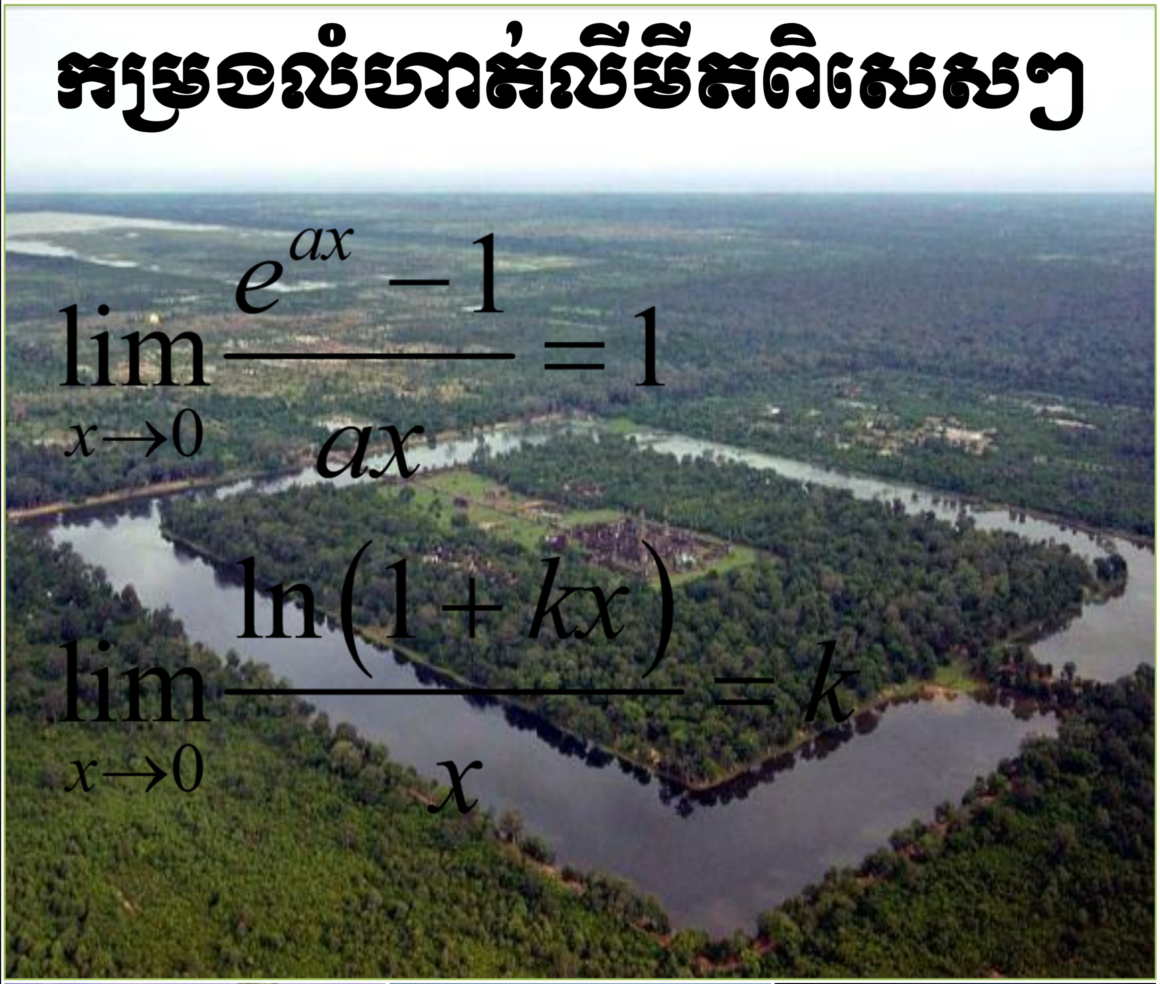
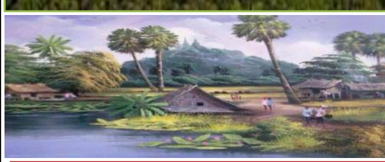
គណនាលីមីតនៃអនុគមន៍

កម្រងលំហាត់លីមីតពិសេសៗ



$$\lim_{x\to 0}\frac{e^{ax}-1}{ax}=1$$

$$\lim_{x\to 0}\frac{\ln(1+kx)}{x}=k$$



2014-2015


រៀបរៀងដោយលោកឈិនវ៉ាន្នី

លីមីតនៃអនុគមន៍

១. **និយមន័យ១** $\lim_{x\to a} f(x)=L \Leftrightarrow \forall\varepsilon>0;\exists\ \delta>0:|x-a|<\delta\Rightarrow|f(x)-L|<\varepsilon$

ឧទាហរណ៍: បង្ហាញថា $\lim_{x\to 2}(2x-1)=3$

តាមនិយមន័យចំពោះគ្រប់ $\varepsilon>0$ គេបាន $|f(x)-L|<\varepsilon$

$$\Leftrightarrow|(2x-1)-3|<\varepsilon\Leftrightarrow|2x-4|<\varepsilon\Leftrightarrow 2|x-2|<\varepsilon\Rightarrow|x-2|<\frac{\varepsilon}{2}$$

គេយក $\delta=\frac{\varepsilon}{2}>0$ នោះគេបាន $|x-2|<\delta$

ដូចនេះ $\lim_{x\to 2}(2x-1)=3$ ពិត

និយមន័យទី២ $\lim_{x\to a} f(x)=+\infty\Leftrightarrow\forall M>0\,;\exists\ \delta>0:|x-a|<\delta\Rightarrow f(x)>M$

ឧទាហរណ៍:បង្ហាញថា $\lim_{x\to 2}\left(\frac{1}{2-x}\right)^2=+\infty$

តាមនិយមន័យ ចំពោះគ្រប់ $M>0$ គេបាន $f(x)>M$

$$\Leftrightarrow\left(\frac{1}{2-x}\right)^2>M\Leftrightarrow(2-x)^2<\frac{1}{M}\Leftrightarrow(x-2)^2<\frac{1}{M}$$

$\Leftrightarrow|x-2|<\frac{1}{\sqrt{M}}$ គេយក $\delta=\frac{1}{\sqrt{M}}>0$ នោះគេបាន $0<|x-2|<\delta$

ដូចនេះ $\lim_{x\to 2}\left(\frac{1}{2-x}\right)^2=+\infty$ ពិត

និយមន័យទី៣ $\lim_{x\to a} f(x)=-\infty\Leftrightarrow\forall M>0\,;\exists\ \delta>0:|x-a|<\delta\Rightarrow f(x)<-M$

ឧទាហរណ៍: បង្ហាញថា $\lim_{x\to 2^-}\left(\frac{3x-1}{x-2}\right)=-\infty$ កាលណា $x\to 2^-$ មានន័យថា $x-2<0$

ឬ $2-x>0$ តាមនិយមន័យចំពោះគ្រប់ $M>0$ គេបាន $f(x)<-M \Leftrightarrow \frac{3x-1}{x-2}<-M \Leftrightarrow \frac{3x-6+5}{x-2}<-M$

$$\Leftrightarrow 3+\frac{5}{x-2}<-M \Leftrightarrow \frac{5}{x-2}<-M-3 \Leftrightarrow \frac{5}{2-x}>M+3$$

$\Leftrightarrow \frac{2-x}{5}<\frac{1}{M+3} \Leftrightarrow 2-x<\frac{5}{M+3}$ គេយក $\delta=\frac{5}{M+3}>0$ នោះគេបាន $2-x<\delta$

ដូចនេះ $\lim_{x\to 2^-}\left(\frac{3x-1}{x-2}\right)=-\infty$ ពិត

និយមន័យទី៤ $\lim_{x\to+\infty} f(x)=L \Leftrightarrow \forall\varepsilon>0;\exists\ N>0: x>N \Rightarrow |f(x)-L|<\varepsilon$

ឧទាហរណ៍:បង្ហាញថា $\lim_{x\to+\infty}\left(\frac{3x+5}{x+2}\right)=3$

តាមនិយមន័យចំពោះគ្រប់ $\varepsilon>0$ គេបាន $|f(x)-L|<\varepsilon \Leftrightarrow \left|\frac{3x+5}{x+2}-3\right|<\varepsilon \Leftrightarrow \left|\frac{3x+5-3x-6}{x+2}\right|<\varepsilon$

$\Leftrightarrow \left|\frac{-1}{x+2}\right|<\varepsilon \Leftrightarrow \frac{1}{|x+2|}<\varepsilon \Leftrightarrow |x+2|>\frac{1}{\varepsilon}$ ដោយ $x\to+\infty$

នោះគេបាន $x+2>\frac{1}{\varepsilon} \Rightarrow x>\frac{1}{\varepsilon}-2$ គេយក $N=\frac{1}{\varepsilon}-2>0$ យើងបាន $x>N$

ដូច្នេះ $\lim_{x\to+\infty}\left(\frac{3x+5}{x+2}\right)=3$ ពិត

និយមន័យទី៥ $\lim_{x\to-\infty} f(x)=L \Leftrightarrow \forall\varepsilon>0;\exists\ N>0: x<-N \Rightarrow |f(x)-L|<\varepsilon$

ឧទាហរណ៍:បង្ហាញថា $\lim_{x\to-\infty}\left(\frac{4x+1}{2x+1}\right)=2$

តាមនិយមន័យចំពោះគ្រប់ $\varepsilon>0$ គេបាន $|f(x)-L|<\varepsilon \Leftrightarrow \left|\frac{4x+1}{2x+1}-2\right|<\varepsilon$

$$\Leftrightarrow \left|\frac{4x+1-4x-2}{2x+1}\right|<\varepsilon \Leftrightarrow \left|\frac{-1}{2x+1}\right|<\varepsilon \Leftrightarrow \frac{1}{|2x+1|}<\varepsilon \Leftrightarrow |2x+1|>\frac{1}{\varepsilon}$$

ដោយ $x\to-\infty$ នោះគេបាន $2x+1<-\frac{1}{\varepsilon}\Rightarrow x<-\left(\frac{1}{2\varepsilon}+\frac{1}{2}\right)$ គេយក $N=\frac{1}{2\varepsilon}+\frac{1}{2}>0$

យើងបាន $x<-N$

ដូច្នេះ $\lim\limits_{x\to-\infty}\left(\frac{4x+1}{2x+1}\right)=2$ ពិត

និយមន័យទី៦ $\lim\limits_{x\to+\infty}f(x)=+\infty\Leftrightarrow\forall M>0;\exists\ N>0:x>N\Rightarrow f(x)>M$

ឧទាហរណ៍:បង្ហាញថា $\lim\limits_{x\to+\infty}\left(\frac{5x^2+4x-1}{x+2}\right)=+\infty$

តាមនិយមន័យ ចំពោះគ្រប់ $M>0$ គេបាន $f(x)>M\Leftrightarrow\frac{5x^2+4x-1}{x+2}>M$

$\Leftrightarrow\frac{5x^2+10x-6x-12+11}{x+2}>M\Leftrightarrow\frac{5x(x+2)-6(x+2)+11}{x+2}>M$

$\Leftrightarrow 5x-6+\frac{11}{x+2}>M$ ដោយ $x\to+\infty$ នោះ $\frac{11}{x+2}\to 0$

គេបាន $5x-6>M\Leftrightarrow 5x>M+6\Rightarrow x>\frac{M+6}{5}$ គេយក $N=\frac{M+6}{5}>0$ យើងបាន $x>N$

ដូចនេះ $\lim\limits_{x\to+\infty}\left(\frac{5x^2+4x-1}{x+2}\right)=+\infty$ ពិត

និយមន័យទី៧ $\lim\limits_{x\to+\infty}f(x)=-\infty\Leftrightarrow\forall M>0;\exists\ N>0:x>N\Rightarrow f(x)<-M$

ឧទាហរណ៍: បង្ហាញថា $\lim\limits_{x\to+\infty}\left(\frac{2x-x^2}{3x+5}\right)=-\infty$

តាមនិយមន័យ ចំពោះគ្រប់ $M>0$ គេបាន $f(x)<-M\Leftrightarrow\frac{2x-x^2}{3x+5}<-M$

$\Leftrightarrow-\frac{x}{3}+\frac{11}{9}-\frac{55}{9(3x+5)}<-M$ ដោយ $x\to+\infty$ នោះ $\frac{55}{9(3x+5)}\to 0$

គេបាន $-\frac{x}{3}+\frac{11}{9}<-M \Leftrightarrow \frac{x}{3}>M+\frac{11}{9} \Leftrightarrow x>3M+\frac{11}{3}$ គេយក $N=3M+\frac{11}{3}>0$

យើងបាន $x>N$

ដូចនេះ $\lim_{x\to+\infty}\left(\frac{2x-x^2}{3x+5}\right)=-\infty$ ពិត

និយមន័យទី៨ $\lim_{x\to-\infty} f(x)=-\infty \Leftrightarrow \forall M>0;\exists\ N>0: x<-N \Rightarrow f(x)<-M$

ឧទាហរណ៍:បង្ហាញថា $\lim_{x\to-\infty} x^3=-\infty$

តាមនិយមន័យចំពោះគ្រប់ $M>0$ គេបាន $f(x)<-M \Leftrightarrow x^3<-M \Leftrightarrow x<\sqrt[3]{-M}$

$\Leftrightarrow x<-\sqrt[3]{M}$ បើគេយក $N=\sqrt[3]{M}>0$ យើងបាន $x<-N$

ដូចនេះ $\lim_{x\to-\infty} x^3=-\infty$ ពិត

និយមន័យទី៩ $\lim_{x\to-\infty} f(x)=+\infty \Leftrightarrow \forall M>0;\exists\ N>0: x<-N \Rightarrow f(x)>M$

ឧទាហរណ៍:បង្ហាញថា $\lim_{x\to-\infty}\sqrt{1-x}=+\infty$

តាមនិយមន័យចំពោះគ្រប់ $M>0$ គេបាន $f(x)>M \Leftrightarrow \sqrt{1-x}>M \Leftrightarrow 1-x>M^2$

$\Leftrightarrow x<1-M^2 \Leftrightarrow x<-\left(M^2-1\right)$ គេយក $N=M^2-1>0$ យើងបាន $x<-N$

ដូចនេះ $\lim_{x\to-\infty}\sqrt{1-x}=+\infty$ ពិត

✡✡ សង្ខេបនិយមន័យខាងលើយើងបាន

+ $\lim_{x\to a} f(x)=L \Leftrightarrow \forall\varepsilon>0\ ;\exists\ \ \delta>0:|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$
+ $\lim_{x\to a} f(x)=\infty \Leftrightarrow \forall M>0\ ;\exists\ \ \delta>0:|x-a|<\delta \Rightarrow |f(x)|>M$
+ $\lim_{x\to\infty} f(x)=L \Leftrightarrow \forall\varepsilon\ ;\exists\ \ N>0:|x|>N \Rightarrow |f(x)-L|<\varepsilon$
+ $\lim_{x\to\infty} f(x)=\infty \Leftrightarrow \forall M>0\ ;\exists\ N>0:|x|>N \Rightarrow |f(x)|<M$

☞ លីមីតឆ្វេងនិងលីមីតស្តាំ

និយមន័យទី១ បើ L គឺជាលីមីតឆ្វេងនៃអនុគមន៍ f កាលណា $x \to a^-$, $(x<a)$

បើចំពោះ $\forall \varepsilon > 0$; $\exists\, \delta > 0$ ដែល $0 < a - x < \delta \Rightarrow |f(x) - L| < \varepsilon$ គេសរសេរ $\lim\limits_{x \to a^-} f(x) = L$

ឧទាហរណ៍:បង្ហាញថា $\lim\limits_{x \to 2^-} \sqrt{x} = \sqrt{2}$

តាមនិយមន័យ ចំពោះគ្រប់ $\varepsilon > 0$ គេបាន $|f(x) - L| < \varepsilon \Leftrightarrow \left|\sqrt{x} - \sqrt{2}\right| < \varepsilon$

$$\Leftrightarrow -\varepsilon < \sqrt{x} - \sqrt{2} < \varepsilon \Leftrightarrow \sqrt{2} - \varepsilon < \sqrt{x} < \varepsilon + \sqrt{2} \Leftrightarrow \left(\sqrt{2} - \varepsilon\right)^2 < x < \left(\varepsilon + \sqrt{2}\right)^2$$

$$\Leftrightarrow \varepsilon^2 - 2\sqrt{2}\varepsilon + 2 < x < \varepsilon^2 + 2\sqrt{2}\varepsilon + 2 \Leftrightarrow \varepsilon^2 - 2\sqrt{2}\varepsilon < x - 2 < \varepsilon^2 + 2\sqrt{2}\varepsilon$$

$\Leftrightarrow 2 - x < 2\sqrt{2}\varepsilon - \varepsilon^2$ បើគេយក $\delta = 2\sqrt{2}\varepsilon - \varepsilon^2 > 0$ យើងបាន $0 < 2 - x < \delta$

ដូចនេះ $\lim\limits_{x \to 2^-} \sqrt{x} = \sqrt{2}$ ពិត

និយមន័យទី២ បើ L គឺជាលីមីតស្តាំនៃអនុគមន៍ f កាលណា $x \to a^+$, $(x > a)$

បើចំពោះ $\forall \varepsilon > 0$; $\exists\, \delta > 0$ ដែល $0 < x - a < \delta \Rightarrow |f(x) - L| < \varepsilon$ គេសរសេរ $\lim\limits_{x \to a^+} f(x) = L$

ឧទាហរណ៍:បង្ហាញថា $\lim\limits_{x \to 4^+} \sqrt{x} = 2$

តាមនិយមន័យ ចំពោះគ្រប់ $\varepsilon > 0$ គេបាន $|f(x) - L| < \varepsilon \Leftrightarrow \left|\sqrt{x} - 2\right| < \varepsilon \Leftrightarrow -\varepsilon < \sqrt{x} - 2 < \varepsilon$

$$\Leftrightarrow 2 - \varepsilon < \sqrt{x} < \varepsilon + 2 \Leftrightarrow (2 - \varepsilon)^2 < x < (\varepsilon + 2)^2$$

$$\Leftrightarrow 4 - 4\varepsilon + \varepsilon^2 < x < \varepsilon^2 + 4\varepsilon + 4 \Leftrightarrow \varepsilon^2 - 4\varepsilon < x - 4 < \varepsilon^2 + 4\varepsilon \Rightarrow x - 4 < \varepsilon^2 + 4\varepsilon$$

គេយក $\delta = \varepsilon^2 + 4\varepsilon > 0$ យើងបាន $0 < x - 4 < \delta$

ដូចនេះ $\lim\limits_{x \to 4^+} \sqrt{x} = 2$ ពិត

លក្ខខណ្ឌចាំបាច់និងគ្រប់គ្រាន់ដើម្បីអោយ $\lim\limits_{x \to a} f(x) = L$ គឺ $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = L$

ឧទាហរណ៍បង្ហាញថា $f(x)=\dfrac{x^2-4}{x-2}$ មានលីមីតស្មើ 4

យើងមាន $\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+}\left(\dfrac{x^2-4}{x-2}\right)=\lim\limits_{x\to 2^+}\dfrac{(x-2)(x+2)}{x-2}=\lim\limits_{x\to 2^+}(x+2)=4$

និង $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-}\dfrac{x^2-4}{x-2}=\lim\limits_{x\to 2^-}\dfrac{(x-2)(x+2)}{x-2}=\lim\limits_{x\to 2^-}(x+2)=4$

ដូចនេះ $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2} f(x)=4$

✡✡ ប្រមាណវិធីលើលីមីតនិងលក្ខណ:

+ $\lim\limits_{x\to a}[f(x)\pm g(x)]=\lim\limits_{x\to a} f(x)\pm\lim\limits_{x\to a} g(x)$
+ $\lim\limits_{x\to a} kf(x)=k\lim\limits_{x\to a} f(x)$ (k *ជាចំនួនថេរ*)
+ $\lim\limits_{x\to a}[f(x)\times g(x)]=[\lim\limits_{x\to a} f(x)]\times[\lim\limits_{x\to a} g(x)]$
+ $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$
+ $\lim\limits_{x\to a}[f(x)]^n=[\lim\limits_{x\to a} f(x)]^n$ (ដែល n ជាចំនួនគត់រ៉ឺឡាទីបវិជ្ជមាន)
+ $\lim\limits_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim\limits_{x\to a} f(x)}$
+ $\lim\limits_{x\to a} k^{f(x)}=k^{\lim\limits_{x\to a} f(x)}$ *និង* $\lim\limits_{x\to a} k=k$
+ $\lim\limits_{x\to a}[\ln f(x)]=\ln[\lim\limits_{x\to a} f(x)]$

✡☞ ទ្រឹស្តីបទ

+ *បើ* $f(x)\le g(x)\le h(x)$ *ចំពោះ* $x\in(\alpha\ ,\ \beta)$ *ហើយ* $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} h(x)=L$

ដែល $a\in(\alpha\ ,\ \beta)$ នោះគេបាន $\lim\limits_{x\to a} g(x)=L$

+ *បើ* $f(x)\ge g(x)$ *ចំពោះ* $x\in(\alpha\ ,\ \beta)$ *បើ* $a\in(\alpha\ ,\ \beta)$ នោះគេបាន $\lim\limits_{x\to a} f(x)\ge\lim\limits_{x\to a} g(x)$
+ *បើ* $f(x)$ មានដែនកំណត់ D , $a\in D$ នោះគេបាន $\lim\limits_{x\to a} f(x)=f(a)$
+ *បើ* $f(x)\ge g(x)$ *ចំពោះ* $x\ge A$ *ដែល* $\lim\limits_{x\to+\infty} g(x)=+\infty$ នោះគេបាន $\lim\limits_{x\to+\infty} f(x)=+\infty$

+ *បើ* $f(x) \le g(x)$ *ចំពោះ* $x \ge A$ *ដែល* $\lim\limits_{x \to +\infty} g(x) = -\infty$ នោះគេបាន $\lim\limits_{x \to +\infty} f(x) = -\infty$

+ *បើ* $f(x) \le g(x) \le h(x)$ *ចំពោះ* $x \ge A$ *ដែល* $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} h(x) = L$ នោះគេបាន $\lim\limits_{x \to +\infty} g(x) = L$

+ *បើ* $f(x) \le g(x)$ *ចំពោះ* $x \ge A$ *ដែល* $\lim\limits_{x \to +\infty} f(x) = L$ និង $\lim\limits_{x \to +\infty} g(x) = \lambda$ នោះគេបាន $L \le \lambda$

+ *បើ* $f(x)$ និង $g(x)$ ជាអនុគមន៍ដែលមានលីមីត $\lim\limits_{x \to a} g(x) = L$ *និង* $\lim\limits_{x \to L} f(x) = f(L)$

នោះគេបាន $\lim\limits_{x \to a} f[g(x)] = f(L)$

បើ $\lim\limits_{x \to a} f(x) = L$ *និង* $\lim\limits_{x \to a} g(x) = M$

+ $\lim\limits_{x \to a} \dfrac{1}{f(x)} = \dfrac{1}{L}$ *កាលណា* $L \ne 0$

+ $\lim\limits_{x \to a} \dfrac{1}{f(x)} = 0$ *កាលណា* $L = \pm\infty$

+ $\lim\limits_{x \to a} \dfrac{1}{f(x)} = +\infty$ *កាលណា* $L \to 0^{+}$

+ $\lim\limits_{x \to a} \dfrac{1}{f(x)} = -\infty$ *កាលណា* $L \to 0^{-}$

+ $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = 0$ *កាលណា* $L \ne 0$ *ហើយ* $m = \infty$

+ $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \infty$ *កាលណា* $L \ne 0$ *ហើយ* $m = 0$

២.លីមីត អនុគមន៍ត្រីកោណមាត្រ អិចស្ប៉ូ និងលោការីត

១. $\lim\limits_{x \to 0} \dfrac{\sin ax}{ax} = \lim\limits_{x \to 0} \dfrac{ax}{\sin ax} = 1$ និង $\lim\limits_{x \to 0} \dfrac{\sin^2 ax}{x^2} = a^2$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$

គេបាន $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x} = \lim\limits_{x \to 0} \left(\dfrac{\sin 3x}{3x} \times 3 \right) = 3 \lim\limits_{x \to 0} \dfrac{\sin 3x}{3x} = 3 \times 1 = 3$

២. $\lim\limits_{x \to 0} \dfrac{1 - \cos ax}{ax} = \lim\limits_{x \to 0} \dfrac{ax}{1 - \cos ax} = 0$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x}$

គេបាន $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x} = \lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{2x} \times 2 = 2 \lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{2x} = 2 \times 0 = 0$

៣. $\lim\limits_{x \to 0} \dfrac{\tan ax}{ax} = \lim\limits_{x \to 0} \dfrac{ax}{\tan ax} = 1$ និង $\lim\limits_{x \to 0} \dfrac{\tan^2 ax}{x^2} = a^2$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to 0}\frac{\tan 5x}{x}$

គេបាន $\lim_{x\to 0}\frac{\tan 5x}{x}=\lim_{x\to 0}\frac{\tan 5x}{5x}\times 5=5\lim_{x\to 0}\frac{\tan 5x}{5x}=5\times 1=5$

៤. $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$ និង $\lim_{x\to 0}\frac{1-\cos^2 ax}{x^2}=\frac{a^2}{2}$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

គេបាន $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}=\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}\times 4=4\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}=4\times\frac{1}{2}=2$

៥. $\lim_{x\to +\infty} e^x=+\infty$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to +\infty}\left(x-2+xe^x\right)$

គេបាន $\lim_{x\to +\infty}\left(x-2+xe^x\right)=\lim_{x\to +\infty}\left(x-2\right)+\lim_{x\to +\infty}\left(xe^x\right)=\left(+\infty-2\right)+\left(+\infty\right)\times\left(+\infty\right)=+\infty$

៦. $\lim_{x\to -\infty} e^x=0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to -\infty}\left(2e^x+e^{2x}+1\right)$

គេបាន $\lim_{x\to -\infty}\left(2e^x+e^{2x}+1\right)=\lim_{x\to -\infty}\left(e^x+1\right)^2=\left(\lim_{x\to -\infty} e^x+1\right)^2=\left(0+1\right)^2=1$

៧. $\lim_{x\to +\infty}\frac{e^x}{x^n}=+\infty\ ,\ n>0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to +\infty}\frac{e^x-x^2+x+2}{x^2}$

គេបាន $\lim_{x\to +\infty}\frac{e^x-x^2+x+2}{x^2}=\lim_{x\to +\infty}\frac{x^2\left(\frac{e^x}{x^2}-1+\frac{1}{x}+\frac{2}{x^2}\right)}{x^2}=\lim_{x\to +\infty}\left(\frac{e^x}{x^2}-1+\frac{1}{x}+\frac{2}{x^2}\right)=+\infty$

៨. $\lim_{x\to +\infty}\frac{x^n}{e^x}=0\ \ ,\ n>0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to+\infty}\left(\frac{e^x-x}{2e^x+1}\right)$

គេបាន $\lim_{x\to+\infty}\left(\frac{e^x-x}{2e^x+1}\right)=\lim_{x\to+\infty}\frac{e^x\left(1-\frac{x}{e^x}\right)}{e^x\left(2+\frac{1}{e^x}\right)}=\lim_{x\to+\infty}\frac{1-\frac{x}{e^x}}{2+\frac{1}{e^x}}=\frac{1-0}{2+0}=\frac{1}{2}$

៩. $\lim_{x\to+\infty}\ln x=+\infty$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to+\infty}\left(\frac{1}{x}-2\ln x\right)$

គេបាន $\lim_{x\to+\infty}\left(\frac{1}{x}-2\ln x\right)=\lim_{x\to+\infty}\frac{1}{x}-2\lim_{x\to+\infty}\ln x=0-2\times(+\infty)=-\infty$

១០. $\lim_{x\to0^+}\ln x=-\infty$

ឧទាហរណ៍ គណនាលីមី $\lim_{x\to0^+}\left(x^2-x-1-5\ln x\right)$

គេបាន $\lim_{x\to0^+}\left(x^2-x-1-5\ln x\right)=\lim_{x\to0^+}\left(x^2-x-1\right)-5\lim_{x\to\infty}\ln x=-1-5\times(-\infty)=+\infty$

១១. $\lim_{x\to0^+}x^n\ln x=0\ ,n>0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to0^+}\left[x\cdot\left(\frac{1}{x}+4x-\ln x\right)\right]$ គេបាន

$\lim_{x\to0^+}\left[x\cdot\left(\frac{1}{x}+4x-\ln x\right)\right]=\lim_{x\to0^+}\left(1+4x^2-x\ln x\right)=1+4\lim_{x\to0^+}x^2-\lim_{x\to0^+}x\ln x=1+4\times0-0=1$

១២. $\lim_{x\to0^+}\frac{\ln x}{x^n}=-\infty\ ,n>0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to0^+}\frac{4x^2+3x-\ln x}{x^2}$

គេបាន $\lim_{x\to0^+}\frac{4x^2+3x-\ln x}{x^2}=\lim_{x\to0^+}\frac{x^2\left(4+\frac{3}{x}-\frac{\ln x}{x^2}\right)}{x^2}=\lim_{x\to0^+}\left(4+\frac{3}{x}-\frac{\ln x}{x^2}\right)=4+\infty+\infty=+\infty$

១៣. $\lim\limits_{x\to+\infty}\dfrac{\ln x}{x^n}=0\ ,\ n>0$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to+\infty}\dfrac{x^2+4x+\ln x}{x^2-4\ln x}$

គេបាន $\lim\limits_{x\to+\infty}\dfrac{x^2+4x+\ln x}{x^2-4\ln x}=\lim\limits_{x\to+\infty}\dfrac{x^2\left(1+\dfrac{4}{x}+\dfrac{\ln x}{x^2}\right)}{x^2\left(1-4\dfrac{\ln x}{x^2}\right)}=\lim\limits_{x\to+\infty}\dfrac{1+\dfrac{4}{x}+\dfrac{\ln x}{x^2}}{1-4\dfrac{\ln x}{x^2}}=\dfrac{1+0+0}{1-0}=1$

១៤. $\lim\limits_{x\to0^+}\dfrac{x^n}{\ln x}=0\ ,n>0$

ឧទារណ៍ គណនាលីមីត $\lim\limits_{x\to0^+}\dfrac{x^2+3x+4\ln x}{x^2-2x-3\ln x}$

គេបាន $\lim\limits_{x\to0^+}\dfrac{x^2+3x+4\ln x}{x^2-2x-3\ln x}=\lim\limits_{x\to0^+}\dfrac{\ln x\left(\dfrac{x^2}{\ln x}+\dfrac{3x}{\ln x}+4\right)}{\ln x\left(\dfrac{x^2}{\ln x}-\dfrac{2x}{\ln x}-3\right)}=\lim\limits_{x\to0^+}\dfrac{\dfrac{x^2}{\ln x}+3\dfrac{x}{\ln x}+4}{\dfrac{x^2}{\ln x}-2\dfrac{x}{\ln x}-3}=\dfrac{0+0+4}{0-0-3}=-\dfrac{4}{3}$

១៥. $\lim\limits_{x\to+\infty}\dfrac{x^n}{\ln x}=+\infty\ ,n>0$

ឧទាហរណ៍ គណនាលីមីត $\lim\limits_{x\to+\infty}\dfrac{x^2+\ln x}{\ln x}$

គេបាន $\lim\limits_{x\to+\infty}\dfrac{x^2+\ln x}{\ln x}=\lim\limits_{x\to+\infty}\dfrac{\ln x\left(\dfrac{x^2}{\ln x}+1\right)}{\ln x}=\lim\limits_{x\to+\infty}\left(1+\dfrac{x^2}{\ln x}\right)=+\infty$

១៦. $\lim\limits_{x\to+\infty}a^x=+\infty\ ,\ a>1$

ឧទាហរណ៍គណនាលីមីត $\lim\limits_{x\to+\infty}\left(4+3^x\right)$ គេបាន

$\lim\limits_{x\to+\infty}\left(4+3^x\right)=4+\lim\limits_{x\to+\infty}3^x=+\infty$

១៧. $\lim\limits_{x\to+\infty}a^x=0\ ,1>a>0$

ឧទាហរណ៍ $\lim_{x\to+\infty}\left(1+2^{-x}\right)=\lim_{x\to+\infty}\left(1+\frac{1}{2^x}\right)=1+0=1$

១៨. $\lim_{x\to-\infty}a^x=0\ ,\ a>1$

ឧទាហរណ៍គណនាលីមីត $\lim_{x\to-\infty}\left(1+2^{x+1}+2^{2x}\right)$ គេបាន

$$\lim_{x\to-\infty}\left(1+2^{x+1}+2^{2x}\right)=\lim_{x\to-\infty}\left(1+2^x\right)^2=\left[\lim_{x\to-\infty}\left(1+2^x\right)\right]^2=(1+0)^2=1$$

១៩. $\lim_{x\to-\infty}a^x=+\infty\ ,1>a>0$

ឧទាហរណ៍ គណនាលីមីត $\lim_{x\to-\infty}\left(1+\frac{1}{2^{x-1}}+\frac{1}{2^{2x}}\right)$

គេបាន $\lim_{x\to-\infty}\left(1+\frac{1}{2^{x-1}}+\frac{1}{2^{2x}}\right)=\lim_{x\to-\infty}\left(1+\frac{1}{2^x}\right)^2=\left[\lim_{x\to-\infty}\left(1+\frac{1}{2^x}\right)\right]^2=(1+\infty)^2=+\infty$

២០. $\lim_{x\to+\infty}\frac{a^x}{x^n}=+\infty\ ,\ a>1$

ឧទាហរណ៍គណនាលីមីត $\lim_{x\to+\infty}\left(\frac{x^2+3^x+4^x}{x^2+2x+1}\right)$ គេបាន

$$\lim_{x\to+\infty}\left(\frac{x^2+3^x+4^x}{x^2+2x+1}\right)=\lim_{x\to+\infty}\frac{x^2\left(1+\frac{3^x}{x^2}+\frac{4^x}{x^2}\right)}{x^2\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}=\lim_{x\to+\infty}\frac{1+\frac{3^x}{x^2}+\frac{4^x}{x^2}}{1+\frac{2}{x}+\frac{1}{x^2}}=\frac{1+\infty+\infty}{1+0+0}=+\infty$$

២១. $\lim_{x\to+\infty}\frac{a^x}{x^n}=0\ ,1>a>0$

ឧទាហរណ៍គណនាលីមីត $\lim_{x\to+\infty}\left(\frac{x^2+3^{-x}+4^{-x}}{x^2-3x+2}\right)$ គេបាន

$$\lim_{x\to+\infty}\left(\frac{x^2+3^{-x}+4^{-x}}{x^2-3x+2}\right)=\lim_{x\to+\infty}\frac{x^2\left(1+\frac{3^{-x}}{x^2}+\frac{4^{-x}}{x^2}\right)}{x^2\left(1-\frac{3}{x}+\frac{2}{x^2}\right)}=\lim_{x\to+\infty}\frac{1+\frac{3^{-x}}{x^2}+\frac{4^{-x}}{x^2}}{1-\frac{3}{x}+\frac{2}{x^2}}=\frac{1+0+0}{1-0+0}=1$$

២២. $\lim\limits_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}=\lim\limits_{x\to0}(1+x)^{\frac{1}{x}}=e$

ឧទាហរណ៍គណនាលីមីត $\lim\limits_{x\to+\infty}\left(\frac{x+8}{x-2}\right)^{x}$

យើងមាន $\frac{x+8}{x-2}=\frac{x+8}{x-2}-1+1=1+\frac{10}{x-2}$

គេបាន $\lim\limits_{x\to+\infty}\left(\frac{x+8}{x-2}\right)^{x}=\lim\limits_{x\to+\infty}\left(1+\frac{10}{x-2}\right)^{x}=\lim\limits_{x\to+\infty}\left[\left(1+\frac{10}{x-2}\right)^{\frac{x-2}{10}}\right]^{\frac{10x}{x-2}}=e^{10}$

ឧទាហរណ៍គណនាលីមីត $\lim\limits_{x\to0}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}$ យើងមាន $\frac{x^2+1}{x^2-1}=1+\frac{x^2+1}{x^2-1}-1=1+\frac{1}{x^2-1}$

គេបាន

$$\lim_{x\to0}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}=\lim_{x\to0}\left(1+\frac{1}{x^2-1}\right)^{x^2}=\lim_{x\to0}\left[\left(1+\frac{1}{x^2-1}\right)^{\left(x^2-1\right)}\right]^{\frac{x^2}{x^2-1}}=e^{\lim\limits_{x\to0}\left(\frac{x^2}{x^2-1}\right)}=e^{0}=1$$

២៣. $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=\lim\limits_{x\to0}\frac{e^{ax}-1}{ax}=1$

ឧទាហរណ៍គណនា $\lim\limits_{x\to0}\left(\frac{e^{2x}+3e^{x}-4}{e^{3x}+2e^{x}-3}\right)$ គេបាន

$$\lim_{x\to0}\left(\frac{e^{2x}-1+3e^{x}-3}{e^{3x}-1+2e^{x}-2}\right)=\lim_{x\to0}\frac{\frac{e^{2x}-1}{x}+3\frac{e^{x}-1}{x}}{\frac{e^{3x}-1}{x}+2\frac{e^{x}-1}{x}}=\frac{\lim\limits_{x\to0}\left(\frac{e^{2x}-1}{2x}\cdot2+3\cdot\frac{e^{x}-1}{x}\right)}{\lim\limits_{x\to0}\left(\frac{e^{3x}-1}{3x}\cdot3+2\cdot\frac{e^{x}-1}{x}\right)}=\frac{1\times2+3\times1}{1\times3+2\times1}=1$$

ឧទាហរណ៍គណនាលីមីត $\lim\limits_{x\to0}\frac{\ln(1+3x)+e^{3x}-1}{\sin4x}$ គេបាន

$$\lim_{x\to0}\frac{\ln(1+3x)+e^{3x}-1}{\sin4x}=\lim_{x\to0}\frac{\frac{\ln(1+3x)+e^{3x}-1}{x}}{\frac{\sin4x}{x}}=\lim_{x\to0}\frac{\frac{\ln(1+3x)}{3x}\cdot3+\frac{e^{3x}-1}{3x}\cdot3}{\frac{\sin4x}{4x}\cdot4}=\frac{1\times3+1\times3}{1\times4}=\frac{3}{2}$$

២៤. $\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$

ឧទាហរណ៍គណនាលីមីត $\lim_{x\to 0}\frac{7^x-8^x}{3^x-4^x}$ គេបាន

$$\lim_{x\to 0}\frac{5^x-4^x}{3^x-4^x}=\lim_{x\to 0}\frac{4^x\cdot\left[\left(\frac{5}{4}\right)^x-1\right]}{4^x\cdot\left[\left(\frac{3}{4}\right)^x-1\right]}=\lim_{x\to 0}\frac{\frac{\left(\frac{5}{4}\right)^x-1}{x}}{\frac{\left(\frac{3}{4}\right)^x-1}{x}}=\frac{\ln 5-\ln 4}{\ln 3-\ln 4}$$

២៥. $\lim_{x\to 0}\frac{(1+x)^n-1}{x}=n$

ឧទាហរណ៍គណនាលីមីត $\lim_{x\to 0}\frac{x^3+3x^2+3x}{x^2+2x}$ គេបាន

$$\lim_{x\to 0}\frac{x^3+3x^2+3x}{x^2+2x}=\lim_{x\to 0}\frac{x^3+3x^2+3x+1-1}{x^2+2x+1-1}=\lim_{x\to 0}\frac{(1+x)^3-1}{(1+x)^2-1}=\lim_{x\to 0}\frac{\frac{(1+x)^3-1}{x}}{\frac{(1+x)^2-1}{x}}=\frac{3}{2}$$

☞ វិធានក្នុងការគណនាលីមីត

១.គណនាលីមីត $\lim_{x\to a}\frac{f(x)}{g(x)}$ មានរាងមិនកំណត់ $\frac{0}{0}$ ដើម្បីគណនាលីមីតរាងមិនកំណត់ $\frac{0}{0}$ បែបនេះគេត្រូវ:

- បំបែកភាគយកនិងភាគបែងអោយមានកត្តារួម $(x-a)$ ឬ $(x-a)^2$............

