Plaidypvs Cheat Sheet

Propositional dL rules

$$\textbf{notR}\ \frac{\Gamma, P \vdash \Delta}{\Gamma \vdash \neg P, \Delta} \qquad \textbf{impliesR}\ \frac{\Gamma,\ P \vdash Q,\ \Delta}{\Gamma \vdash P \to Q,\ \Delta}$$

$$\textbf{notL}\ \frac{\Gamma \vdash P, \Delta}{\Gamma, \neg P \vdash \Delta} \qquad \textbf{impliesL}\ \frac{\Gamma \vdash P,\ \Delta \quad \Gamma,\ Q \vdash \Delta}{\Gamma,\ P \to Q \vdash \Delta}$$

$$\textbf{andR}\ \frac{\Gamma \vdash P, \Delta \quad \Gamma \vdash Q, \Delta}{\Gamma \vdash P \wedge Q, \Delta} \qquad \textbf{iffR}\ \frac{\Gamma, P \vdash Q, \Delta \quad \Gamma, Q \vdash P, \Delta}{\Gamma \vdash P \leftrightarrow Q,\ \Delta}$$

$$\textbf{andL}\ \frac{\Gamma, P, Q \vdash \Delta}{\Gamma, P \wedge Q \vdash \Delta} \qquad \textbf{iffL}\ \frac{\Gamma, P \wedge Q \vdash \Delta \quad \Gamma, \neg P \wedge \neg Q \vdash \Delta}{\Gamma, P \leftrightarrow Q \vdash \Delta}$$

$$\textbf{orR}\ \frac{\Gamma \vdash P,\ Q,\ \Delta}{\Gamma \vdash P \vee Q,\ \Delta} \qquad \textbf{falseL}\ \frac{}{\Gamma,\ \bot \vdash \Delta}$$

$$\textbf{orL}\ \frac{\Gamma,\ P \vdash \Delta \quad \Gamma,\ Q \vdash \Delta}{\Gamma,\ P \vee Q \vdash \Delta} \qquad \textbf{trueR}\ \frac{}{\Gamma \vdash \top,\ \Delta}$$

$$\textbf{cut}\ \frac{\Gamma \vdash C,\ \Delta \quad \Gamma,\ C \vdash \Delta}{\Gamma \vdash \Delta} \qquad \textbf{axiom}\ \frac{}{\Gamma,\ P \vdash P,\ \Delta}$$

$$\textbf{weakR}\ \frac{\Gamma \vdash P, \Delta \quad P \vdash Q}{\Gamma \vdash Q, \Delta} \qquad \textbf{weakL}\ \frac{P, \Gamma \vdash \Delta \quad Q \vdash P}{\Gamma, Q \vdash \Delta}$$

flatten Disjunctively simplifies the dL sequent by applying **trueR**, **falseR**, **orR**, **impliesR**, **notR**, **axiom**, **falseL**.

ground Disjunctively and conjunctively simplifies the dL sequent by applying **flatten** and additional splitting lemmas **andR**, **orL**, and **impliesL**.

inst Instantiates a universal quantifier in the dL-antecedent by applying **forallL** or an existential quantifier in the dL-consequent by applying **existsL**.

skolem Skolemizes an existential quantifier in dL-antecedent by applying **existsR** or an universal quantifier in the dL-consequent by applying **forallR**.

grind Repeatedly uses **ground** and **skolem**, and a number of rewrites related to real expressions. This strategy has the option to use the MetitTarski automatic theorem prover as an outside oracle to discharge the proof if possible.

Quantifier rules

existsR $\dfrac{\Gamma \vdash p(e),\ \Delta}{\Gamma \vdash \exists x : p(x),\ \Delta}$ (any e)

forallL $\dfrac{\Gamma,\ p(e) \vdash \Delta}{\Gamma,\ \forall x : p(x) \vdash \Delta}$ (any e)

forallR $\dfrac{\Gamma \vdash p(y),\ \Delta}{\Gamma \vdash \forall x : p(x), \Delta}$ (y Skolem symbol)

existsL $\dfrac{\Gamma,\ p(y) \vdash \Delta}{\Gamma,\ \exists x : p(x) \vdash \Delta}$ (y Skolem symbol)

Structural rules

moveR $\dfrac{\Gamma \vdash Q,\ P,\ \Delta}{\Gamma \vdash P,\ Q,\ \Delta}$ **hideR** $\dfrac{\Gamma \vdash \Delta}{\Gamma \vdash P,\ \Delta}$

moveL $\dfrac{\Gamma,\ Q,\ P \vdash \Delta}{\Gamma,\ P,\ Q \vdash \Delta}$ **hideL** $\dfrac{\Gamma \vdash \Delta}{\Gamma,\ P \vdash \Delta}$

