Chapter 3 Instructor Notes

Chapter 3 presents the principal topics in the analysis of resistive (DC) circuits. The presentation of node voltage and mesh current analysis is supported by several solved examples and drill exercises, with emphasis placed on developing consistent solution methods, and on reinforcing the use of a systematic approach. The aim of this style of presentation, which is perhaps more detailed than usual in a textbook written for a non-majors audience, is to develop good habits early on, with the hope that the orderly approach presented in Chapter 3 will facilitate the discussion of AC and transient analysis in Chapters 4 and 5. *Make The Connection* sidebars (pp. 75-77) introduce analogies between electrical and thermal circuit elements. These analogies are encountered again in Chapter 5. A brief discussion of the principle of superposition precedes the discussion of Thèvenin and Norton equivalent circuits. Again, the presentation is rich in examples and drill exercises, because the concept of equivalent circuits will be heavily exploited in the analysis of AC and transient circuits in later chapters. The *Focus on Methodology* boxes (p.76 – Node Analysis; p. 86 – Mesh Analysis; pp. 103, 107, 111 – Equivalent Circuits) provide the student with a systematic approach to the solution of all basic network analysis problems.

After a brief discussion of maximum power transfer, the chapter closes with a section on nonlinear circuit elements and load-line analysis. This section can be easily skipped in a survey course, and may be picked up later, in conjunction with Chapter 9, if the instructor wishes to devote some attention to load-line analysis of diode circuits. Finally, those instructors who are used to introducing the op-amp as a circuit element, will find that sections 8.1 and 8.2 can be covered together with Chapter 3, and that a good complement of homework problems and exercises devoted to the analysis of the op-amp as a circuit element is provided in Chapter 8.

The homework problems present a graded variety of circuit problems. Since the aim of this chapter is to teach solution techniques, there are relatively few problems devoted to applications. We should call the instructor's attention to the following end-of-chapter problems: 3.8 and 3.19 on the Wheatstone bridge; 3.21, 3.22, 3.23, on three-wire residential distribution service; 3.24, 3.25, 3.26 on AC three-phase electrical distribution systems; 3.28-3.31 on fuses; 3.62-66 on various nonlinear resistance devices.

Learning Objectives

1. Compute the solution of circuits containing linear resistors and independent and dependent sources using *node analysis*.
2. Compute the solution of circuits containing linear resistors and independent and dependent sources using *mesh analysis*.
3. Apply the *principle of superposition* to linear circuits containing independent sources.
4. Compute *Thévenin and Norton equivalent circuits* for networks containing linear resistors and independent and dependent sources.
5. Use equivalent circuits ideas to compute the *maximum power transfer* between a source and a load.
6. Use the concept of equivalent circuit to determine voltage, current and power for nonlinear loads using *load-line analysis* and analytical methods.

Sections 3.1, 3.2, 3.3, 3.4: Nodal and Mesh Analysis

I

cus on Methodology: Node Voltage Analysis Method

1. Select a reference node(usually ground). This node usually has most elements tied to it. All other nodes will be referenced to this node.
2. Define the remaining *n*–1 node voltages as the independent or dependent variables. Each of the *m* voltage sources in the circuit will be associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.

Focus on Methodology: Mesh Current Analysis Method

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction; known mesh currents (i.e., when a current source is present) will always be defined in the direction of the current source.
2. In a circuit with *n* meshes and *m* current sources, *n*–*m* independent equations will result. The unknown mesh currents are the *n*–*m* independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents..
4. Solve the linear system of *n*–*m* unknowns.

Problem 3.1

Solution:

Known quantities:
Circuit shown in Figure P2.1 with mesh currents: $I_1 = 5$ A, $I_2 = 3$ A, $I_3 = 7$ A.

Find:
The branch currents through:

a) R_1,
b) R_2,
c) R_3.

Analysis:
a) Assume a direction for the current through R_1 (e.g., from node A to node B). Then summing currents at node A:

$$KCL: \quad -I_1 + I_{R1} + I_3 = 0$$

$$I_{R1} = I_1 - I_3 = -2 \text{ A}$$

This can also be done by inspection noting that the assumed direction of the current through R_1 and the direction of I_1 are the same.

b) Assume a direction for the current through R_2 (e.g., from node B to node A). Then summing currents at node B:

$$KCL: \quad I_2 + I_{R2} - I_3 = 0$$

$$I_{R2} = I_3 - I_2 = 4 \text{ A}$$

This can also be done by inspection noting that the assumed direction of the current through R_2 and the direction of I_3 are the same.

c) Only one mesh current flows through R_3. If the current through R_3 is assumed to flow in the same direction, then:

$$I_{R1} = I_3 = 7 \text{ A}.$$

Problem 3.2

Solution:

Known quantities:
Circuit shown in Figure P3.1 with source and node voltages:
$V_{S1} = V_{S2} = 110\ V, \quad V_A = 103\ V, \quad V_B = -107\ V$.

Find:
The voltage across each of the five resistors.

Analysis:
Assume a polarity for the voltages across R_1 and R_2 (e.g., from ground to node A, and from node B to ground). R_1 is connected between node A and ground; therefore, the voltage across R_1 is equal to this node voltage. R_2 is connected between node B and ground; therefore, the voltage across R_2 is equal to the negative of this voltage.

$$V_{R1} = V_A = 103 \text{ V}, \quad V_{R2} = -V_B = +107 \text{ V}$$

The two node voltages are with respect to the ground which is given.
Assume a polarity for the voltage across R_3 (e.g., from node B to node A). Then:

KVL: $\quad V_A + V_{R3} + V_B = 0$

$$V_{R3} = V_A - V_B = 210 \text{ V}$$

Assume polarities for the voltages across R_4 and R_5 (e.g., from node A to ground , and from ground to node B):

KVL: $\quad -V_{S1} + V_{R4} + V_A = 0$

$$V_{R4} = V_{S1} - V_A = 7 \text{ V}$$

KVL: $\quad -V_{S2} - V_B - V_{R5} = 0$

$$V_{R5} = -V_{S2} - V_B = -3 \text{ V}$$

Problem 3.3

Solution:

Known quantities:
Circuit shown in Figure P3.3 with known source currents and resistances, $R_1 = 3\Omega, R_2 = 1\Omega, R_3 = 6\Omega$.

Find:
The currents I_1, I_2 using node voltage analysis.

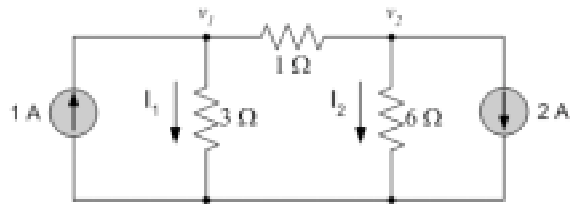
Analysis:

At node 1:

$$v_1 \cdot \left(\frac{1}{3}+1\right)+v_2 \cdot (-1)=1$$

At node 2:

$$v_1 \cdot (-1)+v_2 \cdot \left(1+\frac{1}{6}\right)=-2$$



Solving, we find that:

$$v_1 = -1.5 \text{ V}$$

$$v_2 = -3 \text{ V}$$

Then,

$$i_1 = \frac{v_1}{3} = -0.5 \text{ A}$$

$$i_2 = \frac{v_2}{6} = -0.5 \text{ A}$$

Problem 3.4

Solution:

Known quantities:
Circuit shown in Figure P3.3 with known source currents and resistances, $R_1 = 3\Omega, R_2 = 1\Omega, R_3 = 6\Omega$.

Find:
The currents I_1, I_2 using mesh analysis.

Analysis:

At mesh (a):

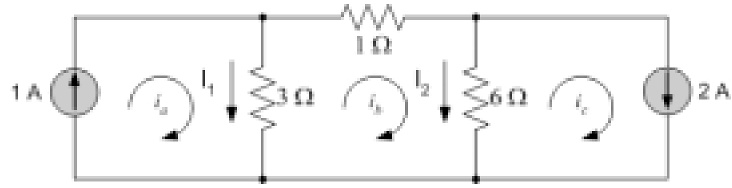
$$i_a = 1 \text{ A}$$

At mesh (b):

$$3(i_b - i_a) + i_b + 6(i_b - i_c) = 0$$

At mesh (c):

$$i_c = 2 \text{ A}$$



Solving, we find that:

$$i_b = 1.5 \text{ A}$$

Then,

$$i_1 = (i_a - i_b) = -0.5 \text{ A}$$

$$i_2 = (i_b - i_c) = -0.5 \text{ A}$$

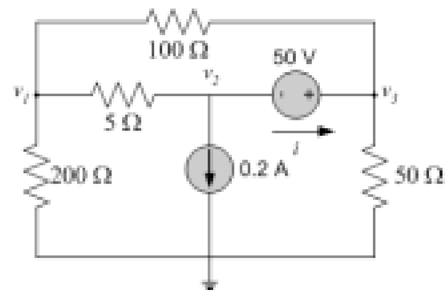
Problem 3.5

Solution:

Known quantities:
Circuit shown in Figure P3.5 with resistance values, current and voltage source values.

Find:
The current, i, through the voltage source using node voltage analysis.

Analysis:



At node 1:

$$\frac{v_1}{200}+\frac{v_1-v_2}{5}+\frac{v_1-v_3}{100}=0$$

At node 2:

$$\frac{v_2-v_1}{5}+i+0.2=0$$

At node 3:

$$-i+\frac{v_3-v_1}{100}+\frac{v_3}{50}=0$$

For the voltage source we have:

$$v_3-v_2=50\text{ V}$$

Solving the system, we obtain:

$v_1=-45.53\text{ V}$, $v_2=-48.69\text{ V}$, $v_3=1.31\text{ V}$ and, finally, $i=491\text{ mA}$.

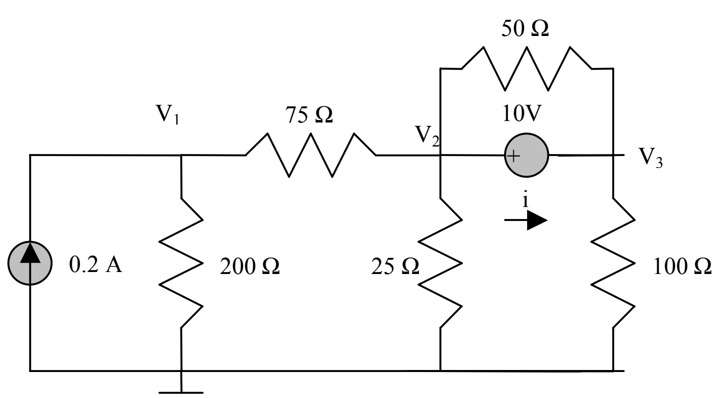
Problem 3.6

Solution:

Known quantities:
The current source value, the voltage source value and the resistance values for the circuit shown in Figure P3.6.

Find:
The three node voltages indicated in Figure P3.6 using node voltage analysis.

Analysis:



At node 1:

$$\frac{v_1}{200}+\frac{v_1-v_2}{75}=0.2\text{ A}$$

At node 2:

$$\frac{v_2-v_1}{75}+\frac{v_2}{25}+\frac{v_2-v_3}{50}+i=0$$

At node 3:

$$-i+\frac{v_3-v_2}{50}+\frac{v_3}{100}=0$$

For the voltage source we have:

$$v_3+10=v_2$$

Solving the system, we obtain:

$v_1=14.24\text{ V}$, $v_2=4.58\text{V}$, $v_3=-5.42\text{ V}$ and, finally, $i=-254\text{ mA}$.

Problem 3.7

Solution:

Known quantities:

The voltage source value, 3 V, and the five resistance values, indicated in Figure P3.7.

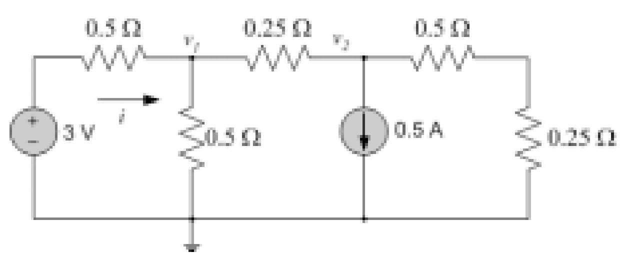
Find:

The current, i, drawn from the independent voltage source using node voltage analysis.

Analysis:

At node 1:

$$\frac{v_1-3}{0.5}+\frac{v_1}{0.5}+\frac{v_1-v_2}{0.25}=0$$



At node 2:

$$\frac{v_2-v_1}{0.25}+\frac{v_2}{0.75}+0.5=0$$

Solving the system, we obtain:

$v_1=1.125\text{ V}$, $v_2=0.75\text{ V}$

Therefore, $i=\dfrac{3-v_1}{0.5}=3.75\text{ A}$.

Problem 3.8

Solution:

Known quantities:

The voltage source value, 15 V, and the four resistance values, indicated in Figure P3.8.

Find:

The voltage at nodes *a* and *b*, V_a and V_b, and their difference, V_a-V_b using node voltage analysis.

Analysis:

Using nodal analysis at the two nodes *a* and *b*, we write the equations

$$\frac{V_b - 15}{18} + \frac{V_b}{20} = 0$$

$$\frac{V_a - 15}{36} + \frac{V_a}{20} = 0$$

Rearranging the equations,

$$38V_b - 300 = 0$$

$$14V_a - 75 = 0$$

Solving for the two unknowns,

$$V_a = 5.36 \text{ V} \quad \text{and} \quad V_b = 7.89 \text{ V}$$

Therefore,

$$V_a - V_b = -2.54 \text{ V}$$

Problem 3.9

Solution:

Known quantities:

The voltage source value, 15 V, and the four resistance values, indicated in Figure P3.8.

Find:

The voltage at nodes *a* and *b*, V_a and V_b , and their difference, $V_a - V_b$ using mesh analysis.

Analysis:

Using mesh analysis at the two meshes *a* and *b*, we write the equations

$$36\left(i_a - i_b\right) + 20\left(i_a - i_b\right) = 15$$

$$18i_b + 20i_b + 20\left(i_b - i_a\right) + 36\left(i_b - i_a\right) = 0$$

Rearranging the equations,

$$i_a = \frac{15}{56} + i_b$$

$$94i_b - 56i_a = 0$$

Solving for the two unknowns,

$$i_a = 662 \text{ mA} \text{ and } i_b = 395 \text{ mA}$$

Therefore,

$$V_a = 20\left(i_b - i_a\right) = 5.36 \text{ V},\ V_b = 20i_b = 7.89 \text{ V} \text{ and } V_a - V_b = -2.54 \text{ V}.$$

Problem 3.10

Solution:

Known quantities:

Circuit of Figure P3.10 with voltage source, V_S , current source, I_S, and all resistances.

Find:

a. The node equations required to determine the node voltages.

b. The matrix solution for each node voltage in terms of the known parameters.

Analysis:

a) Specify the nodes (e.g., A on the upper left corner of the circuit in Figure P3.10, and B on the right corner). Choose one node as the reference or ground node. If possible, ground one of the sources in the circuit. Note that this is possible here. When using KCL, assume all unknown current flow out of the node. The direction of the current supplied by the current source is specified and must flow into node A.

KCL:

$$-I_S + \frac{V_a - V_S}{R_2} + \frac{V_a - V_b}{R_1} = 0$$

$$V_a\left(\frac{1}{R_2} + \frac{1}{R_1}\right) + V_b\left(-\frac{1}{R_1}\right) = I_S + \frac{V_S}{R_2}$$

KCL:

$$\frac{V_b - V_a}{R_1} + \frac{V_b - V_S}{R_3} + \frac{V_b - 0}{R_4} = 0$$

$$V_a\left(-\frac{1}{R_1}\right) + V_b\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{V_S}{R_3}$$

b) Matrix solution:

$$V_a = \frac{\begin{vmatrix} I_S + \dfrac{V_S}{R_2} & -\dfrac{1}{R_1} \\ \dfrac{V_S}{R_3} & \dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4} \end{vmatrix}}{\begin{vmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_1} \\ -\dfrac{1}{R_1} & \dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4} \end{vmatrix}} = \frac{\left(I_S + \dfrac{V_S}{R_2}\right)\left(\dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4}\right) - \left(\dfrac{V_S}{R_3}\right)\left(-\dfrac{1}{R_1}\right)}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\left(\dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4}\right) - \left(-\dfrac{1}{R_1}\right)\left(-\dfrac{1}{R_1}\right)}$$

$$V_b = \frac{\begin{vmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & I_S + \dfrac{V_S}{R_2} \\ -\dfrac{1}{R_1} & \dfrac{V_S}{R_3} \end{vmatrix}}{\begin{vmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_1} \\ -\dfrac{1}{R_1} & \dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4} \end{vmatrix}} = \frac{\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\left(\dfrac{V_S}{R_3}\right) - \left(-\dfrac{1}{R_1}\right)\left(I_S + \dfrac{V_S}{R_2}\right)}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\left(\dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4}\right) - \left(-\dfrac{1}{R_1}\right)\left(-\dfrac{1}{R_1}\right)}$$

Notes:

1. The denominators are the same for both solutions.
2. The main diagonal of a matrix is the one that goes to the right and down.
3. The denominator matrix is the "conductance" matrix and has certain properties:
   a) The elements on the main diagonal [i(row) = j(column)] include all the conductance connected to node i = j.

b) The off-diagonal elements are all negative.
c) The off-diagonal elements are all symmetric, i.e., the i j-th element = j i-th element. This is true only because there are no controlled (dependent) sources in this circuit.
d) The off-diagonal elements include all the conductance connected between node i [row] and node j [column].