- សម្រួលកត្តារួម $(x-a)$ ចោលដើម្បីបំបាត់រាងមិនកំណត់

(ព្រោះកាលណា $x\to a$ នោះ $x\neq a$ ឬ $x-a\neq 0$ ដូច្នេះគេអាចសម្រួលកត្តា $(x-a)$ ចោលបាន)

- ជំនួសតម្លៃ x ដោយ a ទៅក្នុងលីមីតកន្សោមថ្មី

ឧទាហរណ៍: គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1-x}}{3x}$ មានរាងមិនកំណត់ $\dfrac{0}{0}$

វិធីទី១យើងគណនាដោយប្រើកន្សោមឆ្លាស់នោះគេបាន

$$\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}=\lim_{x\to 0}\frac{\left(1-\sqrt[3]{1-x}\right)\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}{3x\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}$$

$$=\lim_{x\to 0}\frac{1-1+x}{3x\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}=\lim_{x\to 0}\frac{1}{3\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}=\frac{1}{9}$$

វិធីទី២ គេតាង $u^3=1-x\Rightarrow x=1-u^3$ កាលណា $x\to 0$ នោះ $u\to 1$ យើងបាន

$$\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}=\lim_{u\to 1}\frac{1-u}{3\left(1-u^3\right)}=\lim_{u\to 1}\frac{(1-u)}{3(1-u)\left(1+u+u^2\right)}=\lim_{u\to 1}\frac{1}{3\left(1+u+u^2\right)}=\frac{1}{9}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1-x}}{3x}=\dfrac{1}{9}$

✡✡ **ចំណាំ** ដើម្បីអោយការបំបែកជាផលគុណកត្តាមានភាពងាយស្រួលយើងត្រូវចាំឯកលក្ខណ:ភាពខាងក្រោម៖

+ $a^2-b^2=(a-b)(a+b)$
+ $a^3-b^3=(a-b)(a^2+ab+b^2)$
+ $a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$
+ $a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+\ldots\ldots\ldots\ldots+ab^{n-2}+b^{n-1})$
+ $a^3+b^3=(a+b)(a^2-ab+b^2)$
+ $a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$
+ $a^n+b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2+\ldots\ldots\ldots\ldots+(-1)^p a^p b^{n-p}+\ldots.+b^{n-1})$ n ជាចំនួនគត់សេស

☛ បើភាគយកឬភាគបែងជាប់រ៉ាឌីកាលគេត្រូវគុណភាគយកនិងភាគបែងជាមួយកន្សោមឆ្លាស់របស់ វាដោយប្រើរូបមន្តខាងក្រោម៖

$$\sqrt[n]{a}-\sqrt[n]{b}=\frac{\sqrt[n]{a^n}-\sqrt[n]{b^n}}{\sqrt[n]{a^{n-1}}+\sqrt[n]{a^{n-2}}\sqrt[n]{b}+\dots\dots\dots\dots\dots\dots\dots+\sqrt[n]{a}\sqrt[n]{b^{n-2}}+\sqrt[n]{b^{n-1}}}$$

២.គណនាលីមីតមានរាងមិនកំណត់ $\frac{\infty}{\infty}$ ដើម្បីគណនាលីមីតប្រភេទនេះគេត្រូវ:

- ទាញយកតួ x ដែលមានដឺក្រេខ្ពស់ជាងគេក្នុង $f(x)$ និង $g(x)$ ចេញជាកត្តា
- សម្រួលកត្តានោះចោល(ព្រោះ $x\to\infty \Rightarrow x\neq 0$)
- អោយតម្លៃ $x\to\infty$ យើងនឹងបានលទ្ធផល

ឧទាហរណ៍:គណនាលីមីត $\lim\limits_{x\to\infty}\frac{x^3-4x^2+5x-1}{3x^3+5x^2-6x+7}$ គេបាន

$$\lim_{x\to\infty}\frac{x^3-4x^2+5x-1}{3x^3+4x^2-6x+7}=\lim_{x\to\infty}\frac{x^3\left(1-\frac{4x^2}{x^3}+\frac{5x}{x^3}-\frac{1}{x^3}\right)}{x^3\left(3+\frac{4x^2}{x^3}-\frac{6x}{x^3}+\frac{7}{x^3}\right)}=\lim_{x\to\infty}\frac{1-\frac{4}{x}+\frac{5}{x^2}-\frac{1}{x^3}}{3+\frac{4}{x}-\frac{6}{x^2}+\frac{7}{x^3}}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{x^3-4x^2+5x-1}{3x^3+5x^2-6x+7}=\frac{1}{3}$

✡ ចំពោះពហុធា $P(x)=a_nx^n+a_{n-1}x^{n-1}+\dots\dots\dots\dots\dots\dots+a^{n-1}x+a^n$ កាលណា $x\to\infty$ នោះ $P(x)\cong a_nx^n$

៣.គណនាលីមីតមានរាងមិនកំណត់ $\infty-\infty$ គេត្រូវដាក់តួដែលមានដឺក្រេធំជាងគេជាកត្តារួមហើយគណនាលីមីតកន្សោមថ្មី។គេច្រើនជួបរាងមិនកំណត់ $\infty-\infty$ នៅពេល $x\to\pm\infty$ បើកន្សោមដែលត្រូវគណនា លីមីតមានជាប់រ៉ាឌីកាលនោះគេប្រើកន្សោមឆ្លាស់ដើម្បីបំបាត់រាងមិនកំណត់ $\infty-\infty$ នេះ។

ឧទាហរណ៍:គណនលីមីត $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)$ គេបាន

$$\lim_{x\to\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)\left(\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}\right)}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}$$

$$=\lim_{x\to\infty}\frac{\left(x^2+5x-1\right)-\left(x^2+3x+2\right)}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}=\lim_{x\to\infty}\frac{x\left(2-\frac{3}{x}\right)}{|x|\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}$$

កាលណា $x\to+\infty$ នោះ $|x|=x$ និងកាលណា $x\to-\infty$ នោះ $|x|=-x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\left(2-\frac{3}{x}\right)}{x\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to+\infty}\frac{2-\frac{3}{x}}{\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}}=1$$

និង $$\lim_{x\to-\infty}\frac{x\left(2-\frac{3}{x}\right)}{-x\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to-\infty}\frac{2-\frac{3}{x}}{-\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}=-1$$

ដូចនេះ $$\lim_{x\to\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)=\begin{cases}1 & \text{បើ } x\to+\infty\\ -1 & \text{បើ } x\to-\infty\end{cases}$$

៤.គណនាលីមីតនៃអនុគមន៍ត្រីកោណមាត្រ

ក) លីមីតនៃអនុគមន៍ត្រីកោណកាលណា $x\to 0$ (x គិតជារ៉ាដ្យង់)គេប្រើរូបមន្តខាងក្រោម

១. $\lim_{x\to 0}\frac{\sin ax}{ax}=\lim_{x\to 0}\frac{ax}{\sin ax}=1$ ២. $\lim_{x\to 0}\frac{1-\cos ax}{ax}=\lim_{x\to 0}\frac{ax}{1-\cos ax}=0$

៣. $\lim_{x\to 0}\frac{\tan ax}{ax}=\lim_{x\to 0}\frac{ax}{\tan ax}=1$ ៤. $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

ចំណាំ

+ បើ x គិតជាដឺក្រេនោះ $\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\tan x}{x}=\frac{\pi}{180}$

+ បើ x គិតក្រាដនោះ $\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\tan x}{x}=\frac{\pi}{200}$

ខ) គណនាលីមីតនៃអនុគមន៍ត្រីកោណមាត្រកាលណា $x \to x_0$ (x គិតជារ៉ាដ្យង់) ដើម្បីគណនាលីមីតរាងមិនកំណត់នៃអនុគមន៍ត្រីកោណមាត្រកាលណា $x \to x_0$ ។

គេត្រូវតាង $u = x - x_0$ (ឬ $u = x_0 - x$) កាលណា $x \to x_0 \Rightarrow u \to 0$

រួចជំនួស $x = u + x_0$ ឬ $x = x_0 - u$ ក្នុងលីមីតដែលគេអោយរួចមកប្រើរូបមន្ត

១. $\lim_{x\to 0}\frac{\sin ax}{ax} = \lim_{x\to 0}\frac{ax}{\sin ax} = 1$ ២. $\lim_{x\to 0}\frac{1-\cos ax}{ax} = \lim_{x\to 0}\frac{ax}{1-\cos ax} = 0$

៣. $\lim_{x\to 0}\frac{\tan ax}{ax} = \lim_{x\to 0}\frac{ax}{\tan ax} = 1$ ៤. $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$

ឧទាហរណ៍:គណនាលីមីតនៃអនុគមន៍ $\lim_{x\to -2}\frac{\tan \pi x}{x+2}$, $\lim_{x\to 1}\left(\frac{1-x^2}{\sin \pi x}\right)$, $\lim_{x\to \frac{\pi}{2}}\left[\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}\right]$

ចម្លើយ

1.តាង $u = x+2 \Rightarrow x = u-2$ កាលណា $x \to -2$ នោះ $u \to 0$

យើងបាន $\lim_{x\to -2}\frac{\tan \pi x}{x+2} = \lim_{u\to 0}\frac{\tan \pi(u-2)}{u} = \lim_{u\to 0}\frac{\tan(\pi u - 2\pi)}{u} = \lim_{u\to 0}\frac{-\tan(2\pi - \pi u)}{u}$

$$= \lim_{u\to 0}\frac{\tan \pi u}{u} = \lim_{u\to 0}\frac{\tan \pi u}{\pi u}\times \pi = 1\times \pi = \pi$$

ដូចនេះ $\lim_{x\to -2}\frac{\tan \pi x}{x+2} = \pi$

2.តាង $u = 1-x \Rightarrow x = 1-u$ កាលណា $x \to 1$ នោះ $u \to 0$

យើងបាន $\lim_{x\to 1}\left(\frac{1-x^2}{\sin \pi x}\right) = \lim_{x\to 1}\left[\frac{(1-x)(1+x)}{\sin \pi x}\right] = \lim_{u\to 0}\left[\frac{u(1+1-u)}{\sin(\pi - \pi u)}\right]$

$$= \lim_{u\to 0}\frac{u(2-u)}{\sin \pi u} = \lim_{u\to 0}\frac{\pi u}{\sin \pi u}\times\frac{2-u}{\pi} = 1\times\frac{2-0}{\pi} = \frac{2}{\pi}$$

ដូចនេះ $\lim_{x \to 1}\left(\frac{1-x^2}{\sin \pi x}\right)=\frac{2}{\pi}$

3.តាង $u=\frac{\pi}{2}-x \Rightarrow x=\frac{\pi}{2}-u$ កាលណា $x \to \frac{\pi}{2}$ នោះ $u \to 0$

យើងបាន $\lim_{x \to \frac{\pi}{2}}\left[\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}\right]=\lim_{u \to 0}\frac{1-\sin\left(\frac{\pi}{2}-u\right)}{u^2}=\lim_{u \to 0}\frac{1-\cos u}{u^2}=\frac{1}{2}$

លំហាត់

I. *គណនាលីមីតរាងមិនកំណត់* $\left(\frac{0}{0}\right)$

1. $\lim\limits_{x\to 1}\frac{x^2-4x+3}{x^3-x^2+2x-2}$ 2. $\lim\limits_{x\to 1}\frac{x^9-3x+2}{x^6+5x-6}$ 3. $\lim\limits_{x\to 2}\frac{x-2}{x^3-x^2-x-2}$ 4. $\lim\limits_{x\to 1}\frac{x^m-1}{x^n-1}$

5. $\lim\limits_{x\to 1}\frac{x^n-1}{x-1}$ 6. $\lim\limits_{x\to -1}\frac{x^7+1}{x^5+1}$ 7. $\lim\limits_{x\to 1}\frac{x+x^2+x^3+\dots\dots+x^n-n}{x+x^2+x^3+\dots\dots+x^m-m}$ 8. $\lim\limits_{x\to a}\frac{x^n-a^n-na^{n-1}x+na^n}{(x-a)^2}$

9. $\lim\limits_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}$ 10. $\lim\limits_{x\to 2}\frac{\sqrt{x+2}-2}{\sqrt{x+7}-3}$ 11. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}$ 12. $\lim\limits_{x\to -1}\frac{\sqrt{1-3x}-2}{\sqrt[3]{5x-3}+\sqrt{7+3x}}$

13. $\lim\limits_{x\to 1}\frac{x^{50}-7x+6}{x^{20}+3x-4}$ 14. $\lim\limits_{x\to -\frac{1}{3}}\frac{3x^2-\frac{1}{3}}{x+\frac{1}{3}}$ 15. $\lim\limits_{x\to -1}\frac{x+1}{\sqrt{x^2+2x+1}}$ 16. $\lim\limits_{x\to 0}\frac{x^2+|x|+\sqrt{x^2}}{x^2-|x|+5x}$

17. $\lim\limits_{x\to 1}\frac{x^{2016}-x^{2015}}{\sqrt{x+3}-2}$ 18. $\lim\limits_{x\to 1}\frac{x+x^2+x^3+\dots\dots+x^{2015}-2015}{x+x^2+x^3+\dots\dots+x^{2017}-2017}$ 19. $\lim\limits_{x\to -2}\frac{\sqrt{3x+10}-2}{\sqrt{2-x}+\sqrt[3]{3x-2}}$

20. $\lim\limits_{x\to 0}\frac{\sqrt[2015]{1+3x}-\sqrt[2017]{1-2x}}{x}$ 21. $\lim\limits_{x\to 0}\frac{\sqrt[2015]{1+3x}-\sqrt[2017]{1-2x}}{\sqrt[2014]{1+2x}-\sqrt[2016]{1-3x}}$ 22. $\lim\limits_{x\to 1}\frac{1-\sqrt[2015]{x}}{1-x}$

23. $\lim\limits_{x\to -\frac{1}{3}}\frac{9x^2-1}{3x+1}$ 24. $\lim\limits_{x\to 1}\frac{x^2+x-2}{x-1}$ 25. $\lim\limits_{x\to -2}\frac{x+2}{x^2+4x+4}$ 26. $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}$ 27. $\lim\limits_{x\to 2}\frac{\sqrt{x^2-4}}{x^3-x^2-x-2}$

28. $\lim\limits_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$ 29. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$ 30. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}+\sqrt{4+x}-3}{x}$ 31. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$

32. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$ 33. $\lim\limits_{x\to 1}\frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$ 34. $\lim\limits_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}$ 35. $\lim\limits_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}$ 36. $\lim\limits_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)$

37. $\lim\limits_{x\to \frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}$ 38. $\lim\limits_{x\to 1}\frac{\sqrt{1+x}+\sqrt{1+x^2}-\sqrt{1+x^3}}{\sqrt{x-1}+\sqrt{1+x^2}-\sqrt{1+x^4}}$ 39. $\lim\limits_{x\to 1}\frac{(x-1)(x^3+x-2)}{x^3-x^2-x+1}$ 40. $\lim\limits_{x\to 2}\frac{\sqrt{x+7}-3}{x-2}$

41. $\lim\limits_{x\to 1}\frac{nx^{n+1}-(n+1)x^n+1}{x^{m+1}-x^m-x+1}$ 42. $\lim\limits_{x\to 1}\frac{x^{2n}-1}{x^{2m}-1}$ 43. $\lim\limits_{x\to 64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}$ 44. $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x-1}}$ 45. $\lim\limits_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

46. $\lim\limits_{x\to 2}\frac{x^2-4}{x-2}$ 47. $\lim\limits_{x\to 1}\frac{x^3-1}{x^2-1}$ 48. $\lim\limits_{x\to 2}\frac{x^2-4}{x^2-3x+2}$ 49. $\lim\limits_{x\to 5}\frac{x^2-7x+10}{x^2-25}$ 50. $\lim\limits_{x\to 2}\frac{x^3-8}{x^2-4}$ 51. $\lim\limits_{x\to -1}\frac{x^2-1}{x^2+3x+2}$

52. $\lim\limits_{x\to 2}\frac{x^2-5x+6}{x^2-12x+20}$ 53. $\lim\limits_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}$ 54. $\lim\limits_{x\to 1}\frac{x^3-3x+2}{x^3-x^2-x-1}$ 55. $\lim\limits_{x\to 1}\frac{x^3-6x^2+11x-6}{x^2-3x+2}$ 56. $\lim\limits_{x\to 1}\frac{4x^6-5x^5+x}{(1-x)^2}$

II.គណនាលីមីតរាងមិនកំណត់ $\left(\frac{\infty}{\infty}\right)$

1. $\lim\limits_{x\to\infty}\frac{x^2-1}{2x^2+x+2}$ 2. $\lim\limits_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}$ 3. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^3+2x-1}}{x+2}$ 4. $\lim\limits_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$ 5. $\lim\limits_{x\to\infty}\frac{2x^2-3x-4}{\sqrt{x^4+2}}$

6. $\lim\limits_{x\to\infty}\frac{4x+1+\sqrt{16x^2+x+1}}{7x}$ 7. $\lim\limits_{x\to\infty}\frac{(2x+3)(3x-5)(x-1)^2}{x^2(2x-3)(4x+3)}$ 8. $\lim\limits_{x\to\infty}\frac{(x-1)(3+2x)(2-x)}{(x^2+1)(1-2x)}$ 9. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

10. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$ 11. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+3}}$ 12. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}$ 13. $\lim\limits_{x\to+\infty}\frac{1+\sqrt[4]{x}}{\sqrt[3]{x^2}}$ 14. $\lim\limits_{x\to\infty}\frac{\sqrt[4]{2+x^4}}{\sqrt[3]{5+27x^3}}$

15. $\lim\limits_{x\to\infty}\frac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2}$ 16. $\lim\limits_{x\to\infty}\frac{2x^2-3x+1}{x^3+2x+5}$ 17. $\lim\limits_{x\to\infty}\frac{3x^2+2x+5}{2x+1}$ 18. $\lim\limits_{|x|\to\infty}\frac{\sqrt{x^2-1}}{x}$ 19. $\lim\limits_{x\to\infty}\frac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6}$

20. $\lim\limits_{x\to\infty}\frac{x^2+3x-5}{2x^2+1}$ 21. $\lim\limits_{x\to\infty}\frac{x^3+5x-7}{x^2+3x-1}$ 22. $\lim\limits_{x\to\infty}\frac{x+5}{2x^2+3x+7}$ 23. $\lim\limits_{x\to\infty}\frac{x^4-5x}{x^2-3x+1}$ 24. $\lim\limits_{x\to\infty}\frac{1+x-3x^3}{1+x^2+3x^3}$

25. $\lim\limits_{x\to\infty}\frac{2x^2+3x+1}{3x^2-x+5}$ 26. $\lim\limits_{x\to\infty}\frac{x^2+3x-8}{x^4-6x+1}$ 27. $\lim\limits_{x\to\infty}\frac{(x-2)(2x+1)(1-4x)}{(3x+4)^3}$ 28. $\lim\limits_{x\to\infty}\frac{4x^3+3x-7}{x^2-3x+5}$ 29. $\lim\limits_{x\to\infty}\frac{1-3x}{2-x}$

30. $\lim\limits_{x\to\infty}\frac{2x^2+3}{x^3-2x+1}$ 31. $\lim\limits_{x\to\infty}\frac{(2x-3)(3x+5)(4x-6)}{3x^3+x-1}$ 32. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x+3}+1+4x}{\sqrt{4x^2+1}+2-x}$

33. $\lim\limits_{x\to\infty}\frac{\sqrt{9x^2+x+1}-\sqrt{4x^2+2x+1}}{x+1}$ 34. $\lim\limits_{x\to\pm\infty}\frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{1+x^2}}$ 35. $\lim\limits_{x\to\pm\infty}\frac{7x}{1+14x+\sqrt{16x^2+x+1}}$

36. $\lim\limits_{x\to\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}$ 37. $\lim\limits_{x\to\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$ 38. $\lim\limits_{x\to\infty}\frac{2x^2-5+\sqrt{x^4-3x+1}}{x-1+\sqrt[3]{4x^6+3x-2}}$ 39. $\lim\limits_{x\to+\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{1+x^2}}{\sqrt[4]{1+x^4}-\sqrt[5]{1+x^4}}$

III.គណនាលីមីតមានរាងមិនកំណត់$(+\infty-\infty)$

1. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)$ 2. $\lim\limits_{x\to\infty}\left(\sqrt[4]{4+x^4}-x\right)$ 3. $\lim\limits_{x\to+\infty}\left(\sqrt{(x+a)(x+b)}-x\right)$

4. $\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ 5. $\lim\limits_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)$ 6. $\lim\limits_{x\to\pm\infty}\left(\sqrt[4]{x^4+4x^3}-\sqrt[3]{x^3+3x^2}-\sqrt{x^2+2x}\right)$

7. $\lim\limits_{x\to1}\left(\frac{3}{\sqrt{x}-1}-\frac{2}{\sqrt[3]{x}-1}\right)$ 8. $\lim\limits_{x\to1}\left(\frac{n}{1-x^n}-\frac{1}{1-x}\right)$ 9. $\lim\limits_{x\to1}\left(\frac{2014}{1-x^{2014}}-\frac{2015}{1-x^{2015}}\right)$ 10. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

11. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-\sqrt{x(x-4)}\right)$ 12. $\lim\limits_{x\to\infty}\left[\sqrt{x^2+x+1}-(ax+b)\right]$ 13. $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+1}-x\right)$ 14. $\lim\limits_{x\to\infty}\left(\sqrt[3]{1+x}-\sqrt[3]{x}\right)$

15. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}\right)$ 16. $\lim\limits_{x\to\infty}\left(\sqrt[4]{1+x^4}-x\right)$ 17. $\lim\limits_{x\to\infty}\left(3x-\sqrt{x^2-x+1}\right)$ 18. $\lim\limits_{x\to\infty}\left(\sqrt{1+x^2}-\sqrt[3]{x^3-1}\right)$

19. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+2}\right)$ 20. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x+5}-\sqrt{1+x^2}\right)$ 21. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x-7}-\sqrt{x^2+4x-1}\right)$

22. $\lim\limits_{x\to+\infty}\left(\sqrt{3x^2+7x+1}-\sqrt{3}\,x\right)$ 23. $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)$ 24. $\lim\limits_{x\to\pm\infty}\left(\dfrac{x^3}{2x^2-1}-\dfrac{x^2}{2x+1}\right)$ 25. $\lim\limits_{x\to0}\left(\dfrac{1}{\sqrt{x}}-\dfrac{x+1}{\sqrt{x}}\right)$

26. $\lim\limits_{x\to1}\left(\dfrac{2}{x^2-1}-\dfrac{1}{x-1}\right)$ 27. $\lim\limits_{x\to-2}\left(\dfrac{1}{x+2}-\dfrac{12}{x^3+8}\right)$ 28. $\lim\limits_{x\to2}\left[\dfrac{1}{x(x-2)^2}-\dfrac{1}{x^2-3x+2}\right]$ 29. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{x+1}-x\right)$

30. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2-4x}\right)$ 31. $\lim\limits_{x\to\infty}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)$ 32. $\lim\limits_{x\to\pm\infty}\left(3x-\sqrt{x^2-x+1}\right)$

IV. គណនាលីមីតនៃអនុគមន៍ត្រីកោណមាត្រ

1. $\lim\limits_{x\to0}\dfrac{\sin 5x}{\sin 3x}$ 2. $\lim\limits_{x\to0}\dfrac{\cos 2x-\cos x}{\sin 2x}$ 3. $\lim\limits_{x\to0}\dfrac{3(1-\cos x)}{5x^2}$ 4. $\lim\limits_{x\to0}\dfrac{2\sin x-\sin 2x}{x^3}$ 5. $\lim\limits_{x\to0}\dfrac{\tan x-\sin x}{x^3}$

6. $\lim\limits_{x\to0}\dfrac{1-\cos kx}{x^2}$ 7. $\lim\limits_{x\to0}\dfrac{\sin 5x\cdot\sin 3x\cdot\sin x}{45x^3}$ 8. $\lim\limits_{x\to0}\dfrac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}$ 9. $\lim\limits_{x\to0}\dfrac{1-\cos(1-\cos 2x)}{x^4}$

10. $\lim\limits_{x\to0}\dfrac{\sin(a+3x)-3\sin(a+2x)+3\sin(a+x)-\sin a}{x^3}$ 11. $\lim\limits_{x\to0}x^2\cdot\sin\dfrac{1}{x}$ 12. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x}{\tan 2x}$ 13. $\lim\limits_{x\to a}\left(a^2-x^2\right)\tan\dfrac{\pi x}{2a}$

14. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan x-1}{2\cos x-\sqrt{2}}$ 15. $\lim\limits_{x\to\infty}x\cdot\sin\dfrac{\pi}{x}$ 16. $\lim\limits_{x\to1}(1-x)\tan\dfrac{\pi x}{2}$ 17. $\lim\limits_{x\to\frac{\pi}{4}}\tan 2x\cdot\tan\left(\dfrac{\pi}{4}-x\right)$ 18. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-2\cos x}{\sin 3x}$

19. $\lim\limits_{x\to1}\dfrac{\sin\pi x}{\sin 3\pi x}$ 20. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\pi}{2}-x\right)\cdot\tan x$ 21. $\lim\limits_{x\to a}\dfrac{\sin(x-a)}{x^3-a^3}$ 22. $\lim\limits_{x\to1}\dfrac{1-x^2}{\sin\pi x}$ 23. $\lim\limits_{x\to\pi}\dfrac{1-\sin\frac{x}{2}}{(\pi-x)^2}$ 24. $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{(x-\pi)^2}$

25. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan^3 x-3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}$ 26. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}$ 27. $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$ 28. $\lim\limits_{x\to a}\dfrac{\tan x-\tan a}{\cos x-\cos a}$ 29. $\lim\limits_{x\to a}\dfrac{\cot x-\cot a}{\tan x-\tan a}$

30. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\cos^2 x-3}$ 31. $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)\cdot\tan x$ 32. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin^2 2x+\cos 2x+1}{\cos 2x+\sin x}$ 33. $\lim\limits_{x\to\frac{\pi}{2}}\tan x\cdot\cot\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

34. $\lim\limits_{x\to a}\sin\left(\dfrac{x-a}{2}\right)\cdot\tan\dfrac{\pi x}{2a}$ 35. $\lim\limits_{x\to+\infty}\left(\sin\sqrt{1+x}-\sin\sqrt{x}\right)$ 36. $\lim\limits_{x\to0}\dfrac{\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\cdot\sin x}{1-\cos\pi x}$

37. $\lim\limits_{x\to+\infty}\dfrac{(1+x)\cdot\sin x}{3+x^2}$ 38. $\lim\limits_{x\to0}\dfrac{(1+x^2)-\cos x}{\tan^2 x}$ 39. $\lim\limits_{x\to0}\sin\left(5\pi+\dfrac{x}{2}\right)\cdot\left(\dfrac{\cos x}{x}-\dfrac{4}{\sin x}\right)$ 40. $\lim\limits_{x\to\frac{\pi}{2}}\left(2x\tan x-\dfrac{\pi}{\cos x}\right)$

41. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin\left(x-\dfrac{\pi}{6}\right)}{\dfrac{\sqrt{3}}{2}-\cos x}$ 42. $\lim\limits_{x\to0}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$ 43. $\lim\limits_{x\to a}\dfrac{\sin^2 x-\sin^2 a}{x^2-a^2}$ 44. $\lim\limits_{x\to0}\dfrac{\sin(x+a)-\sin(a-x)}{x}$

45. $\lim\limits_{x\to0}\dfrac{1-\cos x\cdot\sqrt{\cos 2x}}{\tan x}$ 46. $\lim\limits_{x\to+\infty}2^x\tan\dfrac{\pi}{2^x}$ 47. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{1-\sqrt{2}\sin x}$ 48. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin 7x+\cos 7x}{\sin 9x-\cos 9x}$ 49. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos x-\sin x}{\cos 2x}$

50. $\lim\limits_{x\to2}(x-2)\cdot\tan\dfrac{\pi}{x}$ 51. $\lim\limits_{x\to\frac{\pi}{4}}(\pi-2x)\tan x$ 52. $\lim\limits_{x\to\frac{\pi}{4}}(1-\sin 2x)\cdot\dfrac{\tan 2x}{\tan 4x}$ 53. $\lim\limits_{x\to\frac{\pi}{4}}(1-\sin 2x)\cdot\tan 2x$

54. $\lim\limits_{x\to\pi}\tan x\cdot\tan\dfrac{x}{2}$ 55. $\lim\limits_{x\to\frac{\pi}{4}}\left(4x\tan 2x-\dfrac{\pi}{\cos 2x}\right)$ 56. $\lim\limits_{x\to0}\dfrac{\tan x}{\sqrt[3]{(1-\cos x)^2}}$ 57. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin\left(x+\dfrac{\pi}{2}\right)}{\tan\left(x-\dfrac{\pi}{2}\right)}$

58. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan x}{\sin\left(x-\dfrac{\pi}{4}\right)}$ 59. $\lim\limits_{x\to1}\dfrac{\sin^2\pi x}{x-1}$ 60. $\lim\limits_{x\to0}\dfrac{\sqrt{2x+1}-\sqrt{x+1}}{\sin x}$ 61. $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2\left(1+\sqrt{\cos x}\right)}$ 62. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{\cos\dfrac{\pi}{4}-\cos x}$

63. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin 3x}{1-2\cos x}$ 64. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sqrt{3}\cos x-\sin x}{x-\dfrac{\pi}{3}}$ 65. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cot x}{x-\dfrac{\pi}{2}}$ 66. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin 2x}{\cos 3x}$ 67. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{\tan x-1}$

68. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan x}{1-\cot x}$ 69. $\lim\limits_{x\to\infty}\dfrac{(x+1)\cdot\sin x}{x^2+2}$ 70. $\lim\limits_{x\to0}x\left(\sin\dfrac{1}{x}-\dfrac{1}{\sin x}\right)$ 71. $\lim\limits_{x\to0}\dfrac{\cos x-\sqrt{\cos 2x}}{\sin^2 x}$ 72. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sqrt{2\cos x}-1}{2\cos 2x+1}$

73. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\sqrt{2}\cos x}{1-\sqrt{2}\sin x}$ 74. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin\left(x+\dfrac{\pi}{2}\right)}{\tan\left(x-\dfrac{\pi}{2}\right)}$ 75. $\lim\limits_{x\to0}\dfrac{\cos 3x-\cos x}{\sin 5x+\sin 3x}$ 76. $\lim\limits_{x\to-\frac{\pi}{4}}\left(3+\dfrac{\cos 2x}{\sin x+\cos x}\right)$ 77. $\lim\limits_{x\to0}\dfrac{2\sin 3x}{2x-3\sin 2x}$

78. $\lim\limits_{x\to0}\left(\dfrac{1}{2-2\cos x}-\dfrac{1}{\sin^2 x}\right)$ 79. $\lim\limits_{x\to0}\left(\dfrac{2}{\sin 2x}-\dfrac{1}{\sin x}\right)$ 80. $\lim\limits_{x\to-\frac{\pi}{2}}(1+\sin x)\cdot\tan^2 x$ 81. $\lim\limits_{x\to\pi}(1+\cos x)\cdot\tan\dfrac{x}{2}$

82. $\lim\limits_{x\to\frac{\pi}{2}}\left(\cos\dfrac{x}{2}-\sin\dfrac{x}{2}\right)$ 83. $\lim\limits_{x\to\frac{\pi}{2}}(1-\sin x)\cdot\tan^2 x$ 84. $\lim\limits_{x\to0}\dfrac{\sin x+1-\cos x}{x}$ 85. $\lim\limits_{x\to0}\dfrac{2x-\sin x}{\sqrt{1-\cos x}}$ 86. $\lim\limits_{x\to\pi}\dfrac{\sin x}{1+\cos x}$

87. $\lim\limits_{x\to0}\left(\dfrac{2}{\sin^2 x}-\dfrac{1}{1-\cos x}\right)$ 88. $\lim\limits_{x\to\frac{\pi}{3}}\left(\dfrac{x}{2}-\dfrac{\pi}{3}\cos x\right)\cdot\dfrac{1}{x-\dfrac{\pi}{3}}$ 89. $\lim\limits_{x\to\infty}(3x+1)\sin\dfrac{2\pi x}{x-1}$

90. $\lim\limits_{x\to\infty}(7x+2)\cos\dfrac{\pi x}{2(x+1)}$ 91. $\lim\limits_{x\to1}\dfrac{\sin\pi x^m}{\sin\pi x^n}$ 92. $\lim\limits_{x\to\infty}\left(\log_2 x+\log_2\sin\dfrac{2}{x}\right)$

93. $\lim\limits_{x\to1}\dfrac{1+\cos\pi x}{(x-1)^2}$ 94. $\lim\limits_{x\to\infty}\{\log_3(x+1)-\log_3 x\}$ 95. $\lim\limits_{x\to1}\dfrac{\sqrt[3]{x}-1+\tan\pi x}{x-1}$

96. $\lim\limits_{x\to2}\dfrac{x^2-4+\sin\pi x}{x-2}$ 97. $\lim\limits_{x\to\infty}(2x-1)\sin\dfrac{\pi x}{x+3}$ 98. $\lim\limits_{x\to2}\dfrac{x^3-8+\tan\pi x}{x-2}$

99. $\lim\limits_{x\to0}\dfrac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\cot^2\left(\dfrac{\pi}{2}-x\right)}$ 100. $\lim\limits_{x\to0}\dfrac{2x-3\arcsin x}{2\arcsin x}$

V.គណនាលីមីតរាងមិនកំណត់ខាងក្រោម៖ f

1. $\lim\limits_{x\to\infty}\left(1-\dfrac{1}{x}\right)^x$ 2. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x$ 3. $\lim\limits_{x\to\infty}\left(\dfrac{2x+3}{2x+1}\right)^{x+1}$ 4. $\lim\limits_{x\to\infty}\left(\dfrac{x^2-5x+8}{x^2-6x+3}\right)^x$ 5. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{3x}$ 6. $\lim\limits_{x\to\infty}\left(\dfrac{x}{1+x}\right)^x$

7. $\lim\limits_{x\to\infty}\left(\dfrac{1+x^2}{x^2-1}\right)^{x^2}$ 8. $\lim\limits_{x\to\pm\infty}\left(\dfrac{2x+1}{x-1}\right)^x$ 9. $\lim\limits_{x\to\infty}\left(\dfrac{x^2-6x+5}{x^2-3x+4}\right)^{\frac{x}{4}}$ 10. $\lim\limits_{x\to0}\left(\dfrac{2x+3}{x+1}\right)^{\frac{x}{\sin 3x}}$ 11. $\lim\limits_{x\to\frac{\pi}{2}}(\tan x)^{\cos x}$

12. $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)^{\frac{3}{\cos x}}$ 13. $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{\sin x}}$ 14. $\lim\limits_{x\to0}\dfrac{\ln(1+kx)}{x}$ 15. $\lim\limits_{x\to\infty}x\cdot\ln\left(\dfrac{1+x}{x}\right)$ 16. $\lim\limits_{x\to\infty}\dfrac{\ln\left(1+e^x\right)}{x}$

17. $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}}$ 18. $\lim\limits_{x\to1}\left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$ 19. $\lim\limits_{x\to\infty}\left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$ 20. $\lim\limits_{x\to0}\dfrac{e^{kx}-1}{x}$ 21. $\lim\limits_{x\to0}\dfrac{\sin 2x}{\ln(1+x)}$ 22. $\lim\limits_{x\to0}\dfrac{9^x-7^x}{5^x-3^x}$

23. $\lim\limits_{x\to0}\dfrac{e^{ax}-e^{bx}}{\sin ax-\sin bx}$ 24. $\lim\limits_{x\to0}x^x$ 25. $\lim\limits_{x\to0}\dfrac{1-\cos 2x}{\tan x\cdot\ln(1+2x)}$ 26. $\lim\limits_{x\to0}\dfrac{x^x-1}{x\cdot\ln x}$ 27. $\lim\limits_{x\to0}\dfrac{e^{\sin x}-e^{\tan 2x}}{x}$ 28. $\lim\limits_{x\to2}\left(\dfrac{x}{2}\right)^{\frac{1}{x-2}}$

29. $\lim\limits_{x\to\infty}\left(\dfrac{x+a}{x+b}\right)^{x+c}$ 30. $\lim\limits_{x\to0}(\cos x)^{\frac{1}{x}}$ 31. $\lim\limits_{x\to0}\left(\dfrac{x^2-2x+3}{x^2-3x+2}\right)^{\frac{\sin x}{x}}$ 32. $\lim\limits_{x\to+\infty}\dfrac{(\ln x)^3}{x^2}$ 33. $\lim\limits_{x\to+\infty}\dfrac{(\ln x)^3}{(x+1)^2}$

VI.កំណត់តម្លៃនៃអនុគមន៍និងកំណត់តម្លៃនៃចំនួនថេរ

១.កំណត់អនុគមន៍ដឺក្រេទី២ដែលបំពេញលក្ខខណ្ឌលីមីតទាំងពីរខាងក្រោម:

$$\lim_{x\to+\infty}\frac{f(x)}{x^2+1}=2\quad(i)\quad\&\quad\lim_{x\to1}\frac{f(x)}{x^2-1}=-1\quad(ii)$$

២.កំណត់តម្លៃនៃចំនួនថេរ a និង b ដើម្បីចំនួនទាំងនេះបំពេញលក្ខខណ្ឌដូចខាងក្រោម:

1). $\lim\limits_{x\to -2}\dfrac{x^2+ax-6}{2x^2+3x-2}=b$ 2). $\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+ax}+b}{x^2-1}=\dfrac{1}{2}$ 3). $\lim\limits_{x\to +\infty}\left[\dfrac{x^2+1}{x+1}-(ax+b)\right]=0$

៣.កំណត់តម្លៃនៃចំនួនថេរ a ដើម្បីឲ្យលីមីតខាងក្រោមជាចំនួនថេររួចរកលីមីតនោះផង

1). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x}+a}{x}$ 2). $\lim\limits_{x\to 1}\dfrac{x^2-ax+1}{x-1}$ 3) $\lim\limits_{x\to 2}\dfrac{\sqrt{ax+1}-3}{x-2}$

ដំណោះស្រាយ

I.គណនាលីមីតដែលមានរាងមិនកំណត់ $\left(\dfrac{0}{0}\right)$

វិធាន:ដើម្បីគណនាលីមីត $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}$ ដែលមានរាងមិនកំណត់ $\left(\dfrac{0}{0}\right)$ គេត្រូវ៖

១.បម្លែងភាគយក និងភាគបែងឲ្យបានផលគុណកត្តារួម $(x-x_0)\,,(x-x_0)^2\,,\dots\dots\dots,(x-x_0)^n$ ។

២.សម្រួលកត្តា $(x-x_0)$ ចោលដើម្បីបំបាត់រាងមិនកំណត់

(ព្រោះកាលណា $x\to x_0$ នោះ $x\neq x_0$ ឬ $x-x_0\neq 0$ ដូចនេះគេអាចសម្រួលកត្តា $(x-x_0)$ ចោលនបាន)

៣.គណនាលីមីតកន្សោមថ្មីដោយគ្រាន់តែជំនួសតម្លៃ x ដោយ x_0 ទៅក្នុងលីមីតគេបានលីមីតដែលត្រូវរក។

ចំណាំ:បើភាគយក ឬភាគបែងមានជាប់រ៉ាឌីកាលគេត្រូវគុណភាគយកនិងភាគបែងជាមួយកន្សោមឆ្លាស់របស់វា។

សំគាល់:បើ L ជាចំនួនថេរណាមួយនោះគេបាន $\dfrac{L}{0}=\pm\infty$ ហើយតម្លៃ $\pm\infty$ អាចញែកជាករណីដូចតទៅ

$\dfrac{L}{0^+}=+\infty$ បើ $L>0$; $\dfrac{L}{0^+}=-\infty$ បើ $L<0$; $\dfrac{L}{0^-}=-\infty$ បើ $L>0$, $\dfrac{L}{0^-}=+\infty$ បើ $L<0$

