Tutorial 4

Notations

If $X \sim B(n,p)$, then denote

$$b(x;n,p) = P(X=x) = C_n^k p^x (1-p)^{n-x}$$

and

$$B(x;n,p) = \sum_{k=0}^{x} P(X=k) = \sum_{k=0}^{x} b(x;n,p)$$

which is called the cumulative probabilities

If $X \sim Poi(\lambda)$, then denote

$$f(x;\lambda) = P(X=x) = \frac{\lambda^k e^{-\lambda}}{x!}$$

and

$$F(x;\lambda) = P(X \le x) = \sum_{k=0}^{x} P(X=k) = \sum_{k=0}^{x} f(x;\lambda)$$

If X follows geometric distribution with parameter p, then denote

$$g(x;p) = P(X=x) = p(1-p)^{x-1}$$

and

$$G(x;p) = P(X \le x) = \sum_{k=0}^{x} P(X=k) = \sum_{k=0}^{x} g(x;p)$$

4.2 An experiment consists of four tosses of a coin. Denoting the outcomes *HHTH*, *THTT*, . . . and assuming that all 16 outcomes are equally likely, find the probability distribution for the total number of heads.

4.3 Determine whether the following can be probability distributions of a random variable which can take on only the values 1, 2, 3, and 4.

(a) $f(1) = 0.16$, $f(2) = 0.28$, $f(3) = 0.28$, and $f(4) = 0.28$;

(b) $f(1) = 0.24$, $f(2) = 0.24$, $f(3) = 0.24$, and $f(4) = 0.24$;

(c) $f(1) = 0.37$, $f(2) = 0.31$, $f(3) = 0.34$, and $f(4) = -0.02$.

Tutorial 4

4.4 Check whether the following can define probability distributions, and explain your answers.

(a) $f(x) = \frac{x}{14}$ for $x = 0, 1, 2, 3, 4$

(b) $f(x) = \frac{3 - x^2}{4}$ for $x = 0, 1, 2$

(c) $f(x) = \frac{1}{5}$ for $x = 5, 6, 7, 8, 9$

(d) $f(x) = \frac{2x + 1}{50}$ for $x = 1, 2, 3, 4, 5$

4.5 Given that $f(x) = \frac{k}{2^x}$ is a probability distribution for a random variable that can take on the values $x = 0, 1, 2, 3$, and 4, find k.

4.7 Prove that $b(x; n, p) = b(n - x; n, 1 - p)$.

4.8 Prove that $B(x; n, p) = 1 - B(n - x - 1; n, 1 - p)$.

4.9 Do the assumptions for Bernoulli trials appear to hold? Explain. If the assumptions hold, identify success and the probability of interest.

(a) A TV ratings company will use their electronic equipment to check a sample of homes around the city to see which ones have a set tuned to the mayor's speech on the local channel.

(b) Among 6 nuclear power plants in a state, 2 have had serious violations in last five years. Twoplants will be selected at random, one after the other, and the outcome of interest is a serious violation in the last five years.

4.10 What conditions for the binomial distribution, if any, fail to hold in the following situations?

(a) For each of a company's eight production facilities, record whether or not there was an accident in the past week. The largest facility has three times the number of production workers as the smallest facility.

(b) For each of three shifts, the number of units produced will be compared with the long-term average of 560 and it will be determined whether or not production exceeds 560 units. The second shift will know the result for the first shift before they start working, and the third shift will start with the knowledge of how the first two shifts performed.

4.11 Which conditions for the binomial distribution, if any, fail to hold in the following situations?

(a) The number of persons having a cold at a family reunion attended by 30 persons.

(b) Among 8 projectors in the department office, 2 do not work properly but are not marked defective. Two are selected and the number that do not work properly will be recorded.

Tutorial 4

4.15 Human error is given as the reason for 75% of all accidents in a plant. Use the formula for the binomial distribution to find the probability that human error will be given as the reason for two of the next four accidents.

4.16 If the probability is 0.40 that steam will condense in a thin-walled aluminum tube at 10 atm pressure, use the formula for the binomial distribution to find the probability that, under the stated conditions, steam will condense in 4 of 12 such tubes.