Hybrid program rewrites

boxd $\langle\alpha\rangle P \leftrightarrow \neg\, [\alpha]\, \neg P$

assignb $[x := \ell]\, P = \mathbf{SUB}(\ell)(P)$

assignd $\langle x := \ell_2\rangle\, P = \mathbf{SUB}(\ell)(P)$

testb $[?Q]\, P = Q \rightarrow P$

testd $\langle ?Q\rangle\, P = Q \wedge P$

choiceb $[\alpha_1 \cup \alpha_2]\, P \leftrightarrow [\alpha_1]\, P \wedge [\alpha_2]\, P$

choiced $\langle\alpha_1 \cup \alpha_2\rangle\, P \leftrightarrow \langle\alpha_1\rangle\, P \wedge \langle\alpha_2\rangle\, P$

composeb $[\alpha_1; \alpha_2]\, P \leftrightarrow [\alpha_1]\, [\alpha_2]\, P$

composed $\langle\alpha_1; \alpha_2\rangle\, P \leftrightarrow \langle\alpha_1\rangle\, \langle\alpha_2\rangle\, P$

iterateb $[\alpha^*]\, P = P \wedge [\alpha]\, [\alpha^*]\, P$

iterateb $\langle\alpha^*\rangle\, P = P \vee \langle\alpha\rangle\, \langle\alpha^*\rangle\, P$

anyb $[x := * \,\&\, Q(x)]\, P(x) = \forall x : Q(x) \rightarrow P(x)$

anyd $\langle x := * \,\&\, Q(x)\rangle\, P(x) = \exists x : Q(x) \rightarrow P(x)$

Hybrid program rules

$$\textbf{Mb}\ \frac{\vdash P \to Q}{\Gamma \vdash [\alpha]\, P \to [\alpha]\, Q, \Delta}$$

$$\textbf{Md}\ \frac{\vdash P \to Q}{\Gamma \vdash \langle\alpha\rangle\, P \to \langle\alpha\rangle\, Q, \Delta}$$

$$\textbf{K}\ \frac{\Gamma \vdash [\alpha]\,(P \to Q), \Delta}{\Gamma \vdash [\alpha]\, P \to [\alpha]\, Q, \Delta}$$

$$\textbf{loop}\ \frac{\Gamma \vdash J, \Delta \quad J \vdash [\alpha]\, J \quad J \vdash P}{\Gamma \vdash [\alpha^*]\, P, \Delta}$$

$$\textbf{mbR}\ \frac{\Gamma \vdash [\alpha]\, Q, \Delta \quad Q \vdash P}{\Gamma \vdash [\alpha]\, P, \Delta}$$

$$\textbf{mbL}\ \frac{\Gamma, [\alpha]\, Q \vdash \Delta \quad P \vdash Q}{\Gamma, [\alpha]\, P \vdash \Delta}$$

$$\textbf{ghost}\ \frac{\Gamma \vdash [y := e]\, P, \Delta}{\Gamma \vdash P, \Delta}\ \texttt{fresh?}(y)$$

$$\textbf{Gb}\ \frac{\vdash P}{\Gamma \vdash [\alpha]\, P, \Delta}$$

$$\textbf{Gd}\ \frac{\vdash P}{\Gamma \vdash \langle\alpha\rangle\, P, \Delta}$$

$$\textbf{VRb}\ \frac{\Gamma \vdash P, \Delta}{\Gamma \vdash [\alpha]\, P, \Delta}\ \texttt{fresh?}(P)(\texttt{bv}(\alpha))$$

$$\textbf{VRd}\ \frac{\Gamma \vdash P, \Delta}{\Gamma \vdash \langle\alpha\rangle\, P, \Delta}\ \texttt{fresh?}(P)(\texttt{bv}(\alpha))$$

$$\textbf{mdR}\ \frac{\Gamma \vdash \langle\alpha\rangle\, Q, \Delta \quad Q \vdash P}{\Gamma \vdash \langle\alpha\rangle\, P, \Delta}$$

$$\textbf{mdL}\ \frac{\Gamma, \langle\alpha\rangle\, Q \vdash \Delta \quad P \vdash Q}{\Gamma, \langle\alpha\rangle\, P \vdash \Delta}$$

Differential equation rules

$$\textbf{dinit}\ \frac{\Gamma, Q \vdash [x' = f(x) \,\&\, Q]\, P, \Delta}{\Gamma \vdash [x' = f(x) \,\&\, Q]\, P, \Delta}$$

$$\textbf{dW}\ \frac{Q \vdash P}{\Gamma \vdash [x' = f(x) \,\&\, Q]\, P, \Delta}$$

$$\textbf{dI}\ \frac{Q \vdash [x' := f(x)]\,(P)'}{P \vdash [x' = f(x) \,\&\, Q]\, P, \Delta}$$

$$\textbf{dC}\ \frac{\Gamma \vdash [x' = f(x) \,\&\, Q]\, C, \Delta \quad \Gamma \vdash [x' = f(x) \,\&\, (Q \wedge C)]\, P, \Delta}{P \vdash [x' = f(x) \,\&\, Q]\, P, \Delta}$$

$$\textbf{dG}\ \frac{\Gamma \vdash G,\ G \vdash P,\ \Gamma \vdash \exists y\, [x' = f(x),\ y' = a(x) \cdot y + b(x) \,\&\, Q]\, G, \Delta}{\Gamma \vdash [x' = f(x) \,\&\, Q]\, P, \Delta}$$

$$\textbf{dS}\ \frac{\Gamma \vdash [x' = f(x) \,\&\, Q]\, P}{\Gamma \vdash \forall t \geq 0\, (\forall 0 \leq s \leq t\, Q(y(s))) \to [x := y(t)]\, P}$$