CHAPTER 17

Exercises

E17.1 From Equation 17.5, we have

$$B_{gap} = Ki_a(t)\cos(\theta) + Ki_b(t)\cos(\theta - 120^\circ) + Ki_c(t)\cos(\theta - 240^\circ)$$

Using the expressions given in the Exercise statement for the currents, we have

$$B_{gap} = KI_m\cos(\omega t)\cos(\theta) + KI_m\cos(\omega t - 240^\circ)\cos(\theta - 120^\circ)$$
$$+ KI_m\cos(\omega t - 120^\circ)\cos(\theta - 240^\circ)$$

Then using the identity for the products of cosines, we obtain

$$B_{gap} = \tfrac{1}{2}KI_m[\cos(\omega t - \theta) + \cos(\omega t + \theta) + \cos(\omega t - \theta - 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ) + \cos(\omega t - \theta + 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ)]$$

However we can write

$$\cos(\omega t - \theta) + \cos(\omega t - \theta - 120^\circ) + \cos(\omega t - \theta + 120^\circ) = 0$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

Thus we have

$$B_{gap} = \tfrac{3}{2}KI_m\cos(\omega t + \theta)$$

which can be recognized as flux pattern that rotates clockwise.

E17.2 At 60 Hz, synchronous speed for a four-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{4} = 1800 \text{ rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1750}{1800} = 2.778\%$$

The frequency of the rotor currents is the slip frequency. From Equation

17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

$$f_{\text{slip}} = sf = 0.02778 \times 60 = 1.667 \text{ Hz}$$

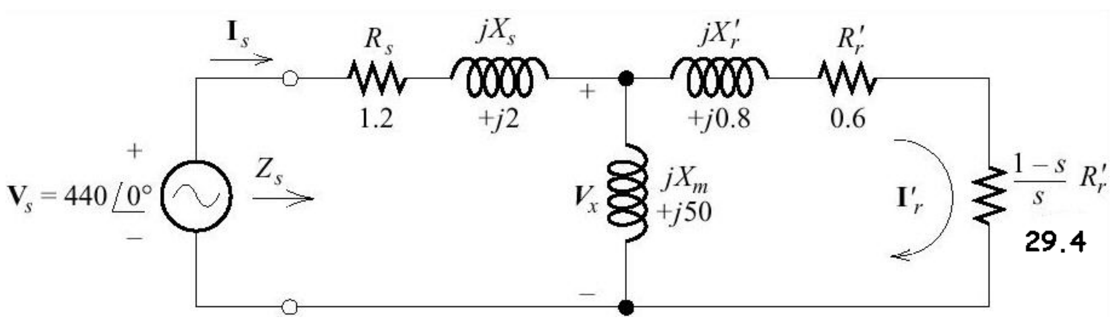
In the normal range of operation, slip is approximately proportional to output power and torque. Thus at half power, we estimate that $s = 2.778/2 = 1.389\%$. This corresponds to a speed of 1775 rpm.

E17.3 Following the solution to Example 17.1, we have:

$$n_s = 1800 \text{ rpm}$$

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1764}{1800} = 0.02$$

The per phase equivalent circuit is:



$$Z_s = 1.2 + j2 + \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$
$$= 22.75 + j15.51$$
$$= 27.53\angle 34.29^\circ$$

$$\text{power factor} = \cos(34.29^\circ) = 82.62\% \text{ lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{27.53\angle 34.29^\circ} = 15.98\angle -34.29^\circ \text{ A rms}$$

For a delta-connected machine, the magnitude of the line current is

$$I_{line} = I_s\sqrt{3} = 15.98\sqrt{3} = 27.68 \text{ A rms}$$

and the input power is

$$P_{\text{in}} = 3I_s V_s \cos\theta = 17.43 \text{ kW}$$

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}_r'$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$
$$= 406.2 - j15.6$$
$$= 406.4\angle -2.2^\circ \quad \text{V rms}$$

$$\mathbf{I}_r' = \frac{\mathbf{V}_x}{j0.8 + 0.6 + 29.4}$$
$$= 13.54\angle -3.727^\circ \quad \text{A rms}$$

The copper losses in the stator and rotor are:

$$P_s = 3R_s I_s^2$$
$$= 3(1.2)(15.98)^2$$
$$= 919.3 \text{ W}$$

and

$$P_r = 3R_r'(I_r')^2$$
$$= 3(0.6)(13.54)^2$$
$$= 330.0 \text{ W}$$

Finally, the developed power is:

$$P_{\text{dev}} = 3 \times \frac{1-s}{s} R_r'(I_r')^2$$
$$= 3(29.4)(13.54)^2$$
$$= 16.17 \text{ kW}$$
$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 15.27 \text{ kW}$$

