Pts ENG 106 Homework #3 Solutions Winter 2013

Present Worth

15 #1. Read the note on p. 192. Is the actual-dollar assumption realistic given the cash flows stated in each of the following three problems: 5.13, 5.33 and 5.45? State your reasoning.

For 5.13: Since flows have no pattern, this is likely an actual-dollar situation. In the real world, we'd have to know the context to say for sure.

For 5.33: The uniform series suggest that constant dollar analysis is used. In an actual dollar scenario, upward adjustments for the future prices of maintenance and renovation would be included.

For 5.45: The uniform series suggest constant \$, although the series are short, so actual \$ might be a possibility if, for example, the costs were payments on a contract. However, if the amounts are in actual \$, the magnitudes will be larger for the second and future copies. We'd therefore need the inflation rate as well as the market interest rate to properly conduct the analysis for this case where the lives of the alternatives are not equal.

5.13: Consider the following set of investment projects, all of which have a three-year investment life:

| n | A | B | C | D |
|---|---|---|---|---|
| 0 | -\$1200 | -\$1800 | -\$1000 | -\$6500 |
| 1 | \$0 | \$600 | -\$1200 | \$2500 |
| 2 | \$0 | \$900 | \$900 | \$1900 |
| 3 | \$3000 | \$1700 | \$3500 | \$2800 |

10 Compute net present worth of each project at i = 10%

$PW(10\%)_A = -1200 + 3000\,(P/F, 10\%, 3) = -1200 + 3000\,(.7513) = \$1{,}053.9$

$PW(10\%)_B = -1800 + 600\,(P/F, 10\%, 1) + 900\,(P/F, 10\%, 2) + 1700\,(P/F, 10\%, 3)$

$= -1800 + 600\,(.9091) + 900\,(.8264) + 1700\,(.7513) = \$776.49$

$PW(10\%)_C = -1000 -1200\,(P/F, 10\%, 1) + 900\,(P/F, 10\%, 2) + 3500\,(P/F, 10\%, 3)$

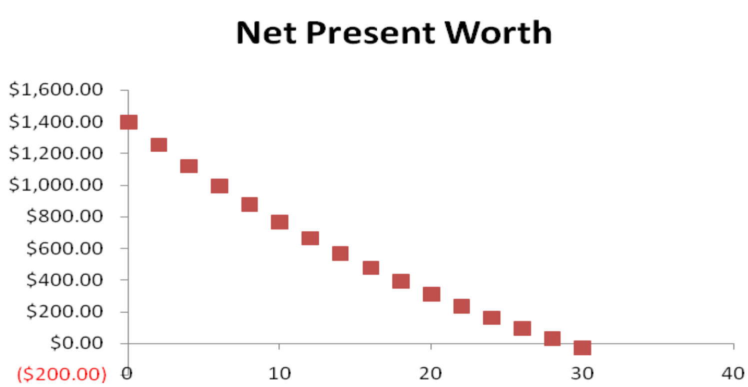
$= -1000 -1200\,(.9091) + 900\,(.8264) + 3500\,(.7513) = \$1282.3$

$PW(10\%)_D = -6500 + 2500\,(P/F, 10\%, 1) + 1900\,(P/F, 10\%, 2) + 2800\,(P/F, 10\%, 3)$

$= -6500 + 2500\,(.9091) + 1900\,(.8264) + 2800\,(.7513) = -\$553.34$

$PW(10\%)_{\text{do nothing}} = 0$

5 W Plot PW of Project B for interest rates from 0% to 30%:




2 - If the projects are independent, choose all with positive NPVs (A, B, and C)

2 - If they are mutually exclusive, with a do nothing alternative, choose the most positive (C). (If they were all negative, choose the do nothing option.)

1 - If they are mutually exclusive, but there is no do nothing alternative, choose highest NPV (C).

20 W 5.33a: A newly constructed bridge costs $10,000,000. The same bridge is estimated to need renovation every 10 years at a cost of $1,000,000. Annual repairs and maintenance are estimated to be $100,000 per year.

If the interest rate is 5%, determine the capitalized equivalent cost of the bridge.

Construction = $P_c$ = 10,000,000

Renovation = $P_r = A/\iota$ = 1,000,000 *(A/F, 5%, 10)/$\iota$ = 1,000,000 *()/.05 = 1,590,000

(alternatively, could calculate effective interest rate for 10 yrs : $\iota_e = (1.05)^{10} - 1$ = 63%

and then calculate $P_r = A/\iota_e$ = 1,590,000, as above)

Maintenance = $P_M = A/\iota$ = 100,000/$\iota$ = 100,000/.05 = 2,000,000

CE(5%) = $P_c$+$P_r$ +$P_M$ = $13,590,000

Repeat the analysis with an interest rate of 10%.

