Q 3 (a) $$V = a A^{\frac{3}{2}}$$

$$\frac{dv}{dt} = -bA$$

In the given solution, ODE is in terms of A, now we shall use V instead of A

$$\frac{dv}{dt} = -b\left(\frac{1}{a}\right)^{\frac{2}{3}} V^{\frac{2}{3}}$$

$$= \alpha V^{\frac{2}{3}} \quad \text{where } \alpha = -b\left(\frac{1}{a}\right)^{\frac{2}{3}}$$

$$V^{-\frac{2}{3}} dV = \alpha dt$$

$$3V^{\frac{1}{3}} = \alpha t + c$$

Very often, initial condition is not given in modelling problem, so we have to set the initial condition.

We assume when $t=0$, $V=V_0$

Hence $c = 3V_0^{\frac{1}{3}}$

$$\therefore \quad 3\left(V^{\frac{1}{3}} - V_0^{\frac{1}{3}}\right) = \alpha t$$

Now find $t_1$, when $V=0$. $t_1 = \frac{3}{\alpha}\left(-V_0^{\frac{1}{3}}\right)$

negative value

(b) Suppose

$$\frac{dv}{dt} = -b A^2 \qquad \boxed{V = a A^{\frac{3}{2}}}$$

instead of $\frac{dv}{dt} = -bA$

$$\therefore \quad \frac{dv}{dt} = (-b)\left(\frac{1}{a}\right)^{\frac{4}{3}} V^{\frac{4}{3}}$$

$$= \beta V^{\frac{4}{3}} \qquad \beta = (-b)\left(\frac{1}{a}\right)^{\frac{4}{3}}$$

$$V^{-\frac{4}{3}} dv = \beta dt$$

$$(-3) V^{-\frac{1}{3}} = \beta t + c$$

when $t = 0$, $V = V_0$

$$\therefore \quad 3\left(\frac{1}{V_0^{\frac{1}{3}}} - \frac{1}{V^{\frac{1}{3}}}\right) = \beta t$$

negative

$$\therefore \quad V \to 0 \iff t = \infty$$

Raindrops always reach the ground

$$\therefore \quad \frac{dv}{dt} = -bA^2 \text{ not correct}$$

Q 4

$$r \frac{d\theta}{dr} = \tan\psi \qquad \psi \neq 90^\circ$$

i.e., $\tan\psi \neq \infty$

$$\frac{dr}{r} = \frac{1}{\tan\psi} d\theta$$

$$\ln r = \frac{\theta}{\tan\psi} + c$$

$$r = e^{c} \, e^{\frac{\theta}{\tan\psi}}$$

Again, initial condition is not given, we should set the initial condition.

Assume when $\theta = 0$, $r = R$

$$\therefore \quad e^{c} = R$$

$$\therefore \quad r = R \, e^{\frac{\theta}{\tan\psi}}$$

Case 1 ; $\psi > 90^\circ$, $\therefore \tan\psi < 0$

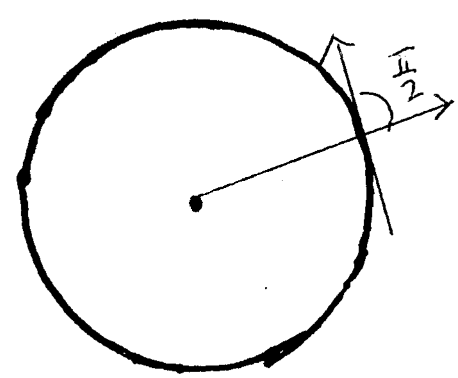
Then $\theta \nearrow \Rightarrow r \swarrow$

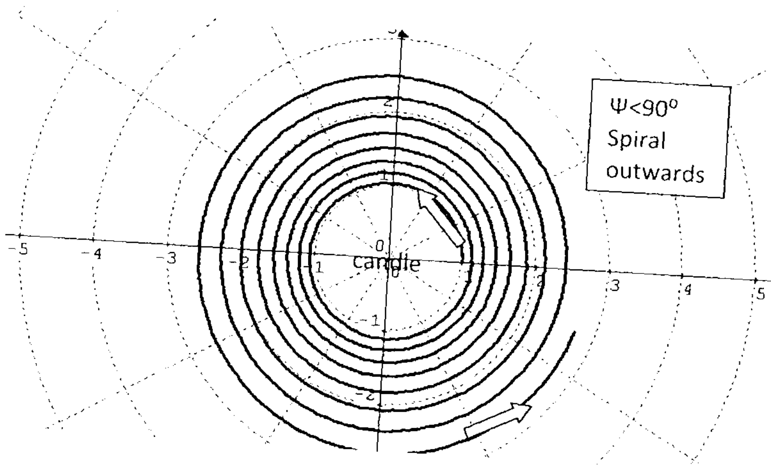
Case 2 : $\psi < 90^\circ$ $\therefore \tan\psi > 0$

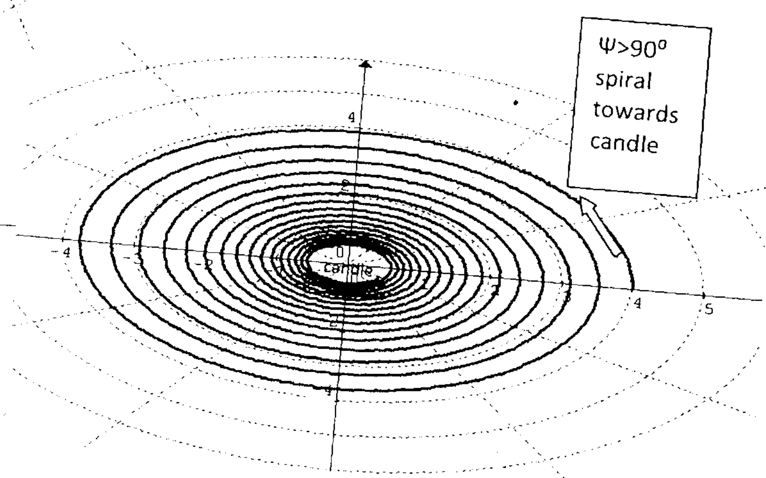
Then $\theta \nearrow \Rightarrow r \nearrow$

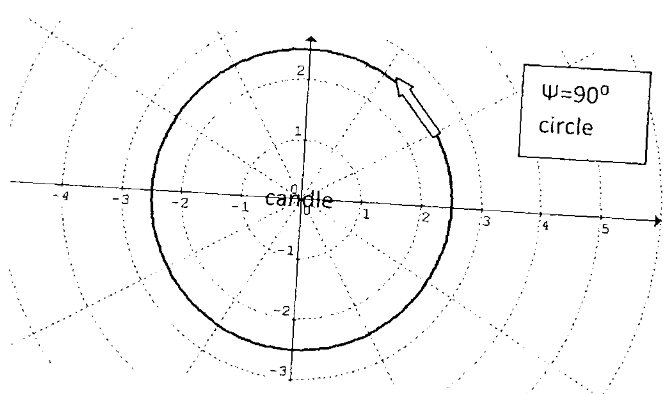
Case 3 $\psi = 90^\circ$

From geometry, we know that $r = R$ which is a circle with radius $R$, centre $(0,0)$




Q 1(d) $y(x) \equiv 0$ is also a solution.