Bell Number – Ans

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Method 1

(refer to code1.cpp) There's a very nice recurrence relation for stirling number:

$$\left\{ \begin{array}{c} n \\ k \end{array} \right\} = \left\{ \begin{array}{c} n-1 \\ k-1 \end{array} \right\} + k \left\{ \begin{array}{c} n-1 \\ k \end{array} \right\}$$

We can understand the above recurrence relation this way:
for an element e from this n elements, if e forms a set itself, then there're $\left\{ \begin{array}{c} n-1 \\ k-1 \end{array} \right\}$ ways for the result of $n-1$ elements to form $k-1$ sets; if e form a set with other elements, then there're k sets for the element e to be in, each of the arrangement has $\left\{ \begin{array}{c} n-1 \\ k \end{array} \right\}$ ways to form sets.

Method 2

(refer to code2.cpp) Another way of solving it, by realizing the recurrence relation:

$$B(n+1) = \sum_{k=0}^{n} \binom{n}{k} B(k)$$

We can understand the above recurrence relation this way:
for the $(n+1)th$ element e, it can be in the same block as the other $n-k$ elements, where $k \in [0, n]$. Choosing out this $(n-k)$ elements we have $\binom{n}{n-k} = \binom{n}{k}$ ways
Then we simply need to use the code to simulate