from Step Response *to* Impulse Response

First-Order System: $y_{step}(t) = K\left[1-\exp(-t/T)\right]u(t)$

$$y_\delta(t) = \frac{d}{dt}y_{step}(t) = \underbrace{K\overbrace{\left[1-\exp(-t/T)\right]}^{\text{Equal to 0 at } t=0}\delta(t)}_{0} + \frac{K}{T}\exp(-t/T)u(t) = \frac{K}{T}\exp(-t/T)u(t)$$

Second-Order System: $y_{step}(t) = K\left[1-\frac{\omega_n}{\omega_d}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]u(t)$

$$y_\delta(t) = \frac{d}{dt}y_{step}(t) = \underbrace{K\overbrace{\left[1-\frac{\omega_n}{\omega_d}\exp(-\sigma t)\sin(\omega_d t+\phi)\right]}^{\text{Equal to 0 at } t=0}\delta(t)}_{0} - K\frac{\omega_n}{\omega_d}\begin{bmatrix} -\sigma\exp(-\sigma t)\sin(\omega_d t+\phi) \\ +\omega_d\exp(-\sigma t)\cos(\omega_d t+\phi)\end{bmatrix}u(t)$$

$$= K\frac{\omega_n^2}{\omega_d}\exp(-\sigma t)\left[\frac{\sigma}{\omega_n}\sin(\omega_d t+\phi) - \frac{\omega_d}{\omega_n}\cos(\omega_d t+\phi)\right]u(t)$$

$$= K\frac{\omega_n^2}{\omega_d}\exp(-\sigma t)\begin{bmatrix} \cos(\phi)\sin(\omega_d t+\phi) \\ -\sin(\phi)\cos(\omega_d t+\phi)\end{bmatrix}u(t) = K\frac{\omega_n^2}{\omega_d}\exp(-\sigma t)\sin(\omega_d t)u(t)$$