CHAPTER 15

Exercises

E15.1 If one grasps the wire with the right hand and with the thumb pointing north, the fingers point west under the wire and curl around to point east above the wire.

E15.2 If one places the fingers of the right hand on the periphery of the clock pointing clockwise, the thumb points into the clock face.

E15.3 $\mathbf{f} = q\mathbf{u} \times \mathbf{B} = (-1.602 \times 10^{-19})10^5 \mathbf{u}_x \times \mathbf{u}_y = -1.602 \times 10^{-14} \mathbf{u}_z$

in which $\mathbf{u}_x$, $\mathbf{u}_y$, and $\mathbf{u}_z$ are unit vectors along the respective axes.

E15.4 $f = i\ell B \sin(\theta) = 10(1)0.5 \sin(90^\circ) = 5\ \text{N}$

E15.5 (a) $\varphi = BA = B\pi r^2 = 0.5\pi(0.05)^2 = 3.927\ \text{mWb}$

$\lambda = N\varphi = 39.27\ \text{mWb turns}$

(b) $e = \dfrac{d\lambda}{dt} = -\dfrac{39.27 \times 10^{-3}}{10^{-3}} = -39.27\ \text{V}$

More information would be needed to determine the polarity of the voltage by use of Lenz's law. Thus the minus sign of the result is not meaningful.

E15.6 $B = \dfrac{\mu I}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 20}{2\pi 10^{-2}} = 4 \times 10^{-4}\ \text{T}$

E15.7 By Ampère's law, the integral equals the sum of the currents flowing through the surface bounded by the path. The reference direction for the currents relates to the direction of integration by the right-hand rule. Thus, for each part the integral equals the sum of the currents flowing upward. Refering to Figure 15.9 in the book, we have

$$\oint_{\text{Path 1}} \mathbf{H} \cdot d\ell = 10\ \text{A} \qquad \oint_{\text{Path 2}} \mathbf{H} \cdot d\ell = 10 - 10 = 0\ \text{A} \qquad \oint_{\text{Path 3}} \mathbf{H} \cdot d\ell = -10\ \text{A}$$

E15.8 Refer to Figure 15.9 in the book. Conceptually the left-hand wire produces a field in the region surrounding it given by

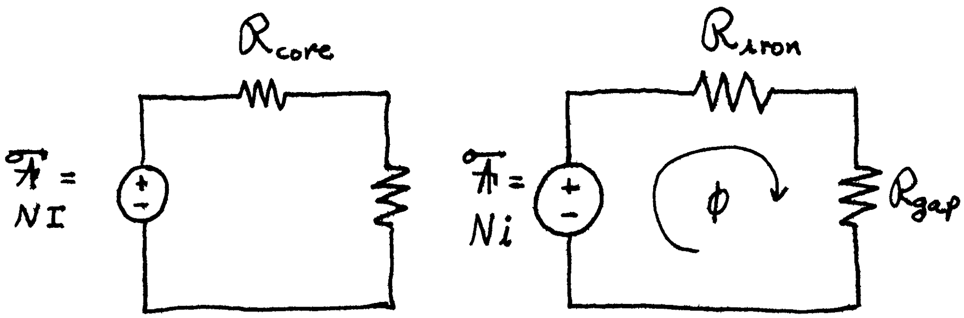
$$B = \frac{\mu I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi 10^{-1}} = 2 \times 10^{-5} \text{ T}$$

By the right-hand rule, the direction of this field is in the direction of Path 1. The field in turn produces a force on the right-hand wire given by

$$f = B\ell i = 2 \times 10^{5}(1)(10) = 2 \times 10^{-4} \text{ N}$$

By the right-hand rule, the direction of the force is such that the wires repel one another.

