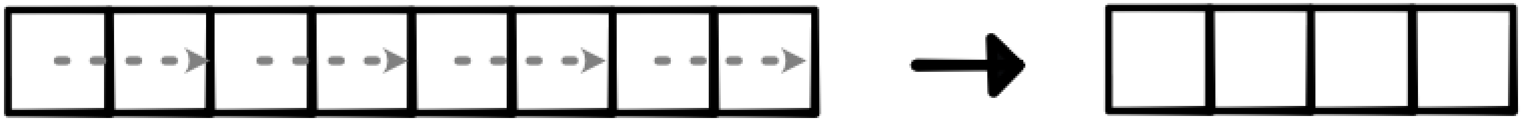
Module 4.3 - Advanced NNs

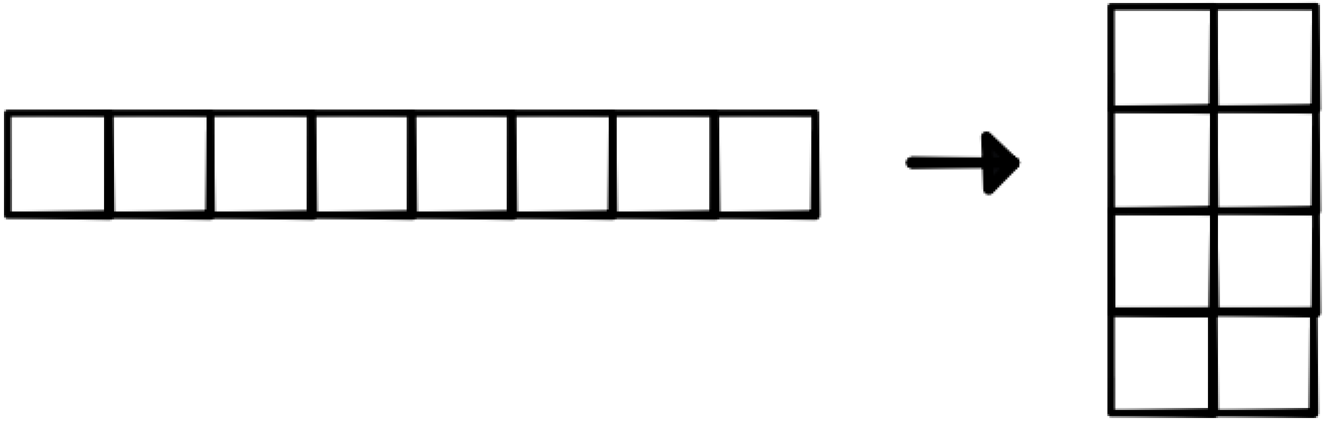
”Pooling”

Reduction applied to each region:



Simple Implementation

- Ensure that it is contiguous
- Use View to "fold" the tensor



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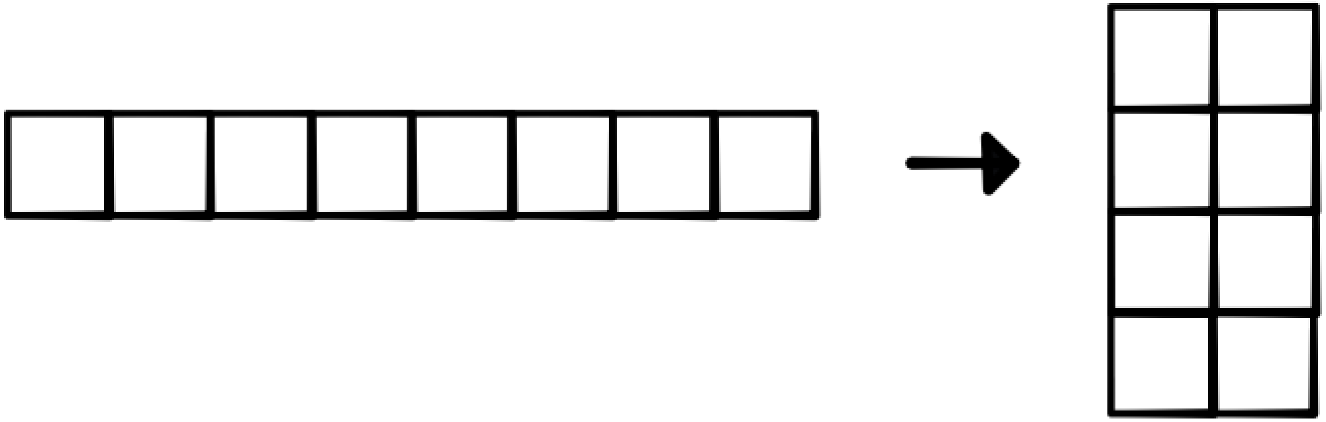
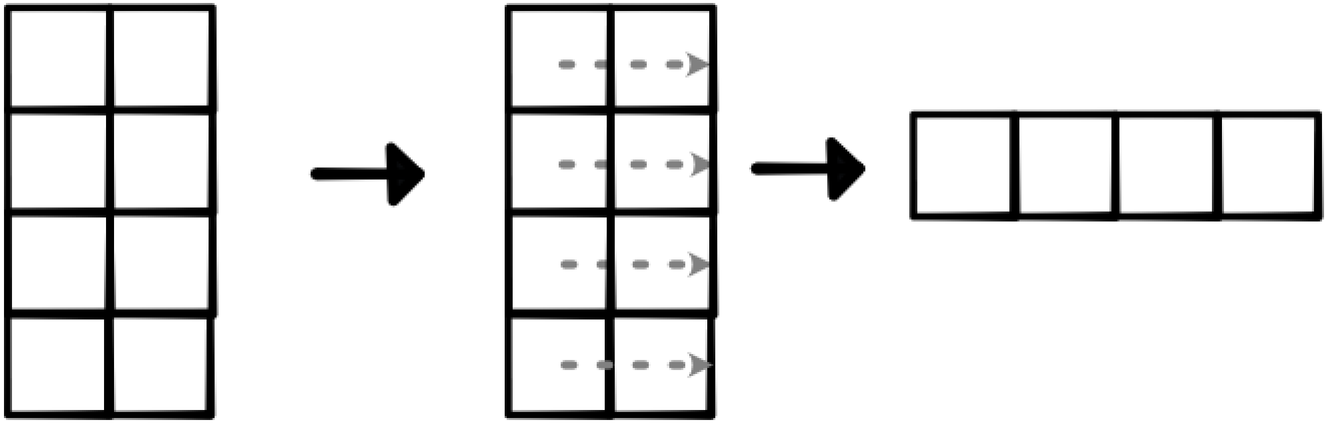
Why does folding work?

- View requires "contiguous" tensor
- View(4, 2) makes strides (2, 1)



Simple Implementation

- Reduce along created fold



Quiz

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Gradient Flow

- Layers that are used get more updates
- Gradient signals which aspect was important
- Can have extra layers

More Reductions

- Heading for a `max` reduction
- Heading for a `softmax` output
- Quick detour

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ReLU, Step, Sigmoid

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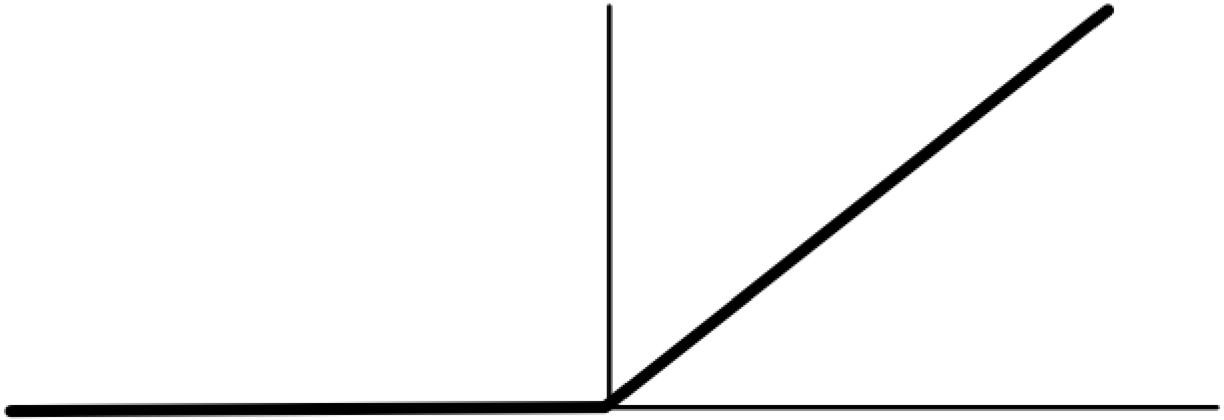
Basic Operations

