NUS National University of Singapore

Dean's Office, Faculty of Science (ANSWER SHEET)

2005 TEST AND ANSWERS

FORM CC1

USE 2B PENCILS ONLY

INSTRUCTIONS

Suggested answers to each question are given in the question paper. Choose an answer and shade the corresponding circle.

EXAMPLES OF SHADING

CORRECT ● INCORRECT ⊗ ✓ ⊖ ⊝

SECTION A STUDENT'S NAME

MODULE : MA1505

YEAR/SEMESTER: 2005

DATE:

SECTION B : MATRICULATION NUMBER

U

0	0	0	0	0	0	A	0	0
1	1	1	1	1	1	B	1	1
2	2	2	2	2	2	E	2	2
3	3	3	3	3	3	H	3	3
4	4	4	4	4	4	J	4	4
5	5	5	5	5	5	L	5	5
6	6	6	6	6	6	M	6	6
7	7	7	7	7	7	N	7	7
8	8	8	8	8	8	R	8	8
9	9	9	9	9	9	U	9	9
						W		
						X		
						Y		

1. Write your matriculation number here.
2. Now SHADE the corresponding circle in the grid for each digit or letter.

S/NO 27890

SECTION C : ANSWERS

(1 2 3 4 5 = A B C D E)

1 ● B C D E	11 A B C D ●	21 A B C D E	31 A B C D E	41 A B C D E
2 A B C D ●	12 A B C D ●	22 A B C D E	32 A B C D E	42 A B C D E
3 A B C ● E	13 A B C D E	23 A B C D E	33 A B C D E	43 A B C D E
4 A B C ● E	14 A B C D E	24 A B C D E	34 A B C D E	44 A B C D E
5 A ● C D E	15 A B C D E	25 A B C D E	35 A B C D E	45 A B C D E
6 A B ● D E	16 A B C D E	26 A B C D E	36 A B C D E	46 A B C D E
7 A ● C D E	17 A B C D E	27 A B C D E	37 A B C D E	47 A B C D E
8 ● B C D E	18 A B C D E	28 A B C D E	38 A B C D E	48 A B C D E
9 A B ● D E	19 A B C D E	29 A B C D E	39 A B C D E	49 A B C D E
10 ● B C D E	20 A B C D E	30 A B C D E	40 A B C D E	50 A B C D E

51 A B C D E	61 A B C D E	71 A B C D E	81 A B C D E	91 A B C D E
52 A B C D E	62 A B C D E	72 A B C D E	82 A B C D E	92 A B C D E
53 A B C D E	63 A B C D E	73 A B C D E	83 A B C D E	93 A B C D E
54 A B C D E	64 A B C D E	74 A B C D E	84 A B C D E	94 A B C D E
55 A B C D E	65 A B C D E	75 A B C D E	85 A B C D E	95 A B C D E
56 A B C D E	66 A B C D E	76 A B C D E	86 A B C D E	96 A B C D E
57 A B C D E	67 A B C D E	77 A B C D E	87 A B C D E	97 A B C D E
58 A B C D E	68 A B C D E	78 A B C D E	88 A B C D E	98 A B C D E
59 A B C D E	69 A B C D E	79 A B C D E	89 A B C D E	99 A B C D E
60 A B C D E	70 A B C D E	80 A B C D E	90 A B C D E	100 A B C D E

1. $f'(x)=2xe^{-x^2}-2x(x^2)e^{-x^2}=2x(1-x^2)e^{-x^2}=2x(1+x)(1-x)e^{-x^2}$

2. $2x+xy'+y+4yy'=0$. at $(-2,-3)$, $2(-2)-2y'-3+4(-3)y'=0 \Rightarrow -4-3-14y'=0 \Rightarrow y'=-\frac{1}{2}$

$\frac{y+3}{x+2}=-\frac{1}{2} \Rightarrow 2y+6=-x-2 \Rightarrow x+2y+8=0$

3. $y=\sin^{-1}(3x^2)\in[-\frac{\pi}{2},\frac{\pi}{2}] \Rightarrow \sin y=3x^2 \Rightarrow (\cos y)y'=6x \Rightarrow y'=\frac{6x}{|\cos y|}=\frac{6x}{\sqrt{1-\sin^2 y}}=\frac{6x}{\sqrt{1-9x^4}}$

4. $y'=2x+\frac{8}{x^2} \Rightarrow y''=2-\frac{16}{x^3}=0 \Rightarrow \frac{16}{x^3}=2 \Rightarrow x^3=8 \Rightarrow x=2$

5. $\lim_{x\to 1}\left[\frac{1}{\ln x}-\left(1+\frac{1}{x-1}\right)\right]=\lim_{x\to 1}\left[\frac{x-1-\ln x}{(x-1)\ln x}\right]-1=\lim_{x\to 1}\left[\frac{1-\frac{1}{x}}{(x-1)\frac{1}{x}+\ln x}\right]-1=\lim_{x\to 1}\left[\frac{x-1}{x-1+x\ln x}\right]-1$