E15.9 The magnetic circuit is:



The reluctance of the iron is:

$$R_{\text{iron}} = \frac{\ell_{\text{iron}}}{\mu_r \mu_0 A_{\text{iron}}} = \frac{27 \times 10^{-2}}{5000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$R_{\text{iron}} = 107.4 \times 10^{3}$$

The reluctance of the air gap is:

$$R_{\text{gap}} = \frac{\ell_{\text{gap}}}{\mu_0 A_{\text{gap}}} = \frac{10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

$$R_{\text{gap}} = 8.842 \times 10^{6}$$

Then we have

$$\varphi = B_{\text{gap}} A_{\text{gap}} = 0.5 \times 9 \times 10^{-4} = 0.45 \text{ mWb}$$

$$i = \frac{(R_{\text{iron}} + R_{\text{gap}})\varphi}{N} = \frac{(107.4 \times 10^{3} + 8.842 \times 10^{6})(0.45 \times 10^{-3})}{1000} = 4.027 \text{ A}$$

E15.10 Refer to Example 15.6 in the book. Neglecting the reluctance of the iron, we have:

$$R_c = 0$$

$$R_a = \frac{\ell_{gap}}{\mu_0 A_a} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 8.842 \times 10^{6}$$

$$R_b = \frac{\ell_{gap}}{\mu_0 A_b} = \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times6.25\times10^{-4}} = 6.366\times10^{6}$$

$$\varphi_a = \frac{Ni}{R_a} = \frac{500\times2}{8.842\times10^{6}} = 113.1\ \mu\text{Wb}$$

$$B_a = \frac{\varphi_a}{A_a} = \frac{113.1\times10^{-6}}{9\times10^{-4}} = 0.1257\ \text{T}$$

compared to 0.1123 T found in the example for an error of 11.9%.

$$\varphi_b = \frac{Ni}{R_b} = \frac{500\times2}{6.366\times10^{6}} = 157.1\ \mu\text{Wb}$$

$$B_b = \frac{\varphi_b}{A_b} = \frac{157.1\times10^{-6}}{6.25\times10^{-4}} = 0.2513\ \text{T}$$

compared to 0.2192T found in the Example for an error of 14.66%.

E15.11

$$\varphi_2 = \frac{N_2 i_2}{R} = \frac{200 i_2}{10^{7}} = 2\times10^{-5}\, i_2$$

$$\lambda_{12} = N_1\varphi_2 = 200\times10^{-5}\, i_2$$

$$M = \frac{\lambda_{12}}{i_2} = 2\ \text{mH}$$

E15.12 By the right-hand rule, clockwise flux is produced by i_1 and counterclockwise flux is produced by i_2. Thus the currents produce opposing fluxes.

If a dot is placed on the top terminal of coil 1, current entering the dot produces clockwise flux. Current must enter the bottom terminal of coil 2 to produce clockwise flux. Thus the corresponding dot should be on the bottom terminal of coil 2.

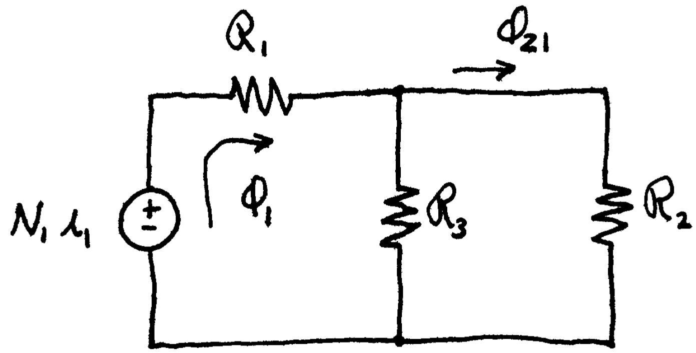
The voltages are given by Equations 15.36 and 15.37 in which we choose the minus signs because the currents produce opposing fluxes. Thus we have

$e_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$ and $e_2 = -M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

E15.13 (a) Using the right-hand rule, we find that the fluxes produced by i_1 and i_2 aid in path 1, aid in path 2, and oppose in path 3.

If a dot is placed on the top terminal of coil 1, the corresponding dot should be on the top terminal of coil 2, because then currents entering the dotted terminals produce aiding flux linkages of the coils.

