EEC130A: Practice Problems for Midterm 1*

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Instructor: Xiaoguang "Leo" Liu (lxgliu@ucdavis.edu)

P-1. A parallel-wire transmission line is constructed of #6 AWG copper wire (diameter $d = 0.162$ in., conductivity $\sigma_c = 5.8 \times 10^7$ S/m) with a 12-inch separation in air. Assuming no leakage between the two wires, find R', L', G', and C'. Assume a working frequency of 1 MHz.

Using Table. 1,

$$Rs = \sqrt{\pi f \mu_c / \sigma_c} = 2.61 \times 10^{-4}\ \Omega$$

$$R' = \frac{2R_s}{\pi d} = 4.04 \times 10^{-2}\ \Omega/\text{m}$$

$$L' = \frac{\mu_0}{\pi} \ln \frac{2D}{d} = 2.0\ \mu\text{H/m}$$

$$C' = \frac{\pi \epsilon}{\ln \dfrac{2D}{d}} = 5.56\ \text{pF/m}$$

$G' = 0$ because the problem states that there is no leakage between the two wires. ∎

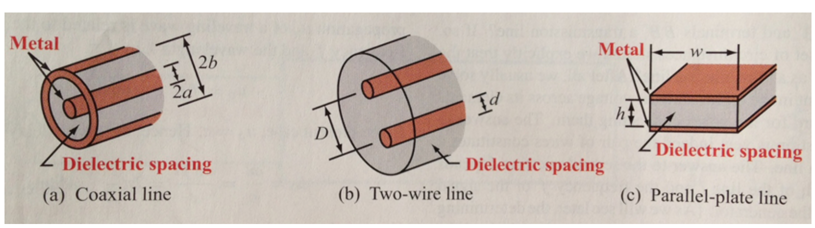


Figure 1: (Fig.2-4 from FAE) A few examples of transmission lines.

*Some problems are adapted from *"The Schaum's Outlines on Electromagnetics"* and *"2008+ Solved Problems in Electromagnetics"*.

Table 1: (Table 2-1 from FAE)Transmission-line parameters R', L', G', and C' for three types of lines.

Table 2-1: Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \simeq \ln(2D/d)$.

P-2. Derive the phasor-form wave equation in terms of current from the telegrapher's equations.

The telegraphers equations are†:

$$-\frac{dV(z)}{dz} = \left(R' + j\omega L'\right) I(z), \tag{1}$$

$$-\frac{dI(z)}{dz} = \left(G' + j\omega C'\right) V(z). \tag{2}$$

Taking the derivative of Eqn. 2 with respect to z, we get

$$-\frac{d^2 I(z)}{dz^2} = \left(G' + j\omega C'\right)\frac{dV(z)}{dz}. \tag{3}$$

Substituting Eqn. 1 into Eqn. 3, we get

$$-\frac{d^2 I(z)}{dz^2} = -\left(G' + j\omega C'\right)\left(R' + j\omega L'\right) I(z). \tag{4}$$

Rearranging Eqn. 4 and defining

$$\gamma = \sqrt{\left(R' + j\omega L'\right)\left(G' + j\omega C'\right)},$$

we get the wave equation

$$\frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0. \tag{5}$$

■

†From now on, we will omit the tilde in the phasors. Any capatalized symbol with a single position variable represents a phasor.

P-3. In lossline transmission lines, we learned that the input reflection coefficient Γ_{in} is related to the load reflection coefficient Γ_L by

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l},$$

where l is the length of the tranmission line. We see that Γ_{in} and Γ_L have the same magnitude but differ in phase by $2\beta l$. For lossy lines, however, both magnitude and phase are different for Γ_{in} and Γ_L. From the general solution to the wave equation, derive an expression for Γ_{in} in terms of Γ_L, β, l, and attenuation constant α.

In the presence of attenuation, the general solution to the wave equation is

$$\begin{aligned} V(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ &= V_0^+ e^{-\alpha z - j\beta z} + V_0^- e^{\alpha z + j\beta z}. \end{aligned}$$

The input impedance at a distance l from the load ($z = -l$) is

$$\begin{aligned} Z_{in} &= \frac{V(l)}{I(l)} \\ &= \frac{V_0^+ \left[e^{\alpha l + j\beta l} + \Gamma_L e^{-\alpha l - j\beta l}\right]}{I_0^+ \left[e^{\alpha l + j\beta l} - \Gamma_L e^{-\alpha l - j\beta l}\right]} \\ &= Z_0 \frac{1 + \Gamma_L e^{-2\alpha l} e^{-j2\beta l}}{1 - \Gamma_L e^{-2\alpha l} e^{-j2\beta l}} \\ &= Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}, \end{aligned} \tag{6}$$

where the input reflection coefficient is given by

$$\Gamma_{in} = \Gamma_L e^{-2\alpha l} e^{-j2\beta l} \tag{7}$$

■

P-4. A parallel-wire line operating at 100 kHz has $Z_0 = 557\ \Omega$, $\alpha = 2.3 \times 10^{-5}$ Np/m, and $\beta = 2.12 \times 10^{-3}$ rad/m. For a matched termination at $z = 0$ and $V_L = 10/0^\circ$ V, (a) give a general expression of $V(z)$, (b) evaluate $V(z)$ at a distance of 10 km from the load.

Since the line is matched, there is no reflected wave. Therefore a general expression for the voltage wave takes the form

$$\tilde{V}(z) = \tilde{V}_0^+ e^{-\alpha z} e^{-j\beta z}.$$

At $z = 0$,

$$\tilde{V}(z) = \tilde{V}_0^+ = 10/0^\circ$$

At $z = -10$ km (notice z becomes negative as we move away from the load)

$$\tilde{V}(z) = 10/0^\circ \times e^{2.3\times10^{-5}\times10\times10^3} e^{j2.12\times10^{-3}\times10\times10^3} = 12.71/135^\circ \qquad V.$$

■

P-5. A 600-Ω transmission line is 150 m long, operates at 400 kHz with $\alpha = 2.4 \times 10^{-3}$ Np/m and $\beta = 0.0212$ rad/m, and supplies a load impedance $Z_L = 300 + j300\ \Omega$. Find the length of line in wavelength, Γ_L, Γ_{in} and Z_{in}. For a received voltage $V(z=0) = 50$ V, find the total voltage at the input $V(z = -150\text{ m})$.

$$\lambda = 2\pi/\beta = 296.4 \text{ m}$$
$$l = 150/296.4 \cdot \lambda = 0.51\lambda$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 + j300 - 600}{300 + j300 + 600} = -0.2 + j0.4 = 0.45\underline{/116.6^\circ}$$

$$\Gamma_{in} = \Gamma_L e^{-2\alpha l} e^{-j2\beta l} = -0.09 + j0.20$$
$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 502\underline{/22.8^\circ}\ \Omega$$

$$V(z) = V_0^+ \left[e^{-\alpha z - j\beta z} + \Gamma_L e^{\alpha z + j\beta z} \right].$$

At $z = 0$,

$$V(z=0) = V_0^+ (1 + \Gamma_L) = 50, \qquad \text{(V)}$$

therefore,

$$V_0^+ = 56.2\underline{/-26.6^\circ} \qquad \text{(V)}.$$

At $z = -150$ m,

$$\begin{aligned} V_{in} &= V_0^+ \left[e^{-j2.4\times10^{-3}(-150) - j0.0212(-150)} + 0.45\underline{/116.6^\circ} e^{j2.4\times10^{-3}(-150) + j0.0212(-150)} \right] \\ &= 75.0\underline{/167.3^\circ} \qquad \text{(V)} \end{aligned}$$

■

P-6. A 15-m length of 300-Ω line must be connected to a 3-m length of 150-Ω line that is terminated in a 150-Ω resister. Assuming all lines are lossless, find the VSWR on the 300-Ω line. In order to match the two sections, a quarter-wavelength line of characteristic impedance Z_0 is added (Fig. 2). Find the appropriate Z_0. Assume a working frequency of 50 MHz.

Since the load and line 2 are matched, the input impedance looking into line 2 is 150 Ω. This is also the load impedance to line 1. The reflection coefficient at $A - A'$ is

$$\Gamma_{AA'} = \frac{150 - 300}{150 + 300} = -1/3.$$

Therefore,

$$VSWR = \frac{1 + 1/3}{1 - 1/3} = 2.$$

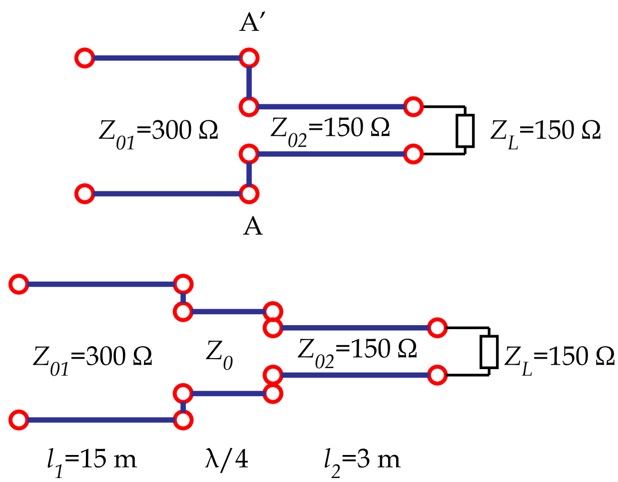


Figure 2: Circuit diagram for Problem. 6.

