APPENDIX A

Exercises

EA.1 Given $Z_1 = 2 - j3$ and $Z_2 = 8 + j6$, we have:

$$Z_1 + Z_2 = 10 + j3$$

$$Z_1 - Z_2 = -6 - j9$$

$$Z_1 Z_2 = 16 - j24 + j12 - j^2 18 = 34 - j12$$

$$Z_1 / Z_2 = \frac{2 - j3}{8 + j6} \times \frac{8 - j6}{8 - j6} = \frac{16 - j12 - j24 + j^2 18}{100} = -0.02 - j0.36$$

EA.2
$$Z_1 = 15\angle 45^\circ = 15\cos(45^\circ) + j15\sin(45^\circ) = 10.6 + j10.6$$
$$Z_2 = 10\angle -150^\circ = 10\cos(-150^\circ) + j10\sin(-150^\circ) = -8.66 - j5$$
$$Z_3 = 5\angle 90^\circ = 5\cos(90^\circ) + j5\sin(90^\circ) = j5$$

EA.3 Notice that Z_1 lies in the first quadrant of the complex plane.

$$Z_1 = 3 + j4 = \sqrt{3^2 + 4^2}\angle \arctan(4/3) = 5\angle 53.13^\circ$$

Notice that Z_2 lies on the negative imaginary axis.

$$Z_2 = -j10 = 10\angle -90^\circ$$

Notice that Z_3 lies in the third quadrant of the complex plane.

$$Z_3 = -5 - j5 = \sqrt{5^2 + 5^2}\angle(180^\circ + \arctan(-5/-5)) = 7.07\angle 225^\circ = 7.07\angle -135^\circ$$

EA.4 Notice that Z_1 lies in the first quadrant of the complex plane.

$$Z_1 = 10 + j10 = \sqrt{10^2 + 10^2}\angle \arctan(10/10) = 14.14\angle 45^\circ = 14.14\exp(j45^\circ)$$

Notice that Z_2 lies in the second quadrant of the complex plane.

$$Z_2 = -10 + j10 = \sqrt{10^2 + 10^2}\angle(180^\circ + \arctan(-10/10))$$
$$= 14.14\angle 135^\circ = 14.14\exp(j135^\circ)$$

EA.5 $Z_1 Z_2 = (10\angle 30^\circ)(20\angle 135^\circ) = (10\times 20)\angle(30^\circ + 135^\circ) = 200\angle(165^\circ)$

$Z_1 / Z_2 = (10\angle 30^\circ)/(20\angle 135^\circ) = (10/20)\angle(30^\circ - 135^\circ) = 0.5\angle(-105^\circ)$

$$Z_1 - Z_2 = (10\angle 30^\circ) - (20\angle 135^\circ) = (8.66 + j5) - (-14.14 + j14.14)$$
$$= 22.8 - j9.14 = 24.6\angle -21.8^\circ$$

$$Z_1 + Z_2 = (10\angle 30^\circ) + (20\angle 135^\circ) = (8.66 + j5) + (-14.14 + j14.14)$$
$$= -5.48 + j19.14 = 19.9\angle 106^\circ$$

Problems

PA.1 Given $Z_1 = 2 + j3$ and $Z_2 = 4 - j3$, we have:

$$Z_1 + Z_2 = 6 + j0$$

$$Z_1 - Z_2 = -2 + j6$$

$$Z_1 Z_2 = 8 - j6 + j12 - j^2 9 = 17 + j6$$

$$Z_1 / Z_2 = \frac{2 + j3}{4 - j3} \times \frac{4 + j3}{4 + j3} = \frac{-1 + j18}{25} = 0.04 + j0.72$$

PA.2 Given that $Z_1 = 1 - j2$ and $Z_2 = 2 + j3$, we have:

$$Z_1 + Z_2 = 3 + j1$$

$$Z_1 - Z_2 = -1 - j5$$

$$Z_1 Z_2 = 2 + j3 - j4 - j^2 6 = 8 - j1$$

$$Z_1 / Z_2 = \frac{1 - j2}{2 + j3} \times \frac{2 - j3}{2 - j3} = \frac{-4 - j7}{13} = -0.3077 - j0.5385$$

PA.3 Given that $Z_1 = 10 + j5$ and $Z_2 = 20 - j20$, we have:

$$Z_1 + Z_2 = 30 - j15$$

$$Z_1 - Z_2 = -10 + j25$$

$$Z_1 Z_2 = 200 - j200 + j100 - j^2 100 = 300 - j100$$

$$Z_1 / Z_2 = \frac{10 + j5}{20 - j20} \times \frac{20 + j20}{20 + j20} = \frac{100 + j300}{800} = 0.125 + j0.375$$

PA.4 (a) $Z_a = 5 - j5 = 7.071\angle -45^\circ = 7.071\exp(-j45^\circ)$

(b) $Z_b = -10 + j5 = 11.18\angle 153.43^\circ = 11.18\exp(j153.43^\circ)$

(c) $Z_c = -3 - j4 = 5\angle -126.87^\circ = 5\exp(-j126.87^\circ)$

(d) $Z_d = -j12 = 12\angle -90^\circ = 12\exp(-j90^\circ)$

PA.5 (a) $Z_a = 5\angle 45^\circ = 5\exp(j45^\circ) = 3.536 + j3.536$

(b) $Z_b = 10\angle 120^\circ = 10\exp(j120^\circ) = -5 + j8.660$

(c) $Z_c = 15\angle -90^\circ = 15\exp(-j90^\circ) = -j15$

(d) $Z_d = -10\angle 60^\circ = 10\exp(-j120^\circ) = -5 - j8.660$

PA.6 (a) $Z_a = 5e^{j30^\circ} = 5\angle 30^\circ = 4.330 + j2.5$

(b) $Z_b = 10e^{-j45^\circ} = 10\angle -45^\circ = 7.071 - j7.071$

(c) $Z_c = 100e^{j135^\circ} = 100\angle 135^\circ = -70.71 + j70.71$

(d) $Z_d = 6e^{j90^\circ} = 6\angle 90^\circ = j6$

PA.7 (a) $Z_a = 5 + j5 + 10\angle 30^\circ = 13.66 + j10$

(b) $Z_b = 5\angle 45^\circ - j10 = 3.536 - j6.464$

(c) $Z_c = \dfrac{10\angle 45^\circ}{3 + j4} = \dfrac{10\angle 45^\circ}{5\angle 53.13^\circ} 2\angle -8.13^\circ = 1.980 - j0.283$

(d) $Z_d = \dfrac{15}{5\angle 90^\circ} = 3\angle -90^\circ = -j3$