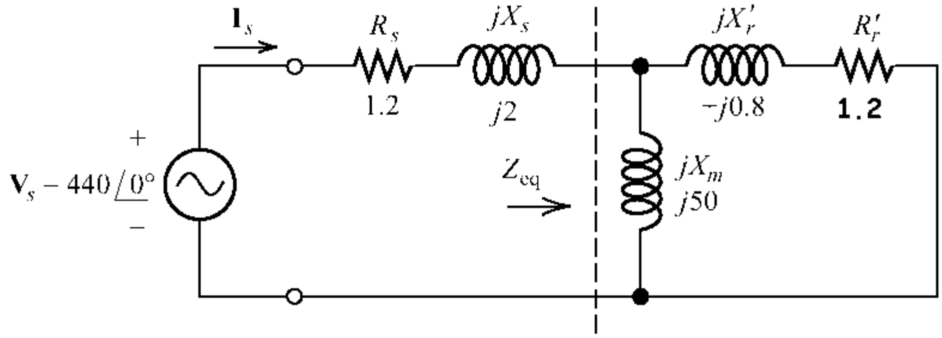
The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 82.66 \text{ newton meters}$$

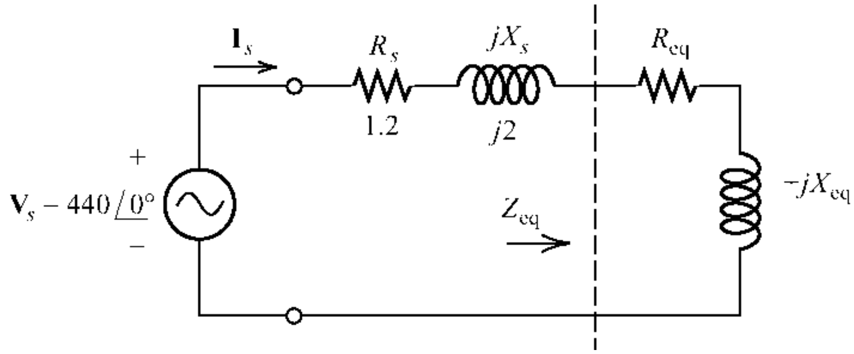
The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.61\%$$

E17.4 The equivalent circuit is:



(a)



$$Z_{eq} = R_{eq} + jX_{eq} = \frac{j50(1.2 + j0.8)}{j50 + 1.2 + j0.8} = 1.162 + j0.8148$$

The impedance seen by the source is:

$$\begin{aligned} Z_s &= 1.2 + j2 + Z_{eq} \\ &= 1.2 + j2 + 1.162 + j0.8148 \\ &= 3.675\angle 50.00^\circ \end{aligned}$$

Thus, the starting phase current is

$$\mathbf{I}_{s,\,\text{starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{3.675\angle 50.00^\circ}$$

$$\mathbf{I}_{s,\,\text{starting}} = 119.7\angle -50.00^\circ \text{ A rms}$$

and for a delta connection, the line current is

$$I_{line,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 119.7\sqrt{3} = 207.3 \text{ A rms}$$

The power crossing the air gap is (three times) the power delivered to the right of the dashed line in the equivalent circuit shown earlier.

$$P_{ag} = 3R_{eq}\left(I_{s,\,\text{starting}}\right)^2 = 49.95 \text{ kW}$$

Finally, the starting torque is found using Equation 17.34.

$$T_{\text{dev, starting}} = \frac{P_{ag}}{\omega_s}$$
$$= \frac{49950}{2\pi 60/2}$$
$$= 265.0 \text{ newton meters}$$

E17.5 This exercise is similar to part (c) of Example 17.4. Thus, we have

$$\frac{\sin\delta_3}{\sin\delta_1} = \frac{P_3}{P_1}$$
$$\frac{\sin\delta_3}{\sin 4.168^\circ} = \frac{200}{50}$$

which yields the new torque angle $\delta_3 = 16.90^\circ$. E_r remains constant in magnitude, thus we have

$$\mathbf{E}_{r3} = 498.9\angle -16.90^\circ \quad \text{V rms}$$
$$\mathbf{I}_{a3} = \frac{\mathbf{V}_a - \mathbf{E}_{r3}}{jX_s} = \frac{480 - 498.9\angle -16.90^\circ}{j1.4} = 103.6\angle -1.045^\circ \quad \text{A rms}$$

The power factor is $\cos(-1.045^\circ) = 99.98\% \text{ lagging}$.