Problem 3.11

Solution:

Known quantities:
Circuit shown in Figure P3.11

$$V_{S1} = V_{S2} = 110 \text{ V}$$

$$R_1 = 500 \text{ m}\Omega \quad R_2 = 167 \text{ m}\Omega$$

$$R_3 = 700 \text{ m}\Omega$$

$$R_4 = 200 \text{ m}\Omega \quad R_5 = 333 \text{ m}\Omega$$

Find:

a. The most efficient way to solve for the voltage across R_3. Prove your case.

b. The voltage across R_3.

Analysis:

a) There are 3 meshes and 3 mesh currents requiring the solution of 3 simultaneous equations. Only one of these mesh currents is required to determine, using Ohm's Law, the voltage across R_3.
In the terminal (or node) between the two voltage sources is made the ground (or reference) node, then three node voltages are known (the ground or reference voltage and the two source voltages). This leaves only two unknown node voltages (the voltages across R_1, V_{R1}, and across R_2, V_{R2}). Both these voltages are required to determine, using KVL, the voltage across R_3, V_{R3}.
A difficult choice. Choose node analysis due to the smaller number of unknowns. Specify the nodes. Choose one node as the ground node. In KCL, assume unknown currents flow out.
b)

$$KCL: \quad \frac{V_{R1} - V_{S1}}{R_4} + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0$$

$$KCL: \quad \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0$$

$$V_{R1}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) + V_{R2}\left(-\frac{1}{R_3}\right) = \frac{V_{S1}}{R_4}$$

$$V_{R1}\left(-\frac{1}{R_3}\right) + V_{R2}\left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3}\right) = -\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}=\frac{1}{500\cdot 10^{-3}}+\frac{1}{700\cdot 10^{-3}}+\frac{1}{200\cdot 10^{-3}}=8.43\,\Omega^{-1}$$

$$\frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{333\cdot 10^{-3}}+\frac{1}{167\cdot 10^{-3}}+\frac{1}{700\cdot 10^{-3}}=10.42\,\Omega^{-1}$$

$$\frac{1}{R_3}=\frac{1}{700\cdot 10^{-3}}=1.43\,\Omega^{-1}$$

$$\frac{V_{S1}}{R_4}=\frac{110}{200\cdot 10^{-3}}=550\text{ A}\qquad \frac{V_{S2}}{R_5}=\frac{110}{333\cdot 10^{-3}}=330\text{ A}$$

$$V_{R1}=\frac{\begin{vmatrix}550 & -1.43\\ -330 & 10.42\end{vmatrix}}{\begin{vmatrix}8.43 & -1.43\\ -1.43 & 10.42\end{vmatrix}}=\frac{(5731)-(472)}{(87.84)-(2.04)}=61.30\text{ V}$$

$$V_{R1}=\frac{\begin{vmatrix}8.429 & 550\\ -1.429 & -330\end{vmatrix}}{85.790}=\frac{(-2782)-(-786)}{85.80}=-23.26\text{ V}$$

KVL: $-V_{R1}+V_{R3}+V_{R2}=0 \qquad V_{R3}=V_{R1}-V_{R2}=84.59\text{ V}$

Problem 3.12

Solution:

Known quantities:

Circuit shown in Figure P3.12

$V_{S2}=\text{kT}\qquad \text{k}=10\text{ V/°C}$

$V_{S1}=24\text{ V}\qquad R_S=R_1=12\text{ k}\Omega$

$R_2=3\text{ k}\Omega\qquad R_3=10\text{ k}\Omega$

$R_4=24\text{ k}\Omega\qquad V_{R3}=-2.524\text{ V}$

The voltage across R_3, which is given, indicates the temperature.

Find:

The temperature, T.

Analysis:

Specify nodes (A between R_1 and R_3, C between R_3 and R_2) and polarities of voltages (V_A from ground to A, V_C from ground to C, and V_{R3} from C to A). When using KCL, assume unknown currents flow out.

KVL:
$$-V_A+V_{R3}+V_C=0$$
$$V_C=V_A-V_{R3}$$

Now write KCL at node C, substitute for V_C, solve for V_A:

KCL: $$\frac{V_C-V_{S1}}{R_2}+\frac{V_C-V_A}{R_3}+\frac{V_C}{R_4}=0$$

$$-\frac{V_A}{R_3}-\frac{V_{S1}}{R_2}+\left(V_A-V_{R3}\right)\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)=0$$

$$V_A=\frac{\frac{V_{S1}}{R_2}+V_{R3}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}{\frac{1}{R_2}+\frac{1}{R_4}}=\frac{\frac{24}{3\cdot 10^3}+\left(-2.524\right)\left(\frac{1}{3\cdot 10^3}+\frac{1}{10\cdot 10^3}+\frac{1}{24\cdot 10^3}\right)}{\frac{1}{3\cdot 10^3}+\frac{1}{24\cdot 10^3}}=18.14\text{ V}$$

$$V_C=V_A-V_{R3}=18.14-\left(-2.524\right)=20.66\text{ V}$$

Now write KCL at node A and solve for V_{S2} and T:

$$KCL:\quad \frac{V_A-V_{S1}}{R_1}+\frac{V_A-V_{S2}}{R_S}+\frac{V_A-V_C}{R_3}=0$$

$$V_{S2}=V_A+\frac{R_S}{R_1}\left(V_A-V_{S1}\right)+\frac{R_S}{R_3}\left(V_A-V_C\right)=$$

$$=18.14+\frac{12\cdot 10^3}{12\cdot 10^3}\left(18.14-24\right)+\frac{12\cdot 10^3}{10\cdot 10^3}\left(18.14-20.66\right)=9.26\text{ V}$$

$$\text{T}=\frac{V_{S2}}{\text{k}}=\frac{9.26}{10}=0.926\text{ °C}$$

Problem 3.13

Solution:

Known quantities:
Circuit shown in Figure P3.13

$V_S=5\text{ V}$ $A_V=70$ $R_1=2.2\text{ k}\Omega$

$R_2=1.8\text{ k}\Omega$ $R_3=6.8\text{ k}\Omega$ $R_4=220\,\Omega$

Find:
The voltage across R_4 using KCL and node voltage analysis.

Analysis:
A node analysis is not a method of choice because the dependent source is [1] a voltage source and [2] a floating source. Both factors cause difficulties in a node analysis. A ground is specified. There are three unknown node voltages, one of which is the voltage across R_4. The dependent source will introduce two additional unknowns, the current through the source and the controlling voltage (across R_1) that is not a node voltage. Therefore 5 equations are required:

$$[1]\,KCL \quad \frac{V_1 - V_S}{R_1} + \frac{V_1 - V_3}{R_3} + \frac{V_1 - V_2}{R_2} = 0$$

$$[2]\,KCL \quad \frac{V_2 - V_1}{R_2} - I_{CS} = 0$$

$$[3]\,KCL \quad \frac{V_3 - V_1}{R_3} + I_{CS} + \frac{V_3}{R_4} = 0$$

$$[4]\,KVL \quad -V_S + V_{R1} + V_1 = 0 \qquad V_{R1} = V_S - V_1$$

$$[5]\,KVL \quad -V_3 - A_V V_{R1} + V_2 = 0 \qquad V_2 = V_3 + A_V V_{R1} = V_3 + A_V \left(V_S - V_1\right)$$

Substitute using Equation [5] into Equations [1], [2] and [3] and eliminate V_2 (because it only appears twice in these equations). Collect terms:

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{A_V}{R_2}\right) + V_3\left(-\frac{1}{R_3} - \frac{1}{R_2}\right) + I_{CS}(0) = \frac{V_S}{R_1} + \frac{V_S A_V}{R_2}$$

$$V_1\left(-\frac{1}{R_2} - \frac{A_V}{R_2}\right) + V_3\left(\frac{1}{R_2}\right) + I_{CS}(-1) = -\frac{V_S A_V}{R_2}$$

$$V_1\left(-\frac{1}{R_3}\right) + V_3\left(\frac{1}{R_3} + \frac{1}{R_4}\right) + I_{CS}(+1) = 0$$

$$\frac{1}{R_2} = \frac{1}{1.8 \cdot 10^3} = 555.6 \cdot 10^{-6}\ \Omega^{-1} \qquad \frac{1}{R_3} = \frac{1}{6.8 \cdot 10^3} = 147.1 \cdot 10^{-6}\ \Omega^{-1}$$

$$\frac{1}{R_3} + \frac{1}{R_2} = \frac{1}{6.8 \cdot 10^3} + \frac{1}{1.8 \cdot 10^3} = 702.6 \cdot 10^{-6}\ \Omega^{-1}$$

$$\frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{6.8 \cdot 10^3} + \frac{1}{0.22 \cdot 10^3} = 4.69 \cdot 10^{-3}\ \Omega^{-1} \qquad \frac{1}{R_2} + \frac{A_V}{R_2} = \frac{1+70}{1.8 \cdot 10^3} = 39.44 \cdot 10^{-3}\ \Omega^{-1}$$

$$\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{A_V}{R_2} = \frac{1}{2.2 \cdot 10^3} + \frac{1}{6.8 \cdot 10^3} + \frac{1+70}{1.8 \cdot 10^3} = 40.05 \cdot 10^{-3}\ \Omega^{-1}$$

$$\frac{V_S A_V}{R_2} = \frac{(5)(70)}{1.8 \cdot 10^3} = 194.4\ \text{mA} \qquad \frac{V_S}{R_1} + \frac{V_S A_V}{R_2} = \frac{5}{2.2 \cdot 10^3} + \frac{(5)(70)}{1.8 \cdot 10^3} = 196.7\ \text{mA}$$

Solving, we have:

$$V_{R4} = V_3 = 5.1\ \text{mV}$$

Note:

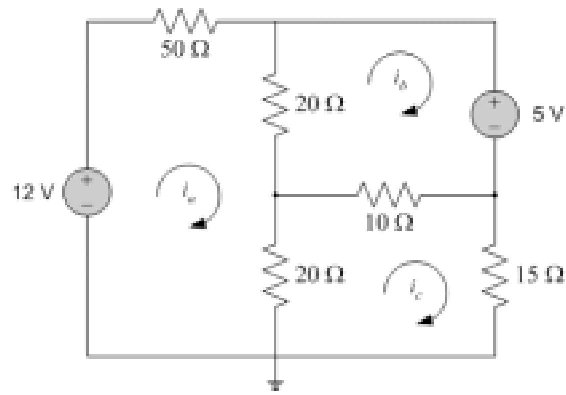
1. This solution was not difficult in terms of theory, but was terribly long and arithmetically cumbersome. This was because the wrong method was used. There are only 2 mesh currents in the circuit; the sources were voltage sources; therefore, a mesh analysis is the method of choice.
2. In general, a node analysis will have fewer unknowns (because one node is the ground or reference node) and will, in such cases, be preferable.

Problem 3.14

Solution:

Known quantities:
The values of the resistors and of the voltage sources (see Figure P3.14).

Find:
The voltage across the $10\ \Omega$ resistor in the circuit of Figure P3.14 using mesh current analysis.



Analysis:

For mesh (a):

$$i_a(50+20+20)-i_b(20)-i_c(20)=12$$

For mesh (b):

$$-i_a(20)+i_b(20+10)-i_c(10)+5=0$$

For mesh (c):

$$-i_a(20)-i_b(10)+i_c(20+10+15)=0$$

Solving,

$$i_a=127.5\text{ mA}$$
$$i_b=-67.8\text{ mA}$$
$$i_c=41.6\text{ mA}$$

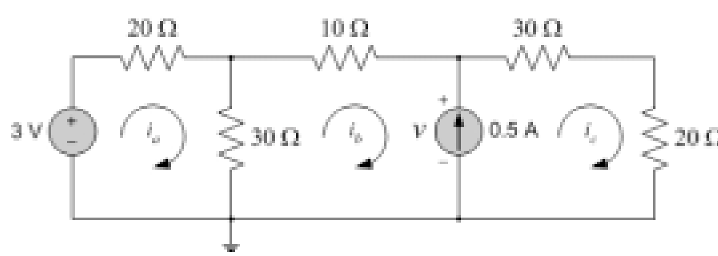
and $v_{R_4}=10\left(i_b-i_c\right)=10\left(-0.109\right)=-1.09\ \text{V}$.

Problem 3.15

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.15.

Find:
The voltage across the current source using mesh current analysis.



Analysis:
For mesh (a):

$$i_a(20+30)+i_b(-30)=3$$

For meshes (b) and (c):

$$i_a(-30)+i_b(10+30)+i_c(30+20)=0$$

For the current source:

$$i_c-i_b=0.5$$

Solving,

$$i_a = -133 \text{ mA},\ i_b = -322 \text{ mA} \text{ and } i_c = 178 \text{ mA}.$$

Therefore,

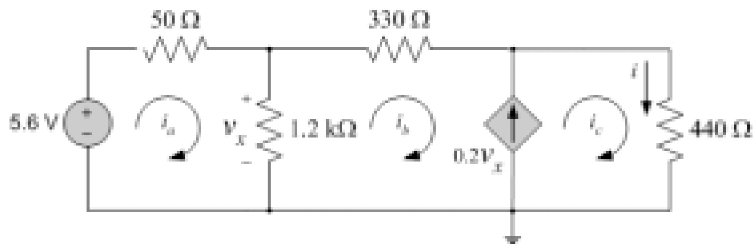
$$v = i_c(30+20) = 8.89 \text{ V}.$$

Problem 3.16

Solution:

Known quantities:
The values of the resistors and of the voltage source in the circuit of Figure P3.16.

Find:
The current i through the resistance R_4 mesh current analysis.



Analysis:
For mesh (a):

$$i_a(50+1200) + i_b(-1200) = 5.6$$

For meshes (b) and (c):

$$i_a(-1200) + i_b(1200+330) + i_c(440) = 0$$

For the current source:

$$i_c - i_b = 0.2v_x = 0.2\left(1200\left(i_a - i_b\right)\right) = 240\left(i_a - i_b\right)$$

Solving,

$$i_a = 136 \text{ mA},\ i_b = 137 \text{ mA} \text{ and } i_c = -106 \text{ mA}.$$

Therefore,

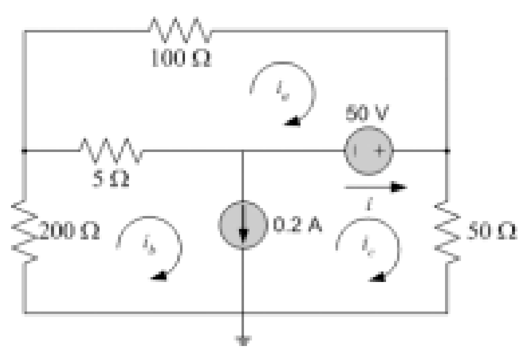
$$i = i_c = -106 \text{ mA}.$$

Problem 3.17

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.5.

Find:
The current through the voltage source using mesh current analysis.