The quarter-wave length line needs to match between $Z_{01} = 300\ \Omega$ and $Z_{0,2} = 150\ \Omega$, therefore it's characteristic impedance needs to be

$$Z_0 = \sqrt{Z_{01} Z_{02}} = 212.1\ \Omega.$$

■

P-7. A 50-Ω slotted line that is 40 cm long is inserted in a 50-Ω lossless line feeding an antenna at 600 MHz. Standing-wave measurements with the antenna in place yield the data of Fig. 3, showing the first voltage maxima point and the first voltage minima point. The scale on the slotted line has the lowest number on the load side. Find the impedance of the antenna, the reflection coefficient due to the load, and the velocity of propagation on the line.

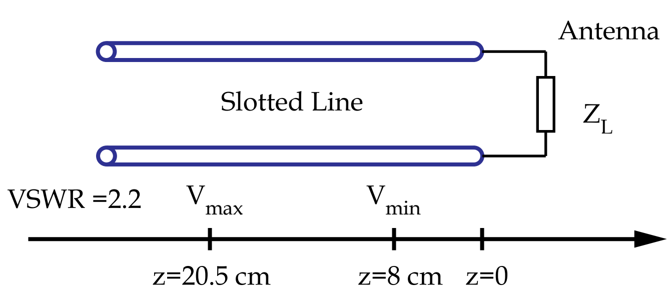


Figure 3: Circuit diagram for Problem. 7.

We know that the separation between adjacent maxima and minima is a quarter of a wavelength. Therefore $\lambda = 4 \times (20.5 - 8) = 50$ cm. At 600 MHz, the propagation velocity (phase velocity) is $u_p = f\lambda = 3 \times 10^8$ m/s (this means that the slotted line has air dielectric).

We can calculate the magnitude of the reflection coefficient from the VSWR measurement.

$$|\Gamma_L| = \frac{VSWR - 1}{VSWR + 1} = 0.375$$

We know that the voltage maxima condition is

$$2\beta d_{max} = \theta_r + 2n\pi$$

Therefore, the phase of the reflection coefficient θ_r is

$$\theta_r = 2\beta d_{max} - 2n\pi = 2\frac{2\pi}{\lambda}d_{max} - 2n\pi = 295.2^\circ \quad (n = 0)$$

Therefore the refection coefficient is $\Gamma_L = 0.375/295.2^\circ$.
The load impedance (antenna impedance) is calculated from the reflection coefficient,

$$Z_L = Z_0\frac{1+\Gamma}{1-\Gamma} = (52.8 - j41.4)\ \Omega$$

∎

P-8. Show that a short section of a shorted transmission line appears as if it's an inductor.

A short circuit is equivalent to a load impedance of $Z_L = 0$.
The input impedance of a short circuited transmission line is

$$Z_{in,SC} = Z_0\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}.$$

With $Z_L = 0$,

$$Z_{in,SC} = jZ_0 \tan\beta l.$$

The input impedance of a capacitor takes a similar form

$$Z_L = j\omega L.$$

∎

P-9. Show that a short section of an open transmission line appears as if it's an capacitor.

An open circuit is equivalent to a load impedance of $Z_L = \infty$.
The input impedance of an open circuited transmission line is

$$Z_{in,OC} = Z_0\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}.$$

With $Z_L = \infty$,

$$Z_{in,OC} = \frac{Z_0}{j \tan \beta l}.$$

The input impedance of a capacitor takes a similar form

$$Z_C = \frac{1}{j\omega C}.$$

■

P-10. A standing wave given by $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$ exist on a short circuited transmission line with characteristic impedance Z_0. Sketch the magnitude of the voltage and current on the transmission line.

For short circuit load, the reflection coefficient is -1. Therefore $V_0^- = -V_0^+$. The total voltage is then given by

$$V(z) = V_0^+ \left(e^{-j\beta z} - e^{j\beta z} \right) = 2V_0^+ \sin \beta z.$$

And the total current is given by

$$I(z) = \frac{V_0^+}{Z_0} \left(e^{-j\beta z} + e^{j\beta z} \right) = 2\frac{V_0^+}{Z_0} \cos \beta z.$$

Plots of the voltage and current are given in Fig. . ■

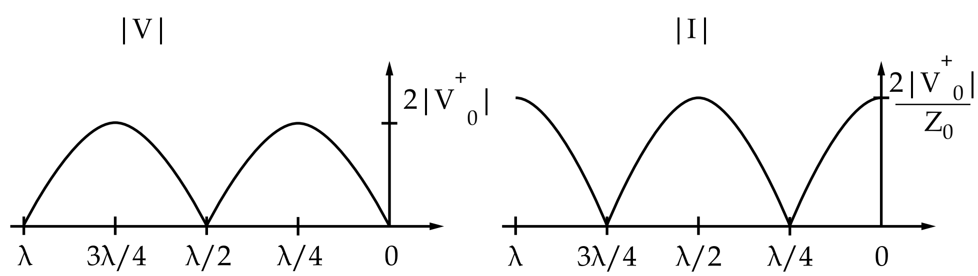


Figure 4: Circuit diagram for Problem. 10.

P-11. A section of lossless coaxial cable having $Z_0 = 50\ \Omega$ and phase velocity $u_p = 2 \times 10^8$ m/s is terminated in a short circuit and operated at a frequency of 10 MHz. Determine the shortest length of the lines such that, at the input terminals, the line appears to be a 100-pF capacitor.

The input impedance of a short circuited transmission line is

$$Z_{in,SC} = jZ_0 \tan \beta l.$$

The input impedance of a capacitor is

$$Z_C = \frac{1}{j\omega C}.$$

In order for them to match,

$$\tan \beta l = \frac{-1}{\omega Z_0 C} = -3.183.$$

Therefore,

$$(\beta l)_{min} = 1.876 \text{ rad},$$
$$l_{min} = 5.97 \text{ m}.$$

■

P-12. Fig. 5 shows a microstrip circuit with a shorted stub 3 mm in length.The effective permitivity of the substrate is $\epsilon_{eff} = 4$ F/m and the microstrip lines are all designed to be 50 Ω. (a) Find the equivalent inductance of the shorted stub at 3 GHz; (b) As frequency increases, the short stub becomes more capacitive than inductive. Find out the frequency at which the short stub appears as an open circuit.

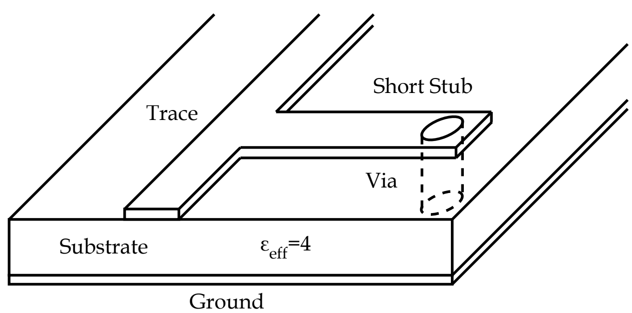


Figure 5: Circuit diagram for Problem. 12.

The propagation wavelength on the microstrip line is

$$\lambda = \frac{u_p}{f} = \frac{c}{f\sqrt{e_{eff}}} = 50 \text{ mm}$$

The input impedance of the short stub is

$$Z_{in,SC} = jZ_0 \tan \beta l = jZ_0 \tan \left(\frac{2\pi}{\lambda} l\right)$$

The equivalent inductance is then given by

$$L = \frac{Z_0 \tan\left(\frac{2\pi}{\lambda} l\right)}{\omega} = \frac{50 \tan\left(\frac{2\pi}{50} \times 3\right)}{2\pi \times 3 \times 10^9} = 1.06\ \mathrm{nH/m}$$

■

P-13. In microstrip circuits working at high frequencies, it is difficult to realize an ideal short circuit because the vias have a finite length, which creates a certain inductance. Sometimes microstrip circuit designers use an open stub to realize a short. For a microstrip circuit working at 30 GHz, find out the shortest length of an open stub that appears as a short. The effective relative permitivity of the microstrip is $\epsilon_{r_{eff}} = 4$.

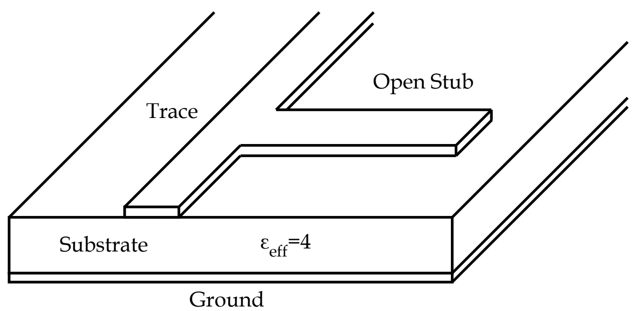


Figure 6: Circuit diagram for Problem. 13.

The propagation wavelength on the microstrip line is

$$\lambda = \frac{u_p}{f} = \frac{c}{f\sqrt{e_{r_{eff}}}} = 5\ \mathrm{mm}$$

We know that a quarter-wavelength line transforms an open to a short, so a $5/4 = 1.25$ mm open stub appears a short.

