Question:

What is the use of trigonometric Fourier series? The coefficients a_k and b_k are more troublesome to use compared to X_k !

Answer:

Complex exponential Fourier series expansion

The complex exponential Fourier series expansion provides a convenient mean for obtaining the frequency domain representation of a periodic signal $x_p(t)$ since the coefficients X_k lead directly to the magnitude and phase spectral plots of the signal. We need only to evaluate one integral, that is

$$X_k = T_p^{-1}\int_0^{T_p} x_p(t)\exp\left(-j2\pi k\,t/T_p\right)dt$$

to obtain X_k. If $x_p(t)$ is <u>real</u>, then $X_k = X_{-k}^*$

Trigonometric Fourier series expansion

The trigonometric Fourier series expansion $x_p(t) = a_0 + 2\sum_{k=1}^{\infty} a_k \cos\left(2\pi k\,t/T_p\right) + b_k \sin\left(2\pi k\,t/T_p\right)$ requires the evaluation of two integrals, that is

$$a_k = T_p^{-1}\int_0^{T_p} x_p(t)\cos\left(2\pi k\,t/T_p\right)dt \quad \text{and} \quad b_k = T_p^{-1}\int_0^{T_p} x_p(t)\sin\left(2\pi k\,t/T_p\right)dt \quad \ldots\ldots \ (\clubsuit)$$

which seems to require twice the effort for evaluating X_k. However, through X_k, the a_k and b_k can also be obtained using

$$a_k = 0.5\left(X_{-k} + X_k\right) \quad \text{and} \quad b_k = -0.5j\left(X_{-k} - X_k\right).$$

In this way, there is no significant increase in computational complexity in evaluating a_k and b_k as compared to X_k.

Clearly, if $x_p(t)$ is <u>real</u>, then a_k and b_k are <u>real</u>, as indicated by (♣). In this case, the trigonometric Fourier series expansion provides a convenient mean for obtaining the time-domain representation of the harmonics of a <u>real</u> periodic signal $x_p(t)$ in terms of real cosine and sine waves of different amplitudes and frequencies.

Hence, we cannot conclude that one Fourier series expansion is more useful than the other. It all depends on what we wish to see, the spectrum or waveforms of harmonics.