Analysis:
For mesh (a):

$$i_a(100+5) + i_b(-5) + 50 = 0$$

For the current source:

$$i_b - i_c = 0.2$$

For meshes (b) and (c):

$$-i_a(5)+i_b(200+5)+i_c(50)=50$$

Solving,

$$i_a = -465 \text{ mA},\ i_b = 226 \text{ mA} \text{ and } i_c = 26 \text{ mA}.$$

Therefore,

$$i = i_c - i_a = 491 \text{ mA}.$$

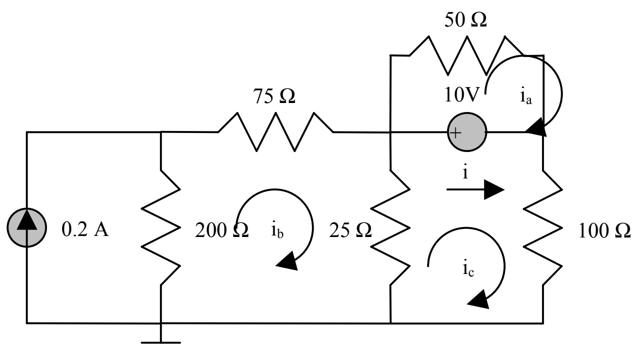
Problem 3.18

Solution:

Known quantities:
The values of the resistors and of the current source in the circuit of Figure P3.6.

Find:
The current through the voltage source in the circuit of Figure P3.6 using mesh current analysis.

Analysis:



For mesh (a):

$$i_a(100)+10=0$$

For mesh (b):

$$i_b(200+75+25)+i_c(-25)+0.2(-200)=0$$

For mesh (c):

$$i_b(-25)+i_c(50+25)=10$$

Solving,

$$i_a = -100 \text{ mA},\ i_b = 148 \text{ mA} \text{ and } i_c = 183 \text{ mA}.$$

Therefore,

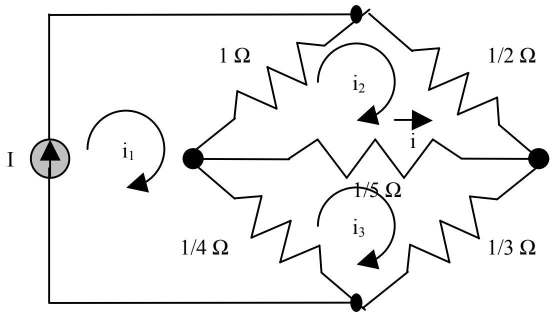
$$i = i_c - i_a = 283 \text{ mA}$$

Problem 3.19

Solution:

Known quantities:
The values of the resistors in the circuit of Figure P3.19.

Find:
The current in the circuit of Figure P3.19 using mesh current analysis.
Analysis:



Since I is unknown, the problem will be solved in terms of this current.

For mesh #1, it is obvious that:

$$i_1 = I$$

For mesh #2:

$$i_1(-1) + i_2\left(1 + \frac{1}{2} + \frac{1}{5}\right) + i_3\left(-\frac{1}{5}\right) = 0$$

For mesh #3:

$$i_1\left(-\frac{1}{4}\right) + i_2\left(-\frac{1}{5}\right) + i_3\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{5}\right) = 0$$

Solving,

$$i_2 = 0.645I$$
$$i_3 = 0.483I$$

Then, $i = i_3 - i_2$

and $i = 0.483I - 0.645I = -0.163I$

Problem 3.20

Solution:

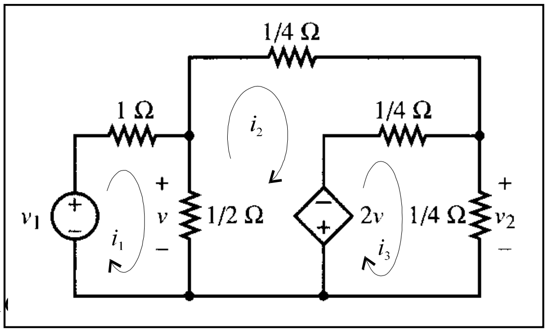
Known quantities:
The values of the resistors of the circuit in Figure P3.20.
Find:

The voltage gain, $A_V = \dfrac{v_2}{v_1}$, in the circuit of Figure P3.20

using mesh current analysis.
Analysis:

Note that $v = \dfrac{i_1 - i_2}{2}$

For mesh #1:



$$i_1\left(1+\frac{1}{2}\right)+i_2\left(-\frac{1}{2}\right)+i_3(0)=v_1$$

For mesh #2:

$$i_1\left(-\frac{1}{2}\right)+i_2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)+i_3\left(-\frac{1}{4}\right)=2v$$

or

$$i_1(-1.5)+i_2(2)+i_3(-0.25)=0$$

For mesh #3:

$$i_1(0)+i_2\left(-\frac{1}{4}\right)+i_3\left(\frac{1}{4}+\frac{1}{4}\right)=-2v$$

or

$$i_1(1)+i_2(-1.25)+i_3(0.5)=0$$

Solving, $i_3=-0.16v_1$

from which $v_2=\frac{1}{4}i_3=-0.04v_1$

and $A_V=\frac{v_2}{v_1}=-0.04$

Problem 3.21

Solution:

Known quantities:

Circuit in Figure P3.21 and the values of the voltage sources, $V_{S1}=V_{S2}=450\text{ V}$, and the values of the 5 resistors:

$$R_1=8\,\Omega \qquad R_2=5\,\Omega$$

$$R_4=R_5=0.25\,\Omega \qquad R_3=32\,\Omega$$

Find:

The voltages across R_1, R_2 and R_3 using KCL and node analysis.

Analysis:

Choose a ground/reference node. The node common to the two voltage sources is the best choice. Specify polarity of voltages and direction of the currents.

KCL: $$\frac{V_{R1}-V_{S1}}{R_4}+\frac{V_{R1}-0}{R_1}+\frac{V_{R1}-V_{R2}}{R_3}=0$$

KCL: $$\frac{V_{R2}-(-V_{S2})}{R_5}+\frac{V_{R2}-0}{R_2}+\frac{V_{R2}-V_{R1}}{R_3}=0$$

Collect terms in terms of the unknown node voltages:

$$V_{R1}\left(\frac{1}{R_4}+\frac{1}{R_1}+\frac{1}{R_3}\right)+V_{R2}\left(-\frac{1}{R_3}\right)=\frac{V_{S1}}{R_4}$$

$$V_{R1}\left(-\frac{1}{R_3}\right)+V_{R2}\left(\frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}\right)=-\frac{V_{S2}}{R_5}$$

Evaluate the coefficients of the unknown node voltages:

$$\frac{V_{S1}}{R_4}=\frac{V_{S2}}{R_5}=\frac{450}{0.25}=1.8\text{ kA} \qquad \frac{1}{R_3}=\frac{1}{32}=0.03125\,\Omega^{-1}$$

$$\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_3}=\frac{1}{0.25}+\frac{1}{8}+\frac{1}{32}=4.14\,\Omega^{-1}$$

$$\frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{0.25}+\frac{1}{5}+\frac{1}{32}=4.23\,\Omega^{-1}$$

$$V_{R1}=\frac{\begin{vmatrix}1800 & -31.25\cdot10^{-3}\\ -1800 & 4.23\end{vmatrix}}{\begin{vmatrix}4.16 & -31.25\cdot10^{-3}\\ -31.25\cdot10^{-3} & 4.23\end{vmatrix}}=429.5\text{ V}$$

$$V_{R2}=\frac{\begin{vmatrix}4.156 & 1800\\ -31.25\cdot10^{-3} & -1800\end{vmatrix}}{17.59}=-422.2\text{ V}$$

KVL:
$$-V_{R1}+V_{R3}+V_{R2}=0$$
$$V_{R3}=V_{R1}-V_{R2}=852.0\text{ V}$$

Problem 3.22

Solution:

Known quantities:

Circuit in Figure P3.22 with the values of the voltage sources, $V_{S1}=V_{S2}=115\text{ V}$, and the values of the 5 resistors:

$$R_1=R_2=5\,\Omega \qquad R_3=10\,\Omega$$
$$R_4=R_5=200\text{ m}\Omega$$

Find:

The new voltages across R_1, R_2 and R_3, in case F_1 "blows" or opens using KCL and node analysis.

Analysis:

Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

KCL: $$0+\frac{V_{R1}-0}{R_1}+\frac{V_{R1}-V_{R2}}{R_3}=0$$

KCL: $$\frac{V_{R2}-(-V_{S2})}{R_5}+\frac{V_{R2}-0}{R_2}+\frac{V_{R2}-V_{R1}}{R_3}=0$$

Collect terms in unknown node voltages:

$$V_{R1}\left(\frac{1}{R_1}+\frac{1}{R_3}\right)+V_{R2}\left(-\frac{1}{R_3}\right)=0$$

$$V_{R1}\left(-\frac{1}{R_3}\right)+V_{R2}\left(\frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}\right)=-\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_3}=\frac{1}{10}=0.1\,\Omega^{-1} \qquad \frac{1}{R_1}+\frac{1}{R_3}=0.3\,\Omega^{-1}$$

$$\frac{V_{S2}}{R_5}=\frac{115}{200\cdot 10^{-3}}=575\text{ A} \qquad \frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}=5.3\,\Omega^{-1}$$

$$V_{R1}=\frac{\begin{vmatrix}0 & -0.1\\ -575 & 5.3\end{vmatrix}}{\begin{vmatrix}0.3 & -0.1\\ -0.1 & 5.3\end{vmatrix}}=\frac{(0)-(57.5)}{(1.59)-(0.01)}=-36.39\text{ V}$$

$$V_{R2}=\frac{\begin{vmatrix}0.3 & 0\\ --0.1 & -575\end{vmatrix}}{1.58}=\frac{(-172.5)-(0)}{1.58}=-109.2\text{ V}$$

KVL:
$$-V_{R1}+V_{R3}+V_{R2}=0$$
$$V_{R3}=V_{R1}-V_{R2}=72.81\text{ V}$$

KVL:
$$-V_{S1}+V_{R4}+V_F+V_{R1}=0 \qquad V_{R4}=I_1R_4=0$$
$$V_F=115-0-(-36.39)=151.4\text{ V}$$

Note the voltages are strongly dependent on the loads (R_1, R_2 and R_3) connected at the time the fuse blows. With other loads, the result will be quite different.

Problem 3.23

Solution:

Known quantities:

Circuit in Figure P3.22 and the values of the voltage sources, $V_{S1}=V_{S2}=120\text{ V}$, and the values of the 5 resistors:

$$R_1=R_2=2\,\Omega \qquad R_3=8\,\Omega$$
$$R_4=R_5=250\text{ m}\Omega$$

Find:

The voltages across R_1, R_2, R_3, and F_1 in case F_1 "blows" or opens using KCL and node analysis.

Analysis:

Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

KCL: $$0+\frac{V_{R1}-0}{R_1}+\frac{V_{R1}-V_{R2}}{R_3}=0$$

KCL: $$\frac{V_{R2}-(-V_{S2})}{R_5}+\frac{V_{R2}-0}{R_2}+\frac{V_{R2}-V_{R1}}{R_3}=0$$

$$V_{R1}\left(\frac{1}{R_1}+\frac{1}{R_3}\right)+V_{R2}\left(-\frac{1}{R_3}\right)=0$$

$$V_{R1}\left(-\frac{1}{R_3}\right)+V_{R2}\left(\frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}\right)=-\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_3}=\frac{1}{8}=0.125\,\Omega^{-1} \qquad \frac{1}{R_1}+\frac{1}{R_3}=0.625\,\Omega^{-1}$$

$$\frac{V_{S2}}{R_5}=\frac{120}{250\cdot10^{-3}}=480\text{ A} \qquad \frac{1}{R_5}+\frac{1}{R_2}+\frac{1}{R_3}=4.625\,\Omega^{-1}$$

$$V_{R1}=\frac{\begin{vmatrix}0 & -0.125\\ -480 & 4.625\end{vmatrix}}{\begin{vmatrix}0.625 & -0.125\\ -0.125 & 4.625\end{vmatrix}}=\frac{(0)-(60)}{(2.89)-(0.016)}=-20.87\text{ V}$$

$$V_{R2}=\frac{\begin{vmatrix}0.625 & 0\\ -0.125 & -480\end{vmatrix}}{2.87}=\frac{(-300)-(0)}{2.87}=-104.35\text{ V}$$

KVL: $$-V_{R1}+V_{R3}+V_{R2}=0$$
$$V_{R3}=V_{R1}-V_{R2}=83.48\text{ V}$$

KVL: $$-V_{S1}+V_{R4}+V_F+V_{R1}=0 \qquad V_{R4}=I_1R_4=0$$
$$V_F=120-0-(-20.87)=140.9\text{ V}$$

Problem 3.24

Solution:

Known quantities:

The values of the voltage sources, $V_{S1}=V_{S2}=V_{S3}=170\text{ V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1}=R_{W2}=R_{W3}=0.7\,\Omega$$
$$R_1=1.9\,\Omega \qquad R_2=2.3\,\Omega \qquad R_3=11\,\Omega$$

Find:

a. The number of unknown node voltages and mesh currents.
b. Unknown node voltages.

Analysis:

If the node common to the three sources is chosen as the ground/reference node, and the series resistances are combined into single equivalent resistances, there is only one unknown node voltage. On the other hand, there are two unknown mesh currents. A node analysis is the method of choice! Specify polarity of voltages and direction of currents.

$$R_{eq1} = R_{W1} + R_1 = 2.6\,\Omega \qquad R_{eq2} = R_{W2} + R_2 = 3.0\,\Omega$$
$$R_{eq3} = R_{W3} + R_3 = 11.7\,\Omega$$

KCL:

$$\frac{V_N - V_{S1}}{R_{eq1}} + \frac{V_N - (-V_{S2})}{R_{eq2}} + \frac{V_N - V_{S3}}{R_{eq3}} = 0$$

$$V_N = \frac{\dfrac{V_{S1}}{R_{eq1}} - \dfrac{V_{S2}}{R_{eq2}} + \dfrac{V_{S3}}{R_{eq3}}}{\dfrac{1}{R_{eq1}} + \dfrac{1}{R_{eq2}} + \dfrac{1}{R_{eq3}}} = \frac{\dfrac{170}{2.6} - \dfrac{170}{3.0} + \dfrac{170}{11.7}}{\dfrac{1}{2.6} + \dfrac{1}{3.0} + \dfrac{1}{11.7}} = 28.94\text{ V}$$

KVL:

$$-V_{S1} + I_1 R_{W1} + I_1 R_1 + V_N = 0$$
$$I_1 = \frac{V_{S1} - V_N}{R_{W1} + R_1} = \frac{170 - 28.94}{2.6} = 54.26\text{ A}$$

Problem 3.25

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170\text{ V}$, the common node voltage, $V_N = 28.94\text{ V}$,and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 0.7\,\Omega$$
$$R_1 = 1.9\,\Omega \qquad R_2 = 2.3\,\Omega \qquad R_3 = 11\,\Omega$$

Find:

The current through and voltage across R_1.

Analysis:

KVL:

$$-V_{S1} + I_1 R_{W1} + I_1 R_1 + V_N = 0$$
$$I_1 = \frac{V_{S1} - V_N}{R_{W1} + R_1} = \frac{170 - 28.94}{2.6} = 54.26\text{ A}$$

OL: $V_{R1} = I_1 R_1 = (54.26)(1.9) = 103.1\text{ V}$

Problem 3.26

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170\text{ V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 0.7\,\Omega$$
$$R_1 = 1.9\,\Omega \qquad R_2 = 2.3\,\Omega \qquad R_3 = 11\,\Omega$$

Find:
The mesh (or loop) equations and any additional equation required to determine the current through R_1 in the circuit shown in Figure P3.24.

Analysis:

KVL: $-V_{S1} + I_1 R_{W1} + I_1 R_1 + (I_1 - I_2)R_2 + (I_1 - I_2)R_{W2} - V_{S2} = 0$

KVL: $V_{S2} + (I_2 - I_1)R_{W2} + (I_2 - I_1)R_2 + I_2 R_3 + I_2 R_{W3} + V_{S3} = 0$

$$I_1(R_1 + R_{W1} + R_2 + R_{W2}) + I_2(-R_2 - R_{W2}) = V_{S1} + V_{S2}$$
$$I_1(-R_2 - R_{W2}) + I_2(R_2 + R_{W2} + R_3 + R_{W3}) = -V_{S2} - V_{S3}$$
$$I_{R1} = I_1 = \frac{\begin{vmatrix} (V_{S1} + V_{S2}) & -(R_2 + R_{W2}) \\ -(V_{S2} + V_{S3}) & (R_2 + R_{W2} + R_3 + R_{W3}) \end{vmatrix}}{\begin{vmatrix} (R_1 + R_{W1} + R_2 + R_{W2}) & -(R_2 + R_{W2}) \\ -(R_2 + R_{W2}) & (R_2 + R_{W2} + R_3 + R_{W3}) \end{vmatrix}}$$

Problem 3.27

Solution:

Known quantities:
The values of the voltage sources, $V_{S1} = 90\text{ V}$, $V_{S2} = V_{S3} = 110\text{ V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 1.3\,\Omega$$
$$R_1 = 7.9\,\Omega \qquad R_2 = R_3 = 3.7\,\Omega$$

Find:
The branch currents, using KVL and loop analysis.