ដើម្បីឲ្យការបម្លែង $f(x)$ និង $g(x)$ ជាផលគុណកត្តាមានភាពងាយស្រួល យើងសូមរំលឹកនូវឯកលក្ខណភាពសំខាន់ៗមួយចំនួន:

1. $a^2-b^2=(a-b)(a+b)$ 2. $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

3. $a^n-b^n=(a-b)\left(a^{n-1}+a^{n-2}b+\dots\dots\dots\dots\dots+ab^{n-2}+b^{n-1}\right)$

4. $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$ 5. $a^5+b^5=(a+b)\left(a^4-a^3b+a^2b^2-ab^3+b^4\right)$

6. $a^n+b^n=(a+b)\left(a^{n-1}-a^{n-2}b+\dots\dots\dots+(-1)^p a^pb^{n-p}+\dots\dots+b^{n-1}\right)$, n ជា ចំនួនគត់សេស

7. $\sqrt{a}-\sqrt{b}=\dfrac{\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}+\sqrt{b}}$

8. $\sqrt[n]{a}-\sqrt[n]{b}=\dfrac{\left(\sqrt[n]{a}-\sqrt[n]{b}\right)\left(\sqrt[n]{a^{n-1}}+\sqrt[n]{a^{n-2}}\cdot\sqrt[n]{b}+\dots\dots\dots\dots\dots+\sqrt[n]{b^{n-1}}\right)}{\sqrt[n]{a^{n-1}}+\sqrt[n]{a^{n-2}}\cdot\sqrt[n]{b}+\dots\dots\dots\dots\dots+\sqrt[n]{b^{n-1}}}$

យើងធ្វើការគណនាលីមីតទៅតាមវិធានខាងលើ:

$$1.\lim_{x\to1}\frac{x^2-4x+3}{x^3-x^2+2x-2}=\lim_{x\to1}\frac{(x-1)(x-3)}{x^2(x-1)+2(x-1)}=\lim_{x\to1}\frac{(x-1)(x-3)}{(x-1)\left(x^2+2\right)}=\lim_{x\to1}\frac{x-3}{x^2+2}=\frac{1-3}{1^2+2}=-\frac{2}{3}$$

$$2.\lim_{x\to1}\frac{x^9-3x+2}{x^6+5x-6}=\lim_{x\to1}\frac{x^9-1-3x+3}{x^6-1+5x-5}=\lim_{x\to1}\frac{(x-1)\left(x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1\right)-3(x-1)}{(x-1)\left(x^5+x^4+x^3+x^2+x+1\right)+5(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)\left(x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1-3\right)}{(x-1)\left(x^5+x^4+x^3+x^2+x+1+5\right)}$$

$$=\lim_{x\to1}\frac{x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1-3}{x^5+x^4+x^3+x^2+x+1+5}=\frac{1^8+1^7+1^6+1^5+1^4+1^3+1^2+1+1-3}{1^5+1^4+1^3+1^2+1+1+5}$$

$$=\frac{6}{11}$$

$$3.\lim_{x\to2}\frac{x-2}{x^3-x^2-x-2}=\lim_{x\to2}\frac{x-2}{x^3-8-x^2+4-x+2}=\lim_{x\to2}\frac{x-2}{(x-2)\left(x^2+2x+4\right)-(x-2)(x+2)-(x-2)}$$

$$=\lim_{x\to2}\frac{x-2}{(x-2)\left[\left(x^2+2x+4\right)-(x+2)-1\right]}=\lim_{x\to2}\frac{1}{\left(x^2+2x+4\right)-(x+2)-1}$$

$$=\frac{1}{\left(2^2+2\times2+4\right)-(2+2)-1}=\frac{1}{7}$$

4. $\lim\limits_{x\to 1}\dfrac{x^m-1}{x^n-1}=\lim\limits_{x\to 1}\dfrac{(x-1)\left(x^{m-1}+x^{m-2}+......................+x+1\right)}{(x-1)\left(x^{n-1}+x^{n-2}+.......................+x+1\right)}$

$$=\lim_{x\to 1}\frac{x^{m-1}+x^{m-2}+......................+x+1}{x^{n-1}+x^{n-2}+.........................+x+1}=\frac{\overbrace{1+1+1+.................+1+1}^{m}}{\underbrace{1+1+1+..................+1+1}_{n}}=\frac{m}{n}$$

5. $\lim\limits_{x\to 1}\dfrac{x^n-1}{x-1}=\lim\limits_{x\to 1}\dfrac{(x-1)\left(x^{n-1}+x^{n-2}+......................+x+1\right)}{x-1}$

$$=\lim_{x\to 1}\left(x^{n-1}+x^{n-2}+...................+x+1\right)=\underbrace{1+1+1+.................+1+1}_{n}=n$$

6. $\lim\limits_{x\to -1}\dfrac{x^7+1}{x^5+1}=\lim\limits_{x\to -1}\dfrac{(x+1)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{(x+1)\left(x^4-x^3+x^2-x+1\right)}$

$$=\lim_{x\to -1}\frac{1-x+x^2-x^3+x^4-x^5+x^6}{1-x+x^2-x^3+x^4}=\frac{1+1+1+1+1+1+1}{1+1+1+1+1}$$

$$=\frac{7}{5}$$

7. $\lim\limits_{x\to1}\dfrac{x+x^2+x^3+\dots\dots\dots+x^n-n}{x+x^2+x^3+\dots\dots\dots+x^m-m}$

$$=\lim_{x\to1}\frac{x+x^2+x^3+\dots\dots+x^n-\left(\overbrace{1+1+1+\dots\dots+1+1}^{n}\right)}{x+x^2+x^3+\dots\dots+x^m-\left(\underbrace{1+1+1+\dots\dots+1+1}_{m}\right)}$$

$$=\lim_{x\to1}\frac{(x-1)+(x^2-1)+(x^3-1)+\dots\dots+(x^n-1)}{(x-1)+(x^2-1)+(x^3-1)+\dots\dots+(x^m-1)}$$

$$=\lim_{x\to1}\frac{(x-1)+(x-1)(x+1)+(x-1)(x^2+x+1)+\dots\dots+(x-1)(x^{n-1}+x^{n-2}+\dots\dots+x+1)}{(x-1)+(x-1)(x+1)+(x-1)(x^2+x+1)+\dots\dots+(x-1)(x^{m-1}+x^{m-2}+\dots\dots+x+1)}$$

$$=\lim_{x\to1}\frac{(x-1)\left[1+(x+1)+(x^2+x+1)+\dots\dots+(1+x+x^2+\dots\dots+x^{n-2}+x^{n-1})\right]}{(x-1)\left[1+(x+1)+(x^2+x+1)+\dots\dots+(1+x+x^2+\dots\dots+x^{m-2}+x^{m-1})\right]}$$

$$=\lim_{x\to1}\frac{1+(1+x)+(1+x+x^2)+\dots\dots+(1+x+x^2+\dots\dots+x^{n-2}+x^{n-1})}{1+(1+x)+(1+x+x^2)+\dots\dots+(1+x+x^2+\dots\dots+x^{m-2}+x^{m-1})}$$

$$=\frac{1+(1+1)+(1+1+1)+\dots\dots+\left(\overbrace{1+1+1+\dots\dots+1+1}^{n}\right)}{1+(1+1)+(1+1+1)+\dots\dots+\left(\underbrace{1+1+1+\dots\dots+1+1}_{m}\right)}$$

$$=\frac{1+2+3+\dots\dots+n}{1+2+3+\dots\dots+m}=\frac{\frac{n(n+1)}{2}}{\frac{m(m+1)}{2}}=\frac{n(n+1)}{m(m+1)}$$

8. $\lim_{x\to a}\frac{x^n-a^n-na^{n-1}x+na^n}{(x-a)^2}$

$=\lim_{x\to a}\frac{(x-a)\left(x^{n-1}+ax^{n-2}+\dots\dots\dots\dots+a^{n-2}x+a^{n-1}\right)-na^{n-1}(x-a)}{(x-a)^2}$

$=\lim_{x\to a}\frac{(x-a)\left[x^{n-1}+ax^{n-2}+\dots\dots\dots\dots+a^{n-2}x+a^{n-1}-na^{n-1}\right]}{(x-a)^2}$

$=\lim_{x\to a}\frac{x^{n-1}+ax^{n-2}+\dots\dots\dots\dots+a^{n-2}x-(n-1)a^{n-1}}{x-a}$

$=\lim_{x\to a}\frac{x^{n-1}+ax^{n-2}+\dots\dots\dots\dots+a^{n-2}x-\left(\overbrace{a^{n-1}+a^{n-1}+\dots\dots\dots\dots+a^{n-1}}^{n-1}\right)}{x-a}$

$=\lim_{x\to a}\frac{\left(x^{n-1}-a^{n-1}\right)+\left(ax^{n-2}-a^{n-1}\right)+\dots\dots\dots\dots+\left(a^{n-2}x-a^{n-1}\right)}{x-a}$

$=\lim_{x\to a}\frac{(x-a)\left(x^{n-2}+ax^{n-3}+\dots\dots+a^{n-3}x+a^{n-2}\right)+a(x-a)\left(x^{n-3}+ax^{n-4}+\dots\dots+a^{n-4}x+a^{n-3}\right)+\dots\dots+a^{n-2}(x-a)}{x-a}$

$=\lim_{x\to a}\frac{(x-a)\left[\left(a^{n-2}+a^{n-3}x+\dots\dots\dots+ax^{n-3}+x^{n-2}\right)+a\left(a^{n-3}+a^{n-4}x+\dots\dots\dots+ax^{n-4}+x^{n-3}\right)+\dots\dots\dots\dots+a^{n-2}\right]}{x-a}$

$=\lim_{x\to a}\left[\left(a^{n-2}+a^{n-3}x+\dots\dots\dots+ax^{n-3}+x^{n-2}\right)+a\left(a^{n-3}+a^{n-4}x+\dots\dots\dots+ax^{n-4}+x^{n-3}\right)+\dots\dots\dots\dots+a^{n-2}\right]$

$=\overbrace{a^{n-2}+a^{n-2}+\dots\dots\dots+a^{n-2}+a^{n-2}}^{(n-1)}+\overbrace{a^{n-2}+a^{n-2}+\dots\dots\dots+a^{n-2}+a^{n-2}}^{(n-2)}+\dots\dots\dots\dots+a^{n-2}$

$=a^{n-2}\left[1+2+3+\dots\dots\dots\dots+(n-2)+(n-1)\right]$

$=\frac{n(n-1)\cdot a^{n-2}}{2}$

9. $\lim_{x\to 1}\dfrac{x^n-nx+n-1}{(x-1)^2}=\lim_{x\to 1}\dfrac{(x^n-1)-(nx-n)}{(x-1)^2}$

$=\lim_{x\to 1}\dfrac{(x-1)(1+x+x^2+.................+x^{n-2}+x^{n-1})-n(x-1)}{(x-1)^2}$

$=\lim_{x\to 1}\dfrac{(x-1)\left[(1+x+x^2+............+x^{n-2}+x^{n-1})-n\right]}{(x-1)^2}$

$=\lim_{x\to 1}\dfrac{(x-1)+(x^2-1)+.................+(x^{n-2}-1)+(x^{n-1}-1)}{(x-1)}$

$=\lim_{x\to 1}\dfrac{(x-1)+(x-1)(x+1)+......+(x-1)(1+x+......+x^{n-4}+x^{n-3})+(x-1)(1+x+......+x^{n-3}+x^{n-2})}{x-1}$

$=\lim_{x\to 1}\dfrac{(x-1)\left[1+(1+x)+........+(1+x+......+x^{n-4}+x^{n-3})+(1+x+........+x^{n-3}+x^{n-2})\right]}{x-1}$

$=\lim_{x\to 1}\left[1+(1+x)+........+(1+x+......+x^{n-4}+x^{n-3})+(1+x+........+x^{n-3}+x^{n-2})\right]$

$=1+(1+1)+...............+\overbrace{1+1+1..........+1+1+1}^{(n-2)}+\overbrace{1+1+1+..........+1+1+1}^{(n-1)}$

$=1+2+3+............+(n-2)+(n-1)$

$=\dfrac{n(n-1)}{2}$

10. $\lim_{x\to 2}\dfrac{\sqrt{x+2}-2}{\sqrt{x+7}-3}=\lim_{x\to 2}\dfrac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)(\sqrt{x+7}+3)}{(\sqrt{x+7}-3)(\sqrt{x+7}+3)(\sqrt{x+2}+2)}$

$=\lim_{x\to 2}\dfrac{(x+2-4)(\sqrt{x+7}+3)}{(x+7-9)(\sqrt{x+2}+2)}=\lim_{x\to 2}\dfrac{(x-2)(\sqrt{x+7}+3)}{(x-2)(\sqrt{x+2}+2)}$

$=\lim_{x\to 2}\dfrac{\sqrt{x+7}+3}{\sqrt{x+2}+2}=\dfrac{\sqrt{2+7}+3}{\sqrt{2+2}+2}=\dfrac{3}{2}$

11. $\lim_{x\to 1}\dfrac{\sqrt[3]{x}-1}{x-1}$ គេតាង $t^3=x$ កាលណា $x\to 1$ នោះ $t\to 1$

គេបាន $\lim_{x\to 1}\dfrac{\sqrt[3]{x}-1}{x-1}=\lim_{t\to 1}\dfrac{t-1}{t^3-1}=\lim_{t\to 1}\dfrac{(t-1)}{(t-1)(t^2+t+1)}=\lim_{t\to 1}\dfrac{1}{t^2+t+1}=\dfrac{1}{3}$

12. $\lim\limits_{x\to -1}\dfrac{\sqrt{1-3x}-2}{\sqrt[3]{5x-3}+\sqrt{7+3x}}$

យើងដឹងថា $\sqrt{1-3x}-2=\dfrac{\left(\sqrt{1-3x}-2\right)\left(\sqrt{1-3x}+2\right)}{\sqrt{1-3x}+2}=\dfrac{1-3x-4}{\sqrt{1-3x}+2}=\dfrac{-3(x+1)}{\sqrt{1-3x}+2}$

$$\sqrt[3]{5x-3}+\sqrt{7+3x}=\sqrt[3]{5x-3}+2+\sqrt{7+3x}-2$$

$$=\frac{\left(\sqrt[3]{5x-3}+2\right)\left(\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4\right)}{\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4}+\frac{\left(\sqrt{7+3x}-2\right)\left(\sqrt{7+3x}+2\right)}{\sqrt{7+3x}+2}$$

$$=\frac{\sqrt[3]{(5x-3)^3}+8}{\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4}+\frac{7+3x-4}{\sqrt{7+3x}+2}$$

$$=\frac{5x-3+8}{\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4}+\frac{3(x+1)}{\sqrt{7+3x}+2}$$

$$=(x+1)\left(\frac{5}{\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4}+\frac{3}{\sqrt{7+3x}+2}\right)$$

យើងបាន

$$\lim_{x\to -1}\frac{\sqrt{1-3x}-2}{\sqrt[3]{5x-3}+\sqrt{7+3x}}=\lim_{x\to -1}\frac{\dfrac{-3(x+1)}{\sqrt{1-3x}+2}}{(x+1)\left(\dfrac{5}{\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4}+\dfrac{3}{\sqrt{7+3x}+2}\right)}$$

$$=\lim_{x\to -1}\frac{\dfrac{-3}{\sqrt{1-3x}+2}}{\dfrac{5}{\sqrt[3]{(5x-3)^2}-2\cdot\sqrt[3]{5x-3}+4}+\dfrac{3}{\sqrt{7+3x}+2}}$$

$$=\frac{\dfrac{-3}{\sqrt{1+3}+2}}{\dfrac{5}{\sqrt[3]{(-5-3)^2}-2\cdot\sqrt[3]{-5-3}+4}+\dfrac{3}{\sqrt{7-3}+2}}=\frac{\dfrac{-3}{4}}{\dfrac{5}{12}+\dfrac{3}{4}}=-\frac{9}{14}$$

$$13.\lim_{x\to1}\frac{x^{50}-7x+6}{x^{20}+3x-4}=\lim_{x\to1}\frac{x^{50}-1-7x+7}{x^{20}-1+3x-3}$$

$$=\lim_{x\to1}\frac{(x-1)\left(1+x+x^2+.........+x^{48}+x^{49}\right)-7(x-1)}{(x-1)\left(1+x+x^2+..........+x^{18}+x^{19}\right)+3(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)\left[\left(1+x+x^2+.........+x^{48}+x^{49}\right)-7\right]}{(x-1)\left[\left(1+x+x^2+..........+x^{18}+x^{19}\right)+3\right]}$$

$$=\lim_{x\to1}\frac{\left(1+x+x^2+.........+x^{48}+x^{49}\right)-7}{\left(1+x+x^2+..........+x^{18}+x^{19}\right)+3}$$

$$=\frac{\overbrace{1+1+1+...................+1+1}^{50}-7}{\underbrace{1+1+1+...................+1+1}_{20}+3}=\frac{43}{23}$$

$$14.\ \lim_{x\to-\frac{1}{3}}\frac{3x^2-\frac{1}{3}}{x+\frac{1}{3}}=\lim_{x\to-\frac{1}{3}}\frac{3\left(x^2-\frac{1}{9}\right)}{\left(x+\frac{1}{3}\right)}=\lim_{x\to-\frac{1}{3}}\frac{3\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)}{\left(x+\frac{1}{3}\right)}=\lim_{x\to-\frac{1}{3}}3\left(x-\frac{1}{3}\right)=3\left(-\frac{1}{3}-\frac{1}{3}\right)=-2$$

$$15.\lim_{x\to-1}\frac{(x+1)}{\sqrt{x^2+2x+1}}=\lim_{x\to-1}\frac{(x+1)}{|x+1|}=\begin{cases}1 & \text{បើ } x\to-1^+ \text{ នោះគេបាន } |x+1|=(x+1)\\ -1 & \text{បើ } x\to-1^- \text{ នោះគេបាន } |x+1|=-(x+1)\end{cases}$$

$$16.\lim_{x\to0}\frac{x^2+|x|+\sqrt{x^2}}{x^2-|x|+5x}=\lim_{x\to0}\frac{x^2+|x|+|x|}{x^2-|x|+5x}$$

⊕ បើ $x\to0^+$ ឬ $x>0$ នោះគេបាន $|x|=x$

យើងបាន $\lim_{x\to0}\frac{x^2+|x|+|x|}{x^2-|x|+5x}=\lim_{\substack{x\to0^+\\x>0}}\frac{x^2+x+x}{x^2-x+5x}=\lim_{\substack{x\to0^+\\x>0}}\frac{x(x+2)}{x(x+4)}=\lim_{\substack{x\to0^+\\x>0}}\frac{x+2}{x+4}=\frac{1}{2}$

⊕ បើ $x\to0^-$ ឬ $x<0$ នោះគេបាន $|x|=-x$

យើងបាន $\lim_{x\to0}\frac{x^2+|x|+|x|}{x^2-|x|+5x}=\lim_{\substack{x\to0^-\\x<0}}\frac{x^2-x-x}{x^2+x+5x}=\lim_{\substack{x\to0^-\\x<0}}\frac{x(x-2)}{x(x+6)}=\lim_{\substack{x\to0^-\\x<0}}\frac{x-2}{x+6}=-\frac{1}{3}$

$$17.\lim_{x\to1}\frac{x^{2016}-x^{2015}}{\sqrt{x+3}-2}=\lim_{x\to1}\frac{x^{2015}(x-1)\left(\sqrt{x+3}+2\right)}{\left(\sqrt{x+3}-2\right)\left(\sqrt{x+3}+2\right)}=\lim_{x\to1}\frac{x^{2015}(x-1)\left(\sqrt{x+3}+2\right)}{(x-1)}=\lim_{x\to1}x^{2015}\left(\sqrt{x+3}+2\right)=4$$

18. $\lim\limits_{x\to 1}\dfrac{x+x^2+x^3+\dots\dots\dots\dots+x^{2015}-2015}{x+x^2+x^3+\dots\dots\dots\dots+x^{2017}-2017}$

$$=\lim_{x\to 1}\frac{x+x^2+x^3+\dots\dots\dots\dots+x^{2015}-\overbrace{(1+1+1+1+\dots\dots+1+1)}^{2015}}{x+x^2+x^3+\dots\dots\dots\dots+x^{2017}-\underbrace{(1+1+1+\dots\dots\dots\dots+1+1)}_{2017}}$$

$$=\lim_{x\to 1}\frac{(x-1)+(x^2-1)+(x^3-1)+\dots\dots\dots\dots+(x^{2015}-1)}{(x-1)+(x^2-1)+(x^3-1)+\dots\dots\dots\dots+(x^{2017}-1)}$$

$$=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)+(x-1)(x^2+x+1)+\dots\dots+(x-1)(x^{2014}+x^{2013}+\dots\dots+x+1)}{(x-1)+(x-1)(x+1)+(x-1)(x^2+x+1)+\dots\dots+(x-1)(x^{2016}+x^{2015}+\dots\dots+x+1)}$$

$$=\lim_{x\to 1}\frac{(x-1)\left[1+(1+x)+(1+x+x^2)+\dots\dots\dots\dots+(1+x+x^2+\dots\dots+x^{2012}+x^{2013}+x^{2014})\right]}{(x-1)\left[1+(1+x)+(1+x+x^2)+\dots\dots\dots\dots+(1+x+x^2+\dots\dots+x^{2014}+x^{2015}+x^{2016})\right]}$$

$$=\lim_{x\to 1}\frac{1+(1+x)+(1+x+x^2)+\dots\dots\dots\dots+(1+x+x^2+\dots\dots+x^{2012}+x^{2013}+x^{2014})}{1+(1+x)+(1+x+x^2)+\dots\dots\dots\dots+(1+x+x^2+\dots\dots+x^{2014}+x^{2015}+x^{2016})}$$

$$=\frac{1+(1+1)+(1+1+1^2)+\dots\dots\dots\dots+\overbrace{(1+1+1^2+\dots\dots+1^{2012}+1^{2013}+1^{2014})}^{2015}}{1+(1+1)+(1+1+1^2)+\dots\dots\dots\dots+\underbrace{(1+1+1^2+\dots\dots+1^{2014}+1^{2015}+1^{2016})}_{2017}}$$

$$=\frac{1+2+3+\dots\dots\dots\dots+2015}{1+2+3+\dots\dots\dots\dots+2017}=\frac{2015\times 2016}{2017\times 2018}$$

19. $\lim\limits_{x\to -2}\dfrac{\sqrt{3x+10}-2}{\sqrt{2-x}+\sqrt[3]{3x-2}}$ យើងដឹងថា

$\oplus$ $$\sqrt{3x+10}-2=\frac{(\sqrt{3x+10}-2)(\sqrt{3x+10}+2)}{\sqrt{3x+10}+2}=\frac{3x+10-4}{\sqrt{3x+10}+2}=\frac{3(x+2)}{\sqrt{3x+10}+2}$$

$\oplus$ $$\sqrt{2-x}+\sqrt[3]{3x-2}=\sqrt{2-x}-2+\sqrt[3]{3x-2}+2$$

$$=\frac{(\sqrt{2-x}-2)(\sqrt{2-x}+2)}{\sqrt{2-x}+2}+\frac{(\sqrt[3]{3x-2}+2)\left(\sqrt[3]{(3x-2)^3}-2\cdot\sqrt[3]{3x-2}+4\right)}{\sqrt[3]{(3x-2)^3}-2\cdot\sqrt[3]{3x-2}+4}$$

$$=\frac{\sqrt{(2-x)^2}-4}{\sqrt{2-x}+2}+\frac{\sqrt[3]{(3x-2)^3}+8}{\sqrt[3]{(3x-2)^3}-2\cdot\sqrt[3]{3x-2}+4}=(x+2)\left(\frac{3}{\sqrt[3]{(3x-2)^3}-2\cdot\sqrt[3]{3x-2}+4}-\frac{1}{\sqrt{2-x}+2}\right)$$

យើងបាន

$$\lim_{x\to-2}\frac{\sqrt{3x+10}-2}{\sqrt{2-x}+\sqrt[3]{3x-2}}=\lim_{x\to-2}\frac{\dfrac{3(x+2)}{\sqrt{3x+10}+2}}{(x+2)\left(\dfrac{3}{\sqrt[3]{(3x-2)^2}-2\cdot\sqrt[3]{3x-2}+4}-\dfrac{1}{\sqrt{2-x}+2}\right)}$$

$$=\lim_{x\to-2}\frac{\dfrac{3}{\sqrt{3x+10}+2}}{\dfrac{3}{\sqrt[3]{(3x-2)^2}-2\cdot\sqrt[3]{3x-2}+4}-\dfrac{1}{\sqrt{2-x}+2}}$$

$$=\frac{\dfrac{3}{\sqrt{-6+10}+2}}{\dfrac{3}{\sqrt[3]{(-6-2)^2}-2\cdot\sqrt[3]{-6-2}+4}-\dfrac{1}{\sqrt{2+2}+2}}$$

$$=\frac{\dfrac{3}{4}}{\dfrac{3}{4+4+4}-\dfrac{1}{4}}=\frac{3}{0}=\infty$$

20. $\lim_{x\to0}\dfrac{\sqrt[2015]{1+3x}-\sqrt[2017]{1-2x}}{x}$

$$=\lim_{x\to0}\frac{\sqrt[2015]{1+3x}-1-\sqrt[2017]{1-2x}+1}{x}$$

$$=\lim_{x\to0}\frac{\sqrt[2015]{(1+3x)^{2015}}-1^{2015}}{x\left(\sqrt[2015]{(1+3x)^{2014}}+\sqrt[2015]{(1+3x)^{2013}}+................+\sqrt[2015]{1+3x}+1\right)}$$

$$-\lim_{x\to0}\frac{\sqrt[2017]{(1-2x)^{2017}}-1^{2017}}{x\left(\sqrt[2017]{(1-2x)^{2016}}+\sqrt[2017]{(1-2x)^{2015}}+................+\sqrt[2017]{1-2x}+1\right)}$$

$$=\lim_{x\to0}\frac{3x}{x\left(1+\sqrt[2015]{1+3x}+......+\sqrt[2015]{(1+3x)^{2013}}+\sqrt[2015]{(1+3x)^{2014}}\right)}$$

$$-\lim_{x\to0}\frac{-2x}{x\left(1+\sqrt[2017]{1-2x}+............+\sqrt[2017]{(1-2x)^{2015}}+\sqrt[2017]{(1-2x)^{2016}}\right)}$$

$$=\lim_{x\to 0}\frac{3}{1+\sqrt[2015]{1+3x}+\dots\dots\dots\dots+\sqrt[2015]{(1+3x)^{2013}}+\sqrt[2015]{(1+3x)^{2014}}}$$

$$-\lim_{x\to 0}\frac{-2}{1+\sqrt[2017]{1-2x}+\dots\dots\dots\dots+\sqrt[2017]{(1-2x)^{2015}}+\sqrt[2017]{(1-2x)^{2016}}}$$

$$=\frac{3}{\underbrace{1+1+1+\dots\dots\dots\dots+1+1}_{2015}}+\frac{2}{\underbrace{1+1+1+\dots\dots\dots\dots+1+1}_{2017}}$$

$$=\frac{3}{2015}+\frac{2}{2017}$$

21. $$\lim_{x\to 0}\frac{\sqrt[2015]{1+3x}-\sqrt[2017]{1-2x}}{\sqrt[2014]{1+2x}-\sqrt[2016]{1-3x}}=\lim_{x\to 0}\frac{\frac{\sqrt[2015]{1+3x}-\sqrt[2017]{1-2x}}{x}}{\frac{\sqrt[2014]{1+2x}-\sqrt[2016]{1-3x}}{x}}$$

តាមលំហាត់ទី២០យើងបាន $\lim_{x\to 0}\frac{\frac{\sqrt[2015]{1+3x}-\sqrt[2017]{1-2x}}{x}}{\frac{\sqrt[2014]{1+2x}-\sqrt[2016]{1-3x}}{x}}=\frac{\frac{3}{2015}+\frac{2}{2017}}{\frac{2}{2014}+\frac{3}{2016}}$ ។

22. $\lim_{x\to 1}\frac{1-\sqrt[2015]{x}}{1-x}$ គេតាង $x=k^{2015}$ កាលណា $x\to 1$ នោះ $k\to 1$

យើងបាន

$$\lim_{x\to 1}\frac{1-\sqrt[2015]{x}}{1-x}=\lim_{k\to 1}\frac{1-k}{1-k^{2015}}$$

$$=\lim_{k\to 1}\frac{(1-k)}{(1-k)(1+k+k^2+\dots\dots\dots\dots+k^{2013}+k^{2014})}$$

$$=\lim_{k\to 1}\frac{1}{1+k+k^2+\dots\dots\dots\dots+k^{2013}+k^{2014}}$$

$$=\frac{1}{\underbrace{1+1+1^2+\dots\dots\dots\dots+1^{2013}+1^{2014}}_{2015}}$$

$$=\frac{1}{2015}$$

23. $$\lim_{x\to -\frac{1}{3}}\frac{9x^2-1}{3x+1}=\lim_{x\to -\frac{1}{3}}\frac{(3x+1)(3x-1)}{(3x+1)}=\lim_{x\to -\frac{1}{3}}(3x-1)=-2$$

$$24.\lim_{x\to1}\frac{x^2+x-2}{x-1}=\lim_{x\to1}\frac{x^2-1+x-1}{(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)(x+1)+(x-1)}{(x-1)}=\lim_{x\to1}\frac{(x-1)\left[(x+1)+1\right]}{(x-1)}$$

$$=\lim_{x\to1}(x+2)=3$$

$$25.\lim_{x\to-2}\frac{x+2}{x^2+4x+4}=\lim_{x\to-2}\frac{(x+2)}{(x+2)(x+2)}=\lim_{x\to-2}\frac{1}{(x+2)}=\frac{1}{0}=\infty$$

$$26.\lim_{x\to1}\frac{\sqrt{x+3}-2}{x-1}=\lim_{x\to1}\frac{\left(\sqrt{x+3}-2\right)\left(\sqrt{x+3}+2\right)}{(x-1)\left(\sqrt{x+3}+2\right)}$$

$$=\lim_{x\to1}\frac{(x+3-4)}{(x-1)\left(\sqrt{x+3}+2\right)}=\lim_{x\to1}\frac{1}{\sqrt{x+3}+2}=\frac{1}{4}$$

$$27.\lim_{x\to2}\frac{\sqrt{x^2-4}}{x^3-x^2-x-2}=\lim_{x\to2}\frac{x^2-4}{\left(x^3-8-x^2+4-x+2\right)\sqrt{x^2-4}}$$

$$=\lim_{x\to2}\frac{(x-2)(x+2)}{\left[(x-2)\left(x^2+2x+4\right)-(x-2)(x+2)-(x-2)\right]\sqrt{x^2-4}}$$

$$=\lim_{x\to2}\frac{(x-2)(x+2)}{(x-2)\left[\left(x^2+2x+4\right)-(x+2)-1\right]\sqrt{x^2-4}}$$

$$=\lim_{x\to2}\frac{x+2}{\left(x^2+x+1\right)\sqrt{x^2-4}}=\frac{4}{7\times0}=\infty$$

$$28.\lim_{x\to4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\lim_{x\to4}\frac{\left(\sqrt{2x+1}-3\right)\left(\sqrt{2x+1}+3\right)\left(\sqrt{x-2}+\sqrt{2}\right)}{\left(\sqrt{x-2}-\sqrt{2}\right)\left(\sqrt{x-2}+\sqrt{2}\right)\left(\sqrt{2x+1}+3\right)}$$

$$=\lim_{x\to4}\frac{(2x+1-9)\left(\sqrt{x-2}+\sqrt{2}\right)}{(x-2-2)\left(\sqrt{2x+1}+3\right)}$$

$$=\lim_{x\to4}\frac{2(x-4)\left(\sqrt{x-2}+\sqrt{2}\right)}{(x-4)\left(\sqrt{2x+1}+3\right)}=\lim_{x\to4}\frac{2\left(\sqrt{x-2}+\sqrt{2}\right)}{\sqrt{2x+1}+3}$$

$$=\frac{2\sqrt{2}}{2\times3}=\frac{\sqrt{2}}{3}$$

29. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$ គេតាង $1+x=k^6 \Rightarrow x=k^6-1$ កាលណា $x\to 0$ នោះ $k\to 1$

យើងបាន

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}=\lim_{k\to 1}\frac{k^3-k^2}{k^6-1}$$
$$=\lim_{k\to 1}\frac{k^2(k-1)}{(k-1)\left(1+k+k^2+k^3+k^4+k^5\right)}$$
$$=\lim_{k\to 1}\frac{k^2}{1+k+k^2+k^3+k^4+k^5}$$
$$=\frac{1^2}{1+1+1^2+1^3+1^4+1^5}=\frac{1}{6}$$

30. $$\lim_{x\to 0}\frac{\sqrt{1+x}+\sqrt{x+4}-3}{x}=\lim_{x\to 0}\frac{\sqrt{1+x}-1+\sqrt{x+4}-2}{x}$$
$$=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}+\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$$
$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}{x\left(\sqrt{1+x}+1\right)}+\lim_{x\to 0}\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}+2\right)}{x\left(\sqrt{x+4}+2\right)}$$
$$=\lim_{x\to 0}\frac{1+x-1}{x\left(\sqrt{x+1}+1\right)}+\lim_{x\to 0}\frac{x+4-4}{x\left(\sqrt{x+4}+2\right)}$$
$$=\lim_{x\to 0}\frac{1}{\sqrt{1+x}+1}+\lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}$$
$$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

31. $$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}=\lim_{x\to 1}\frac{\left(\sqrt[3]{x}-1\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)\left(\sqrt[3]{(4x+4)^2}+2\cdot\sqrt[3]{4x+4}+4\right)}{\left(\sqrt[3]{4x+4}-2\right)\left(\sqrt[3]{(4x+4)^2}+2\cdot\sqrt[3]{4x+4}+4\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}$$
$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt[3]{(4x+4)^2}+2\cdot\sqrt[3]{4x+4}+4\right)}{(4x+4-8)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}$$
$$=\lim_{x\to 1}\frac{\sqrt[3]{(4x+4)^2}+2\cdot\sqrt[3]{4x+4}+4}{4\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=\frac{4+4+4}{4\times 2}=\frac{3}{2}$$

$$32.\lim_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$$

$$=\lim_{x\to1}\frac{\sqrt[3]{x-2}+1+\sqrt[3]{1-x+x^2}-1}{x^2-1}$$

$$=\lim_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}+\lim_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}$$

$$=\lim_{x\to1}\frac{\left(\sqrt[3]{x-2}+1\right)\left(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1\right)}{\left(x^2-1\right)\left(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1\right)}+\lim_{x\to1}\frac{\left(\sqrt[3]{1-x+x^2}-1\right)\left(\sqrt[3]{\left(1-x+x^2\right)^2}+\sqrt[3]{1-x+x^2}+1\right)}{\left(x^2-1\right)\left(\sqrt[3]{\left(1-x+x^2\right)^2}+\sqrt[3]{1-x+x^2}+1\right)}$$

$$=\lim_{x\to1}\frac{(x-2+1)}{(x-1)(x+1)\left(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1\right)}+\lim_{x\to1}\frac{\left(1-x+x^2-1\right)}{(x-1)(x+1)\left(\sqrt[3]{\left(1-x+x^2\right)^2}+\sqrt[3]{1-x+x^2}+1\right)}$$

$$=\lim_{x\to1}\frac{1}{(x+1)\left(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1\right)}+\lim_{x\to1}\frac{x}{(x+1)\left(\sqrt[3]{\left(1-x+x^2\right)^2}+\sqrt[3]{1-x+x^2}+1\right)}$$

$$=\frac{1}{2}+\frac{1}{2\times3}=\frac{2}{3}$$

33. $\lim\limits_{x\to1}\dfrac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$ គេតាង $x=k^{n\cdot m}$ កាលណា $x\to1$ នោះ $k\to1$ យើងបាន

$$\lim_{x\to1}\frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}=\lim_{k\to1}\frac{k^m-1}{k^n-1}$$

$$=\lim_{k\to1}\frac{(k-1)\left(1+k+k^2+\dots\dots\dots\dots\dots+k^{m-2}+k^{m-1}\right)}{(k-1)\left(1+k+k^2+\dots\dots\dots\dots\dots+k^{n-2}+k^{n-1}\right)}$$

$$=\lim_{x\to1}\frac{1+k+k^2+\dots\dots\dots\dots\dots+k^{m-2}+k^{m-1}}{1+k+k^2+\dots\dots\dots\dots\dots+k^{n-2}+k^{n-1}}$$

$$=\frac{\overbrace{1+1+1^2+\dots\dots\dots\dots\dots+1^{m-2}+1^{m-1}}^{m}}{\underbrace{1+1+1^2+\dots\dots\dots\dots\dots+1^{n-2}+1^{n-1}}_{n}}=\frac{m}{n}$$

34. $\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}$ *គេតាង* $k^3=1-x\Rightarrow x=1-k^3$ *កាលណា* $x\to 0$ *នោះ* $k\to 1$

យើងបាន

$$\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}=\lim_{k\to 1}\frac{1-k}{3\left(1-k^3\right)}$$

$$=\lim_{k\to 1}\frac{(1-k)}{3(1-k)\left(1+k+k^2\right)}$$

$$=\lim_{k\to 1}\frac{1}{3\left(1+k+k^2\right)}=\frac{1}{9}$$

35. $$\lim_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}=\lim_{x\to -1}\frac{\left(\sqrt[3]{x}+1\right)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)\left(\sqrt{x^2+3}+2\right)}{\left(\sqrt{x^2+3}-2\right)\left(\sqrt{x^2+3}+2\right)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}$$

$$=\lim_{x\to -1}\frac{(x+1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2+3-4\right)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}$$

$$=\lim_{x\to -1}\frac{(x+1)\left(\sqrt{x^2+3}+2\right)}{(x+1)(x-1)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}$$

$$=\lim_{x\to -1}\frac{\sqrt{x^2+3}+2}{(x-1)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}$$

$$=\frac{4}{-2\times 3}=-\frac{2}{3}$$

36. $$\lim_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)=-\frac{1}{4}+1=\frac{3}{4}$$

37. $$\lim_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}=\lim_{x\to\frac{1}{2}}\frac{(2x-1)\left(4x^2+2x+1\right)}{6x^2-3x-2x+1}$$

$$=\lim_{x\to\frac{1}{2}}\frac{(2x-1)\left(4x^2+2x+1\right)}{(2x-1)(3x-1)}=\lim_{x\to\frac{1}{2}}\frac{4x^2+2x+1}{(3x-1)}=\frac{1+1+1}{\frac{3}{2}-1}=6$$

38. $$\lim_{x\to 1}\frac{\sqrt{1+x}+\sqrt{1+x^2}-\sqrt{1+x^3}}{\sqrt{x-1}+\sqrt{x^2+1}-\sqrt{x^4+1}}=\frac{\sqrt{2}}{0}=\infty$$

39. $\lim\limits_{x\to1}\dfrac{(x-1)(x^3+x-2)}{x^3-x^2-x+1}=\lim\limits_{x\to1}\dfrac{(x-1)(x^3+x-2)}{(x-1)(x^2-1)}$

$$=\lim_{x\to1}\frac{(x-1)^2(x^2+x+2)}{(x-1)^2(x+1)}=\lim_{x\to1}\frac{x^2+x+2}{x+1}=2$$

40. $\lim\limits_{x\to2}\dfrac{\sqrt{x+7}-3}{(x-2)}=\lim\limits_{x\to2}\dfrac{(\sqrt{x+7}-3)(\sqrt{x+7}+3)}{(x-2)(\sqrt{x+7}+3)}$

$$=\lim_{x\to2}\frac{(x+7-9)}{(x-2)(\sqrt{x+7}+3)}=\lim_{x\to2}\frac{1}{\sqrt{x+7}+3}=\frac{1}{6}$$

