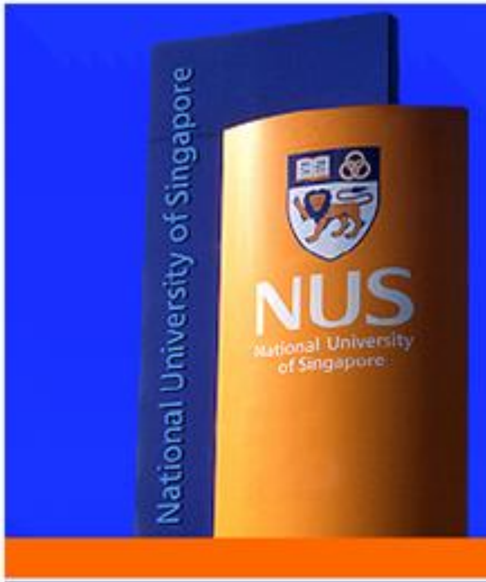


Singapore's
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# *CG1108*
# *Electrical Engineering*

## *Lecture 2*

**André-Marie Ampère**

**(20 January 1775 – 10 June 1836)**

**was a French physicist and mathematician who is generally regarded as one of the main discoverers of electromagnetism. In honor of his achievements, the basic unit of electric current, the ampere, is named after him.**

## *Lecture Outline*

- Revision
- Node Voltage Analysis
- Mesh Current Analysis

# *Lecture 1 Review*

- EE handles Information and Energy
- Voltage – energy per unit charge

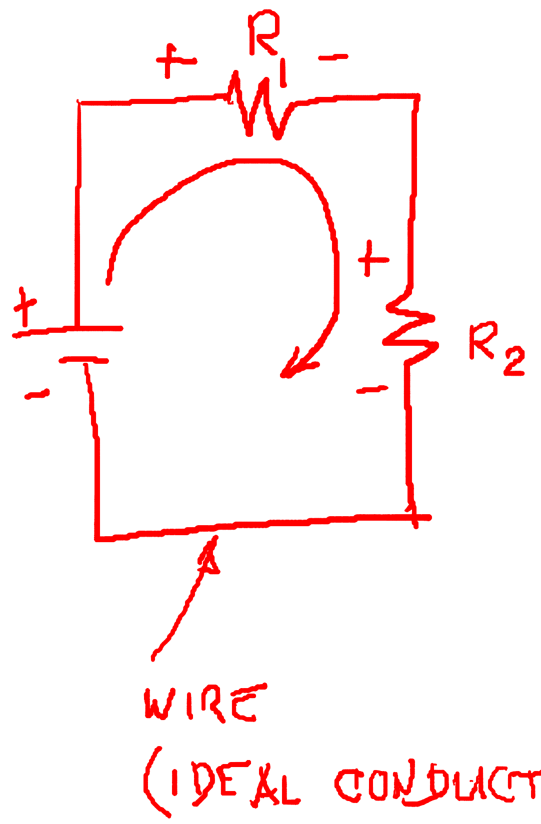
  Polarity – energy gained, lost
- Current – charge per unit time

  Direction
- Power – energy per unit time

  Voltage x Current

# *Lecture 1 Review*

## Passive sign convention



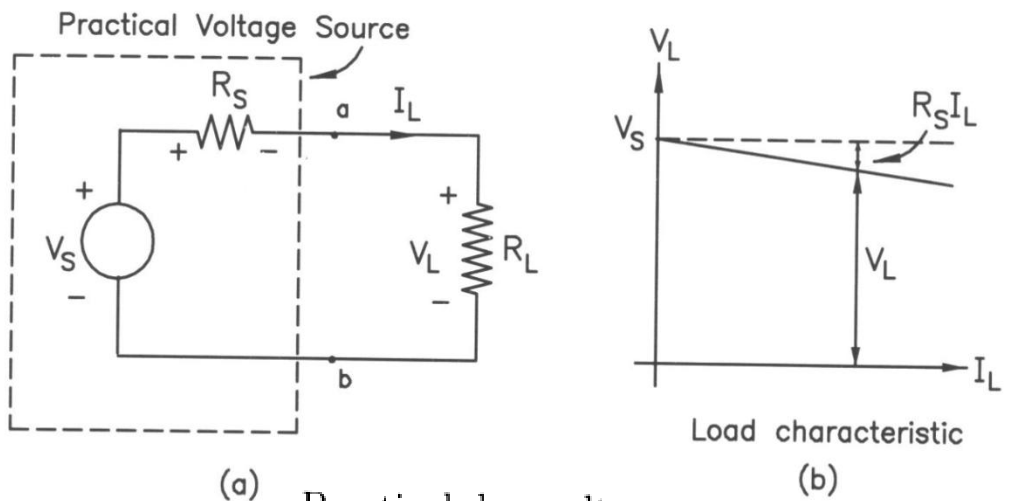

$$I = \frac{\Delta Q}{\Delta t}$$

| AWG gauge | Conductor dia. (in) | Conductor dia. (mm) | Ohms / 1000 ft | Ohms / km |
|---|---|---|---|---|
| 0 | 0.32 | 8.25 | 0.10 | 0.32 |
| 1 | 0.29 | 7.35 | 0.12 | 0.41 |
| 2 | 0.26 | 6.54 | 0.16 | 0.51 |
| 3 | 0.23 | 5.83 | 0.20 | 0.65 |
| 4 | 0.20 | 5.19 | 0.25 | 0.82 |
| 5 | 0.18 | 4.62 | 0.31 | 1.03 |
| 6 | 0.16 | 4.11 | 0.40 | 1.30 |
| 7 | 0.14 | 3.67 | 0.50 | 1.63 |
| 8 | 0.13 | 3.26 | 0.63 | 2.06 |
| 9 | 0.11 | 2.91 | 0.79 | 2.60 |
| 10 | 0.10 | 2.59 | 1.00 | 3.28 |
| 11 | 0.09 | 2.30 | 1.26 | 4.13 |
| 12 | 0.08 | 2.05 | 1.59 | 5.21 |
| 13 | 0.07 | 1.83 | 2.00 | 6.57 |
| 14 | 0.06 | 1.63 | 2.53 | 8.28 |
| 15 | 0.06 | 1.45 | 3.18 | 10.44 |
| 16 | 0.05 | 1.29 | 4.02 | 13.17 |
| 17 | 0.05 | 1.15 | 5.06 | 16.61 |
| 18 | 0.04 | 1.02 | 6.39 | 20.94 |
| 19 | 0.04 | 0.91 | 8.05 | 26.41 |
| 20 | 0.03 | 0.81 | 10.15 | 33.29 |
| 21 | 0.03 | 0.72 | 12.80 | 41.98 |
| 22 | 0.03 | 0.65 | 16.14 | 52.94 |
| 23 | 0.02 | 0.57 | 20.36 | 66.78 |
| 24 | 0.02 | 0.51 | 25.67 | 84.20 |
| 25 | 0.02 | 0.45 | 32.37 | 106.17 |

$$R = \frac{\rho l}{A}$$

| Material | Resistivity (Ohm-meter) |
|---|---|
| Aluminum | $2.733 \times 10^{-8}$ |
| Copper | $1.725 \times 10^{-8}$ |
| Gold | $2.271 \times 10^{-8}$ |
| Iron | $9.98 \times 10^{-8}$ |
| Nickel | $7.20 \times 10^{-8}$ |
| Platinum | $10.8 \times 10^{-8}$ |
| Carbon | $3.5 \times 10^{-8}$ |

# Practical Circuit Elements - Practical Voltage Source




Practical d.c. voltage source

As the load current $I_L$ increases, the load voltage $V_L$ decreases because of the voltage drop across the internal resistance $R_S$ of the source.

$$V_L = V_S - I_L R_S$$

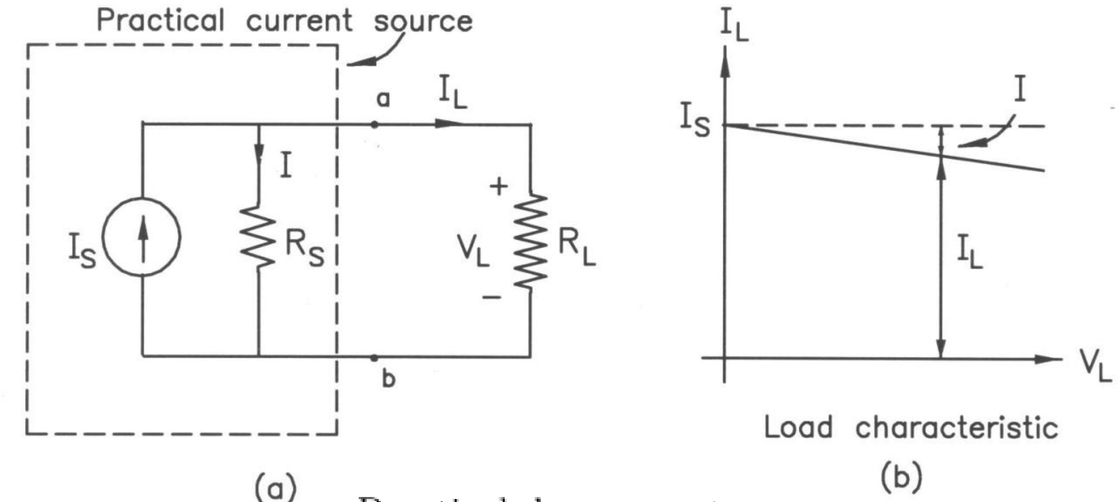
Using voltage division method, $V_L$ is given by

$$V_L = V_S \times \frac{R_L}{R_S + R_L}$$

Note that the load current $I_L$ increases when $R_L$ decreases.