Alternatively, we could work out the math. We know that the input impedance of an open stub is

$$Z_{in,OC} = \frac{Z_0}{j \tan \beta l}.$$

In order for it to appear as a short, $Z_{in,OC}$ must be 0, which means that βl must be $\pi/4 + n\pi$, $n = 0, 1, 2, \ldots$. Therefore, the shortest length of an open stub that appears as a short is $l = \lambda/4$. ■

P-14. Show that for a lossless transmission line having a purely resistive load R_L, show that

$$VSWR = \frac{\max(Z_L, Z_0)}{\min(Z_L, Z_0)}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

If $Z_L > Z_0$,

$$|\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \dfrac{Z_L - Z_0}{Z_L + Z_0}}{1 - \dfrac{Z_L - Z_0}{Z_L + Z_0}} = \frac{Z_L}{Z_0} = \frac{\max(Z_L, Z_0)}{\min(Z_L, Z_0)}$$

If $Z_L < Z_0$,

$$|\Gamma| = \frac{Z_0 - Z_L}{Z_0 + Z_L}$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \dfrac{Z_0 - Z_L}{Z_0 + Z_L}}{1 - \dfrac{Z_0 - Z_L}{Z_0 + Z_L}} = \frac{Z_0}{Z_L} = \frac{\max(Z_L, Z_0)}{\min(Z_L, Z_0)}$$

Note: this provide a quick way to estimate VSWR for real load impedances. ■

P-15. One method of determining the characteristics of a transmission line (with length l) is to measure the input impedance $Z_{in,SC}$ when the line is terminated with a short circuit and the input impedance $Z_{in,OC}$ when the line is terminated with an open circuit. (a) Show that you can determine Z_0 and β from $Z_{in,SC}$ and $Z_{in,OC}$. (b) Given $Z_{in,SC} = 62.0/37.7^\circ$, $Z_{in,OC} = 141.9/-84.1^\circ$ and $l = 2$ miles, find Z_0, α, and β.

(a) The input impedance of a short circuited transmission line is

$$Z_{in,SC} = jZ_0 \tan \beta l.$$

The input impedance of an open circuited transmission line is

$$Z_{in,OC} = \frac{Z_0}{j \tan \beta l}.$$

We notice that

$$Z_{in,SC} Z_{in,OC} = Z_0^2.$$

Therefore

$$Z_0 = \sqrt{Z_{in,SC} Z_{in,OC}}.$$

We also notice that

$$Z_{in,SC}/Z_{in,OC} = -\tan^2 \beta l.$$

Therefore

$$\beta = \frac{\arctan\left(\sqrt{-Z_{in,SC}/Z_{in,OC}}\right)}{l}.$$

(b) For a lossy transmission line, the input impedance transformation expression is slightly different (note the use of "tanh" instead of tan and the disappearance of "j"),

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}.$$

It follows that

$$Z_{in,SC} = Z_0 \tanh \gamma l,$$

and

$$Z_{in,OC} = \frac{Z_0}{\tanh \gamma l}.$$

$$Z_0 = \sqrt{Z_{in,SC} Z_{in,OC}} = \sqrt{62.0/37.7^\circ \cdot 141.9/-84.1^\circ} = 93.8/-23.2^\circ. \qquad \Omega$$

$$\gamma = \frac{\tanh^{-1}\left(\sqrt{\frac{62.0/37.7^\circ}{141.9/-84.1^\circ}}\right)}{2 \times 1.609 \times 10^3} = 1.74 \times 10^{-4} + j7.46 \times 10^{-5}.$$

$$\alpha = 1.74 \times 10^{-4} \qquad \text{Np/m.}$$
$$\beta = 7.46 \times 10^{-5} \qquad \text{rad/m.}$$

■

P-16. A 70-Ω high-frequency lossless line is used at a frequency where $\lambda = 80$ cm with a load at $x = 0$ of $(140 + j91)$ Ω. Use the Smith chart to find the following: Γ_L, VSWR, distance to the first voltage maximum from the load, distance to the first voltage minimum from the load, the impedance at $|V|_{max}$, the impedance at $|V|_{min}$, the input impedance for a section of line that is 54 cm long, and the input admittance.

Fig. is the Smith chart plot for Problem 16.

The normalized impedance is

$$z_L = \frac{140 + j91}{70} = 2 + j1.3.$$

Identify the position (Point **A**) of this load impedance on the Smith chart by finding the right constant-r_L circle ($r_L = 2$) and the right constant-x_L circle ($x_L = 1.3$).

Draw a line from the origin to point **A** and extend to intersect with the unit circle at $\mathbf{A}'$. Find the magnitude of the Γ_L by measuring the length of $\bar{\mathbf{OA}}$ with respect to the radius of the unit circle. In this case, $|\Gamma_L| = 0.5$. Find the phase of Γ_L by reading $\mathbf{A}'$ on the phase scale. In this case, $\theta_r = 29^\circ$. Therefore, $\Gamma_L = 0.5/29^\circ$.

Draw a circle centered at the origin with a radius of $\bar{\mathbf{OA}}$. This is the constant-SWR circle. The constant-SWR circle intersects with Γ_r axis at $\mathbf{P_{max}}$ (closer to the open circuit point) and $\mathbf{P_{min}}$ (closer to the short circuit point). Point $\mathbf{P_{max}}$ represents the input

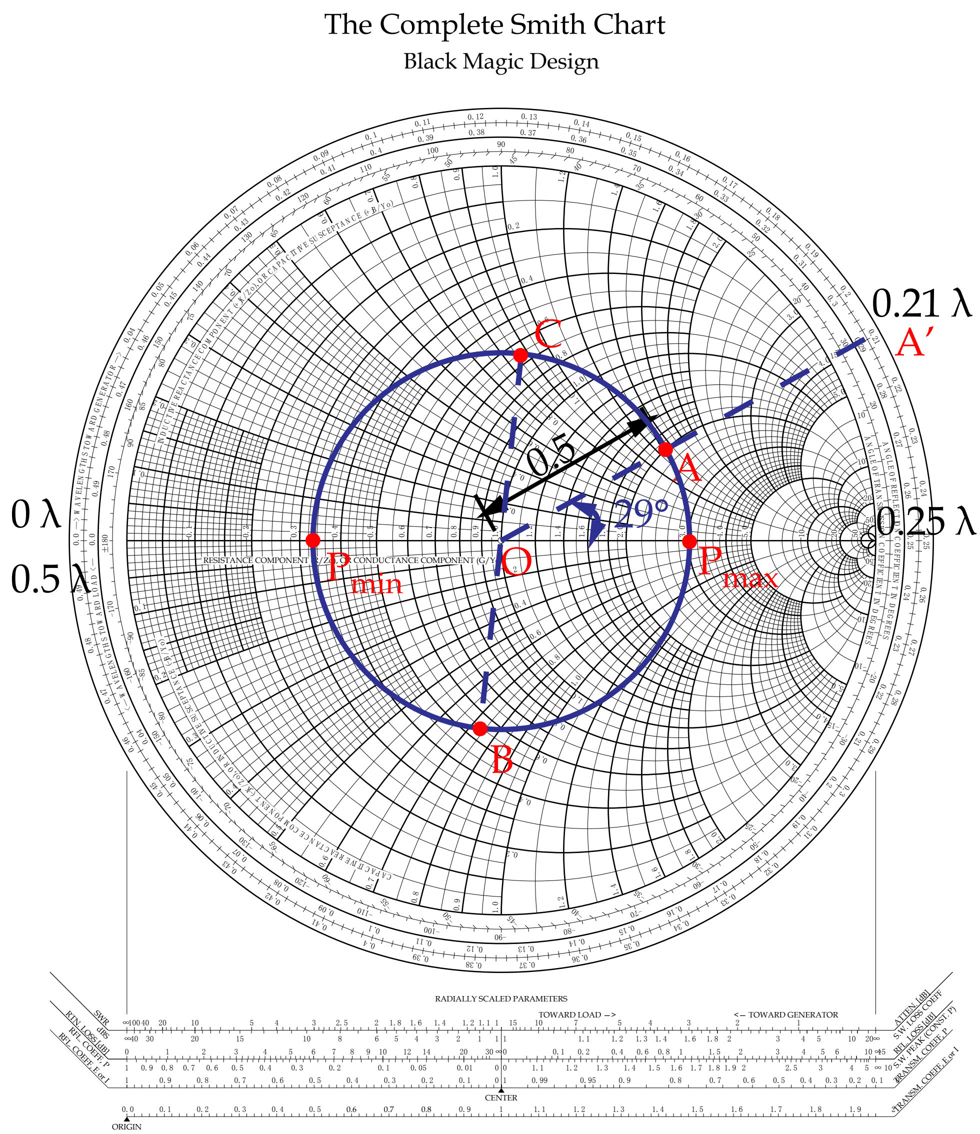


Figure 7: Smith chart plot for Problem 16.

impedance at the first voltage maximum. In this case, $Z_{vmax} = Z_0(3 + j0) = 210\ \Omega$. Point $\mathbf{P_{min}}$ represents the input impedance at the first voltage maximum. In this case, $Z_{vmin} = Z_0(0.33 + j0) = 23.3\ \Omega$.