41. $\lim\limits_{x\to1}\dfrac{nx^{n+1}-(n+1)x^n+1}{x^{m+1}-x^m-x+1}$

$$=\lim_{x\to1}\frac{nx^n(x-1)-(x^n-1)}{x^m(x-1)-(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)\left[nx^n-(1+x+x^2+..............+x^{n-2}+x^{n-1})\right]}{(x-1)(x^m-1)}$$

$$=\lim_{x\to1}\frac{\overbrace{x^n+x^n+...........+x^n}^{n}-(1+x+x^2+..............+x^{n-2}+x^{n-1})}{(x-1)(1+x+x^2+..............+x^{m-2}+x^{m-1})}$$

$$=\lim_{x\to1}\frac{(x^n-1)+(x^n-x)+(x^n-x^2)+..................+(x^n-x^{n-2})+(x^n-x^{n-1})}{(x-1)(1+x+x^2+.............+x^{m-2}+x^{m-1})}$$

$$=\lim_{x\to1}\frac{x^{n-1}(x-1)+x^{n-2}(x^2-1)+...............+x(x^{n-1}-1)+(x^n-1)}{(x-1)(1+x+x^2+.............+x^{m-2}+x^{m-1})}$$

$$=\lim_{x\to1}\frac{(x-1)\left[x^{n-1}+x^{n-2}(x+1)+.....+x(1+x+x^2+......+x^{n-2})+(1+x+x^2+.......+x^{n-1})\right]}{(x-1)(1+x+x^2+.............+x^{m-2}+x^{m-1})}$$

$$=\lim_{x\to1}\frac{x^{n-1}+x^{n-2}(x+1)+.......+x(1+x+x^2+.....+x^{n-2})+(1+x+x^2+.......+x^{n-1})}{1+x+x^2+.............+x^{m-2}+x^{m-1}}$$

$$=\frac{1^{n-1}+1^{n-2}\cdot(1+1)+\dots\dots+1\cdot\overbrace{\left(1+1+1^2+\dots+1^{n-2}\right)}^{(n-1)}+\overbrace{\left(1+1+1^2+\dots+1^{n-1}\right)}^{n}}{\underbrace{1+1+1^2+\dots\dots+1^{m-2}+1^{m-1}}_{m}}$$

$$=\frac{1+2+3+\dots\dots+(n-1)+n}{m}=\frac{n(n+1)}{2m}$$

42. $\lim\limits_{x\to1}\frac{x^{2n}-1}{x^{2m}-1}$ គេតាង $x=\sqrt{k}$ កាលណា $x\to1$ នោះ $k\to1$

យើងបាន

$$\lim_{x\to1}\frac{x^{2n}-1}{x^{2m}-1}=\lim_{k\to1}\frac{k^n-1}{k^m-1}$$
$$=\lim_{k\to1}\frac{(k-1)\left(1+k+k^2+\dots\dots+k^{n-1}\right)}{(k-1)\left(1+k+k^2+\dots\dots+k^{m-1}\right)}$$
$$=\lim_{k\to1}\frac{1+k+k^2+\dots\dots+k^{n-1}}{1+k+k^2+\dots\dots+k^{m-1}}$$
$$=\frac{\overbrace{1+1+1+\dots\dots+1+1+1}^{n}}{\underbrace{1+1+1+\dots\dots+1+1}_{m}}=\frac{n}{m}$$

43. $\lim\limits_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}$ យើងតាង $x=k^6$ កាលណា $x\to64$ នោះ $k\to2$

យើងបាន $\lim\limits_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}=\lim\limits_{k\to2}\frac{k^3-8}{k^2-4}=\lim\limits_{k\to2}\frac{(k-2)\left(k^2+2k+4\right)}{(k-2)(k+2)}=\lim\limits_{k\to2}\frac{k^2+2k+4}{k+2}=\frac{4+4+4}{4}=3$

44. $$\lim_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x-1}}=\lim_{x\to1}\frac{\left(\sqrt{x+3}-\sqrt{3x+1}\right)\left(\sqrt{x+3}+\sqrt{3x+1}\right)\sqrt{x-1}}{(x-1)\left(\sqrt{x+3}+\sqrt{3x+1}\right)}$$
$$=\lim_{x\to1}\frac{(x+3-3x-1)\sqrt{x-1}}{(x-1)\left(\sqrt{x+3}+\sqrt{3x+1}\right)}$$
$$=\lim_{x\to1}\frac{-2(x-1)\sqrt{x-1}}{(x-1)\left(\sqrt{x+3}+\sqrt{3x+1}\right)}=\lim_{x\to1}\frac{-2\sqrt{x-1}}{\left(\sqrt{x+3}+\sqrt{3x+1}\right)}=\frac{-2\times0}{4}=0$$

45. $\lim\limits_{x\to1}\dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

$$=\lim_{x\to1}\frac{\sqrt[3]{7+x^3}-2-\sqrt{3+x^2}+2}{x-1}$$

$$=\lim_{x\to1}\frac{\sqrt[3]{7+x^3}-2}{x-1}-\lim_{x\to1}\frac{\sqrt{3+x^2}-2}{x-1}$$

$$=\lim_{x\to1}\frac{\left(\sqrt[3]{7+x^3}-2\right)\left(\sqrt[3]{\left(7+x^3\right)^2}+2\cdot\sqrt[3]{7+x^3}+4\right)}{(x-1)\left(\sqrt[3]{\left(7+x^3\right)^2}+2\cdot\sqrt[3]{7+x^3}+4\right)}-\lim_{x\to1}\frac{\left(\sqrt{3+x^2}-2\right)\left(\sqrt{3+x^2}+2\right)}{(x-1)\left(\sqrt{3+x^2}+2\right)}$$

$$=\lim_{x\to1}\frac{\left(7+x^3-8\right)}{(x-1)\left(\sqrt[3]{\left(7+x^3\right)^2}+2\cdot\sqrt[3]{7+x^3}+4\right)}-\lim_{x\to1}\frac{\left(3+x^2-4\right)}{(x-1)\left(\sqrt{3+x^2}+2\right)}$$

$$=\lim_{x\to1}\frac{(x-1)\left(x^2+x+1\right)}{(x-1)\left(\sqrt[3]{\left(7+x^3\right)^2}+2\cdot\sqrt[3]{7+x^3}+4\right)}-\lim_{x\to1}\frac{(x-1)(x+1)}{(x-1)\left(\sqrt{3+x^2}+2\right)}$$

$$=\lim_{x\to1}\frac{x^2+x+1}{\sqrt[3]{\left(7+x^3\right)^2}+2\cdot\sqrt[3]{7+x^3}+4}-\lim_{x\to1}\frac{x+1}{\sqrt{3+x^2}+2}$$

$$=\frac{1+1+1}{4+4+4}-\frac{1+1}{2+2}=-\frac{1}{4}$$

46. $\lim\limits_{x\to2}\dfrac{x^2-4}{x-2}=\lim\limits_{x\to2}\dfrac{(x-2)(x+2)}{(x-2)}=\lim\limits_{x\to2}(x+2)=4$

47. $\lim\limits_{x\to1}\dfrac{x^3-1}{x^2-1}=\lim\limits_{x\to1}\dfrac{(x-1)\left(x^2+x+1\right)}{(x-1)(x+1)}=\lim\limits_{x\to1}\dfrac{x^2+x+1}{x+1}=\dfrac{3}{2}$

48. $\lim\limits_{x\to2}\dfrac{x^2-4}{x^2-3x+2}=\lim\limits_{x\to2}\dfrac{(x-2)(x+2)}{(x-1)(x-2)}=\lim\limits_{x\to2}\dfrac{x+2}{x-1}=4$

49. $\lim\limits_{x\to5}\dfrac{x^2-7x+10}{x^2-25}=\lim\limits_{x\to5}\dfrac{(x-5)(x-2)}{(x-5)(x+5)}=\lim\limits_{x\to5}\dfrac{x-2}{x+5}=\dfrac{3}{10}$

50. $\lim\limits_{x\to2}\dfrac{x^3-8}{x^2-4}=\lim\limits_{x\to2}\dfrac{(x-2)\left(x^2+2x+4\right)}{(x-2)(x+2)}=\lim\limits_{x\to2}\dfrac{x^2+2x+4}{x+2}=\dfrac{4+4+4}{2+2}=3$

51. $\lim\limits_{x\to -1}\frac{x^2-1}{x^2+3x+2}=\lim\limits_{x\to -1}\frac{(x+1)(x-1)}{(x+1)(x+2)}=\lim\limits_{x\to -1}\frac{x-1}{x+2}=-2$

52. $\lim\limits_{x\to 2}\frac{x^2-5x+6}{x^2-12x+20}=\lim\limits_{x\to 2}\frac{(x-2)(x-3)}{(x-2)(x-10)}=\lim\limits_{x\to 2}\frac{x-3}{x-10}=\frac{1}{8}$

53. $\lim\limits_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}=\lim\limits_{x\to -2}\frac{x(x+1)(x+2)}{(x+2)(x-3)}=\lim\limits_{x\to -2}\frac{x(x+1)}{x-3}=-\frac{2}{5}$

54. $\lim\limits_{x\to 1}\frac{x^3-3x+2}{x^3-x^2-x-1}=\frac{1-3+2}{1-1-1-1}=0$

55. $\lim\limits_{x\to 1}\frac{x^3-6x^2+11x-6}{x^2-3x+2}=\lim\limits_{x\to 1}\frac{x^2(x-1)-5x(x-1)+6(x-1)}{(x-1)(x-2)}=\lim\limits_{x\to 1}\frac{(x-1)(x^2-5x+6)}{(x-1)(x-2)}=\lim\limits_{x\to 1}\frac{x^2-5x+6}{x-2}=-2$

56. $\lim\limits_{x\to 1}\frac{4x^6-5x^5+x}{(1-x)^2}=\lim\limits_{x\to 1}\frac{x(4x^5-5x^4+1)}{(1-x)^2}$

$$=\lim_{x\to 1}\frac{x\left[4x^4(x-1)-(x^4-1)\right]}{(x-1)^2}$$
$$=\lim_{x\to 1}\frac{x(x-1)\left[4x^4-(1+x+x^2+x^4)\right]}{(x-1)^2}$$
$$=\lim_{x\to 1}\frac{x(x-1)\left[(x^4-1)+(x^4-x)+(x^4-x^2)+(x^4-x^3)\right]}{(x-1)^2}$$
$$=\lim_{x\to 1}\frac{x(x-1)^2\left[(1+x+x^2+x^3)+x(1+x+x^2)+x^2(x+1)+x^3\right]}{(x-1)^2}$$
$$=\lim_{x\to 1}x\left[(1+x+x^2+x^3)+x(1+x+x^2)+x^2(x+1)+x^3\right]$$
$$=1\times\left[(1+1+1^2+1^3)+1\times(1+1+1^2)+1^2\times(1+1)+1^3\right]$$
$$=4+3+2+1=10$$

II. គណនាលីមីតមានរាងមិនកំណត់ $\left(\frac{\infty}{\infty}\right)$

វិធាន:ដើម្បីគណនាលីមីត $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}$ មានរាងមិនកំណត់ $\left(\frac{\infty}{\infty}\right)$ គេត្រូវ៖

១.ទាញយកដឺក្រេភាគយក និងភាគបែងដែលខ្ពស់ជាងគេជាកត្តារួម

២.សម្រួលកត្តារួមចោល(ព្រោះ $x\to\infty$ នោះ $x\neq 0$)

៣.គណនាលីមីតកន្សោមថ្មីគ្រាន់តែជំនួស x ដោយ ∞ ចូលទៅ យើងនឹងបានលទ្ធផលលីមីតត្រូវរក

សំគាល់:បើ L ជាចំនួនថេរណាមួយនោះគេបាន $\frac{L}{\pm\infty}=0$ ។

យើងធ្វើការគណនាលីមីតទៅតាមវិធានខាងលើ ដូចខាងក្រោម៖

$$1.\lim_{x\to\infty}\frac{x^2-1}{2x^2+x+2}=\lim_{x\to\infty}\frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left(2+\frac{1}{x}+\frac{2}{x^2}\right)}=\lim_{x\to\infty}\frac{1-\frac{1}{x^2}}{2+\frac{1}{x}+\frac{2}{x^2}}=\frac{1-0}{2+0+0}=\frac{1}{2}$$

$$2.\lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}=\lim_{x\to\infty}\frac{x^3\left(1+\frac{1}{x^2}\right)}{x^4\left(1-\frac{3}{x^2}+\frac{1}{x^4}\right)}=\lim_{x\to\infty}\frac{1+\frac{1}{x^2}}{x\left(1-\frac{3}{x^2}+\frac{1}{x^4}\right)}=\frac{1+0}{\infty\cdot(1-0+0)}=0$$

$$3.\lim_{x\to\infty}\frac{\sqrt[3]{x^3+2x-1}}{x+2}=\lim_{x\to\infty}\frac{\sqrt[3]{x^3\left(1+\frac{2}{x^2}-\frac{1}{x^3}\right)}}{x\left(1+\frac{2}{x}\right)}=\lim_{x\to\infty}\frac{x\cdot\sqrt[3]{1+\frac{2}{x^2}-\frac{1}{x^3}}}{x\left(1+\frac{2}{x}\right)}=\lim_{x\to\infty}\frac{\sqrt[3]{1+\frac{2}{x^2}-\frac{1}{x^3}}}{1+\frac{2}{x}}=\frac{\sqrt[3]{1+0-0}}{1+0}=1$$

$$4.\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}=\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x\cdot\left(1+\frac{\sqrt{x+\sqrt{x}}}{x}\right)}}=\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x}\times\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}=\frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}=1$$

5. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x-4}{\sqrt{x^4+2}}=\lim\limits_{x\to\infty}\dfrac{x^2\left(2-\dfrac{3}{x}-\dfrac{4}{x^2}\right)}{\sqrt{x^4\left(1+\dfrac{2}{x^4}\right)}}=\lim\limits_{x\to\infty}\dfrac{x^2\left(2-\dfrac{3}{x}-\dfrac{4}{x^2}\right)}{x^2\times\sqrt{1+\dfrac{2}{x^4}}}=\lim\limits_{x\to\infty}\dfrac{2-\dfrac{3}{x}-\dfrac{4}{x^2}}{\sqrt{1+\dfrac{2}{x^4}}}=\dfrac{2-0-0}{\sqrt{1+0}}=2$

6. $\lim\limits_{x\to\infty}\dfrac{4x+1+\sqrt{16x^2+x+1}}{7x}=\lim\limits_{x\to\infty}\dfrac{4x+1+\sqrt{x^2\left(16+\dfrac{1}{x}+\dfrac{1}{x^2}\right)}}{7x}=\lim\limits_{x\to\infty}\dfrac{4x+1+|x|\cdot\sqrt{16+\dfrac{1}{x}+\dfrac{1}{x^2}}}{7x}$

$\oplus$ បើ $x\to-\infty$ នោះ $|x|=-x$ យើងបាន

$$\lim_{x\to\infty}\frac{4x+1+|x|\cdot\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}{7x}=\lim_{x\to-\infty}\frac{4x+1-x\cdot\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}{7x}=\lim_{x\to-\infty}\frac{x\left(4+\frac{1}{x}-\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}\right)}{7x}$$

$$=\lim_{x\to-\infty}\frac{4+\frac{1}{x}-\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}{7}=\frac{4+0-\sqrt{16+0+0}}{7}=0$$

$\oplus$ បើ $x\to+\infty$ *នោះ* $|x|=x$ យើងបាន

$$\lim_{x\to\infty}\frac{4x+1+|x|\cdot\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}{7x}=\lim_{x\to+\infty}\frac{4x+1+x\cdot\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}{7x}=\lim_{x\to+\infty}\frac{x\left(4+\frac{1}{x}+\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}\right)}{7x}$$

$$=\lim_{x\to+\infty}\frac{4+\frac{1}{x}+\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}{7}=\frac{4+0+\sqrt{16+0+0}}{7}=\frac{8}{7}$$

ដូចនេះ $\lim\limits_{x\to\infty}\dfrac{4x+1+\sqrt{16x^2+x+1}}{7x}=\begin{cases}\dfrac{8}{7} & \text{បើ } x\to+\infty\\ 0 & \text{បើ } x\to-\infty\end{cases}$

7. $\lim\limits_{x\to\infty}\dfrac{(2x+3)(3x-5)(x-1)^2}{x^2(2x-3)(4x+3)}=\lim\limits_{x\to\infty}\dfrac{x\left(2+\dfrac{3}{x}\right)\cdot x\cdot\left(3-\dfrac{5}{x}\right)\cdot x^2\left(1-\dfrac{1}{x}\right)^2}{x^2\cdot x\left(2-\dfrac{3}{x}\right)\cdot x\cdot\left(4+\dfrac{3}{x}\right)}$

$$=\lim_{x\to\infty}\frac{\left(2+\frac{3}{x}\right)\left(3-\frac{5}{x}\right)\left(1-\frac{1}{x}\right)^2}{\left(2-\frac{3}{x}\right)\left(4+\frac{3}{x}\right)}$$

$$=\frac{(2+0)(3-0)(1-0)^2}{(2-0)(4+0)}=\frac{3}{4}$$

$$8.\lim_{x\to\infty}\frac{(x-1)(3+2x)(2-x)}{(x^2+1)(1-2x)}=\lim_{x\to\infty}\frac{x\cdot\left(1-\frac{1}{x}\right)\cdot x\cdot\left(\frac{3}{x}+2\right)\cdot x\cdot\left(\frac{2}{x}-1\right)}{x^2\cdot\left(1+\frac{1}{x^2}\right)\cdot x\cdot\left(\frac{1}{x}-2\right)}$$

$$=\lim_{x\to\infty}\frac{\left(1-\frac{1}{x}\right)\left(\frac{3}{x}+2\right)\left(\frac{2}{x}-1\right)}{\left(1+\frac{1}{x^2}\right)\left(\frac{1}{x}-2\right)}=\frac{(1-0)(0+2)(0-1)}{(1+0)(0-2)}=1$$

$$9.\ \lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x\left(1+\frac{\sqrt{x}}{x}\right)}}=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x}\cdot\sqrt{1+\sqrt{\frac{1}{x}}}}=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}=\frac{1}{\sqrt{1+\sqrt{0}}}=1$$

$$10.\ \lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt{x\left(1+\frac{\sqrt{x+\sqrt{x}}}{x}\right)}}{\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}\cdot\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}{\sqrt{x}}=\lim_{x\to+\infty}\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}=\sqrt{1+\sqrt{0+\sqrt{0}}}=1$$

$11.\ \lim_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+3}}$ គេតាង $x=k^{12}$ កាលណា $x\to+\infty$ នោះ $k\to+\infty$ គេបាន

$$\lim_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+3}}=\lim_{k\to+\infty}\frac{k^6+k^4+k^3}{\sqrt{2k^{12}+3}}=\lim_{k\to+\infty}\frac{k^6\left(1+\frac{1}{k^2}+\frac{1}{k^3}\right)}{k^6\cdot\sqrt{2+\frac{3}{k^{12}}}}$$

$$=\lim_{k\to+\infty}\frac{1+\frac{1}{k^2}+\frac{1}{k^3}}{\sqrt{2+\frac{3}{k^{12}}}}=\frac{1+0+0}{\sqrt{2+0}}=\frac{1}{\sqrt{2}}$$

$$12.\ \lim_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}=\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}+2x}{\sqrt{x^2\left(1+\frac{4}{x^2}\right)}+x}=\lim_{x\to\infty}\frac{|x|\cdot\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2x}{|x|\cdot\sqrt{1+\frac{4}{x^2}}+x}$$

$\oplus$ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2x}{|x|\cdot\sqrt{1+\frac{4}{x^2}}+x}=\lim_{x\to+\infty}\frac{x\cdot\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2x}{x\cdot\sqrt{1+\frac{4}{x^2}}+x}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2}{\sqrt{1+\frac{4}{x^2}}+1}=\frac{\sqrt{1+0-0}+2}{\sqrt{1+0}+1}=\frac{3}{2}$$

$\oplus$ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2x}{|x|\cdot\sqrt{1+\frac{4}{x^2}}+x}=\lim_{x\to-\infty}\frac{-x\cdot\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2x}{-x\cdot\sqrt{1+\frac{4}{x^2}}+x}$$

$$=\lim_{x\to-\infty}\frac{2-\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}}{1-\sqrt{1+\frac{4}{x^2}}}=\frac{2-\sqrt{1+0-0}}{1-\sqrt{1+0}}=\infty$$

ដូចនេះ $\lim_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}=\left[\begin{array}{l}\frac{3}{2}\ \text{បើ}\ x\to+\infty\\ \infty\ \text{បើ}\ x\to-\infty\end{array}\right.$

13. $\lim_{x\to+\infty}\frac{1+\sqrt[4]{x}}{\sqrt[3]{x^2}}$ គេតាង $x=k^{12}$ កាលណា $x\to+\infty$ នោះ $k\to+\infty$ យើងបាន

$$\lim_{x\to+\infty}\frac{1+\sqrt[4]{x}}{\sqrt[3]{x^2}}=\lim_{k\to+\infty}\frac{1+k^3}{k^8}=\lim_{k\to+\infty}\frac{k^3\left(1+\frac{1}{k^3}\right)}{k^8}=\lim_{k\to+\infty}\frac{1+\frac{1}{k^3}}{k^5}=\frac{1+0}{+\infty}=0$$

14. $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{2+x^4}}{\sqrt[3]{5+27x^3}}=\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{x^4\left(1+\dfrac{2}{x^4}\right)}}{\sqrt[3]{x^3\left(27+\dfrac{5}{x^3}\right)}}=\lim\limits_{x\to\infty}\dfrac{|x|\cdot\sqrt[4]{1+\dfrac{2}{x^4}}}{x\cdot\sqrt[3]{27+\dfrac{5}{x^3}}}$

⊕ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\sqrt[4]{1+\frac{2}{x^4}}}{x\cdot\sqrt[3]{27+\frac{5}{x^3}}}=\lim_{x\to+\infty}\frac{x\cdot\sqrt[4]{1+\frac{2}{x^4}}}{x\cdot\sqrt[3]{27+\frac{5}{x^3}}}=\lim_{x\to+\infty}\frac{\sqrt[4]{1+\frac{2}{x^4}}}{\sqrt[3]{27+\frac{5}{x^3}}}=\frac{\sqrt[4]{1+0}}{\sqrt[3]{27+0}}=\frac{1}{3}$$

⊕ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\sqrt[4]{1+\frac{2}{x^4}}}{x\cdot\sqrt[3]{27+\frac{5}{x^3}}}=\lim_{x\to-\infty}\frac{-x\cdot\sqrt[4]{1+\frac{2}{x^4}}}{x\cdot\sqrt[3]{27+\frac{5}{x^3}}}=\lim_{x\to-\infty}\frac{-\sqrt[4]{1+\frac{2}{x^4}}}{\sqrt[3]{27+\frac{5}{x^3}}}=\frac{-\sqrt[4]{1+0}}{\sqrt[3]{27+0}}=-\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{2+x^4}}{\sqrt[3]{5+27x^3}}=\left[\begin{array}{ll}\dfrac{1}{3} & \text{បើ } x\to+\infty\\ -\dfrac{1}{3} & \text{បើ } x\to-\infty\end{array}\right.$

15. $\lim\limits_{x\to\infty}\dfrac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2}=\lim\limits_{x\to\infty}\dfrac{x^3\left(8+\dfrac{12}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}\right)}{x^3\left(6+\dfrac{3}{x}-\dfrac{5}{x^2}+\dfrac{2}{x^3}\right)}$

$$=\lim_{x\to\infty}\frac{8+\frac{12}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{6+\frac{3}{x}-\frac{5}{x^2}+\frac{2}{x^3}}=\frac{8+0+0+0}{6+0-0+0}=\frac{4}{3}$$

16. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{x^3+2x+5}=\lim\limits_{x\to\infty}\dfrac{x^2\left(2-\dfrac{3}{x}+\dfrac{1}{x^2}\right)}{x^3\left(1+\dfrac{2}{x^2}+\dfrac{5}{x^3}\right)}=\lim\limits_{x\to\infty}\dfrac{2-\dfrac{3}{x}+\dfrac{1}{x^2}}{x\left(1+\dfrac{2}{x^2}+\dfrac{5}{x^3}\right)}=\dfrac{2-0+0}{\infty\cdot(1+0+0)}=0$

17. $\lim\limits_{x\to\infty}\dfrac{3x^2+2x+5}{2x+1}=\lim\limits_{x\to\infty}\dfrac{x^2\left(3+\dfrac{2}{x}+\dfrac{5}{x^2}\right)}{x\left(2+\dfrac{1}{x}\right)}=\lim\limits_{x\to\infty}\dfrac{x\left(3+\dfrac{2}{x}+\dfrac{5}{x^2}\right)}{2+\dfrac{1}{x}}=\dfrac{\infty\cdot(3+0+0)}{2+0}=\infty$

18. $\lim\limits_{|x|\to\infty}\dfrac{\sqrt{x^2-1}}{x}=\lim\limits_{|x|\to\infty}\dfrac{\sqrt{x^2\left(1-\dfrac{1}{x^2}\right)}}{x}=\lim\limits_{|x|\to\infty}\dfrac{|x|\cdot\sqrt{1-\dfrac{1}{x^2}}}{x}$

$\oplus$ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{|x|\to\infty}\frac{|x|\cdot\sqrt{1-\dfrac{1}{x^2}}}{x}=\lim_{x\to+\infty}\frac{x\cdot\sqrt{1-\dfrac{1}{x^2}}}{x}=\lim_{x\to+\infty}\sqrt{1-\frac{1}{x^2}}=\sqrt{1-0}=1$$

$\oplus$ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{|x|\to\infty}\frac{|x|\cdot\sqrt{1-\dfrac{1}{x^2}}}{x}=\lim_{x\to-\infty}\frac{-x\cdot\sqrt{1-\dfrac{1}{x^2}}}{x}=\lim_{x\to-\infty}\left(-\sqrt{1-\frac{1}{x^2}}\right)=-\sqrt{1-0}=-1$$

ដូចនេះ $\lim\limits_{|x|\to\infty}\dfrac{\sqrt{x^2-1}}{x}=\left[\begin{array}{l}1 \ \text{បើ}\ x\to+\infty\\ -1\ \text{បើ}\ x\to-\infty\end{array}\right.$

19. $\lim\limits_{x\to\infty}\dfrac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6}=\lim\limits_{x\to\infty}\dfrac{x^3\left(1-\dfrac{4}{x}+\dfrac{5}{x^2}-\dfrac{1}{x^3}\right)}{x^3\left(2+\dfrac{3}{x}-\dfrac{4}{x^2}+\dfrac{6}{x^3}\right)}=\lim\limits_{x\to\infty}\dfrac{1-\dfrac{4}{x}+\dfrac{5}{x^2}-\dfrac{1}{x^3}}{2+\dfrac{3}{x}-\dfrac{4}{x^2}+\dfrac{6}{x^3}}=\dfrac{1-0+0-0}{2+0-0+0}=\dfrac{1}{2}$

20. $\lim\limits_{x\to\infty}\dfrac{x^2+3x-5}{2x^2+1}=\lim\limits_{x\to\infty}\dfrac{x^2\left(1+\dfrac{3}{x}-\dfrac{5}{x^2}\right)}{x^2\left(2+\dfrac{1}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{1+\dfrac{3}{x}-\dfrac{5}{x^2}}{2+\dfrac{1}{x^2}}=\dfrac{1+0-0}{2+0}=\dfrac{1}{2}$

21. $\lim\limits_{x\to\infty}\dfrac{x^3+5x-7}{x^2+3x-1}=\lim\limits_{x\to\infty}\dfrac{x^3\left(1+\dfrac{5}{x^2}-\dfrac{7}{x^3}\right)}{x^2\left(1+\dfrac{3}{x}-\dfrac{1}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{x\left(1+\dfrac{5}{x^2}-\dfrac{7}{x^3}1+\dfrac{5}{x^2}-\dfrac{7}{x^3}\right)}{1+\dfrac{3}{x}-\dfrac{1}{x^2}}=\dfrac{\infty\cdot(1+0-0)}{1+0-0}=\infty$

22. $\lim\limits_{x\to\infty}\dfrac{x+5}{2x^2+3x+7}=\lim\limits_{x\to\infty}\dfrac{x\left(1+\dfrac{5}{x}\right)}{x^2\left(2+\dfrac{3}{x}+\dfrac{7}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{1+\dfrac{5}{x}}{x\left(2+\dfrac{3}{x}+\dfrac{7}{x^2}\right)}=\dfrac{1+0}{\infty\cdot(2+0+0)}=0$

23. $\lim\limits_{x\to\infty}\dfrac{x^4-5x}{x^2-3x+1}=\lim\limits_{x\to\infty}\dfrac{x^4\left(1-\dfrac{5}{x^3}\right)}{x^2\left(1-\dfrac{3}{x}+\dfrac{1}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{x^2\left(1-\dfrac{5}{x^3}\right)}{1-\dfrac{3}{x}+\dfrac{1}{x^2}}=\dfrac{\infty\cdot(1-0)}{1-0+0}=\infty$

$$24.\ \lim_{x\to\infty}\frac{1+x-3x^3}{1+x^2+3x^3}=\lim_{x\to\infty}\frac{x^3\left(\frac{1}{x^3}+\frac{1}{x^2}-3\right)}{x^3\left(\frac{1}{x^3}+\frac{1}{x}+3\right)}=\lim_{x\to\infty}\frac{\frac{1}{x^3}+\frac{1}{x^2}-3}{\frac{1}{x^3}+\frac{1}{x}+3}=\frac{0+0-3}{0+0+3}=1$$

$$25.\ \lim_{x\to\infty}\frac{2x^2+3x+1}{3x^2-x+5}=\lim_{x\to\infty}\frac{x^2\left(2+\frac{3}{x}+\frac{1}{x^2}\right)}{x^2\left(3-\frac{1}{x}+\frac{5}{x^2}\right)}=\lim_{x\to\infty}\frac{2+\frac{3}{x}+\frac{1}{x^2}}{3-\frac{1}{x}+\frac{5}{x^2}}=\frac{2+0+0}{3-0+0}=\frac{2}{3}$$

$$26.\ \lim_{x\to\infty}\frac{x^2+3x-8}{x^4-6x+1}=\lim_{x\to\infty}\frac{x^2\left(1+\frac{3}{x}-\frac{8}{x^2}\right)}{x^4\left(1-\frac{6}{x^3}+\frac{1}{x^4}\right)}=\lim_{x\to\infty}\frac{1+\frac{3}{x}-\frac{8}{x^2}}{x^2\left(1-\frac{6}{x^3}+\frac{1}{x^4}\right)}=\frac{1+0-0}{\infty\cdot(1-0+0)}=0$$

$$27.\ \lim_{x\to\infty}\frac{(x-2)(2x+1)(1-4x)}{(3x+4)^3}=\lim_{x\to\infty}\frac{x\left(1-\frac{2}{x}\right)\cdot x\left(2+\frac{1}{x}\right)\cdot x\left(\frac{1}{x}-4\right)}{x^3\left(3+\frac{4}{x}\right)^3}=\lim_{x\to\infty}\frac{\left(1-\frac{2}{x}\right)\left(2+\frac{1}{x}\right)\left(\frac{1}{x}-4\right)}{\left(3+\frac{4}{x}\right)^3}$$

$$=\frac{(1-0)(2+0)(0-4)}{(3+0)^3}=-\frac{8}{27}$$

$$28.\ \lim_{x\to\infty}\frac{4x^3+3x-7}{x^2-3x+5}=\lim_{x\to\infty}\frac{x^3\left(4+\frac{3}{x^2}-\frac{7}{x^3}\right)}{x^2\left(1-\frac{3}{x}+\frac{5}{x^2}\right)}=\lim_{x\to\infty}\frac{x\left(4+\frac{3}{x^2}-\frac{7}{x^3}\right)}{1-\frac{3}{x}+\frac{5}{x^2}}=\frac{\infty\cdot(4+0-0)}{1-0+0}=\infty$$

$$29.\ \lim_{x\to\infty}\frac{1-3x}{2-x}=\lim_{x\to\infty}\frac{x\left(\frac{1}{x}-3\right)}{x\left(\frac{2}{x}-1\right)}=\lim_{x\to\infty}\frac{\frac{1}{x}-3}{\frac{2}{x}-1}=\frac{0-3}{0-1}=3$$

$$30.\ \lim_{x\to\infty}\frac{2x^2+3}{x^3-2x+1}=\lim_{x\to\infty}\frac{x^2\left(2+\frac{3}{x^2}\right)}{x^3\left(1-\frac{2}{x^2}+\frac{1}{x^3}\right)}=\lim_{x\to\infty}\frac{2+\frac{3}{x^2}}{x\left(1-\frac{2}{x^2}+\frac{1}{x^3}\right)}=\frac{2+0}{\infty\cdot(1-0+0)}=0$$

31. $\lim\limits_{x\to\infty}\dfrac{(2x-3)(3x+5)(4x-6)}{3x^3+x-1}=\lim\limits_{x\to\infty}\dfrac{x\cdot\left(2-\dfrac{3}{x}\right)\cdot x\cdot\left(3+\dfrac{5}{x}\right)\cdot x\cdot\left(4-\dfrac{6}{x}\right)}{x^3\left(3+\dfrac{1}{x^2}-\dfrac{1}{x^3}\right)}$

$$=\lim_{x\to\infty}\frac{\left(2-\dfrac{3}{x}\right)\left(3+\dfrac{5}{x}\right)\left(4-\dfrac{6}{x}\right)}{3+\dfrac{1}{x^2}-\dfrac{1}{x^3}}=\frac{(2-0)(3+0)(4-0)}{3+0-0}=8$$

32. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2x+3}+1+4x}{\sqrt{4x^2+1}+2-x}=\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2\left(1+\dfrac{2}{x}+\dfrac{3}{x}\right)}+x\left(4+\dfrac{1}{x}\right)}{\sqrt{x^2\left(4+\dfrac{1}{x^2}\right)}+x\left(\dfrac{2}{x}-1\right)}=\lim\limits_{x\to\infty}\dfrac{|x|\cdot\sqrt{3+\dfrac{1}{x^2}+\dfrac{1}{x^3}}+x\left(4+\dfrac{1}{x}\right)}{|x|\cdot\sqrt{4+\dfrac{1}{x^2}}+x\left(\dfrac{2}{x}-1\right)}$

⊕ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\sqrt{3+\dfrac{1}{x^2}-\dfrac{1}{x^3}}+x\left(4+\dfrac{1}{x}\right)}{|x|\cdot\sqrt{4+\dfrac{1}{x^2}}+x\left(\dfrac{2}{x}-1\right)}=\lim_{x\to+\infty}\frac{x\cdot\left(\sqrt{3+\dfrac{1}{x^2}+\dfrac{1}{x^3}}+4+\dfrac{1}{x}\right)}{x\cdot\left(\sqrt{4+\dfrac{1}{x^2}}+\dfrac{2}{x}-1\right)}=\lim_{x\to+\infty}\frac{\sqrt{3+\dfrac{1}{x^2}+\dfrac{1}{x^3}}+4+\dfrac{1}{x}}{\sqrt{4+\dfrac{1}{x^2}}+\dfrac{2}{x}-1}$$

$$=\frac{\sqrt{3+0+0}+4+0}{\sqrt{4+0}+0-1}=4+\sqrt{3}$$

⊕ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\sqrt{3+\dfrac{1}{x^2}-\dfrac{1}{x^3}}+x\left(4+\dfrac{1}{x}\right)}{|x|\cdot\sqrt{4+\dfrac{1}{x^2}}+x\left(\dfrac{2}{x}-1\right)}=\lim_{x\to-\infty}\frac{x\cdot\left(-\sqrt{3+\dfrac{1}{x^2}+\dfrac{1}{x^3}}+4+\dfrac{1}{x}\right)}{x\cdot\left(-\sqrt{4+\dfrac{1}{x^2}}+\dfrac{2}{x}-1\right)}=\lim_{x\to-\infty}\frac{-\sqrt{3+\dfrac{1}{x^2}+\dfrac{1}{x^3}}+4+\dfrac{1}{x}}{-\sqrt{4+\dfrac{1}{x^2}}+\dfrac{2}{x}-1}$$

$$=\frac{-\sqrt{3+0+0}+4+0}{-\sqrt{4+0}+0-1}=\frac{\sqrt{3}-4}{3}$$

ដូចនេះ $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2x+3}+1+4x}{\sqrt{4x^2+1}+2-x}=\left[\begin{array}{ll}4+\sqrt{3} & \text{បើ } x\to+\infty\\ \dfrac{\sqrt{3}-4}{3} & \text{បើ } x\to-\infty\end{array}\right.$

33. $$\lim_{x\to\infty}\frac{\sqrt{9x^2+x+1}-\sqrt{4x^2+2x+1}}{x+1}=\lim_{x\to\infty}\frac{\sqrt{x^2\left(9+\frac{1}{x}+\frac{1}{x^2}\right)}-\sqrt{x^2\left(4+\frac{2}{x}+\frac{1}{x^2}\right)}}{x\left(1+\frac{1}{x}\right)}$$

$$=\lim_{x\to\infty}\frac{|x|\cdot\left(\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}$$

⊕ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\left(\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}=\lim_{x\to+\infty}\frac{x\cdot\left(\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}}{1+\frac{1}{x}}$$

$$=\frac{\sqrt{9+0+0}-\sqrt{4+0+0}}{1+0}=1$$

⊕ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to\infty}\frac{|x|\cdot\left(\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}=\lim_{x\to-\infty}\frac{-x\cdot\left(\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}$$

$$=\lim_{x\to-\infty}\frac{-\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}}{1+\frac{1}{x}}$$

$$=\frac{-\sqrt{9+0+0}+\sqrt{4+0+0}}{1+0}=-1$$

ដូចនេះ $$\lim_{x\to\infty}\frac{\sqrt{9x^2+x+1}-\sqrt{4x^2+2x+1}}{x+1}=\begin{cases}1 & \text{បើ } x\to+\infty\\ -1 & \text{បើ } x\to-\infty\end{cases}$$

34. $$\lim_{x\to\pm\infty}\frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{1+x^2}}=\lim_{x\to\pm\infty}\frac{\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}+\sqrt{x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)}}{x+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}$$

$$=\lim_{x\to\pm\infty}\frac{|x|\cdot\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)}{x+|x|\cdot\sqrt{1+\frac{1}{x^2}}}$$

⊕ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\cdot\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)}{x+x\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}{1+\sqrt{1+\frac{1}{x^2}}}=\frac{\sqrt{1+0+0}+\sqrt{1-0+0}}{1+\sqrt{1+0}}=1$$

⊕ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{-x\cdot\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)}{x-x\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to-\infty}\frac{-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}{1-\sqrt{1+\frac{1}{x^2}}}=\frac{-\sqrt{1+0+0}-\sqrt{1-0+0}}{1-\sqrt{1+0}}=\infty$$

ដូចនេះ $\lim_{x\to\pm\infty}\frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{1+x^2}}=\begin{cases}1 & \text{បើ } x\to+\infty\\ \infty & \text{បើ } x\to-\infty\end{cases}$

35. $$\lim_{x\to\pm\infty}\frac{7x}{1+14x+\sqrt{16x^2+x+1}}=\lim_{x\to\pm\infty}\frac{7x}{x\left(14+\frac{1}{x}\right)+\sqrt{x^2\left(16+\frac{1}{x}+\frac{1}{x^2}\right)}}$$

$$=\lim_{x\to\pm\infty}\frac{7x}{x\cdot\left(14+\frac{1}{x}\right)+|x|\cdot\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}$$

⊕ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{7x}{x\left(14+\frac{1}{x}\right)+x\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{7x}{x\left(14+\frac{1}{x}+\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to+\infty}\frac{7}{14+\frac{1}{x}+\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}$$

$$=\frac{7}{14+0+\sqrt{16+0+0}}=\frac{7}{18}$$

⊕ បើ $x \to -\infty$ នោះគេបាន $|x| = -x$ យើងបាន

$$\lim_{x\to-\infty} \frac{7x}{x\left(14+\frac{1}{x}\right) - x\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{7x}{x\left(14+\frac{1}{x}-\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}\right)} = \lim_{x\to-\infty} \frac{7}{14+\frac{1}{x}-\sqrt{16+\frac{1}{x}+\frac{1}{x^2}}}$$

$$= \frac{7}{14+0-\sqrt{16+0+0}} = \frac{7}{10}$$

ដូចនេះ $$\lim_{x\to\pm\infty} \frac{7x}{1+14x+\sqrt{16x^2+x+1}} = \begin{cases} \frac{7}{18} & \text{បើ } x\to+\infty \\ \frac{7}{10} & \text{បើ } x\to-\infty \end{cases}$$

36. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}$ គេតាង $x = k^{12}$ កាលណា $x\to\infty$ នោះ $k\to\infty$ យើងបាន

$$\lim_{x\to\infty} \frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}} = \lim_{k\to\infty} \frac{k^6+k^4+k^3}{\sqrt{2k^{12}+1}} = \lim_{k\to\infty} \frac{k^6\left(1+\frac{1}{k^2}+\frac{1}{k^3}\right)}{k^6\cdot\sqrt{2+\frac{1}{k}}}$$

$$= \lim_{k\to\infty} \frac{1+\frac{1}{k^2}+\frac{1}{k^3}}{\sqrt{2+\frac{1}{k}}} = \frac{1+0+0}{\sqrt{2+0}} = \frac{\sqrt{2}}{2}$$

37. $$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}} = \lim_{x\to\infty} \frac{\sqrt{x\left(1+\frac{\sqrt{x+\sqrt{x}}}{x}\right)}}{\sqrt{x\left(1+\frac{1}{x}\right)}} = \lim_{x\to\infty} \frac{\sqrt{x}\cdot\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}{\sqrt{x}\cdot\sqrt{1+\frac{1}{x}}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}{\sqrt{1+\frac{1}{x}}} = \frac{\sqrt{1+\sqrt{0+\sqrt{0}}}}{\sqrt{1+0}} = 1$$

38. $\lim\limits_{x\to\infty}\dfrac{2x^2-5+\sqrt{x^4-3x+1}}{x-1+\sqrt[3]{4x^6+3x-2}}=\lim\limits_{x\to\infty}\dfrac{2x^2-5+\sqrt{x^4\left(1-\dfrac{3}{x^3}+\dfrac{1}{x^4}\right)}}{x-1+\sqrt[3]{x^6\left(4+\dfrac{3}{x^5}-\dfrac{2}{x^6}\right)}}=\lim\limits_{x\to\infty}\dfrac{2x^2-5+x^2\sqrt{1-\dfrac{3}{x^3}+\dfrac{1}{x^4}}}{x-1+x^2\cdot\sqrt[3]{4+\dfrac{3}{x^5}-\dfrac{2}{x^6}}}$

$$=\lim_{x\to\infty}\frac{x^2\left(2-\dfrac{5}{x^2}+\sqrt{1-\dfrac{3}{x^2}+\dfrac{1}{x^4}}\right)}{x^2\left(\dfrac{1}{x}-\dfrac{1}{x^2}+\sqrt[3]{4+\dfrac{3}{x^5}-\dfrac{2}{x^6}}\right)}=\lim_{x\to\infty}\frac{2-\dfrac{5}{x^2}+\sqrt{1-\dfrac{3}{x^2}+\dfrac{1}{x^4}}}{\dfrac{1}{x}-\dfrac{1}{x^2}+\sqrt[3]{4+\dfrac{3}{x^5}-\dfrac{2}{x^6}}}$$

$$=\frac{2-0+\sqrt{1-0+0}}{0-0+\sqrt[3]{4+0-0}}=\frac{3}{\sqrt[3]{4}}$$

39. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{1+x^2}-\sqrt[3]{1+x^2}}{\sqrt[4]{1+x^4}-\sqrt[5]{1+x^4}}$ គេតាង $x=k^{15}$ កាលណា $x\to+\infty$ នោះ $k\to+\infty$

យើងបាន

$$\lim_{x\to+\infty}\frac{\sqrt{1+x^2}-\sqrt[3]{1+x^2}}{\sqrt[4]{1+x^4}-\sqrt[5]{1+x^4}}=\lim_{k\to+\infty}\frac{\sqrt{1+k^{30}}-\sqrt[3]{1+k^{30}}}{\sqrt[4]{1+k^{60}}-\sqrt[5]{1+k^{60}}}=\lim_{k\to+\infty}\frac{k^{15}\cdot\sqrt{1+\dfrac{1}{k^{30}}}-k^{10}\cdot\sqrt{1+\dfrac{1}{k^{30}}}}{k^{15}\cdot\sqrt[4]{1+\dfrac{1}{k^{60}}}-k^{12}\cdot\sqrt[5]{1+\dfrac{1}{k^{60}}}}$$

$$=\lim_{k\to+\infty}\frac{k^{15}\left(\sqrt{1+\dfrac{1}{k^{30}}}-\dfrac{1}{k^5}\cdot\sqrt{1+\dfrac{1}{k^{30}}}\right)}{k^{15}\cdot\left(\sqrt[4]{1+\dfrac{1}{k^{60}}}-\dfrac{1}{k^3}\cdot\sqrt[5]{1+\dfrac{1}{k^{60}}}\right)}$$

$$=\lim_{k\to+\infty}\frac{\sqrt{1+\dfrac{1}{k^{30}}}-\dfrac{1}{k^5}\cdot\sqrt{1+\dfrac{1}{k^{30}}}}{\sqrt[4]{1+\dfrac{1}{k^{60}}}-\dfrac{1}{k^3}\cdot\sqrt[5]{1+\dfrac{1}{k^{60}}}}=\frac{\sqrt{1+0}-\dfrac{1}{+\infty}\cdot\sqrt{1+0}}{\sqrt[4]{1+0}-\dfrac{1}{+\infty}\cdot\sqrt[5]{1+0}}=1$$

III. គណនាលីមីតមានរាងមិនកំណត់ $(+\infty-\infty)$

វិធាន: ដើម្បីគណនាលីមីតរាងមិនកំណត់ $(+\infty-\infty)$

⊕ បើ $P(x)$ ជាកន្សោមពហុធា គេត្រូវ៖

១. ដាក់តួដែលមានដឺក្រេធំជាងគេជាកត្តារួម

២. គណនាលីមីតនៃកន្សោមថ្មី

⊕ បើ $P(x)$ ជាកន្សោមរ៉ាឌីកាល់ គេត្រូវគុណនិងចែកកន្សោមនោះជាមួយកន្សោមឆ្លាស់របស់វា។

$$1.\lim_{x\to\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)\left(\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}\right)}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}$$

$$=\lim_{x\to\infty}\frac{x^2+5x-1-x^2-3x-2}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}=\lim_{x\to\infty}\frac{2x-3}{\sqrt{x^2\left(1+\frac{5}{x}-\frac{1}{x^2}\right)}+\sqrt{x^2\left(1+\frac{3}{x}+\frac{2}{x^2}\right)}}$$

$$=\lim_{x\to\infty}\frac{x\left(2-\frac{3}{x}\right)}{|x|\cdot\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}$$

⊕ បើ $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\cdot\left(2-\frac{3}{x}\right)}{x\cdot\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to+\infty}\frac{2-\frac{3}{x}}{\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}}=\frac{2-0}{\sqrt{1+0-0}+\sqrt{1+0+0}}=1$$

⊕ បើ $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x\cdot\left(2-\frac{3}{x}\right)}{-x\cdot\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to-\infty}\frac{2-\frac{3}{x}}{-\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}=\frac{2-0}{-\left(\sqrt{1+0-0}+\sqrt{1+0+0}\right)}=-1$$

ដូចនេះ $\lim_{x\to\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)=\begin{cases}1 & \text{បើ } x\to+\infty\\ -1 & \text{បើ } x\to-\infty\end{cases}$

$$2.\lim_{x\to\infty}\left(\sqrt[4]{4+x^4}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt[4]{4+x^4}-x\right)\left(\sqrt[4]{\left(4+x^4\right)^3}+x\cdot\sqrt[4]{\left(4+x^4\right)^2}+x^2\cdot\sqrt[4]{4+x^4}+x^3\right)}{\sqrt[4]{\left(4+x^4\right)^3}+x\cdot\sqrt[4]{\left(4+x^4\right)^2}+x^2\cdot\sqrt[4]{4+x^4}+x^3}$$

$$=\lim_{x\to\infty}\frac{4+x^4-x^4}{\sqrt[4]{x^{12}\left(1+\frac{4}{x^4}\right)^3}+x\cdot\sqrt[4]{x^8\left(1+\frac{4}{x^4}\right)^2}+x^2\cdot\sqrt[4]{x^4\left(1+\frac{4}{x^4}\right)}+x^3}$$

$$=\lim_{x\to\infty}\frac{4}{x^3\left(1+\sqrt[4]{\left(1+\frac{4}{x^4}\right)^3}+\sqrt[4]{\left(1+\frac{4}{x^4}\right)^2}+\sqrt[4]{1+\frac{4}{x^4}}\right)}$$

$$=\frac{4}{\infty\cdot\left(1+\sqrt[4]{(1+0)^3}+\sqrt[4]{(1+0)^2}+\sqrt[4]{1+0}\right)}=0$$

$$3.\lim_{x\to+\infty}\left(\sqrt{(x+a)(x+b)}-x\right)=\lim_{x\to+\infty}\frac{\left(\sqrt{(x+a)(x+b)}-x\right)\left(\sqrt{(x+a)(x+b)}+x\right)}{\sqrt{(x+a)(x+b)}+x}$$

$$=\lim_{x\to+\infty}\frac{(x+a)(x+b)-x^2}{x\sqrt{\left(1+\frac{a}{x}\right)\left(1+\frac{b}{x}\right)}+x}=\lim_{x\to+\infty}\frac{x^2+x(a+b)+ab-x^2}{x\left(\sqrt{\left(1+\frac{a}{x}\right)\left(1+\frac{b}{x}\right)}+1\right)}$$

$$=\lim_{x\to+\infty}\frac{x\left(a+b+\frac{ab}{x}\right)}{x\left(\sqrt{\left(1+\frac{a}{x}\right)\left(1+\frac{b}{x}\right)}+1\right)}=\lim_{x\to+\infty}\frac{a+b+\frac{ab}{x}}{\sqrt{\left(1+\frac{a}{x}\right)\left(1+\frac{b}{x}\right)}+1}$$

$$=\frac{a+b+0}{\sqrt{(1+0)(1+0)}+1}=\frac{a+b}{2}$$

4. $\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)=\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$

$$=\lim_{x\to+\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt{x}\cdot\sqrt{1+\sqrt{\frac{1}{x}}}}{\sqrt{x}\left(\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+1\right)}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{1+\sqrt{\frac{1}{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+1}=\frac{\sqrt{1+\sqrt{0}}}{\sqrt{1+\sqrt{0+\sqrt{0}}}+1}=\frac{1}{2}$$

5. $\lim\limits_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)=\lim\limits_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-x-\sqrt{x^2-2x}+x\right)$

$$=\lim_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-x\right)-\lim_{x\to+\infty}\left(\sqrt{x^2-2x}-x\right)$$

តែ $\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt[3]{x^3+3x^2}-x\right)\left(\sqrt[3]{\left(x^3+3x^2\right)^2}+x\cdot\sqrt[3]{x^3+3x^2}+x^2\right)}{\sqrt[3]{\left(x^3+3x^2\right)^2}+x\cdot\sqrt[3]{x^3+3x^2}+x^2}=\lim\limits_{x\to+\infty}\dfrac{x^3+3x^2-x^3}{x^2\cdot\left(\sqrt[3]{\left(1+\frac{3}{x}\right)^2}+\sqrt[3]{1+\frac{3}{x}}+1\right)}$

$$=\lim_{x\to+\infty}\frac{3}{\sqrt[3]{\left(1+\frac{3}{x}\right)^2}+\sqrt[3]{1+\frac{3}{x}}+1}$$

$$=\frac{3}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1}=1$$

និង $\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt{x^2-2x}-x\right)\left(\sqrt{x^2-2x}+x\right)}{\sqrt{x^2-2x}+x}=\lim\limits_{x\to+\infty}\dfrac{x^2-2x-x^2}{x\left(\sqrt{1-\frac{2}{x}}+1\right)}=\lim\limits_{x\to+\infty}\dfrac{-2}{\sqrt{1-\frac{2}{x}}+1}=\dfrac{-2}{\sqrt{1-0}+1}=-1$

ដូចនេះ $\lim\limits_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)=2$

6. $\lim\limits_{x\to\pm\infty}\left(\sqrt[4]{x^4+4x^3}-\sqrt[3]{x^3+3x^2}-\sqrt{x^2+2x}\right)=\lim\limits_{x\to\pm\infty}\left(\sqrt[4]{x^4\left(1+\frac{4}{x}\right)}-\sqrt[3]{x^3\left(1+\frac{3}{x}\right)}-\sqrt{x^2\left(1+\frac{2}{x}\right)}\right)$

$$=\lim_{x\to\pm\infty}\left(|x|\cdot\sqrt[4]{1+\frac{4}{x}}-x\cdot\sqrt[3]{1+\frac{3}{x}}-|x|\cdot\sqrt{1+\frac{2}{x}}\right)$$

$\oplus$ កាលណា $x\to+\infty$ នោះ $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\left(x\cdot\sqrt[4]{1+\frac{4}{x}}-x\cdot\sqrt[3]{1+\frac{3}{x}}-x\cdot\sqrt{1+\frac{2}{x}}\right)=\lim_{x\to+\infty}x\cdot\left(\sqrt[4]{1+\frac{4}{x}}-\sqrt[3]{1+\frac{3}{x}}-\sqrt{1+\frac{2}{x}}\right)$$

$$=(+\infty)\times\left(\sqrt[4]{1+\frac{4}{+\infty}}-\sqrt[3]{1+\frac{3}{+\infty}}-\sqrt{1+\frac{2}{+\infty}}\right)=-\infty$$

$\oplus$ កាលណា $x\to-\infty$ នោះ $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\left(-x\cdot\sqrt[4]{1+\frac{4}{x}}-x\cdot\sqrt[3]{1+\frac{3}{x}}+x\cdot\sqrt{1+\frac{2}{x}}\right)=\lim_{x\to-\infty}x\cdot\left(-\sqrt[4]{1+\frac{4}{x}}-\sqrt[3]{1+\frac{3}{x}}+\sqrt{1+\frac{2}{x}}\right)$$

$$=(-\infty)\times\left(-\sqrt[4]{1+\frac{4}{-\infty}}-\sqrt[3]{1+\frac{3}{-\infty}}+\sqrt{1+\frac{2}{-\infty}}\right)=+\infty$$

ដូចនេះ $\lim_{x\to\pm\infty}\left(\sqrt[4]{x^4+4x^3}-\sqrt[3]{x^3+3x^2}-\sqrt{x^2+2x}\right)=\mp\infty$

7. $\lim_{x\to1}\left(\frac{3}{\sqrt{x}-1}-\frac{2}{\sqrt[3]{x}-1}\right)$ គេតាង $t^6=x$ កាលណា $x\to1$ នោះ $t\to1$ យើងបាន

$$\lim_{x\to1}\left(\frac{3}{\sqrt{x}-1}-\frac{2}{\sqrt[3]{x}-1}\right)=\lim_{t\to1}\left(\frac{3}{t^3-1}-\frac{2}{t^2-1}\right)$$

$$=\lim_{t\to1}\frac{3\left(t^2-1\right)-2\left(t^3-1\right)}{\left(t^2-1\right)\left(t^3-1\right)}=\lim_{t\to1}\frac{(t-1)\left[3(t+1)-2\left(t^2+t+1\right)\right]}{(t-1)^2(t+1)\left(t^2+t+1\right)}$$

$$=\lim_{t\to1}\frac{3t+3-2t^2-2t-2}{(t-1)(t+1)\left(t^2+t+1\right)}=\lim_{t\to1}\frac{-2t^2+t+1}{(t-1)(t+1)\left(t^2+t+1\right)}$$

$$=\lim_{t\to1}\frac{(t-1)(-2t-1)}{(t-1)(t+1)\left(t^2+t+1\right)}=\lim_{t\to1}\frac{(-2t-1)}{(t+1)\left(t^2+t+1\right)}=\frac{-2-1}{(1+1)(1+1+1)}=-\frac{1}{2}$$

ដូចនេះ $\lim_{x\to1}\left(\frac{3}{\sqrt{x}-1}-\frac{2}{\sqrt[3]{x}-1}\right)=-\frac{1}{2}$

$$8.\ \lim_{x\to1}\left(\frac{n}{1-x^n}-\frac{1}{1-x}\right)=\lim_{x\to1}\frac{n(1-x)-\left(1-x^n\right)}{(1-x)\left(1-x^n\right)}$$

$$=\lim_{x\to1}\frac{n(1-x)-(1-x)\left(1+x+\dots\dots\dots+x^{n-1}\right)}{(1-x)^2\left(1+x+x^2+\dots\dots\dots\dots\dots\dots+x^{n-1}\right)}$$

$$=\lim_{x\to1}\frac{(1-x)\left[n-\left(1+x+\dots\dots\dots\dots\dots+x^{n-1}\right)\right]}{(1-x)^2\left(1+x+x^2+\dots\dots\dots\dots\dots+x^{n-1}\right)}$$

$$=\lim_{x\to1}\frac{(1-x)+\left(1-x^2\right)+\dots\dots\dots\dots\dots\dots\dots+\left(1-x^{n-1}\right)}{(1-x)\left(1+x+x^2+\dots\dots\dots\dots\dots\dots\dots\dots\dots+x^{n-1}\right)}$$

$$=\lim_{x\to1}\frac{(1-x)\left[1+(1+x)+\dots\dots\dots+\left(1+x+\dots\dots\dots+x^{n-2}\right)\right]}{(1-x)\left(1+x+x^2+\dots\dots\dots\dots\dots+x^{n-1}\right)}$$

$$=\lim_{x\to1}\frac{1+(1+x)+\dots\dots\dots\dots+\left(1+x+\dots\dots\dots\dots+x^{n-2}\right)}{1+x+x^2+\dots\dots\dots\dots\dots+x^{n-1}}$$

$$=\frac{1+(1+1)+\dots\dots\dots\dots\dots\dots+(\overbrace{1+1+\dots\dots\dots\dots\dots+1+1}^{(n-1)\text{ដង}})}{\underbrace{1+1+1\dots\dots\dots\dots\dots\dots+1+1+1}_{n\text{ដង}}}$$

$$=\frac{1+2+3+\dots\dots\dots\dots\dots+(n-1)}{n}=\frac{n-1}{2}$$

$$9.\ \lim_{x\to1}\left(\frac{2014}{1-x^{2014}}-\frac{2015}{1-x^{2015}}\right)=\lim_{x\to1}\left(\frac{2014}{1-x^{2014}}-\frac{1}{1-x}-\frac{2015}{1-x^{2015}}+\frac{1}{1-x}\right)$$

$$=\lim_{x\to1}\left(\frac{2014}{1-x^{2014}}-\frac{1}{1-x}\right)-\lim_{x\to1}\left(\frac{2015}{1-x^{2015}}-\frac{1}{1-x}\right)$$

$$=\frac{2014-1}{2}-\frac{2015-1}{2}=-\frac{1}{2}$$ (ព្រោះតាមលំហាត់ទី 8)

$$10.\ \lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\sqrt{x^2+x}+x}$$

$$=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2\left(1+\frac{1}{x}\right)}+x}=\lim_{x\to\infty}\frac{x}{x+|x|\cdot\sqrt{1+\frac{1}{x}}}$$

⊕ បើកាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x}{x+x\cdot\sqrt{1+\frac{1}{x}}}=\lim_{x\to+\infty}\frac{x}{x\cdot\left(1+\sqrt{1+\frac{1}{x}}\right)}=\lim_{x\to+\infty}\frac{1}{1+\sqrt{1+\frac{1}{x}}}=\frac{1}{1+\sqrt{1+\frac{1}{+\infty}}}=\frac{1}{2}$$

⊕ បើកាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x}{x-x\cdot\sqrt{1+\frac{1}{x}}}=\lim_{x\to-\infty}\frac{x}{x\cdot\left(1-\sqrt{1+\frac{1}{x}}\right)}=\lim_{x\to-\infty}\frac{1}{1-\sqrt{1+\frac{1}{x}}}=\frac{1}{1-\sqrt{1+\frac{1}{-\infty}}}=\frac{1}{0}=\infty$$

11. $$\lim_{x\to\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2-4x}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+2x}-\sqrt{x^2-4x}\right)\left(\sqrt{x^2+2x}+\sqrt{x^2-4x}\right)}{\sqrt{x^2+2x}+\sqrt{x^2-4x}}$$

$$=\lim_{x\to\infty}\frac{x^2+2x-x^2+4x}{\sqrt{x^2\left(1+\frac{2}{x}\right)}+\sqrt{x^2\left(1-\frac{4}{x}\right)}}=\lim_{x\to\infty}\frac{6x}{|x|\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}\right)}$$

⊕ បើកាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{6x}{x\cdot\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}\right)}=\lim_{x\to+\infty}\frac{6}{\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}}=\frac{6}{\sqrt{1+\frac{2}{+\infty}}+\sqrt{1-\frac{4}{+\infty}}}=3$$

⊕ បើកាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{6x}{-x\cdot\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}\right)}=\lim_{x\to-\infty}\frac{-6}{\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}}=\frac{-6}{\sqrt{1+\frac{2}{-\infty}}+\sqrt{1-\frac{4}{-\infty}}}=-3$$

12. $$\lim_{x\to\infty}\left[\sqrt{x^2+x+1}-(ax+b)\right]=\lim_{x\to\infty}\frac{\left[\sqrt{x^2+x+1}-(ax+b)\right]\left[\sqrt{x^2+x+1}+(ax+b)\right]}{\sqrt{x^2+x+1}+(ax+b)}$$

$$=\lim_{x\to\infty}\frac{x^2+x+1-(ax+b)^2}{\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}+x\left(a+\frac{b}{x}\right)}=\lim_{x\to\infty}\frac{x^2\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{x\left(a+\frac{b}{x}\right)+|x|\cdot\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}$$

$\oplus$ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x^2\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{x\left(a+\frac{b}{x}\right)+x\cdot\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{x^2\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{x\left(a+\frac{b}{x}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to+\infty}\frac{x\cdot\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{a+\frac{b}{x}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}$$

$\oplus$ បើ $1>a$ នោះគេបាន $\lim\limits_{x\to+\infty}\dfrac{x\cdot\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{a+\frac{b}{x}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\dfrac{(+\infty)(1-a^2)}{(a+1)}=(+\infty)(1-a)=+\infty$

$\oplus$ បើ $a>1$ នោះគេបាន $\lim\limits_{x\to+\infty}\dfrac{x\cdot\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{a+\frac{b}{x}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\dfrac{(+\infty)(1-a^2)}{(a+1)}=(+\infty)(1-a)=-\infty$

$\oplus$ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x^2\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{x\left(a+\frac{b}{x}\right)-x\cdot\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\lim_{x\to-\infty}\frac{x^2\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{x\left(a+\frac{b}{x}-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to-\infty}\frac{x\cdot\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{a+\frac{b}{x}-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}$$

$\otimes$ បើ $1>a$ នោះគេបាន $\lim\limits_{x\to-\infty}\dfrac{x\cdot\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{a+\frac{b}{x}-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\dfrac{(-\infty)(1-a^2)}{(a-1)}=(-\infty)(-1-a)=+\infty$

$\otimes$ បើ $a>1$ នោះគេបាន $\lim\limits_{x\to-\infty}\dfrac{x\cdot\left[1+\frac{1}{x}+\frac{1}{x^2}-\left(a+\frac{b}{x}\right)^2\right]}{a+\frac{b}{x}-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\dfrac{(-\infty)(1-a^2)}{(a-1)}=(-\infty)(-1-a)=+\infty$

13. $$\lim_{x\to\infty}\left(\sqrt[3]{x^3+1}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt[3]{x^3+1}-x\right)\left(\sqrt[3]{\left(x^3+1\right)^2}+x\cdot\sqrt[3]{x^3+1}+x^2\right)}{\sqrt[3]{\left(x^3+1\right)^2}+x\cdot\sqrt[3]{x^3+1}+x^2}$$

$$=\lim_{x\to\infty}\frac{x^3+1-x^3}{\sqrt[3]{\left(x^3+1\right)^2}+x\cdot\sqrt[3]{x^3+1}+x^2}=\lim_{x\to\infty}\frac{1}{\sqrt[3]{\left(x^3+1\right)^2}+x\cdot\sqrt[3]{x^3+1}+x^2}=0$$

14. $$\lim_{x\to\infty}\left(\sqrt[3]{1+x}-\sqrt[3]{x}\right)=\lim_{x\to\infty}\frac{\left(\sqrt[3]{1+x}-\sqrt[3]{x}\right)\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{x(1+x)}+\sqrt[3]{x^2}\right)}{\sqrt[3]{(1+x)^2}+\sqrt[3]{x(1+x)}+\sqrt[3]{x^2}}$$

$$=\lim_{x\to\infty}\frac{1+x-x}{\sqrt[3]{(1+x)^2}+\sqrt[3]{x(1+x)}+\sqrt[3]{x^2}}=0$$

15. $$\lim_{x\to\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}\right)\left(\sqrt{x^2-2x-1}+\sqrt{x^2-7x+3}\right)}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+3}}$$

$$=\lim_{x\to\infty}\frac{x^2-2x-1-x^2+7x-3}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+3}}=\lim_{x\to\infty}\frac{5x-4}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+3}}$$

$$=\lim_{x\to\infty}\frac{x\cdot\left(5-\frac{4}{x}\right)}{|x|\cdot\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{3}{x^2}}\right)}$$

⊕ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\cdot\left(5-\frac{4}{x}\right)}{x\cdot\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{3}{x^2}}\right)}=\lim_{x\to+\infty}\frac{5-\frac{4}{x}}{\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{3}{x^2}}}=\frac{5}{2}$$

⊕ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x\cdot\left(5-\frac{4}{x}\right)}{-x\cdot\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{3}{x^2}}\right)}=\lim_{x\to+\infty}\frac{-\left(5-\frac{4}{x}\right)}{\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{3}{x^2}}}=-\frac{5}{2}$$

16. $$\lim_{x\to\infty}\left(\sqrt[4]{1+x^4}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt[4]{1+x^4}-x\right)\left(\sqrt[4]{\left(1+x^4\right)^3}+x\cdot\sqrt[4]{\left(1+x^4\right)^2}+x^2\cdot\sqrt[4]{1+x^4}+x^3\right)}{\sqrt[4]{\left(1+x^4\right)^3}+x\cdot\sqrt[4]{\left(1+x^4\right)^2}+x^2\cdot\sqrt[4]{1+x^4}+x^3}$$

$$=\lim_{x\to\infty}\frac{1+x^4-x^4}{\sqrt[4]{\left(1+x^4\right)^3}+x\cdot\sqrt[4]{\left(1+x^4\right)^2}+x^2\cdot\sqrt[4]{1+x^4}+x^3}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt[4]{\left(1+x^4\right)^3}+x\cdot\sqrt[4]{\left(1+x^4\right)^2}+x^2\cdot\sqrt[4]{1+x^4}+x^3}=0$$

17. $$\lim_{x\to\infty}\left(3x-\sqrt{x^2-x+1}\right)=\lim_{x\to\infty}\frac{\left(3x-\sqrt{x^2-x+1}\right)\left(3x+\sqrt{x^2-x+1}\right)}{3x+\sqrt{x^2-x+1}}$$

$$=\lim_{x\to\infty}\frac{9x^2-x^2+x-1}{3x+|x|\cdot\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{x^2\left(8+\frac{1}{x}-\frac{1}{x^2}\right)}{3x+|x|\cdot\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}$$

$\oplus$ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x^2\left(8+\frac{1}{x}-\frac{1}{x^2}\right)}{x\left(3+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to+\infty}\frac{x\cdot\left(8+\frac{1}{x}-\frac{1}{x^2}\right)}{3+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}=+\infty$$

$\oplus$ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x^2\left(8+\frac{1}{x}-\frac{1}{x^2}\right)}{x\left(3-\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to-\infty}\frac{x\cdot\left(8+\frac{1}{x}-\frac{1}{x^2}\right)}{3-\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}=-\infty$$

18. $\lim_{x\to\infty}\left(\sqrt{1+x^2}-\sqrt[3]{x^3-1}\right)=\lim_{x\to\infty}\left(\sqrt{1+x^2}-x+x-\sqrt[3]{x^3-1}\right)$

$$=\lim_{x\to\infty}\left(\sqrt{1+x^2}-x\right)+\lim_{x\to\infty}\left(x-\sqrt[3]{x^3-1}\right)$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{1+x^2}-x\right)\left(\sqrt{1+x^2}+x\right)}{\sqrt{1+x^2}+x}+\lim_{x\to\infty}\frac{\left(x-\sqrt[3]{x^3-1}\right)\left(x^2+x\cdot\sqrt[3]{x^3-1}+\sqrt[3]{\left(x^3-1\right)^2}\right)}{x^2+x\cdot\sqrt[3]{x^3-1}+\sqrt[3]{\left(x^3-1\right)^2}}$$

$$=\lim_{x\to\infty}\frac{1+x^2-x^2}{\sqrt{1+x^2}+x}+\lim_{x\to\infty}\frac{x^3-x^3+1}{x^2+x\cdot\sqrt[3]{x^3-1}+\sqrt[3]{\left(x^3-1\right)^2}}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+x^2}+x}+\lim_{x\to\infty}\frac{1}{x^2+x\cdot\sqrt[3]{x^3-1}+\sqrt[3]{\left(x^3-1\right)^2}}=0$$

19. $\lim_{x\to\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+2}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+2}\right)\left(\sqrt{x^2-2x-1}+\sqrt{x^2-7x+2}\right)}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+2}}$

$$=\lim_{x\to\infty}\frac{x^2-2x-1-x^2+7x-2}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+2}}=\lim_{x\to\infty}\frac{5x-3}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+2}}$$

$$=\lim_{x\to\infty}\frac{x\cdot\left(5-\frac{3}{x}\right)}{|x|\cdot\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{2}{x^2}}\right)}$$

⊕ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\cdot\left(5-\frac{3}{x}\right)}{x\cdot\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to+\infty}\frac{5-\frac{3}{x}}{\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{2}{x^2}}}=\frac{5}{2}$$

⊕ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x\cdot\left(5-\frac{3}{x}\right)}{-x\cdot\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to+\infty}\frac{-\left(5-\frac{3}{x}\right)}{\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{2}{x^2}}}=-\frac{5}{2}$$

20. $$\lim_{x\to\infty}\left(\sqrt{x^2+2x+5}-\sqrt{1+x^2}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+2x+5}-\sqrt{1+x^2}\right)\left(\sqrt{x^2+2x+5}+\sqrt{1+x^2}\right)}{\sqrt{x^2+2x+5}+\sqrt{1+x^2}}$$

$$=\lim_{x\to\infty}\frac{x^2+2x+5-1-x^2}{|x|\cdot\left(\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}=\lim_{x\to\infty}\frac{x\left(2+\frac{4}{x}\right)}{|x|\cdot\left(\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}$$

$\oplus$ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\cdot\left(2+\frac{4}{x}\right)}{x\cdot\left(\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}=\lim_{x\to+\infty}\frac{2+\frac{4}{x}}{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x^2}}}=1$$

$\oplus$ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x\cdot\left(2+\frac{4}{x}\right)}{-x\cdot\left(\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}=\lim_{x\to-\infty}\frac{-\left(2+\frac{4}{x}\right)}{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x^2}}}=-1$$

21. $$\lim_{x\to\infty}\left(\sqrt{x^2+4x-7}-\sqrt{x^2+4x-1}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+4x-7}-\sqrt{x^2+4x-1}\right)\left(\sqrt{x^2+4x-7}+\sqrt{x^2+4x-1}\right)}{\sqrt{x^2+4x-7}+\sqrt{x^2+4x-1}}$$

$$=\lim_{x\to\infty}\frac{x^2+4x-7-x^2-4x+1}{\sqrt{x^2+4x-7}+\sqrt{x^2+4x-1}}=\lim_{x\to\infty}\frac{-6}{\sqrt{x^2+4x-7}+\sqrt{x^2+4x-1}}=0$$

22. $$\lim_{x\to+\infty}\left(\sqrt{3x^2+7x+1}-\sqrt{3}x\right)=\lim_{x\to+\infty}\frac{\left(\sqrt{3x^2+7x+1}-\sqrt{3}x\right)\left(\sqrt{3x^2+7x+1}+\sqrt{3}x\right)}{\sqrt{3x^2+7x+1}+\sqrt{3}x}$$

$$=\lim_{x\to+\infty}\frac{3x^2+7x+1-3x^2}{x\sqrt{3+\frac{7}{x}+\frac{1}{x^2}}+\sqrt{3}x}=\lim_{x\to+\infty}\frac{x\left(7+\frac{1}{x}\right)}{x\left(\sqrt{3+\frac{7}{x}+\frac{1}{x^2}}+\sqrt{3}\right)}$$

$$=\lim_{x\to+\infty}\frac{7+\frac{1}{x}}{\sqrt{3+\frac{7}{x}+\frac{1}{x^2}}+\sqrt{3}}=\frac{7}{2\sqrt{3}}$$

23. $$\lim_{x\to\pm\infty}\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)=\lim_{x\to\pm\infty}\frac{\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)\left(\sqrt{x^2+4x+7}+\sqrt{x^2-4}\right)}{\sqrt{x^2+4x+7}+\sqrt{x^2-4}}$$

$$=\lim_{x\to\pm\infty}\frac{x^2+4x+7-x^2+4}{\sqrt{x^2+4x+7}+\sqrt{x^2-4}}$$

$$=\lim_{x\to\pm\infty}\frac{4x+11}{|x|\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}$$

⊕ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{x\left(4+\frac{11}{x}\right)}{x\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}=\lim_{x\to+\infty}\frac{4+\frac{11}{x}}{\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}}=\frac{4}{2}=2$$

⊕ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{x\left(4+\frac{11}{x}\right)}{-x\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}=\lim_{x\to-\infty}\frac{4+\frac{11}{x}}{-\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}=\frac{4}{-2}=-2$$

24. $$\lim_{x\to\pm\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)=\lim_{x\to\pm\infty}\frac{2x^4+x^3-2x^4+x^2}{\left(2x^2-1\right)\left(2x+1\right)}$$

$$=\lim_{x\to\pm\infty}\frac{x^3+x^2}{x^3\left(2-\frac{1}{x^2}\right)\left(2+\frac{1}{x}\right)}=\lim_{x\to\pm\infty}\frac{x^3\left(1+\frac{1}{x}\right)}{x^3\left(2-\frac{1}{x^2}\right)\left(2+\frac{1}{x}\right)}$$

$$=\lim_{x\to\pm\infty}\frac{1+\frac{1}{x}}{\left(2-\frac{1}{x^2}\right)\left(2+\frac{1}{x}\right)}=\frac{1}{4}$$

25. $\lim\limits_{x\to 0}\left(\frac{1}{\sqrt{x}}-\frac{x+1}{\sqrt{x}}\right)=\lim\limits_{x\to 0}\frac{1-x-1}{\sqrt{x}}=\lim\limits_{x\to 0}\frac{-\sqrt{x}\times\sqrt{x}}{\sqrt{x}}=\lim\limits_{x\to 0}\left(-\sqrt{x}\right)=0$

26. $\lim\limits_{x\to 1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)=\lim\limits_{x\to 1}\frac{2-(x+1)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\frac{-(x-1)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\left(-\frac{1}{x+1}\right)=-\frac{1}{2}$

27. $\lim\limits_{x\to -2}\left(\frac{1}{x+2}-\frac{12}{x^3+8}\right)=\lim\limits_{x\to -2}\left[\frac{1}{x+2}-\frac{12}{(x+2)(x^2-2x+4)}\right]$

$$=\lim_{x\to -2}\frac{x^2-2x+4-12}{(x+2)(x^2-2x+4)}=\lim_{x\to -2}\frac{x^2-2x-8}{(x+2)(x^2-2x+4)}$$

$$=\lim_{x\to -2}\frac{(x+2)(x-4)}{(x+2)(x^2-2x+4)}=\lim_{x\to -2}\frac{x-4}{x^2-2x+4}=\frac{-2-4}{4+4+4}=-\frac{1}{2}$$

28. $\lim\limits_{x\to 2}\left[\frac{1}{x(x-2)^2}-\frac{1}{x^2-3x+2}\right]=\lim\limits_{x\to 2}\left[\frac{1}{x(x-2)^2}-\frac{1}{(x-1)(x-2)}\right]$

$$=\lim_{x\to 2}\frac{(x-1)-x(x-2)}{x(x-1)(x-2)^2}=\lim_{x\to 2}\frac{-x^2+3x-1}{x(x-1)(x-2)^2}$$

$$=\frac{-4+6-1}{2\times 1\times 0}=\infty$$

29. $\lim\limits_{x\to\infty}\left(\frac{x^3}{x+1}-x\right)=\lim\limits_{x\to\infty}\frac{x^3-x(x+1)}{x+1}=\lim\limits_{x\to\infty}\frac{x^3-x^2-x}{x\left(1+\frac{1}{x}\right)}$

$$=\lim_{x\to\infty}\frac{x^3\left(1-\frac{1}{x}-\frac{1}{x^2}\right)}{x\left(1+\frac{1}{x}\right)}=\lim_{x\to\infty}\frac{x^2\cdot\left(1-\frac{1}{x}-\frac{1}{x^2}\right)}{1+\frac{1}{x}}=\frac{\infty\times(1-0-0)}{1+0}=\infty$$

30. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2-4x}\right)=\lim\limits_{x\to\infty}\frac{\left(\sqrt{x^2+2x}-\sqrt{x^2-4x}\right)\left(\sqrt{x^2+2x}+\sqrt{x^2-4x}\right)}{\sqrt{x^2+2x}+\sqrt{x^2-4x}}$

$$=\lim_{x\to\infty}\frac{x^2+2x-x^2+4x}{\sqrt{x^2+2x}+\sqrt{x^2-4x}}=\lim_{x\to\infty}\frac{6x}{|x|\cdot\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}\right)}$$

⊕ កាលណា $x\to+\infty$ នោះគេបាន $|x|=x$ យើងបាន

$$\lim_{x\to+\infty}\frac{6x}{x\cdot\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}\right)}=\lim_{x\to+\infty}\frac{6}{\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}}=\frac{6}{2}=3$$

⊕ កាលណា $x\to-\infty$ នោះគេបាន $|x|=-x$ យើងបាន

$$\lim_{x\to-\infty}\frac{6x}{-x\cdot\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}\right)}=\lim_{x\to-\infty}\frac{-6}{\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{4}{x}}}=\frac{-6}{2}=-3$$

31. $\lim_{x\to\infty}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)$

$$=\lim_{x\to\infty}\frac{\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)\left(\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}\right)}{\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}}$$

$$=\lim_{x\to\infty}\frac{(x+1)^2-(x-1)^2}{\sqrt[3]{x^4\cdot\left(1+\frac{1}{x}\right)^4}+\sqrt[3]{x^4\cdot\left(1+\frac{1}{x}\right)^2\left(1-\frac{1}{x}\right)^2}+\sqrt[3]{x^4\cdot\left(1-\frac{1}{x}\right)^4}}$$

$$=\lim_{x\to\infty}\frac{x^2+2x+1-x^2+2x-1}{\sqrt[3]{x^4\cdot\left(1+\frac{1}{x}\right)^4}+\sqrt[3]{x^4\cdot\left(1+\frac{1}{x}\right)^2\left(1-\frac{1}{x}\right)^2}+\sqrt[3]{x^4\cdot\left(1-\frac{1}{x}\right)^4}}$$

$$=\lim_{x\to\infty}\frac{4x}{x\cdot\left(\sqrt[3]{x\cdot\left(1+\frac{1}{x}\right)^4}+\sqrt[3]{x\cdot\left(1+\frac{1}{x}\right)^2\left(1-\frac{1}{x}\right)^2}+\sqrt[3]{x\cdot\left(1-\frac{1}{x}\right)^4}\right)}$$

$$=\lim_{x\to\infty}\frac{4}{\sqrt[3]{x\cdot\left(1+\frac{1}{x}\right)^4}+\sqrt[3]{x\cdot\left(1+\frac{1}{x}\right)^2\left(1-\frac{1}{x}\right)^2}+\sqrt[3]{x\cdot\left(1-\frac{1}{x}\right)^4}}=\frac{4}{\infty}=0$$