- As $R_L \to 0\ \Omega$, $I_L \to \frac{V_S}{R_S}$ and $V_L \to 0$.
- As $R_L \to \infty\ \Omega$, $I_L \to 0$ and $V_L \to V_S$.

# Practical Circuit Elements - Practical Current Source




Practical d.c. current source

The load current $I_L$ and the load voltage $V_L$ are given by the following equations:

$$I_L = I_S \times \frac{R_S}{R_S + R_L}$$

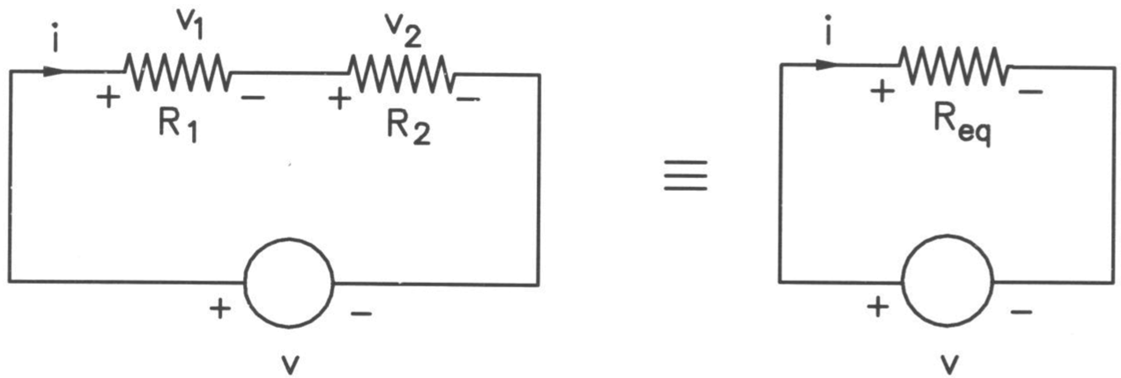
$$V_L = R_L I_L = I_S \times \frac{R_S R_L}{R_S + R_L}$$

Note that the load current $I_L$ increases when $R_L$ decreases.

- As $R_L \to 0\,\Omega$, $I_L \to I_S$ and $V_L \to 0$.

- As $R_L \to \infty\,\Omega$, $I_L \to 0$ and $V_L \to I_S R_S$.

# Series Connection of Resistors




- Find the voltages across the resistors $R_1$, and $R_2$ of the circuit above

$$\begin{aligned} v_1 &= i\,R_1 \\ v_2 &= i\,R_2 \\ v &= v_1 + v_2 = i\,R_1 + i\,R_2 \\ &= i\,(R_1 + R_2) = i\,R_{eq} \end{aligned}$$

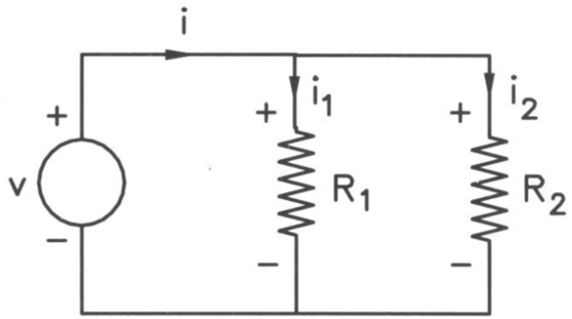
$$\Longrightarrow \qquad i = \frac{v}{R_1 + R_2}$$

- Substituting for *i* in the equations above, we get:

$$\begin{aligned} v_1 &= i\,R_1 = \frac{R_1}{R_1 + R_2}\,v \\ v_2 &= i\,R_2 = \frac{R_2}{R_1 + R_2}\,v \end{aligned}$$

The applied voltage *v* is divided between these two resistors in proportion to their resistances. This is known as the **Voltage Division Method.**

# *Parallel Connection of Resistors*




- Let us determine the currents $i_1$, and $i_2$ in the resistors $R_1$, and $R_2$

$$i_1 = \frac{v}{R_1} = \frac{1}{R_1} \times \frac{R_1 R_2}{R_1 + R_2} i = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{v}{R_2} = \frac{1}{R_2} \times \frac{R_1 R_2}{R_1 + R_2} i = \frac{R_1}{R_1 + R_2} i$$

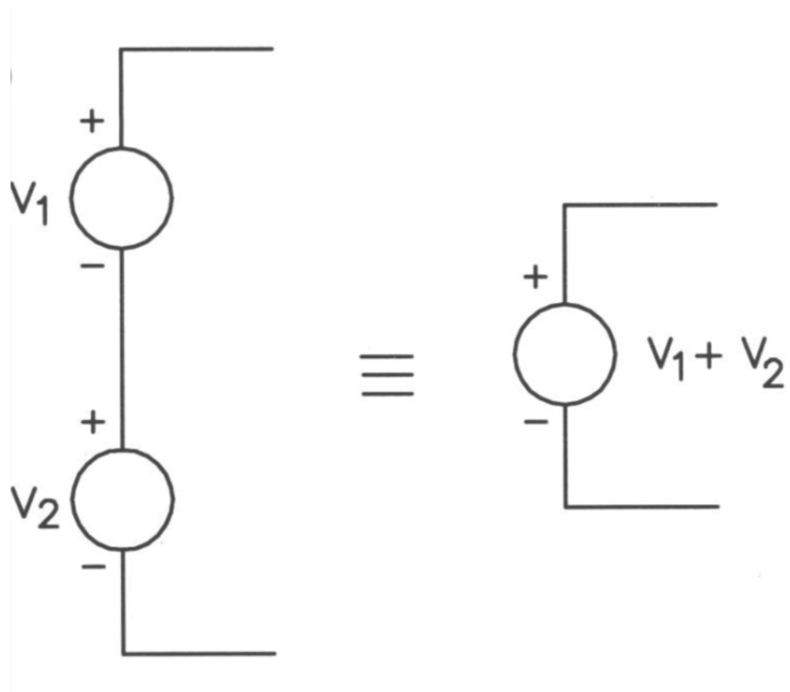
The current divides in inverse proportion of the resistance. This is known as the **Current Division Method.**

- The ratio of the currents in the two parallel paths is:

$$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

# *Source Combination*

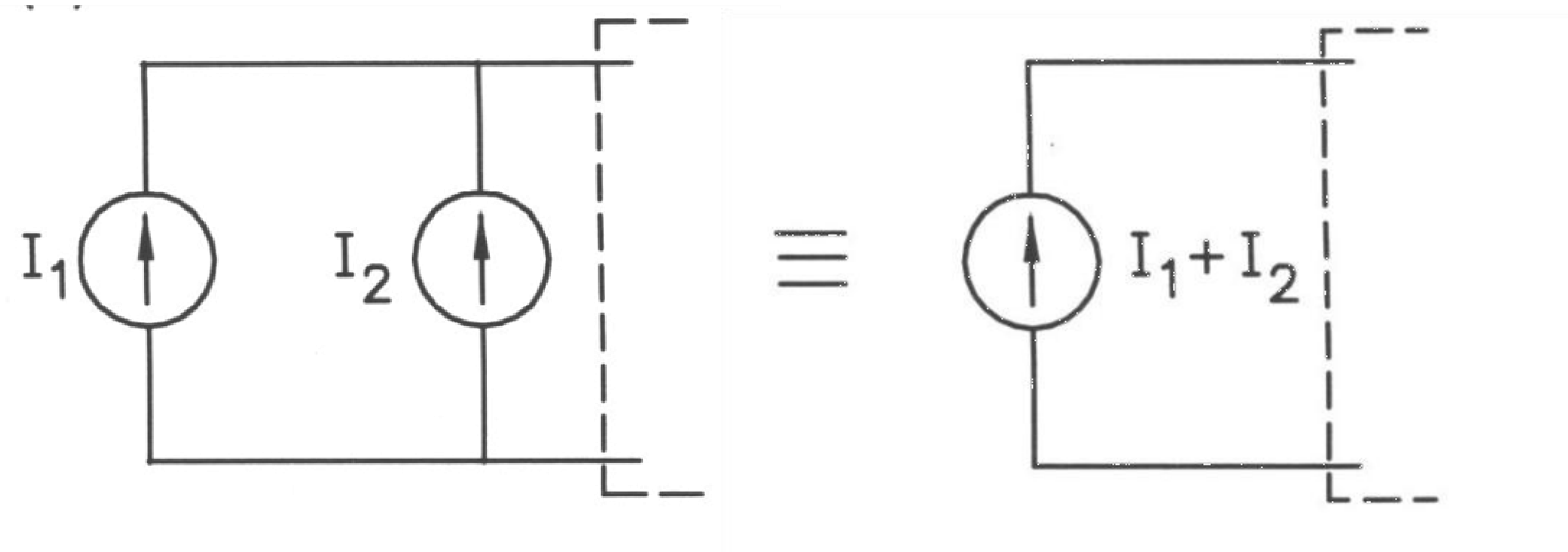
If two or more *voltage sources* are connected in *series*:




…they may be replaced by a *single voltage source* whose voltage is the *algebraic sum of the voltages of all these sources*.