4.17 During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of chips receive a thick enough coating, use Table 1 to find the probabilities that, among 15 chips:

(a) at least 12 will have thick enough coatings;

(b) at most 6 will have thick enough coatings;

(c) exactly 10 will have thick enough coatings.

4.18 The probability that the noise level of a wide-band amplifier will exceed 2 dB is 0.05. Use Table 1 to find the probabilities that among 12 such amplifiers the noise level of

(a) one will exceed 2 dB;

(b) at most two will exceed 2 dB;

(c) two or more will exceed 2 dB.

4.19 An agricultural cooperative claims that 90% of the watermelons shipped out are ripe and ready to eat. Find the probabilities that among 18 watermelons shipped out

(a) all 18 are ripe and ready to eat;

(b) at least 16 are ripe and ready to eat;

(c) at most 14 are ripe and ready to eat.

4.20 A quality-control engineer wants to check whether (in accordance with specifications) 95% of the electronic components shipped by his company are in good working condition. To this end, he randomly selects 15 from each large lot ready to be shipped and passes the lot if the selected components are all in good working condition; otherwise, each of the components in the lot is checked. Find the probabilities that the quality-control engineer will commit the error of

(a) holding a lot for further inspection even though 95% of the components are in good working condition;

(b) letting a lot pass through without further inspection even though only 90% of the components are in good working condition;

(c) letting a lot pass through without further inspection even though only 80% of the components are in good condition.

4.21 A food processor claims that at most 10% of her jars of instant coffee contain less coffee than claimed on the label. To test this claim, 16 jars of her instant coffee are randomly selected and the contents are weighed; her claim is accepted if fewer than 3 of the jars contain less coffee than claimed on the label. Find the probabilities that the food processor's claim will be accepted when the actual percentage of her jars containing less coffee than claimed on the label is

(a) 5%; (b) 10%; (c) 15%; (d) 20%.

4.50 Prove that for the Poisson distribution

$$\frac{f(x+1;\lambda)}{f(x;\lambda)} = \frac{\lambda}{x+1}$$

for $x = 0, 1, 2, \ldots$.

4.57 The number of gamma rays emitted per second by a certain radioactive substance is a random variable having the Poisson distribution with $\lambda = 5.8$. If a recording instrument becomes inoperative when there are more than 12 rays per second, what is the probability that this instrument becomes inoperative during any given second?

4.58 A consulting engineer receives, on average, 0.7 requests per week. If the number of requests follows a Poisson process, find the probability that

(a) in a given week, there will be at least 1 request;

(b) in a given 4-week period there will be at least 3 requests.

4.59 At a checkout counter customers arrive at an average of 1.5 per minute. Find the probabilities that

(a) at most 4 will arrive in any given minute;

(b) at least 3 will arrive during an interval of 2 minutes;

(c) at most 15 will arrive during an interval of 6 minutes.

4.61 In a "torture test," a light switch is turned on and off until it fails. If the probability that the switch will fail any time it is turned on or off is 0.001, what is the probability that the switch will fail *after* it has been turned on or off 1,200 times? Assume that the conditions underlying the geometric distribution are met. [*Hint*: Use the formula for the value of an infinite geometric progression.]

Tutorial 4

4.62 An automated weight monitor can detect underfilled cans of beverages with probability 0.98. What is the probability it fails to detect an underfilled can for the first time when it encounters the 10th underfilled can?

4.63 A company fabricates special-purpose robots, and records show that the probability is 0.10 that one of its new robots will require repairs during confirmation tests. What is the probability that the eighth robot it builds in a month is the first one to require repairs?

4.65 During an assembly process, parts arrive just as they are needed. However, at one station, the probability is 0.01 that a defective part will arrive in a one-hour period. Find the probability that

(a) exactly 1 defective part arrives in a 4-hour span;

(b) 1 or more defective parts arrive in a 4-hour span;

(c) exactly 1 defective part arrives in a 4-hour span and exactly 1 defective part arrives in the next 4-hour span.