32. $\lim\limits_{x\to\pm\infty}\left(3x-\sqrt{x^2-x+1}\right)=\lim\limits_{x\to\pm\infty}\dfrac{\left(3x-\sqrt{x^2-x+1}\right)\left(3x+\sqrt{x^2-x+1}\right)}{3x+\sqrt{x^2-x+1}}$

$$=\lim_{x\to\pm\infty}\frac{9x^2-x^2+x-1}{3x+\sqrt{x^2-x+1}}=\lim_{x\to\pm\infty}\frac{x^2\left(9-\frac{1}{x}+\frac{1}{x^2}\right)}{3x+\sqrt{x^2-x+1}}=\pm\infty$$

IV.គណនាលីមីតនៃអនុគមន៍ត្រីកោណមាត្រ

រូបមន្តដែលត្រូវចង់ចាំរួមមាន៖

$\oplus\ \sin^2 a+\cos^2 a=1$ $\quad\oplus\ \sin 2a=2\sin a\cos a$ $\quad\oplus\ \cos 2a=\cos^2 a-\sin^2 a=2\cos^2 a-1=1-2\sin^2 a$

$\oplus\ 1-\cos a=2\sin^2\dfrac{a}{2}$ $\quad\oplus\ \sin 3a=3\sin a-4\sin^3 a$ $\quad\oplus\ \cos 3a=4\cos^3 a-3\cos a$ $\quad\oplus\ \tan 3a=\dfrac{3\tan a-\tan^3 a}{1-3\tan^2 a}$

$\oplus\ \tan 2a=\dfrac{2\tan a}{1-\tan^2 a}$ $\quad\oplus\sin a+\sin b=2\sin\dfrac{a+b}{2}\cdot\cos\dfrac{a-b}{2}$ $\quad\oplus\sin a-\sin b=2\sin\dfrac{a-b}{2}\cdot\cos\dfrac{a+b}{2}$

$\oplus\ \cos a+\cos b=2\cos\dfrac{a+b}{2}\cdot\cos\dfrac{a-b}{2}$ $\quad\oplus\ \cos a-\cos b=-2\sin\dfrac{a-b}{2}\cdot\sin\dfrac{a+b}{2}$

$\oplus\ \tan a\pm\tan b=\dfrac{\sin(a\pm b)}{\cos a\cdot\cos b}$ $\quad\oplus\ \cot a\pm\cot b=\dfrac{\sin(b\pm a)}{\sin a\cdot\sin b}$ $\quad\oplus\ \tan a=\dfrac{\sin 2a}{1+\cos 2a}$ $\quad\oplus\ \tan\dfrac{a}{2}=\dfrac{\sin a}{1+\cos a}$

$\oplus\ \sin(a\pm b)=\sin a\cdot\cos b\pm\sin b\cdot\cos a$ $\quad\oplus\ \cos(a\pm b)=\cos a\cdot\cos b\mp\sin a\cdot\sin b$

$\oplus\ \tan(a\pm b)=\dfrac{\tan a\pm\tan b}{1\mp\tan a\cdot\tan b}$ $\quad\oplus\ \dfrac{1}{\cos^2 a}=1+\tan^2 a$ $\quad\oplus\ 1+\cot^2 a=\dfrac{1}{\sin^2 a}$

$\oplus\ \sin(-\alpha)=-\sin\alpha$ $\quad\oplus\ \cos(-\alpha)=\cos\alpha$ $\quad\oplus\ \tan(-\alpha)=-\tan\alpha$ $\quad\oplus\ \cot(-\alpha)=-\cot\alpha$

$\oplus\ \sin(\pi\mp\alpha)=\pm\sin\alpha$ $\quad\oplus\ \cos(\pi\pm\alpha)=-\cos\alpha$ $\quad\oplus\ \tan(\pi\pm\alpha)=\pm\tan\alpha$ $\quad\oplus\ \cot(\pi\pm\alpha)=\pm\cot\alpha$

$\oplus\ \sin\left(\dfrac{\pi}{2}\pm\alpha\right)=\cos\alpha$ $\quad\oplus\ \cos\left(\dfrac{\pi}{2}\pm\alpha\right)=\mp\sin\alpha$ $\quad\oplus\ \tan\left(\dfrac{\pi}{2}\pm\alpha\right)=\mp\cot\alpha$ $\quad\oplus\ \cot\left(\dfrac{\pi}{2}\pm\alpha\right)=\mp\tan\alpha$

$\oplus\ \sin(\alpha+2k\pi)=\sin\alpha$ $\quad\oplus\ \cos(\alpha+2k\pi)=\cos\alpha$

$\oplus\ \tan(\alpha+2k\pi)=\tan(\alpha+k\pi)=\tan\alpha$ $\quad\oplus\ \cot(\alpha+2k\pi)=\cot(\alpha+k\pi)=\cot\alpha\ \ ;k\in\mathbb{Z}$

រូបមន្តលីមីតនៃអនុគមន៍ត្រីកោណមាត្រ:

លីមីតត្រីកោណមាត្រទាក់ទងនឹងអថេរដូចជា: $x\to 0$, $x\to x_0$; $x\to\infty$ ។

$\otimes$ *បើរាងមិនកំណត់កើតឡើងត្រូវនឹង* $x\to 0$ *នោះគេប្រើ:*

$$\lim_{x\to0}\frac{\sin x}{x}=\lim_{x\to0}\frac{x}{\sin x}=\lim_{x\to0}\frac{\sin ax}{ax}=\lim_{x\to0}\frac{ax}{\sin ax}=1\ ;\ \lim_{x\to0}\frac{\tan x}{x}=\lim_{x\to0}\frac{x}{\tan x}=\lim_{x\to0}\frac{\tan ax}{ax}=\lim_{x\to0}\frac{ax}{\tan ax}=1$$

$$\lim_{x\to0}\frac{1-\cos x}{x}=\lim_{x\to0}\frac{1-\cos kx}{kx}=0\ ;\ \lim_{x\to0}\frac{1-\cos kx}{x^2}=\frac{k^2}{2}$$

⊗ បើរាងមិនកំណត់កើតឡើងត្រូវនឹង $x\to x_0$ នោះគេត្រូវតាង $k=x-x_0$ ឬ $k=x_0-x$ កាលណា $x\to x_0$ នោះ $k\to0$ បន្ទាប់មកប្រើរូបមន្តមិនកំណត់ខាងលើ។

⊗ បើរាងមិនកំណត់កើតឡើងត្រូវនឹង $x\to\infty$ នោះគេត្រូវតាង $k=\frac{1}{x}$ កាលណា $x\to\infty$ នោះ $k\to0$ បន្ទាប់មកប្រើរូបមន្តមិនកំណត់ខាងលើ។

យើងធ្វើការគណនាលីមីតខាងលើទៅតាមវិធាននិងរូបមន្តដែលបានបង្ហាញខាងលើ

1. $\lim_{x\to0}\frac{\sin5x}{\sin3x}=\lim_{x\to0}\frac{\sin5x}{x}\times\frac{x}{\sin3x}=\left(\lim_{x\to0}\frac{\sin5x}{5x}\times5\right)\times\left(\lim_{x\to0}\frac{3x}{\sin3x}\times\frac{1}{3}\right)=5\times\frac{1}{3}=\frac{5}{3}$

2. $\lim_{x\to0}\frac{\cos2x-\cos x}{\sin2x}=\lim_{x\to0}\frac{\cos2x-1+1-\cos x}{\sin2x}$

$$=\lim_{x\to0}\frac{(1-\cos x)-(1-\cos2x)}{\sin2x}=\lim_{x\to0}\left[\frac{(1-\cos x)-(1-\cos2x)}{x}\right]\times\frac{x}{\sin2x}$$

$$=\left[\lim_{x\to0}\frac{1-\cos x}{x}-\lim_{x\to0}\frac{1-\cos2x}{2x}\times2\right]\times\left(\lim_{x\to0}\frac{2x}{\sin2x}\times\frac{1}{2}\right)$$

$$=(0-0\times2)\times1\times\frac{1}{2}=0$$

3. $\lim_{x\to0}\frac{3(1-\cos x)}{5x^2}=\frac{3}{5}\cdot\lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{3}{5}\times\frac{1}{2}=\frac{3}{10}$

4. $\lim_{x\to0}\frac{2\sin x-\sin2x}{x^3}=\lim_{x\to0}\frac{2\sin x-2\sin x\cos x}{x^3}=2\lim_{x\to0}\frac{\sin x\cdot(1-\cos x)}{x\cdot x^2}$

$$=2\left(\lim_{x\to0}\frac{\sin x}{x}\right)\times\left(\lim_{x\to0}\frac{1-\cos x}{x^2}\right)=2\times1\times\frac{1}{2}=1$$

5. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}=\lim\limits_{x\to 0}\frac{\sin x(1-\cos x)}{x^3\cos x}=\lim\limits_{x\to 0}\frac{\sin x}{x}\times\frac{1-\cos x}{x^2}\times\frac{1}{\cos x}$

$$=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\times\left(\lim_{x\to 0}\frac{1-\cos x}{x^2}\right)\times\left(\lim_{x\to 0}\frac{1}{\cos x}\right)=1\times\frac{1}{2}\times 1=\frac{1}{2}$$

6. $\lim\limits_{x\to 0}\frac{1-\cos kx}{x^2}=\lim\limits_{x\to 0}\frac{1-\cos kx}{(kx)^2}\times k^2=k^2\times\lim\limits_{x\to 0}\frac{1-\cos kx}{(kx)^2}=k^2\times\frac{1}{2}=\frac{k^2}{2}$

7. $\lim\limits_{x\to 0}\frac{\sin 5x\cdot\sin 3x\cdot\sin x}{45x^3}=\lim\limits_{x\to 0}\frac{\sin 5x}{5x}\cdot\frac{\sin 3x}{3x}\cdot\frac{\sin x}{x}\cdot\frac{1}{3}$

$$=\left(\lim_{x\to 0}\frac{\sin 5x}{5x}\right)\cdot\left(\lim_{x\to 0}\frac{\sin 3x}{3x}\right)\cdot\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\cdot\frac{1}{3}=1\times 1\times 1\times\frac{1}{3}=\frac{1}{3}$$

8. $\lim\limits_{x\to 0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}=\lim\limits_{x\to 0}\frac{\sin(a+2x)+\sin a-2\sin(a+x)}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{a+2x+a}{2}\right)\cdot\cos\left(\frac{a+2x-a}{2}\right)-2\sin(a+x)}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin(a+x)\cdot\cos x-2\sin(a+x)}{x^2}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\left[-2\sin(a+x)\right]=\frac{1}{2}\times(-2\sin a)=-\sin a$$

9. $\lim\limits_{x\to 0}\frac{1-\cos(1-\cos 2x)}{x^4}=\lim\limits_{x\to 0}\frac{1-\cos(1-\cos 2x)}{(1-\cos 2x)^2}\times\frac{(1-\cos 2x)^2}{(4x^2)^2}\times 16$

$$=\lim_{x\to 0}\frac{1-\cos(1-\cos 2x)}{(1-\cos 2x)^2}\cdot\left[\frac{1-\cos 2x}{(2x)^2}\right]^2\times 16=\frac{1}{2}\times\left(\frac{1}{2}\right)^2\times 16=2$$

10. $\lim\limits_{x\to 0}\dfrac{\sin(a+3x)-3\sin(a+2x)+3\sin(a+x)-\sin a}{x^3}$

$$=\lim_{x\to 0}\frac{\sin(a+3x)-\sin a+3\sin(a+x)-3\sin(a+2x)}{x^3}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{a+3x-a}{2}\right)\cdot\cos\left(\frac{a+3x+a}{2}\right)+6\sin\left(\frac{a+x-a-2x}{2}\right)\cdot\cos\left(\frac{a+x+a+2x}{2}\right)}{x^3}$$

$$=\lim_{x\to 0}\frac{2\cos\left(\frac{2a+3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right]}{x^3}$$

$$=\lim_{x\to 0}\frac{2\cos\left(\frac{2a+3x}{2}\right)\left(3\sin\frac{x}{x}-4\sin^3\left(\frac{x}{2}\right)-3\sin\frac{x}{2}\right)}{x^3}$$

$$=-\lim_{x\to 0}\left[\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right]^3\times\cos\left(\frac{2a+3x}{2}\right)=-1\times\cos a=-\cos a$$

11. $\lim\limits_{x\to 0}x^2\cdot\sin\dfrac{1}{x}=\lim\limits_{x\to 0}x\cdot\dfrac{\sin\frac{1}{x}}{\frac{1}{x}}=\left(\lim\limits_{x\to 0}x\right)\cdot\left(\lim\limits_{x\to 0}\dfrac{\sin\frac{1}{x}}{\frac{1}{x}}\right)=0\times 1=0$

12. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x}{\tan 2x}$ គេតាង $k=x-\dfrac{\pi}{2}\Rightarrow x=\dfrac{\pi}{2}+k$ កាលណា $x\to\dfrac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\cos x=\cos\left(\dfrac{\pi}{2}+k\right)=-\sin k$ និង $\tan 2x=\tan(\pi+2k)=\tan 2k$

យើងបាន

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\tan 2x}=\lim_{k\to 0}\frac{-\sin k}{\tan 2k}=-\lim_{k\to 0}\frac{\sin k}{k}\times\frac{2k}{\tan 2k}\times\frac{1}{2}$$

$$=-\frac{1}{2}\left(\lim_{k\to 0}\frac{\sin k}{k}\right)\cdot\left(\lim_{k\to 0}\frac{2k}{\tan 2k}\right)=-\frac{1}{2}\times 1\times 1=-\frac{1}{2}$$

13. $\lim_{x\to a}\left(a^2-x^2\right)\cdot\tan\frac{\pi x}{2a}$ គេតាង $k=a-x\Rightarrow x=a-k$ កាលណា $x\to a$ នោះ $k\to 0$

ដោយ $\tan\frac{\pi x}{2a}=\tan\left(\frac{\pi}{2}-\frac{\pi k}{2a}\right)=\frac{1}{\tan\frac{\pi k}{2a}}$ និង $a^2-x^2=(a-x)(a+x)$ យើងបាន

$$\lim_{x\to a}\left(a^2-x^2\right)\tan\frac{\pi x}{2a}=\lim_{k\to 0}k(2a-k)\left(\frac{1}{\tan\frac{\pi k}{2a}}\right)=\lim_{k\to 0}(2a-k)\times\frac{\frac{\pi k}{2a}}{\tan\frac{\pi k}{2a}}\times\frac{2a}{\pi}$$

$$=\frac{2a}{\pi}\left[\lim_{k\to 0}(2a-k)\right]\times\left(\lim_{k\to 0}\frac{\frac{\pi k}{2a}}{\tan\frac{\pi k}{2a}}\right)=\frac{2a}{\pi}\times 2a\times 1=\frac{4a^2}{\pi}$$

14. $$\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{2\cos x-\sqrt{2}}=\lim_{x\to\frac{\pi}{4}}\frac{\tan x-\tan\frac{\pi}{4}}{2\left(\cos x-\cos\frac{\pi}{4}\right)}=\frac{1}{2}\cdot\lim_{x\to\frac{\pi}{4}}\frac{\frac{\sin\left(x-\frac{\pi}{4}\right)}{\cos x\cdot\cos\frac{\pi}{4}}}{-2\sin\left(\frac{x-\frac{\pi}{4}}{2}\right)\cdot\sin\left(\frac{x+\frac{\pi}{4}}{2}\right)}$$

$$=-\frac{1}{2}\cdot\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(\frac{x-\frac{\pi}{4}}{2}\right)\cdot\cos\left(\frac{x-\frac{\pi}{4}}{2}\right)}{\sin\left(\frac{x-\frac{\pi}{4}}{2}\right)\cdot\sin\left(\frac{x+\frac{\pi}{4}}{2}\right)\cdot\cos x\cdot\cos\frac{\pi}{4}}=-\frac{1}{2}\cdot\lim_{x\to\frac{\pi}{4}}\frac{\cos\left(\frac{x-\frac{\pi}{4}}{2}\right)}{\sin\left(\frac{x+\frac{\pi}{4}}{2}\right)\cdot\cos x\cdot\cos\frac{\pi}{4}}$$

$$=-\frac{1}{2}\times\frac{1}{\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{4}\cdot\cos\frac{\pi}{4}}=-\frac{1}{2}\times\frac{1}{\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}}=-\sqrt{2}$$

15. $\lim\limits_{x\to\infty} x\cdot\sin\dfrac{\pi}{x}=\lim\limits_{x\to\infty}\dfrac{\sin\dfrac{\pi}{x}}{\dfrac{1}{x}}$ គេតាង $k=\dfrac{1}{x}$ កាលណា $x\to\infty$ នោះ $k\to 0$

យើងបាន $\lim\limits_{x\to\infty} x\cdot\sin\dfrac{\pi}{x}=\lim\limits_{k\to 0}\dfrac{\sin\pi k}{k}=\lim\limits_{k\to 0}\dfrac{\sin\pi k}{\pi k}\times\pi=1\times\pi=\pi$

16. $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi x}{2}$ គេតាង $k=1-x\Rightarrow x=1-k$ កាលណា $x\to 1$ នោះ $k\to 0$

ដោយ $\tan\dfrac{\pi x}{2}=\tan\left(\dfrac{\pi}{2}-\dfrac{\pi k}{2}\right)=\dfrac{1}{\tan\dfrac{\pi k}{2}}$

យើងបាន $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi x}{2}=\lim\limits_{k\to 0}\dfrac{k}{\tan\dfrac{\pi k}{2}}=\lim\limits_{k\to 0}\dfrac{\dfrac{\pi k}{2}}{\tan\dfrac{\pi k}{2}}\times\dfrac{2}{\pi}=1\times\dfrac{2}{\pi}=\dfrac{2}{\pi}$

17. $\lim\limits_{x\to\frac{\pi}{4}}\tan 2x\cdot\tan\left(\dfrac{\pi}{4}-x\right)$ គេតាង $k=\dfrac{\pi}{4}-x\Rightarrow x=\dfrac{\pi}{4}-k$ កាលណា $x\to\dfrac{\pi}{4}$ នោះ $k\to 0$

ដោយ $\tan 2x=\tan\left(\dfrac{\pi}{2}-2k\right)=\dfrac{1}{\tan 2k}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{4}}\tan 2x\cdot\tan\left(\dfrac{\pi}{4}-x\right)=\lim\limits_{k\to 0}\dfrac{\tan k}{\tan 2k}=\lim\limits_{k\to 0}\dfrac{\tan k}{k}\times\dfrac{2k}{\tan 2k}\cdot\dfrac{1}{2}=1\times 1\times\dfrac{1}{2}=\dfrac{1}{2}$

18. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-2\cos x}{\sin 3x}$ គេតាង $k=x-\dfrac{\pi}{3}\Rightarrow x=\dfrac{\pi}{3}+k$ កាលណា $x\to\dfrac{\pi}{3}$ នោះ $k\to 0$

ដោយ $\sin 3x=\sin(\pi+3k)=-\sin 3k$ និង

$\cos x=\cos\left(\dfrac{\pi}{3}+k\right)=\cos\dfrac{\pi}{3}\cdot\cos k-\sin\dfrac{\pi}{3}\cdot\sin k=\dfrac{1}{2}\cdot\cos k-\dfrac{\sqrt{3}}{2}\cdot\sin k$

យើងបាន

$$\lim_{x\to\frac{\pi}{3}}\frac{1-2\cos x}{\sin 3x}=\lim_{k\to 0}\frac{1-2\left(\frac{1}{2}\cdot\cos k-\frac{\sqrt{3}}{2}\cdot\sin k\right)}{\sin 3k}=\lim_{k\to 0}\frac{1-\cos k+\sqrt{3}\cdot\sin k}{\sin 3k}$$

$$=\lim_{k\to 0}\left(\frac{1-\cos k+\sqrt{3}\cdot\sin k}{k}\right)\cdot\frac{k}{\sin 3k}$$

$$=\left(\lim_{k\to 0}\frac{3k}{\sin 3k}\cdot\frac{1}{3}\right)\left(\lim_{k\to 0}\frac{1-\cos k}{k}+\sqrt{3}\cdot\lim_{k\to 0}\frac{\sin k}{k}\right)$$

$$=\left(1\times\frac{1}{3}\right)\cdot\left(0+\sqrt{3}\times 1\right)=\frac{\sqrt{3}}{3}$$

19. $\lim_{x\to 1}\frac{\sin\pi x}{\sin 3\pi x}$ គេតាង $k=1-x\Rightarrow x=1-k$ កាលណា $x\to 1$ នោះ $k\to 0$

ដោយ $\sin\pi x=\sin\left(\pi-\pi k\right)=\cos\pi k$ និង

$\sin 3\pi x=\sin\left[2\pi+\left(\pi-3\pi k\right)\right]=\sin\left(\pi-3\pi k\right)=\cos 3\pi k$

យើងបាន $\lim_{x\to 1}\frac{\sin\pi x}{\sin 3\pi k}=\lim_{k\to 0}\frac{\cos k}{\cos 3\pi k}=1$

20. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\cdot\tan x$ គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\tan x=\tan\left(\frac{\pi}{2}-k\right)=\frac{1}{\tan k}$

យើងបាន $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\times\tan x=\lim_{k\to 0}\frac{k}{\tan k}=1$

21. $\lim_{x\to a}\frac{\sin\left(x-a\right)}{x^3-a^3}=\lim_{x\to a}\frac{\sin\left(x-a\right)}{\left(x-a\right)\left(x^2+ax+a^2\right)}$ គេតាង $k=x-a\Rightarrow k+a$ កាលណា $x\to a$ នោះ $k\to 0$

យើងបាន $\lim_{x\to a}\frac{\sin\left(x-a\right)}{x^3-a^3}=\lim_{k\to 0}\frac{\sin k}{k\left[\left(k+a\right)^2+\left(k+a\right)a+a^2\right]}=1\times\frac{1}{a^2+a^2+a^2}=\frac{1}{3a^2}$

22. $\lim\limits_{x\to 1}\dfrac{1-x^2}{\sin\pi x}=\lim\limits_{x\to 1}\dfrac{(1-x)(1+x)}{\sin(\pi-\pi x)}=\lim\limits_{x\to 1}\dfrac{(1-x)(1+x)}{\sin\pi(1-x)}$

គេតាង $k=1-x\Rightarrow x=1-k$ កាលណា $x\to 1$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to 1}\frac{(1-x)(1+x)}{\sin\left[\pi(1-x)\right]}=\lim_{k\to 0}\frac{k(1+1-k)}{\sin\pi k}=\lim_{k\to 0}\frac{\pi k}{\sin\pi k}\times\frac{(2-k)}{\pi}=1\times\frac{(2-0)}{\pi}=\frac{2}{\pi}$$

23. $\lim\limits_{x\to\pi}\dfrac{1-\sin\frac{x}{2}}{(\pi-x)^2}=\lim\limits_{x\to\pi}\dfrac{1-\cos\left(\frac{\pi}{2}-\frac{x}{2}\right)}{(\pi-x)^2}=\lim\limits_{x\to\pi}\dfrac{1-\cos\left(\frac{\pi-x}{2}\right)}{(\pi-x)^2}$

គេតាង $k=\pi-x$ កាលណា $x\to\pi$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\pi}\frac{1-\cos\left(\frac{\pi-x}{2}\right)}{(\pi-x)^2}=\lim_{k\to 0}\frac{1-\cos\frac{k}{2}}{k^2}=\lim_{k\to 0}\frac{1-\cos\frac{k}{2}}{\left(\frac{k}{2}\right)^2}\times\frac{1}{4}=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$$

24. $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{(x-\pi)^2}=\lim\limits_{x\to\pi}\dfrac{1-\cos(\pi-x)}{(\pi-x)^2}$ គេតាង $k=\pi-x$ កាលណា $x\to\pi$ នោះ $k\to 0$

យើងបាន $\lim\limits_{x\to\pi}\dfrac{1-\cos(\pi-x)}{(\pi-x)^2}=\lim\limits_{k\to 0}\dfrac{1-\cos k}{k^2}=\dfrac{1}{2}$

25. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan^3 x-3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan x\left(\tan^2 x-\sqrt{3^2}\right)}{\cos\left(x+\frac{\pi}{6}\right)}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\tan x\left(\tan^2 x-\tan^2\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{6}\right)}$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\tan x\left(\tan x-\tan\frac{\pi}{3}\right)\left(\tan x+\tan\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{6}\right)}=\lim_{x\to\frac{\pi}{3}}\frac{\tan x}{\cos\left(x+\frac{\pi}{6}\right)}\cdot\frac{\sin\left(x-\frac{\pi}{3}\right)}{\cos x\cdot\cos\frac{\pi}{3}}\cdot\frac{\sin\left(x+\frac{\pi}{3}\right)}{\cos x\cdot\cos\frac{\pi}{3}}$$

គេតាង $k=x-\frac{\pi}{3}\Rightarrow x=k+\frac{\pi}{3}$ កាលណា $x\to\frac{\pi}{3}$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\frac{\pi}{3}}\frac{\tan^3 x-3\tan x}{\cos\left(x+\frac{\pi}{6}\right)}=\lim_{k\to 0}\frac{\tan\left(k+\frac{\pi}{3}\right)\cdot\sin k\cdot\sin\left(\frac{\pi}{3}+k+\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}+k+\frac{\pi}{6}\right)\cdot\cos^2\left(\frac{\pi}{3}+k\right)\cdot\cos^2\frac{\pi}{3}}$$

$$=\lim_{k\to 0}\frac{\tan\left(k+\frac{\pi}{3}\right)\cdot\sin k\cdot\sin\left(\frac{2\pi}{3}+k\right)}{\cos\left(\frac{\pi}{2}+k\right)\cdot\cos^2\left(\frac{\pi}{3}+k\right)\cdot\cos^2\frac{\pi}{3}}$$

$$=\lim_{k\to 0}\frac{\tan\left(k+\frac{\pi}{3}\right)\cdot\sin k\cdot\sin\left(\frac{2\pi}{3}+k\right)}{-\sin k\cdot\cos^2\left(\frac{\pi}{3}+k\right)\cdot\cos^2\frac{\pi}{3}}$$

$$=-\lim_{k\to 0}\frac{\tan\left(k+\frac{\pi}{3}\right)\cdot\sin\left(\frac{2\pi}{3}+k\right)}{\cos^2\left(\frac{\pi}{3}+k\right)\cdot\cos^2\frac{\pi}{3}}=-\frac{\sqrt{3}\cdot\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^2}=-24$$

26. $$\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{2\cdot\left(\frac{1}{2}-\cos x\right)}=\frac{1}{2}\cdot\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{\pi}{3}-x\right)}{\cos\frac{\pi}{3}-\cos x}$$

$$=\frac{1}{2}\lim_{x\to\frac{\pi}{3}}\frac{2\sin\left(\frac{\frac{\pi}{3}-x}{2}\right)\cdot\cos\left(\frac{\frac{\pi}{3}-x}{2}\right)}{2\sin\left(\frac{\frac{\pi}{3}-x}{2}\right)\cdot\sin\left(\frac{x+\frac{\pi}{3}}{2}\right)}=\frac{1}{2}\lim_{x\to\frac{\pi}{3}}\frac{\cos\left(\frac{\frac{\pi}{3}-x}{2}\right)}{\sin\left(\frac{x+\frac{\pi}{3}}{2}\right)}$$

$$=\frac{1}{2}\cdot\frac{1}{\sin\frac{\pi}{3}}=\frac{\sqrt{3}}{3}$$

27. $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}=\lim\limits_{x\to a}\dfrac{2\sin\left(\dfrac{x-a}{2}\right)\cdot\cos\left(\dfrac{x+a}{2}\right)}{(x-a)}$

$$=\lim_{x\to a}\frac{\sin\left(\dfrac{x-a}{2}\right)\cdot\cos\left(\dfrac{x+a}{2}\right)}{\left(\dfrac{x-a}{2}\right)}=1\times\cos a=\cos a$$

28. $\lim\limits_{x\to a}\dfrac{\tan x-\tan a}{\cos x-\cos a}=\lim\limits_{x\to a}\dfrac{\sin(x-a)}{\cos x\cdot\cos a}\cdot\dfrac{1}{\left[-2\sin\left(\dfrac{x-a}{2}\right)\cdot\sin\left(\dfrac{x+a}{2}\right)\right]}$

$$=\lim_{x\to a}\frac{2\sin\left(\dfrac{x-a}{2}\right)\cdot\cos\left(\dfrac{x-a}{2}\right)}{\cos x\cdot\cos a}\cdot\frac{1}{\left[-2\sin\left(\dfrac{x-a}{2}\right)\cdot\sin\left(\dfrac{x+a}{2}\right)\right]}$$

$$=-\lim_{x\to a}\frac{\cos\left(\dfrac{x-a}{2}\right)}{\cos x\cdot\cos a}\cdot\frac{1}{\sin\left(\dfrac{x+a}{2}\right)}=-\frac{2}{\sin 2a\cdot\cos a}$$

29. $\lim\limits_{x\to a}\dfrac{\cot x-\cot a}{\tan x-\tan a}=\lim\limits_{x\to a}\dfrac{\tan a-\tan x}{\tan x\cdot\tan a(\tan x-\tan a)}=-\dfrac{1}{\tan^2 a}$

30. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\cos^2 x-3}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\cdot\left(\sin x-\frac{1}{2}\right)}{4\cdot\left(\cos^2 x-\frac{\sqrt{3^2}}{4}\right)}=\dfrac{1}{2}\cdot\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin x-\sin\frac{\pi}{6}}{\cos^2 x-\cos^2\frac{\pi}{6}}$

$$=\frac{1}{2}\cdot\lim_{x\to\frac{\pi}{6}}\frac{2\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)\cdot\cos\left(\frac{x+\frac{\pi}{6}}{2}\right)}{-2\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)\cdot\sin\left(\frac{x+\frac{\pi}{6}}{2}\right)\left(\cos x+\cos\frac{\pi}{6}\right)}$$

$$=-\frac{1}{2}\lim_{x\to\frac{\pi}{6}}\cot\left(\frac{x+\frac{\pi}{6}}{2}\right)\cdot\frac{1}{\cos x+\cos\frac{\pi}{6}}=-\frac{1}{2}\cdot\sqrt{3}\cdot\frac{1}{\sqrt{3}}=-\frac{1}{2}$$

31. $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)\cdot\tan x$ គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\cos x=\cos\left(\frac{\pi}{2}-k\right)=\sin k$ និង $\tan x=\tan\left(\frac{\pi}{2}-k\right)=\frac{1}{\tan k}$ យើងបាន

$$\lim_{x\to\frac{\pi}{2}}(1+\cos x)\cdot\tan x=\lim_{k\to 0}\frac{1+\sin k}{\tan k}=\lim_{k\to 0}\left(\frac{1}{k}+\frac{\sin k}{k}\right)\cdot\frac{k}{\tan k}=1\cdot\left(1+\frac{1}{0}\right)=\infty$$

32. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin^2 2x+\cos 2x+1}{\cos 2x+\sin x}=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\cos^2 2x+\cos 2x+1}{1-2\sin^2 x+\sin x}=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos^2 2x-\cos 2x-2}{2\sin^2 x-\sin x-1}$

$$=\lim_{x\to\frac{\pi}{2}}\frac{(1+\cos 2x)(\cos 2x-2)}{(2\sin x+1)(\sin x-1)}$$

គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

គេបាន $\cos 2x=\cos(\pi-2k)=-\cos 2k$ និង $\sin x=\sin\left(\frac{\pi}{2}-k\right)=\cos k$ យើងបាន:

$$\lim_{x\to\frac{\pi}{2}}\frac{(1+\cos 2x)(\cos 2x-2)}{(2\sin x+1)(\sin x-1)}=\lim_{k\to 0}\frac{(1-\cos 2k)(-\cos 2k-2)}{(1+2\cos x)(\cos x-1)}$$

$$=\lim_{k\to 0}\left[\frac{(1-\cos 2k)}{(2k)^2}\cdot\frac{k^2}{(1-\cos x)}\cdot\frac{4\cdot(2+\cos 2k)}{(1+2\cos k)}\right]$$

$$=\lim_{k\to 0}\frac{4\cdot(2+\cos 2k)}{1+2\cos k}=4$$

33. $\lim_{x\to\frac{\pi}{2}}\tan x\cdot\cot\left(\frac{\pi}{4}+\frac{x}{2}\right)$ គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\tan x=\tan\left(\frac{\pi}{2}-k\right)=\frac{1}{\tan k}$ និង $\cot\left(\frac{\pi}{4}+\frac{x}{2}\right)=\cot\left(\frac{\pi}{4}+\frac{\pi}{4}-\frac{k}{2}\right)=\tan\frac{k}{2}$

យើងបាន: $\lim_{x\to\frac{\pi}{2}}\tan x\cdot\cot\left(\frac{\pi}{4}+\frac{x}{2}\right)=\lim_{k\to 0}\frac{\tan\frac{k}{2}}{\tan k}=\lim_{k\to 0}\frac{\tan\frac{k}{2}}{\left(\frac{k}{2}\right)}\cdot\frac{k}{\tan k}\cdot\frac{1}{2}=1\times 1\times\frac{1}{2}=\frac{1}{2}$

34. $$\lim_{x\to a}\sin\left(\frac{x-a}{2}\right)\cdot\tan\left(\frac{\pi x}{2a}\right)=\lim_{x\to a}\sin\left(\frac{x-a}{2}\right)\cdot\cot\left(\frac{\pi}{2}-\frac{\pi x}{2a}\right)$$

$$=\lim_{x\to a}\sin\left(\frac{x-a}{2}\right)\cdot\cot\left[\frac{\pi(a-x)}{2a}\right]$$

គេតាង $k=x-a$ កាលណា $x\to a$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to a}\sin\left(\frac{x-a}{2}\right)\cdot\cot\left[\frac{\pi(a-x)}{2a}\right]=\lim_{k\to 0}\sin\left(\frac{k}{2}\right)\cdot\frac{1}{\tan\left(-\frac{\pi k}{2a}\right)}$$

$$=-\lim_{k\to 0}\frac{\sin\frac{k}{2}}{\frac{k}{2}}\cdot\frac{\frac{\pi k}{2a}}{\tan\left(\frac{\pi k}{2a}\right)}\cdot\frac{a}{\pi}=-\frac{a}{\pi}$$

35. $\lim\limits_{x\to+\infty}\left(\sin\sqrt{1+x}-\sin\sqrt{x}\right)$

យើងមាន $\sin\sqrt{1+x}\le 1$ និង $\sin\sqrt{x}\le 1$ ចំពោះ $x\to+\infty$ គេបាន $\sin\sqrt{1+x}-\sin\sqrt{x}\le 0$ នោះគេបាន

$\lim\limits_{x\to+\infty}\left(\sin\sqrt{1+x}-\sin\sqrt{x}\right)=0$

36. $\lim\limits_{x\to 0}\dfrac{\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\cdot\sin x}{1-\cos\pi x}$

$$=\lim_{x\to 0}\frac{(x-1+1+x)\cdot\sin x}{\left[\sqrt[3]{(x-1)^2}-\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right](1-\cos\pi x)}$$

$$=\lim_{x\to 0}\frac{2x\cdot\sin x}{\left[\sqrt[3]{(x-1)^2}-\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right](1-\cos\pi x)}$$

$$=\lim_{x\to 0}\frac{(\pi x)^2}{1-\cos\pi x}\cdot\frac{\sin x}{x}\cdot\frac{2}{\pi^2\cdot\left(\sqrt[3]{(x-1)^2}-\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right)}$$

$$=2\times 1\times\frac{2}{\pi^2\cdot(1+1+1)}=\frac{4}{3\pi^2}$$

37. $\lim\limits_{x\to+\infty}\dfrac{(1+x)\cdot\sin x}{3+x^2}$

ដោយ $\lim\limits_{x\to+\infty}\sin x$ គ្មានលីមីតតែយើងដឹងថាចំពោះ $\forall x\in\mathbb{R}$, $-1\le\sin x\le 1$

គេបាន $-\dfrac{1+x}{3+x^2}\le\dfrac{(1+x)\cdot\sin x}{3+x^2}\le\dfrac{1+x}{3+x^2}$

$$\Rightarrow\lim_{x\to+\infty}\frac{-(1+x)}{3+x^2}\le\lim_{x\to+\infty}\frac{(1+x)\cdot\sin x}{3+x^2}\le\lim_{x\to+\infty}\frac{(1+x)}{3+x^2}$$

$$\Leftrightarrow 0\le\lim_{x\to+\infty}\frac{(1+x)\cdot\sin x}{3+x^2}\le 0\Rightarrow\lim_{x\to+\infty}\frac{(1+x)\cdot\sin x}{3+x^2}=0$$

38. $\lim\limits_{x\to 0}\dfrac{x^2+1-\cos x}{\tan^2 x}=\lim\limits_{x\to 0}\left[\dfrac{x^2}{\tan^2 x}\cdot\left(1+\dfrac{1-\cos x}{x^2}\right)\right]=1^2\cdot\left(1+\dfrac{1}{2}\right)=\dfrac{3}{2}$

39. $\lim\limits_{x\to 0}\sin\left(5\pi+\frac{x}{2}\right)\cdot\left(\frac{\cos x}{x}-\frac{4}{\sin x}\right)$

តាម $\sin(2\pi k+\alpha)=\sin\alpha$ យើងបាន $\sin\left(5\pi+\frac{x}{2}\right)=-\sin\frac{x}{2}$

យើងបាន: $\lim\limits_{x\to 0}\sin\left(5\pi+\frac{x}{2}\right)\cdot\left(\frac{\cos x}{x}-\frac{4}{\sin x}\right)=\lim\limits_{x\to 0}\sin\left(\frac{x}{2}\right)\cdot\left(\frac{4}{\sin x}-\frac{\cos x}{x}\right)$

$$=\lim_{x\to 0}\frac{\sin\frac{x}{2}}{2x\cdot\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}\cdot(4x-\cos x\cdot\sin x)$$

$$=\lim_{x\to 0}\frac{1}{2\cos\left(\frac{x}{2}\right)}\cdot\left(4-\frac{\sin 2x}{2x}\right)=\frac{1}{2}\cdot(4-1)=\frac{3}{2}$$

40. $\lim\limits_{x\to\frac{\pi}{2}}\left(2x\cdot\tan x-\frac{\pi}{\cos x}\right)$ គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\tan x=\tan\left(\frac{\pi}{2}-k\right)=\frac{1}{\tan k}$ និង $\cos x=\cos\left(\frac{\pi}{2}-k\right)=\sin k$ យើងបាន

$$\lim_{x\to\frac{\pi}{2}}\left(2x\cdot\tan x-\frac{\pi}{\cos x}\right)=\lim_{k\to 0}\left(\frac{\pi-2k}{\tan k}-\frac{\pi}{\sin k}\right)=\lim_{k\to 0}\left(\frac{\pi}{\tan k}-\frac{2k}{\tan k}-\frac{\pi}{\sin k}\right)$$

$$=-2\lim_{k\to 0}\frac{k}{\tan k}+\pi\lim_{k\to 0}\frac{\cos k-1}{\sin k}$$

$$=-2-\pi\lim_{k\to 0}\left(\frac{1-\cos k}{k}\times\frac{k}{\sin k}\right)=-2-\pi\cdot 0\cdot 1=-2$$

41. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2}-\cos x}=\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\cos\frac{\pi}{6}-\cos x}=\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{-2\sin\left(\frac{\frac{\pi}{6}-x}{2}\right)\cdot\sin\left(\frac{\frac{\pi}{6}+x}{2}\right)}$

គេតាង $k=\frac{\pi}{6}-x \Rightarrow x=\frac{\pi}{6}-k$ កាលណា $x\to\frac{\pi}{6}$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2}-\cos x}=\lim_{k\to 0}\frac{\sin(-k)}{-2\sin\left(\frac{k}{2}\right)\cdot\sin\left(\frac{\frac{\pi}{6}+\frac{\pi}{6}-k}{2}\right)}=\lim_{k\to 0}\frac{\sin\left(\frac{k}{2}\right)\cdot\cos\left(\frac{k}{2}\right)}{\sin\left(\frac{k}{2}\right)\cdot\sin\left(\frac{\frac{\pi}{3}-k}{2}\right)}$$

$$=\lim_{k\to 0}\frac{\cos\frac{k}{2}}{\sin\left(\frac{\frac{\pi}{3}-k}{2}\right)}=\frac{1}{\frac{1}{2}}=2$$