# *Source Combination*

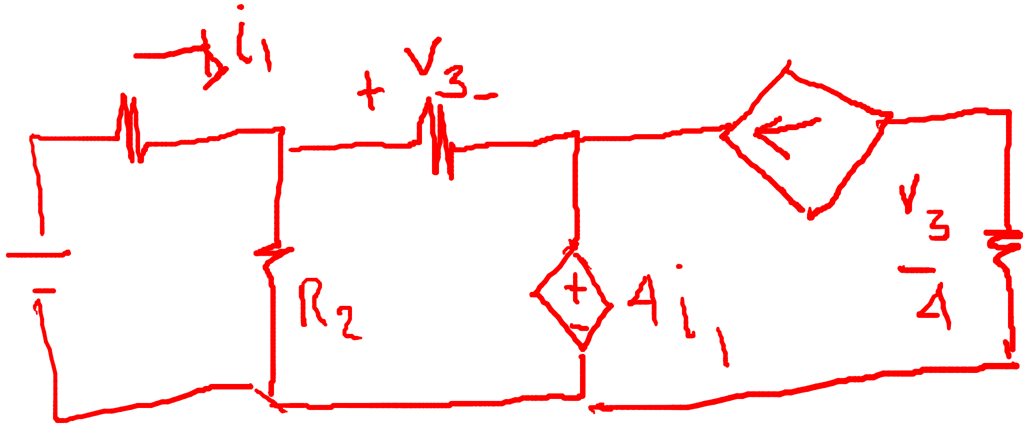
If *two or more current sources* are connected in *parallel*:




…they may be replaced by a *single current source* whose current is the *algebraic sum of the currents of all these sources*.

# *Lecture 1 Review*

- Different types of dependent source
  - Voltage controlled voltage source
  - Current controlled voltage source
  - Voltage controlled current source
  - Current controlled current source



# *Lecture 1 Review*

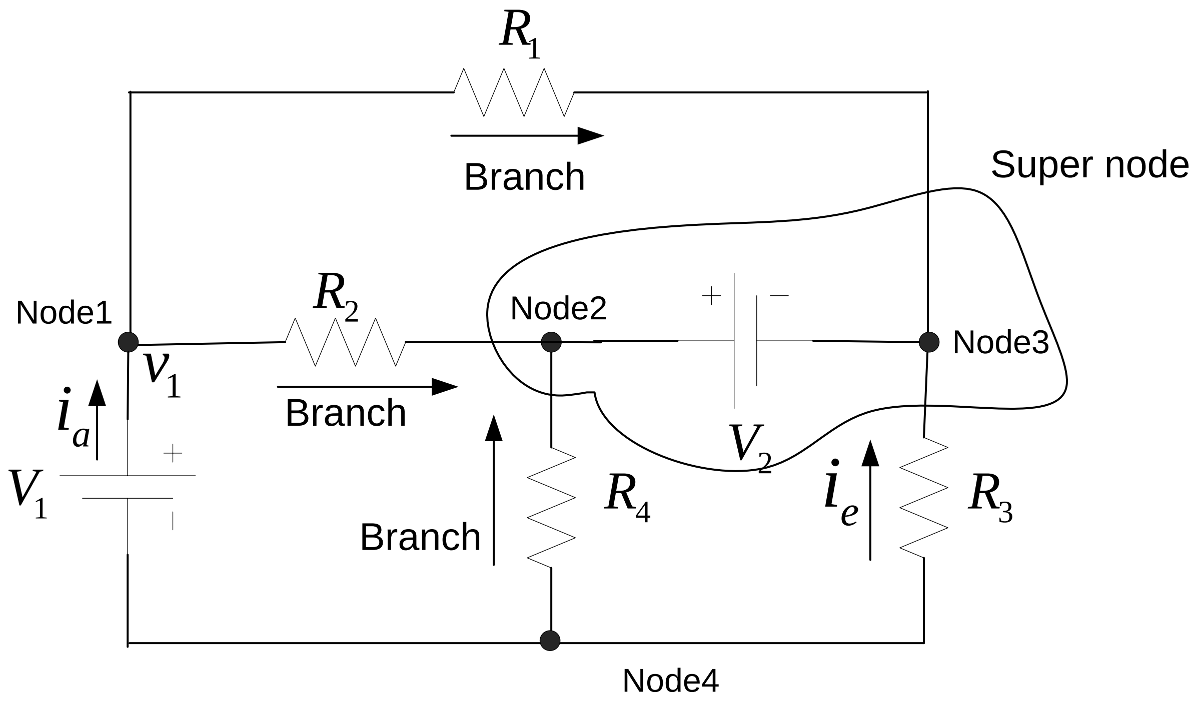
- Resistance, and Ohm's law
- Power in resistors

$$V = iR$$

$$P = iV = \frac{V^2}{R} = i^2 R$$

# *Lecture 1 Review*

- A super node is a closed surface enclosing part of a circuit. It may contain some sources and other nodes.

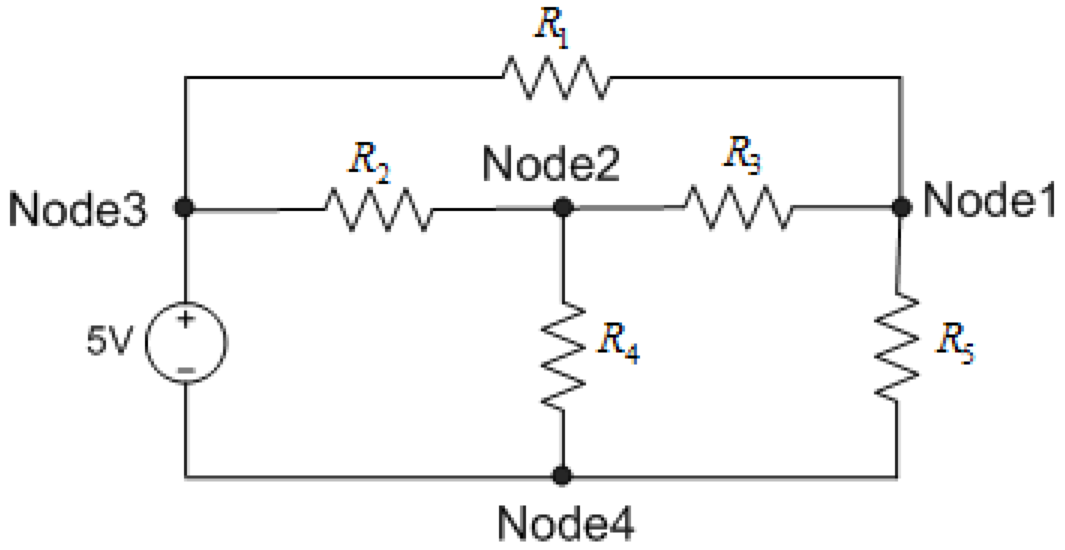



- A super-node is a theoretical construct that can be used to solve a circuit.
- A voltage source can be viewed on a wire as a point source voltage in relation to other point voltages located at various nodes in the circuit, relative to a ground node assigned a zero or negative charge.

*CG1108 AY2010/11*

Lecture 2

# Case: Node analysis with one source and resistors




- If voltages at all nodes are known, then all the voltages and currents can be obtained.

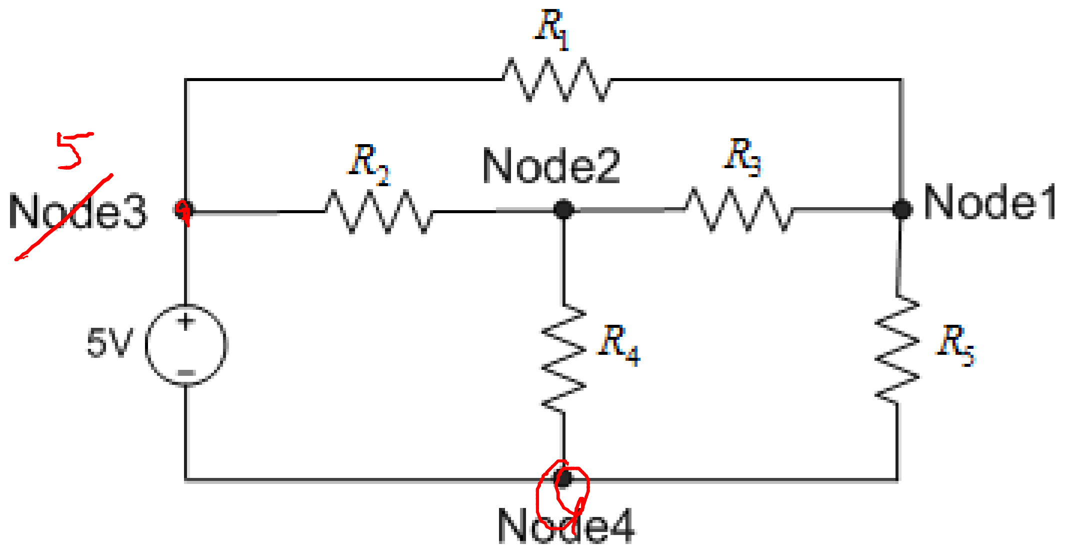
## *Steps of Node Voltage Analysis method*

