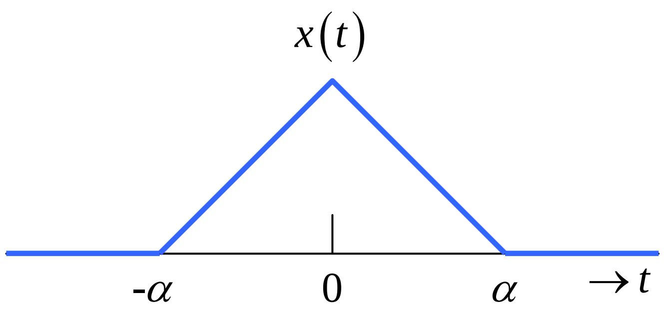
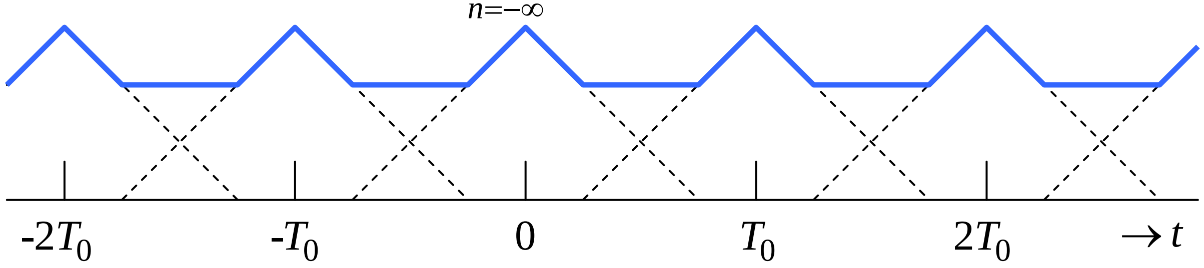
from Fourier Series *to* Fourier Transform



$$x_{T_0}(t) = \sum_{n=-\infty}^{\infty} x(t - nT_0)$$



Fourier series coefficients of x_{T_0}:

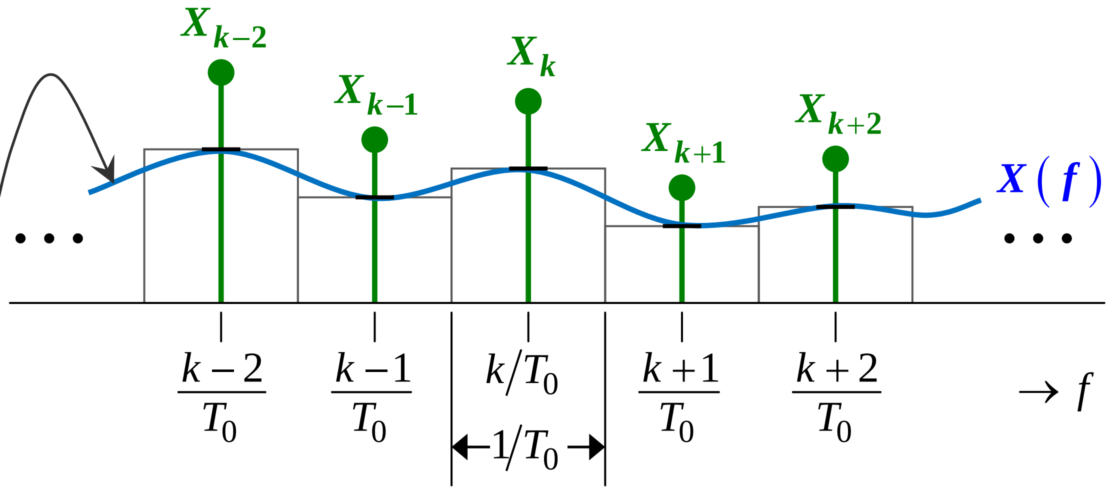
$$X_k = \frac{1}{T_0}\int_{-0.5T_0}^{0.5T_0} x_{T_0}(t)\exp\left(-j2\pi\frac{k}{T_0}t\right)dt. \quad \cdots\cdots (1)$$

Note that:

$$\lim_{T_0\to\infty} x_{T_0}(t) = x(t). \quad \cdots\cdots (2)$$

Define a continuous frequency function $X(f)$ such that

$$X\left(\frac{k}{T_0}\right) = X_k T_0, \quad \forall k. \quad \cdots\cdots (3)$$



Combining (1) and (3):

$$X\left(\frac{k}{T_0}\right) = X_k T_0 = \int_{-0.5T_0}^{0.5T_0} x_{T_0}(t)\exp\left(-j2\pi\frac{k}{T_0}t\right)dt. \tag{4}$$

In the limit $\left(T_0 \to \infty,\ k \to \infty\right)$, let

$$\lim_{\substack{T_0\to\infty \\ k\ \to\infty}} \frac{k}{T_0} \to \tilde{f}\ . \tag{5}$$

Taking (4) to the limit $\left(T_0 \to \infty,\ k \to \infty\right)$ and applying (2) and (5):

$$\underbrace{\lim_{\substack{T_0\to\infty \\ k\ \to\infty}} X\left(\frac{k}{T_0}\right)}_{X\left(\tilde{f}\right)} = \underbrace{\lim_{\substack{T_0\to\infty \\ k\ \to\infty}} \int_{-0.5T_0}^{0.5T_0} x_{T_0}(t)\exp\left(-j2\pi\frac{k}{T_0}t\right)dt}_{\int_{-\infty}^{\infty} x(t)\exp\left(-j2\pi\tilde{f}t\right)dt} \tag{6}$$

Replacing $\tilde{f}$ with f in (6), we get:

$$\underbrace{\boldsymbol{X(f) = \int_{-\infty}^{\infty} x(t)\exp\left(-j2\pi ft\right)dt}}_{\textit{Fourier transform of } \boldsymbol{x(t)}}$$