42. $$\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}=\lim_{x\to 0}\frac{1+\sin x-1+\sin x}{\tan x\cdot\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{2\sin x}{\tan x\cdot\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=2\lim_{x\to 0}\left[\frac{\sin x}{x}\cdot\frac{x}{\tan x}\cdot\frac{1}{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}\right]$$

$$=2\cdot 1\cdot 1\cdot\frac{1}{1+1}=1$$

43. $$\lim_{x\to a}\frac{\sin^2 x-\sin^2 a}{x^2-a^2}=\lim_{x\to a}\frac{(\sin x-\sin a)(\sin x+\sin a)}{(x-a)(x+a)}$$

$$=\lim_{x\to a}\frac{2\sin\left(\frac{x-a}{2}\right)\cdot\cos\left(\frac{x+a}{2}\right)(\sin x+\sin a)}{\left(\frac{x-a}{2}\right)\cdot 2\cdot(x+a)}$$

$$=\lim_{x\to a}\frac{\cos\left(\frac{x+a}{2}\right)\cdot(\sin x+\sin a)}{x+a}=\frac{\sin 2a}{2a}$$

44. $\lim\limits_{x\to 0}\dfrac{\sin(x+a)-\sin(a-x)}{x}=\lim\limits_{x\to 0}\dfrac{2\sin\left(\dfrac{x+a-a+x}{2}\right)\cdot\cos\left(\dfrac{x+a+a-x}{2}\right)}{x}$

$$=\lim_{x\to 0}\frac{2\sin x\cdot\cos a}{x}=2\cos a$$

45. $\lim\limits_{x\to 0}\dfrac{1-\cos x\cdot\sqrt{\cos 2x}}{\tan x}=\lim\limits_{x\to 0}\dfrac{1-\cos^2 x\cdot\cos 2x}{\tan x\cdot\left(1+\cos x\cdot\sqrt{\cos 2x}\right)}$

$$=\lim_{x\to 0}\frac{1-\left(1-2\sin^2 x\right)\cdot\cos x}{\tan x\cdot\left(1+\cos x\cdot\sqrt{\cos 2x}\right)}=\lim_{x\to 0}\frac{1-\cos x+2\sin^2 x\cdot\cos x}{\tan x\cdot\left(1+\cos x\cdot\sqrt{\cos 2x}\right)}$$

$$=\lim_{x\to 0}\left(\frac{1-\cos x}{x}+\frac{\sin x}{x}\cdot\sin 2x\right)\cdot\frac{x}{\tan x}\cdot\frac{1}{\left(1+\cos x\cdot\sqrt{\cos 2x}\right)}$$

$$=(0+1\cdot 0)\cdot 1\cdot\frac{1}{2}=0$$

46. $\lim\limits_{x\to+\infty}2^x\cdot\tan\left(\dfrac{\pi}{2^x}\right)$ គេតាង $k=\dfrac{1}{2^x}\Rightarrow 2^x=\dfrac{1}{k}$ កាលណា $x\to+\infty$ នោះ $k\to 0$

យើងបាន $\lim\limits_{x\to+\infty}2^x\cdot\tan\left(\dfrac{\pi}{2^x}\right)=\lim\limits_{k\to 0}\dfrac{\tan\pi k}{k}=\lim\limits_{k\to 0}\dfrac{\tan\pi k}{\pi k}\cdot\pi=\pi$ ។

47. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{1-\sqrt{2}\sin x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin\left(\dfrac{\pi}{2}-2x\right)}{\sqrt{2}\left(\dfrac{\sqrt{2}}{2}-\sin x\right)}=\dfrac{\sqrt{2}}{2}\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin\left[2\left(\dfrac{\pi}{4}-x\right)\right]}{\sin\left(\dfrac{\pi}{4}\right)-\sin x}$

$$=\frac{\sqrt{2}}{2}\lim_{x\to\frac{\pi}{4}}\frac{\sin\left[2\left(\dfrac{\pi}{4}-x\right)\right]}{2\sin\left(\dfrac{\frac{\pi}{4}-x}{2}\right)\cdot\cos\left(\dfrac{\frac{\pi}{4}+x}{2}\right)}$$

គេតាង $k=\dfrac{\pi}{4}-x\Rightarrow x=\dfrac{\pi}{4}-k$ កាលណា $x\to\dfrac{\pi}{4}$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{1-\sqrt{2}\sin x}=\frac{\sqrt{2}}{2}\lim_{k\to 0}\frac{\sin 2k}{2\sin\left(\frac{k}{2}\right)\cdot\cos\left(\frac{\frac{\pi}{4}+\frac{\pi}{4}-k}{2}\right)}$$

$$=\frac{\sqrt{2}}{2}\lim_{k\to 0}\frac{\sin 2k}{2k}\cdot\frac{\left(\frac{k}{2}\right)}{\sin\left(\frac{k}{2}\right)}\cdot\frac{2}{\cos\left(\frac{\frac{\pi}{2}-k}{2}\right)}=\frac{\sqrt{2}}{2}\cdot 1\cdot 1\cdot\frac{2}{\cos\left(\frac{\pi}{4}\right)}=2$$

48. $$\lim_{x\to\frac{\pi}{4}}\frac{\sin 7x+\cos 7x}{\sin 9x-\cos 9x}=\lim_{x\to\frac{\pi}{4}}\frac{\sin 7x+\sin\left(\frac{\pi}{2}-7x\right)}{\sin 9x-\sin\left(\frac{\pi}{2}-9x\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2\sin\left(\frac{7x+\frac{\pi}{2}-7x}{2}\right)\cdot\cos\left(\frac{7x-\frac{\pi}{2}+7x}{2}\right)}{2\sin\left(\frac{9x-\frac{\pi}{2}+9x}{2}\right)\cdot\cos\left(\frac{9x+\frac{\pi}{2}-9x}{2}\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(\frac{\pi}{4}\right)\cdot\cos\left(7x-\frac{\pi}{4}\right)}{\sin\left(9x-\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{\cos\left(7x-\frac{\pi}{4}\right)}{\sin\left(9x-\frac{\pi}{4}\right)}$$

គេតាង $k=\frac{\pi}{4}-x\Rightarrow x=\frac{\pi}{4}-k$ កាលណា $x\to\frac{\pi}{4}$ នោះ $k\to 0\to$ យើងបាន

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos\left(7x-\frac{\pi}{4}\right)}{\sin\left(9x-\frac{\pi}{4}\right)}=\lim_{k\to0}\frac{\cos\left(\frac{7\pi}{4}-7k-\frac{\pi}{4}\right)}{\sin\left(\frac{9\pi}{4}-9k-\frac{\pi}{4}\right)}=\lim_{k\to0}\frac{\cos\left(\frac{3\pi}{2}-7k\right)}{\sin\left(2\pi-9k\right)}=\lim_{k\to0}\frac{-\sin7k}{-\sin9k}$$

$$=\lim_{k\to0}\frac{\sin7k}{7k}\cdot\frac{9k}{\sin9k}\cdot\frac{7}{9}=1\cdot1\cdot\frac{7}{9}=\frac{7}{9}$$

49. $\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos2x}=\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{(\cos x-\sin x)\cdot(\cos x+\sin x)}=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x+\sin x}=\frac{1}{\sqrt{2}}$

50. $\lim_{x\to2}(x-2)\cdot\tan\left(\frac{\pi}{x}\right)=\lim_{x\to2}\frac{x-2}{\tan\left(\frac{\pi}{2}-\frac{\pi}{x}\right)}=\lim_{x\to2}\frac{x-2}{\tan\left[\frac{\pi}{2x}(x-2)\right]}$

គេតាង $k=x-2\Rightarrow x=2+k$ កាលណា $x\to2$ នោះ $k\to0$ យើងបាន

$$\lim_{x\to2}(x-2)\cdot\tan\frac{\pi}{x}=\lim_{k\to0}\frac{k}{\tan\left[\frac{\pi k}{2(k+2)}\right]}=\lim_{k\to0}\frac{\frac{\pi k}{2(k+2)}}{\tan\left[\frac{\pi k}{2(k+2)}\right]}\cdot\frac{2(k+2)}{\pi}=1\cdot\frac{2(0+2)}{\pi}=\frac{4}{\pi}$$

51. $\lim_{x\to\frac{\pi}{4}}(\pi-2x)\cdot\tan x=\left(\pi-2\cdot\frac{\pi}{4}\right)\cdot\tan\frac{\pi}{4}=\frac{\pi}{2}$

52. $\lim_{x\to\frac{\pi}{4}}(1-\sin2x)\cdot\frac{\tan2x}{\tan4x}$ គេតាង $k=\frac{\pi}{4}-x\Rightarrow x=\frac{\pi}{4}-k$ កាលណា $x\to\frac{\pi}{4}$ នោះ $k\to0$

ដោយ $\sin2x=\sin\left(\frac{\pi}{2}-2k\right)=\cos2k$ និង

$\tan2x=\tan\left(\frac{\pi}{2}-2x\right)=\frac{1}{\tan2k}$, $\tan4x=\tan(\pi-4k)=-\tan4k$

យើងបាន $\lim_{x\to\frac{\pi}{4}}(1-\sin2x)\cdot\frac{\tan2x}{\tan4x}=\lim_{k\to0}(1-\cos2k)\left(\frac{1}{-\tan2k\cdot\tan4k}\right)$

$$= -\lim_{k\to 0}\left(\frac{1-\cos 2k}{4k^2}\right)\cdot\left(\frac{4k}{\tan 4k}\cdot\frac{2k}{\tan 2k}\cdot\frac{1}{2}\right) = -\frac{1}{2}\cdot 1\cdot 1\cdot\frac{1}{2} = -\frac{1}{4}$$

53. $\lim_{x\to\frac{\pi}{4}}(1-\sin 2x)\cdot\tan 2x$ គេតាង $k = \frac{\pi}{4} - x \Rightarrow x = \frac{\pi}{4} - k$ កាលណា $x \to \frac{\pi}{4}$ នោះ $k \to 0$

ដោយ $\sin 2x = \sin\left(\frac{\pi}{2} - 2k\right) = \cos 2k$ និង $\tan 2x = \tan\left(\frac{\pi}{2} - 2x\right) = \frac{1}{\tan 2k}$ យើងបាន

$$\lim_{x\to\frac{\pi}{4}}(1-\sin 2x)\cdot\tan 2x = \lim_{k\to 0}\frac{1-\cos 2k}{\tan 2k} = \lim_{k\to 0}\left(\frac{1-\cos 2k}{4k^2}\right)\cdot\frac{2k}{\tan 2k}\cdot 2k = \frac{1}{2}\cdot 1\cdot 2\cdot 0 = 0$$

54. $\lim_{x\to\pi}\tan x\cdot\tan\frac{x}{2}$ គេតាង $k = \pi - x \Rightarrow x = \pi - k$ កាលណា $x \to \pi$ នោះ $k \to 0$

តែ $\tan x = \tan(\pi - k) = -\tan k$ និង $\tan\frac{x}{2} = \tan\left(\frac{\pi}{2} - \frac{k}{2}\right) = \frac{1}{\tan\left(\frac{k}{2}\right)}$ យើងបាន

$$\lim_{x\to\pi}\tan x\cdot\tan 2x = \lim_{k\to 0}\frac{-\tan k}{\tan\left(\frac{k}{2}\right)} = -\lim_{k\to 0}\frac{\tan k}{k}\cdot\frac{\frac{k}{2}}{\tan\frac{k}{2}}\cdot 2 = -1\cdot 1\cdot 2 = -2$$

55. $\lim_{x\to\frac{\pi}{4}}\left(4x\cdot\tan 2x - \frac{\pi}{\cos 2x}\right)$ គេតាង $k = \frac{\pi}{4} - x \Rightarrow x = \frac{\pi}{4} - k$ កាលណា $x \to \frac{\pi}{4}$ នោះ $k \to 0$

តែ $\tan 2x = \tan\left(\frac{\pi}{2} - 2x\right) = \frac{1}{\tan 2k}$ និង $\cos 2x = \cos\left(\frac{\pi}{2} - 2k\right) = \sin 2k$ យើងបាន

$$\lim_{x\to\frac{\pi}{4}}\left(4x\cdot\tan 2x - \frac{\pi}{\cos 2x}\right) = \lim_{k\to 0}\left[\frac{4\left(\frac{\pi}{4} - k\right)}{\tan 2k} - \frac{\pi}{\sin 2k}\right] = \lim_{k\to 0}\left[\frac{(\pi - 4k)}{\tan 2k} - \frac{\pi}{\sin 2k}\right]$$

$$= -2\lim_{k\to 0}\frac{2k}{\tan 2k} + \lim_{k\to 0}\left(\frac{\pi\cdot\cos 2k}{\sin 2k} - \frac{\pi}{\sin 2k}\right) = -2\cdot 1 - \pi\lim_{k\to 0}\left[\left(\frac{1-\cos 2k}{2k}\right)\cdot\frac{2k}{\sin 2k}\right] = -2 - \pi\cdot 0\cdot 1 = -2$$

56. $\lim_{x\to 0}\frac{\tan x}{\sqrt[3]{(1-\cos x)^2}}=\lim_{x\to 0}\frac{\tan x}{\sqrt[3]{4\left(\sin\frac{x}{2}\right)^4}}=\lim_{x\to 0}\frac{2\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)\cdot\cos x\cdot\sqrt[3]{4\sin\frac{x}{2}}}$

$$=\lim_{x\to 0}\frac{2\cos\left(\frac{x}{2}\right)}{\cos x\cdot\sqrt[3]{\sin\frac{x}{2}}}=\frac{2\cdot 1}{1\cdot\sqrt[3]{4\cdot 0}}=\infty$$

57. $\lim_{x\to\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}+x\right)}{\tan\left(x-\frac{\pi}{2}\right)}=\lim_{x\to\frac{\pi}{2}}\left[-\tan x\cdot\cos x\right]=-\lim_{x\to\frac{\pi}{2}}\sin x=-1$

58. $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin\left(x-\frac{\pi}{4}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{\tan\frac{\pi}{4}-\tan x}{\sin\left(x-\frac{\pi}{4}\right)}=\lim_{x\to\frac{\pi}{4}}\frac{\sin\left(\frac{\pi}{4}-x\right)}{\sin\left(x-\frac{\pi}{4}\right)\cdot\cos x\cdot\cos\frac{\pi}{4}}$

$$=-\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x\cdot\cos\frac{\pi}{4}}=-2$$

59. $\lim_{x\to 1}\frac{\sin^2\pi x}{x-1}=\lim_{x\to 1}\frac{\left[\sin\pi(1-x)\right]^2}{x-1}$ គេតាង $k=1-x\Rightarrow x=1-k$

កាលណា $x\to 1$ នោះ $k\to 0$

យើងបាន $\lim_{x\to 1}\frac{\sin^2\pi x}{x-1}=\lim_{k\to 0}\frac{\sin^2\pi k}{-k}=-\lim_{k\to 0}\frac{\sin\pi k}{\pi k}\cdot\sin\pi k=-1\cdot 0=0$

60. $\lim_{x\to 0}\frac{\sqrt{2x+1}-\sqrt{x+1}}{\sin x}=\lim_{x\to 0}\frac{(2x+1-x-1)}{\sin x\cdot\left(\sqrt{2x+1}+\sqrt{x+1}\right)}$

$$=\lim_{x\to 0}\frac{x}{\sin x}\cdot\frac{1}{\left(\sqrt{2x+1}+\sqrt{x+1}\right)}=1\cdot\frac{1}{1+1}=\frac{1}{2}$$

61. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2\cdot\left(1+\sqrt{\cos x}\right)}=\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}\cdot\dfrac{1}{1+\sqrt{\cos x}}=\dfrac{1}{2}\cdot\dfrac{1}{2}=\dfrac{1}{4}$

62. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{\cos\frac{\pi}{4}-\cos x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin 2\left(\frac{\pi}{4}-x\right)}{-2\sin\left(\dfrac{\frac{\pi}{4}-x}{2}\right)\cdot\sin\left(\dfrac{\frac{\pi}{4}+x}{2}\right)}$

គេតាង $k=\dfrac{\pi}{4}-x\Rightarrow x=\dfrac{\pi}{4}-k$ កាលណា $x\to\dfrac{\pi}{4}$ នោះ $k\to 0$ យើងបាន

$$\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos 2x}{\cos\frac{\pi}{4}-\cos x}=-\dfrac{1}{2}\lim\limits_{k\to 0}\dfrac{\sin 2k}{\sin\frac{k}{2}\cdot\sin\left(\dfrac{\frac{\pi}{4}+\frac{\pi}{4}-k}{2}\right)}=-\cdot\lim\limits_{k\to 0}\dfrac{\sin 2k}{2k}\cdot\dfrac{\frac{k}{2}}{\sin\frac{k}{2}}\cdot\dfrac{2}{\sin\left(\dfrac{\frac{\pi}{2}-k}{2}\right)}$$

$$=-1\cdot 1\cdot\dfrac{2}{\sin\frac{\pi}{4}}=-2\sqrt{2}$$

63. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin 3x}{1-2\cos x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin(\pi-3x)}{2\left(\frac{1}{2}-\cos x\right)}=\dfrac{1}{2}\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left[3\left(\frac{\pi}{3}-x\right)\right]}{\cos\frac{\pi}{3}-\cos x}$

$$=\dfrac{1}{2}\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left[3\left(\frac{\pi}{3}-x\right)\right]}{-2\sin\left(\dfrac{\frac{\pi}{3}-x}{2}\right)\cdot\sin\left(\dfrac{\frac{\pi}{3}+x}{2}\right)}=-\dfrac{1}{4}\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left[3\left(\frac{\pi}{3}-x\right)\right]}{\sin\left(\dfrac{\frac{\pi}{3}-x}{2}\right)\cdot\sin\left(\dfrac{\frac{\pi}{3}+x}{2}\right)}$$

គេតាង $k=\dfrac{\pi}{3}-x\Rightarrow x=\dfrac{\pi}{3}-k$ កាលណា $x\to\dfrac{\pi}{3}$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin 3x}{1-2\cos x}=-\frac{1}{4}\lim_{k\to 0}\frac{\sin 3k}{\sin\frac{k}{2}\cdot\sin\left(\frac{\frac{\pi}{3}+\frac{\pi}{3}-k}{2}\right)}=-\frac{3}{2}\lim_{k\to 0}\frac{\sin 3k}{3k}\cdot\frac{\frac{k}{2}}{\sin\frac{k}{2}}\cdot\frac{1}{\sin\left(\frac{\pi}{3}-\frac{k}{2}\right)}$$

$$=-\frac{3}{2}\cdot 1\cdot 1\cdot\frac{1}{\sin\frac{\pi}{3}}=-\sqrt{3}$$

64. $$\lim_{x\to\frac{\pi}{3}}\frac{\sqrt{3}\cos x-\sin x}{x-\frac{\pi}{3}}=\lim_{x\to\frac{\pi}{3}}\frac{2\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)}{x-\frac{\pi}{3}}=\lim_{x\to\frac{\pi}{3}}\frac{-2\sin\left(x-\frac{\pi}{3}\right)}{x-\frac{\pi}{3}}=-2$$

(គេយក $k=\frac{\pi}{3}-x$ កាលណា $x\to\frac{\pi}{3}$ នោះ $k\to 0$)

65. $$\lim_{x\to\frac{\pi}{2}}\frac{\cot x}{x-\frac{\pi}{2}}=\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{2}-x\right)}{-\left(\frac{\pi}{2}-x\right)}=-1$$ (យក $k=\frac{\pi}{2}-x$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$)

66. $\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\cos 3x}$ គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

តែ $\sin 2x=\sin(\pi-2k)=\sin 2k$ និង $\cos 3x=\cos\left(\frac{3\pi}{2}-3k\right)=-\sin 3k$ យើងបាន

$$\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\cos 3x}=\lim_{k\to 0}\frac{\sin 2k}{-\sin 3k}=-\lim_{k\to 0}\frac{\sin 2k}{2k}\cdot\frac{3k}{\sin 3k}\cdot\frac{2}{3}=-1\cdot 1\cdot\frac{2}{3}=-\frac{2}{3}$$

67. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{\tan x-1}=\sqrt{2}\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\sin\frac{\pi}{4}}{\tan x-\tan\frac{\pi}{4}}$

$$=\sqrt{2}\lim_{x\to\frac{\pi}{4}}\frac{2\sin\left(\frac{x-\frac{\pi}{4}}{2}\right)\cdot\cos\left(\frac{x+\frac{\pi}{4}}{2}\right)}{\sin\left(x-\frac{\pi}{4}\right)}\cdot\cos x\cdot\cos\frac{\pi}{4}$$

តាង $k=x-\frac{\pi}{4}\Rightarrow x=\frac{\pi}{4}+k$ កាលណា $x\to\frac{\pi}{4}$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{\tan x-1}=2\lim_{k\to 0}\frac{\sin\frac{k}{2}\cdot\cos\left(\frac{\frac{\pi}{4}+\frac{\pi}{4}+k}{2}\right)}{\sin k}\cdot\cos\left(k+\frac{\pi}{4}\right)$$

$$=\lim_{k\to 0}\frac{k}{\sin k}\cdot\frac{\sin\frac{k}{2}}{\frac{k}{2}}\cdot\cos\left(\frac{\pi}{4}+\frac{k}{2}\right)\cdot\cos\left(k+\frac{\pi}{4}\right)=1\cdot 1\cdot\frac{1}{2}=\frac{1}{2}$$

68. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan x}{1-\cot x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan\frac{\pi}{4}-\tan x}{\cot\frac{\pi}{4}-\cot x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin\left(\frac{\pi}{4}-x\right)\cdot\sin x\cdot\sin\frac{\pi}{4}}{\sin\left(x-\frac{\pi}{4}\right)\cdot\cos x\cdot\cos\frac{\pi}{4}}=-\lim\limits_{x\to\frac{\pi}{4}}\tan x=-1$

69. $\lim\limits_{x\to\infty}\dfrac{(x+1)\sin x}{2+x^2}$ ដោយ $\lim\limits_{x\to\infty}\sin x$ គ្មានលីមីត

ចំពោះគ្រប់ $x\in\mathbb{R}$ គេបាន $-1\le\sin x\le 1\Rightarrow-\dfrac{(x+1)}{2+x^2}\le\dfrac{(x+1)\cdot\sin x}{2+x^2}\le\dfrac{x+1}{2+x^2}$

$$\Rightarrow \lim_{x\to\infty}\frac{-(x+1)}{2+x^2}\le \lim_{x\to\infty}\frac{(x+1)\cdot\sin x}{2+x^2}\le \lim_{x\to\infty}\frac{x+1}{2+x^2}$$

$$\Leftrightarrow 0\le \lim_{x\to\infty}\frac{(x+1)\cdot\sin x}{2+x^2}\le 0 \Rightarrow \lim_{x\to\infty}\frac{(x+1)\cdot\sin x}{2+x^2}=0$$

70. $\lim_{x\to0} x\cdot\left(\sin\frac{1}{x}-\frac{1}{\sin x}\right)=\lim_{x\to0} x\cdot\sin\frac{1}{x}-\lim_{x\to0}\frac{x}{\sin x}=\lim_{x\to0} x\cdot\sin\frac{1}{x}-1$

ដោយ $-1\le\sin\frac{1}{x}\le1\Rightarrow -x\le\sin\frac{1}{x}\le x\Rightarrow \lim_{x\to0}(-x)\le\lim_{x\to0}x\cdot\sin\frac{1}{x}\le\lim_{x\to0}x$

$$\Leftrightarrow 0\le\lim_{x\to0}x\cdot\sin\frac{1}{x}\le0\Rightarrow\lim_{x\to0}x\cdot\sin\frac{1}{x}=0$$

យើងបាន $\lim_{x\to0} x\cdot\left(\sin\frac{1}{x}-\frac{1}{\sin x}\right)=-1$

71. $\lim_{x\to0}\frac{\cos x-\sqrt{\cos 2x}}{\sin^2 x}=\lim_{x\to0}\frac{\cos^2 x-\cos 2x}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}=\lim_{x\to0}\frac{\cos^2 x-2\cos^2 x+1}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$

$$=\lim_{x\to0}\frac{1-\cos^2 x}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}=\lim_{x\to0}\frac{1}{\cos x+\sqrt{\cos 2x}}=\frac{1}{2}$$

72. $\lim_{x\to\frac{\pi}{3}}\frac{\sqrt{2\cos x}-1}{2\cos 2x+1}=\lim_{x\to\frac{\pi}{3}}\frac{2\cos x-1}{(2\cos 2x+1)\left(\sqrt{2\cos x}+1\right)}$

គេតាង $k=\frac{\pi}{3}-x\Rightarrow x=\frac{\pi}{3}-k$ កាលណា $x\to\frac{\pi}{3}$ នោះ $k\to0$

តែ $2\cos x-1=2\cos\left(\frac{\pi}{3}-k\right)-1=2\cos k\cdot\cos\frac{\pi}{3}+2\sin x\sin\frac{\pi}{3}-1=\cos k-1+\sqrt{3}\sin k$

$$2\cos 2x+1=2\cos\left(\frac{2\pi}{3}-2k\right)+1=2\cos 2k\cdot\cos\frac{2\pi}{3}+2\sin 2k\cdot\sin 2k+1$$

$$=1-\cos 2k+\sqrt{3}\sin 2k$$

យើងបាន

$$\lim_{x\to\frac{\pi}{3}}\frac{2\cos x-1}{(2\cos 2x+1)\left(\sqrt{2\cos x}+1\right)}=\lim_{k\to 0}\frac{\cos k-1+\sqrt{3}\sin k}{\left(1-\cos 2k+\sqrt{3}\sin 2k\right)\left(\sqrt{\cos k+\sqrt{3}\sin x}+1\right)}$$

$$=\lim_{k\to 0}\frac{\frac{\sqrt{3}\sin k}{k}-\frac{1-\cos k}{k}}{\left(\frac{1-\cos 2k}{2k}\cdot 2+2\sqrt{3}\cdot\frac{\sin 2k}{2k}\right)\left(\sqrt{\cos k+\sqrt{3}\sin k}+1\right)}$$

$$=\frac{\sqrt{3}\cdot 1-0}{\left(0\cdot 2+2\sqrt{3}\cdot 1\right)(1+1)}=\frac{1}{4}$$

73. $\lim_{x\to\frac{\pi}{4}}\frac{1-\sqrt{2}\cos x}{1-\sqrt{2}\sin x}$ គេតាង $k=\frac{\pi}{4}-x\Rightarrow x=\frac{\pi}{4}-k$ កាលណា $x\to\frac{\pi}{4}$ នោះ $k\to 0$

តែ $1-\sqrt{2}\cos x=1-\sqrt{2}\cos\left(\frac{\pi}{4}-k\right)=1-\sqrt{2}\cos k\cdot\cos\frac{\pi}{4}-\sqrt{2}\sin k\cdot\sin\frac{\pi}{4}=1-\cos k-\sin k$

$1-\sqrt{2}\sin x=1-\sqrt{2}\sin\left(\frac{\pi}{4}-k\right)=1-\sqrt{2}\sin\frac{\pi}{4}\cdot\cos k+\sqrt{2}\cos\frac{\pi}{4}\cdot\sin k=1-\cos k+\sin k$

យើងបាន

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\sqrt{2}\cos x}{1-\sqrt{2}\sin x}=\lim_{k\to 0}\frac{1-\cos k-\sin k}{1-\cos k+\sin k}$$

$$=\lim_{k\to 0}\left(\frac{1-\cos k}{k}-\frac{\sin k}{k}\right)\left(\frac{1}{\frac{1-\cos k}{k}+\frac{\sin k}{k}}\right)=(0-1)\left(\frac{1}{0+1}\right)=-1$$

74. $\lim_{x\to\frac{\pi}{2}}\frac{\sin\left(x+\frac{\pi}{2}\right)}{\tan\left(x-\frac{\pi}{2}\right)}$ គេតាង $k=x-\frac{\pi}{2}\Rightarrow x=\frac{\pi}{2}+k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

យើងបាន $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin\left(x+\frac{\pi}{2}\right)}{\tan\left(x-\frac{\pi}{2}\right)}=\lim\limits_{k\to 0}\dfrac{\sin(\pi+k)}{\tan k}=\lim\limits_{k\to 0}\dfrac{-\sin k}{\tan k}=-\lim\limits_{k\to 0}\dfrac{\sin k}{k}\cdot\dfrac{k}{\tan k}=-1\times 1=-1$

75. $$\lim_{x\to 0}\frac{\cos 3x-\cos x}{\sin 5x+\sin 3x}=\lim_{x\to 0}\frac{1-\cos x-1+\cos 3x}{\sin 5x+\sin 3x}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x}-\frac{1-\cos 3x}{x}}{\frac{\sin 5x}{x}+\frac{\sin 3x}{x}}$$

$$=\lim_{x\to 0}\frac{\frac{1-\cos x}{x}-3\cdot\frac{1-\cos 3x}{3x}}{5\cdot\frac{\sin 5x}{5x}+3\cdot\frac{\sin 3x}{3x}}=\frac{0-3\cdot 0}{5\cdot 1+3\cdot 1}=0$$

76. $$\lim_{x\to-\frac{\pi}{4}}\left(3+\frac{\cos 2x}{\sin x+\cos x}\right)=\lim_{x\to-\frac{\pi}{4}}\left(3+\frac{\cos 2x}{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)}\right)$$

គេតាង $k=x+\frac{\pi}{4}\Rightarrow x=k-\frac{\pi}{4}$ កាលណា $x\to-\frac{\pi}{4}$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to-\frac{\pi}{4}}\left(3+\frac{\cos 2x}{\sin x+\cos x}\right)=\lim_{k\to 0}\left[3+\frac{\cos\left(2x-\frac{\pi}{2}\right)}{\sqrt{2}\sin k}\right]=\lim_{k\to 0}\left(3+\frac{\sin 2k}{\sqrt{2}\sin k}\right)$$

$$=\lim_{k\to 0}\left(3+\frac{2\sin k\cdot\cos k}{\sqrt{2}\sin k}\right)=\lim_{k\to 0}\left(3+\sqrt{2}\cos k\right)=3+\sqrt{2}$$

77. $$\lim_{x\to 0}\frac{2\sin 3x}{2x-3\sin 2x}=\lim_{x\to 0}\frac{\frac{2\sin 3x}{2x}}{1-3\cdot\frac{\sin 2x}{2x}}=\lim_{x\to 0}\left(3\cdot\frac{\sin 3x}{3x}\right)\left(\frac{1}{1-3\cdot\frac{\sin 2x}{2x}}\right)$$

$$=3\cdot 1\cdot\frac{1}{1-3\cdot 1}=-\frac{3}{2}$$

78. $\lim\limits_{x\to 0}\left(\frac{1}{2-2\cos x}-\frac{1}{\sin^2 x}\right)$

$$=\lim_{x\to 0}\left[\frac{1}{2(1-\cos x)}-\frac{1}{1-\cos^2 x}\right]=\lim_{x\to 0}\frac{1+\cos x-2}{2\sin^2 x}$$

$$=\lim_{x\to 0}\frac{-(1-\cos x)}{2\sin^2 x}=-\frac{1}{2}\lim_{x\to 0}\left[\left(\frac{1-\cos^2 x}{x^2}\right)\cdot\frac{x^2}{\sin^2 x}\right]=-\frac{1}{2}\cdot\frac{1}{2}\cdot 1^2=-\frac{1}{4}$$

79. $\lim\limits_{x\to 0}\left(\frac{2}{\sin 2x}-\frac{1}{\sin x}\right)=\lim\limits_{x\to 0}\frac{1-\cos x}{\sin x\cdot\cos x}=\lim\limits_{x\to 0}\left[\left(\frac{1-\cos x}{x}\right)\cdot\frac{x}{\sin x}\cdot\frac{1}{\cos x}\right]=0\cdot 1\cdot 1=0$

80. $\lim\limits_{x\to -\frac{\pi}{2}}(1+\sin x)\cdot\tan^2 x$ គេតាង $k=x+\frac{\pi}{2}\Rightarrow x=-\frac{\pi}{2}+k$

កាលណា $x\to -\frac{\pi}{2}$ នោះ $k\to 0$

តែ $\sin x=\sin\left(k-\frac{\pi}{2}\right)=-\cos k$ និង $\tan x=\tan\left(k-\frac{\pi}{2}\right)=-\frac{1}{\tan k}$

យើងបាន $\lim\limits_{x\to -\frac{\pi}{2}}(1+\sin x)\cdot\tan^2 x=\lim\limits_{k\to 0}(1-\cos k)\cdot\frac{1}{\tan^2 k}=\lim\limits_{k\to 0}\left(\frac{1-\cos k}{k^2}\right)\cdot\frac{k^2}{\tan^2 k}=\frac{1}{2}\cdot 1^2=\frac{1}{2}$

81. $\lim\limits_{x\to \pi}(1+\cos\pi)\cdot\tan\frac{x}{2}$ យើងតាង $k=\pi-x\Rightarrow x=\pi-k$ កាលនា $x\to\pi$ នោះ $k\to 0$

ដោយ $\cos x=\cos(\pi-k)=-\cos k$ និង $\tan\frac{x}{2}=\tan\left(\frac{\pi}{2}-\frac{k}{2}\right)=\frac{1}{\tan\frac{k}{2}}$

យើងបាន $\lim\limits_{x\to \pi}(1+\cos\pi)\cdot\tan\frac{x}{2}=\lim\limits_{k\to 0}(1-\cos k)\cdot\frac{1}{\tan\frac{k}{2}}=\lim\limits_{k\to 0}\left(\frac{1-\cos k}{k}\right)\cdot\frac{\frac{k}{2}}{\tan\frac{k}{2}}\cdot 2=0\cdot 1\cdot 2=0$

82. $\lim\limits_{x\to \frac{\pi}{2}}\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)=\lim\limits_{x\to \frac{\pi}{2}}\sqrt{2}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)=\sqrt{2}\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right)=0$

83. $\lim_{x\to\frac{\pi}{2}}(1-\sin x)\cdot\tan^2 x$ គេតាង $k=\frac{\pi}{2}-x \Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

តែ $\sin x=\sin\left(\frac{\pi}{2}-k\right)=\cos k$ និង $\tan x=\tan\left(\frac{\pi}{2}-k\right)=\frac{1}{\tan k}$

យើងបាន $\lim_{x\to\frac{\pi}{2}}(1-\sin x)\cdot\tan^2 x=\lim_{k\to 0}(1-\cos k)\cdot\frac{1}{\tan^2 k}=\lim_{k\to 0}\left(\frac{1-\cos k}{k^2}\right)\cdot\frac{k^2}{\tan^2 k}=\frac{1}{2}\cdot 1^2=\frac{1}{2}$

84. $\lim_{x\to 0}\frac{\sin x+1-\cos x}{x}=\lim_{x\to 0}\left(\frac{\sin x}{x}+\frac{1-\cos x}{x}\right)=1+0=1$

85. $\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}=\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{2\sin^2\frac{x}{2}}}=\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{2}\left|\sin\frac{x}{2}\right|}$

បើ $x\to 0^+$ ឬ $x>0$ នោះគេបាន $\left|\sin\frac{x}{2}\right|=\sin\frac{x}{2}$ យើងបាន

$$\lim_{x\to 0^+}\frac{2x-\sin x}{\sqrt{2}\sin\frac{x}{2}}=\frac{\sqrt{2}}{2}\cdot\lim_{x\to 0^+}\left(2-\frac{\sin x}{x}\right)\cdot\frac{\frac{x}{2}}{\sin\frac{x}{2}}\cdot 2=\frac{\sqrt{2}}{2}\cdot(2-1)\cdot 1\cdot 2=\sqrt{2}$$

បើ $x\to 0^-$ ឬ $x<0$ នោះគេបាន $\left|\sin\frac{x}{2}\right|=-\sin\frac{x}{2}$ យើងបាន

$$\lim_{x\to 0^-}\frac{2x-\sin x}{-\sqrt{2}\sin\frac{x}{2}}=-\frac{\sqrt{2}}{2}\cdot\lim_{x\to 0^-}\left(2-\frac{\sin x}{x}\right)\cdot\frac{\frac{x}{2}}{\sin\frac{x}{2}}\cdot 2=-\frac{\sqrt{2}}{2}\cdot(2-1)\cdot 1\cdot 2=-\sqrt{2}$$

ដូចនេះ $\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}=\left[\begin{array}{ll}\sqrt{2} & ,\ x>0\\ -\sqrt{2} & ,\ x<0\end{array}\right.$

86. $\lim_{x\to\pi}\frac{\sin x}{1+\cos x}=\lim_{x\to\pi}\frac{\sin(\pi-x)}{1-\cos(\pi-x)}$ គេយក $k=\pi-x$ កាលណា $x\to\pi$ នោះ $k\to 0$

យើងបាន $\lim_{x\to\pi}\frac{\sin x}{1+\cos x}=\lim_{k\to 0}\frac{\sin k}{1-\cos k}=\lim_{k\to 0}\left[\frac{\sin k}{k}\cdot\left(\frac{k}{1-\cos k}\right)\right]=1\cdot 0=0$

87. $\lim_{x\to 0}\left(\frac{2}{\sin^2 x}-\frac{1}{1-\cos x}\right)=\lim_{x\to 0}\left(\frac{2}{1-\cos^2 x}-\frac{1}{1-\cos x}\right)=\lim_{x\to 0}\frac{2-1-\cos x}{1-\cos^2 x}$

$$=\lim_{x\to 0}\frac{1-\cos x}{1-\cos^2 x}=\lim_{x\to 0}\frac{1}{1+\cos x}=\frac{1}{2}$$

88. $\lim_{x\to\frac{\pi}{3}}\left(\frac{x}{2}-\frac{\pi}{3}\cos x\right)\cdot\frac{1}{x-\frac{\pi}{3}}$ គេតាង $k=x-\frac{\pi}{3}\Rightarrow x=k+\frac{\pi}{3}$ កាលណា $x\to\frac{\pi}{3}$ នោះ $k\to 0$

ដោយ $\cos x=\cos\left(\frac{\pi}{3}+k\right)=\frac{1}{2}\cos k-\frac{\sqrt{3}}{2}\sin k$

យើងបាន $\lim_{x\to\frac{\pi}{3}}\left(\frac{x}{2}-\frac{\pi}{3}\cos x\right)\cdot\frac{1}{x-\frac{\pi}{3}}=\lim_{k\to 0}\left[\frac{k+\frac{\pi}{3}}{2}-\frac{\pi}{3}\cdot\left(\frac{1}{2}\cos k-\frac{\sqrt{3}}{2}\sin k\right)\right]\cdot\frac{1}{k}$

$$=\lim_{k\to 0}\left(\frac{k}{2}+\frac{\pi}{6}-\frac{\pi}{6}\cos k+\frac{\sqrt{3}\pi}{6}\sin k\right)\cdot\frac{1}{k}$$

$$=\lim_{k\to 0}\left[\frac{1}{2}+\frac{\pi}{6}\cdot\left(\frac{1-\cos k}{k}\right)+\frac{\sqrt{3}\pi}{6}\cdot\frac{\sin k}{k}\right]$$

$$=\frac{1}{2}+\frac{\pi}{6}\cdot 0+\frac{\sqrt{3}\pi}{6}=\frac{1}{2}+\frac{\sqrt{3}\pi}{6}$$

89. $\lim_{x\to\infty}(3x+1)\cdot\sin\left(\frac{2\pi x}{x-1}\right)=\lim_{x\to\infty}\left[3(x-1)+4\right]\cdot\sin\left[2\pi\left(1+\frac{1}{x-1}\right)\right]$

គេតាង $k=\frac{1}{x-1}\Rightarrow x-1=\frac{1}{k}$ កាលណា $x\to\infty$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\infty}(3x+1)\sin\frac{2\pi x}{x-1}=\lim_{k\to 0}\left(\frac{3}{k}+4\right)\cdot\sin\left(2\pi+2\pi k\right)$$

$$=\lim_{k\to 0}\left[\frac{\sin 2\pi k}{2\pi k}\cdot 6\pi+4\sin 2\pi k\right]=6\cdot 1+4\cdot 0=6$$

