2006/2007 SEMESTER 1 MID-TERM TEST

MA1505 MATHEMATICS I

October 2, 2006

SESSION 1 : 6:00 - 7:00pm

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This test paper consists of **TWELVE (12)** multiple choice questions and comprises **Seven (7)** printed pages.

2. Answer all 12 questions. 1 mark for each correct answer. No penalty for wrong answers. Full mark is 12.

3. All answers (Choices A, B, C, D, E) are to be submitted using the pink form (FORM CC1).

4. Use **only 2B pencils** for FORM CC1.

5. On FORM CC1 (section B), **write** your **matriculation number** and **shade** the corresponding numbered circles carefully. Your FORM CC1 will be graded by a computer and it will record a **ZERO** for your score if your matriculation number is not correct.

6. Write your full name in section A of FORM CC1.

7. Only circles for answers 1 to 12 are to be shaded.

8. For each answer, the circle corresponding to your choice should be properly shaded. If you change your answer later, you must make sure that the original answer is properly erased.

9. For each answer, **do not shade more than one circle**. The answer for a question with more than one circle shaded will be marked wrong.

10. **Do not fold** FORM CC1.

11. Submit FORM CC1 before you leave the test hall.

1. Let $f(x) = \ln \frac{1+\sin x}{1-\sin x}$, where $0 < x < \frac{\pi}{2}$. Then $f'(x) =$

(A) $2\cos x$

(B) $2\cot x$

(C) $2\sec x$

(D) $2\sin x$

(E) $2\tan x$

2. If $y^2 - 2y\sqrt{1+x^2} + x^2 = 0$, then $\frac{dy}{dx} =$

(A) $\frac{x}{\sqrt{1+x^2}}$

(B) $\frac{x}{1+x^2}$

(C) $\frac{2x}{\sqrt{1+x^2}}$

(D) $\frac{2x}{1+x^2}$

(E) $\frac{-2x}{(1+x^2)^2}$

3. A girl 5 feet tall is running at the rate of 12 feet/second and passes under a street light 20 feet above the ground. Find how rapidly the length of her shadow is increasing when she is 20 feet past the base of the street light.

(A) 20 feet/second

(B) 16 feet/second

(C) 12 feet/second

(D) 4 feet/second

(E) 2 feet/second

4. Evaluate $\lim_{x\to 0} \frac{(1-e^x)\tan x}{x\ln(1+kx)}$, where k is a positive constant.

(A) k

(B) $-e$

(C) $-\frac{e}{k}$

(D) $\frac{e}{k}$

(E) $-\frac{1}{k}$

5. Evaluate $\int_n^{n+1} x^2 (n-x)^{10}\, dx$, where n is a constant.

(A) $-\frac{n^2}{11}+\frac{n}{6}+\frac{1}{13}$

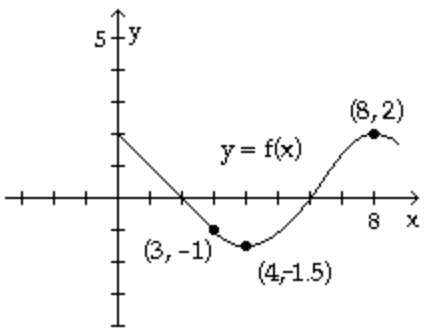
(B) $\frac{n^2}{11}+\frac{n}{6}+\frac{1}{13}$

(C) $\frac{n^2}{11}-\frac{n}{6}+\frac{1}{13}$

(D) $\frac{n^2}{11}+\frac{n}{6}-\frac{1}{13}$

(E) None of the above

6. Let $f(x)$ be a differentiable function whose graph is shown in the figure. (Note that the function is linear for $0 \le x \le 3$.) The position, measured from the origin in meters, at time t seconds, of a particle moving along the x-axis is given by the formula $s = \int_0^t f(x)\, dx$. What is the position of the particle at $t = 3$ seconds?



(A) 2m

(B) 1.5m

(C) 0.5m

(D) 1m

(E) 3m

7. Let R be the region in the first quadrant bounded above by the line $y = 1$, below by the curve $y = \sqrt{\sin 6x}$, on the left by the y-axis and on the right by the point of intersection of the line $y = 1$ and the curve $y = \sqrt{\sin 6x}$. Find the volume of the solid generated by revolving R about the line $y = 0$.