$=\lim_{x\to 1}\left[\frac{1}{1+x(\frac{1}{x})+\ln x}\right]-1=\frac{1}{2+0}-1=-\frac{1}{2}$

6. $f'(x)=\frac{(4+x^2)(1)-x(2x)}{(4+x^2)^2}=\frac{4-x^2}{(4+x^2)^2}=0 \Rightarrow x=-2\in[-3,1]$. $f(-2)=\frac{-6}{24}=-\frac{1}{4}<f(-3)=\frac{-6}{26}<f(1)=\frac{1}{5}$

7. $\int(1-\cos^2 x)^2\sin x\,dx=-\int 1-2\cos^2 x+\cos^4 x\,d(\cos x)=-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+C$

8. $\int_0^1\ln(1+x^2)\,dx=x\ln(1+x^2)\Big|_0^1-\int_0^1\frac{2x^2}{1+x^2}dx=\ln 2-\int_0^1\frac{2(1+x^2-1)}{1+x^2}dx$

$=\ln 2-2+\int_0^1\frac{2}{1+x^2}dx=\ln 2-2+\int_{t=0}^{t=\frac{\pi}{4}}\frac{2}{1+\tan^2 t}d(\tan t)=\ln 2-2+\int_0^{\frac{\pi}{4}}\frac{2}{\sec^2 t}\sec^2 t\,dt$

$=\ln 2-2+(\frac{\pi}{4})2=\ln 2+\frac{\pi}{2}-2$

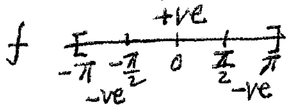
sgn	Diff	Int
	$\ln(1+x^2)$	x
$-$	$\frac{2x}{1+x^2}$	$\frac{2x^2}{1+x^2}$
$-$	1	$2-2\tan^{-1}x$

9. $\int_{x=1}^{x=2}\frac{-1}{x\sqrt{1+\frac{1}{x^2}}}d(\frac{1}{x})=\int_1^{\frac{1}{2}}\frac{-2u}{2\sqrt{1+u^2}}du=\int_{(u=1)}^{(u=\frac{1}{2})}\frac{-1}{2\sqrt{1+u^2}}d(u^2)=\int_1^{\frac{1}{4}}\frac{-1}{2\sqrt{1+t}}dt=\left[-\sqrt{1+t}\right]_1^{\frac{1}{4}}=\left[\sqrt{2}-\sqrt{1+\frac{1}{4}}\right]=\sqrt{2}-\frac{\sqrt{5}}{2}$

10. $\int_0^{\frac{\pi}{4}}(\tan^{n-2}x)(1+\tan^2 x)\,dx=\int_0^{\frac{\pi}{4}}\tan^{n-2}x\sec^2 x\,dx=\int_{x=0}^{x=\frac{\pi}{4}}\tan^{n-2}x\,d(\tan x)=\frac{\tan^{n-1}x}{n-1}\Big|_0^{\frac{\pi}{4}}=\frac{1}{n-1}$

11. $f(x)=2\cos x-\sin 2x=0 \Rightarrow (2\cos x)(1-\sin x)=0 \Rightarrow \cos x=0$ or $\sin x=1 \Rightarrow x=\pm\frac{\pi}{2}$

$\lim_{x\to\frac{\pi}{2}}\frac{f(x)-f(\frac{\pi}{2})}{x-\frac{\pi}{2}}=f'(\frac{\pi}{2})=(-2\sin x-2\cos 2x)\Big|_{x=\frac{\pi}{2}}=-2-2(-1)=0$



$f'(-\frac{\pi}{2})=2-2(-1)=4\neq 0 \Rightarrow f(x)=(x+\frac{\pi}{2})g_1(x),\ g_1(-\frac{\pi}{2})\neq 0$

$f(0)=2>0 \Rightarrow f\leq 0$ on $[-\pi,-\frac{\pi}{2}]$ and $f\geq 0$ on $[-\frac{\pi}{2},\frac{\pi}{2}]$.

$f''(x)=-2\cos x+4\sin 2x$. $f''(\frac{\pi}{2})=0$. $f^{(3)}(x)=2\sin x+8\cos 2x$, $f^{(3)}(\frac{\pi}{2})=2+8(-1)\neq 0$

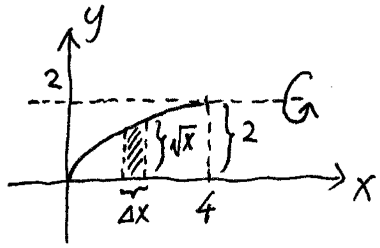
$\therefore f(x)=(x-\frac{\pi}{2})^3g_2(x),\ g_2(\frac{\pi}{2})\neq 0 \Rightarrow f\leq 0$ on $[\frac{\pi}{2},\pi]$. $\int f(x)\,dx=2\sin x+\frac{1}{2}\cos 2x+C$

