Mean Lifetime and Shielding Factor Determination For Cosmic Ray Muons in the Elliott Building

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The goals of this study was two-fold: experimentally determine the mean lifetime of cosmic ray muons using a trio of scintillator particle detectors and quantitatively describe the attenuation of the flux of the same muons through the Elliott building on the UVIC campus floor-by-floor. The first goal was performed successfully in two cases: with no extensions to the base apparatus and with a copper target imposed between detectors 2 and 3. The resultant muon lifetimes were found to be $\tau_{noCu} = (2.00 \pm 0.09)\mu s$ and $\tau_{Cu} = (2.23 \pm 0.07)\mu s$. We combined these values to produce a less uncertain lifetime value of $\tau = (2.12 \pm 0.1)\mu s$. This is consistent with the literature and can be reported as a statistically significant 1σ result. The loss of muon flux was modelled such that two key parameters were determined. These being the attenuation coefficient for concrete shielding factor forflux due to attenuation with values of $\mu_{concrete} = (2.2 \pm 0.3) \cdot 10^{-1} m^{-1}$ and $A = 0.68 \pm 0.04$ respectively. The latter was found to be largely discrepant with the predictions from literature suggesting that a missing parameter was omitted from the analysis encouraging a need for further study.

I. INTRODUCTION

A muon is a fundamental particle that is a member of the Lepton family [1]. They are similar in property to an electron, with the same charge of $-e_0$ and spin-$\frac{1}{2}$ but exhibit a much larger mass on the order of $200m_e$ [1]. We find that muons are abundant in nature due to high energy processes occurring in the upper atmosphere between cosmic ray particles arriving from the depths of the universe, and nuclei in the sky [2]. When these collisions occur, a shower of ionized particles, radiation, and more exotic materials cascade onto the surface of the Earth [3]. The processes that creates (anti)muons can be written as:

$$\pi^- \to \mu^- + \bar{\nu}_\mu \;\; and \;\; \pi^+ \to \mu^+ + \nu_\mu \qquad (1)$$

Where muons ($\mu^\pm$) are created from decaying pions ($\pi^\pm$) yielding muon neutrinos (ν_μ) as a secondary product [4]. The progenitor of the muons studied here are these pions, which are created first in the high energy collisions in the upper atmosphere. Pions consist of a quark-antiquark pair (denoted $^+/^-/^0$ depending on the specific composition) and are a result of a cosmic ray striking a proton or nucleus. This collision overcomes the binding force on the nucleons producing quarks and anti-quarks. These combine to produce pions which subsequently decay giving us the muons of interest that eventually reach the Earth's surface.

We know that muons are an unstable product [4] of the above process and as such have an associated decay time. We present here a method to determine this value and compare it to the accepted one found here [1].

We also demonstrate a method with which to quantify the change in relative flux of cosmic ray muons through the depth of the building where the experiment takes place [5]. As muons travel vertically through the floors of the building, we expect an attenuation of the muon events per unit time [6] [7]. To perform the analysis of this process we must make a number of operational assumptions outlined in the methods section in conjunction with the statistical assumptions of the ergodicity of cosmic ray events.

Theory:

Muon Lifetime Determination

Muons and antimuons are unstable such that they decay into other constituents. This is governed by the weak interaction producing electrons, positrons, and neutrinos [2]. The processes involved are [2]:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \qquad (2)$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \qquad (3)$$

As this is a random decay process, we can model it accordingly, assuming that both process (2) and (3) are described in combination:

$$N = N_0 e^{-\frac{t}{\tau}} + B_g \qquad (4)$$

Characterized by a decay factor τ, N is a function of time, t and the background B_g that must be removed. Of particular interest from the above equation is the expectation value of the distribution. We may find this using the following:

$$E[t] = \int_0^\infty tN(t)dt = \int_0^\infty tN_0 e^{-\frac{t}{\tau}} dt = \tau \quad (5)$$

And so the mean lifetime of a cosmic ray muon is the decay factor τ.

