Problem 4.48 With reference to Fig. 4-19, find $\mathbf{E}_1$ if $\mathbf{E}_2 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}2$ (V/m), $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 18\varepsilon_0$, and the boundary has a surface charge density $\rho_s = 3.54 \times 10^{-11}$ (C/m^2). What angle does $\mathbf{E}_2$ make with the z-axis?

Solution: We know that $\mathbf{E}_{1t} = \mathbf{E}_{2t}$ for any 2 media. Hence, $\mathbf{E}_{1t} = \mathbf{E}_{2t} = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2$. Also, $(\mathbf{D}_1 - \mathbf{D}_2) \cdot \hat{\mathbf{n}} = \rho_s$ (from Table 4.3). Hence, $\varepsilon_1(\mathbf{E}_1 \cdot \hat{\mathbf{n}}) - \varepsilon_2(\mathbf{E}_2 \cdot \hat{\mathbf{n}}) = \rho_s$, which gives

$$E_{1z} = \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} = \frac{3.54 \times 10^{-11}}{2\varepsilon_0} + \frac{18(2)}{2} = \frac{3.54 \times 10^{-11}}{2 \times 8.85 \times 10^{-12}} + 18 = 20 \quad \text{(V/m)}.$$

Hence, $\mathbf{E}_1 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}20$ (V/m). Finding the angle $\mathbf{E}_2$ makes with the z-axis:

$$\mathbf{E}_2 \cdot \hat{\mathbf{z}} = |\mathbf{E}_2| \cos\theta, \qquad 2 = \sqrt{9+4+4}\cos\theta, \qquad \theta = \cos^{-1}\left(\frac{2}{\sqrt{17}}\right) = 61^\circ.$$

Problem 4.52 Determine the force of attraction in a parallel-plate capacitor with $A = 5$ cm^2, $d = 2$ cm, and $\varepsilon_r = 4$ if the voltage across it is 50 V.

Solution: From Eq. (4.131),

$$\mathbf{F} = -\hat{\mathbf{z}}\,\frac{\varepsilon A|\mathbf{E}|^2}{2} = -\hat{\mathbf{z}}\,2\varepsilon_0(5\times 10^{-4})\left(\frac{50}{0.02}\right)^2 = -\hat{\mathbf{z}}\,55.3\times 10^{-9} \quad \text{(N)}.$$

Problem 4.54 An electron with charge $Q_e = -1.6 \times 10^{-19}$ C and mass $m_e = 9.1 \times 10^{-31}$ kg is injected at a point adjacent to the negatively charged plate in the region between the plates of an air-filled parallel-plate capacitor with separation of 1 cm and rectangular plates each 10 cm^2 in area (Fig. P4.54). If the voltage across the capacitor is 10 V, find the following:

(a) The force acting on the electron.

(b) The acceleration of the electron.

(c) The time it takes the electron to reach the positively charged plate, assuming that it starts from rest.

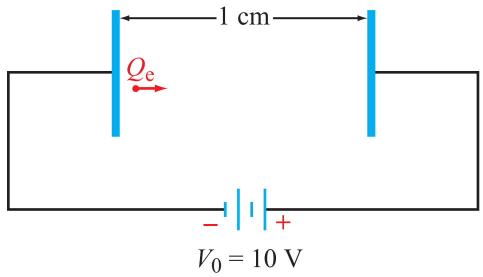


Figure P4.54: Electron between charged plates of Problem 4.54.

Solution:

(a) The electric force acting on a charge Q_e is given by Eq. (4.14) and the electric field in a capacitor is given by Eq. (4.112). Combining these two relations, we have

$$F = Q_e E = Q_e \frac{V}{d} = -1.6 \times 10^{-19} \frac{10}{0.01} = -1.6 \times 10^{-16} \quad \text{(N)}.$$

The force is directed from the negatively charged plate towards the positively charged plate.

(b)

$$a = \frac{F}{m} = \frac{1.6 \times 10^{-16}}{9.1 \times 10^{-31}} = 1.76 \times 10^{14} \quad (\text{m/s}^2).$$

(c) The electron does not get fast enough at the end of its short trip for relativity to manifest itself; classical mechanics is adequate to find the transit time. From classical mechanics, $d = d_0 + u_0 t + \frac{1}{2}at^2$, where in the present case the start position is $d_0 = 0$, the total distance traveled is $d = 1$ cm, the initial velocity $u_0 = 0$, and the acceleration is given by part (b). Solving for the time t,

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.01}{1.76 \times 10^{14}}} = 10.7 \times 10^{-9} \text{ s} = 10.7 \quad \text{(ns)}.$$

