General Relativity HW 9 Quiz Name_______________

You know the drill!

1. Two observers in two rockets are hovering above a Schwarzschild black hole of mass *M*. They hover at a fixed radius *r* such that

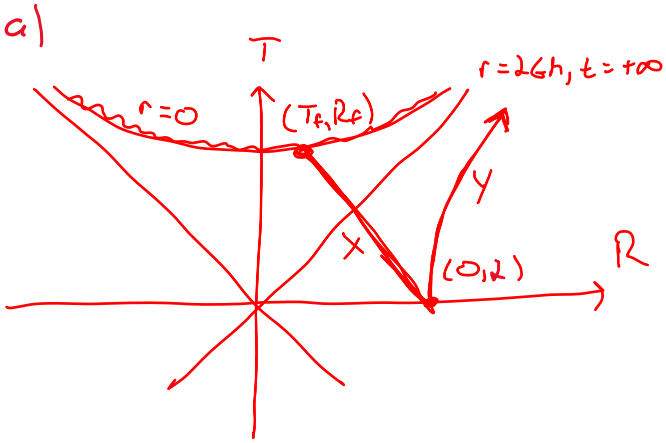
$$\left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} = 2$$

with fixed angular position. The first observer leaves this position at $t = 0$ and travels into the black hole **on a straight line path in a Kruskal diagram** until destroyed in the singularity at the point where the singularity crosses the line $R = \sqrt{3}$ where R is the Kruskal radial coordinate (Note *r*,*t* are Schwarzschild coordinate values). The other observer continues to hover at radius *r*.

a) On a Kruskal diagram, sketch the worldlines of the two observers.

b) Is the observer who goes into the black hole following a timelike worldline?

a)



For $r = 4GM$, $t = 0$:

$$T_i = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} \sinh\left(\frac{t}{4GM}\right) = 0$$

$$R_i = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} \cosh\left(\frac{t}{4GM}\right) = 2$$

The $r=0$ singularity is at: $T^2 - R^2 = 1$

so when $R_f = \sqrt{3} \Rightarrow T_f^2 = 4 \Rightarrow T_f = \pm 2$

Based on picture $T_f = +2$

b) $m = \dfrac{T_f - T_i}{R_f - R_i} = \dfrac{2 - 0}{\sqrt{3} - 2} > 1$ which is timelike!

2. Argue that once inside of a Schwarzschild black hole, that any angular motion will only make your journey to the singularity even shorter than without it.

Consider: $$d\tau^2 = \left(1-\frac{2GM}{r}\right)dt^2 - \left(1-\frac{2GM}{r}\right)^{-1}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta\, d\phi^2$$

$$1 = \left(1-\frac{2GM}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - \left(1-\frac{2GM}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\theta}{d\tau}\right)^2 - r^2\sin^2\theta\left(\frac{d\phi}{d\tau}\right)^2$$

$$\frac{dr}{d\tau} = \sqrt{\left(\frac{2GM}{r}-1\right) + \left(1-\frac{2GM}{r}\right)^2\left(\frac{dt}{d\tau}\right)^2 + r^2\left(\frac{2GM}{r}-1\right)\left(\frac{d\theta}{d\tau}\right)^2 + r^2\sin^2\theta\left(\frac{2GM}{r}-1\right)\left(\frac{d\phi}{d\tau}\right)^2}$$

All of the terms on the right are >0 for $r<2GM$, so making any of them $=0$ will lower $\frac{dr}{d\tau}$ which means that one will travel more slowly towards $r=0$.