Names (First only, no student numbers):

Before you begin you should have read and worked through Lab 2.

All questions should be done by hand (not by computer) and show your steps.

1. Error terms in the backwards Euler

    (a) Derive the error term for the backward difference formula using Taylor series, and
        hence show that it is first order.

    (b) How does the constant in front of the leading order error term differ from that for
        the forward difference formula? Relate this back to the results plotted in the Lab
        (Figure error), where these two formulae were used to derive difference schemes for
        the heat conduction problem. (i.e., explain why backward euler is an overestimate
        and forward euler is an underestimate of the exact solution)

2. For the equation $y = sin(\lambda t)$ derive the following:

   (a) dy/dt $(= y'(t))$

   (b) Write down an implicit scheme approximation for $y'(t_i)$

   (c) Using the approximation $sin(\alpha - \beta) \approx sin(\alpha) - sin(\beta)$ (true when α and/or β follow a particular condition) show that your implicit scheme is consistent with the analytical solution.

   (d) What are the conditions under which the approximation $sin(\alpha - \beta) \approx sin(\alpha) - sin(\beta)$ holds? (do an internet search if none of your group knows offhand). What do these conditions mean (qualitatively) for the accuracy of the implicit scheme from section 2b?