Attenuation Factor

The differential count rate of a process through a medium can be modelled relatively easily if we only assume two *linear* terms of attenuation and growth outlined in [8]. Our initial differential equation is:

$$\frac{d\bar{N}}{dx} = -\mu\bar{N} + \lambda\bar{N} = (\lambda - \mu)\bar{N} \quad (6)$$

Where x is the depth in the media, μ is the attenuation coefficient, and λ is the growth factor while $\bar{N}$ denotes N affected by attenuation. This can be solved:

$$\frac{d\bar{N}}{\bar{N}} = (\lambda - \mu)dx \quad (7)$$

$$\int \frac{d\bar{N}}{\bar{N}} = \int (\lambda - \mu)dx \quad (8)$$

$$ln\bar{N} + ln\bar{N}_0 = (\lambda - \mu)x \quad (9)$$

$$\implies \bar{N} = \bar{N}_0 e^{(\lambda-\mu)x} \quad (10)$$

We may obtain a rough estimate of the attenuation coefficient by measuring the count rates as a function of the depth through our material. For the rest of the analysis we will assume that the growth factor through concrete is negligible allowing us to find $\mu_{concrete}$. This assumption leads us to *under*estimate the value of $|\mu|$. In a reality, we would expect a growth factor that is nonzero as the quantity of interest here is the count rate. Incident high energy muons on the material could interact with particles in the material causing other products to be created (potentially more than one) yielding a growth in counts but not energy.

Relativity Considerations

Since we have already determined using equation [5] that the muon lifetime will be finite, there necessarily exists a finite length that they can travel as a function of their speed. Since we are interested in the amount a flux of muons decreases through some height, it is of use to determine if the height of the building meaningfully impacts the penetration depth of a muon before decay. We may convert a decay in time to a decay in space using the following [3]:

$$N(h) = N_0 e^{-\frac{h}{v_{muons}\cdot\tau}} = N_0 e^{-\frac{h}{h_{eff}}} \quad (11)$$

To further complicate things, the rate at which cosmic ray muons travel is on the order of $v_{muons} = 0.994c$ [2] where c is the speed of light. We then must take into account relativistic corrections. Since h_{eff} is a length scale, we must have that in the relativistic regime:

$$h_{eff}^{rel} = \gamma h_{eff} = \frac{h_{eff}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (12)$$

Where γ is the Lorentz factor. Using this correction alongside the value for v_{muons} and the mean muon lifetime found in analysis, we may produce the curves found in figure 1. By inspection, over the height of a building, both non-relativistic and relativistic muons are largely unaffected by their decay times.

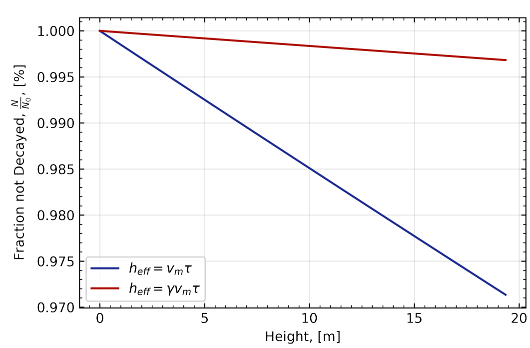

FIG. 1. Corrections to the fractional transmission of muons via the decay due to their finite lifetime. Both are stretched exponential curves that appear linear. The effect, even without relativistic considerations, are minute over the height of the Elliott building.

II. MATERIALS AND METHOD

Apparatus Description

To perform the measurements of interest for this experiment, a trio of muon detectors were used. Each detector consisted of a paddle made of scintillating organic material that act as targets for cosmic ray muons. When a muon is incident, it ionizes some of the material [9] which in turn emit scintillation light that is directed using a wave-guide into a photomultiplier tube (PMT) which multiplies the signal [9] and can be picked up by a data acquisition setup recording an event as a count.

Calibration

To calibrate a counting detector such as this, we note that the efficiency of the apparatus will be some function of the operating voltage of the PMTs. As the voltage increase, so does the counts made by the detector, including dark counts (signals generated without the presence of a muon). To mitigate the background noise, we may use two detectors coincidenced with each other to confirm that a muon event happens when both detectors fire. In doing so, we expect a counting function of voltage for a coincidenced detector to increase with voltage to a plateau where only *true* counts are being made. This process was done for all permutations of the three detectors to determine the optimal voltages for each, shown in Appendix D. The operational regime was estimated in regions where both combination of curves appeared plateaued. This gave operating voltages of $V_{CH1} = (1.8 \pm 0.05)KV$, $V_{CH2} = (1.7 \pm 0.05)KV$, and $V_{CH3} = (1.7 \pm 0.05)KV$.

