Solutions to Tutorial 10

7.54 This is the large sample setting.

(a) The alternative hypothesis is the result we intend to establish.

1. *Null hypothesis* $H_0 : \mu = 2.0$

   Alternative hypothesis $H_1 : \mu < 2.0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* Using a normal approximation for the distribution of the sample mean, we reject the null hypothesis when

$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \; < \; -z_\alpha.$$

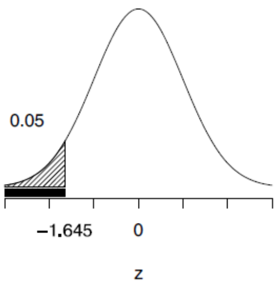
   Since $\alpha = .05$ and $z_{.05} = 1.645$, the null hypothesis must be rejected if $Z < -1.645$.

4. *Calculations:* $\mu_0 = 2.0$, $\bar{x} = 1.865$, $s = 1.250$ and $n = 52$ so

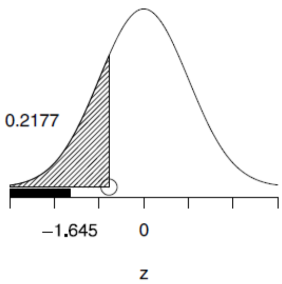
$$Z = \frac{1.865 - 2.0}{1.250/\sqrt{52}} = -0.78$$

5. *Decision:* Because $-0.78 > -1.645$, the null hypothesis that $\mu = 2.0$ is not rejected at level .05. The P-value $= P[Z < -0.78] = .218$ confirms that the evidence against the null hypothesis, $\mu = 2.0$, is absent.

(b) We could have failed to reject the null hypothesis when the mean labor time used is less than 2.0 hours.



(a) Rejection Region



(b) P-value for Exercise 7.54

7.55 This is the large sample setting.

(a) The alternative hypothesis is the result we intend to establish.

1. *Null hypothesis* $H_0 : \mu = 3.6$

   Alternative hypothesis $H_1 : \mu < 3.6$

2. *Level of significance:* $\alpha = 0.025$.

3. *Criterion:* Using a normal approximation for the distribution of the sample mean, we reject the null hypothesis when

$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \quad < \quad - z_\alpha.$$

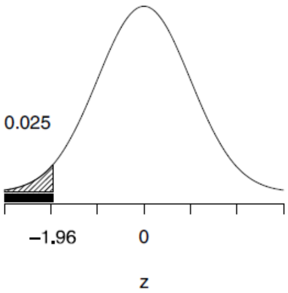
Since $z_{.025} = 1.96$, the null hypothesis must be rejected if $Z < -1.96$.

4. *Calculations:* The observed $\bar{x} = 2.467$, $s = 3.057$ and $n = 45$. Since $\mu_0 = 3.6$,

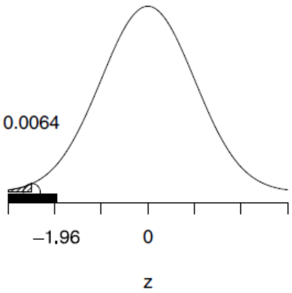
$$Z = \frac{2.467 - 3.6}{3.057/\sqrt{45}} = -2.49$$

5. *Decision:* Because $-2.49 < -1.96$, we reject the null hypothesis that $\mu = 3.6$ at level .025. The P-value $= P[Z < -2.49] = .006$ confirms that the evidence against the null hypothesis, $\mu = 3.6$, is somewhat strong.

(b) We could have rejected the null hypothesis that the mean number of unremovable defects is 3.6 and falsely concluded that it is less.



(a) Rejection Region



(b) P-value for Exercise 7.55

7.57 This is the small sample setting.

(a) The alternative hypothesis is the result we intend to establish concerning the key performance indicator.

