antenna port

36.211 Table 6.3.4.2.3-1:

Codebook index	Number of layers υ	
	1	2
0	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
1	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$
3	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$	-

Random Selection

$$\begin{bmatrix} y_{bf}(i) \\ \tilde{y}_{bf}(i) \end{bmatrix} = \frac{1}{\sqrt{2}}\left(W_1(i)y^{(7)}(i) + W_2(i)y^{(8)}(i) \right)$$

< Dual-layer random beamforming (antenna ports 7 and 8) : 36.101 B.4.2>

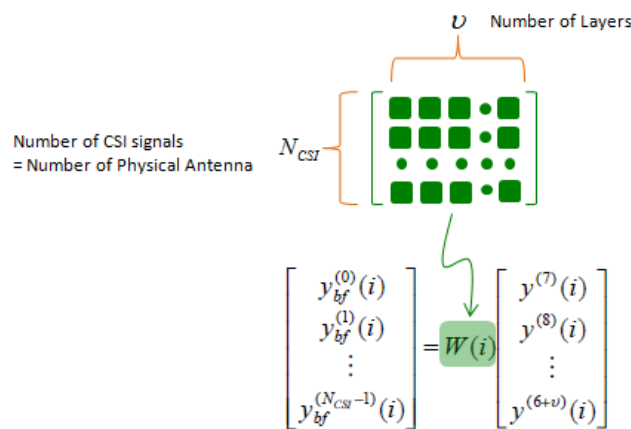TM8, DCI Format 2B dual layer would fall into this type.

36.211 Table 6.3.4.2.3-1:

Codebook index	Number of layers υ	
	1	**2**
0	$\dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	$\dfrac{1}{\sqrt{2}}\begin{bmatrix}1&0\\0&1\end{bmatrix}$
1	$\dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$	$\dfrac{1}{2}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$
2	$\dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$	$\dfrac{1}{2}\begin{bmatrix}1&1\\j&-j\end{bmatrix}$
3	$\dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}$	-

Random Selection

$$\begin{bmatrix} y_{bf}(i) \\ \tilde{y}_{bf}(i) \end{bmatrix} = W(i) \begin{bmatrix} y^{(7)}(i) \\ y^{(8)}(i) \end{bmatrix}$$

< Generic beamforming model (antenna ports 7-14) : 36.101 B.4.3 >

TM9, DCI Format 2C dual layer would fall into this type.

$$\begin{bmatrix} y_{bf}^{(0)}(i) \\ y_{bf}^{(1)}(i) \\ \vdots \\ y_{bf}^{(N_{CSI}-1)}(i) \end{bmatrix} = W(i) \begin{bmatrix} y^{(7)}(i) \\ y^{(8)}(i) \\ \vdots \\ y^{(6+\upsilon)}(i) \end{bmatrix}$$

Unliked other model like B.4.1 and B.4.2, the definition of W(i) is not always obvious. The 36.101 B.4.3 says "The precoder matrix W(i) is specific to a test case.", it means we have to define a specific test case (situation) and then try to define the W(i).

Example 1 : 8 x 2 (8 Antenna, 2 Layers)

In this example, I will look into a case where two layered data is transmitted through 8 Tx antenna. Two UE specific reference signal p7, p8 are subject to BeamForming and 8 CSI RS ports (p15~p22) will be transmitted for CSI estimation on UE side. Overall mapping between antenna ports and physical antenna is as shown below.

< Mapping between Antenna port and physical antenna >
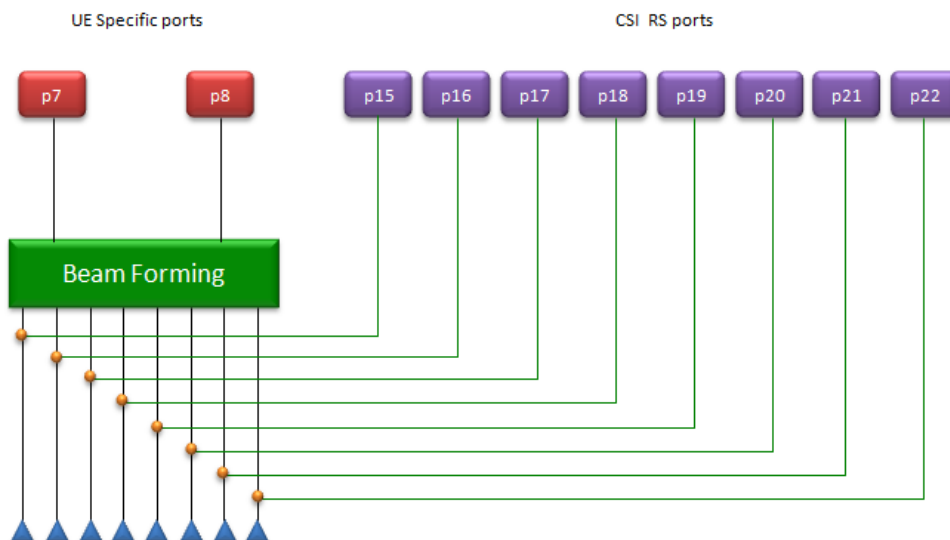
First, it seems obvious that we need something to distribute the two layered data over 8 physical antenna and it can be easily understood that we need following form of matrix.