Analysis:
Three equations are required. Voltages will be summed around the 2 loops that are meshes, and KCL at the common node between the resistances. Assume directions of the branch currents and the associated polarities of the voltages. After like terms are collected:

KVL: $-V_{S1} + I_1 R_{W1} + I_1 R_1 + (I_1 - I_2)R_2 + (I_1 - I_2)R_{W2} - V_{S2} = 0$

KVL: $V_{S2} + (I_2 - I_1)R_{W2} + (I_2 - I_1)R_2 + I_2 R_3 + I_2 R_{W3} - V_{S3} = 0$

KCL: $I_3 = I_1 - I_2$

Plugging in the given parameters results in the following system of equations:

$$14.2 I_1 - 5.0 I_2 = 200$$
$$5.0 I_1 - 10.0 I_2 = 0$$
$$I_3 = I_1 - I_2$$

Solving the system of equations gives:

$$I_1 = 17.09\text{ A}$$
$$I_2 = 8.55\text{ A}$$

$I_3 = 8.55$ A

Hence, the assumed polarity of the second and third branch currents is actually reversed.

Problem 3.28

Solution:

Known quantities:
The values of the voltage sources, $V_{S1} = V_{S2} = 115$ V , and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 \quad R_2 \quad 5$$

$$R_3 \quad 10 \qquad\qquad R_4 \quad R_5 \quad 200\ \text{m}$$

Find:
The voltages across R_1, R_2 and R_3, under normal conditions, i.e., no blown fuses using KVL and a mesh analysis.

Analysis:
KVL:

$$I_1\ R_1 \quad R_4 \quad R_1 I_3 \quad V_{s1}$$

$$I_2\ R_2 \quad R_5 \quad R_2 I_3 \quad V_{s2}$$

$$-I_1 R_1 \quad I_2 R_2 \quad R_1 \quad R_2 \quad R_3\ I_3 \quad 0$$

Rearranging the above equations:

$$5.2I_1 - 5I_3 = 115$$
$$5.2I_2 - 5I_3 = 115$$
$$5I_1 + 5I_2 - 20I_3 = 0$$

$$I_1 = I_2 \qquad I_1 = 2I_3$$

$$I_1 = I_2 = \frac{\begin{vmatrix} 115 & 0 & -5 \\ 115 & 5.2 & -5 \\ 0 & 5 & -20 \end{vmatrix}}{\begin{vmatrix} 5.2 & 0 & -5 \\ 0 & 5.2 & -5 \\ 5 & 5 & -20 \end{vmatrix}} = \frac{-11960}{-280.8} = 42.6\ \text{A} \qquad\qquad I_3 = 21.3\ \text{A}$$

$$V_{R1} = R_1\left(I_1 - I_3\right) = 106.5\ \text{V}$$
$$V_{R2} = R_2\left(I_3 - I_2\right) = -106.5\ \text{V}$$
$$V_{R3} = R_3 I_3 = 213\ \text{V}$$

Problem 3.29

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = 110 \text{ V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 = 100\,\Omega \qquad R_2 = 22\,\Omega \qquad R_3 = 70\,\Omega$$

$$R_4 = R_5 = 13\,\Omega$$

Find:

The voltage across R_1 using KVL and mesh analysis.

Analysis:

KVL:

$$I_1(R_1 + R_4) - R_1 I_3 = V_{S1}$$

$$I_2(R_2 + R_5) - R_2 I_3 = V_{S2}$$

$$-I_1 R_1 - I_2 R_2 + (R_1 + R_2 + R_3) I_3 = 0$$

Rearranging the above equations:

$$113 I_1 - 100 I_3 = 110$$

$$35 I_2 - 22 I_3 = 110$$

$$100 I_1 + 22 I_2 - 192 I_3 = 0$$

$$I_1 = \frac{\begin{vmatrix} 110 & 0 & -100 \\ 110 & 35 & -22 \\ 0 & 22 & -192 \end{vmatrix}}{\begin{vmatrix} 113 & 0 & -100 \\ 0 & 35 & -22 \\ 100 & 22 & -192 \end{vmatrix}} = \frac{-935660}{-354668} = 2.64 \text{ A} \qquad I_3 = 1.88 \text{ A}$$

$$V_{R1} = R_1 (I_1 - I_3) = 75.89 \text{ V}$$

Problem 3.30

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = 115 \text{ V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 = 5\ \Omega \qquad R_2 = 5\ \Omega \qquad R_3 = 10\ \Omega$$

$$R_4 = R_5 = 0.2\ \Omega$$

Find:

The voltage across R_1 using KVL and mesh analysis.

Analysis:
Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

$$I_1 = 0$$

KVL: $-V_{S2} + (I_2 - I_3)R_2 + I_2 R_5 = 0$

KVL: $I_3 R_1 + I_3 R_3 + (I_3 - I_2)R_2 = 0$

$$I_2(R_2 + R_5) + I_3(-R_2) = V_{S2}$$
$$I_2(-R_2) + I_3(R_1 + R_2 + R_3) = 0$$

$$R_2 + R_5 = 5.2\,\Omega$$
$$R_1 + R_2 + R_3 = 20\,\Omega$$

$$I_2 = \frac{\begin{vmatrix} 115 & -5 \\ 0 & 20 \end{vmatrix}}{\begin{vmatrix} 5.2 & -5 \\ -5 & 20 \end{vmatrix}} = \frac{(2300)-(0)}{(104)-(25)} = 29.11 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 5.2 & 115 \\ -5 & 0 \end{vmatrix}}{\begin{vmatrix} 5.2 & -5 \\ -5 & 20 \end{vmatrix}} = \frac{(0)-(-575)}{79} = 7.28 \text{ A}$$

$$V_{R1} = I_{R1}R_1 = -I_3 R_1 = -36.39 \text{ V}$$
$$V_{R2} = I_{R2}R_2 = (I_3 - I_2)R_2 = -109.16 \text{ V}$$
$$V_{R3} = I_{R3}R_3 = I_3 R_3 = 72.78 \text{ V}$$

KVL: $-V_{S1} + I_1 R_4 + V_F + V_{R1} = 0$

$$V_F = 151.39 \text{ V}$$

Problem 3.31

Solution:

Known quantities:
The values of the voltage sources, $V_{S1} = V_{S2} = 115 \text{ V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 = 4\,\Omega \qquad R_2 = 7.5\,\Omega \qquad R_3 = 12.5\,\Omega$$
$$R_4 = R_5 = 1\,\Omega$$

Find:
The voltages across R_1, R_2, R_3, and across the open fuse using KVL and mesh analysis.

Analysis:
Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

$$I_1 = 0$$

KVL:

$$I_2(R_2 + R_5) - R_2 I_3 = V_{S2}$$
$$-I_2 R_2 + (R_1 + R_2 + R_3) I_3 = 0$$

Rearranging the above equations:

$$8.5I_2 - 7.5I_3 = 115$$
$$7.5I_2 - 24I_3 = 0$$

$$I_2 = \frac{\begin{vmatrix} 115 & -7.5 \\ 0 & -24 \end{vmatrix}}{\begin{vmatrix} 8.5 & -7.5 \\ 7.5 & -24 \end{vmatrix}} = \frac{-2760}{-147.75} = 18.68 \text{ A} \qquad I_3 = 5.84 \text{ A}$$

$$V_{R1} = -I_3 R_1 = -23.35 \text{ V}$$
$$V_{R2} = R_2(I_3 - I_2) = -96.3 \text{ V}$$
$$V_{R3} = R_3 I_3 = 73 \text{ V}$$
$$V_F = V_{S1} - V_{R1} = 138.35 \text{ V}$$

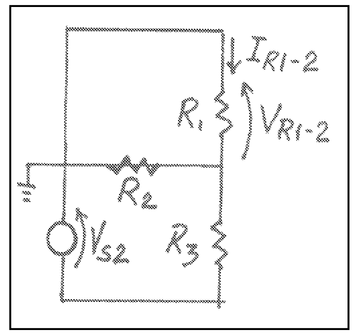
Section 3.5: Superposition

Problem 3.32

Solution:

Known quantities:
The values of the voltage sources, $V_{S1} = 110\ \text{V}$, $V_{S2} = 90\ \text{V}$ and the values of the 3 resistors in the circuit of Figure P3.32:

$$R_1 = 560\,\Omega \qquad R_2 = 3.5\ k\Omega \qquad R_3 = 810\,\Omega$$



Find:
The current through R_1 due only to the source V_{S2}.

Analysis:
Suppress V_{S1}. Redraw the circuit. Specify polarity of V_{R1}. Choose ground.

KCL: $$\frac{-V_{R1-2}-0}{R_1}+\frac{-V_{R1-2}-0}{R_2}+\frac{-V_{R1-2}-(-V_{S2})}{R_3}=0$$

$$V_{R1-2}=\frac{\frac{V_{S2}}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{\frac{90}{810}}{\frac{1}{560}+\frac{1}{3500}+\frac{1}{810}}=33.61\ \text{V}$$

OL: $$I_{R1-2}=\frac{V_{R1-2}}{R_1}=\frac{33.61}{560}=60.02\ \text{mA}$$

Problem 3.33

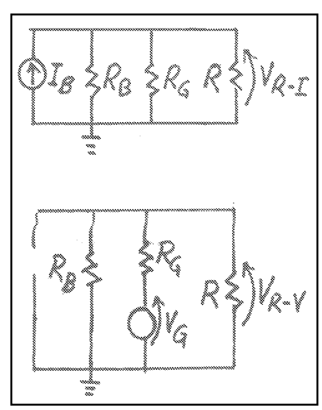
Solution:

Known quantities:
The values of the current source, of the voltage source and of the resistors in the circuit of Figure P3.33:

$$I_B = 12\ \text{A} \qquad R_B = 1\,\Omega$$
$$V_G = 12\ \text{V} \qquad R_G = 0.3\,\Omega$$
$$R = 0.23\,\Omega$$

Find:
The voltage across R_1 using superposition.

Analysis:
Specify a ground node and the polarity of the voltage across R. Suppress the voltage source by replacing it with a short circuit. Redraw the circuit.



KCL: $$-I_B+\frac{V_{R-I}}{R_B}+\frac{V_{R-I}}{R_G}+\frac{V_{R-I}}{R}=0$$

$$V_{R-I} = \frac{I_B}{\frac{1}{R_B}+\frac{1}{R_G}+\frac{1}{R}} = \frac{12}{\frac{1}{1}+\frac{1}{0.3}+\frac{1}{0.23}} = 1.38 \text{ V}$$

Suppress the current source by replacing it with an open circuit.

KCL: $$\frac{V_{R-V}}{R_B}+\frac{V_{R-V}-V_G}{R_G}+\frac{V_{R-V}}{R}=0$$

$$V_{R-V} = \frac{\frac{V_G}{R_G}}{\frac{1}{R_B}+\frac{1}{R_G}+\frac{1}{R}} = \frac{\frac{12}{0.3}}{\frac{1}{1}+\frac{1}{0.3}+\frac{1}{0.23}} = 4.61 \text{ V}$$

$$V_R = V_{R-I}+V_{R-V} = 5.99 \text{ V}$$

Note: Superposition essentially doubles the work required to solve this problem. The voltage across R can easily be determined using a single KCL.

Problem 3.34

Solution:

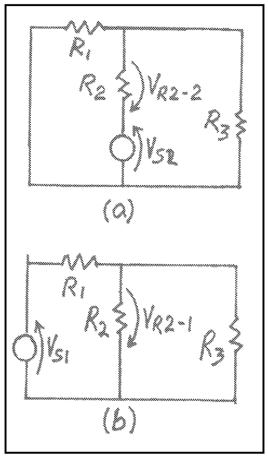
Known quantities:
The values of the voltage sources and of the resistors in the circuit of Figure P3.34:

$$V_{S1} = V_{S2} = 12 \text{ V}$$

$$R_1 = R_2 = R_3 = 1 \text{ k}\Omega$$

Find:
The voltage across R_2 using superposition.

Analysis:
Specify the polarity of the voltage across R_2. Suppress the voltage source V_{S1} by replacing it with a short circuit. Redraw the circuit.



$$R_{eq} = R_1 \| R_3 = \frac{1}{2} 1 \text{ k}\Omega = 0.5 \text{ k}\Omega$$

$$V_{R2-2} = V_{S2}\frac{R_2}{R_2+R_{eq}} = \frac{(12)(1000)}{1000+500} = 8 \text{ V}$$

Suppress the voltage source V_{S2} by replacing it with a short circuit. Redraw the circuit.

$$R_{eq} = R_2 \| R_3 = \frac{1}{2} 1 \text{ k}\Omega = 0.5 \text{ k}\Omega$$

$$V_{R2-1} = -V_{S1}\frac{R_{eq}}{R_1+R_{eq}} = \frac{(12 \text{ V})(0.5 \text{ k}\Omega)}{1 \text{ k}\Omega + 0.5 \text{ k}\Omega} = -4 \text{ V}$$

$$V_{R2} = V_{R2-1}+V_{R2-2} = -4 \text{ V} + 8 \text{ V} = 4 \text{ V}$$

Note: Although superposition is necessary to solve some circuits, it is a very inefficient and very cumbersome way to solve a circuit. This method should, if at all possible, be avoided. It must be used when the sources in a circuit are AC sources with different frequencies, or where some sources are DC and others are AC.

Problem 3.35

Solution:

Known quantities:
The values of the voltage sources and of the resistors in the circuit of Figure P3.35:

$$V_{S1} = V_{S2} = 450 \text{ V}$$

$$R_1 = 7\,\Omega \qquad R_2 = 5\,\Omega \qquad R_3 = 10\,\Omega \qquad R_4 = R_5 = 1\,\Omega$$

Find:
The component of the current through R_3 that is due to V_{S2}, using superposition.

Analysis:

Suppress V_{S1} by replacing it with a short circuit. Redraw the circuit. A solution using equivalent resistances looks reasonable. R_1 and R_4 are in parallel:

$$R_{14} = \frac{R_1 R_4}{R_1 + R_4} = \frac{(7)(1)}{7+1} = 0.875\,\Omega$$

R_{14} is in series with R_3:

$$R_{143} = R_{14} + R_3 = 0.875 + 10 = 10.875\,\Omega$$

$$R_{eq} = R_5 + \left(R_2 \| R_{143}\right) = R_5 + \frac{R_2 R_{143}}{R_2 + R_{143}} = 1 + \frac{(5)(10.875)}{5 + 10.875} = 4.425\,\Omega$$

OL: $$I_S = \frac{V_{S2}}{R_{eq}} = \frac{450}{4.425} = 101.695 \text{ A}$$

CD: $$I_{R3-2} = \frac{I_S R_2}{R_2 + R_{143}} = \frac{(101.695)(5)}{5 + 10.875} = 32.03 \text{ A}$$

Problem 3.36

Solution:

Known quantities:
The values of the voltage sources and of the resistors in the circuit of Figure P3.24:

$$V_{S1} = V_{S2} = V_{S3} = 170 \text{ V}$$

$$R_{W1} = R_{W2} = R_{W3} = 0.7\,\Omega$$

$$R_1 = 1.9\,\Omega \qquad R_2 = 2.3\,\Omega \qquad R_3 = 11\,\Omega$$

Find:
The current through R_1, using superposition.