1. *Null hypothesis* $H_0 : \mu = 107$
   Alternative hypothesis $H_1 : \mu \neq 107$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* The population is normal and the sample size is small so we use the test statistic

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

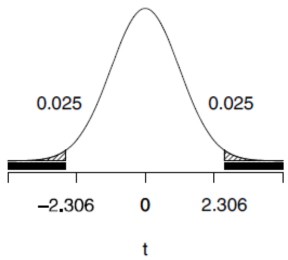
Since $t_{\alpha/2} = t_{.025} = 2.306$, for $n - 1 = 9 - 1 = 8$ degrees of freedom, the null hypothesis must be rejected if $t < -2.306$ or $t > 2.306$.

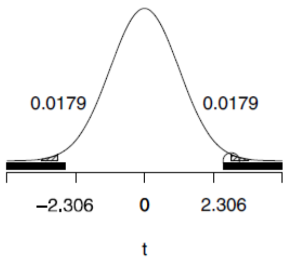
4. *Calculations:* $\mu_0 = 107$ and we find that $\bar{x} = 114.0$ and $s = 8.34$ so

$$t = \frac{114.0 - 107}{8.34/\sqrt{9}} = 2.52$$

5. *Decision:* Because $2.52 > 2.306$, we reject the null hypothesis that $\mu = 107$ at level .025. The P-value $= P[t < -2.52] + P[t > 2.52] = .036$ confirms that the evidence against the null hypothesis, $\mu = 107$, is somewhat strong.

(b) We could have rejected the null hypothesis that the mean key performance indicator is 107 and falsely concluded that it is different from 107.





7.59 1. *Null hypothesis* $H_0 : \mu = 1.3$
Alternative hypothesis $H_1 : \mu > 1.3$

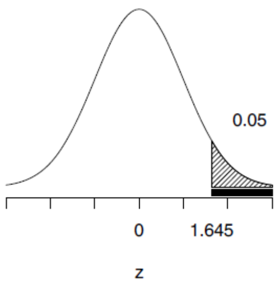
2. *Level of significance:* $\alpha = 0.05$.
3. *Criterion:* We Use the large sample normal approximation for the distribution of the sample mean and reject the null hypothesis when

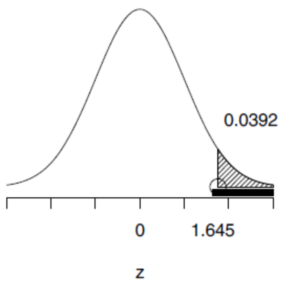
$$Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} > z_\alpha.$$

Since $\alpha = .05$ and $z_{.05} = 1.645$, the null hypothesis must be rejected if $Z > 1.645$.
4. *Calculations:* $\mu_0 = 1.3$, $\bar{x} = 1.4707$, $s = 0.5235$, and $n = 35$

$$Z = \frac{1.4707 - 1.3}{0.5235/\sqrt{29}} = 1.76$$

5. *Decision:* Because $1.76 > 1.645$, the null hypothesis that $\mu = 1.3$ is rejected. at level .05. The P-value $= P[Z > 1.76] = .039$ as shown in the figure. The evidence against the null hypothesis, $\mu = 1.3$, is moderately strong.





Solutions to Tutorial 10

7.60 1. *Null hypothesis* $H_0 : \mu = 1000$

Alternative hypothesis $H_1 : \mu > 1000$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* Since the sample is large, we will use the normal approximation to the distribution of the mean substituting S for σ. We reject the null hypothesis when

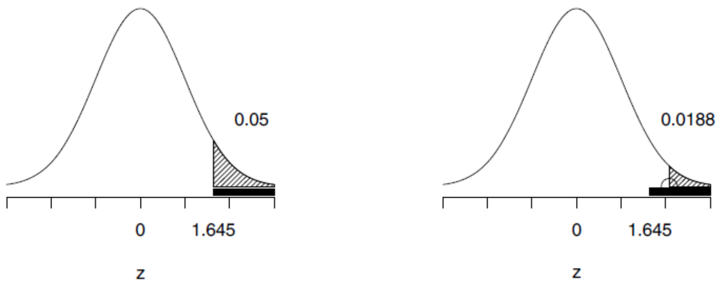
$$Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} > z_\alpha.$$

Since $\alpha = .05$ and $z_{.05} = 1.645$, the null hypothesis must be rejected if $Z > 1.645$.