$P_c$ = 10,000,000

$P_r = A/\iota$ = 1,000,000 *(A/F, 10%, 10)/$\iota$ = 1,000,000 *()/.1 = 627,000

$P_M = A/\iota$ = 100,000/$\iota$ = 100,000/.1 = 1,000,000

CE(10%) = $P_c$+$P_r$ +$P_M$ = $11,627,000

(As interest rate increases, CE decreases. But note that market $\iota$ and inflation rate are coupled, so the actual-dollar amounts on the time line would also probably be larger if the market rate was higher due only to inflation.)

20 5.45 Consider the following two mutually exclusive investment projects:

| | A | | B | |
|---|---|---|---|---|
| n | Cash Flow | Salvage Value | Cash Flow | Salvage Value |
| 0 | -$12,000 | | -$10,000 | |
| 1 | -$2,000 | $6,000 | -$2,100 | $6000 |
| 2 | -$2,000 | $4,000 | -$2,100 | $3,000 |
| 3 | -$2,000 | $3,000 | -$2,100 | $1,000 |
| 4 | -$2,000 | $2,000 | | |
| 5 | -$2,000 | $2,000 | | |

Salvage values represent the net proceeds (after tax) from disposal of the assets if they are sold at the end of year listed. Both projects will be available (and can be repeated) with the same costs and salvage values for an indefinite period. Assume constant dollars and inflation free interest rates.

(a) With an infinite planning horizon, which project is a better choice at MARR = 12%

Infinite planning horizon means find a common period of service to work with (15 years).

**Project A**

$PW(12\%)_{cycle}$= -$12,000 – $2,000(P/A, 12%, 5) + $2,000(P/F,12%,5)

= -$12,000 – $2,000(3.6048) + $2,000(0.5674) = -$18,075

12% over five years MARR: (1.12)^5 – 1 = 76.23%

$PW(12\%)_{total}$= -$18,075(1+P/A,76.23%,2) [N=2 because three cycles occur]

= -$18,075(1+P/A,76.23%,2) = -$18,075(1.88943) = -$34,151

**Project B**

$PW(12\%)_{cycle}$= -$10,000 – $2,100(P/A, 12%, 3) + $1,000(P/F,12%,3)

= -$12,000 – $2,100(2.4018) + $1,000(0.7118) =-$14,332.

12% over three years MARR: (1.12)^5 – 1 = 40.49%

$PW(12\%)_{total}$= -$14,332(1+P/A,40.49%,4) [N=4 because 5 cycles of equipment purchase occurs.]

= -$14,332(1+P/A,40.49%,4) = -14,332(2.8352) = -$40,642

You should choose the project with the highest value (lowest negative value), so choose A.

(b) With an 10 year planning horizon, which project is a better choice at MARR = 12%

**Project A**

$PW(12\%)_{cycle}$= -\$12,000 – \$2,000(P/A, 12%, 5) + \$2,000(P/F,12%,5)
= -\$12,000 – \$2,000(3.6048) + \$2,000(0.5674) = -\$18,075
$PW(12\%)_{total}$= -\$18,075 + -\$18,075(P/F,12%,5) [two cycles]
= -\$18,075 + -\$18,075(0.5674) = -\$28,330.

**Project B**

$PW(12\%)_{cycle}$= -\$10,000 – \$2,100(P/A, 12%, 3) + \$1,000(P/F,12%,3)
= -\$12,000 – \$2,100(2.4018) + \$1,000(0.7118) =-\$14,332.
$PW(12\%)_{total}$
= -\$14,332[1+(P/F,12%,3)+(P/F,12%,6) ] - [10,000(P/F,12%,9)] – [2100-6000](P/F,12%,10)
1$^{st}$ year 4$^{th}$ year 7$^{th}$ year 10$^{th}$ year
=-\$14,332[1+(0.7118)+(0.5066) ] - [10,000(0.3606)] – [2100-6000](.3220)=-\$34,145.
Project A still has a higher value, so choose A despite the shorter planning horizon.

<u>Annual Equivalent Worth</u>

10 #5. Read notes on p.232. Is the actual-dollar assumption realistic for the cash flows stated in 6.25 and #7 below? State your reasoning.
For 6.25: It is possible that the service contract amount is fixed in actual dollars over the full ten-year period, and the income-tax saving could well be in actual \$. Labor costs are likely to change over time, so the salary of \$40k is probably in today's (constant) \$ rather than in actual \$ of each of the ten years.
For #7: In the real world, it's very likely that the year-end installment payments would be in actual \$.