The angle between $\bar{\mathbf{OA}}$ and $\bar{\mathbf{OP}}_{\mathbf{max}}$ represents the distance between the load to the first voltage maximum d_{max}. Read from the WAVELENGTH TO GENERATOR scale that $d_{max} = (0.25 - 0.21)\lambda = 0.04\lambda$. The distance between the load to the first voltage minimum d_{min} is simply $d_{min} = d_{max} + \lambda/4 = 0.29\lambda$.

Draw a vertical line from P_{max} to intersect with the VSWR scale at the bottom of the Smith chart. Read out $VSWR = 3.0$.

A section of 54 cm long transmission line is $54/80 \cdot \lambda = 0.675\lambda$. Move point **A** by 0.675λ along the constant-SWR circle to arrive at point B, which represents the normalized input impedance z_{in}. Read from the corresponding constant-r_L and constant-x_L circles that $z_{in} = 0.56 - j0.71$. Therefore, $Z_{in} = 39.2 - j49.7$.

The normalized admittance y_{in} can be found by rotating point **B** by 180° to point **C**. Read from the corresponding constant-r_L and constant-x_L circles that $y_{in} = 0.68 + j0.87$. Therefore, $Y_{in} = y_{in}/Z_0 = 9.71 - j12.4$ mS.

■

P-17. The high-frequency lossless transmission system shown in Fig. 8 operates at 700 MHz with a phase velocity for each line section of 2.1×10^8 m/s. Use the Smith chart to find the VSWR on each section of line and the input impedance to line #1.

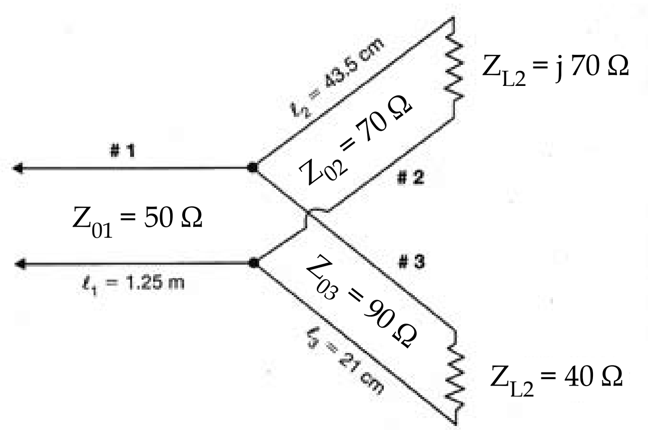


Figure 8: Circuit diagram for Problem 17.

Fig. is the Smitch chart plot for Problem. 17.

For all three lines, wavelength is $\lambda = u_p/f = (2.1 \times 10^8)/(7 \times 10^8) = 30cm$.

For line #2, the electrical length is $(43.5/30)\lambda = 1.45\lambda$. The normalized load impedance is $z_{L2} = j70/70 = j$. Find z_{L2} on the Smith chart at Point **A**. Move along the const-SWR circle (in this case it's simply the unit circle) by 1.45λ to point **B**. Read $z_{in,2} = j0.51$ from point **B**. Therefore, $Z_{in,2} = Z_{02}z_{in,2} = +j35.7\ \Omega$.

Similarly, for line #3, the electrical length is $(21/30)\lambda = 0.7\lambda$. The normalized load impedance is $z_{L3} = 40/90 = 0.44$. Find z_{L3} on the Smith chart at Point **C**. Move along the const-SWR circle by 0.7λ to point **D**. Read $z_{in,3} = 1.62 + j0.86$ from point **B**. Therfore, $Z_{in,3} = Z_{03}z_{in,3} = 145.8 + j77.4\ \Omega$.

Therefore the load impedance for line #1 is $Z_{in,2}//Z_{in,2}$. Normalized both $Z_{in,2}$ and $Z_{in,2}$ to Z_{01}, find each admittance, add the admittance and find the total normalized load impedance z_{L1}.

$$z_{in,21} = Z_{in,2}/Z_{01} = j35.7/50 = j0.714 \qquad \text{(Point } \mathbf{E}\text{)}$$

$$y_{in,21} = -j1.41 \qquad \text{(Point } \mathbf{F}\text{)}$$

$$z_{in,31} = Z_{in,3}/Z_{01} = (145.8 + j77.4)/50 = 2.92 + j1.55 \qquad \text{(Point } \mathbf{G}\text{)}$$

$$y_{in,31} = 0.27 - j0.14 \qquad \text{(Point } \mathbf{H}\text{)}$$

$$y_{L1} = y_{in,21} + y_{in,31} = 0.27 - j1.55 \qquad \text{(Point } \mathbf{I}\text{)}$$

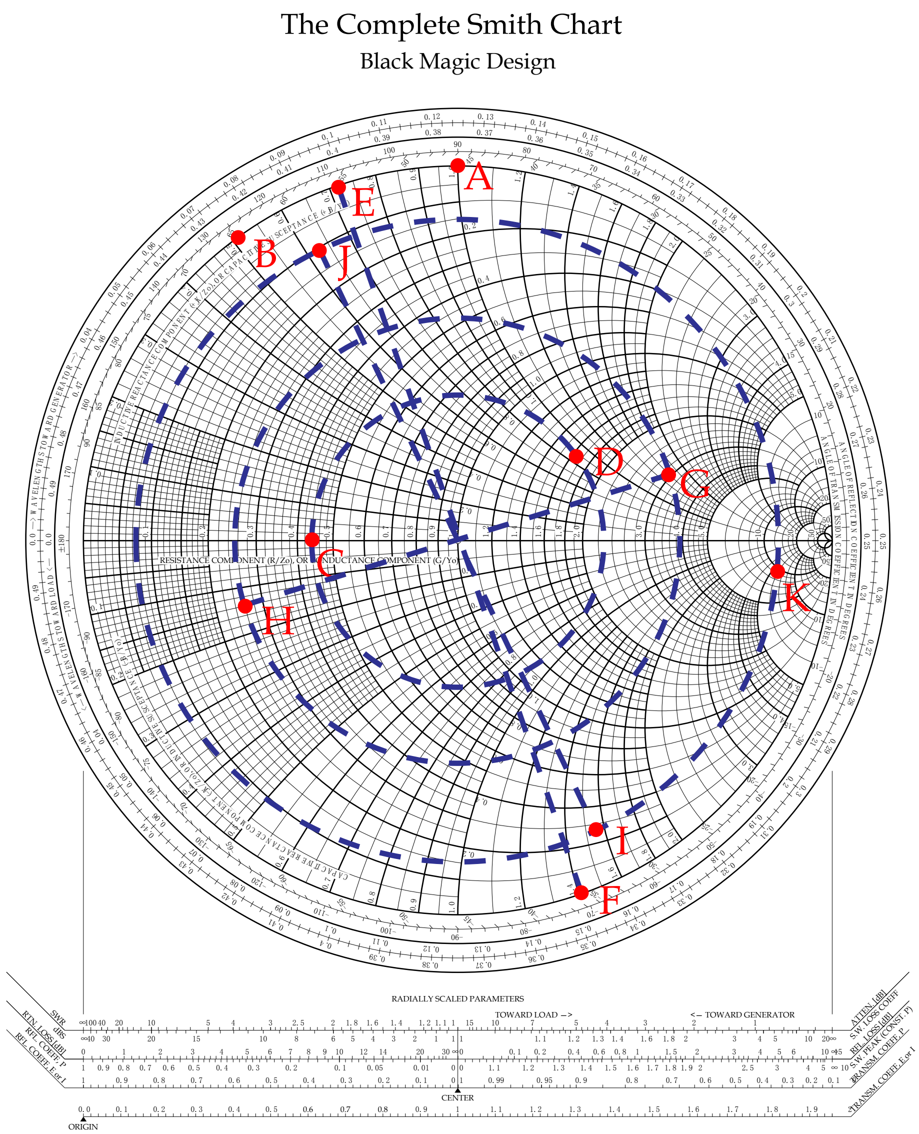


Figure 9: Smith chart plot for Problem 17.

$$z_{L1} = 0.1 + j0.63 \qquad \text{(Point } \mathbf{J}\text{)}$$

The electrical length of line #1 is $(1.25/0.3)\lambda = 4.167\lambda$. Move Point **J** by 4.167λ towards the generator to Point **K** and read out the normalized input impedance $z_{in} = 9.5 - j6.3$. The input impedance to line #1 is then

$$Z_{in} = Z_{01} z_{in} = 50(9.5 - j6.3) = 475 - j315\ \Omega.$$

■

P-18. A high-frequency 50-Ω lossless line is 141.6 cm long, with a relative dielectric constant $\epsilon_r = 2.49$. At 500 MHz the input impedance of the terminated line is measured as $Z_{in} = (20 + j25)\ \Omega$. (a) Use the Smith chart to find the value of the terminating load. (b) After the impedance measurement an 8-pF lossless capacitor is connected in parallel with the line at a distance of 8.5 cm from the load. Find the VSWR on the main line.

Fig. is the Smith chart plot for Problem. 18.