Determining the Muon Lifetime

After the detector was properly calibrated, we determined the lifetime of a muon by utilizing all three detectors in conjunction with the DAQ board and software from [10].

The procedure here begins with assuming that a specific type of event for the three detectors corresponds to a muon *decay* such that the timing of this event will allow us to find the decay timescale. If a muon enters the top detector and then the middle, we can be quite sure that it is a true muon event and so all coincidenced signals from detectors 1 and 2 were recorded. Further, if the muon enters detector 2 and never reaches detector 3, we can roughly assume that the muon has decayed somewhere in detector 2. If it stops inside of the detector, it will produce a second signal in 2 but will never reach 3. The decay products may make it to the third detector initiating a signal but this will only effect the number of total instances of the $1 + 2 + \bar{3}$ event we obtain. If we take a collection of all points that are coincidenced for 1,2 but not 3 we get a sample of possible decays that can be used to determine the lifetime.

The above was completed twice for two different cases: one with the addition of a copper target and one without. The copper target was placed between the second and third detectors and served to block some of the flux from reaching detector 3, increasing the amount of data points. The simpler case was undertaken from October 16th at 5pm to October 19th at 12 pm for 60 hours while the inclusion of the copper target was performed between October 12th at 5pm to October 16th at 11am for a total of 90 hours.

Floor-by-Floor Methodology

Of interest for our analysis is the change in flux of the cosmic ray muons as a function of the floors of the Elliott building. To perform this stage of the experiment, only two of the detectors were used to determine coincidence events combined with the DAQ board and software from [10] as well.

The resulting flux of muons for 30 minute intervals was acquired from all five floors of the building as well as a recording outside the building to obtain the expected baseline flux with no attenuation. The position of each measurement was constrained to where outlets were available and as such, slightly differ from a straight vertical line inside the building seen in the appendix. All data was taken in largely the same weather on the same day on October 19th throughout the evening.

III. RESULTS AND ANALYSIS

The Lifetime of a Muon

In order to find the lifetime of the muon using equation (4), we first needed to center the data around a t_0. This was chosen to be $t_0 = 16 \cdot \Delta t$, removing all of the leading extraneous data before the decay. The resulting decay curves can be found in figure (3). The delay bin size was set to $1000ns$, which was likely too large, leading to a strong exponential fit for both cases. The much smaller number of counts/bin for the no copper case is due both to the $2/3T$ sample size and the copper doing its job.

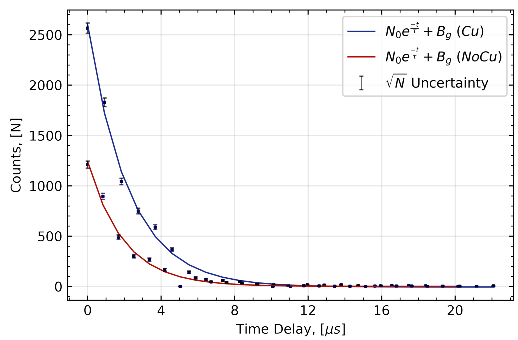

FIG. 2. Counts in each bin as a function of the time delay with bin size of 1000 ns. Both data taken without a copper target and with one are presented. This data was fitted with an exponential curve so as to determine the mean muon lifetime. The fits correspond to $\chi^2 = 386$ with $\nu = 24$ and $\chi^2 = 424$ with $\nu = 24$ respectively where both p-values are < 0.05 indicating a solid treatment of the model to the data.

The fit on this data can be found in Appendix B. We end up with two values for the lifetime which are $\tau_{noCu} = (2.00 \pm 0.09)\mu s$ and $\tau_{Cu} = (2.23 \pm 0.07)\mu s$. We may combine these two results to obtain a value with less inherent uncertainty as $\tau = 2.1 \pm 0.1\mu s$. The accepted value for this quantity, found here [1], is $\tau_{acc} = 2.1969811 \pm 0.0000022\mu s$. It is easy to see that the lifetime without a target found here is inconsistent with this result and the experimental uncertainty purely due to the fitting algorithm. That being said, with the inclusion of the copper target, we find that τ_{Cu} is consistent with the accepted result. Further, our combined measurement is both consistent with the literature and can be reported as a 1σ result.

Muon Flux Attenuation

To obtain meaningful results from the attenuation experiment, we make a number of significant assumptions. These are split into two categories. The first is detector assumptions that include ignoring any separation between the two detectors involved, assuming a constant gain from the PMT regardless of temperature, time of use etc. and assuming that we only detect muons with the device.