(A) $\frac{1}{4}\pi - \frac{1}{18}$

(B) $\frac{1}{12}\pi^2 + \frac{1}{6}\pi$

(C) $\frac{1}{12}\pi^2 - \frac{1}{6}\pi$

(D) $\frac{1}{4}\pi^2 + \frac{1}{18}\pi$

(E) $\frac{1}{4}\pi^2 - \frac{1}{18}\pi$

8. $\int_0^4 |x\,(x-1)\,(x-2)|\,dx =$

(A) $\frac{23}{2}$

(B) $\frac{39}{2}$

(C) $\frac{25}{2}$

(D) $\frac{45}{2}$

(E) $\frac{33}{2}$

9. $\int_0^{\pi/3} \left(1 + \tan^6 x\right) dx =$

(A) $\frac{3}{\pi}$

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}\pi}{5}$

(D) $\frac{\pi}{3}$

(E) $\frac{9\sqrt{3}}{5}$

10. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{9^n (n!)^3}{(3n)!} x^n$.

(A) 9

(B) $\frac{1}{9}$

(C) 3

(D) $\frac{1}{3}$

(E) 0

11. Find the Taylor series of $f(x) = \frac{1}{(x-1)^2}$ at $a = 3$.

(A) $\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{2^{n+2}} (x-3)^n$

(B) $\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{2^{n+3}} (x-3)^n$

(C) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{n-1}{2^{n+2}} (x-3)^n$

(D) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{n+2}} (x-3)^n$

(E) None of the above

12. Let $\frac{d}{dx}\left\{x^{10}(e^x - 1)\right\} = \sum_{n=0}^{\infty} a_n x^n$. Then $a_{12} =$

(A) $\frac{13}{6}$

(B) $\frac{11}{6}$

(C) $\frac{13}{3}$

(D) $\frac{11}{3}$

(E) $\frac{13}{2}$

END OF PAPER

National University of Singapore
Department of Mathematics

2006-2007 Semester 1 MA1505 Mathematics I Mid-Term Test Session 1 Answers

Question	1	2	3	4	5	6	7	8	9	10	11	12
Answer	C	A	D	E	B	B	C	E	E	C	A	A

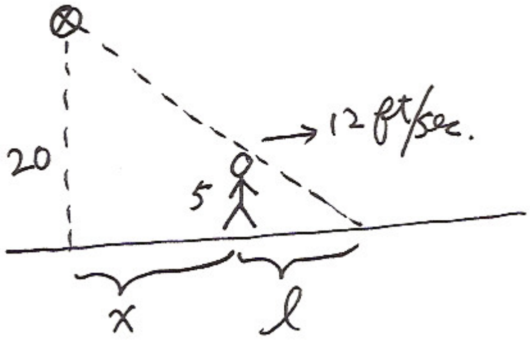
Session 1 Hints and Solutions

1). $f(x) = \ln(1+\sin x) - \ln(1-\sin x)$ ...

2). $2yy' - 2y'\sqrt{1+x^2} - \frac{2xy}{\sqrt{1+x^2}} + 2x = 0 \Rightarrow y'(y-\sqrt{1+x^2}) = \frac{x(y-\sqrt{1+x^2})}{\sqrt{1+x^2}}$

$$\Rightarrow y' = \frac{x}{\sqrt{1+x^2}}. \quad \left(\begin{array}{l} \because y^2 - 2y\sqrt{1+x^2} + x^2 = 0 \\ \therefore y - \sqrt{1+x^2} \neq 0 \end{array}\right)$$

3).



We have $\frac{dx}{dt} = 12$ ft/sec.

Using similar Δ's:

$$\frac{l}{5} = \frac{l+x}{20}$$

$\therefore 3l = x$

$$\therefore \frac{dl}{dt} = \frac{1}{3}\frac{dx}{dt} = 4 \text{ ft/sec.}$$

4). $$\lim_{x\to 0} \frac{(1-e^x)\tan x}{x\ln(1+kx)} = \left\{\lim_{x\to 0} \frac{1-e^x}{\ln(1+kx)}\right\}\left\{\lim_{x\to 0}\frac{\tan x}{x}\right\}$$

$$= \left\{\lim_{x\to 0} \frac{-e^x}{\frac{k}{1+kx}}\right\}\left\{\lim_{x\to 0}\frac{\sec^2 x}{1}\right\}$$