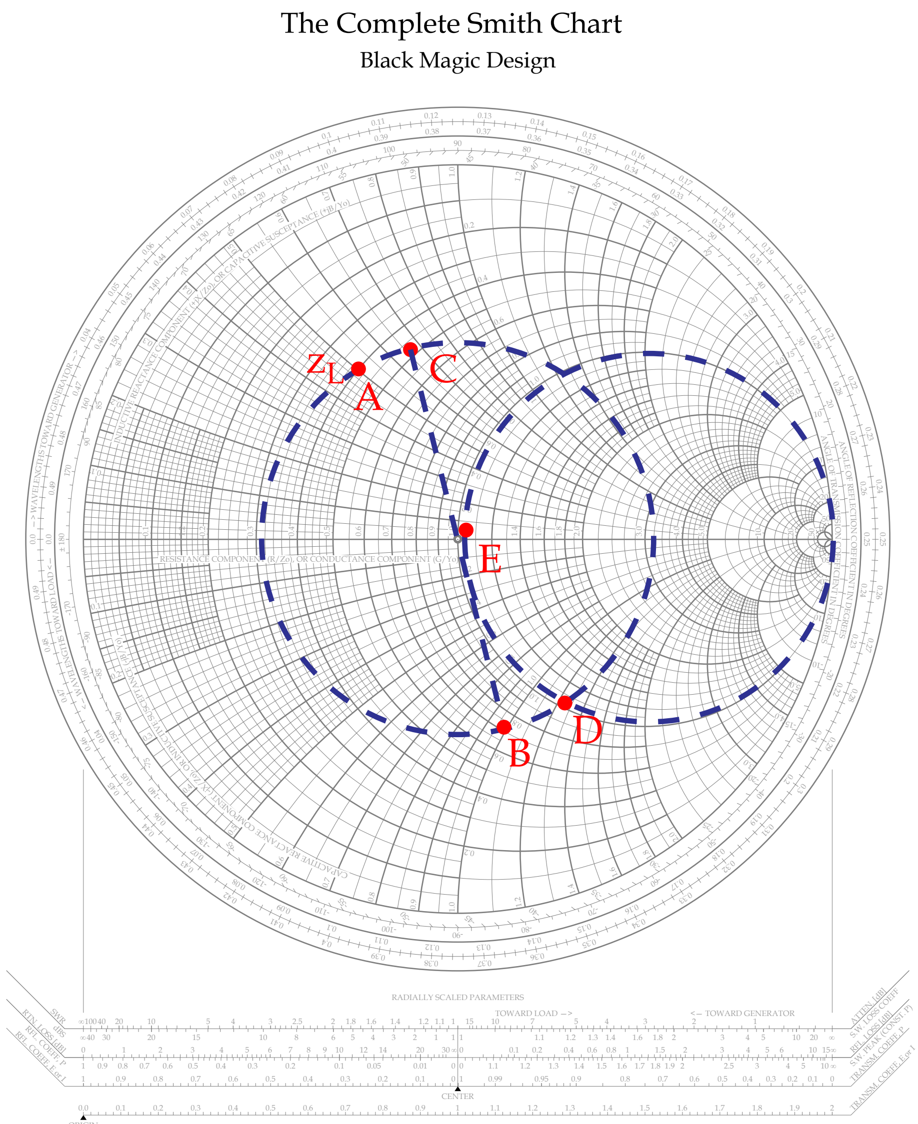


Figure 10: Smith chart plot for Problem 18.

(a) The phase velocity is

$$u_p = \frac{c}{\sqrt{\epsilon_{eff}}} = \frac{3 \times 10^8}{\sqrt{2.49}} = 1.9 \times 10^8 \qquad \text{m/s}.$$

The wavelength is

$$\lambda = \frac{u_p}{f} = \frac{1.9 \times 10^8}{5 \times 10^8} = 38 \text{ cm}.$$

The electrical length of the line is $(141.6/38)\lambda = 3.726\lambda$. The normalized input impedance is $z_{in} = (20 + j25)/50 = 0.4 + j0.4$. Locate z_{in} on the Smith chart at Point **A**. Move Point **A** towards the load (that is, counterclockwise) by 3.726λ to Point **B**. Read the normalized load impedance from Point **B** to be $z_L = 0.72 - j0.98$. The load impedance is then $Z_L = Z_0 z_L = (36 - j49)\ \Omega$.

(b) Since the capacitor is connected in parallel, it is easier to work with admittance. Find the admittance $y_L = 0.48 + j0.67$ by moving Point **B** by 180° to point **C**. Next, move Point **C** by $(8.5/38)\lambda = 0.224\lambda$ towards the generator on the const-SWR circle to Point **D**. Adding a shunt capacitor is equivalent to moving clockwise on the const-r_L circle (Note that we are working with admittance and the admittance of a capacitor has a positive value). The normalized admittance of a capacitor is

$$y_c = \frac{j\omega C}{Y_0} = j2\pi f C Z_0 = j2\pi(5\times10^8)(8\times10^{-12})50 = j1.26$$

Move Point **D** by 1.26 along the const-r_L circle to Point **E**.

The final normalized input impedance is $y_{in} = 1.04 + j0.04$, which is very close to the matched condition. In fact, the VSWR has decreased from 3.2 for the load to 1.04 for the input. This can also be verified by proximity of Point **E** to the origin. ■

□

The following problems from the textbook are also considered practice problems.
1.15, 2.6, 2.16, 2.32, 2.34, 2.52, 2.58, 2.61, 2.66

Problem 1.15 A laser beam traveling through fog was observed to have an intensity of 1 (μW/m^2) at a distance of 2 m from the laser gun and an intensity of 0.2 (μW/m^2) at a distance of 3 m. Given that the intensity of an electromagnetic wave is proportional to the square of its electric-field amplitude, find the attenuation constant α of fog.

Solution: If the electric field is of the form

$$E(x,t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x),$$

then the intensity must have a form

$$\begin{aligned} I(x,t) &\approx [E_0 e^{-\alpha x} \cos(\omega t - \beta x)]^2 \\ &\approx E_0^2 e^{-2\alpha x} \cos^2(\omega t - \beta x) \end{aligned}$$

or

$$I(x,t) = I_0 e^{-2\alpha x} \cos^2(\omega t - \beta x)$$

where we define $I_0 \approx E_0^2$. We observe that the magnitude of the intensity varies as $I_0 e^{-2\alpha x}$. Hence,

$$\begin{aligned} &\text{at } x = 2 \text{ m}, \qquad I_0 e^{-4\alpha} = 1 \times 10^{-6} \quad (\text{W/m}^2), \\ &\text{at } x = 3 \text{ m}, \qquad I_0 e^{-6\alpha} = 0.2 \times 10^{-6} \quad (\text{W/m}^2). \end{aligned}$$

$$\begin{aligned} \frac{I_0 e^{-4\alpha}}{I_0 e^{-6\alpha}} &= \frac{10^{-6}}{0.2 \times 10^{-6}} = 5 \\ e^{-4\alpha} \cdot e^{6\alpha} &= e^{2\alpha} = 5 \\ \alpha &= 0.8 \quad (\text{NP/m}). \end{aligned}$$

Problem 2.6 A coaxial line with inner and outer conductor diameters of 0.5 cm and 1 cm, respectively, is filled with an insulating material with $\varepsilon_{\mathrm{r}} = 4.5$ and $\sigma = 10^{-3}$ S/m. The conductors are made of copper.

(a) Calculate the line parameters at 1 GHz.

Solution: **(a)** Given

$$a = (0.5/2) \text{ cm} = 0.25 \times 10^{-2} \text{ m},$$
$$b = (1.0/2) \text{ cm} = 0.50 \times 10^{-2} \text{ m},$$

combining Eqs. (2.5) and (2.6) gives

$$\begin{aligned} R' &= \frac{1}{2\pi}\sqrt{\frac{\pi f \mu_{\mathrm{c}}}{\sigma_{\mathrm{c}}}}\left(\frac{1}{a}+\frac{1}{b}\right) \\ &= \frac{1}{2\pi}\sqrt{\frac{\pi(10^9 \text{ Hz})(4\pi\times 10^{-7} \text{ H/m})}{5.8\times 10^7 \text{ S/m}}}\left(\frac{1}{0.25\times 10^{-2} \text{ m}}+\frac{1}{0.50\times 10^{-2} \text{ m}}\right) \\ &= 0.788\ \Omega/\text{m}. \end{aligned}$$

From Eq. (2.7),

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times 10^{-7} \text{ H/m}}{2\pi}\ln 2 = 139 \text{ nH/m}.$$

From Eq. (2.8),

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi\times 10^{-3} \text{ S/m}}{\ln 2} = 9.1 \text{ mS/m}.$$

From Eq. (2.9),

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_{\mathrm{r}}\varepsilon_0}{\ln(b/a)} = \frac{2\pi\times 4.5\times(8.854\times 10^{-12} \text{ F/m})}{\ln 2} = 362 \text{ pF/m}.$$

(b) Solution via Module 2.2:

Problem 2.16 A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since Z_0 is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega\text{/m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega\text{/m}} = 0.5 \text{ mS/m}.$$

Problem 2.32 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30^\circ) \quad \text{(V)}$$

and $Z_g = 150\ \Omega$. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)\ \Omega$, determine:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

(e) The time-domain input voltage $v_i(t)$.