1. Select a reference node (Usually the ground of a voltage source.)
2. Define the remaining (n-1) node voltages as unknown variables.
3. Apply KCL at each node, expressing each current in terms of the adjacent node voltages.
4. Solve the linear system equations

## *Developing the equations*

- We shall learn how to choose the unknown variables and write the system of linear equations
- We shall not solve the equations
- We shall keep that for another day

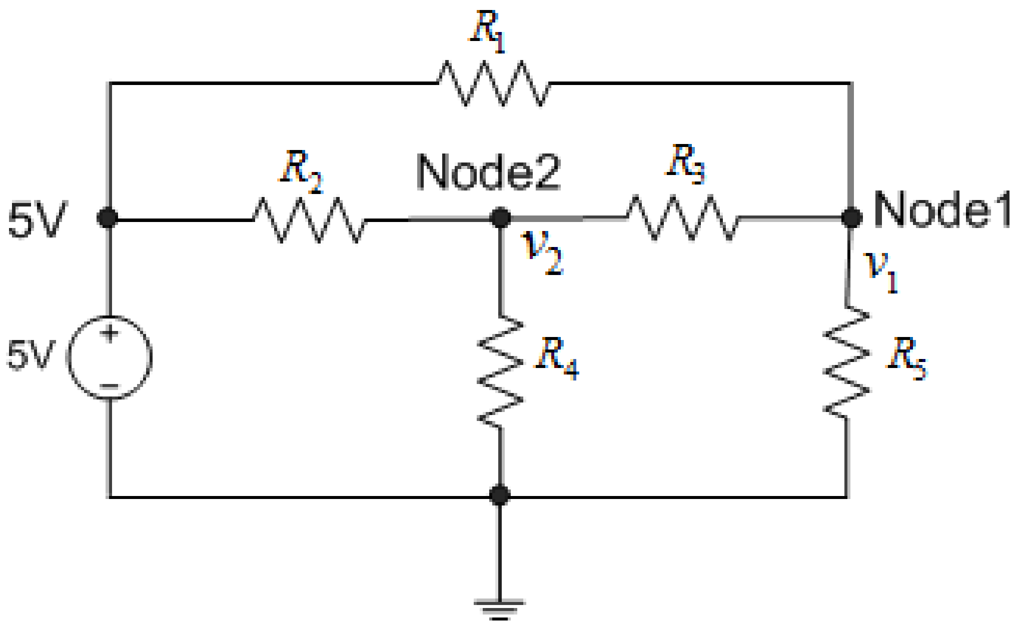
# Case: Node analysis with one source and resistors




- Reference node
- Identifying unknown node voltage as variables.
- To find the current flowing out of a node *n* through a resistance toward node *k*, we subtract the voltage at node *k* from the voltage at node *n* and divide the difference by the resistance.

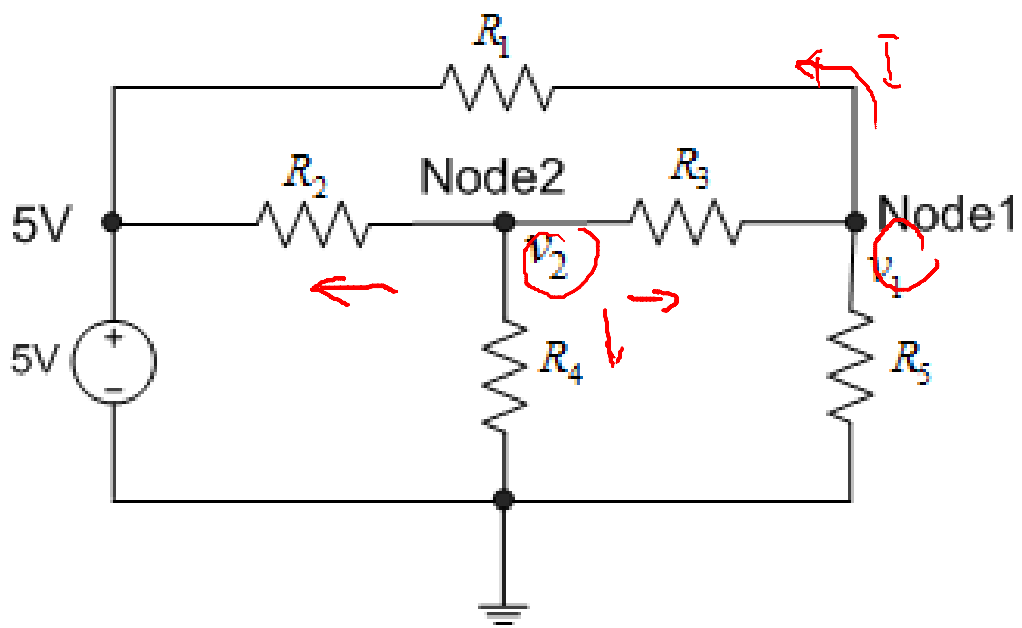
$$\frac{V_n - V_k}{R}$$

# Case: Node analysis with one source and resistors




- Total nodes 4
- Negative of the voltage source taken as reference node
- Positive of voltage source as a node with known value
- (4-2) nodes with unknown voltages

# Case: Node analysis with one source and resistors




@ NODE 1

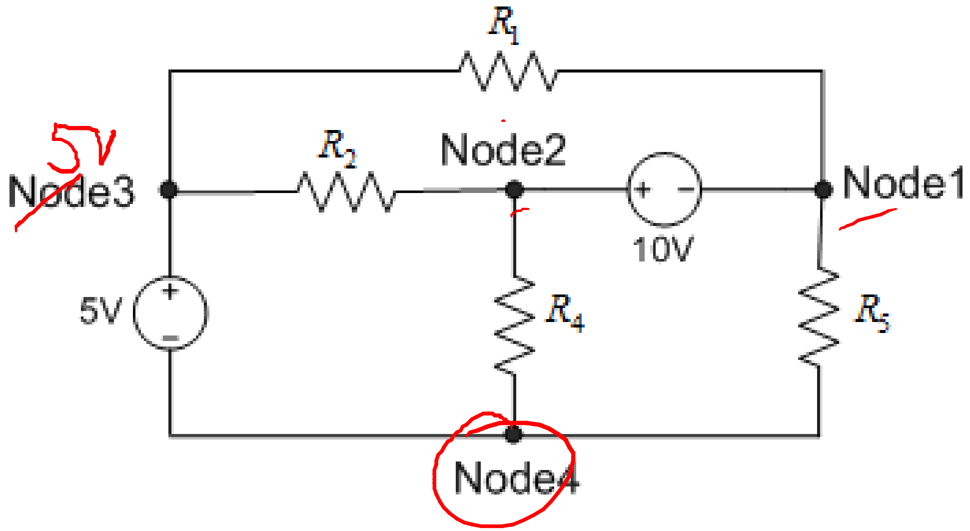
$$\frac{V_1 - 5}{R_1} + \frac{V_1 - V_2}{R_3} + \frac{V_1}{R_5} = 0$$

$$\boxed{\frac{V_N - V_K}{R}}$$

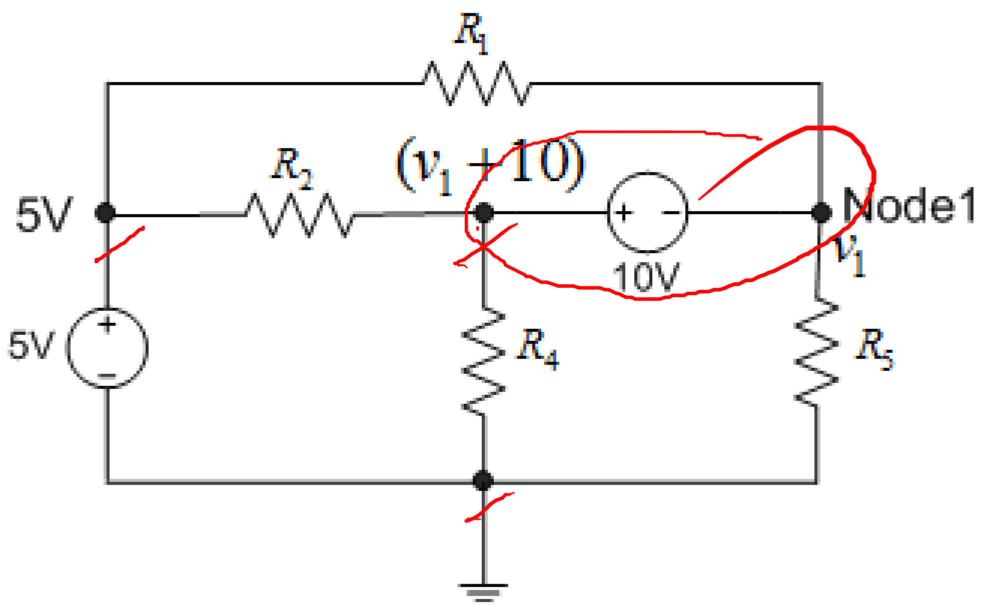
@ NODE 2

$$\frac{V_2 - 5}{R_2} + \frac{V_2 - \cancel{V_4}^{\,0}}{R_4} + \frac{V_2 - V_1}{R_3} = 0$$