E17.6 We follow the approach of Example 17.5. Thus as in the example, we have

$$I_{a1} = \frac{P_{dev}}{3V_a \cos\theta_1} = \frac{74600}{3(240)0.85} = 121.9 \text{ A}$$
$$\theta_1 = \cos^{-1}(0.85) = 31.79^\circ$$
$$\mathbf{I}_{a1} = 121.9\angle -31.79^\circ \quad \text{A rms}$$
$$\mathbf{E}_{r1} = \mathbf{V}_a - jX_s\mathbf{I}_{a1} = 416.2\angle -20.39^\circ \quad \text{V rms}$$

The phasor diagram is shown in Figure 17.24a

For 90% leading power factor, the power angle is $\theta_3 = \cos^{-1}(0.9) = 25.84^\circ$. The new value of the current magnitude is

$$I_{a3} = \frac{P_{dev}}{3V_{a3}\cos(\theta_3)} = 115.1 \quad \text{A rms}$$

and the phasor current is

$$\mathbf{I}_{a3} = 115.1\angle 25.84^\circ \quad \text{A rms}$$

Thus we have

$$\mathbf{E}_{r3} = \mathbf{V}_a - jX_s\mathbf{I}_{a3} = 569.0\angle -14.77^\circ \quad \text{V rms}$$

The magnitude of E_r is proportional to the field current, so we have:

$$I_{f3} = I_{f1}\frac{E_{r3}}{E_{r1}} = 10\times\frac{569.0}{416.2} = 13.67 \text{ A dc}$$

E17.7 The phasor diagram for $\delta = 90^\circ$ is shown in Figure 17.27. The developed power is given by

$$P_{\max} = 3V_a I_a \cos(\theta)$$

However from the phasor diagram, we see that

$$\cos(\theta) = \frac{E_r}{X_s I_a}$$

Substituting, we have

$$P_{\max} = \frac{3V_a E_r}{X_s}$$

The torque is

$$T_{\max} = \frac{P_{\max}}{\omega_m} = \frac{3V_a E_r}{\omega_m X_s}$$

Answers for Selected Problems

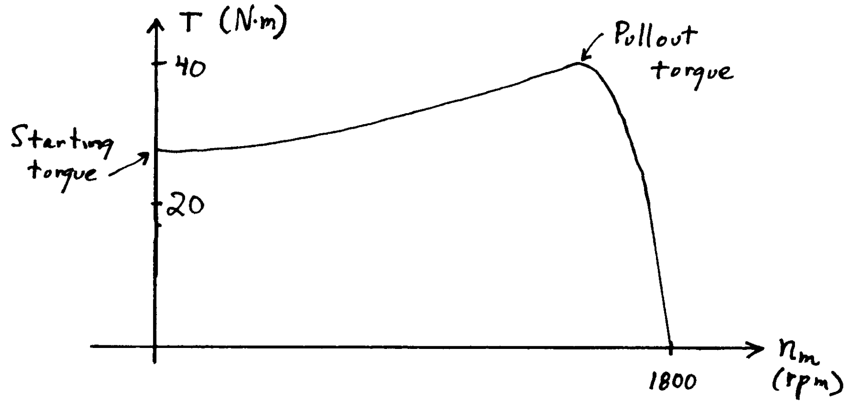
P17.1* $P = 8$ pole motor
$s = 5.55\%$

P17.7* $f = 86.8$ Hz
$I = 5.298$ A

P17.10* As frequency is reduced, the reactances X_s, X_m, and X_r' of the machine become smaller. (Recall that $X = \omega L$.) Thus the applied voltage must be reduced to keep the currents from becoming too large, resulting in magnetic saturation and overheating.

P17.13* $B_{\text{four-pole}} = B_m\cos(\omega t - 2\theta)$
$B_{\text{six-pole}} = B_m\cos(\omega t - 3\theta)$

P17.16*



$I_{line} = 16.3$ A rms

Typically the starting current is 5 to 7 times the full-load current.

P17.20* $I_{line,\text{starting}} = 115.0$ A rms

$T_{\text{dev, starting}} = 40.8$ newton meters

Comparing these results to those of the example, we see that the starting current is reduced by a factor of 2 and the starting torque is reduced by a factor of 4.

P17.23* Neglecting rotational losses, the slip is zero with no load, and the motor runs at synchronous speed which is 1800 rpm.

The power factor is 2.409%.

$I_{line} = 10$ A rms

P17.26* The motor runs at synchronous speed which is 1200 rpm.