Solution:

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30^\circ) \text{ V},$$
$$\widetilde{V}_g = 5e^{-j30^\circ} \text{ V}.$$

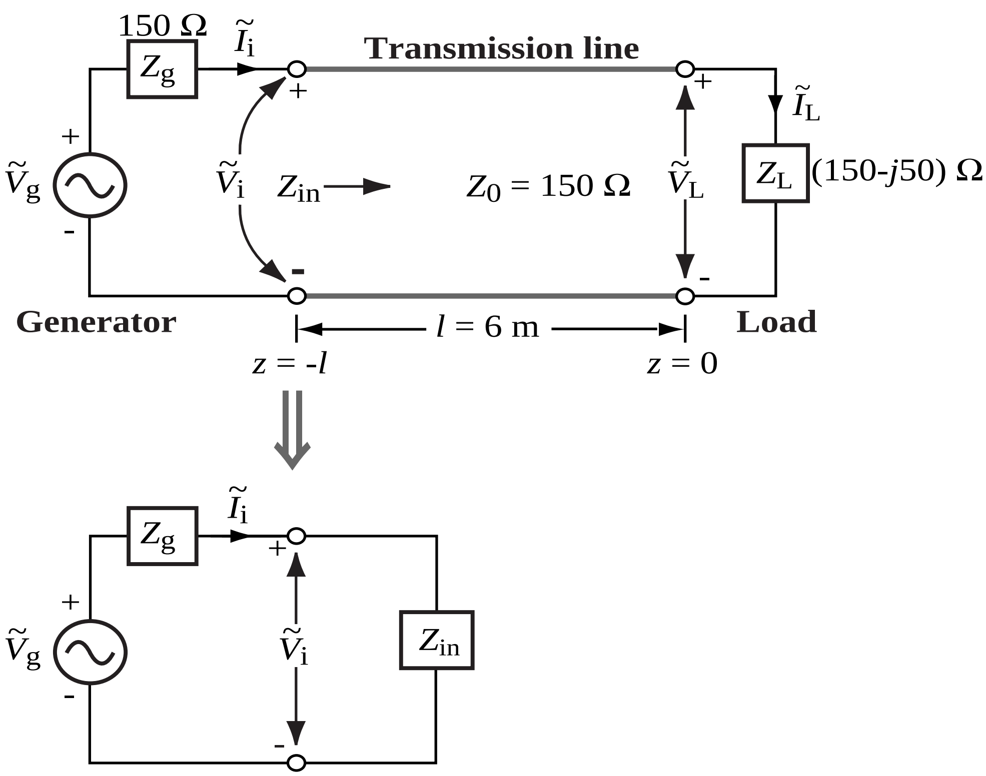


Figure P2.32: Circuit for Problem 2.32.

(a)

$$
\begin{aligned}
u_{\mathrm{p}} &= \frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{3\times 10^8}{\sqrt{2.25}} = 2\times 10^8 \quad \text{(m/s)},\\
\lambda &= \frac{u_{\mathrm{p}}}{f} = \frac{2\pi u_{\mathrm{p}}}{\omega} = \frac{2\pi\times 2\times 10^8}{8\pi\times 10^7} = 5 \ \text{ m},\\
\beta &= \frac{\omega}{u_{\mathrm{p}}} = \frac{8\pi\times 10^7}{2\times 10^8} = 0.4\pi \quad \text{(rad/m)},\\
\beta l &= 0.4\pi\times 6 = 2.4\pi \quad \text{(rad)}.
\end{aligned}
$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \dfrac{Z_{\mathrm{L}} - Z_0}{Z_{\mathrm{L}} + Z_0} = \dfrac{150 - j50 - 150}{150 - j50 + 150} = \dfrac{-j50}{300 - j50} = 0.16\,e^{-j80.54^\circ}$.

(c)

$$
\begin{aligned}
Z_{\mathrm{in}} &= Z_0\left[\frac{Z_{\mathrm{L}} + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]\\
&= 150\left[\frac{(150 - j50) + j150\tan(0.4\pi)}{150 + j(150 - j50)\tan(0.4\pi)}\right] = (115.70 + j27.42)\ \Omega.
\end{aligned}
$$

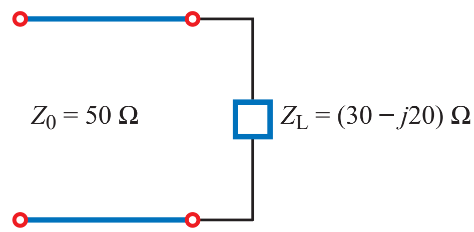
(d)

$$
\begin{aligned}
\widetilde{V}_{\mathrm{i}} = \frac{\widetilde{V}_{\mathrm{g}} Z_{\mathrm{in}}}{Z_{\mathrm{g}} + Z_{\mathrm{in}}} &= \frac{5e^{-j30^\circ}(115.7 + j27.42)}{150 + 115.7 + j27.42}\\
&= 5e^{-j30^\circ}\left(\frac{115.7 + j27.42}{265.7 + j27.42}\right)\\
&= 5e^{-j30^\circ}\times 0.44\,e^{j7.44^\circ} = 2.2\,e^{-j22.56^\circ} \quad \text{(V)}.
\end{aligned}
$$

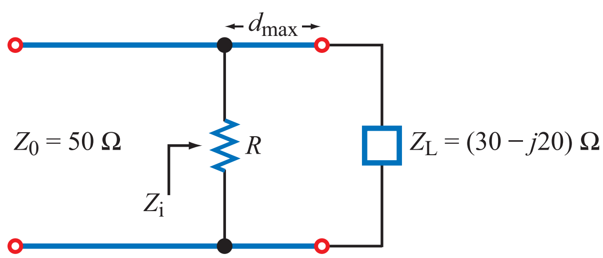
(e)

$$
v_{\mathrm{i}}(t) = \mathfrak{Re}[\widetilde{V}_{\mathrm{i}} e^{j\omega t}] = \mathfrak{Re}[2.2\,e^{-j22.56^\circ} e^{j\omega t}] = 2.2\cos(8\pi\times 10^7 t - 22.56^\circ) \ \text{ V}.
$$

Problem 2.34 A 50-Ω lossless line is terminated in a load impedance $Z_L = (30 - j20)$ Ω.



(a)



(b)

Figure P2.34: Circuit for Problem 2.34.

(a) Calculate Γ and S.

(b) It has been proposed that by placing an appropriately selected resistor across the line at a distance d_{max} from the load (as shown in Fig. P2.34(b)), where d_{max} is the distance from the load of a voltage maximum, then it is possible to render $Z_i = Z_0$, thereby eliminating reflection back to the end. Show that the proposed approach is valid and find the value of the shunt resistance.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j20 - 50}{30 - j20 + 50} = \frac{-20 - j20}{80 - j20} = \frac{-(20 + j20)}{80 - j20} = 0.34e^{-j121^\circ}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.34}{1 - 0.34} = 2.$$

(b) We start by finding d_{max}, the distance of the voltage maximum nearest to the load. Using (2.70) with $n = 1$,

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} = \left(\frac{-121^\circ \pi}{180^\circ}\right)\frac{\lambda}{4\pi} + \frac{\lambda}{2} = 0.33\lambda.$$

Applying (2.79) at $d = d_{\max} = 0.33\lambda$, for which $\beta l = (2\pi/\lambda) \times 0.33\lambda = 2.07$ radians, the value of Z_{in} before adding the shunt resistance is:

$$\begin{aligned} Z_{\text{in}} &= Z_0 \left(\frac{Z_{\text{L}} + jZ_0 \tan \beta l}{Z_0 + jZ_{\text{L}} \tan \beta l} \right) \\ &= 50 \left(\frac{(30 - j20) + j50 \tan 2.07}{50 + j(30 - j20) \tan 2.07} \right) = (102 + j0)\ \Omega. \end{aligned}$$

Thus, at the location A (at a distance $d_{\max}$ from the load), the input impedance is purely real. If we add a shunt resistor R in parallel such that the combination is equal to Z_0, then the new Z_{in} at any point to the left of that location will be equal to Z_0.

Hence, we need to select R such that

$$\frac{1}{R} + \frac{1}{102} = \frac{1}{50}$$

or $R = 98\ \Omega$.

Problem 2.52 On a lossless transmission line terminated in a load $Z_L = 100\ \Omega$, the standing-wave ratio was measured to be 2.5. Use the Smith chart to find the two possible values of Z_0.

Solution: Refer to Fig. P2.52. $S = 2.5$ is at point $L1$ and the constant SWR circle is shown. z_L is real at only two places on the SWR circle, at $L1$, where $z_L = S = 2.5$, and $L2$, where $z_L = 1/S = 0.4$. so $Z_{01} = Z_L/z_{L1} = 100\ \Omega/2.5 = 40\ \Omega$ and $Z_{02} = Z_L/z_{L2} = 100\ \Omega/0.4 = 250\ \Omega$.