< Beamforming Matrix converting two user ports to 8 physical antenna >

$$\begin{bmatrix} y_{bf}^{(0)}(i) \\ y_{bf}^{(1)}(i) \\ y_{bf}^{(2)}(i) \\ y_{bf}^{(3)}(i) \\ y_{bf}^{(4)}(i) \\ y_{bf}^{(5)}(i) \\ y_{bf}^{(6)}(i) \\ y_{bf}^{(7)}(i) \end{bmatrix} = W^{(2)}(i) \begin{bmatrix} y^{(7)}(i) \\ y^{(8)}(i) \end{bmatrix}$$
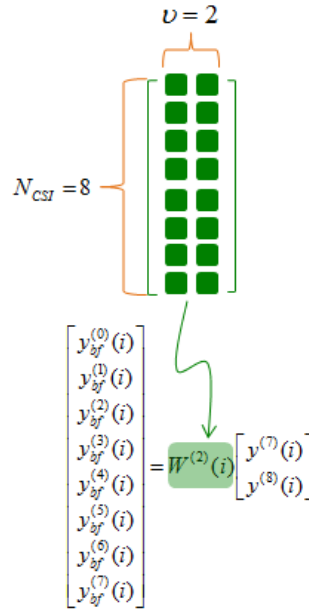
Now we have to figure out (or define) values of each matrix element. For this, we have to define our model more specifically.

36.213 7.2.4 Precoding Matrix Indicator (PMI) definition says as follows and this example is the case described by the underlined parts.

For transmission modes 4, 5 and 6, precoding feedback is used for channel dependent codebook based precoding and relies on UEs reporting precoding matrix indicator (PMI). For transmission mode 8, the UE shall report PMI if configured with PMI/RI reporting. For transmission mode 9, the UE shall report PMI if configured with PMI/RI reporting and the number of CSI-RS ports is larger than 1. A UE shall report PMI based on the feedback modes described in 7.2.1 and 7.2.2. For other transmission modes, PMI reporting is not supported

The exact value of the matrix is determined by UE PMI report and network construct the matrix W(i) based on following table.

36.213 Table 7.2.4-2 Codebook for 2-layer CSI reporting using antenna ports 15 to 22

i_1	i_2			
	0	**1**	**2**	**3**
$0-15$	$W_{2i_1,2i_1,0}^{(2)}$	$W_{2i_1,2i_1,1}^{(2)}$	$W_{2i_1+1,2i_1+1,0}^{(2)}$	$W_{2i_1+1,2i_1+1,1}^{(2)}$
i_1	i_2			
	4	**5**	**6**	**7**
$0-15$	$W_{2i_1+2,2i_1+2,0}^{(2)}$	$W_{2i_1+2,2i_1+2,1}^{(2)}$	$W_{2i_1+3,2i_1+3,0}^{(2)}$	$W_{2i_1+3,2i_1+3,1}^{(2)}$
i_1	i_2			
	8	**9**	**10**	**11**
$0-15$	$W_{2i_1,2i_1+1,0}^{(2)}$	$W_{2i_1,2i_1+1,1}^{(2)}$	$W_{2i_1+1,2i_1+2,0}^{(2)}$	$W_{2i_1+1,2i_1+2,1}^{(2)}$
i_1	i_2			
	12	**13**	**14**	**15**
$0-15$	$W_{2i_1,2i_1+3,0}^{(2)}$	$W_{2i_1,2i_1+3,1}^{(2)}$	$W_{2i_1+1,2i_1+3,0}^{(2)}$	$W_{2i_1+1,2i_1+3,1}^{(2)}$
where $W_{m,m',n}^{(2)} = \dfrac{1}{4}\begin{bmatrix} v_m & v_{m'} \\ \varphi_n v_m & -\varphi_n v_{m'} \end{bmatrix}$				

The first thing that confuses me was "Where is i_1 and i_2 is defined ?" and what do they mean (indicate) ? These are described in 36.213 section 7.2.1 "Wideband feedback -> Mode 1-2 description" as shown below. Of course it will take a long time to understand real meaning of these variable. (First you have to understand everything on CQI/RI Feedback type page first just to understand these i_1 and i_2)

The UE shall report the selected precoding matrix indicator for each set S subband except with 8 CSI-RS ports configured for transmission modes 9 and 10 in which case a first precoding matrix indicator i_1 is reported for the set S subbands and a second precoding matrix indicator i_2 is reported for each set S subband.

Following is more detailed information from 36.212 regarding the i1 and i2 report from UE.