# Case: Having an ideal voltage source between two nodes

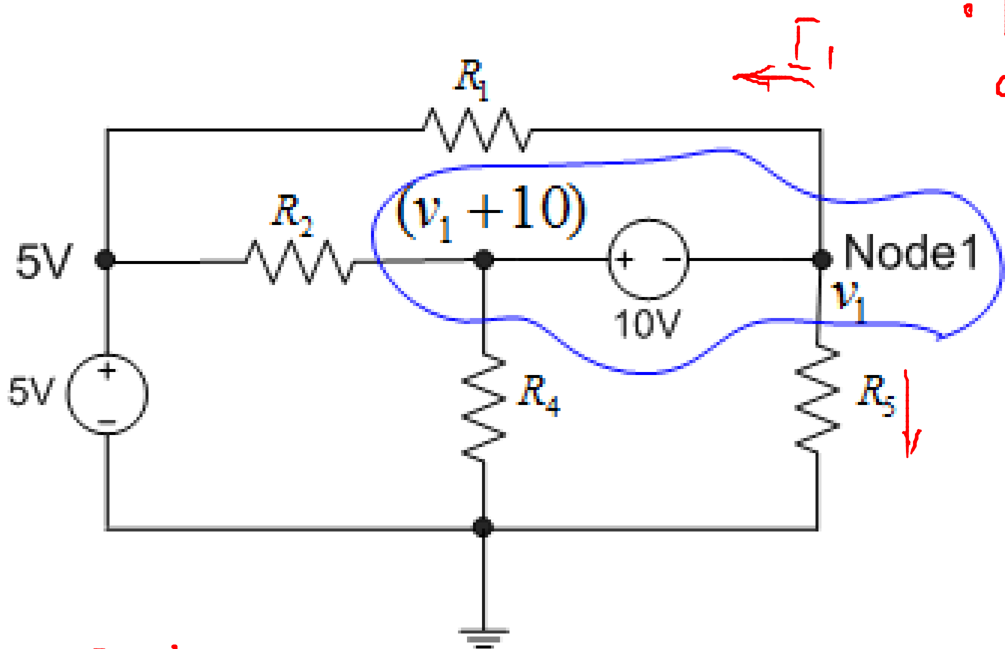


## *Case: Having an ideal voltage source between two nodes*




- Total nodes 4
- Negative of the 5V as reference node
- Positive of 5V as known
- Negative of the 10V as v1
- Positive of 5V as known variable
- (4-1-1-1) unknown node voltages

## Case: Having an ideal voltage source between two nodes



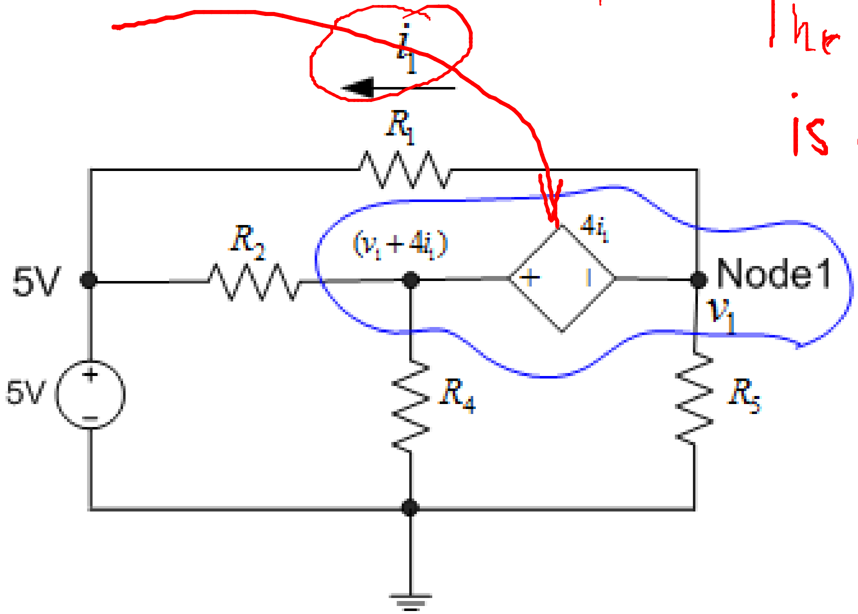

- Treat the dependent source as a normal voltage source
- Replace the control variable in terms of the node voltage.

@ NODE 1

$$\frac{V_1 - 5}{R_1} + \frac{V_1}{R_5} + \frac{V_1 + 10 - 5}{R_2} + \frac{V_1 + 10}{R_4} = 0$$

# Case: Having a dependent voltage source in the circuit

Treat as a normal voltage source



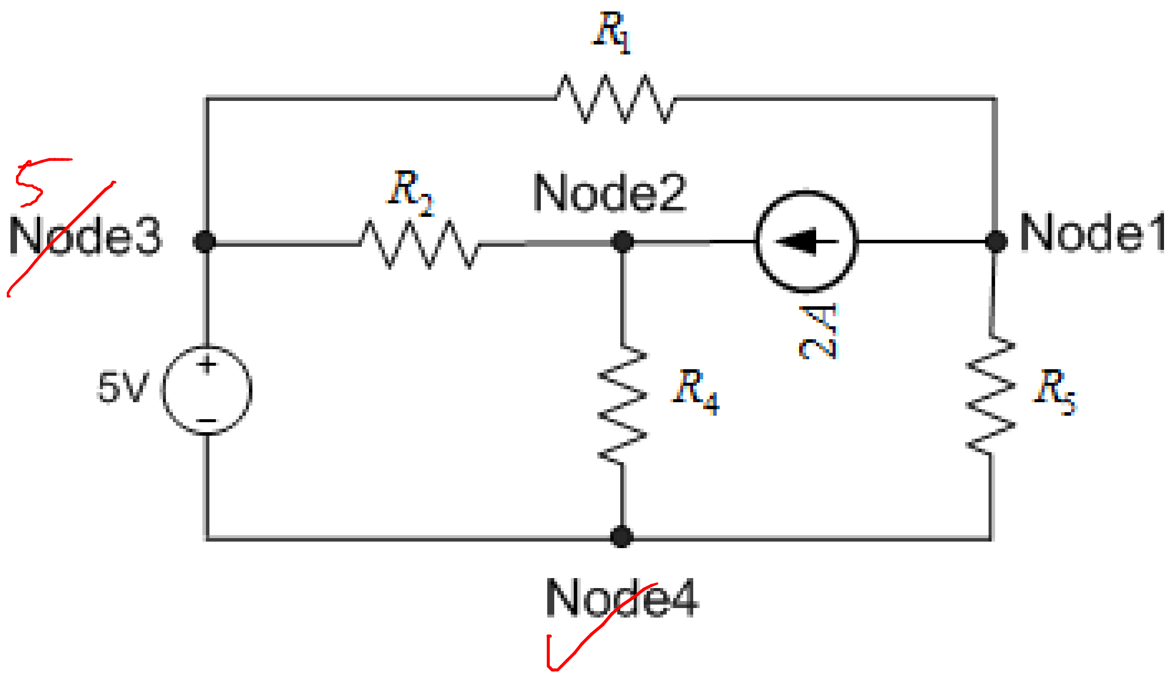

The current leaving node 1 is denoted as:

$$i_1 = \frac{V_1 - 5}{R_1}$$

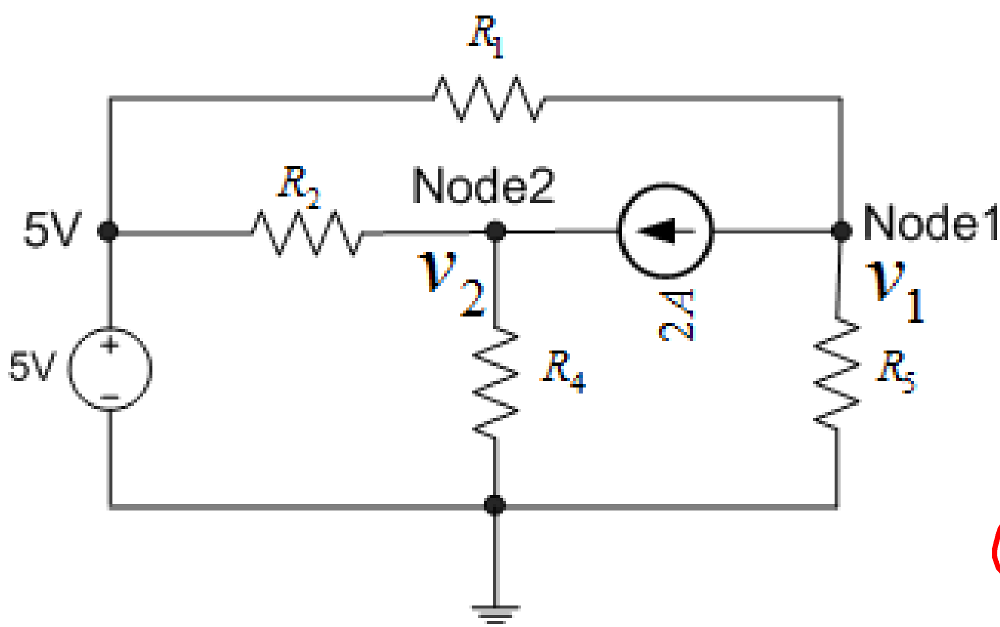
@ NODE 1

$$\frac{V_1 - 5}{R_1} + \frac{V_1}{R_5} + \frac{V_1 + 4i_1 - 5}{R_2} + \frac{V_1 + 4i_1}{R_4} = 0$$