More interesting assumptions arise when we consider the environment. We will simplify the analysis such that the only active attenuation is the concrete in the building. This means that all interactions with the air and other materials is ignored. We also assume that the angular contributions of the distribution of materials is not only symmetric but constant floor-by-floor. Lastly, to convert flux per floor to flux through concrete, we assume that the contribution from each floor is approximately 6 inches ($\approx$ 15 cm) of concrete motivated here [11].

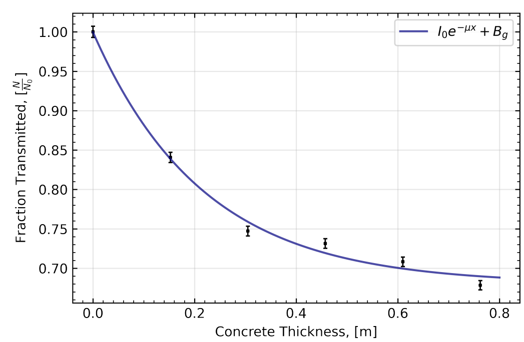

FIG. 3. Transmission fraction of muons with $N_0 = N_{outside}$ as a function of the estimated thickness of cumulative concrete fitted with an attenuating exponential decay. The fitting here results in a $\chi^2 = 9.8$ with $\nu = 5$ yielding a p-value less than 0.05 which tells us that the model is likely accurately describing the scenario.

In fitting the data, we find a fairly well-defined exponential decay such as the one outlined in equation (10). The fit gives us an attenuation coefficient for concrete of $\mu_{concrete} = (2.2 \pm 0.3) \cdot 10^{-1} m^{-1}$. This value was largely absent from literature, however, comparisons to known attenuation fractions can be done as follows. It is useful for us then to find this value, which can be seen in figure (4) where each point also corresponds to a floor of the building. We report here an attenuation fraction or shielding factor of $A = 0.68 \pm 0.04$. A shielding factor in this context is a slightly unintuitive value that corresponds to the normalized fraction of *transmitted* flux of an incident radiation. It has been reported that the shielding factor for a five-story building made of concrete such as the Elliott building will have a value of $A = 0.97$. This is a significant discrepancy.

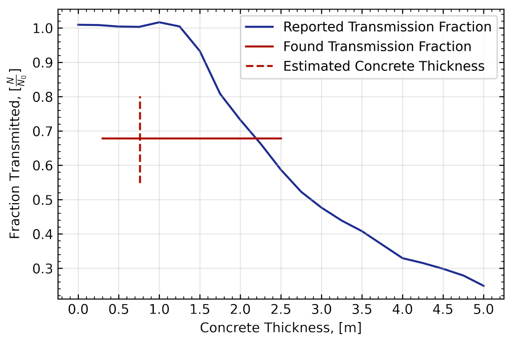

FIG. 4. Reported transmission fraction as a function of concrete thickness from [6] . The rough estimate of the thickness of the Elliott concrete if plotted in conjunction with the found transmission fraction from figure 3.

We can also compare our result to that of [6], where the attenuation fraction for concrete slabs at sea level were obtained for muons. In figure (5) we see the data from [6], which once again disagree significantly. Investigating this further, it appears like the estimate of the thickness of the Elliott concrete slabs may have been much too small. We can quantify this by comparing our shielding factor with a curve that describes the fractional transmission of muons through varying concrete thickness. As see in figure (5), the estimate concrete thickness in metres is overlaid on a plot of the shielding factor as a function of concrete thickness. To get the $A = 0.68$ result reported here, the intersection between this horizontal line and the curve may yield a better estimate of the actual cumulative concrete thickness of the building. A shielding factor of A 0.68 corresponds to a thickness of $2.2m$ for the data obtained from [6].

IV. DISCUSSION

In this analysis, we report both a determination of the lifetime of a cosmic ray muon as well as the attenuation coefficient and fractions for a 5 story building, assuming concrete morphology.

In the finding of the muon lifetime, we find two values of $\tau_{noCu} = (2.0 \pm 0.09)\mu s$ and $\tau_{Cu} = (2.23 \pm 0.07)\mu s$. The result with the copper target is consistent with the experimental uncertainty and the accepted value presented here [1]. Uncertainty on the two values is wholly contributed by the goodness of fit associated with the time delay-count curve.