- Introduced in Module-0
- Widely used in ML
- What is it?

Simple Function: ReLU

Main "activation" function

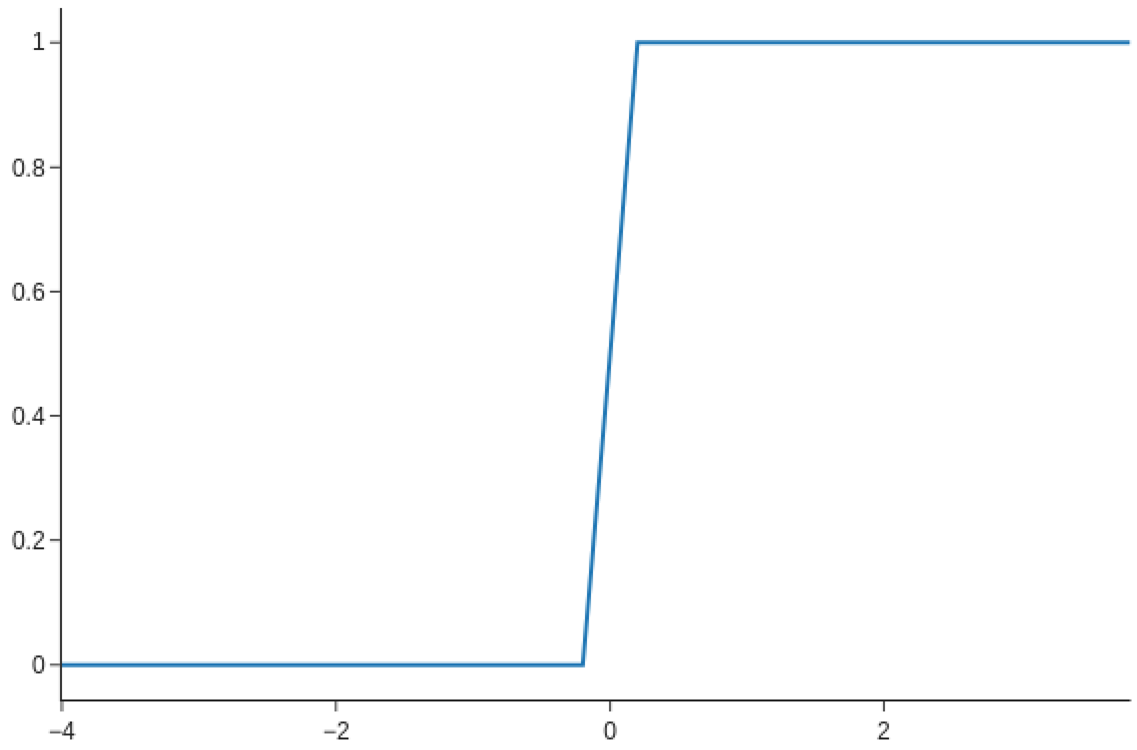


Primarily used to split the data.

Simple Function: Step

Step function $f(x) = x > 0$ determines correct answer

Derivative of ReLU



ReLU

Mathematically,

$$\mathrm{ReLU}(x) = \max\{0, x\}$$

Simplest `max` function.