# Case : Having an independent current source in the circuit



## Case : Having an independent current source in the circuit



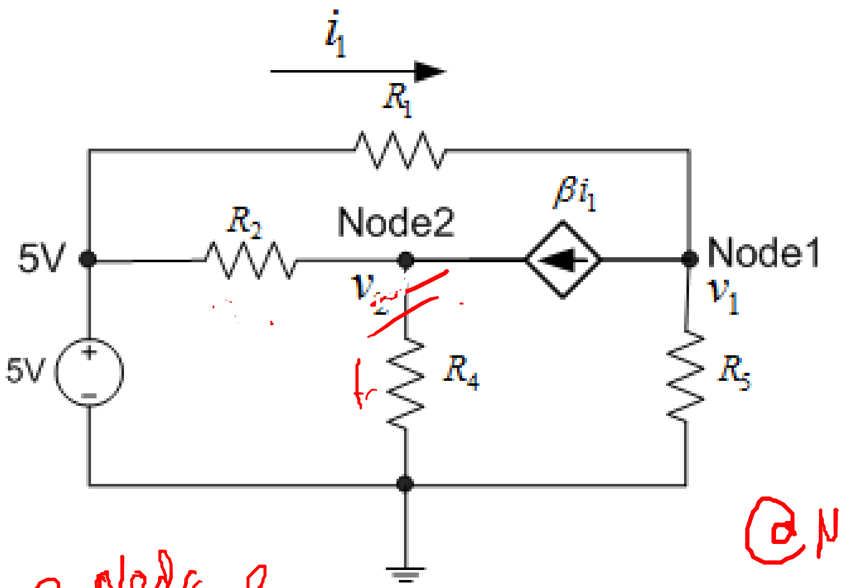

@ Node 1

$$\frac{V_1 - 5}{R_1} + \frac{V_1}{R_5} + 2 = 0 \quad (1)$$

@ Node 2

$$\frac{V_2 - 5}{R_2} + \frac{V_2}{R_4} - 2 = 0 \quad (2)$$

# Case : Having a dependent current source in the circuit




Writing the equations

$$i_1 = \frac{5 - V_1}{R_1}$$

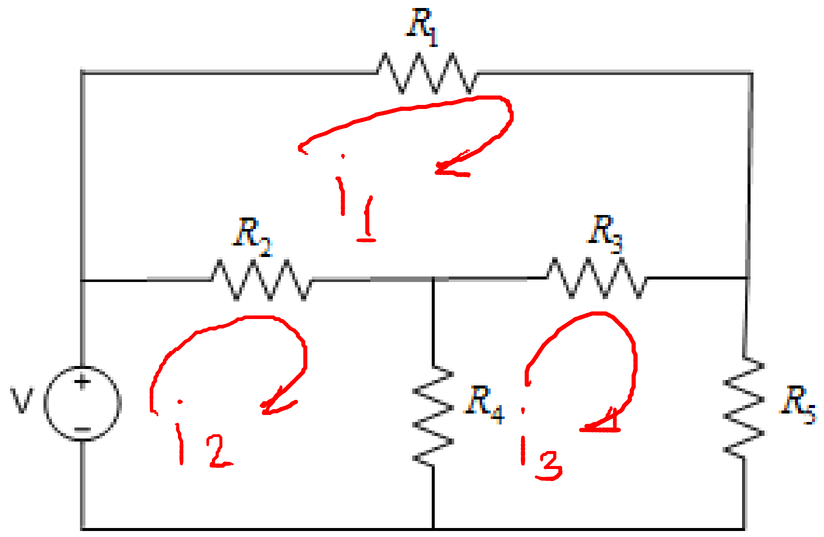
@ NODE 1

$$\frac{V_1 - 5}{R_1} + \beta i_1 + \frac{V_1}{R_5} = 0$$

@ Node 2

$$\frac{V_2 - 5}{R_2} + \frac{V_2}{R_4} - \beta i_1 = 0$$

# *Intro to mesh current Analysis*

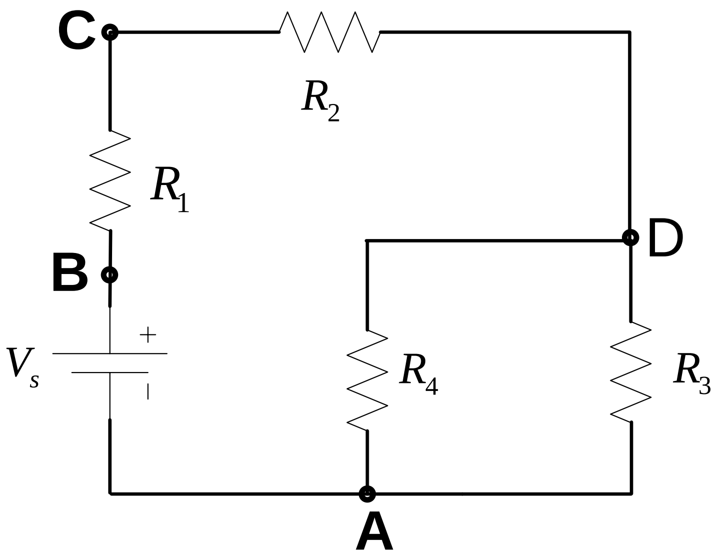


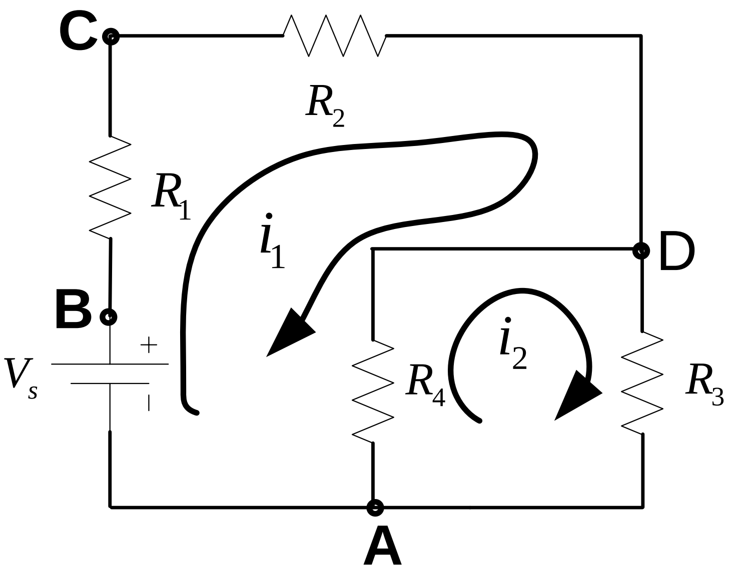

- If currents in the meshes are known, then all the other currents and voltages can be obtained.

## *Steps of mesh current analysis*

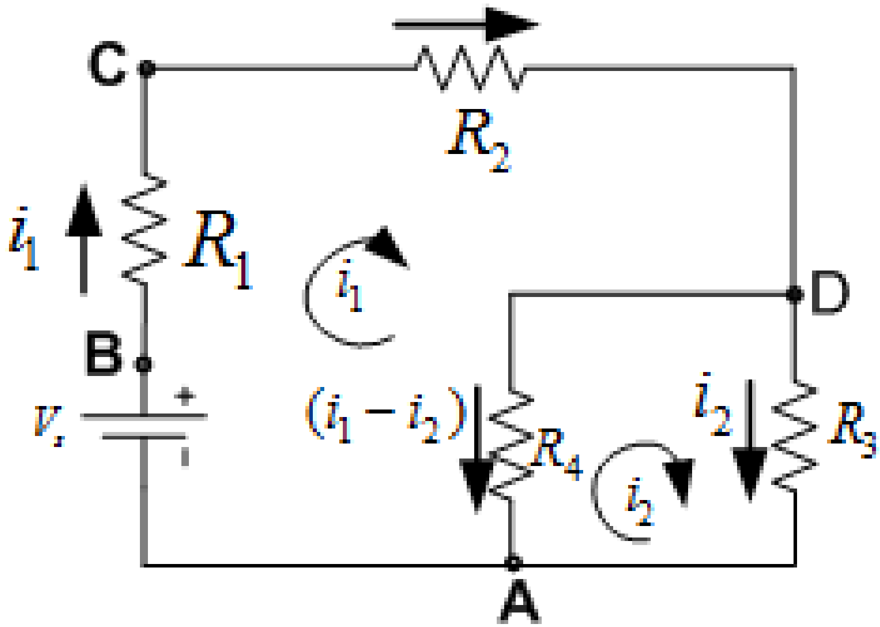
1. Define each mesh current in the clockwise direction
2. Find the branch current and voltages in terms of the mesh currents.
3. Apply KVL to each mesh to obtain an independent equation
4. Solve the linear system of equations.