(b) For $i_2 = 0$, the magnetic circuit is:



Then the reluctance seen by the source is

$$R_{total} = R_1 + \frac{1}{1/R_2 + 1/R_3} = 1.5 \times 10^6$$

$$\varphi_1 = \frac{N_1 i_1}{R_{total}} \qquad \lambda_{11} = N_1 \varphi_1 = \frac{N_1^2 i_1}{R_{total}}$$

$$L_1 = \frac{\lambda_{11}}{i_1} = \frac{N_1^2}{R_{total}} = 6.667 \text{ mH}$$

The flux φ_1 splits equally between paths 2 and 3. Thus we have $\varphi_{21} = \varphi_1 / 2$. Then

$$\lambda_{21} = N_2 \varphi_{21} = \frac{N_1 N_2 i_1}{2R_{total}} \text{ and } M = \frac{\lambda_{21}}{i_1} = \frac{N_1 N_2}{2R_{total}} = 10 \text{ mH}$$

Similarly, we find $L_2 = 60 \text{ mH}$.

(c) Because the currents produce aiding flux linkages, the mutual term carries a + sign.

E15.14 The energy lost per cycle is $W_{cycle} = (40 \text{ J/m}^3) \times (200 \times 10^{-6} \text{ m}^3) = 8 \text{ mJ}$, and the power loss is $P = W_{cycle} f = 8 \times 10^{-3} \times 60 = 0.48 \text{ W}$.

E15.15 $H_{gap} = \dfrac{NI}{\ell_{gap}} = \dfrac{1000}{0.5\times 10^{-2}} = 200\times 10^{3}$ A/m

$$B_{gap} = \mu_0 H_{gap} = 0.2513 \text{ T}$$

$$W = W_v \times \text{Volume} = \frac{B_{gap}^2}{2\mu_0}(2\times 10^{-2}\times 3\times 10^{-2}\times 0.5\times 10^{-2}) = 0.0754 \text{ J}$$

E15.16 Refer to Figure 15.26c in the book.

$$\mathbf{I}_2 = \frac{V_s'}{R_s' + Z_L} = \frac{100\angle 0^\circ}{10 + 10 + j20} = 3.536\angle -45^\circ$$

$$\mathbf{V}_2 = Z_L\mathbf{I}_2 = (10 + j20)\mathbf{I}_2 = 79.06\angle 18.43^\circ \text{ V}$$

$$P_L = I_{2\text{rms}}^2 R_L = \left(\frac{3.536}{\sqrt{2}}\right)^2 (10) = 62.51 \text{ W}$$

E15.17 $R_L' = \left(\dfrac{N_1}{N_2}\right)^2 R_L = \left(\dfrac{1}{4}\right)^2 400 = 25\ \Omega$

$$\mathbf{I}_1 = \frac{100\angle 0^\circ}{R_s + R_L'} = 1.538\angle 0^\circ$$

$$\mathbf{I}_2 = \left(\frac{N_1}{N_2}\right)\mathbf{I}_1 = 0.3846\angle 0^\circ$$

$$\mathbf{V}_2 = R_L\mathbf{I}_2 = 153.8\angle 0^\circ$$

$$P_L = R_L' I_{1\text{rms}}^2 = R_L I_{2\text{rms}}^2 = 29.60 \text{ W}$$

E15.18 For maximum power transfer, we need

$$R_s = R_L'$$

However we have

$$R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{N_1}{N_2}\right)^2 400$$

Thus we have

$$R_s = 40 = \left(\frac{N_1}{N_2}\right)^2 400$$

Solving we find

$$\frac{N_1}{N_2} = \frac{1}{\sqrt{10}}$$

Answers for Selected Problems

P15.5* r = 66.67 cm

P15.6* v_{ab} is negative

P15.11* $B = 0.6$ T

P15.15* $\phi = 0.0314$ Wb
$\lambda = 0.157$ Wb turns
$e = 157$ V

P15.16* $\mu_r = 1592$

P15.24* Magnetomotive force $F = Ni$ in a magnetic circuit is analogous to a voltage source in an electrical circuit. Reluctance R is analogous to electrical resistance. Magnetic flux φ is analogous to electrical current.