We find that the resolution in the time delay data leaves something to be desired with a bin size of 1000ns. In future experiments, this value will need to be far larger. We report a shielding factor of $A = 0.68 \pm 0.04$ for muon flux as a function of the floors of the Elliott building. This is in significant disagreement with two sources found here [6] [7]. Uncertainty in this value is due to the $\sqrt{N}$ uncertainty associated with the floor data counts before normalization which does very little to overcome the discrepancy with [7]. It is likely, then, that one or more of the original assumptions are not the case.

The simplest assumption to reconcile is the estimate thickness of the depth of material overcome by the incident muons. An increase in the floor-by-floor thickness is motivated by the data in figure (5) from [6].

Beyond this, dealing with the incongruity becomes muddier. Of likely significant effect is the angular anisotropy of the building shape from floor-to-floor. Since we assumed that all flux is normal to the detectors, the non-linear accumulation of more and more material as the floors are traversed downwards should result in a larger A then reported here [6] which was measured next to a slab. This, once again, should be systematic and likely insignificant.

We also failed to include contributions from other materials as well as the air and so $A_{concrete}$ is really $A_{Elliott}$. But the analysis presented in [7] seemingly accounts for that and still gets a much smaller value.

A further consideration here is the type of concrete we are considering as the makeup of the building. The paper here [6] does their analysis with simple concrete blocks with no added reinforcement. It may very well be the case that the increased reinforcement of the building (such as rebar etc.) is also an important consideration and the conclusion drawn from figure (5) would yield a less thick resut.

To mitigate any bias, as well as systematic errors in gain, the floor data was taken in a random order and analyzed later. It is then of the confusion of this author as to why we report such a large effect on the muon flux. It is possible that some other material is present in the building apart from concrete that does a much better job at attenuating muons which is missing from the two conflicting reports above.

To reconcile these differences, it begs a more thorough data collection run from floor-to-floor.

V. CONCLUSION

By modelling the distribution of time delays for subsequent events on the second detector in a stack of three while vetoing the third, it was possible to obtain an experimental value for the lifetime of a cosmic ray muon. A value without the inclusion of a copper target was found to be $\tau_{noCu} = (2.00 \pm 0.09)\mu s$ while the inclusion of a copper target used as a stop between detectors 2 and 3 resulted in a value of $\tau_{Cu} = (2.23 \pm 0.07)\mu s$. Only the second value is consistent with the experimental uncertainty and the accepted value reported here [1]. Using a simple attenuation model, it was also possible to describe quantitatively, the relative change in flux of muons incident on the Elliott building on the UVIC campus in a floor-by-floor analysis. In doing so, values for the attenuation coefficient of concrete and the attenuation fraction were determined such that $\mu_{concrete} = (2.2 \pm 0.3) \cdot 10^{-1} m^{-1}$ and $A_{Elliott} = 0.68 \pm 0.04$. We found that the measured values here differ wildly from the reported data here [7] and here [6]. It is likely the case that another variable is unaccounted for, such as a better attenuating material present in the building. Regardless, the author is unable to explain the large discrepancy which suggests a need for further inquiry.

[1] J. Beringer *et al.*, PR **D86** (2012).

[2] L. Liu, Physics Department, Massachusetts Institute of Technology **2139** (2007).

[3] E. Covarrubias, California State University Stanislaus (2018).

[4] M. D. of Physics, `https://www.physics.rutgers.edu/~eandrei/389/muon-mit.pdf` (2007), [Accessed: 23-October-2023].

[5] M. Segger, `https://uvac.uvic.ca/Architecture_Exhibits/UVic_campus/buildings/Elliott/` (2012), [Accessed: 20-October-2023].

[6] D. T. Ordinario, "Neutron/muon correlation functions to improve neutron capabilities outside nuclear facilities," (2016).

[7] W.-L. Chen and R.-J. Sheu, Radiat Prot Dosimetry **179**, 233 (2018).

[8] R. N. Altameemi, N. S. Abdul Hamid, W. M. A. Wan Mohd Kamil, and S. M. Saleh Ahmed, Journal of Radiation Research and Applied Sciences **13**, 721 (2020).

[9] E. Knoff, *Plateauing Cosmic Ray Detectors to Achieve Optimum Operating Voltage*, Tech. Rep. (Northwestern University, 2007) [Accessed: 9-October 2023].

