NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 9 **SEMESTER II, AY 2010/2011**

1. Let $A(x) = \sum_n a_n x^n$ and $B(x) = \sum_n b_n x^n$ be formal power series such that $A(x)B(x) = 0$. If $a_0 \neq 0$, prove that $B(x) = 0$.

2. Let $A(x) = \sum_n a_n x^n$ be the generating function for the sequence a_n. If $\dfrac{A(x)}{1-x}$ is the generating function for the sequence c_n, find an expression for c_n.

3. Let $C(x) = \sum_n c_n x^n$ be the generating function for the sequence c_n, where $c_n = \sum_{i=0}^{n} i^2$. Find a close form expression for $C(x)$. Hence or otherwise, show that $c_n = \binom{n+2}{3} + \binom{n+1}{3}$.

4. Find the coefficients of x^9 and x^{14} in the expansion of $(1+x+x^2+x^3+x^4+x^5)^4$.

5. Find the # of selections of 10 letters from "C, O, M, B, I, N, A", if the letter O must appear at least once and the letter B cannot appear more than three times.

6. Prove that the number of partitions of n into parts where each part appears at least twice is equal to the number of partitions of n into parts where the size each part does not have remainders 1 or 5 when divided by 6.

Answers

4. 140; 80 5. 4543