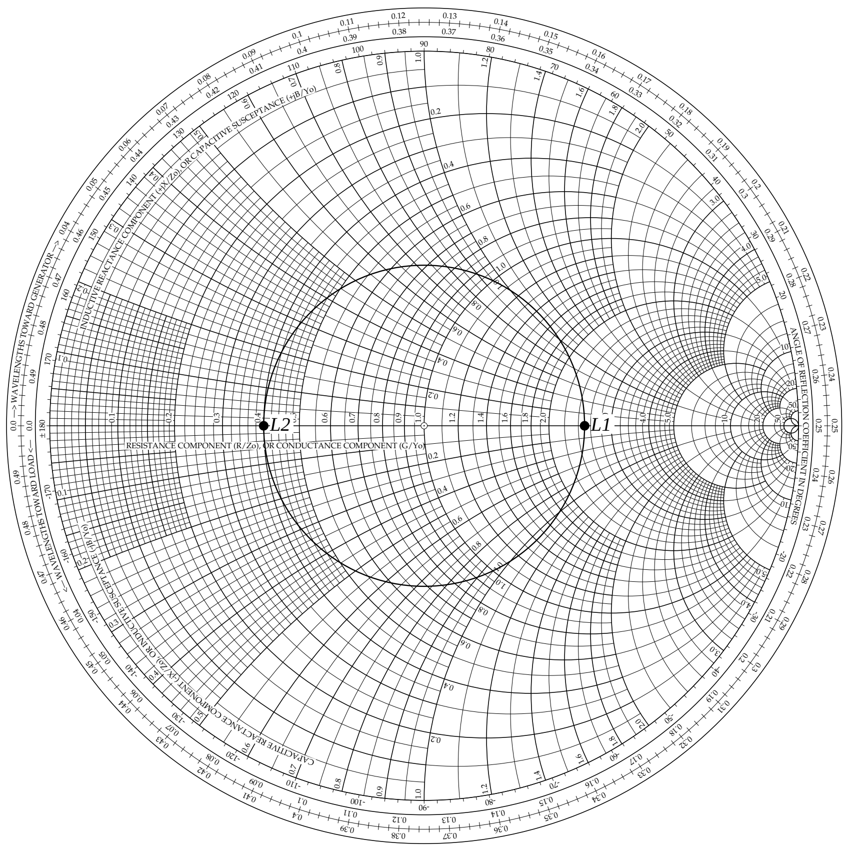


Figure P2.52: Solution of Problem 2.52.

Problem 2.58 A lossless 100-Ω transmission line $3\lambda/8$ in length is terminated in an unknown impedance. If the input impedance is $Z_{\text{in}} = -j2.5\ \Omega$,

(a) Use the Smith chart to find Z_{L}.

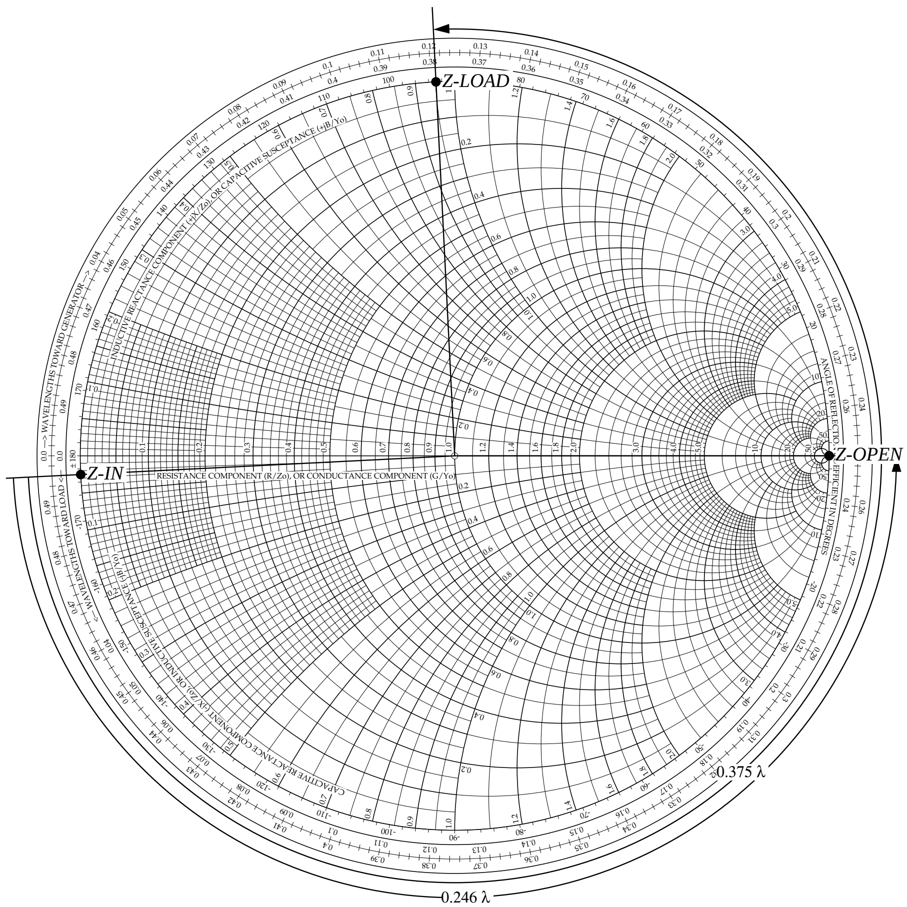


Figure P2.58: Solution of Problem 2.58.

Solution: Refer to Fig. P2.58. $z_{\text{in}} = Z_{\text{in}}/Z_0 = -j2.5\ \Omega/100\ \Omega = 0.0 - j0.025$ which is at point *Z-IN* and is at 0.004λ on the wavelengths to load scale.

(a) Point *Z-LOAD* is 0.375λ toward the load from the end of the line. Thus, on the wavelength to load scale, it is at $0.004\lambda + 0.375\lambda = 0.379\lambda$.

$$Z_{\text{L}} = z_{\text{L}} Z_0 = (0 + j0.95) \times 100\ \Omega = j95\ \Omega.$$

Problem 2.61 Using a slotted line on a 50-Ω air-spaced lossless line, the following measurements were obtained: $S = 1.6$ and $|\widetilde{V}|_{\max}$ occurred only at 10 cm and 24 cm from the load. Use the Smith chart to find Z_{L}.

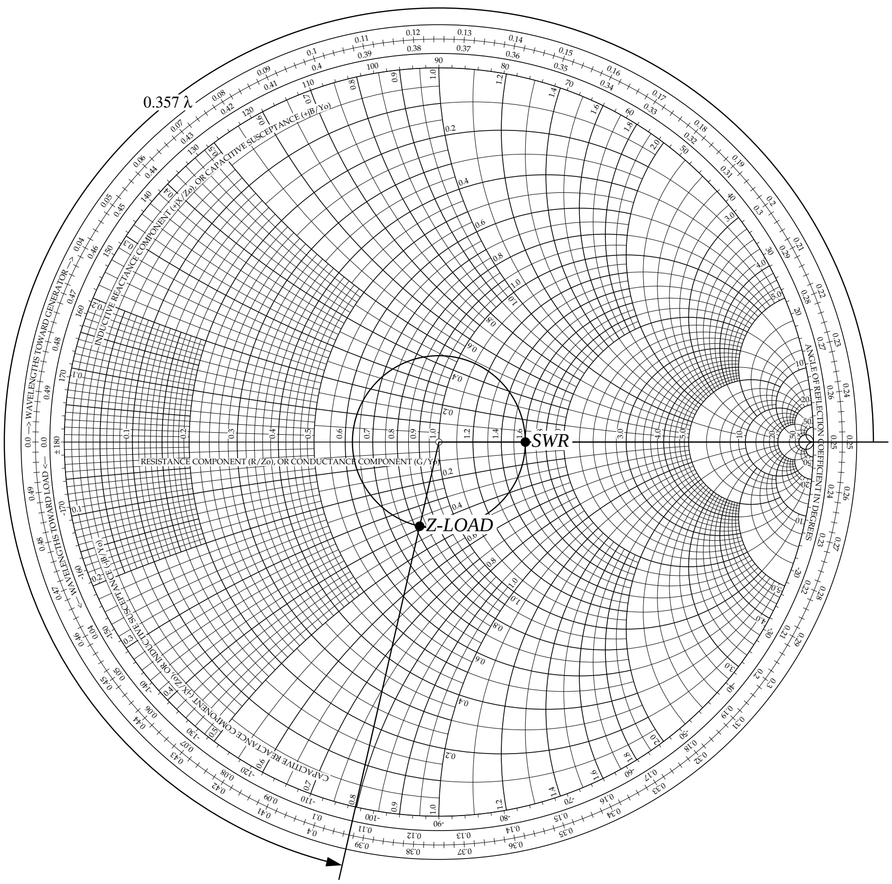


Figure P2.61: Solution of Problem 2.61.

Solution: Refer to Fig. P2.61. The point *SWR* denotes the fact that $S = 1.6$. This point is also the location of a voltage maximum. From the knowledge of the locations of adjacent maxima we can determine that $\lambda = 2(24 \text{ cm} - 10 \text{ cm}) = 28$ cm. Therefore, the load is $\frac{10 \text{ cm}}{28 \text{ cm}}\lambda = 0.357\lambda$ from the first voltage maximum, which is at 0.250λ on the WTL scale. Traveling this far on the SWR circle we find point *Z-LOAD* at $0.250\lambda + 0.357\lambda - 0.500\lambda = 0.107\lambda$ on the WTL scale, and here

$$z_{\mathrm{L}} = 0.82 - j0.39.$$

Therefore $Z_L = z_L Z_0 = (0.82 - j0.39) \times 50\ \Omega = (41.0 - j19.5)\ \Omega$.