Analysis:

$$R_{eq1} = R_{W1} + R_1 = 2.6\,\Omega \qquad R_{eq2} = R_{W2} + R_2 = 3\,\Omega \qquad R_{eq3} = R_{W3} + R_3 = 11.7\,\Omega$$

Specify the direction of I_1. Suppress V_{S2} and V_{S3}. Redraw circuit.

$$R_{eq} = R_{eq1} + \frac{R_{eq2}R_{eq3}}{R_{eq2} + R_{eq3}} = 4.99\,\Omega$$

$$I_{I-1} = \frac{V_{S1}}{R_{eq}} = 34.08 \text{ A}$$

Suppress V_{S1} and V_{S3}. Redraw circuit.

$$KCL: \quad \frac{V_A - (-V_{S2})}{R_{eq2}} + \frac{V_A}{R_{eq1}} + \frac{V_A}{R_{eq3}} = 0$$

$$V_A = -\frac{V_{S2}}{1 + \frac{R_{eq2}}{R_{eq1}} + \frac{R_{eq2}}{R_{eq3}}} = -70.54 \text{ V}$$

$$I_{I-2} = -\frac{V_A}{R_{eq1}} = 27.13 \text{ A}$$

Suppress V_{S1} and V_{S2}. Redraw circuit.

$$KCL: \quad \frac{V_A - (-V_{S3})}{R_{eq3}} + \frac{V_A - 0}{R_{eq1}} + \frac{V_A - 0}{R_{eq2}} = 0$$

$$V_A = -\frac{V_{S3}}{1 + \frac{R_{eq3}}{R_{eq1}} + \frac{R_{eq3}}{R_{eq2}}} = 18.09 \text{ V}$$

$$I_{I-3} = -\frac{V_A}{R_{eq1}} = -6.96 \text{ A}$$

$$I = I_{I-1} + I_{I-2} + I_{I-3} = 54.25 \text{ A}$$

Note: Superposition should be used only for special conditions, as stated in the solution to Problem 3.34. In the problem above a better method is:

a. mesh analysis using *KVL* (2 unknowns)

b. node analysis using *KCL* (1 unknown but current must be obtained using *OL*).

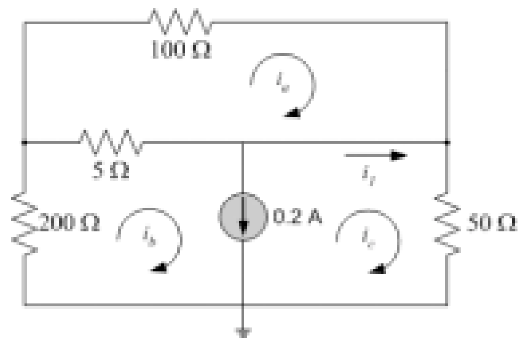
Problem 3.37

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.5.

Find:
The current through the voltage source using superpoistion.

Analysis:
(1) Suppress voltage source *V*. Redraw the circuit.



For mesh (a):

$$i_a(100+5)+i_b(-5)=0$$

For the current source:

$$i_b - i_c = 0.2$$

For meshes (b) and (c):

$$-i_a(5)+i_b(200+5)+i_c(50)=0$$

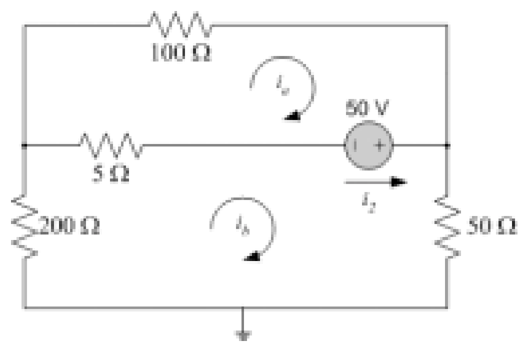
Solving,

$$i_a = 2\text{ mA},\ i_b = 39\text{ mA} \text{ and } i_c = -161\text{ mA}.$$

Therefore,

$$i_1 = i_c - i_a = -163\text{ mA}.$$

(2) Suppress current source *I*. Redraw the circuit.
For mesh (a):



$$i_a(100+5)+i_b(-5)+50=0$$

For mesh (b):

$$-i_a(5)+i_b(200+5+50)=50$$

Solving,

$$i_a = -467\text{mA and } i_b = 187\text{ mA}.$$

Therefore,

$$i_2 = i_b - i_a = 654\text{ mA}.$$

Using the principle of superposition,

$$i = i_1 + i_2 = 491\text{ mA}$$

Problem 3.38

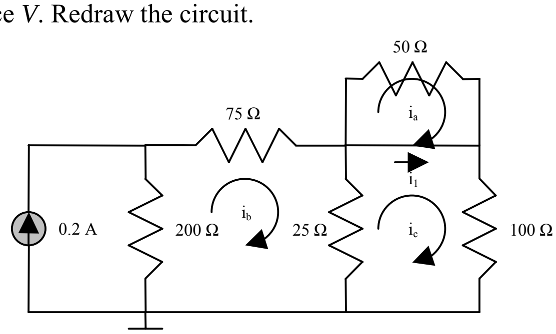
Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.6.

Find:
The current through the voltage source using superposition.

Analysis:

(1) Suppress voltage source *V*. Redraw the circuit.



For mesh (a):

$$i_a = 0$$

For mesh (b):

$$i_b(200+75+25)+i_c(-25)-40=0$$

For mesh (c):

$$i_b(-25)+i_c(25+100)=0$$

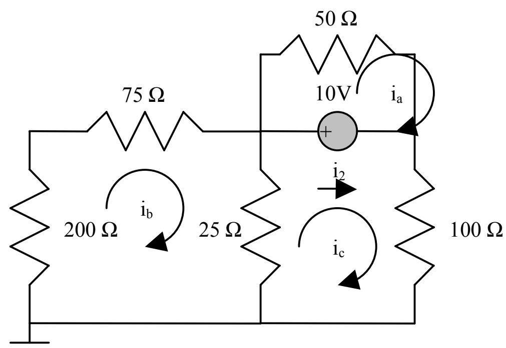
Solving,

$$i_b = 136 \text{ mA} \text{ and } i_c = 27 \text{ mA}.$$

Therefore,

$$i_1 = i_c = 27 \text{ mA}.$$

(2) Suppress current source *I*. Redraw the circuit.



For mesh (a):

$$i_a(50)-10=0$$

For mesh (b):

$$i_b(200+75+25)+i_c(-25)=0$$

For mesh (c):

$$i_b(-25)+i_c(25+100)=-10$$

Solving,

$$i_a = 200 \text{ mA},\ i_b = -6.8 \text{ mA} \text{ and } i_c = -81 \text{ mA}.$$

Therefore,

$$i_2 = i_c - i_a = -281 \text{ mA}.$$

Using the principle of superposition,

$$i = i_1 + i_2 = -254 \text{ mA}$$

Problem 3.39

Solution:

Known quantities:
The voltage source value, 3 V, and the five resistance values, indicated in Figure P3.7.

Find:
The current, i, drawn from the independent voltage source using superposition.

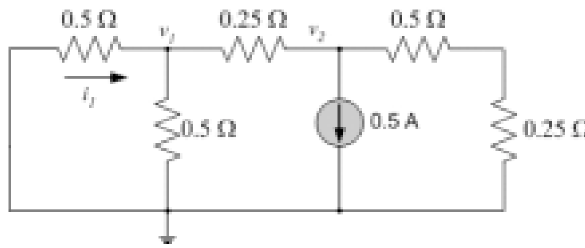
Analysis:
(1) Suppress voltage source V. Redraw the circuit.

At node 1:

$$\frac{v_1}{0.5}+\frac{v_1}{0.5}+\frac{v_1-v_2}{0.25}=0$$

At node 2:

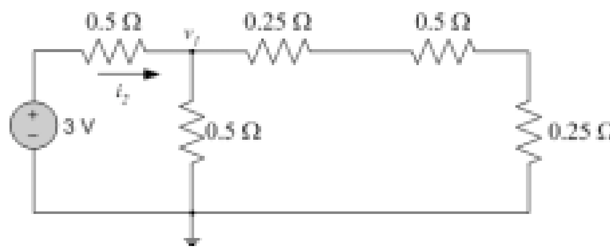
$$\frac{v_2-v_1}{0.25}+\frac{v_2}{0.75}+0.5=0$$



Solving the system, we obtain:

$$v_1=-0.075\text{ V},\ v_2=-0.15\text{ V}$$

Therefore, $i_1=-\frac{v_1}{0.5}=150\text{ mA}$.



(2) Suppress current source I. Redraw the circuit.

At node 1:

$$\frac{v_1-3}{0.5}+\frac{v_1}{0.5}+\frac{v_1}{(0.25+0.5+0.25)}=0$$

Solving,

$$v_1=1.2\text{ V}$$

Therefore, $i_1=\frac{3-v_1}{0.5}=3.6\text{ A}$.

Using the principle of superposition,

$$i=i_1+i_2=3.75\text{ A}$$

Problem 3.40

Solution:

Known quantities:
Circuit in Figure P3.12

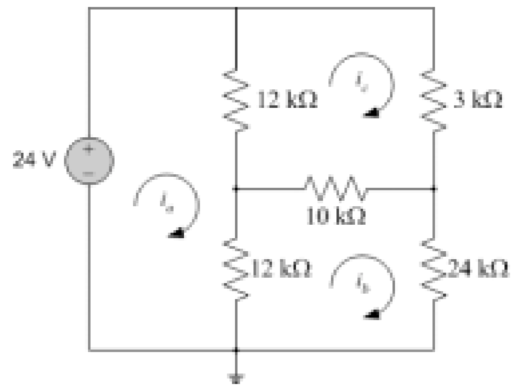
$$V_{S2}=\text{kT}\qquad \text{k}=10\text{ V/°C}$$
$$V_{S1}=24\text{ V}\qquad R_S=R_1=12\text{ k}\Omega$$
$$R_2=3\text{ k}\Omega\qquad R_3=10\text{ k}\Omega$$
$$R_4=24\text{ k}\Omega\qquad V_{R3}=-2.524\text{ V}$$

The voltage across R_3, which is given, indicates the temperature.

Find:

The temperature, T using superposition.

Analysis:



(1) Suppress voltage source V_{S2}. Redraw the circuit.

For mesh (a):

$$i_a(24k)+i_b(-12k)+i_c(-12k)=24$$

For mesh (b):

$$i_a(-12k)+i_b(46k)+i_c(-10k)=0$$

For mesh (c):

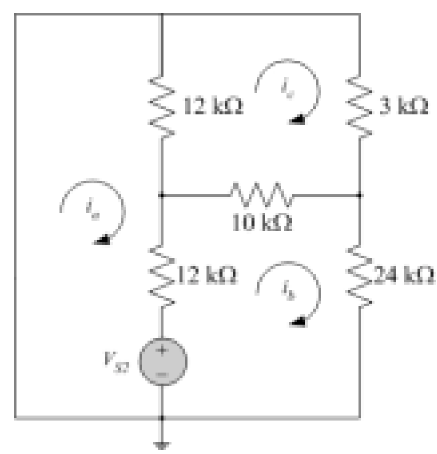
$$i_a(-12k)+i_b(-10k)+i_c(25k)=0$$

Solving,

$$i_a = 2.08\text{ mA},\ i_b = 0.83\text{ mA} \text{ and } i_c = 1.33\text{ mA}.$$

Therefore,

$$V_{R3,S2} = 10000(i_b - i_c) = -5\text{ V}.$$



(2) Suppress voltage source V_{S1}. Redraw the circuit.

For mesh (a):

$$i_a(24k)+i_b(-12k)+i_c(-12k)+10T=0$$

For mesh (b):

$$i_a(-12k)+i_b(46k)+i_c(-10k)=10T$$

For mesh (c):

$$i_a(-12k)+i_b(-10k)+i_c(25k)=0$$

Solving,

$$i_a = -0.52T\text{ mA},\ i_b = 0.029T\text{ mA} \text{ and } i_c = -0.2381T\text{ mA}.$$

Therefore,

$$V_{R3,S1} = 10000(i_b - i_c) = 2.671T\text{ V}.$$

Using the principle of superposition,

$$V_{R3} = V_{R3,S2} + V_{R3,S1} = -5 + 2.671T = -2.524\text{ V}$$

Therefore,

$$T = 0.926\ ^\circ\text{C}.$$

Problem 3.41

Solution:

Known quantities:

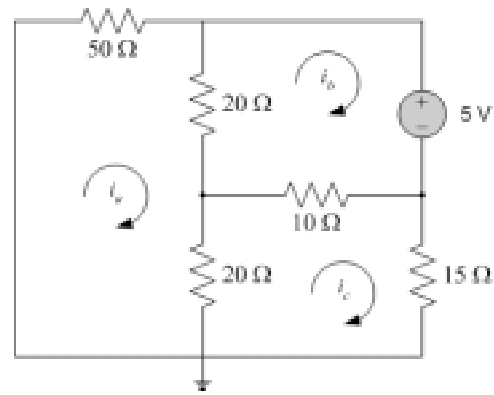
The values of the resistors and of the voltage sources (see Figure P3.14).

Find:

The voltage across the $10\ \Omega$ resistor in the circuit of Figure P3.14 using superposition.

Analysis:

(1) Suppress voltage source V_{S1}. Redraw the circuit.



For mesh (a):

$$i_a(50+20+20)-i_b(20)-i_c(20)=0$$

For mesh (b):

$$-i_a(20)+i_b(20+10)-i_c(10)+5=0$$

For mesh (c):

$$-i_a(20)-i_b(10)+i_c(20+10+15)=0$$

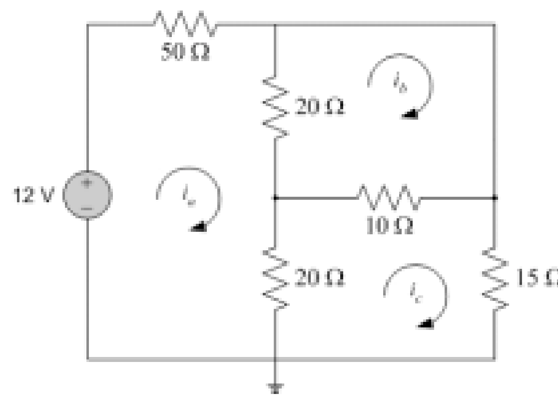
Solving,

$i_a=-73.8\text{ mA}$, $i_b=-245\text{ mA}$ and $i_c=-87.2\text{ mA}$.

Therefore,

$$V_{10\Omega,S1}=10(i_b-i_c)=-1.578\text{ V}.$$

(2) Suppress voltage source V_{S2}. Redraw the circuit.



For mesh (a):

$$i_a(50+20+20)-i_b(20)-i_c(20)=12$$

For mesh (b):

$$-i_a(20)+i_b(20+10)-i_c(10)=0$$

For mesh (c):

$$-i_a(20)-i_b(10)+i_c(20+10+15)=0$$

Solving,

$i_a=201\text{ mA}$, $i_b=177\text{ mA}$ and $i_c=129\text{ mA}$.

Therefore,

$$V_{10\Omega,S1}=10(i_b-i_c)=0.48\text{ V}.$$

Using the principle of superposition,

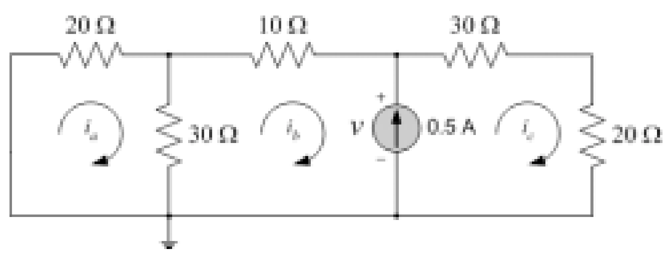
$$V_{10\Omega}=V_{10\Omega,S2}+V_{10\Omega,S1}=-1.09\text{ V}.$$

Problem 3.42

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.15.

Find:
The voltage across the current source using superposition.



Analysis:
(1) Suppress voltage source. Redraw the circuit.