# Case: Voltage source with resistor network



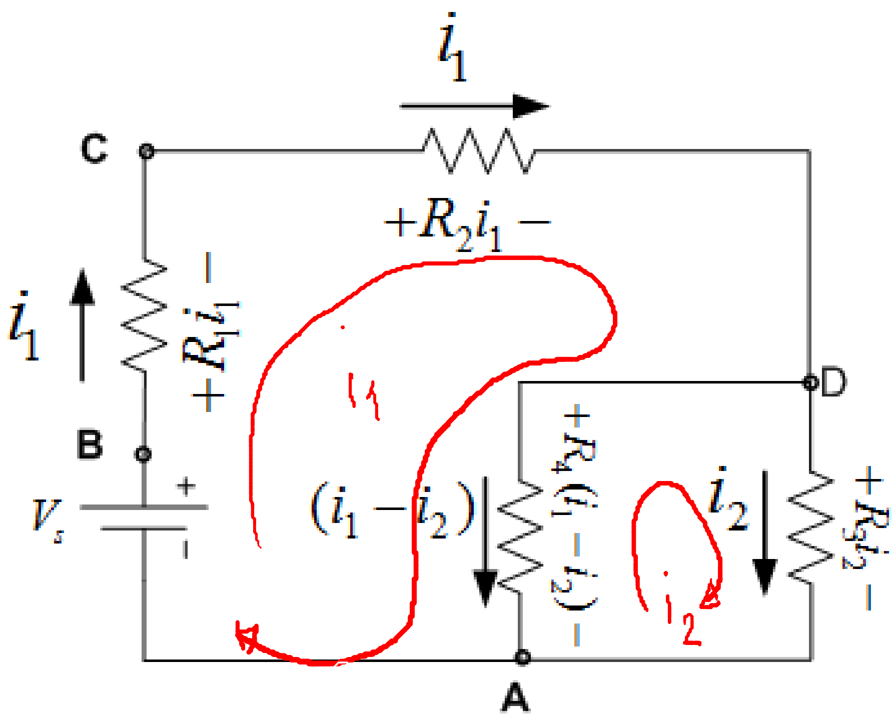



# *Case: Voltage source with resistor network*




Finding branch currents in terms of mesh currents

# Case: Voltage source with resistor network




Writing the equations

$$V_S - R_1 i_1 - R_2 i_1 - R_4(i_1 - i_2) = 0$$

$$R_4(i_1 - i_2) - R_3 i_2 = 0$$

Branch voltages from the branch currents

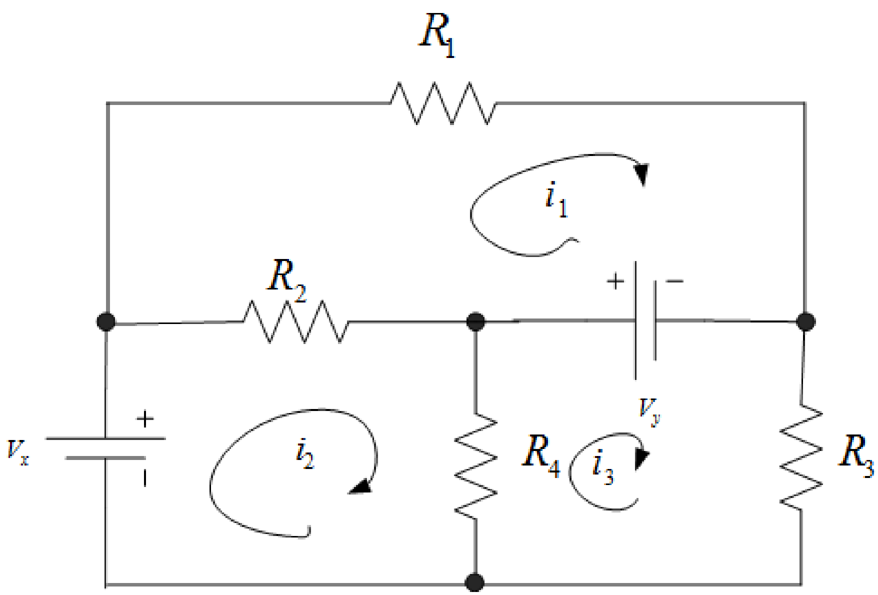
ABCDE

$-V_s + R_1 i_1 + R_2 i_1 + R_4(i_1 - i_2) = 0$ — (1)

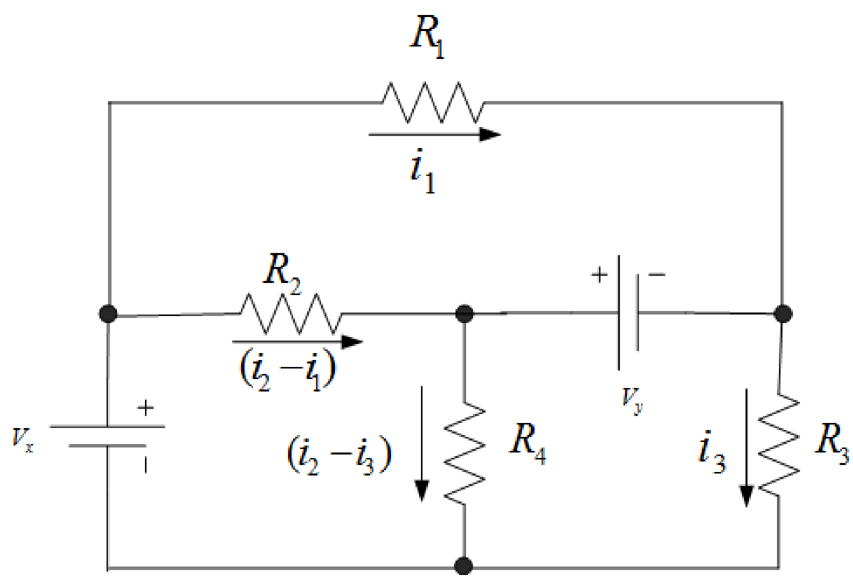
ADA

$R_3 i_2 - R_4(i_1 - i_2) = 0$ — (2)