4. *Calculations:* $\mu_0 = 1000$, $\bar{x} = 1038$, $s = 146$, and $n = 64$

$$Z = \frac{1038 - 1000}{146/\sqrt{64}} = 2.08$$

5. *Decision:* Because $2.08 > 1.645$, the null hypothesis that $\mu = 1000$ is rejected. at level .05. The P-value $= P[Z > 2.08] = .019$ as shown in the figure. The evidence against the null hypothesis, $\mu = 1000$, is quite strong.



(a) Rejection region (b) P-value for Exercise 7.60

7.61 1. *Null hypothesis* $H_0 : \mu = 30.0$

Alternative hypothesis $H_1 : \mu \neq 30.0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* Since the sample is small, we can not use the normal approximation. If it is reasonable to assume that the data are from a distribution that is nearly normal, we can use the t statistic

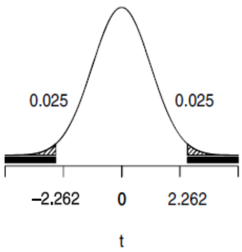
$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

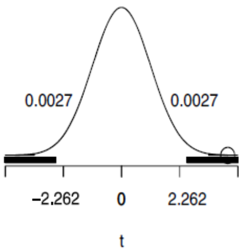
Since the alternative hypothesis is two-sided, the critical region is defined by $t < -t_{.025}$ or $t > t_{.025}$ where $t_{.025}$ with 9 degrees of freedom is 2.262.

4. *Calculations:* $\mu_0 = 30.0,\ \bar{x} = 30.91,\ s = .778$, and $n = 10$ so

$$t = \frac{30.91 - 30.0}{.788/\sqrt{10}} = 3.652$$

5. *Decision:* Because $3.652 > 2.262$, we reject the null hypothesis at the .05 level of significance and conclude that the mean thickness of paper is different from 30.0mm. A computer calculation gives the The P-value $= P[t < -3.652] + P[t > 3.652] = .0054$ in the figure.





(a) Rejection region (b) P-value for Exercise 7.61

7.63 1. *Null hypothesis* $H_0 : \mu = 14.0$
Alternative hypothesis $H_1 : \mu \neq 14.0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* Assuming the population is normal, we can use the t statistic.

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

Since the alternative hypothesis is two-sided, the critical region is defined by $t < -t_{.025}$ or $t > t_{.025}$ where $t_{.025}$ with 4 degrees of freedom is 2.776.

4. *Calculations:* In this case, $\mu_0 = 14.0$, $n = 5$, $\bar{x} = 14.4$ and $s = .158$ so

$$t = \frac{14.4 - 14.0}{.158/\sqrt{5}} = 5.66.$$

5. *Decision:* Because $5.66 > 2.776$, we reject the null hypothesis in favor of the alternative hypothesis $\mu \neq 14.0$ at the .05 level of significance. From the $t-$table, the P-value is less than .005. A computer program gives the P-value 0.0048 in the figure.

Solutions to Tutorial 10

7.66 (a) The value 320 nm is outside of the 95 % confidence interval. Consquently, at level $\alpha = 0.05$, we reject the null hypothesis $H_0 : \mu = 320$ in favor of the two-sided alternative.

(b) The value 310 nm lies inside the 95 % confidence interval. Consquently, at level $\alpha = 0.05$, we fail to reject the null hypothesis $H_0 : \mu = 320$.

(c) If $\alpha = 0.02$ the confidence interval would be centered at the same value but would be even wider. Therefore the value 310 nm also lies inside the 98 % confidence interval. At level $\alpha = 0.02$, we fail to reject the null hypothesis $H_0 : \mu = 320$.