Problem 2.66 A 200-Ω transmission line is to be matched to a computer terminal with $Z_{\mathrm{L}} = (50 - j25)$ Ω by inserting an appropriate reactance in parallel with the line. If $f = 800$ MHz and $\varepsilon_{\mathrm{r}} = 4$, determine the location nearest to the load at which inserting:

(a) A capacitor can achieve the required matching, and the value of the capacitor.

(b) An inductor can achieve the required matching, and the value of the inductor.

Solution:

(a) After entering the specified values for Z_{L} and Z_0 into Module 2.6, we have z_{L} represented by the red dot in Fig. P2.66(a), and y_{L} represented by the blue dot. By moving the cursor a distance $d = 0.093\lambda$, the blue dot arrives at the intersection point between the SWR circle and the $S = 1$ circle. At that point

$$y(d) = 1.026126 - j1.5402026.$$

To cancel the imaginary part, we need to add a reactive element whose admittance is positive, such as a capacitor. That is:

$$\begin{aligned}\omega C &= (1.54206) \times Y_0 \\ &= \frac{1.54206}{Z_0} = \frac{1.54206}{200} = 7.71 \times 10^{-3},\end{aligned}$$

which leads to

$$C = \frac{7.71 \times 10^{-3}}{2\pi \times 8 \times 10^8} = 1.53 \times 10^{-12} \text{ F}.$$

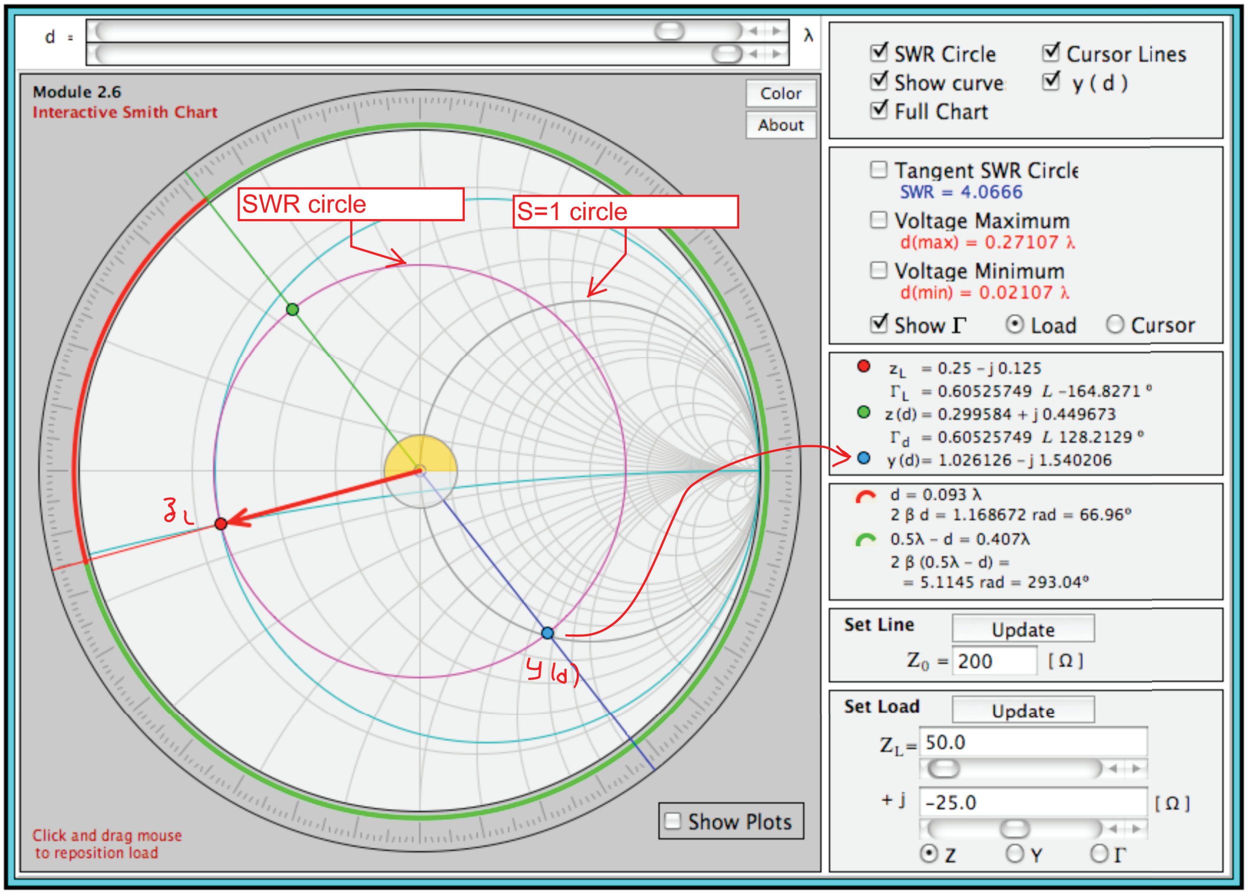


Figure P2.66(a)

(b) Repeating the procedure for the second intersection point [Fig. P2.66(b)] leads to

$$y(d) = 1.000001 + j1.520691,$$

at $d_2 = 0.447806\lambda$.

To cancel the imaginary part, we add an inductor in parallel such that

$$\frac{1}{\omega L} = \frac{1.520691}{200},$$

from which we obtain

$$L = \frac{200}{1.52 \times 2\pi \times 8 \times 10^8} = 2.618 \times 10^{-8} \text{ H}.$$

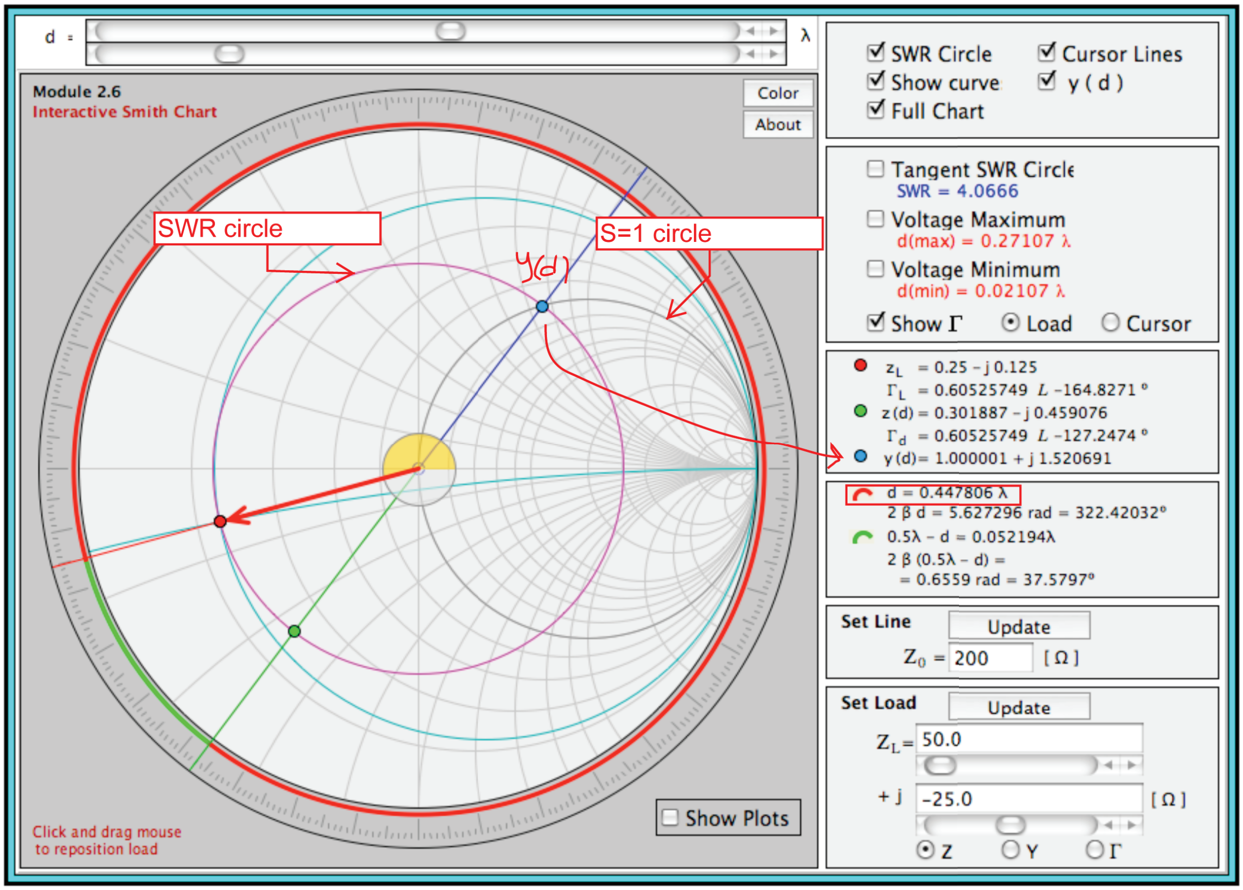


Figure P2.66(b)