Problem 4.56 Figure P4.56(a) depicts a capacitor consisting of two parallel, conducting plates separated by a distance d. The space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2. The objective of this problem is to show that the capacitance C of the configuration shown in Fig. P4.56(a) is equivalent to two capacitances in parallel, as illustrated in Fig. P4.56(b), with

$$C = C_1 + C_2 \tag{19}$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d} \tag{20}$$

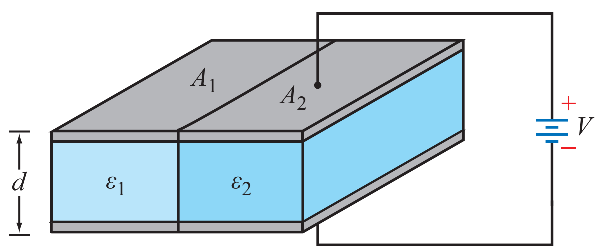
$$C_2 = \frac{\varepsilon_2 A_2}{d} \tag{21}$$

To this end, proceed as follows:

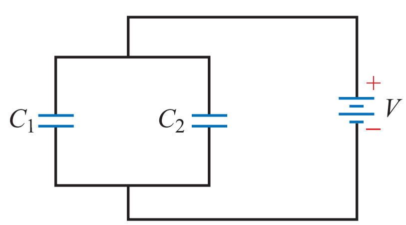
(a) Find the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ in the two dielectric layers.

(b) Calculate the energy stored in each section and use the result to calculate C_1 and C_2.

(c) Use the total energy stored in the capacitor to obtain an expression for C. Show that (19) is indeed a valid result.



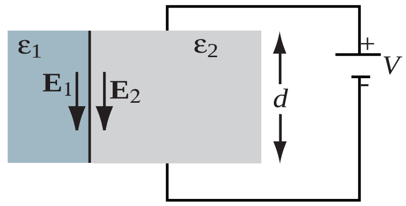
(a)



(b)

Figure P4.56: (a) Capacitor with parallel dielectric section, and (b) equivalent circuit.

Solution:



(c)

Figure P4.56: (c) Electric field inside of capacitor.

(a) Within each dielectric section, **E** will point from the plate with positive voltage to the plate with negative voltage, as shown in Fig. P4-56(c). From $V = Ed$,

$$E_1 = E_2 = \frac{V}{d}.$$

(b)

$$W_{e_1} = \frac{1}{2}\varepsilon_1 E_1^2 \cdot \mathscr{V} = \frac{1}{2}\varepsilon_1 \frac{V^2}{d^2} \cdot A_1 d = \frac{1}{2}\varepsilon_1 V^2 \frac{A_1}{d}.$$

But, from Eq. (4.121),

$$W_{e_1} = \frac{1}{2} C_1 V^2.$$

Hence $C_1 = \varepsilon_1 \dfrac{A_1}{d}$. Similarly, $C_2 = \varepsilon_2 \dfrac{A_2}{d}$.

(c) Total energy is

$$W_e = W_{e_1} + W_{e_2} = \frac{1}{2}\frac{V^2}{d}(\varepsilon_1 A_1 + \varepsilon_2 A_2) = \frac{1}{2} C V^2.$$

Hence,

$$C = \frac{\varepsilon_1 A_1}{d} + \frac{\varepsilon_2 A_2}{d} = C_1 + C_2.$$

Problem 5.2 When a particle with charge q and mass m is introduced into a medium with a uniform field $\mathbf{B}$ such that the initial velocity of the particle $\mathbf{u}$ is perpendicular to $\mathbf{B}$(Fig. P5.2), the magnetic force exerted on the particle causes it to move in a circle of radius a. By equating $\mathbf{F}_m$ to the centripetal force on the particle, determine a in terms of q, m, u, and $\mathbf{B}$.

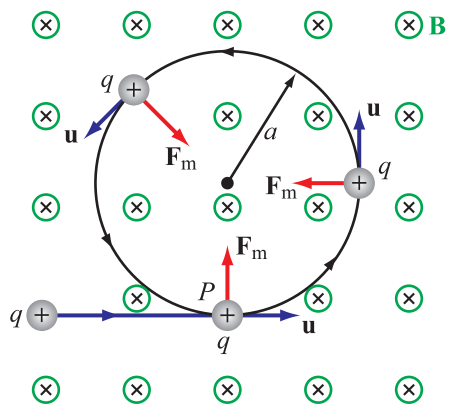


Figure P5.2: Particle of charge q projected with velocity $\mathbf{u}$ into a medium with a uniform field $\mathbf{B}$ perpendicular to $\mathbf{u}$ (Problem 5.2).

Solution: The centripetal force acting on the particle is given by $F_c = mu^2/a$. Equating F_c to F_m given by Eq. (5.4), we have $mu^2/a = quB\sin\theta$. Since the magnetic field is perpendicular to the particle velocity, $\sin\theta = 1$. Hence, $a = mu/qB$.