For mesh (a):

$$i_a(20+30)+i_b(-30)=0$$

For meshes (b) and (c):

$$i_a(-30)+i_b(10+30)+i_c(30+20)=0$$

For the current source:

$$i_c-i_b=0.5$$

Solving,

$$i_a=-208\text{ mA},\ i_b=-347\text{ mA and } i_c=153\text{ mA}.$$

Therefore,

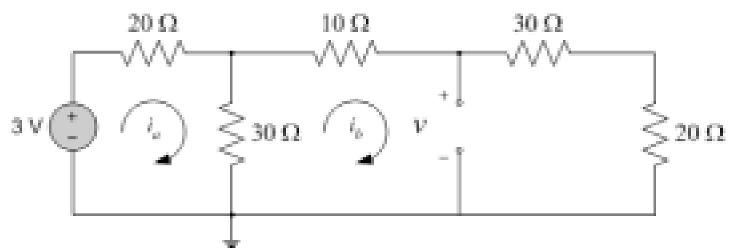
$$v_V=i_c(30+20)=7.65\text{ V}.$$

(2) Suppress current source. Redraw the circuit.
For mesh (a):

$$i_a(20+30)+i_b(-30)=3$$

For mesh (b):

$$i_a(-30)+i_b(90)=0$$



Solving,

$$i_a=75\text{ mA and } i_b=25\text{ mA}.$$

Therefore,

$$v_I=i_b(30+20)=1.25\text{ V}.$$

Using the principle of superposition,

$$v=v_V+v_I=8.9\text{ V}.$$

Section 3.6: Equivalent circuits

Focus on Methodology: Computation of equivalent resistance of a one-port network that does not contain dependent sources

1. Remove the load, leaving the load terminals open circuited.
2. Zero all independent voltage and current sources
3. Compute the total resistance between load terminals, *with the load removed*. This resistance is equivalent to that which would be encountered by a current source connected to the circuit in place of the load.

Focus on Methodology: Computing the Thevenin voltage

1. Remove the load, leaving the load terminals open circuited.
2. Define the open-circuit voltage v_{OC} across the open load terminals.
3. Apply any preferred method (e.g.: nodal analysis) to solve for v_{OC}.
4. The Thevenin voltage is $v_T = v_{OC}$.

Focus on Methodology: Computing the Norton current

1. Replace the load with a short circuit.
2. Define the short-circuit current i_{SC} to be the Norton equivalent current.
3. Apply any preferred method (e.g.: nodal analysis) to solve for i_{SC}.
4. The Norton current is $i_N = i_{SC}$.

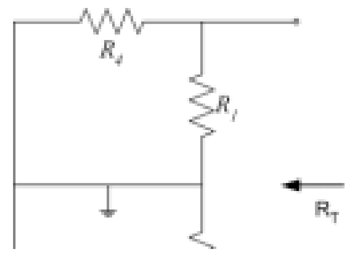
Problem 3.43

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.1).

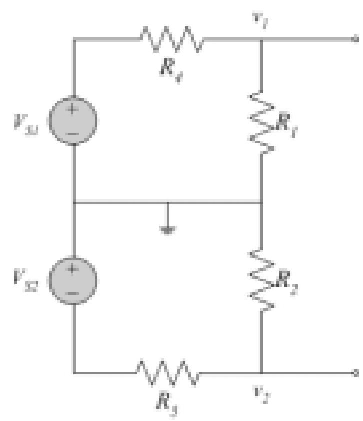
Find:
The Thévenin equivalent resistance seen by resistor R_3, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.



$$R_T = R_1 \parallel R_4 + R_2 \parallel R_5 = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5}$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1}{R_1} + \frac{v_1 - V_{S1}}{R_4} = 0$$

For node #2:

$$\frac{v_2}{R_2}+\frac{v_2+V_{S2}}{R_5}=0$$

Solving the system,

$$v_1=\frac{R_1}{R_1+R_4}V_{S1}$$

$$v_2=-\frac{R_2}{R_2+R_5}V_{S2}$$

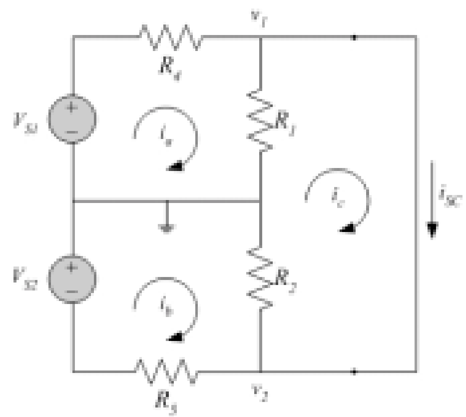
Therefore,

$$v_{OC}=v_1-v_2=\frac{R_1}{R_1+R_4}V_{S1}+\frac{R_2}{R_2+R_5}V_{S2}$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(R_1+R_4)-R_1i_c=V_{S1}$$



For mesh (b):

$$i_b(R_2+R_5)-R_2i_c=V_{S2}$$

For mesh (c):

$$-R_1i_a-R_2i_b+i_c(R_1+R_2)=0$$

Solving the system,

$$i_a=\frac{(R_1R_2+R_1R_5+R_2R_5)V_{S1}+R_1R_2V_{S2}}{R_1R_4(R_2+R_5)+R_2R_5(R_1+R_4)}$$

$$i_b=\frac{R_1R_2V_{S1}+(R_1R_2+R_1R_4+R_2R_4)V_{S2}}{R_1R_4(R_2+R_5)+R_2R_5(R_1+R_4)}$$

$$i_c=\frac{R_1(R_2+R_5)V_{S1}+R_2(R_1+R_4)V_{S2}}{R_1R_4(R_2+R_5)+R_2R_5(R_1+R_4)}$$

Therefore,

$$i_{SC}=i_c=\frac{R_1(R_2+R_5)V_{S1}+R_2(R_1+R_4)V_{S2}}{R_1R_4(R_2+R_5)+R_2R_5(R_1+R_4)}$$

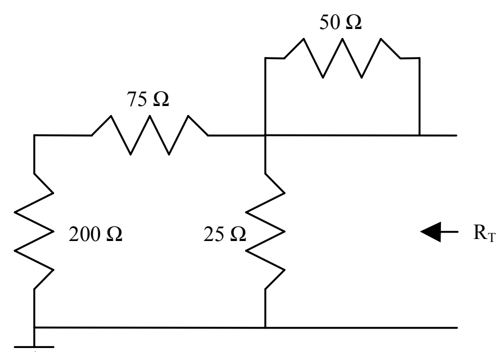
Problem 3.44

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.6).

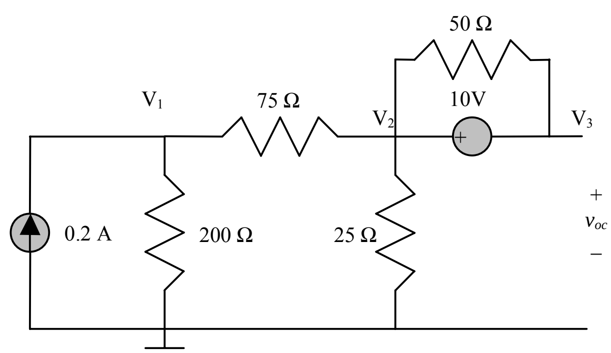
Find:
The Thévenin equivalent resistance seen by resistor R_5, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.



$$R_T = 25\,\Omega \,||\, \left(75\,\Omega + 200\,\Omega\right) = 22.92\,\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1}{200} + \frac{v_1 - v_2}{75} = 0.2$$

For node #2:

$$\frac{v_2 - v_1}{75} + \frac{v_2}{25} + \frac{v_2 - v_3}{50} + i_{10V} = 0$$

For node #3:

$$\frac{v_3 - v_2}{50} = i_{10V}$$

For the voltage source:

$$v_3 + 10 = v_2$$

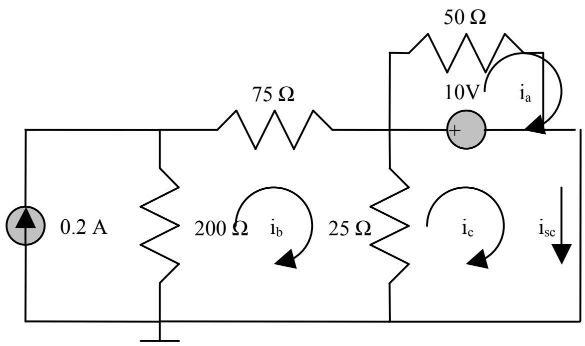
Solving the system,

$$v_1 = 13.33 \text{ V}, \; v_2 = 3.33 \text{ V} \text{ and } v_3 = -6.67 \text{ V}.$$

Therefore,

$$v_{OC} = v_3 = -6.67 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(50) = 10$$

For mesh (b):

$$i_b(300) - i_c(25) = 40$$

For mesh (c):

$$i_b(25) - i_c(25) = 10$$

Solving the system,

$$i_a = 200\text{ mA},\ i_b = 109\text{ mA and } i_c = -291\text{ mA}.$$

Therefore,

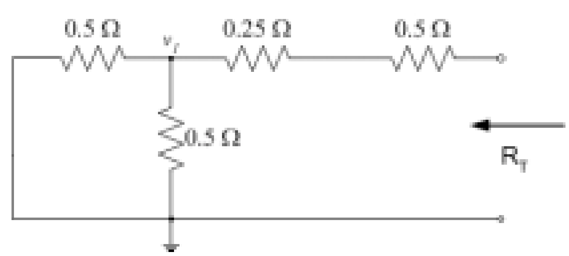
$$i_{SC} = i_c = -291\text{ mA}.$$

Problem 3.45

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.7).

Find:
The Thévenin equivalent resistance seen by resistor R_5, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

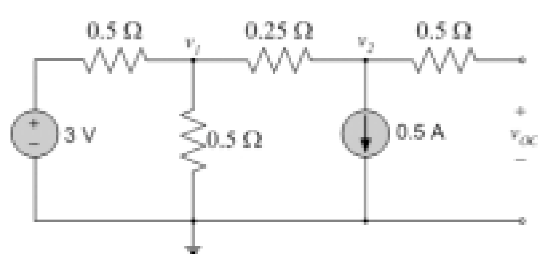


Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 0.5\,\Omega + 0.25\,\Omega + (0.5\,\Omega \,||\, 0.5\,\Omega) = 1\,\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} + \frac{v_1 - v_2}{0.25} = 0$$

For node #2:

$$\frac{v_2 - v_1}{0.25} + 0.5 = 0$$

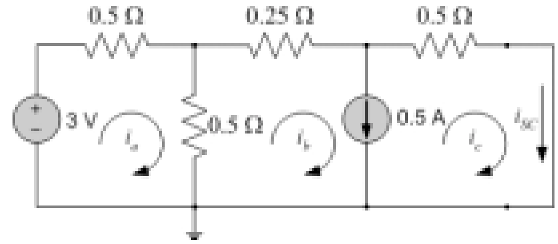
Solving the system,

$v_1 = 1.375 \text{ V}$ and $v_2 = 1.25 \text{ V}$.

Therefore,

$$v_{OC} = v_2 = 1.25 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(0.5+0.5) - i_b(0.5) = 3$$

For meshes (b) and (c):

$$-i_a(0.5) + i_b(0.5+0.25) + i_c(0.5) = 0$$

For the current source:

$$i_b - i_c = 0.5$$

Solving the system,

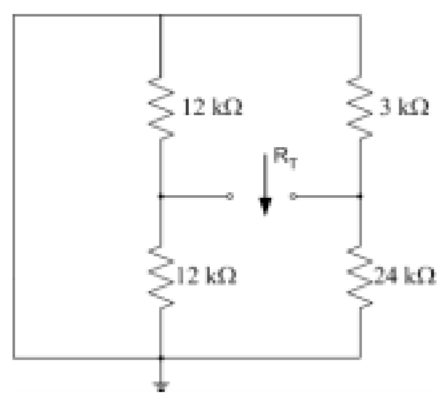
$i_a = 3.875 \text{ A}$, $i_b = 1.75 \text{ A}$ and $i_c = 1.25 \text{ A}$.

Therefore,

$$i_{SC} = i_c = 1.25 \text{ A}.$$

Problem 3.46

Solution:



Known quantities:
The schematic of the circuit (see Figure P3.12).

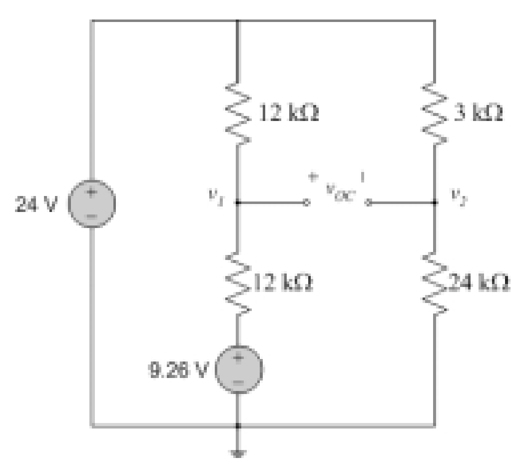
Find:
The Thévenin equivalent resistance seen by resistor R_3 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

Assumption:
As in P3.12, we assume $T = 0.926$ °C, so that $V_{S2} = 9.26 \text{ V}$.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 12 \text{ k}\Omega \,||\, 12 \text{ k}\Omega + 3 \text{ k}\Omega \,||\, 24 \text{ k}\Omega = 8.67 \text{ k}\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1 - 24}{12000} + \frac{v_1 - 9.26}{12000} = 0$$

For node #2:

$$\frac{v_2 - 24}{3000} + \frac{v_2}{24000} = 0$$

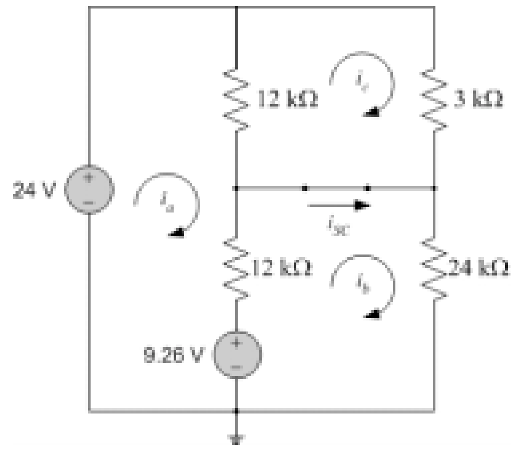
Solving the system,

$v_1 = 16.63 \text{ V}$ and $v_2 = 21.33 \text{ V}$.

Therefore,

$$v_{OC} = v_1 - v_2 = -4.7 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(24k)-i_b(12k)-i_c(12k)=24-9.26$$

For mesh (b):

$$-i_a(12k)+i_b(36k)=9.26$$

For mesh (c):

$$-i_a(12k)+i_c(15k)=0$$

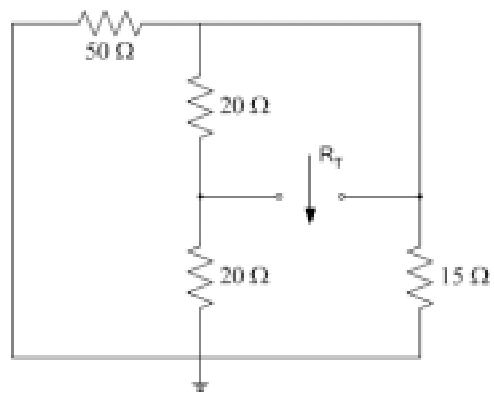
Solving the system,

$$i_a=1.71\text{ mA},\ i_b=0.83\text{ mA} \text{ and}$$
$$i_c=1.37\text{ mA}.$$

Therefore,

$$i_{SC}=i_b-i_c=-0.54\text{ mA}.$$

Problem 3.47



Solution:

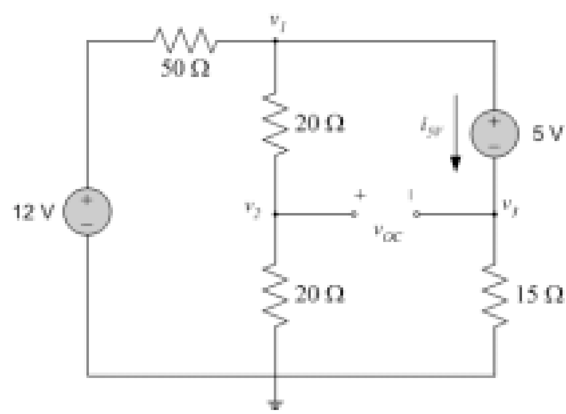
Known quantities:
The schematic of the circuit (see Figure P3.14).