P15.25* $\ell_{\text{core}} = 500$ cm

P15.28* $\phi_b = 105.6\ \mu$Wb

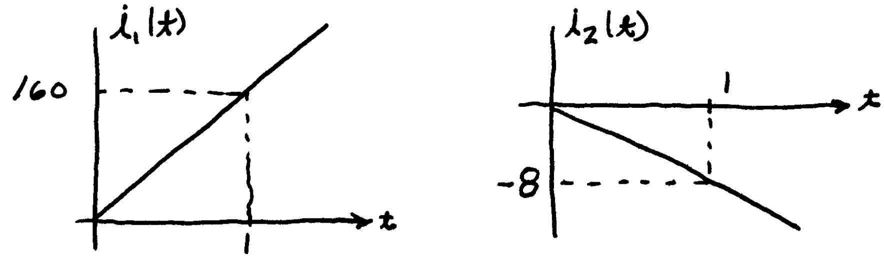
P15.34*
$$\phi = \frac{NI}{\dfrac{\ell_g}{\mu_0 \pi (d + \ell_g) L} + \dfrac{x}{\mu_0 \pi (d/2)^2}}$$

P15.37* 800 mH

P15.38* $L = 0.3183$ H
$R = 78.54 \times 10^4$
$\mu_r = 405.3$

P15.45* $e_1 = 471.3 \sin(377t)$
$e_2 = 565.5 \sin(377t)$

P15.48*



P15.52* Two causes of core loss are hysteresis and eddy currents. To minimize loss due to hysteresis, we should select a material having a thin hysteresis loop (as shown in Figure 15.21 in the book). To minimize loss due to eddy currents, we laminate the core. The laminations are insulated from one another so eddy currents cannot flow between them.

If the frequency of operation is doubled, the power loss due to hysteresis doubles and the power loss due to eddy currents is quadrupled.

P15.53* $P = 28.88$ W

P15.59* If residential power was distributed at 12 V (rather than 120 V) higher currents (by an order of magnitude) would be required to deliver the same amounts of power. This would require much larger wire sizes to avoid excessive power loss in the resistances of the conductors.

On the other hand, if residential power was distributed at 12 kV, greater safety hazards would result.

P15.62* If we tried to make the 25 Ω load look like 100 Ω by adding 75 Ω in series, 75% of the power delivered by the source would be dissipated in the 75-Ω resistance. On the other hand, when using the transformer, virtually all of the power taken from the source is delivered to the load. Thus, from the standpoint of efficiency, the transformer is a much better choice.

P15.67* (a) The dots should be placed on the top end of coil 2 and on the right-hand end of coil 3.

(b) $\mathbf{V}_2 = 50\angle 0^\circ$ $\quad\quad$ $\mathbf{I}_2 = 10\angle 0^\circ$

$\mathbf{V}_3 = 100\angle 0^\circ$ $\quad\quad$ $\mathbf{I}_3 = 10\angle 0^\circ$

(c) $N_1\mathbf{I}_1 - N_2\mathbf{I}_2 - N_3\mathbf{I}_3 = 0$

$\mathbf{I}_1 = 15\angle 0^\circ$

P15.70* The equivalent circuit of a real transformer is shown in Figure 15.28 in the book. The resistances R_1 and R_2 account for the resistance of the wires used to wind the coils of the transformer. L_1 and L_2 account for flux produced by each coil that does not link the other coil. L_m accounts for the current needed to set up the mutual flux in the core. Finally, R_c accounts for core losses due to eddy currents and hysteresis.

P15.73* $\text{Efficiency} = 83.06\%$

$\text{Percent regulation} = 0.625\%$

P15.77* The voltage across a transformer coil is approximately equal to

$$N\frac{d\varphi}{dt}$$

in which N is the number of turns and $\varphi = BA$ is the flux in the core. If B is reduced in magnitude, either N or the core area A would need to be increased to maintain the same voltage rating. In either case, more material (i.e., iron for the core or copper for the windings) is needed for the transformer.

On the other hand if the peak value of B is much higher than the saturation point, much more magnetizing current is required resulting in higher losses.

Practice Test

T15.1 (a) We have $f = i\ell B\sin(\theta) = 12(0.2)0.3\sin(90^\circ) = 0.72\,\text{N}$. ($\theta$ is the angle between the field and the wire.) The direction of the force is that of i**l** $\times$ **B** in which the direction of the vector **l** is the positive direction of the current (given as the positive x direction). Thus, the force is directed in the negative y direction.

(b) The current and the field are in the same direction so $\theta = 0$ and the force is zero. Direction does not apply for a vector of zero magnitude.