Step

Mathematically,

$$\text{step}(x) = x > 0 = \arg\max\{0, x\}$$

Simplest `argmax` function.

Relationship

Step is derivative of ReLU

$$\mathrm{ReLU}'(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{ow} \end{cases}$$

$$\mathrm{step}(x) = \mathrm{ReLU}'(x)$$

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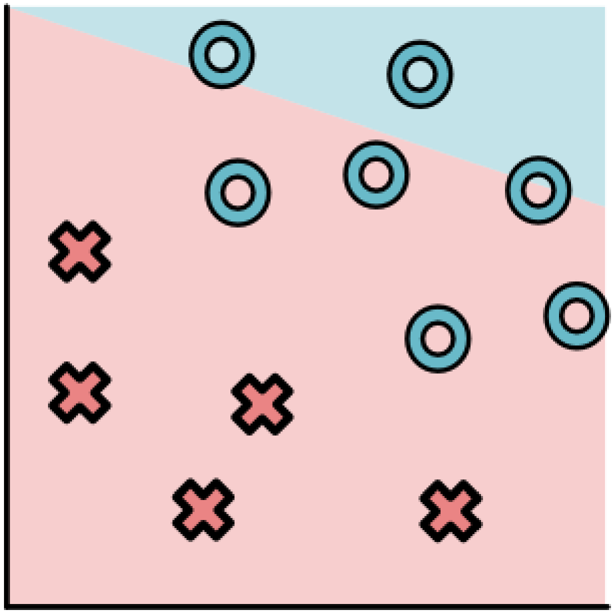

Loss of step tells us how many points are wrong.

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Derivative of Step?

Mathematically,

$$\text{step}'(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 0 & \text{ow} \end{cases}$$

Not a useful function to differentiate

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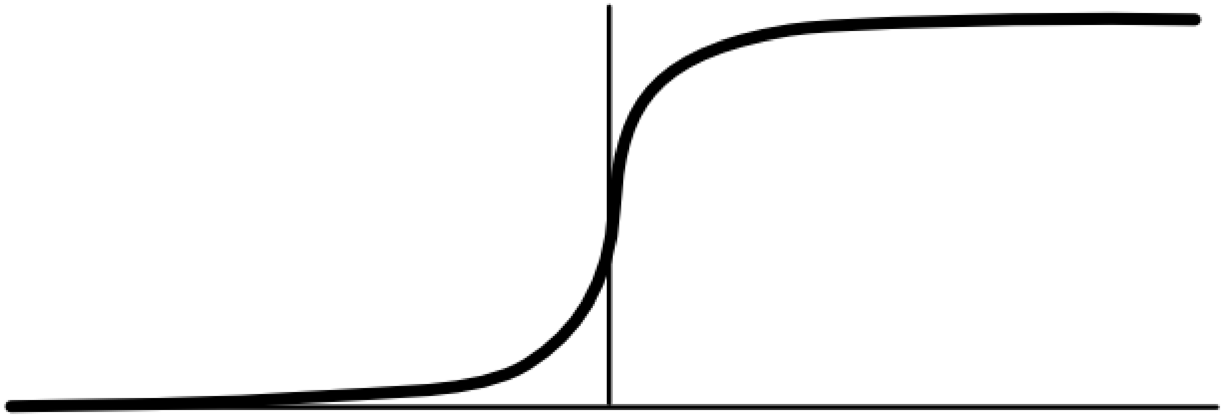
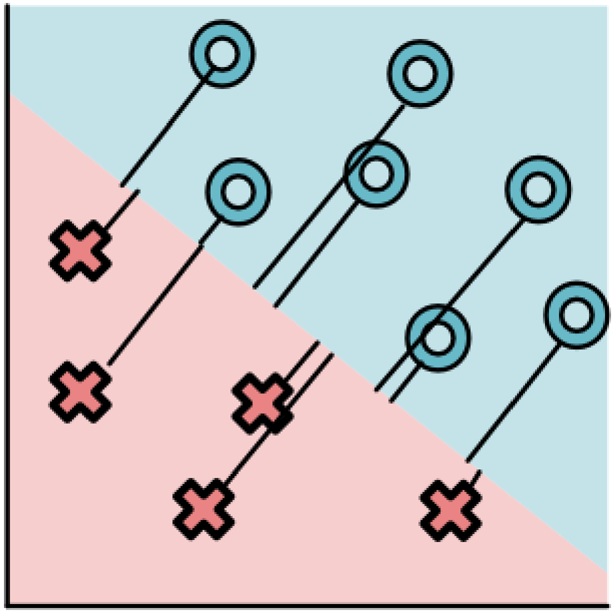
Altenative Function: Sigmoid