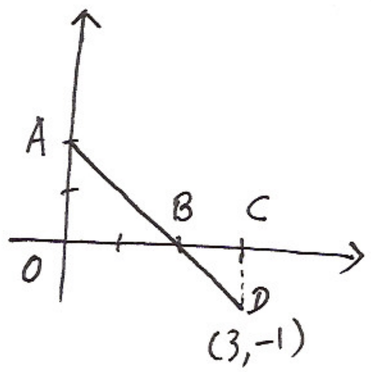
$$= -\frac{1}{k}.$$

5). $$\int_n^{n+1} x^2(n-x)^{10}dx = \int_0^{-1}(n-u)^2 u^{10}(-du) \quad (\text{let } u = n-x)$$

$$= \int_{-1}^{0}(n^2u^{10} - 2nu^{11} + u^{12})\,du$$

⋮

6). Position at $t=3$ is given by



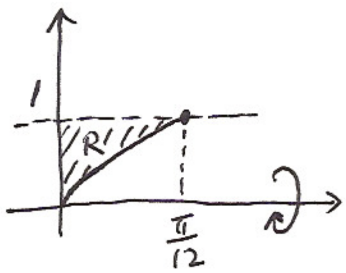
$$S=\int_0^3 f(x)\,dx$$

$= \text{area}\,(\Delta OAB)$

$\quad - \text{area}\,(\Delta BCD)$

$$= \tfrac{1}{2}(2)(2)-\tfrac{1}{2}(1)(1) = \underline{\underline{1.5}}$$

7). Solving $\sqrt{\sin 6x}=1 \Rightarrow$ the first point of intersection in the first quadrant is $x=\frac{\pi}{12}$.



$$\text{Volume} = \int_0^{\pi/12} \pi\left\{1^2-\left(\sqrt{\sin 6x}\right)^2\right\}dx$$

$$= \pi\int_0^{\pi/12}(1-\sin 6x)\,dx$$

$$= \pi\left[x+\frac{1}{6}\cos 6x\right]_0^{\pi/12} = \underline{\underline{\pi\left(\frac{\pi}{12}-\frac{1}{6}\right)}}$$

8). $x(x-1)(x-2)$ −   +   −   +

0   1   2

$$\int_0^4 |x(x-1)(x-2)|\,dx = \int_0^1 x(x-1)(x-2)\,dx - \int_1^2 x(x-1)(x-2)\,dx$$

$$+\int_2^4 x(x-1)(x-2)\,dx$$

$\vdots$

$$= \underline{\underline{\frac{33}{2}}}$$

9). $\displaystyle\int_0^{\pi/3}(1+\tan^6 x)\,dx = \int_0^{\pi/3}\{1+(\tan^2 x)^3\}\,dx$

$$= \int_0^{\pi/3}(1+\tan^2 x)(\tan^4 x-\tan^2 x+1)\,dx$$

$$= \int_0^{\pi/3}(\tan^4 x-\tan^2 x+1)\,d(\tan x)$$

$$= \left[\frac{1}{5}\tan^5 x-\frac{1}{3}\tan^3 x+\tan x\right]_0^{\pi/3}$$

$$= \underline{\underline{\frac{9}{5}\sqrt{3}}}$$

10). $$\left|\frac{\dfrac{9^{n+1}[(n+1)!]^3}{(3n+3)!}x^{n+1}}{\dfrac{9^n(n!)^3}{(3n)!}x^n}\right| = \frac{9(n+1)^3}{(3n+3)(3n+2)(3n+1)}|x| \longrightarrow \frac{1}{3}|x|$$

$$\frac{1}{3}|x|<1 \iff \underline{\underline{|x|<3}}$$

11). $\displaystyle\frac{1}{1-r}=\sum_{n=0}^{\infty} r^n \Rightarrow \frac{1}{(1-r)^2}=\sum_{n=1}^{\infty} n r^{n-1}$ (differentiate with respect to r)

$$= \sum_{n=0}^{\infty}(n+1)r^n$$

$$\therefore \frac{1}{(x-1)^2} = \frac{1}{[2+(x-3)]^2} = \frac{1}{2^2}\frac{1}{\left\{1-\dfrac{-(x-3)}{2}\right\}^2} = \frac{1}{2^2}\sum_{n=0}^{\infty}(n+1)(-1)^n\frac{(x-3)^n}{2^n}$$

$$= \underline{\underline{\sum_{n=0}^{\infty}\frac{(-1)^n(n+1)}{2^{n+2}}(x-3)^n}}$$

12). $\displaystyle\frac{d}{dx}\{x^{10}(e^x-1)\} = \frac{d}{dx}\left\{x^{10}\left[\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)-1\right]\right\} = \frac{d}{dx}\left\{x^{10}\sum_{n=1}^{\infty}\frac{x^n}{n!}\right\}$

$$= \frac{d}{dx}\sum_{n=1}^{\infty}\frac{x^{n+10}}{n!} = \sum_{n=1}^{\infty}\frac{n+10}{n!}x^{n+9}$$

Put $n=3 \Rightarrow a_{12} = \dfrac{13}{3!} = \underline{\underline{\dfrac{13}{6}}}$