

Remarks on T8

Q1

$$\frac{d^4 y}{dx^4} = \frac{-W(x)}{EI}$$

where $W(x)$ is weight (load) **per unit length** of the beam downward direction

By given, weight=0, except at the point $x=A$.

NOTE THAT weight=0, NOT weight per unit length=0

Weight over very small interval $[A-\tau, A+\tau]$ is $W(A)2\tau$

Weight at the point $A = \lim_{\tau \rightarrow 0} W(A)2\tau$

By given, weight at the point $A = Mg$

Hence

$$\lim_{\tau \rightarrow 0} W(A)2\tau = Mg$$

We can use Dirac delta function $\delta(x-A)$ to represent $W(x)$ (=weight **per unit length**),
where $W(x)=0$ when $x \neq A$, and **weight=1 unit at $x=A$**

Therefore, $Mg \delta(x-A)$ represent

$W(x)$ (=weight **per unit length**)
where $W(x)=0$ when $x \neq A$, and **weight= Mg at $x=A$**

Hence we have

$$\frac{d^4 y}{dx^4} = \frac{-Mg \delta(x - A)}{EA}$$

We need to use Laplace transform to solve the above ODE

Note that

$$L\left(\frac{d^4 y}{dx^4}\right) = s^4 L(y) - s^3 y(0) - s^2 y'(0) - sy''(0) - y'''(0)$$

We know that

$$y(0) = 0, y'(0) = 0, y''(L) = 0, y'''(L) = 0$$

However we don't need $y''(L), y'''(L)$

We need $y''(0), y'''(0)$ which are given in Q1

Q2

$$V(t) = RI(t) + L \frac{dI(t)}{dt} + \frac{1}{C} \int_0^t I(x) dx$$

“switch the gadget on and off at $t=0$,
thus firing a short burst of voltage “ means

$$V(t) = A\delta(t)$$

where A is unknown