NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

ST2334 PROBABILITY AND STATISTICS

(Semester 1: AY 2006–2007)

December 2006 — Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **TWELVE (12)** printed pages.
2. Answer **ALL** the questions for TOTAL 60 marks.
3. Read the questions **CAREFULLY**, and label some important quantities clearly at your own convenience.
4. This is a **CLOSED** book examination. Candidates may bring in **TWO** A4 size (210×297mm) help sheets written on both sides.
5. **Decimal answers with more than 4 decimal places** should be given to at least **FOUR (4)** decimal places.

Matriculation No: ______________________

Question	1	2	3	4	Total
Max. marks	6	21	21	12	60
Marks scored					

1. **[6 marks]** Suppose that two defective refrigerators have been included in a shipment of six refrigerators. The buyer begins to test the six refrigerators one at a time.

   (a) What is the probability that no more than four refrigerators need to be tested to locate both of the defective refrigerators? **[3 marks]**

   (b) Given that exactly one of the two defective refrigerators has been located in the first two tests, what is the probability that the remaining defective refrigerator is found in the third or fourth test? **[3 marks]**

2. **[21 marks]** Let Y_1 and Y_2 denote the lengths of life, in hours, for components of types I and II, respectively, in an electronic system. Suppose the joint density of Y_1 and Y_2 is given by

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} \left[\dfrac{1}{b^a(a-1)!}\right]^2 y_1^{a-1} y_2^{a-1} \, \mathrm{e}^{-(y_1+y_2)/b}, & y_1 > 0, \quad y_2 > 0 \\ 0, & \text{elsewhere,} \end{cases} \tag{1}$$

where $a, b > 0$.

(a) Show that the marginal pdf of Y_1 is $f_{Y_1}(y_1) = \dfrac{1}{b^a(a-1)!} y_1^{a-1} \, \mathrm{e}^{-y_1/b}$, $y_1 > 0$. [Hint: Use the fact that any density same as $f_{Y_1}(y_1)$ is a proper pdf in y_1.] **[3 marks]**

(b) Argue why Y_1 and Y_2 are independent. **[3 marks]**

(c) Show that $E(Y_1) = ab$ and $V(Y_1) = ab^2$. [Hint: $\int_0^\infty x^{a-1}\,\mathrm{e}^{-x/b}\,\mathrm{d}x = b^a(a-1)!$ for any positive integer a and b.] **[5 marks]**

(d) The cost C of replacing the two components depends upon the length of life of the two components and is given by $C = 50 + 2Y_1 + 4Y_2$. Find $E(C)$ and $V(C)$. [Hint: Apply the fact that Y_1 and Y_2 are independent.] **[4 marks]**

(e) Suppose $b = 2$ in (1). To estimate a, we collect lengths of life, $X_1, X_2, \ldots, X_N$, of N components of type I.

(i) Use all lengths of life $X_1, \ldots, X_N$ to suggest and verify a statistic as an unbiased estimator of a. **[3 marks]**

(ii) What is the standard error of your estimator in (i)? **[3 marks]**

3. **[21 marks]** The length of life Y (in hundreds of hours) for fuses of a certain type is modeled by a distribution with pdf

$$f_Y(y) = \begin{cases} (1/3)\,\mathrm{e}^{-y/3}, & y > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Suppose two fuses having independent lengths of life Y_1 and Y_2 identically distributed as Y are installed inside an equipment. To operate the equipment, at least one of the two installed fuses has to function properly. One fuse is in a primary system, and the other is in a backup system that comes into use only if the primary system fails. Replacements of the fuses will only be made after failure of all fuses in the equipment.

(a) Find the probability that the total operating time of the equipment (after replacing two new fuses) before any new replacement is less than 100 hours. **[6 marks]**

(b) To reduce downtime of the equipment, another 30 new and independent fuses with lengths of life distributed as Y have been installed. They are coming into use one-by-one after each failure. Approximate the probability that the total operating time of the equipment before any new replacement is less than 3000 hours. [Hint: Get $E(Y)$ and $V(Y)$ based on the results of question 2(c) at page 4.] **[6 marks]**

(c) In part (b), what is the probability that there are no more than 3 of the fuses operating for longer than 400 hours? **[3 marks]**

(d) Approximate the probability in part (c) by

(i) Poisson approximation. **[2 marks]**

(ii) Central Limit Theorem. **[4 marks]**

4. **[12 marks]** Suppose the weekly study time (in minutes) of students enrolled in a Statistics module after lectures is approximately normally distributed.

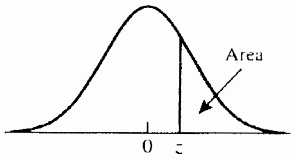
   (a) What is the proportion of students in this module whose weekly study time spent after lecture is within one standard deviation of the average study time of the whole module? **[2 marks]**

   (b) The Professor is interested in knowing the average weekly study time of students in the module. He obtains the weekly study times of a random group of 25 students with an average of 49.2 minutes and a standard deviation of 25 minutes. Construct a 90% confidence interval for the average weekly study time of students in this module. **[3 marks]**

(c) Interpret the interval you get in (b). **[3 marks]**

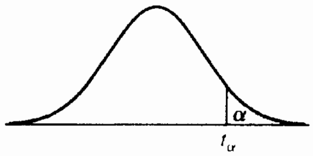
(d) Suppose the normality assumption of the weekly study time is not valid and that the Professor is interested in the weekly study information of a group of another 50 students. Approximate the probability that the average weekly study time of this group of students exceeds the population average weekly study time by 1 minute. **[4 marks]**

Normal curve areas
Standard normal probability in right-hand
tail (for negative values of z areas are found by symmetry)



	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	d.f.
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

[END OF PAPER]