90. $$\lim_{x\to\infty}(7x+2)\cdot\cos\frac{\pi x}{2(x+1)}=\lim_{x\to\infty}\left[7(x+1)-5\right]\cdot\cos\left[\frac{\pi}{2}\left(1-\frac{1}{x+1}\right)\right]$$

គេតាង $k=\frac{1}{x+1}\Rightarrow x+1=\frac{1}{k}$ កាលណា $x\to\infty$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to\infty}(7x+2)\cos\frac{\pi x}{2(x+1)}=\lim_{x\to\infty}\left(\frac{7}{k}-5\right)\cdot\cos\left(\frac{\pi}{2}-\frac{\pi k}{2}\right)$$

$$=\lim_{k\to 0}\left(\frac{\sin\frac{\pi k}{2}}{\frac{\pi k}{2}}\cdot\frac{7\pi}{2}-5\sin\frac{\pi k}{2}\right)=1\cdot\frac{7\pi}{2}-5\cdot 0=\frac{7\pi}{2}$$

91. $$\lim_{x\to 1}\frac{\sin\pi x^m}{\sin\pi x^n}=\lim_{x\to 1}\frac{\sin\left[\pi\left(1-x^m\right)\right]}{\sin\left[\pi\left(1-x^n\right)\right]}=\lim_{x\to 1}\left\{\frac{\sin\left[\pi\left(1-x^m\right)\right]}{\pi\left(1-x^m\right)}\cdot\frac{\pi\left(1-x^n\right)}{\sin\left[\pi\left(1-x^n\right)\right]}\cdot\frac{\pi\left(1-x^m\right)}{\pi\left(1-x^n\right)}\right\}$$

$$=\lim_{x\to 1}\frac{1-x^m}{1-x^n}=\lim_{x\to 1}\frac{1+x+x^2+\dots\dots\dots\dots+x^{m-1}}{1+x+x^2+\dots\dots\dots\dots+x^{n-1}}=\frac{\overbrace{1+1+1+\dots\dots\dots+1}^{m}}{\underbrace{1+1+1+\dots\dots\dots+1}_{n}}=\frac{m}{n}$$

92. $$\lim_{x\to\infty}\left(\log_2 x+\log_2\sin\frac{2}{x}\right)=\lim_{x\to\infty}\left[\log_2\left(x\cdot\sin\frac{2}{x}\right)\right]=\log_2\left[\lim_{x\to\infty}x\cdot\sin\frac{2}{x}\right]$$

$$=\log_2\left[\lim_{x\to\infty}\frac{\sin\frac{2}{x}}{\frac{2}{x}}\cdot 2\right]=\log_2 2=1$$

(គេតាង $k=\frac{2}{x}$ កាលណា $x\to\infty$ នោះ $k\to 0$)

93. $\lim\limits_{x\to 1}\dfrac{1+\cos\pi x}{(1-x)^2}=\lim\limits_{x\to 1}\dfrac{1-\cos\left[\pi(1-x)\right]}{(1-x)^2}$ យើងតាង $k=1-x$ កាលណា $x\to 1$ នោះ $k\to 0$

យើងបាន $\lim\limits_{x\to 1}\dfrac{1+\cos\pi x}{(1-x)^2}=\lim\limits_{k\to 0}\dfrac{1-\cos\pi k}{k^2}=\lim\limits_{k\to 0}\dfrac{1-\cos\pi k}{(\pi k)^2}\cdot\pi^2=\dfrac{1}{2}\cdot\pi^2=\dfrac{\pi^2}{2}$

94. $\lim\limits_{x\to\infty}\left[\log_3(x+1)-\log_3 x\right]=\lim\limits_{x\to\infty}\left[\log_3\left(\dfrac{x+1}{x}\right)\right]=\log_3\left[\lim\limits_{x\to\infty}\dfrac{x+1}{x}\right]=\log_3 1=0$

95. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1+\tan\pi x}{x-1}=\lim\limits_{x\to 1}\left(\dfrac{\sqrt[3]{x}-1}{x-1}+\dfrac{\tan\pi x}{x-1}\right)$

យើងតាង $k=x-1\Rightarrow x=k+1$ កាលណា $x\to 1$ នោះ $k\to 0$ តែ $\tan\pi x=\tan(\pi+\pi k)=\tan\pi k$

យើងបាន

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1+\tan\pi x}{x-1}=\lim_{k\to 0}\left(\frac{\sqrt[3]{1+k}-1}{k}+\frac{\tan\pi k}{k}\right)$$

$$=\lim_{k\to 0}\frac{1+k-1}{k\left(\sqrt[3]{(1+k)^2}+\sqrt[3]{1+k}+1\right)}+\lim_{k\to 0}\frac{\tan\pi k}{\pi k}\cdot\pi$$

$$=\lim_{k\to 0}\frac{1}{\sqrt[3]{(1+k)^2}+\sqrt[3]{1+k}+1}+1\cdot\pi=\pi+\frac{1}{3}$$

96. $\lim\limits_{x\to 2}\dfrac{x^2-4+\sin\pi x}{x-2}=\lim\limits_{x\to 2}\left(x+2+\dfrac{\sin\pi x}{x-2}\right)=4+\lim\limits_{x\to 2}\dfrac{\sin\pi x}{x-2}$

គេតាង $k=x-2\Rightarrow x=2+k$ កាលណា $x\to 2$ នោះ $k\to 0$ យើងបាន

$$\lim_{x\to 2}\frac{\sin\pi x}{x-2}=\lim_{k\to 0}\frac{\sin(2\pi+\pi k)}{k}=\lim_{k\to 0}\frac{\sin\pi k}{\pi k}\cdot\pi=1\cdot\pi=\pi$$

ដូចនេះ $\lim\limits_{x\to 2}\dfrac{x^2-4+\sin\pi x}{x-2}=4+\pi$

97. $\lim\limits_{x\to\infty}(2x-1)\cdot\sin\dfrac{\pi x}{x+3}=\lim\limits_{x\to\infty}\left[2(x+3)-5\right]\cdot\sin\left[\pi\left(1-\dfrac{3}{x+3}\right)\right]$

គេតាង $k=\dfrac{1}{x+3}\Rightarrow x+3=\dfrac{1}{k}$ កាលណា $x\to\infty$ នោះ $k\to 0$

យើងបាន

$$\lim_{x\to\infty}(2x-1)\cdot\sin\frac{\pi x}{x+3}=\lim_{k\to0}\left(\frac{2}{k}-5\right)\cdot\sin(\pi-3\pi k)$$
$$=\lim_{k\to0}\left(\frac{\sin3\pi k}{3\pi k}\cdot6\pi-5\sin3\pi k\right)=1\cdot6\pi-5\cdot0=6\pi$$

98. $\lim\limits_{x\to2}\dfrac{x^3-8+\tan\pi x}{x-2}=\lim\limits_{x\to2}\left(x^2+2x+4+\dfrac{\tan\pi x}{x-2}\right)=12+\lim\limits_{x\to2}\dfrac{\tan\pi x}{x-2}$

គេតាង $k=x-2\Rightarrow x=2+k$ កាលណា $x\to2$ នោះ $k\to0$ យើងបាន

$$\lim_{x\to2}\frac{\tan\pi x}{x-2}=\lim_{k\to0}\frac{\tan(2\pi+\pi k)}{k}=\lim_{k\to0}\frac{\tan\pi k}{\pi k}\cdot\pi=1\cdot\pi=\pi$$

ដូចនេះ $\lim\limits_{x\to2}\dfrac{x^3-8+\tan\pi x}{x-2}=12+\pi$

99. $\lim\limits_{x\to0}\dfrac{\sqrt{1+x\sin x}-\sqrt{\cos2x}}{\cot^2\left(\dfrac{\pi}{2}-x\right)}=\lim\limits_{x\to0}\dfrac{1+x\sin x-\cos2x}{\tan^2x\cdot\left(\sqrt{1+x\sin x}+\sqrt{\cos2x}\right)}$

$$=\lim_{x\to0}\left[\left(\frac{1-\cos2x}{(2x)^2}\cdot4+\frac{\sin x}{x}\right)\cdot\frac{x^2}{\tan^2x}\cdot\frac{1}{\left(\sqrt{1+x\sin x}+\sqrt{\cos2x}\right)}\right]$$
$$=\left(\frac{1}{2}\cdot4+1\right)\cdot1^2\cdot\frac{1}{2}=\frac{3}{2}$$

100. $\lim\limits_{x\to0}\dfrac{2x-3\arcsin x}{2\arcsin x}$ គេតាង $k=\arcsin x\Rightarrow x=\sin k$ កាលណា $x\to0$ នោះ $k\to0$

យើងបាន $\lim\limits_{x\to0}\dfrac{2x-3\arcsin x}{\arcsin x}=\lim\limits_{k\to0}\dfrac{2\sin k-3k}{k}=\lim\limits_{k\to0}\left(\dfrac{\sin k}{k}\cdot2-3\right)=2\cdot1-3=-1$ ។

V.គណនាលីមីតរាងមិនកំណត់ 1^{∞} , ∞^{0} , $0\times\infty$, 0^{0}

ដើម្បីគណនាលីមីតដែលមានរាងមិនកំណត់ដូចខាងលើគេត្រូវប្រើរូបមន្តដូចខាងក្រោម:

$a)$ $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{x}=\lim\limits_{x\to0}(1+x)^{\frac{1}{x}}=e$ $\quad b)$ $\lim\limits_{x\to0}\dfrac{\ln(1+x)}{x}=\lim\limits_{x\to0}\dfrac{e^{x}-1}{x}=1$

$c)$ $\lim\limits_{x\to0}\dfrac{a^{x}-1}{x}=\ln a$

បើរាងមិនកំណត់កើតឡើងមានរាង $0\times\infty$ ឬ 0^{0} គេត្រូវបំលែងអោយបានជារាងមិនកំណត់ $\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$

1. $\lim\limits_{x\to\infty}\left(1-\dfrac{1}{x}\right)^{x}$ គេតាង $k=-\dfrac{1}{x}\Rightarrow x=-\dfrac{1}{k}$ កាលណា $x\to\infty$ នោះ $k\to0$

យើងបាន $\lim\limits_{x\to\infty}\left(1-\dfrac{1}{x}\right)^{x}=\lim\limits_{k\to0}(1+k)^{\frac{-1}{k}}=\left[\lim\limits_{k\to0}(1+k)^{\frac{1}{k}}\right]^{-1}=e^{-1}=\dfrac{1}{e}$

2. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{x}=\lim\limits_{x\to\infty}\left[\left(1+\dfrac{1}{\frac{x}{2}}\right)^{\frac{x}{2}}\right]^{2}=e^{2}$

3. $\lim\limits_{x\to\infty}\left(\dfrac{2x+3}{2x-1}\right)^{x}=\lim\limits_{x\to\infty}\left(1+\dfrac{2x+3}{2x-1}-1\right)^{x+1}=\lim\limits_{x\to\infty}\left(1+\dfrac{4}{2x-1}\right)^{x+1}$

$$=\lim\limits_{x\to\infty}\left[\left(1+\dfrac{1}{\frac{2x-1}{4}}\right)^{\frac{2x-1}{4}}\right]^{\frac{4(x+1)}{2x-1}}=e^{\lim\limits_{x\to\infty}\frac{4(x+1)}{2x-1}}=e^{2}$$

4. $$\lim_{x\to\infty}\left(\frac{x^2-5x+8}{x^2-6x+3}\right)^x=\lim_{x\to\infty}\left(1+\frac{x^2-5x+8}{x^2-6x+3}-1\right)^x=\lim_{x\to\infty}\left(1+\frac{x+5}{x^2-6x+3}\right)^x$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x^2-6x+3}{x+5}}\right)^{\frac{x^2-6x+3}{x+5}}\right]^{\frac{x(x+5)}{x^2-6x+3}}=e^{\lim\limits_{x\to\infty}\frac{x(x+5)}{x^2-6x+3}}=e$$

5. $$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}}\right]^6=e^6$$

6. $$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x=\lim_{x\to\infty}\left(1+\frac{x}{1+x}-1\right)^x$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{-(x+1)}\right)^{-(x+1)}\right]^{\frac{x}{-(x+1)}}=e^{\lim\limits_{x\to\infty}\frac{x}{-(x+1)}}=e^{-1}$$

7. $$\lim_{x\to\infty}\left(\frac{1+x^2}{x^2-1}\right)^{x^2}=\lim_{x\to\infty}\left(1+\frac{1+x^2}{x^2-1}-1\right)^{x^2}=\lim_{x\to\infty}\left(1+\frac{2}{x^2-1}\right)^{x^2}$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x^2-1}{2}}\right)^{\frac{x^2-1}{2}}\right]^{\frac{2x^2}{x^2-1}}=e^{\lim\limits_{x\to\infty}\frac{2x^2}{x^2-1}}=e^2$$

8. $$\lim_{x\to\pm\infty}\left(\frac{2x+1}{x-1}\right)^{x} = \lim_{x\to\pm\infty}\left(1+\frac{2x+1}{x-1}-1\right)^{x}$$

$$= \lim_{x\to\pm\infty}\left[\left(1+\frac{1}{\frac{x-1}{x+2}}\right)^{\frac{x-1}{x+2}}\right]^{\frac{x(x+2)}{x-1}} = e^{\lim\limits_{x\to\pm\infty}\frac{x(x+2)}{x-1}} = \pm\infty$$

9. $$\lim_{x\to\infty}\left(\frac{x^2-6x+5}{x^2-3x+4}\right)^{\frac{x}{4}} = \lim_{x\to\infty}\left(1+\frac{x^2-6x+5}{x^2-3x+4}-1\right)^{\frac{x}{4}} = \lim_{x\to\infty}\left(1+\frac{1-3x}{x^2-3x+4}\right)^{\frac{x}{4}}$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x^2-3x+4}{1-3x}}\right)^{\frac{x^2-3x+4}{1-3x}}\right]^{\frac{x(1-3x)}{4(x^2-3x+4)}} = e^{\lim\limits_{x\to\infty}\frac{x(1-3x)}{4(x^2-3x+4)}} = \sqrt[4]{e^{-3}}$$

10. $$\lim_{x\to 0}\left(\frac{2x+3}{x+1}\right)^{\frac{x}{\sin 3x}} = \lim_{x\to 0}\left(\frac{2x+3}{x+1}\right)^{\lim\limits_{x\to 0}\frac{3x}{\sin 3x}\cdot\frac{1}{3}} = \sqrt[3]{3}$$

11. $\lim\limits_{x\to\frac{\pi}{2}}(\tan x)^{\cos x}$ គេតាង $k=\frac{\pi}{2}-x \Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\cos x=\cos\left(\frac{\pi}{2}-k\right)=\sin k$ និង $\tan x=\tan\left(\frac{\pi}{2}-k\right)=\cot k$ យើងបាន

$$\lim_{x\to\frac{\pi}{2}}(\tan x)^{\cos x} = \lim_{k\to 0}(\cot k)^{\sin k} = \lim_{k\to 0}\left(\frac{1}{\tan k}\right)^{\sin k} = \frac{1}{\lim\limits_{k\to 0}(\tan k)^{\sin k}}$$

ម្យ៉ាងទៀត

$$\lim_{k\to 0}(\tan k)^{\sin k}=\lim_{k\to 0}\left(1+\frac{\sin k-\cos k}{\cos k}\right)^{\sin k}=\lim_{k\to 0}\left[\left(1+\frac{\sin k-\cos k}{\cos k}\right)^{\frac{1}{\frac{\sin k-\cos k}{\cos k}}}\right]^{\frac{\sin k(\sin k-\cos k)}{\cos k}}$$

$$=e^{\lim\limits_{k\to 0}\frac{\sin k(\sin k-\cos k)}{\cos k}}=e^{0}=1$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{2}}(\tan x)^{\cos x}=1$

12. $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)^{\frac{3}{\cos x}}$ គេតាង $k=\frac{\pi}{2}-x\Rightarrow x=\frac{\pi}{2}-k$ កាលណា $x\to\frac{\pi}{2}$ នោះ $k\to 0$

ដោយ $\cos x=\cos\left(\frac{\pi}{2}-k\right)=\sin k$ យើងបាន $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)^{\frac{3}{\cos x}}=\lim\limits_{k\to 0}(1+\sin k)^{\frac{3}{\sin k}}=e^{3}$

13. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}}=e$

14. $\lim\limits_{x\to 0}\frac{\ln(1+kx)}{x}=\lim\limits_{x\to 0}\ln(1+kx)^{\frac{1}{x}}=\ln\left[\lim\limits_{x\to 0}(1+kx)^{\frac{1}{kx}\cdot k}\right]=\ln e^{k}=k$

15. $\lim\limits_{x\to\infty}x\cdot\ln\left(\frac{1+x}{x}\right)=\lim\limits_{x\to\infty}\ln\left(\frac{1+x}{x}\right)^{x}=\ln\left[\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]=\ln e=1$

16. $\lim\limits_{x\to\infty}\frac{\ln(1+e^{x})}{x}=\lim\limits_{x\to\infty}\ln(1+e^{x})^{\frac{1}{x}}=\ln\left[\lim\limits_{x\to\infty}(1+e^{x})^{\frac{1}{x}}\right]$

$\oplus$ បើ $x\to-\infty$ នោះយើងបាន $\frac{1}{x}\to 0$ និង $e^{x}\to 0$ នាំឲ្យ $\ln\left[\lim\limits_{x\to-\infty}(1+e^{x})^{\frac{1}{x}}\right]=\ln 1=0$

$\oplus$ បើ $x\to+\infty$ គេបាន $\lim\limits_{x\to+\infty}\frac{\ln(1+e^{x})}{x}$ អនុវត្តតាមទ្រឹស្តីបទឡូពីតាល់យើងបាន

$$\lim_{x\to+\infty}\frac{\ln(1+e^{x})}{x}=\lim_{x\to+\infty}\frac{e^{x}}{1+e^{x}}=1$$

17. $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}}=\lim\limits_{x\to0}\left[(1+\sin x)^{\frac{1}{\sin x}\cdot\sin x}\right]^{\frac{1}{x}}=\lim\limits_{x\to0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\frac{\sin x}{x}}=e$

18. $\lim\limits_{x\to1}\left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}=\left(\dfrac{2}{3}\right)^{\lim\limits_{x\to1}\frac{1-\sqrt{x}}{1-x}}=\left(\dfrac{2}{3}\right)^{\lim\limits_{x\to1}\frac{(1-x)}{(1-x)(1+\sqrt{x})}}=\sqrt{\dfrac{2}{3}}$

19. $\lim\limits_{x\to\infty}\left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}=\lim\limits_{x\to\infty}\left(1+\dfrac{1+x}{2+x}-1\right)^{\frac{(1-x)}{(1-x)(1+\sqrt{x})}}=\lim\limits_{x\to\infty}\left(1+\dfrac{1}{-x-2}\right)^{\frac{1}{1+\sqrt{x}}}$

$=\lim\limits_{x\to\infty}\left[\left(1+\dfrac{1}{-x-2}\right)^{-(x+2)}\right]^{\frac{1}{-(x+2)\cdot(1+\sqrt{x})}}=e^{\lim\limits_{x\to\infty}\frac{1}{(-x-2)\cdot(1+\sqrt{x})}}=e^0=1$

20. $\lim\limits_{x\to0}\dfrac{e^{kx}-1}{x}=\lim\limits_{x\to0}\dfrac{e^{kx}-1}{kx}\cdot k=k$

21. $\lim\limits_{x\to0}\dfrac{\sin 2x}{\ln(1+x)}=\lim\limits_{x\to0}\left[\dfrac{\sin 2x}{2x}\cdot\dfrac{x}{\ln(1+x)}\cdot 2\right]=1\cdot1\cdot2=2$

22. $\lim\limits_{x\to0}\dfrac{9^x-7^x}{5^x-3^x}=\lim\limits_{x\to0}\dfrac{7^x\left[\left(\frac{9}{7}\right)^x-1\right]}{3^x\left[\left(\frac{5}{3}\right)^x-1\right]}=\lim\limits_{x\to0}\dfrac{\left(\frac{7}{3}\right)^x\cdot\dfrac{\left[\left(\frac{9}{7}\right)^x-1\right]}{x}}{\dfrac{\left(\frac{5}{3}\right)^x-1}{x}}=\dfrac{\ln9-\ln7}{\ln5-\ln3}$

23. $\lim\limits_{x\to0}\dfrac{e^{ax}-e^{bx}}{\sin ax-\sin bx}=\lim\limits_{x\to0}\dfrac{e^{ax}-1-e^{bx}+1}{\sin ax-\sin bx}=\lim\limits_{x\to0}\dfrac{\dfrac{e^{ax}-1}{ax}\cdot a-\dfrac{e^{bx}-1}{bx}\cdot b}{\dfrac{\sin ax}{ax}\cdot a-\dfrac{\sin bx}{bx}\cdot b}=\dfrac{a-b}{a-b}=1$

24. $\lim\limits_{x\to0}x^x=\lim\limits_{x\to0}[1+(x-1)]^x=\lim\limits_{x\to0}\left\{[1+(x-1)]^{\frac{1}{x-1}}\right\}^{x(x-1)}=e^{\lim\limits_{x\to0}x(x-1)}=e^0=1$

25. $\lim\limits_{x\to0}\dfrac{1-\cos 2x}{\tan x\cdot\ln(1+2x)}=\lim\limits_{x\to0}\left[\left(\dfrac{1-\cos 2x}{4x^2}\right)\cdot\dfrac{x}{\tan x}\cdot\dfrac{2x}{\ln(1+2x)}\cdot2\right]=\dfrac{1}{2}\cdot1\cdot1\cdot2=1$

26. $\lim_{x\to 0}\frac{x^x-1}{x\cdot\ln x}=\lim_{x\to 0}\frac{e^{x\cdot\ln x}-1}{x\cdot\ln x}=1$

27. $\lim_{x\to 0}\frac{e^{\sin x}-e^{\tan 2x}}{x}=\lim_{x\to 0}\left[\left(\frac{e^{\sin x}-1}{\sin x}\right)\cdot\frac{\sin x}{x}-\left(\frac{e^{\tan 2x}-1}{\tan 2x}\right)\cdot\frac{\tan 2x}{2x}\cdot 2\right]=1\cdot 1-1\cdot 2=-1$

28. $\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}}=\lim_{x\to 2}\left(1+\frac{x-2}{2}\right)^{\frac{1}{x-2}}$ គេតាង $k=x-2$ កាលណា $x\to 2$ នោះ $k\to 0$

យើងបាន $\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}}=\lim_{k\to 0}\left(1+\frac{k}{2}\right)^{\frac{1}{k}}=\lim_{k\to 0}\left[\left(1+\frac{k}{2}\right)^{\frac{2}{k}}\right]^{\frac{1}{2}}=\sqrt{e}$

29. $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+c}=\lim_{x\to\infty}\left(1+\frac{a-b}{x+b}\right)^{x+c}=\lim_{x\to\infty}\left[\left(1+\frac{a-b}{x+b}\right)^{\frac{x+b}{a-b}}\right]^{\frac{(x+c)(a-b)}{x+b}}=e^{\lim_{x\to\infty}\frac{(x+c)(a-b)}{(x+b)}}=e^{a-b}$

30. $\lim_{x\to 0}(\cos x)^{\frac{1}{x}}=\lim_{x\to 0}\left(1-2\sin^2\frac{x}{2}\right)^{\frac{1}{x}}=\lim_{x\to 0}\left[\left(1-2\sin^2\frac{x}{2}\right)^{\frac{-1}{2\sin^2\frac{x}{2}}}\right]^{\frac{-2\sin^2\frac{x}{2}}{x}}=e^{\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\left(-\sin\frac{x}{2}\right)}=e^{1\cdot 0}=1$

31. $\lim_{x\to 0}\left(\frac{x^2-2x+3}{x^2-3x+2}\right)^{\frac{\sin x}{x}}=\left(\frac{3}{2}\right)^{\lim_{x\to 0}\frac{\sin x}{x}}=\frac{3}{2}$

32. $\lim_{x\to +\infty}\frac{(\ln x)^3}{x^2}$ គេតាង $x=k^{\frac{3}{2}}$ នោះគេបាន

$$\lim_{x\to +\infty}\frac{(\ln x)^3}{x^2}=\lim_{k\to +\infty}\frac{\left(\ln k^{\frac{3}{2}}\right)^3}{\left(k^{\frac{3}{2}}\right)^2}=\lim_{k\to +\infty}\left[\left(\frac{3}{2}\right)^3\cdot\left(\frac{\ln k}{k}\right)^3\right]=\left(\frac{3}{2}\right)^3\cdot 0=0$$

33. $\lim\limits_{x\to+\infty}\dfrac{(\ln x)^3}{(x+1)^3}=\lim\limits_{x\to+\infty}\left[\left(\dfrac{\ln x}{x}\right)^3\cdot\left(\dfrac{x}{x+1}\right)^3\right]=0\cdot1=0$ ។

VI.*កំណត់តម្លៃនៃអនុគមន៍និងកំណត់តម្លៃនៃចំនួនថេរ*

១.កំណត់អនុគមន៍ដឺក្រេទី២ដែលបំពេញលក្ខខណ្ឌលីមីតទាំងពីរ

$$\lim_{x\to+\infty}\frac{f(x)}{x^2+1}=2 \quad (i) \quad \& \quad \lim_{x\to1}\frac{f(x)}{x^2-1}=-1 \quad (ii)$$

យើងតាងអនុគមន៍ $f(x)=ax^2+bx+c$ *ដែល* $a\neq0$

តាមលក្ខខណ្ឌ (i) *យើងមាន* $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x^2+1}=2\Leftrightarrow\lim\limits_{x\to+\infty}\dfrac{ax^2+bx+c}{x^2+1}=1\Rightarrow a=2$

តាមលក្ខខណ្ឌ (ii) *យើងបាន*

$\lim\limits_{x\to1}\dfrac{f(x)}{x^2-1}=-1\Leftrightarrow\lim\limits_{x\to1}\dfrac{2x^2+bx+c}{x^2-1}=-1$ *ដោយលីមីតភាគបែង* $\lim\limits_{x\to1}(x^2-1)=0$ *ដើម្បីលីមីតជាចំនួនកំណត់លុះត្រាតែលីមីតភាគយក* $\lim\limits_{x\to1}(2x^2+bx+c)=0\Leftrightarrow2+b+c=0\Rightarrow c=-b-2$

ហេតុនេះយើងបាន

$$\lim_{x\to1}\frac{2x^2+bx-b-2}{x^2-1}=-1\Leftrightarrow\lim_{x\to1}\frac{(x-1)(2x+2+b)}{(x-1)(x+1)}=-1\Leftrightarrow\lim_{x\to1}\frac{2x+2+b}{x+1}=-1$$

$\Leftrightarrow 2+2+b=-2\Rightarrow b=-6$

ចំពោះ $b=-6\Rightarrow c=-b-2=6-2=4$

ដូច្នេះយើងបាន $f(x)=2x^2-6x+4$

២.កំណត់តម្លៃនៃចំនួនថេរ a *និង* b *ដើម្បីចំនួនទាំងនេះបំពេញលក្ខខណ្ឌលីមីត*

1). $\lim\limits_{x\to-2}\dfrac{x^2+ax-6}{2x^2+3x-2}=b$

កាលណា $x \to -2$ នោះយើងបានលីមីតភាគបែងខិតទៅរកសូន្យដើម្បីឲ្យលីមីតជាចំនួនកំណត់លុះត្រាតែលីមីតភាគយកខិតទៅរកសូន្យដែរគេបាន

$$\lim_{x\to-2}\left(2x^2+3x-2\right)=0 \Leftrightarrow \lim_{x\to-2}\left(x^2+ax-6\right)=0 \Leftrightarrow 4-2a-6=0 \Rightarrow a=-1$$

ចំពោះ $a=-1$ យើងបាន $\lim_{x\to-2}\frac{x^2-x-6}{2x^2+3x-2}=\lim_{x\to-2}\frac{(x-3)(x+2)}{(2x-1)(x+2)}=\lim_{x\to-2}\frac{x-3}{2x-1}=1$

ដូច្នេះ $a=-1 \;\;\&\;\; b=1$។

2). $\lim_{x\to-1}\frac{\sqrt{x^2+ax}+b}{x^2-1}=\frac{1}{2}$ កាលណា $x\to-1$ នោះលីមីតភាគបែងខិតទៅរកសូន្យដើម្បីលីមីតជាចំនួនកំនត់លុះត្រាតែ លីមីតភាគយកខិតទៅរកសូន្យដែរ។

យើងបាន $\lim_{x\to-1}\left(x^2-1\right)=0 \Leftrightarrow \lim_{x\to-1}\left(\sqrt{x^2+ax}+b\right)=0 \Leftrightarrow \sqrt{1-a}+b=0 \Rightarrow b=-\sqrt{1-a}$

គេបាន $\lim_{x\to-1}\frac{\sqrt{x^2+ax}+b}{x^2-1}=\frac{1}{2} \Leftrightarrow \lim_{x\to-1}\frac{\sqrt{x^2+ax}-\sqrt{1-a}}{(x-1)(x+1)}=\frac{1}{2}$

$$\Leftrightarrow \lim_{x\to-1}\frac{x^2+ax-1+ax}{(x+1)(x-1)\left(\sqrt{x^2+ax}+\sqrt{1-a}\right)}=\frac{1}{2} \Leftrightarrow \lim_{x\to-1}\frac{x-1+a}{(x-1)\left(\sqrt{x^2+ax}+\sqrt{1-a}\right)}=\frac{1}{2}$$

$$\Leftrightarrow \frac{a-2}{-2\left(\sqrt{1-a}+\sqrt{1-a}\right)}=\frac{1}{2} \Leftrightarrow a-2=-2\sqrt{1-a} \Leftrightarrow a^2-4a+4=4-4a \Rightarrow a=0$$

ដូចនេះ $a=0 \;\;\&\;\; b=-1$។

3). $\lim_{x\to+\infty}\left[\frac{x^2+1}{x+1}-(ax+b)\right]=0$ ដើម្បីឲ្យលីមីតស្មើសូន្យលុះត្រាតែដឺក្រេភាគបែងធំជាងដឺក្រេភាគយក

គេបាន $\lim_{x\to+\infty}\left[\frac{x^2+1}{x+1}-(ax+b)\right]=0 \Leftrightarrow \lim_{x\to+\infty}\frac{(1-a)x^2-(a+b)x+1-b}{x+1}=0$

យើងទាញបាន $\begin{cases}1-a=0\\ a+b=0\end{cases} \Rightarrow \begin{cases}a=1\\ b=-1\end{cases}$

ដូច្នេះ $a=1 \quad \& \ b=-1$ ។

៣.កំណត់តម្លៃនៃចំនួនថេរ a ដើម្បីឲ្យលីមីតជាចំនួនថេរនិងគណនាលីមីត

1). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x}+a}{x}$ ដោយលីមីតភាគបែងខិតទៅរកសូន្យកាលណា $x\to 0$ ដើម្បីលីមីតជាចំនួនថេរលុះត្រាតែលីមីតភាគយកខិតទៅរកសូន្យដែរ។

យើងបាន $\lim\limits_{x\to 0} x=0 \Leftrightarrow \lim\limits_{x\to 0}\left(\sqrt{1+3x}+a\right)=0 \Leftrightarrow 1+a=0 \Rightarrow =-1$

គេបាន $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x}-1}{x}=\lim\limits_{x\to 0}\dfrac{1+3x-1}{x\left(\sqrt{1+3x}+1\right)}=\lim\limits_{x\to 0}\dfrac{3}{\sqrt{1+3x}+1}=\dfrac{3}{2}$

ដូច្នេះ $a=-1$ និង $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x}-1}{x}=\dfrac{3}{2}$

2). $\lim\limits_{x\to 1}\dfrac{x^2-ax+1}{x-1}$ ដោយ $\lim\limits_{x\to 1}(x-1)=0$ ដើម្បីលីមីតជាចំនួនកំណត់លុះត្រាតែ

$\lim\limits_{x\to 1}\left(x^2-ax+1\right)=0 \Leftrightarrow 1-a+1=0 \Rightarrow a=2$

យើងបាន $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x-1}=\lim\limits_{x\to 1}(x-1)=0$

ដូចនេះ $a=2$ និង $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x-1}=0$

3) $\lim\limits_{x\to 2}\dfrac{\sqrt{ax+1}-3}{x-2}$ ដោយ $\lim\limits_{x\to 2}(x-2)=0$ ដើម្បីលីមីតជាចំនួនកំណត់លុះត្រាតែ

$\lim\limits_{x\to 2}\left(\sqrt{ax+1}-3\right)=0 \Leftrightarrow \sqrt{2a+1}-3=0 \Leftrightarrow 2a+1=9 \Rightarrow a=4$

យើងបាន $\lim_{x\to 2}\frac{\sqrt{4x+1}-3}{x-2}=\lim_{x\to 2}\frac{4x+1-9}{(x-2)\left(\sqrt{4x+1}+3\right)}=\lim_{x\to 2}\frac{4}{\sqrt{4x+1}+3}=\frac{2}{3}$

ដូចនេះ $a=4$ និង $\lim_{x\to 2}\frac{\sqrt{4x+1}-3}{x-2}=\frac{2}{3}$

គណនាលីមីតនៃអនុគមន៍ទៅតាមលក្ខខណ្ឌដែលដឹងដូចខាងក្រោម:

1). គេឲ្យ $\lim_{x\to 1}\frac{2f(x)-3}{x-1}=-1$ គណនាលីមីត $\lim_{x\to 1}f(x)$

គេតាង $A=\frac{2f(x)-3}{x-1}\Leftrightarrow 2f(x)-3=A(x-1)\Rightarrow f(x)=\frac{A(x-1)+3}{2}$

ដោយ $\lim_{x\to 1}\frac{2f(x)-1}{x-1}=-1\Leftrightarrow \lim_{x\to 1}A=-1$

យើងបាន $\lim_{x\to 1}f(x)=\lim_{x\to 1}\frac{A(x-1)+3}{2}=\frac{-1\cdot(1-1)+3}{2}=\frac{3}{2}$

ដូច្នេះ $\lim_{x\to 1}f(x)=\frac{3}{2}$

2). គេឲ្យ $\lim_{x\to 1}\frac{f(x)-1}{x-1}=3$ គណនាលីមីត $\lim_{x\to 1}\frac{x^2f(x)-1}{x^2-1}$

យើងតាង $A=\frac{f(x)-1}{x-1}\Rightarrow f(x)=A(x-1)+1$ គេបាន $\lim_{x\to 1}A=3$

យើងបាន

$$\lim_{x\to 1}\frac{x^2f(x)-1}{x^2-1}=\lim_{x\to 1}\frac{x^2\left[A(x-1)+1\right]-1}{x^2-1}=\lim_{x\to 1}\frac{Ax^2(x-1)+x^2-1}{x^2-1}$$

$$=1+\lim_{x\to 1}\frac{Ax^2(x-1)}{(x-1)(x+1)}=1+\frac{3\cdot 1}{2}=\frac{5}{2}$$

ដូចនេះ $\lim_{x\to 1}\frac{x^2f(x)-1}{x^2-1}=\frac{5}{2}$

3). គេឲ្យ $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=8$ និង $\lim\limits_{x\to+\infty}\left[f(x)-5x\right]=2$

គណនាលីមីត $\lim\limits_{x\to+\infty}\dfrac{f(x)+2x}{xf(x)-5x^2+4}$

យើងមានកន្សោម $\dfrac{f(x)+2x}{xf(x)-5x^2+4}=\dfrac{x\left[\dfrac{f(x)}{x}+2\right]}{x\left[f(x)-5x+\dfrac{4}{x}\right]}=\dfrac{\dfrac{f(x)}{x}+2}{f(x)-5x+\dfrac{4}{x}}$

យើងបាន $\lim\limits_{x\to+\infty}\dfrac{f(x)+2x}{xf(x)-5x^2+4}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{f(x)}{x}+2}{f(x)-5x+\dfrac{4}{x}}=\dfrac{8+2}{2+0}=5$

ដូចនេះ $\lim\limits_{x\to+\infty}\dfrac{f(x)+2x}{xf(x)-5x^2+4}=5$

4). គេឲ្យ $x+4\le f(x)\le\dfrac{x^2-4}{x-2}+2$ គណនាលីមីត $\lim\limits_{x\to2}f(x)$

យើងបាន $\lim\limits_{x\to2}(x+4)\le\lim\limits_{x\to2}f(x)\le\lim\limits_{x\to2}\left(\dfrac{x^2-4}{x-2}+2\right)\Leftrightarrow 6\le\lim\limits_{x\to2}f(x)\le\lim\limits_{x\to2}(x+2+2)=6$

ដូចនេះ $\lim\limits_{x\to2}f(x)=6$

5). គេឲ្យ $\sin5x-x^3\le xf(x)\le\sin4x+x$ ចំពោះ $\forall x\in\mathbb{R}$ គណនាលីមីត $\lim\limits_{x\to0}f(x)$

យើងមាន $\sin5x-x^3\le xf(x)\le\sin4x+x\Leftrightarrow\dfrac{\sin5x}{x}-x^2\le f(x)\le\dfrac{\sin4x}{x}+1$

គេបាន $\lim\limits_{x\to0}\left(\dfrac{\sin5x}{5x}\cdot5-x^2\right)\le\lim\limits_{x\to0}f(x)\le\lim\limits_{x\to0}\left(\dfrac{\sin4x}{4x}\cdot4+1\right)\Leftrightarrow5\le\lim\limits_{x\to0}f(x)\le5$

ដូចនេះ $\lim\limits_{x\to0}f(x)=5$

6). គេឲ្យ $\lim_{x\to1} f(x)=2$ គណនាលីមីត $\lim_{x\to1}\frac{\sqrt{f(x)+2}+\sqrt{f(x)+7}-5}{f(x)-2}$

គេបាន

$$\begin{aligned}\lim_{x\to1}\frac{\sqrt{f(x)+2}+\sqrt{f(x)+7}-5}{f(x)-2}&=\lim_{x\to1}\frac{\sqrt{f(x)+2}-2+\sqrt{f(x)+7}-3}{f(x)-2}\\&=\lim_{x\to1}\left[\frac{f(x)+2-4}{(f(x)-2)\left(\sqrt{f(x)+2}+2\right)}+\frac{f(x)+7-9}{(f(x)-2)\left(\sqrt{f(x)+7}+3\right)}\right]\\&=\lim_{x\to1}\left(\frac{1}{\sqrt{f(x)+2}+2}+\frac{1}{\sqrt{f(x)+7}+3}\right)=\frac{1}{4}+\frac{1}{6}=\frac{5}{12}\end{aligned}$$

7). គេឲ្យ $\lim_{x\to0}\frac{f(x)-\sqrt{x}}{x}=1$ និង $\lim_{x\to0}\frac{g(x)-1}{x}=2$

គណនាលីមីត $\lim_{x\to0}\frac{f(x)\cdot g(x)-\sqrt{x}}{x}$

យើងតាង $A=\frac{f(x)-\sqrt{x}}{x}\Rightarrow f(x)=Ax+\sqrt{x}$ និង $B=\frac{g(x)-1}{x}\Rightarrow g(x)=Bx+1$

នោះគេបាន $f(x)\cdot g(x)=\left(Ax+\sqrt{x}\right)(Bx+1)$ ហើយ $\lim_{x\to0}A=1$ និង $\lim_{x\to0}B=2$

យើងបាន

$$\begin{aligned}\lim_{x\to0}\frac{f(x)\cdot g(x)-\sqrt{x}}{x}&=\lim_{x\to0}\frac{\left(Ax+\sqrt{x}\right)(Bx+1)-\sqrt{x}}{x}\\&=\lim_{x\to0}\frac{ABx^2+Bx\sqrt{x}+Ax+\sqrt{x}-\sqrt{x}}{x}\\&=\lim_{x\to0}\left(ABx+B\sqrt{x}+A\right)=1\cdot2\cdot0+2\cdot0+1=1\end{aligned}$$