$\int_{-\pi}^{\pi}|f|=-\int_{-\pi}^{-\frac{\pi}{2}}f+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f+\int_{\frac{\pi}{2}}^{\pi}-f=\int_{-\frac{\pi}{2}}^{-\pi}f+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f+\int_{\pi}^{\frac{\pi}{2}}f=\left[2(0)+\frac{1}{2}-(-2-\frac{1}{2})\right]+\left[2-\frac{1}{2}-(-2-\frac{1}{2})\right]+\left[2-\frac{1}{2}-\frac{1}{2}\right]$

$=3+4+1=8$

12. Vol of typical disk with a hole $=\pi\{2^2-(2-\sqrt{x})^2\}\Delta x$

$=\pi(4\sqrt{x}-x)\Delta x$



Vol of solid of revolution $=\int_0^4\pi(4\sqrt{x}-x)\,dx$

$=\pi\left[\frac{8}{3}x^{3/2}-\frac{1}{2}x^2\right]_0^4$

$=\underline{\underline{\frac{40}{3}\pi}}$

1. Let $f(x) = x^2 e^{-x^2}$. Then $f'(x) =$

(A) $2x(1+x)(1-x)e^{-x^2}$

(B) $x\left(2+x^2\right)e^{-x^2}$

(C) $2x\left(1+x^2\right)e^{-x^2}$

(D) $2x\left(1-2x^2\right)e^{-x^2}$

(E) $x\left(2-x^2\right)e^{-x^2}$

2. Find the equation of the tangent to the curve $x^2+xy+2y^2 = 28$ at the point $(-2,-3)$.

(A) $3x-2y=0$

(B) $x-2y-4=0$

(C) $2x-y+1=0$

(D) $x+y-5=0$

(E) $x+2y+8=0$

3. $\frac{d}{dx}\sin^{-1}\left(3x^2\right) =$

(A) $\frac{6x}{\sqrt{1-3x^2}}$

(B) $\frac{3x^2}{\sqrt{1-9x^2}}$

(C) $\frac{3x^2}{\sqrt{1-9x^4}}$

(D) $\frac{6x}{\sqrt{1-9x^4}}$

(E) $\frac{6x}{\sqrt{1+9x^2}}$

4. The graph of the function $y = x^2 - \frac{8}{x}$ for $x \in (0, \infty)$ has a point of inflection when $x =$

(A) 1
(B) 3
(C) $\sqrt{5}$
(D) 2
(E) 4

5. Evaluate $\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{x}{x-1} \right)$.

(A) ∞
(B) $-\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $-\frac{2}{3}$
(E) $\frac{4}{5}$

6. Let $f(x) = \frac{x}{4+x^2}$, $x \in [-3, 1]$. Let M and m denote the absolute maximum value and absolute minimum value of f respectively. Then

(A) $M = \frac{1}{5}$, $m = -\frac{3}{13}$
(B) $M = \frac{1}{4}$, $m = -\frac{3}{13}$
(C) $M = \frac{1}{5}$, $m = -\frac{1}{4}$
(D) $M = \frac{1}{4}$, $m = -\frac{1}{4}$
(E) $M = \frac{3}{13}$, $m = -\frac{3}{13}$

7. $\int \sin^5 x \mathrm{d}x =$

(A) $-\cos x - \frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(B) $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(C) $\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(D) $-\cos x + \frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(E) $\cos x + \frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

8. $\displaystyle\int_0^1 \ln\left(1+x^2\right)\mathrm{d}x =$

(A) $\ln 2 + \frac{\pi}{2} - 2$

(B) $\ln 2 + \frac{\pi}{4} - 2$

(C) $\ln 2 - \frac{\pi}{2} + \frac{1}{2}$

(D) $\ln 2 + \frac{\pi}{2} - \frac{1}{2}$

(E) $\ln 2 - \frac{\pi}{4} + 2$

9. $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{1+x^2}}dx =$

(A) $\frac{\sqrt{5}}{2}$

(B) $\sqrt{5} - \sqrt{2}$

(C) $\sqrt{2} - \frac{\sqrt{5}}{2}$

(D) $2\sqrt{5}$

(E) $2\sqrt{2} - \sqrt{5}$

10. For each positive integer $n \geq 3$, define $f(n) = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$. Then $f(n) + f(n-2) =$

(A) $\frac{1}{n-1}$
(B) $\frac{1}{n+1}$
(C) $\frac{1}{n-2}$
(D) $\frac{1}{n}$
(E) $\frac{1}{n(n-2)}$

11. Find the area of the region bounded by the curves $y = 2\cos x$ and $y = \sin 2x$ for $x \in [-\pi, \pi]$.

(A) $\frac{8}{3}$
(B) 1
(C) $\frac{1}{2}$
(D) 3
(E) 8

12. Let R denote the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$. Find the volume generated by revolving the region R about the line $y = 2$.

(A) $\frac{133\pi}{10}$
(B) $\frac{53\pi}{4}$
(C) $\frac{66\pi}{5}$
(D) $\frac{27\pi}{2}$
(E) $\frac{40\pi}{3}$

END OF PAPER