[10] S. Hansen *et al.*, *QuarkNet Cosmic Ray Muon Detector User's Manual Series "6000" DAQ*, Tech. Rep. (FermiLab, 2010) [Accessed: 9-October 2023].

[11] M. I. Hamakareem, *How thick should a concrete slab be?*, Tech. Rep., [Accessed: 21-October 2023].

[12] J. Autran, D. Munteanu, T. Saad Saoud, and S. Moindjie, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **903**, 77 (2018).

APPENDIX A: Environment Data

TABLE I. Experimental values for the descriminator threshold and the plateaued operating voltages for the three detectors

	$CH1$	$CH2$	$CH3$
Threshold [mV]	500 ± 0.5	500 ± 0.5	500 ± 0.5
Operating Voltage [kV]	1.8 ± 0.05	1.7 ± 0.05	$1.7 \pm .05$

TABLE II. Values of note involved in the experiment

Signal Delay [x10 ns]	50 ± 0.5
Gate Width [x10 ns]	1000 ± 0.5
Muon Count Rate[12] [$\frac{counts}{cm^2}/s$]	1 ± 0.5
Area of Detector [cm^2]	$(1.4 \pm 0.05) \cdot 10^3$

APPENDIX B: Fitting Parameters

TABLE III. Exponential fitting parameters for the three instances of a needed decay fit

	Time Delay Determination	$N = N_0 e^{\frac{-t}{\tau}} + B_g$	
	N_0 [counts]	τ [μs]	B_g [counts]
No Copper Target	$(1.2 \pm 0.03) \cdot 10^3$	(2.0 ± 0.09)	1 ± 8
Copper Target	$(2.6 \pm 0.04) \cdot 10^3$	$2.23 \pm 0.07) \cdot 10^3$	-5 ± 12
	Floor-by-Floor Attenuation Fitting	$I = I_0 e^{-\mu x} + B_g$	
	I_0 [counts]	μ [m^{-1}]	B_g [counts]
	$(6.3 \pm 0.3) \cdot 10^3$	$(2.2 \pm 0.3) \cdot 10^{-1}$	$(1.34 \pm 0.03) \cdot 10^4$

Appendix C: Floor-by-Floor Measurement Locations

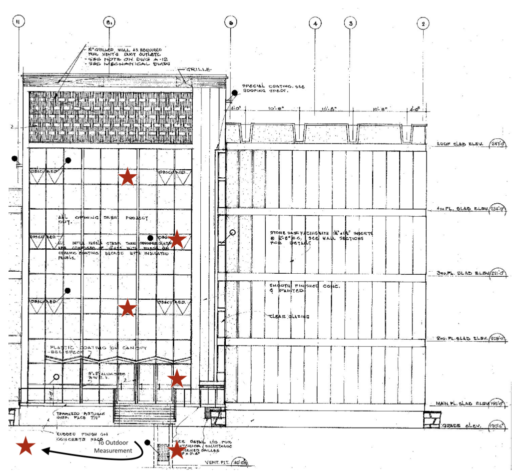

FIG. 5. Schematic of the Elliott building used to estimate the floor-by-floor height of the building. A rough position of where in the building each data run was undertaken is highlighted by the red stars.

Appendix D: Detector Plateaus

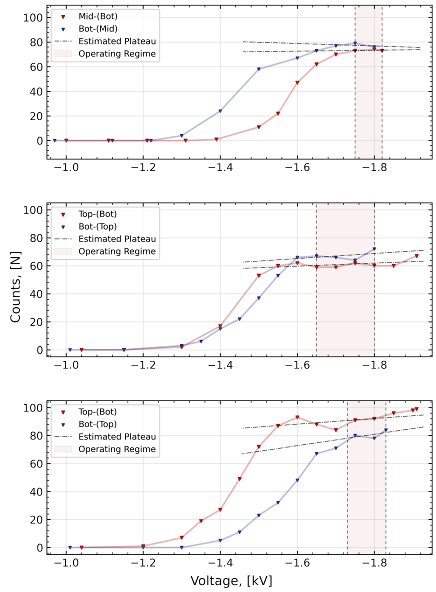

FIG. 6. Plateau curves for the three scintillater detectors used. Each detector was plateaued with respect to both of the others to obtain an operating voltage regime highlighted in red. Estimated fits have been placed on each plateau region with the proceeding label acting as the variable voltage.