# Case: Two Voltage sources

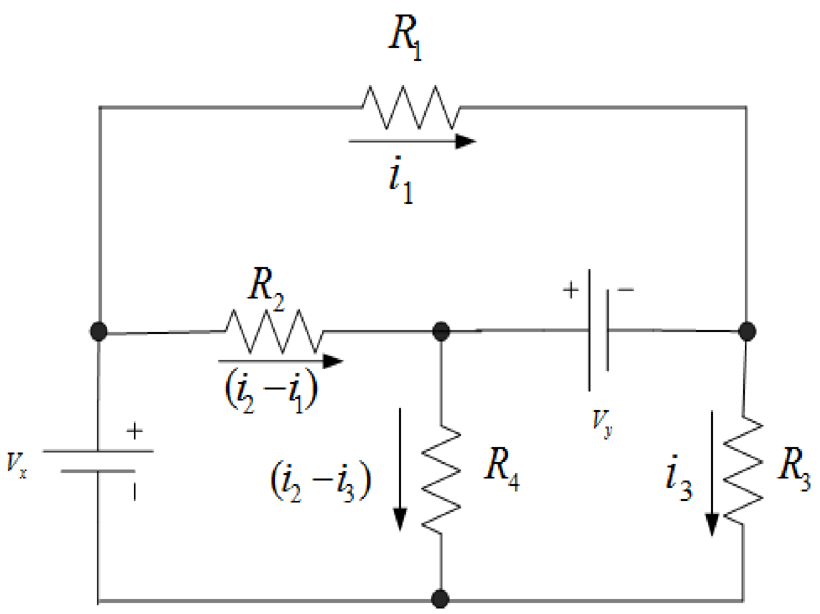



Mesh currents

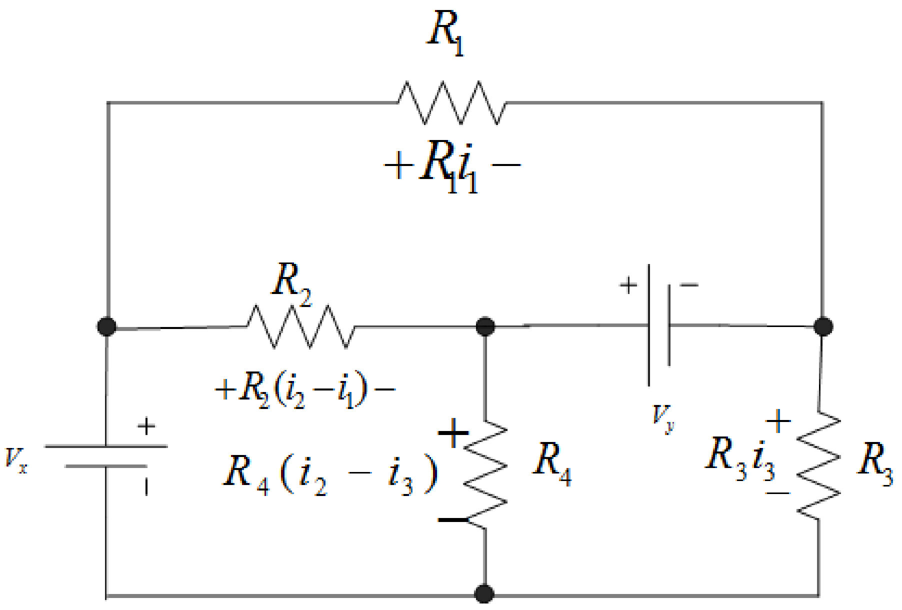



Branch currents

# Case: Two Voltage sources

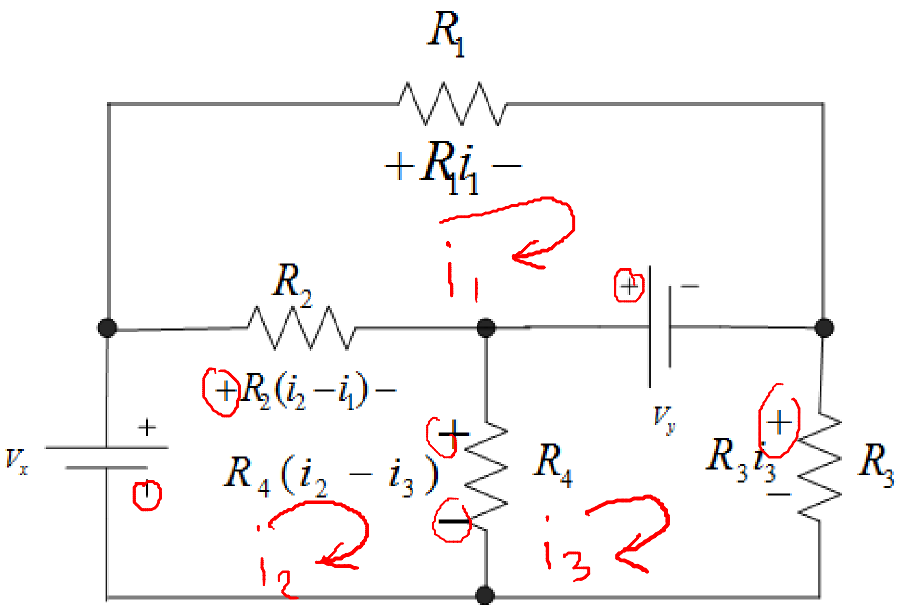



Branch currents




Branch voltages

# *Case: Two Voltage sources*



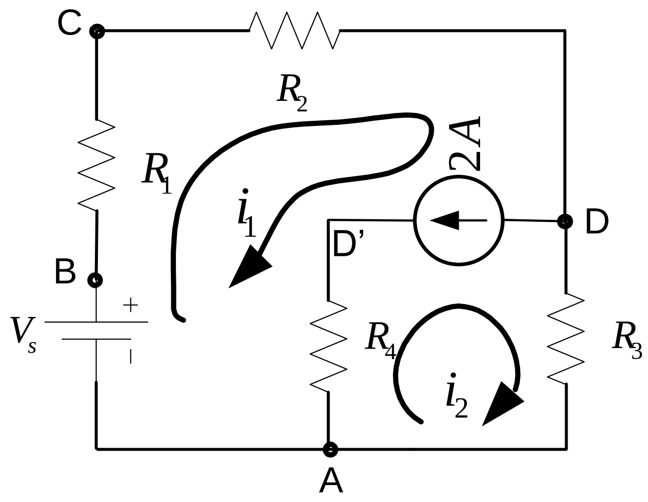

$$R_1 i_1 - V_y - R_2(i_2 - i_1) = 0$$

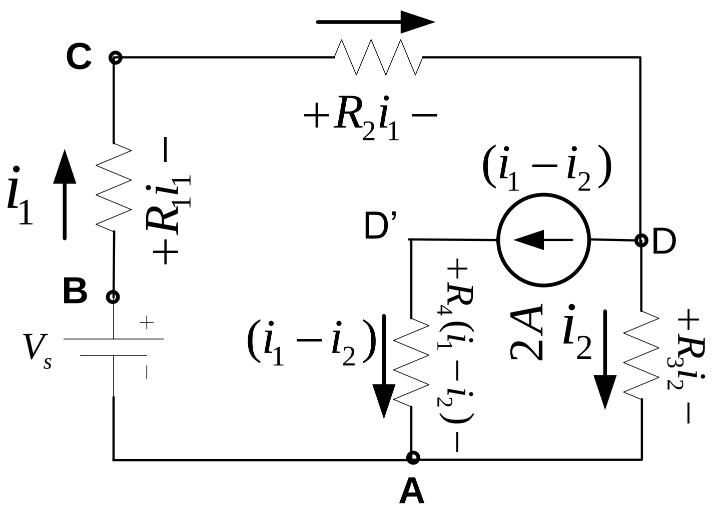
$$-V_x + R_2(i_2 - i_1) + R_4(i_2 - i_3) = 0$$

$$V_y + R_3 i_3 - R_4(i_2 - i_3) = 0$$

Branch voltages

# *Case: a current source in one of the branches*



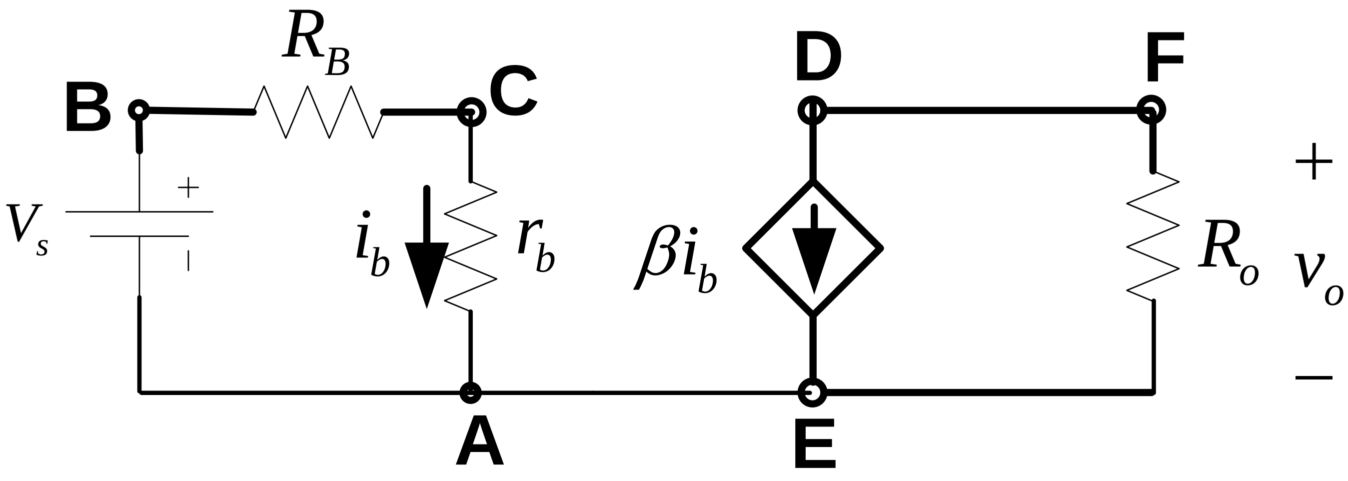




*$Vs - R_1 i_1 - R_2 i_1 - R_3 i_2 = 0$*

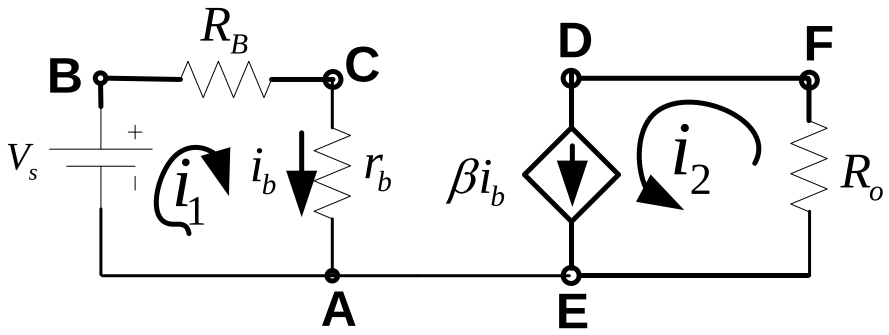
$$i_1 - i_2 = 2$$

We cannot write the voltage across a current source, in terms of the mesh currents

# Case: with dependent current source



# Case: with dependent current source




$$-V_S + R_B i_1 + r_b i_1 = 0$$

$$\beta i_1 = i_2$$