Find:
The Thévenin equivalent resistance seen by resistor R_4, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_4 is the load.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T=R_2\,||\,(R_3+(R_1\,||\,R_5))=20\,\Omega\,||\,(20\,\Omega+(50\,\Omega\,||\,15\,\Omega))=12.24\,\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1-12}{50}+\frac{v_1-v_2}{20}+i_{5V}=0$$

For node #2:

$$\frac{v_2-v_1}{20}+\frac{v_2}{20}=0$$

For node #3:

$$\frac{v_3}{15}-i_{5V}=0$$

For the 5-V voltage source:

$$v_1-v_3=5$$

Solving the system,

$$v_1=5.14\text{ V},\ v_2=2.57\text{ V},\ v_1=0.13\text{ V} \text{ and } i_{5V}=8.95\text{ mA}.$$

Therefore,

$$v_{OC} = v_2 - v_3 = 2.44 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(90) - i_b(20) - i_c(20) = 12$$

For mesh (b):

$$-i_a(20) + i_b(20) + 5 = 0$$

For mesh (c):

$$-i_a(20) + i_c(35) = 0$$

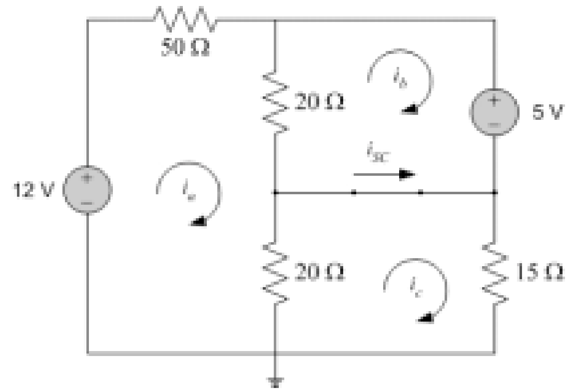
Solving the system,

$$i_a = 119.5 \text{ mA},\ i_b = -130.5 \text{ mA} \text{ and}$$

$$i_c = 68.3 \text{ mA}.$$

Therefore,

$$i_{SC} = i_c - i_b = 198.8 \text{ mA}.$$

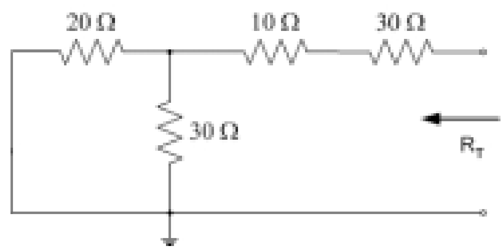


Problem 3.48

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.15).

Find:
The Thévenin equivalent resistance seen by resistor R_5, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.



Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 30\ \Omega + 10\ \Omega + (20\ \Omega \,\|\, 30\ \Omega) = 52\ \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 3}{20} + \frac{v_1}{30} + \frac{v_1 - v_2}{10} = 0$$

For node #2:

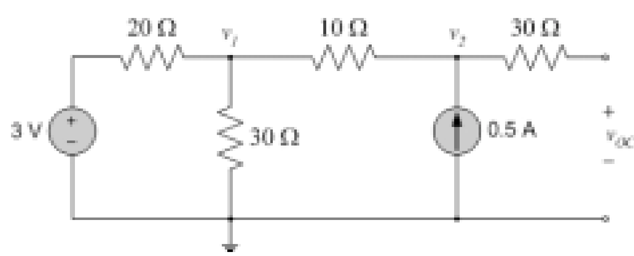
$$\frac{v_2 - v_1}{10} = 0.5$$

Solving the system,

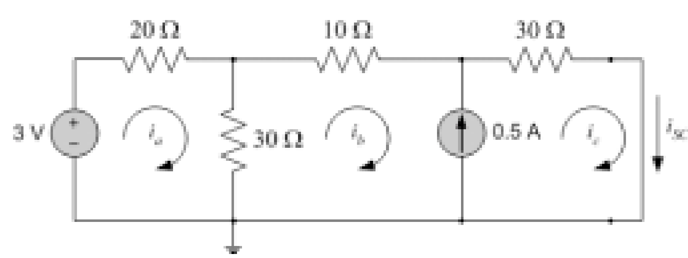
$$v_1 = 7.8 \text{ V} \text{ and } v_2 = 12.8 \text{ V}.$$

Therefore,

$$v_{OC} = v_2 = 12.8 \text{ V}.$$



(3) Replace the load with a short circuit.
Redraw the circuit.



For mesh (a):

$$i_a(20+30)-i_b(30)=3$$

For meshes (b) and (c):

$$-i_a(30)+i_b(30+10)+i_c(30)=0$$

For the current source:

$$i_c-i_b=0.5$$

Solving the system,

$$i_a=-92\text{ mA},\ i_b=-254\text{ mA and } i_c=246\text{ mA}.$$

Therefore,

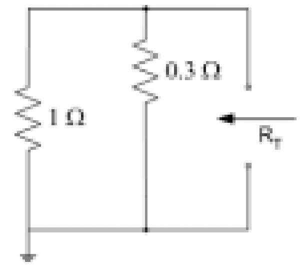
$$i_{SC}=i_c=246\text{ mA}.$$

Problem 3.49

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.33).

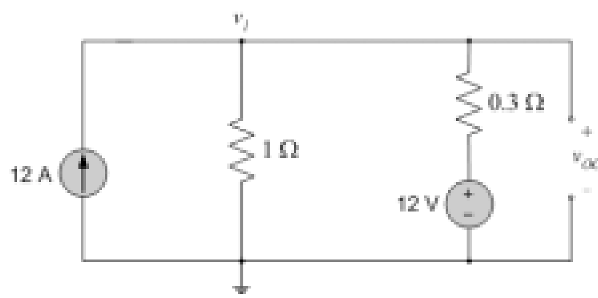
Find:
The Thévenin equivalent resistance seen by resistor R, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R is the load.



Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T=1\,\Omega\,||\,0.3\,\Omega=0.23\,\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1}{1}+\frac{v_1-12}{0.3}=12$$

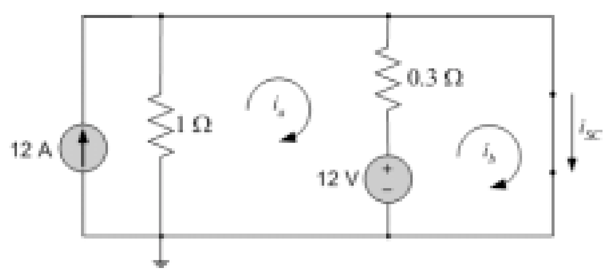
Solving,

$$v_1=12\text{ V}.$$

Therefore,

$$v_{OC}=v_1=12\text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(1+0.3)-i_b(0.3)-12(1)+12=0$$

For mesh (b):

$$-i_a(0.3)+i_b(0.3)=12$$

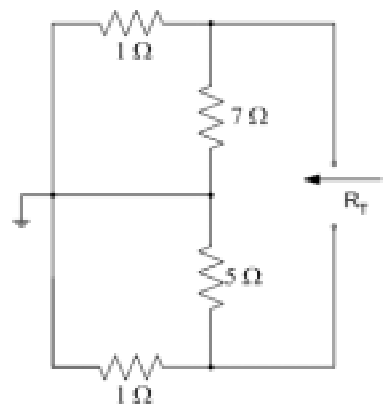
Solving the system,

$$i_a=12\text{ A and } i_b=52\text{ A}.$$

Therefore,

$$i_{SC} = i_b = 52 \text{ A} .$$

Problem 3.50



Solution:

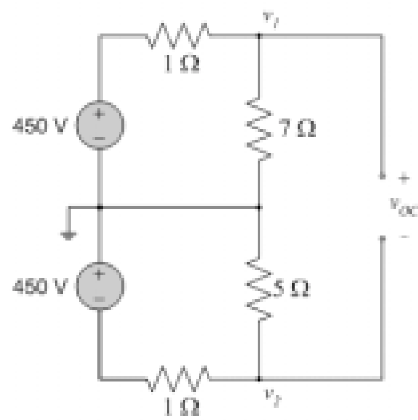
Known quantities:
The schematic of the circuit (see Figure P3.35).

Find:
The Thévenin equivalent resistance seen by resistor R_3, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 1\,\Omega \,||\, 7\,\Omega + 1\,\Omega \,||\, 5\,\Omega = 1.71\,\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1 - 450}{1} + \frac{v_1}{7} = 0$$

For node #2:

$$\frac{v_2 + 450}{1} + \frac{v_2}{5} = 0$$

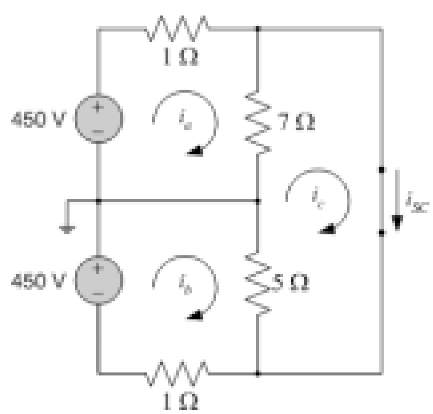
Solving the system,

$$v_1 = 393.75 \text{ V} \text{ and } v_2 = -375 \text{ V} .$$

Therefore,

$$v_{OC} = v_1 - v_2 = 768.75 \text{ V} .$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(1+7) - i_c(7) = 450$$

For mesh (b):

$$i_b(5+1) - i_c(5) = 450$$

For mesh (c):

$$-i_a(7) - i_b(5) + i_c(7+5) = 0$$

Solving the system,

$$i_a = 450 \text{ A} ,\ i_b = 450 \text{ A} \text{ and } i_c = 450 \text{ A} .$$

Therefore,

$$i_{SC} = i_c = 450 \text{ A} .$$

Problem 3.51

Solution:

Known quantities:

The values of the voltage source, $V_S = 110 \text{ V}$, and the values of the 4 resistors in the circuit of Figure P3.51:

$$R_1 = R_2 = 930 \text{ m}\Omega \qquad R_3 = 100 \text{ m}\Omega$$
$$R_S = 19 \text{ m}\Omega$$

Find:

The change in the voltage across the total load, when the customer connects the third load R_3 in parallel with the other two loads.

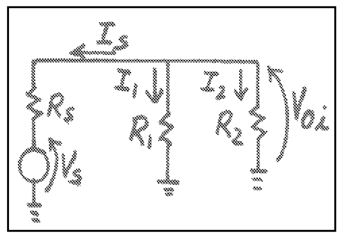
Analysis:

Choose a ground. If the node at the bottom is chosen as ground (which grounds one terminal of the ideal source), the only unknown node voltage is the required voltage. Specify directions of the currents and polarities of voltages.

Without R_3:

KCL: $I_S + I_1 + I_2 = 0$

OL:



$$\frac{V_{RS}}{R_S} + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} = 0$$

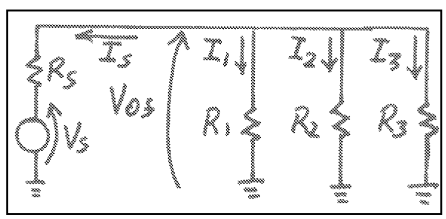
$$\frac{V_{Oi} - V_S}{R_S} + \frac{V_{Oi} - 0}{R_1} + \frac{V_{Oi} - 0}{R_2} = 0$$

$$V_{Oi} = \frac{\frac{V_S}{R_S}}{\frac{1}{R_S} + \frac{1}{R_1} + \frac{1}{R_2}} \frac{R_S}{R_S} = \frac{110}{1.04086} = 105.7 \text{ V}$$

With R_3:

KCL: $I_S + I_1 + I_2 + I_3 = 0$

OL:



$$\frac{V_{RS}}{R_S} + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} + \frac{V_{R3}}{R_3} = 0$$

$$\frac{V_{Of} - V_S}{R_S} + \frac{V_{Of} - 0}{R_1} + \frac{V_{Of} - 0}{R_2} + \frac{V_{Of} - 0}{R_3} = 0$$

$$V_{Of} = \frac{\frac{V_S}{R_S}}{\frac{1}{R_S} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \frac{R_S}{R_S} = \frac{110}{1.04086 + 0.19} = 89.37 \text{ V}$$

Therefore, the voltage decreased by: $\Delta V_o = V_{Of} - V_{Oi} = -16.33 \text{ V}$

Notes:

1. "Load" to an EE usually means current rather than resistance.
2. Additional load reduces the voltage supplied to the customer because of the additional voltage dropped across the losses in the distribution system.

Problem 3.52

Solution:

Known quantities:
The values of the voltage source, $V_S = 450 \text{ V}$, and the values of the 4 resistors in the circuit of Figure P3.52

$$R_1 = R_2 = 1.3\,\Omega \qquad R_3 = 500 \text{ m}\Omega$$
$$R_S = 19 \text{ m}\Omega$$

Find:
The change in the voltage across the total load, when the customer connects the third load R_3 in parallel with the other two loads.

Analysis:
See Solution to Problem 3.40 for a detailed mathematical analysis.

$$\Delta V_o = V_{Of} - V_{Oi} = -15.6 \text{ V}$$

Problem 3.53

Solution:

Known quantities:
The circuit shown in Figure P3.53, the values of the terminal voltage, V_T, before and after the application of the load, respectively $V_T = 20 \text{ V}$ and $V_T = 18 \text{ V}$, and the value of the load resistor $R_L = 2.7 \text{ k}\Omega$.

Find:
The internal resistance and the voltage of the ideal source.

Analysis:

KVL: $-V_S + I_T R_S + V_T = 0$

If $I_T = 0$: $V_S = V_T = 20 \text{ V}$

If $V_T = 18 \text{ V}$:

OL: $I_T = \dfrac{V_T}{R_L} = 6.67 \text{ mA}$ and $R_S = \dfrac{V_S - V_T}{I_T} = 300\,\Omega$

Note that R_S is an equivalent resistance, representing the various internal losses of the source and is not physically a separate component. V_S is the voltage generated by some internal process. The source voltage can be measured directly by reducing the current supplied by the source to zero, i.e., no-load or open-circuit conditions. The source resistance cannot be directly measured; however, it can be determined, as was done above, using the interaction of the source with an external load.

Problem 3.54

Solution:

Known quantities:

The values of the voltage source, $V_{CC} = 20\text{ V}$, and the values of the 2 resistors in the circuit of Figure P3.54:

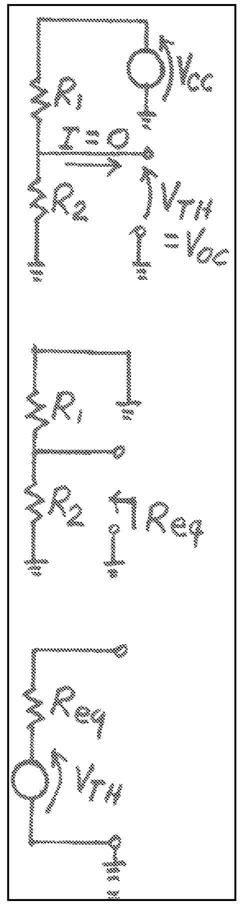
$$R_1 = 1.3\text{ M}\Omega \qquad R_2 = 220\text{ k}\Omega$$

Find:

The Thévenin equivalent circuit with respect to the port shown in Figure P3.54.

Analysis:

The Thévenin equivalent voltage is the open circuit voltage [with $I = 0$] between the terminals of the port. Specify the polarity of the voltage.



KCL:

$$\frac{V_{TH} - V_{CC}}{R_1} + \frac{V_{TH}}{R_2} + I = 0$$

$$V_{TH} = \frac{\frac{V_{CC}}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} = 2.895\text{ V}$$

Note that, since $I = 0$, R_1 and R_2 are effectively in series, using a VD relation would be easier.

Suppress the ideal, independent voltage source, by shorting it. Determine the equivalent resistance with respect to the terminals of the port.

$$R_{EQ} = R_1 \| R_2 = \frac{(1.3 \cdot 10^6)(220 \cdot 10^3)}{1.3 \cdot 10^6 + 220 \cdot 10^3} = 188.2\text{ k}\Omega$$

Problem 3.55

Solution:

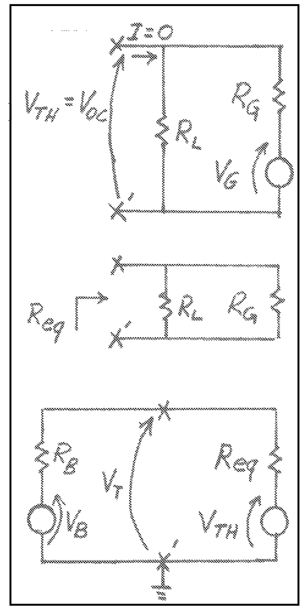
Known quantities:

The values of the battery voltage, $V_B = 11\text{ V}$, the value of the generator voltage, $V_G = 12\text{ V}$, and the values of the 3 resistors in the circuit of Figure P3.55:

$$R_B = 0.7\,\Omega \qquad R_G = 0.3\,\Omega \qquad R_L = 7.2\,\Omega$$

Find:

a. The Thévenin equivalent of the circuit to the right of the terminal pair of port x-x'.
b. The terminal voltage of the battery, i.e., the voltage between x and x'.



