

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA 1505 Mathematics I
Tutorial 4

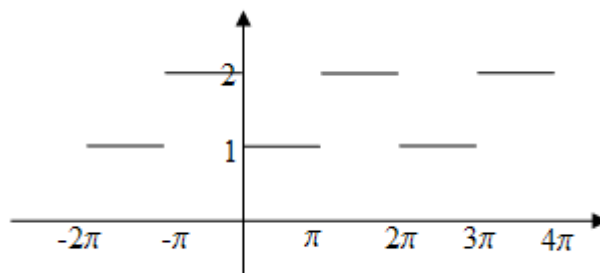
1. Show that the Fourier series for

$$f(x) = \frac{1}{2}(x + |x|), -\pi < x < \pi; \quad f(x + 2\pi) = f(x)$$

is given by

$$\frac{\pi}{4} + \sum_1^{\infty} \left\{ \frac{(-1)^n - 1}{\pi n^2} \cos nx + \frac{(-1)^{n+1}}{n} \sin nx \right\}.$$

2. Find the Fourier series that represent the following graph:



Ans. $\frac{3}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$

3. Sketch the graph of the following periodic function. Determine whether it is an even or odd function, and find its Fourier series.

$$f(x) = \begin{cases} 1 & -\pi/2 < x < \pi/2 \\ 0 & \pi/2 < x < 3\pi/2 \end{cases}, \quad f(x + 2\pi) = f(x).$$

Ans. $\frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos(2n-1)x}{2n-1}$

4. Find the Fourier series of a half-wave rectifier

$$u(t) = \begin{cases} 0 & -\pi/w < t < 0 \\ \sin wt & 0 < t < \pi/w \end{cases}$$

and $u(t + \frac{2\pi}{w}) = u(t)$.

Ans. $\frac{1}{\pi} + \frac{1}{2} \sin wt - \frac{2}{\pi} \left(\frac{1}{1 \cdot 3} \cos 2wt + \frac{1}{3 \cdot 5} \cos 4wt + \dots \right)$

5. Find the Fourier series for the function

$$f(x) = \begin{cases} -2 - x & -2 < x < 0 \\ 2 - x & 0 < x < 2 \end{cases}$$

and $f(x+4) = f(x)$.

Ans. $\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{2}$

6. Find the sine and cosine Fourier half range expansion for

$$f(x) = x, \quad 0 < x < \pi.$$

Also, sketch the corresponding expansions of f .

Ans. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2 \sin nx}{n}; \quad \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2((-1)^n - 1)}{\pi} \frac{\cos nx}{n^2}$