< 36.212 - Table 5.2.3.3.1-3A: UCI fields for joint report of RI and i1 (transmission mode 9 configured with PMI/RI reporting with 2/4/8 antenna ports and transmission mode 10 configured with PMI/RI reporting with 2/4/8 antenna ports) >

Field	Bit width					
	2 antenna ports	4 antenna ports		8 antenna ports		
		Max 2 layers	Max 4 layers	Max 2 layers	Max 4 layers	Max 8 layers
Rank indication	1	1	2	4	5	5
i1	-	-	-			

< 36.212 - Table 5.2.3.3.2-2B: UCI fields for channel quality feedback for UE-selected sub-band reports (transmission mode 9 configured with PMI/RI reporting with 8 antenna ports and transmission mode 10 configured with PMI/RI reporting with 8 antenna ports) >

Field	Bit width			
	2 antenna ports		4 antenna ports	
	Rank = 1	Rank = 2	Rank = 1	Rank > 1
Wide-band CQI	0	0	0	0
Sub-band CQI	4	4	4	4
Spatial differential CQI	0	3	0	3
Wide-band i2	0	0	0	0
Sub-band i2	0	0	0	0
Sub-band label	1 or 2	1 or 2	1 or 2	1 or 2

< 36.212 - Table 5.2.3.3.2-2B: UCI fields for channel quality feedback for UE-selected sub-band reports (transmission mode 9 configured with PMI/RI reporting with 8 antenna ports and transmission mode 10 configured with PMI/RI reporting with 8 antenna ports) >

Field	Bit width									
	8 antenna ports									
	Rank = 1		Rank = 2, 3		Rank = 4		Rank = 5, 6, 7		Rank = 8	
	PTI=0	PTI=1	PTI=0	PTI=1	PTI=0	PTI=1	PTI=0	PTI=1	PTI=0	PTI=1
Wide-band CQI	4	0	4	0	4	0	4	0	4	0
Sub-band CQI	0	4	0	4	0	4	0	4	0	4
Spatial differential CQI	0	0	3	3	3	3	3	3	3	3
Wide-band i2	4	0	4	0	3	0	0	0	0	0
Sub-band i2	0	4	0	2	0	2	0	0	0	0
Sub-band label	0	1 or 2	0	1 or 2	0	1 or 2	0	1 or 2	0	1 or 2

Now let's look into more details of W(i) matrix. Let's construct the generic form of W(i) matrix for this case. The matrix is defined at the bottom of the table shown above.
You would notice that W(i) is made up of following two components.

With Reference to 36.213 7.2.4

$$\varphi_n = e^{j\pi n/2}$$

$$v_m = \begin{bmatrix} 1 \\ e^{j2\pi m/32} \\ e^{j4\pi m/32} \\ e^{j6\pi m/32} \end{bmatrix}$$

With the two components, you can contruct the generic form of W(i) matrix as shown below. For each specific case, we have to figure out n,m',n to fill out the matrix with specific values. and n,m',n is determined by the PMI report and Table 7.2.4-2.

With Reference to 36.213 Table 7.2.4-2

$$W_{m,m',n}^{(2)} = \frac{1}{4}\begin{bmatrix} v_m & v_{m'} \\ \varphi_n v_m & -\varphi_n v_{m'} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ e^{j2\pi m/32} & e^{j2\pi m'/32} \\ e^{j4\pi m/32} & e^{j4\pi m'/32} \\ e^{j6\pi m/32} & e^{j6\pi m'/32} \\ e^{j2\pi n} & -e^{j2\pi n} \\ e^{j2\pi n}e^{j2\pi m/32} & -e^{j2\pi n}e^{j2\pi m'/32} \\ e^{j2\pi n}e^{j4\pi m/32} & -e^{j2\pi n}e^{j4\pi m'/32} \\ e^{j2\pi n}e^{j6\pi m/32} & -e^{j2\pi n}e^{j6\pi m'/32} \end{bmatrix}$$

< CSI RS Configuration >

As described above, the key issue of Beamforming implementation is how to figure out proper BeamForming matrix for each transmission. As in a common MIMO technology, we can think of roughly a couple different approach as below.

i) Open Loop Method : This is based on the assumption that a Network 'SOMEHOW' knows the proper beamforming matrix without any information from UE. Ideally we can think of two approaches as below. (Practically the method a) would not be of much meaning)

a) Applying the same Beamforming matrix for all the transmission

b) Applying the dynamically chaning Beamforming matrix but not based on specific UE Report

ii) Close Loop Method : This is the method where a Network generate the proper beamforming matrix based on specific report from UE. For this purpose, network transmit a specific pilot signal called (CSI-RS) and UE evaluate the it's receiving signal quality based on the received CSI-RS and report the result to Network. (For the CSI-RS configuration being used in LTE for this purpose, refer to CSI-RS and Periodicity section)

YouTube :

- BeamForming
- 802.11ac: Why Beamforming?
- Demystifying Beamforming - Steven Glapa, Ruckus Wireless
- Antenna array part-I
- Antenna parameters
- Various forms of Antenna array

More Readings :

These are not my posting, but I would like to recommend these for further understanding and giving you different perspective.

- All Beamforming Solutions Are Not Equal.
- Smart Antennas桺eamforming
- TD-LTE 8-antenna dual-stream beamforming technology
- Trend of Multiple Antenna Beamforming Technology in Mobile Wireless Communication Field