Used to determine the loss function





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Soft (arg)max?

Would be nice to have a version that with a useful derivative

$$\mathrm{sigmoid}(x) = \mathrm{softmax}\{0, x\}$$

Useful soft version of argmax.

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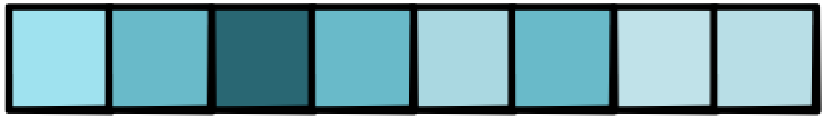
Max, Argmax, Softmax

Challenge

How do we generalize sigmoid to multiple outputs?



Max reduction

- Max is a binary associative operator
- $\max(a, b)$ returns max value
- Generalized $\mathrm{ReLU}(a) = \max(a, 0)$

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Max Pooling

- Common to apply pooling with max
- Sets pooled value to "most active" in block
- Forward code is easy to implement

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Max Backward

- Unlike sum, max throws away other values
- Only top value gets used
- Backward needs to know this.

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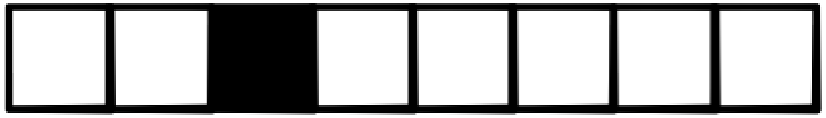
Argmax

- Function that returns `argmax`, one-hot

- Generalizes step



Max Backward

- First compute `argmax`

- Only send gradient to `argmax` gradinput

- Everything else is 0

Ties

- What if there are two or more argmax's?
- Max is non-differentiable, like `ReLU(0)`.
- Short answer: Ignore, pick one

HW

- When writing tests for max, ties will break finite-differences
- Suggestion: perturb your input by adding a small amount of random noise.

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Soft argmax?

- Need a soft version of argmax.
- Generalizes sigmoid for our new loss function
- Standard name -> softmax

Softmax

$$\operatorname{softmax}(\mathbf{x}) = \frac{\exp \mathbf{x}}{\sum_i \exp x_i}$$

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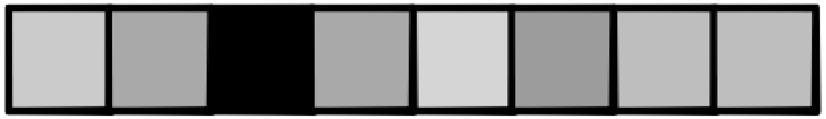
Sigmoid is Softmax

$$\operatorname{softmax}([0, x])[1] = \frac{\exp x}{\exp x + \exp 0} = \sigma(x)$$

