

NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

(Semester I: 2004–2005)

ST2334 PROBABILITY AND STATISTICS

Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This is not an open book examination.
2. Candidates are allowed to use the following material during the examination.
 - ST2334 Summary Sheet.
 - An A4 sheet of handwritten notes (two sided).
 - Scientific Calculator.
 - Book of Statistical tables (provided).
3. This examination paper contains **FIVE (5)** questions and comprises **FOURTEEN (14)** printed pages.
4. Answer **ALL** the questions. The number in [] indicates the number of marks allocated for that part. The total number of marks for this paper is 90.
5. Write answers in the spaces provided. Candidates may write answers on the three blank pages, given at the end of this paper, if the space provided for any of the questions is not sufficient.
6. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Matriculation No.:

--	--	--	--	--	--	--	--	--	--

Question No.	1	2	3	4	5	Total
Score						

Question 1.

The nicotine contents, in milligrams, for a simple random sample of 8 cigarettes of a certain brand are as follows:

1.09 2.03 1.82 1.37 1.90 1.52 1.68 1.47

Assume that the nicotine content follows a normal distribution.

(a) Find the mean, median and the standard deviation of the given data. [4 marks]

(b) Find a 95% confidence interval for the true mean nicotine content in the cigarettes of the brand. [6 marks]

Question 1 continued ...

- (c) Test whether the mean nicotine content is at most 1.5 milligrams per cigarette. Clearly write down the hypotheses, specify the p-value for the test, and write down your conclusion at 1% level of significance. [7 marks]

(A total of 17 marks for Question 1.)

Question 2 is on the next page

Question 2.

It is claimed that more than 50% of the adult residents in a small town own mobile phones. In order to test this it is decided to take a random sample of 20 adults residents in the town and agree with the the claim if more than 13 individuals in the sample own mobile phones.

- (a) Write down the null and the alternative hypotheses and find α , the probability of type I error. [4 marks]

- (b) Find the power of the test if, in fact, 60% of the adults residents in the town own mobile phones. [5 marks]

(A total of 9 marks for Question 2.)

Question 3 is on the next page

Question 3.

The following give a summary of the insulin-binding capacity, measured in pmol/mg protein, for two independent sample of diabetic rats. The first sample of rats were treated with a low dose of insulin and the second sample of rats were treated with a high dose of insulin:

Sample with low dose	$n_1 = 8$	$\bar{x}_1 = 1.98$	$s_1 = 0.51$
Sample with high dose	$n_2 = 13$	$\bar{x}_2 = 1.30$	$s_2 = 0.35$

- (a) Find a 95% confidence interval for the variance of the insulin binding capacity of diabetic rates treated with low dose of insulin. [7 marks]

Question 3 continued ...

- (b) Find a 95% confidence interval for the ratio of the variances of the insulin binding capacity of the two groups of diabetic rates. [7 marks]

- (c) At 5% significance level test whether there is any difference between the variances of the insulin binding capacity of the two groups of diabetic rates. Clearly write down the hypotheses and your conclusion. [4 marks]

Question 3 continued ...

- (d) Taking into account the conclusion in (c), test whether there is any significance difference between the mean insulin binding capacities of the two groups of diabetic rates. Use 1% level of significance. Clearly write down the hypotheses and your conclusion.

[10 marks]

- (e) State the assumption(s) required for the validity of the test procedure in (d).

[2 marks]

(A total of 30 marks for Question 3.)

Question 4 is on the next page

Question 4.

The lifetime (in hours), Y , of an electronic component is a random variable with Exponential distribution. The probability density function (pdf) of the random variable is given by

$$f_Y(y) = \begin{cases} \frac{1}{50}e^{-\frac{y}{50}}, & \text{if } y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

The mean and variance of Y are given by $E(Y) = 50$, and $Var(Y) = 2500$ respectively. Let A be the event that a component lasts at least 100 hours and B be the event that a component lasts at least 110 hours.

(a) Find $P(B|A)$. [3 marks.]

(b) Derive the moment generating function of Y . [5 marks]

Question 4 continued ...

(c) Find the probability density function of Y^2 . [5 marks]

(d) Suppose Y_1, Y_2, \dots, Y_{50} denote the life time of ⁵⁰~~30~~ randomly chosen electronic components with the common life time pdf f_Y . Let $\bar{Y} = \frac{1}{50}(Y_1 + Y_2 + \dots + Y_{50})$. [6 marks]

(i) Find the $E(\bar{Y})$ and $Var(\bar{Y})$.

(ii) Using the Central Limit Theorem evaluate $P(\bar{Y} \geq 52)$.

(A total of 19 marks for Question 4.)

Question 5 is on the next page

Question 5.

Consider the random variables X and Y with the joint probability density function

$$f(x, y) = \begin{cases} e^{y-x}, & 0 < 2y < x < \infty \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Draw the region where $f(x, y) > 0$. [2 marks.]

(b) Find the marginal probability density function of X . [6 marks.]

Question 5 continued ...

(c) Find $P(Y < 2|X = 3)$. [4 marks.]

(d) Are X and Y independent? Justify your answer. [3 marks.]

(A total of 15 marks for Question 5.)

END OF QUESTIONS

Next three pages are blank pages

Blank Page 1

Blank Page 2

Blank Page 3

END OF EXAMINATION