20 6.25 A firm is considering purchasing several controllers to automate their operations. The equipment will initially cost \$150k, and the labor to install it will cost \$45k. A service contract to maintain the equipment will cost \$5k per year. A trained machine operator will have to be hired at an annual salary of \$40k. The equipment will also give an approximate \$15k annual income-tax savings (cash inflow). How much will this investment have to increase the annual revenues after taxes in order for the firm to break even? The equipment is estimated to have a life of 10 years with no salvage value. The firm's MARR is 12%.

AEC of the investment = \$195k(A/P, 12%, 10) + \$5k + \$40k - \$15k = \$64.5k
So the revenues would need to increase by this amount for the firm to break even.

10 #7. An industrial firm can purchase a certain machine for \$40,000. A down payment of \$4,000 is required, and the balance can be paid in five equal year-end installments at 7% interest on the unpaid balance. As an alternative, the machine can be purchased for \$36,000 in cash. If the firm's MARR is 10%, determine which alternative should be accepted, using the annual-equivalence method.

**Down Payment Scheme**: A = \$36,000 (A/P,7%, 5) = \$8,780.
AEC(10%) = yearly cost = \$4000(A/P,10%,5)+8,780 = \$9,835/year
**Full Purchase Scheme**: AEC(10%) = \$36,000 (A/P,10%, 5) = \$9,497/year
The full purchase is a better decision because it has a lower annual equiv cost.

20 W #8. Using AE worth as the basis for comparison, repeat problem 5.39. Start with the given data (not with your PW results from above) and use the "shortcuts" inherent in the AE method where possible. Assume the cash flows are stated in constant dollars and the MARR of 12% is inflation-free. These are some of the necessary conditions for using AE to compare the first copies of repeated projects that have unequal lives. Note that for part b, the planning horizon must be used explicitly. (As a check on your results, for each part (a or b) the ratios of AE to PW should be the same for all the alternatives.)

Part (a): We must still implicitly use a common analysis period, but by symmetry we can just compare the AEs of the first copies of the alternatives – 5 years for Project A versus 3 years for Project B – as long as the AEs are stated in constant (yr 0) $ so the amounts will repeat throughout the LCM of 15 years..

**Project A**

$AEW(12\%)_{cycle}$= -$12,000(A/P, 12%, 5) – $2,000 + $2,000(A/F,12%,5)

= -$12,000(.2774) – $2,000 + $2,000(.1574) = -$5,014

**Project B**

$AEW(12\%)_{cycle}$= -$10,000(A/P, 12%, 3) – $2,100 + $1,000(A/F,12%,3)

= -$10,000(.4163) – $2,100 + $1,000(0.2963) = -$5,966.7

You should choose the project with the highest value (lowest negative value), so choose A.

Part (b): Because the 10-yr period of need is not an LCM of the lives, we need to explicitly use 10-year cash flows and convert them to AEs over 10 years, although clever people will see ways of shortcutting the process, e.g., for Project A the AE for 10 years is the same as for the first copy over 5 years.

**Project A**

$AEW(12\%)_{5\text{-yr cycle}}$= -$12,000(A/P, 12%, 5) – $2,000 + $2,000(A/F,12%,5)

= (same as in Part (a))

$AEW(12\%)_{10\text{ years}}$= $AEW(12\%)_{cycle}$

**Project B**

$AEW(12\%)_{3\text{-yr cycle}}$= -$10,000(A/P, 12%, 3) – $2,100 + $1,000(A/F,12%,3)

= (same as in Part (a))

A 9-yr uniform series, each of this amount, covers the first 9 year of need. Note that this series is not spread over the full ten years. Also, to cover the need during year 10, we must include flows for a truncated fourth copy. Converting all these three pieces into a uniform series can be done in many ways. Here is one:

$AEW(12\%)_{10\text{ yr s}}$= [$AEW(12\%)_{3\text{-yr cycle}}$(P/A, 12%, 9) -$10,000(P/F, 12%, 9)](A/P, 12%, 10) – (2100-6000) (A/F,12%,10)

=[ -$5,966.7* (5.3282)-$10,000*(0.3606)]*(0.1770)-(-$3,900*(.0570))

= -$6,043.1

Project A still has a higher value, so choose A despite the shorter planning horizon.

135 (Total pts)