Softmax

Softmax



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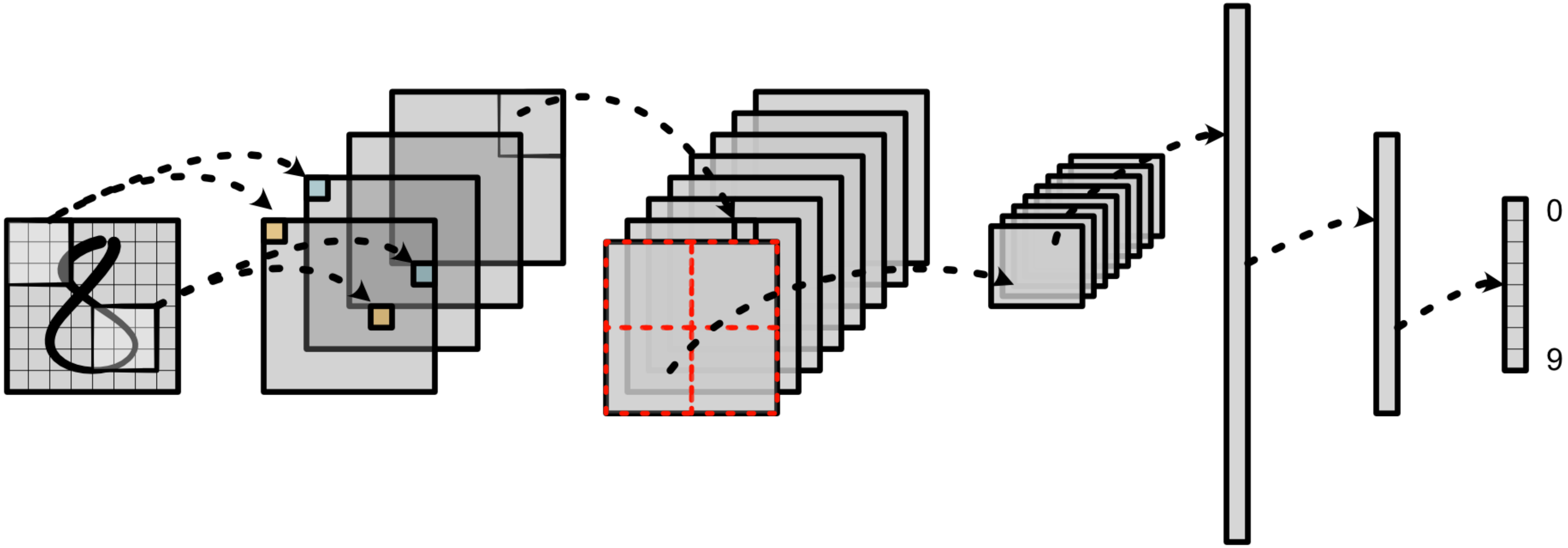
Review

- ReLU -> Max
- Step -> Argmax
- Sigmoid -> Softmax

Softmax

Network

Network



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Softmax Layer

- Produces a probability distribution over outputs (Sum to 1)
- Derivative similar to sigmoid
- Lots of interesting practical properties

Softmax in Context

- Not a map!
- Gradient spreads out from one point to all.

Softmax

- (Colab) [https://colab.research.google.com/drive/1EB7MI_3gzAR1g

Soft Gates

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New Methods

- Sigmoid and softmax produce distributions
- Can be used to "control" information flow

Example

Returns a combination of x and y

$$f(x, y, r) = x * \sigma(r) + y * (1 - \sigma(r))$$

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Gradient is controlled

$$\begin{aligned}
f'_x(x,y,r) &= \sigma(r) \\
f'_y(x,y,r) &= (1-\sigma(r)) \\
f'_r(x,y,r) &= (x-y)\sigma'(r)
\end{aligned}$$

Neural Network Gates

Learn which one of the previous layers is most useful.

$$
\begin{aligned}
r &= NN_1 \\
x &= NN_2 \\
y &= NN_3
\end{aligned}
$$

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Gradient Flow

- Layers that are used get more updates
- Gradient signals which aspect was important
- Can have extra layers

Selecting Choices

- Gating gives us a binary choice
- What if we want to select between many elements?
- Softmax!

Softmax Gating

Combines many elements of X based on R

$$f(X, R) = X \times softmax(R)$$

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Softmax Gating

- Brand name: Attention

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Example: Translation

- Show example

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Example: GPT-3

- Show example

QA

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