Analysis:

a. Specify the polarity of the voltage:

$$VD: \quad V_{TH} = \frac{V_G R_L}{R_G + R_L} = \frac{(12)(7.2)}{0.3+7.2} = 11.52 \text{ V}$$

Suppress generator source:

$$R_{EQ} = R_L \| R_G = \frac{(7.2)(0.3)}{0.3+7.2} = 288 \text{ m}\Omega$$

b. Specify the polarity of the terminal voltage. Choose a ground.

$$KCL: \quad \frac{V_T - V_B}{R_B} + \frac{V_T - V_{TH}}{R_{EQ}} = 0$$

$$V_T = \frac{\dfrac{V_B}{R_B} + \dfrac{V_{TH}}{R_{EQ}}}{\dfrac{1}{R_B} + \dfrac{1}{R_{EQ}}} = 11.37 \text{ V}$$

Problem 3.56

Solution:

The values of the battery voltage, $V_B = 11 \text{ V}$, the value of the generator voltage, $V_G = 12 \text{ V}$, and the values of the 3 resistors in the circuit of Figure P3.56:

$$R_B = 0.7\,\Omega \qquad R_G = 0.3\,\Omega \qquad R_L = 7.2\,\Omega$$

Find:

a. The Thévenin equivalent of the circuit to the left of the terminal pair of port y-y'.
b. The terminal voltage of the generator, i.e., the voltage between y and y'.

Analysis:

a. Specify the polarity of the Thévenin equivalent voltage:

$$VD: \quad V_{TH} = \frac{V_B R_L}{R_B + R_L} = \frac{(11)(7.2)}{0.7+7.2} = 10.03 \text{ V}$$

Suppress generator source:

$$R_T = R_L \| R_B = \frac{(7.2)(0.7)}{0.7+7.2} = 638 \text{ m}\Omega$$

b. Specify the polarity of the terminal voltage. Choose a ground.

$$KCL: \quad \frac{V_T - V_G}{R_G} + \frac{V_T - v_T}{R_T} = 0$$

$$V_T = \frac{\dfrac{V_G}{R_G} + \dfrac{v_T}{R_T}}{\dfrac{1}{R_G} + \dfrac{1}{R_T}} = 11.37 \text{ V}$$

Section 3.7: Maximum power transfer

Problem 3.57

Solution:

Known quantities:
The values of the voltage and of the resistor in the equivalent circuit of Figure P3.57:

$$V_{TH} = 12 \text{ V}$$
$$R_{eq} = 8\,\Omega$$

Assumptions:
Assume the conditions for maximum power transfer exist.

Find:

a. The value of R_L.

b. The power developed in R_L.

c. The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

Analysis:

a. For maximum power transfer:

$$R_L = R_{eq} = 8\,\Omega$$

b. *VD*:

$$V_{RL} = \frac{V_{TH} R_L}{R_{eq} + R_L} = \frac{(12)(8)}{8+8} = 6 \text{ V}$$

$$P_{RL} = \frac{V^2{}_{RL}}{R_L} = \frac{(6)^2}{8} = 4.5 \text{ W}$$

c. $$\eta = \frac{P_0}{P_S} = \frac{P_{RL}}{P_{\mathrm{Re}q} + P_{RL}} = \frac{I_S^2 R_L}{I_S^2 R_{eq} + I_S^2 R_L} = \frac{R_L}{R_{eq} + R_L} = 0.5 = 50\%$$

Problem 3.58

Solution:

Known quantities:
The values of the voltage and of the resistor in the equivalent circuit of Figure P3.57:

$$V_{TH} = 300 \text{ V}$$
$$R_{eq} = 600\,\Omega$$

Assumptions:
Assume the conditions for maximum power transfer exist.

Find:

a. The value of R_L.

b. The power developed in R_L.

c. The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

Analysis:

a. For maximum power transfer:

$$R_L = R_{eq} = 600 \ \Omega$$

b. *VD*: $$V_{RL} = \frac{V_{TH} R_L}{R_{eq} + R_L} = \frac{(35)(600)}{600+600} = 17.5 \text{ V}$$

$$P_{RL} = \frac{V^2{}_{RL}}{R_L} = \frac{(17.5)^2}{600} = 510.4 \text{ mW}$$

c. $$\eta = \frac{P_0}{P_S} = \frac{P_{RL}}{P_{\text{Req}} + P_{RL}} = \frac{I_S^2 R_L}{I_S^2 R_{eq} + I_S^2 R_L} = \frac{R_L}{R_{eq} + R_L} = 0.5 = 50\%$$

Problem 3.59

Solution:

Known quantities:

The values of the voltage source, $V_S = 12 \text{ V}$, and of the resistance representing the internal losses of the source, $R_S = 0.3\,\Omega$, in the circuit of Figure P3.59.

Find:

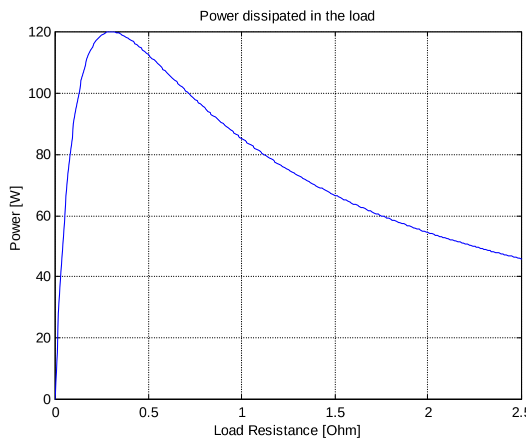
a. Plot the power dissipated in the load as a function of the load resistance. What can you conclude from your plot?
b. Prove, analytically, that your conclusion is valid in all cases.

Analysis:

$$-V_S = IR_S + IR = 0$$

a. *KVL*: $$I = \frac{V_S}{R_S + R} \qquad P_R = I^2 R$$

R [Ω]	I [A]	P_R [W]
0	40	0.0
0.1	30	90.0
0.3	20	120.0
0.9	10	90.0
2.1	5	52.5



b. $$P_R = I^2 R = \frac{V_S^2 R}{(R+R_S)^2} = V_S^2 R (R+R_S)^{-2}$$

$$\frac{dP_R}{dR} = V_S^2 (1)(R+R_S)^{-2} + V_S^2 (R)(-2)(R+R_S)^{-3}(1) = 0$$

$$(R+R_S)^1 - 2R = 0 \quad \Rightarrow \quad R = R_S$$

Section 3.8: Nonlinear circuit elements

Problem 3.60

Solution:

Known quantities:
The two nonlinear resistors, in the circuit of Figure P3.60, are characterized by:

$$i_a = 2v_a^3 \qquad i_b = v_b^3 + 10v_b$$

Find:
The node voltage equations in terms of v_1 and v_2.

Analysis:

At node 1, $\frac{v_1}{1} + i_a = 1 \Rightarrow v_1 + 2v_a^3 = 1$

At node 2, $i_b - i_a = 26 \Rightarrow v_b^3 + 10v_b - 2v_a^3 = 26$

But $v_a = v_1 - v_2$ and $v_b = v_2$. Therefore, the node equations are

$$v_1 + 2(v_1 - v_2)^3 = 1$$

and

$$v_2^3 + 10v_2 - 2(v_1 - v_2)^3 = 26$$

Problem 3.61

Solution:

Known quantities:
The characteristic curve I-V shown in Figure P3.61, and the values of the voltage, $V_T = 15 \text{ V}$, and of the resistance, $R_T = 200\ \Omega$, in the circuit of Figure P3.61.

Find:
a. The operating point of the element that has the characteristic curve shown in Figure P3.61.
b. The incremental resistance of the nonlinear element at the operating point of part a.
c. If V_T were increased to 20 V, find the new operating point and the new incremental resistance.

Analysis:

a. *KVL*:

$$-15 + 200I + V = 0$$
$$-15 + 200\left(0.0025V^2\right) + V = 0$$

Solving for V and I,

$$I = 52.2 \text{ mA} \qquad V = 4.57 \text{ V} \quad \text{or} \quad -6.57 \text{ V}$$

The second voltage value is physically impossible.

b. $R_{\text{inc}} = 10(0.0522)^{-0.5} = 43.8\,\Omega$

c. $I = 73 \text{ mA} \qquad V = 5.40 \text{ V} \qquad R_{\text{inc}} = 37\,\Omega$

Problem 3.62

Solution:

Known quantities:

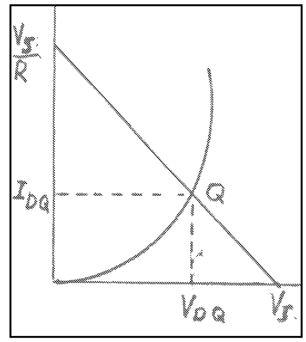
The characteristic curve I-V shown in Figure P3.62, and the values of the voltage, $V_S = 450\text{ V}$, and of the resistance, $R = 9\ \Omega$, in the circuit of Figure P3.62.

Find:

The current through and the voltage across the nonlinear device.

Analysis:

The I-V characteristic for the nonlinear device is given. Plot the circuit I-V characteristic, i.e., the DC load line.



KVL: $-V_S + I_D R + V_D = 0$

$$I_D = \frac{V_S - V_D}{R} = \frac{450 - V_D}{9} =$$

$$= 0\text{ A} \quad \text{if} \quad V_D = 450\text{ V}$$

$$= 50\text{ A} \quad \text{if} \quad V_D = 0$$

The DC load line [circuit characteristic] is linear. Plotting the two intercepts above and connecting them with a straight line gives the DC load line. The solution for V and I is at the intersection of the device and circuit characteristics:

$$I_{DQ} \approx 26\text{ A} \quad V_{DQ} \approx 210\text{ V}.$$

Problem 3.63

Solution:

Known quantities:

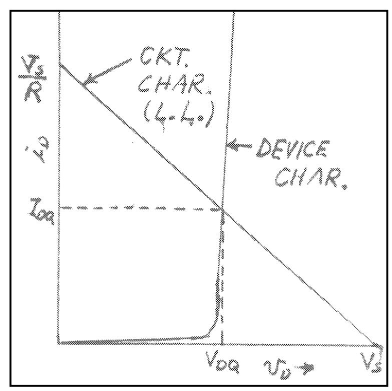
The I-V characteristic shown in Figure P3.63, and the values of the voltage, $V_S = V_T = 1.5\text{ V}$, and of the resistance, $R = R_{eq} = 60\ \Omega$, in the circuit of Figure P3.63.

Find:

The current through and the voltage across the nonlinear device.

Analysis:

The solution is at the intersection of the device and circuit characteristics. The device I-V characteristic is given. Determine and plot the circuit I-V characteristic.



KVL: $-V_S + I_D R + V_D = 0$

$$I_D = \frac{V_S - V_D}{R} = \frac{1.5\text{ V} - \text{V}_\text{D}}{60\,\Omega} =$$

$$= 0\text{ A} \quad \text{if} \quad V_D = 1.5\text{ V}$$

$$= 25\text{ mA} \quad \text{if} \quad V_D = 0$$

The DC load line [circuit characteristic] is linear. Plotting the two intercepts above and connecting them with a straight line gives the DC load line. The solution is at the intersection of the device and circuit characteristics, or "Quiescent", or "Q", or "DC operating" point:

$$I_{DQ} \approx 12\text{ mA} \quad V_{DQ} \approx 0.77\text{ V}.$$

Problem 3.64

Solution:

Known quantities:
The I-V characteristic shown in Figure P3.64 as a function of pressure.

$$V_S = V_T = 2.5 \text{ V} \qquad R = R_{eq} = 125\,\Omega$$

Find:
The DC load line, the voltage across the device as a function of pressure, and the current through the nonlinear device when $p = 30$ psig.

Analysis:
Circuit characteristic [DC load line]:

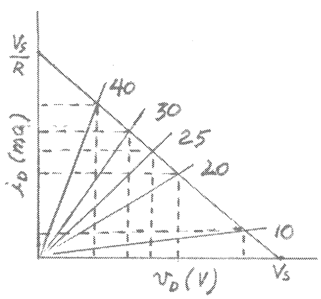
KVL: $-V_S + I_D R + V_D = 0$

$$I_D = \frac{V_S - V_D}{R} = \frac{2.5\text{ V} - V_D}{125\,\Omega} =$$

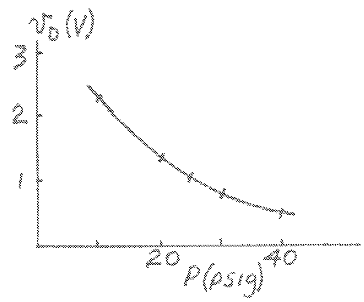
$$= 0 \text{ A} \qquad \text{if} \quad V_D = 2.5 \text{ V}$$

$$= 20 \text{ mA} \qquad \text{if} \quad V_D = 0$$

The circuit characteristic is a linear relation. Plot the two intercepts and connect with a straight line to plot the DC load line. Solutions are at the intersections of the circuit with the device characteristics, i.e.:



p [psig]	V_D [V]
10	2.14
20	1.43
25	1.18
30	0.91
40	0.60



The plot is nonlinear.

At $p = 30$ psig: $V_D = 1.08$ V $\qquad I_D = 12.5$ mA .

Problem 3.65

Solution:

Known quantities:
The I-V characteristic of the nonlinear device in the circuit shown in Figure P3.65:

$$I_D = I_0 e^{\frac{v_D}{V_T}}$$

$$I_0 = 10^{-15} \text{ A} \qquad V_T = 26 \text{ mV}$$

$$V_S = V_T = 1.5 \text{ V}$$

$$R = R_{ea} = 60\,\Omega$$

Find:
An expression for the DC load line. The voltage across and current through the nonlinear device.

Analysis:
Circuit characteristic [DC load line]:

KVL: $-V_S + I_D R + V_D = 0$

$$[1] \qquad I_D = \frac{V_S - V_D}{R} = \frac{1.5 - V_D}{60}$$

$$[2] \qquad V_D = V_T \ln\left(\frac{I_D}{I_0}\right) = 0.026 \cdot \ln\left(\frac{I_D}{10^{-15}}\right)$$

Iterative procedure:
Initially guess $V_D = 750 \text{ mV}$. Note this voltage must be between zero and the value of the source voltage.
Then:

a. Use Equation [1] to compute a new I_D.
b. Use Equation [2] to compute a new V_D.
c. Iterate, i.e., go step a. and repeat.

V_D [mV]	I_D [mA]
750	12.5
784.1	11.93
782.9	11.95
782.9	11.95
…	…

$$I_{DQ} \approx 11.95 \text{ mA} \quad V_{DQ} \approx 782.9 \text{ mV}.$$

Problem 3.66

Solution:

Known quantities:
The I-V characteristic shown in Figure P3.66 as a function of pressure.

$$V_S = V_T = 2.5 \text{V} \qquad R = R_{eq} = 125\ \Omega$$

Find:
The DC load line, and the current through the nonlinear device when $p = 40$ psig.

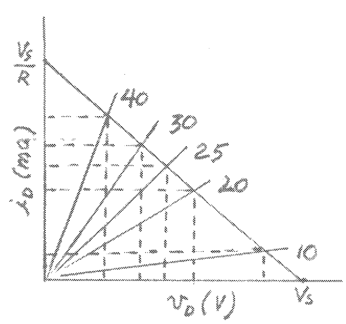
Analysis:
Circuit characteristic [DC load line]:

KVL: $-V_S + I_D R + V_D = 0$

$$I_D = \frac{V_S - V_D}{R} = \frac{2.5 \text{ V} - V_D}{125\ \Omega} =$$

$$= 0 \text{ A} \qquad \text{if} \quad V_D = 2.5 \text{ V}$$

$$= 20 \text{ mA} \qquad \text{if} \quad V_D = 0$$



The circuit characteristic is a linear relation that can be plotted by plotting the two intercepts and connecting them with a straight line. Solutions are at the intersections of the circuit and device characteristics.

At $p = 40$ psig: $\qquad V_D = 0.60 \text{ V} \qquad I_D = 15.2 \text{ mA}$