T15.2 The flux linking the coil is

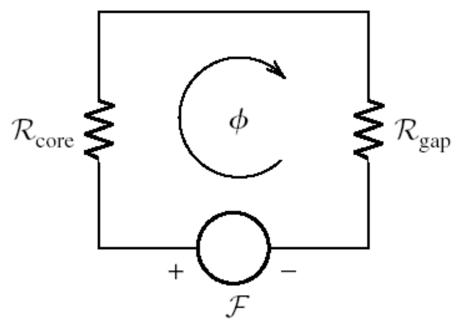
$\phi = BA = 0.7[\sin(120\pi t)](0.25)^2 = 43.75\times10^{-3}\sin(120\pi t)$ Wb

The induced voltage is

$$v = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = 10\times43.75\times10^{-3}\times120\pi\cos(120\pi t) = 164.9\cos(120\pi t)\text{ V}$$

T15.3 $e = Blu = 0.4\times0.2\times15 = 1.2$ V

T15.4 (a) The magnetic circuit is:



The permeability of the core is:

$$\mu_{core} = \mu_r\mu_0 = 1500\times4\pi\times10^{-7} = 1.885\times10^{-3}$$

The reluctance of the core is given by

$$R_{core} = \frac{\ell_{core}}{\mu_{core}A_{core}} = \frac{33.7\times10^{-2}}{1.885\times10^{-3}\times2\times10^{-2}\times3\times10^{-2}} = 298.0\times10^3$$

To account for fringing, we add the gap length to the width and depth of the gap.

$$R_{gap} = \frac{0.3\times10^{-2}}{4\pi\times10^{-7}\times2.3\times10^{-2}\times3.3\times10^{-2}} = 3.145\times10^6$$

The equivalent reluctance seen by the source is:

$$R_{eq} = R_{core} + R_{gap} = 3.443\times10^6$$

The flux is :

$$\phi = \frac{F}{R_{eq}} = \frac{4\times350}{3.443\times10^6} = 406.6\times10^{-6}\text{ Wb}$$

Finally, the flux density in the gap is approximately

$$B_{gap} = \frac{\phi}{A_{gap}} = \frac{406.6\times10^{-6}}{2.3\times10^{-2}\times3.3\times10^{-2}} = 0.5357\text{ T}$$

(b) The inductance is

$$L = \frac{N^2}{R_{eq}} = \frac{350^2}{3.443 \times 10^6} = 35.58\,\text{mH}$$

T15.5 The two mechanisms by which power is converted to heat in an iron core are hysteresis and eddy currents. To minimize loss due to hysteresis, we choose a material for which the plot of B versus H displays a thin hysteresis loop. To minimize loss due to eddy currents, we make the core from laminated sheets or from powered iron held together by an insulating binder. Hysteresis loss is proportional to frequency and eddy-current loss is proportional to the square of frequency.

T15.6 (a) With the switch open, we have $I_{2\text{rms}} = 0$, $I_{1\text{rms}} = 0$ and the voltage across R_s is zero. Therefore, we have $V_{1\text{rms}} = 120$ V and $V_{2\text{rms}} = (N_2/N_1)V_{1\text{rms}} = 1200$ V. (The dots affect the phases of the voltages but not their rms values. Thus, $V_{2\text{rms}} = -1200$ V would not be considered to be correct.)
(b) With the switch closed, the impedance seen looking into the primary is $R_L' = (N_1/N_2)^2 R_L = 10\,\Omega$. Then, using the voltage division principle, we have $V_{1rms} = 120\dfrac{R_L'}{R_s + R_L'} = 114.3$ V. Next, $V_{2\text{rms}} = (N_2/N_1)V_{1\text{rms}} = 1143$ V.
The primary current is $I_{1rms} = 120/(10.5) = 11.43$ A. The secondary current is $I_{2\text{rms}} = (N_1/N_2)I_{1\text{rms}} = 1.143$ A.

T15.7 Core loss is nearly independent of load, while loss in the coil resistances is nearly proportional to the square of the rms load current. Thus, for a transformer that is lightly loaded most of the time, core loss is more significant. Transformer B would be better from the standpoint of total energy loss and operating costs.