General Relativity HW 8 Quiz Name_____

You know the drill!

1. What is the metric for the interior solution of an infinite body of vacuum energy? That is, the energy-momentum tensor is simply proportional to the metric, i.e. $T_{\mu\nu} = k g_{\mu\nu}$.

First note that $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \Rightarrow G_{\mu\nu} = 8\pi G\left(\underbrace{-\frac{\Lambda}{8\pi G}}_{k} g_{\mu\nu}\right)$

so the solution to the equation $G_{\mu\nu} = 8\pi G T_{\mu\nu} = 8\pi G k g_{\mu\nu}$

is the same as $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$ w/ $k = -\frac{\Lambda}{8\pi G}$!

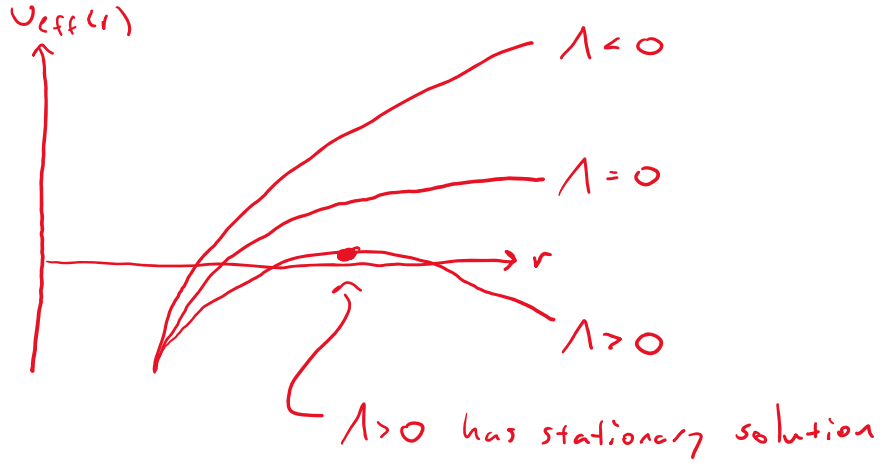
$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{8\pi G k r^2}{3}\right)dt^2 + \left(1 - \frac{2GM}{r} + \frac{8\pi G k r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2$$

but since we do not have a source other than Λ, we need to take $M \to 0$, hence

$$ds^2 = -\left(1 + \frac{8\pi G k r^2}{3}\right)dt^2 + \left(1 + \frac{8\pi G k r^2}{3}\right)^{-1} dt^2 + r^2 d\Omega^2$$

2. For which of $\Lambda = +, -, 0$ is it possible to place an object at rest a certain finite distance from the center of a spherically symmetric source and have it remain at rest? What is going on?

Recall that for $L=0$ we have $V_{eff}(r) = \frac{1}{2} - \frac{1}{r} - \frac{\Lambda r^2}{6}$



What's going on is that gravity for a localized source is attractive, but gravity from a negative vacuum source is repulsive and so when both are present they can balance each other out.