The power factor is 1.04%.

$I_{line} = 98.97$ A rms

P17.29* $P_{ag} = 9.893$ kW

$P_{dev} = 9.773$ kW

$P_{out} = 9.373$ kW

$\eta = 91.51\%$

P17.32* $P_{rot} = 76.12$ W

P17.35* 1. Use an electronic system to convert 60-Hz power into three-phase ac of variable frequency. Start with a frequency of one hertz or less and then gradually increase the frequency.

2. Use a prime mover to bring the motor up to synchronous speed before connecting the source.

3. Start the motor as an induction motor relying on the amortisseur conductors to produce torque.

P17.38* (a) Field current remains constant. The field circuit is independent of the ac source and the load.

(b) Mechanical speed remains constant assuming that the pull-out torque has not been exceeded.

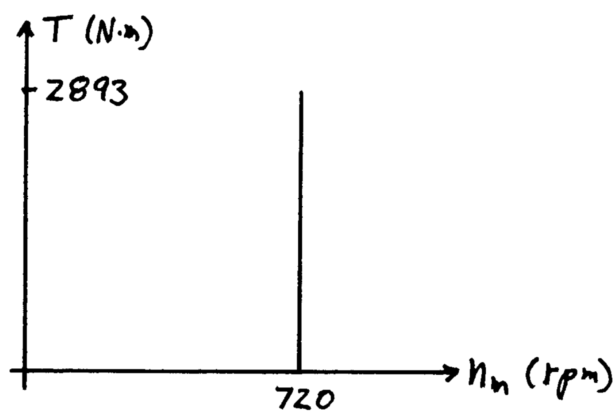
(c) Output torque increases by a factor of 1/0.75 = 1.333.

(d) Armature current increases in magnitude.

(e) Power factor decreases and becomes lagging.

(f) Torque angle increases.

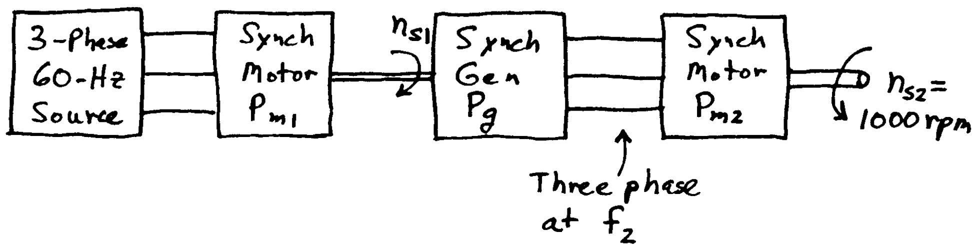
P17.41*



P17.44* $I_{f2} = 5.93\,A$

P17.47* (a) $f_{gen} = 50\text{ Hz}$

(b)



One solution is:

$$P_g = 10 \qquad P_{m1} = 12 \qquad \text{and} \qquad P_{m2} = 6$$

for which $f_2 = 50\text{ Hz}$.

Another solution is:

$$P_g = 10 \qquad P_{m1} = 6 \qquad \text{and} \qquad P_{m2} = 12$$

for which $f_2 = 100\text{ Hz}$.

P17.50* (a) power factor $= 76.2\%$ lagging

(b) $Z = 11.76\angle 40.36^\circ\ \Omega$

(c) Since the motor runs just under 1800 rpm, evidently we have a four-pole motor.

P17.53* The percentage drop in voltage is 7.33%.

Practice Test

T17.1 (a) The magnetic field set up in the air gap of a four-pole three-phase induction motor consists of four magnetic poles spaced 90° from one another in alternating order (i.e., north-south-north-south). The field points from the stator toward the rotor under the north poles and in the opposite direction under the south poles. The poles rotate with time at synchronous speed around the axis of the motor.

(b) The air gap flux density of a two-pole machine is given by Equation 17.12 in the book:

$$B_{gap} = B_m \cos(\omega t - \theta)$$

in which B_m is the peak field intensity, ω is the angular frequency of the three-phase source, and θ is angular displacement around the air gap. This describes a field having two poles: a north pole corresponding to $\omega t - \theta = 0$ and a south pole corresponding to $\omega t - \theta = \pi$. The location of either pole moves around the gap at an angular speed of $\omega_s = \omega$.

For a four pole machine, the field has four poles rotating at an angular speed of $\omega_s = \omega/2$ and is given by

$$B_{gap} = B_m \cos(\omega t - \theta / 2)$$

in which B_m is the peak field intensity, ω is the angular frequency of the three-phase source, and θ denotes angular position around the gap.

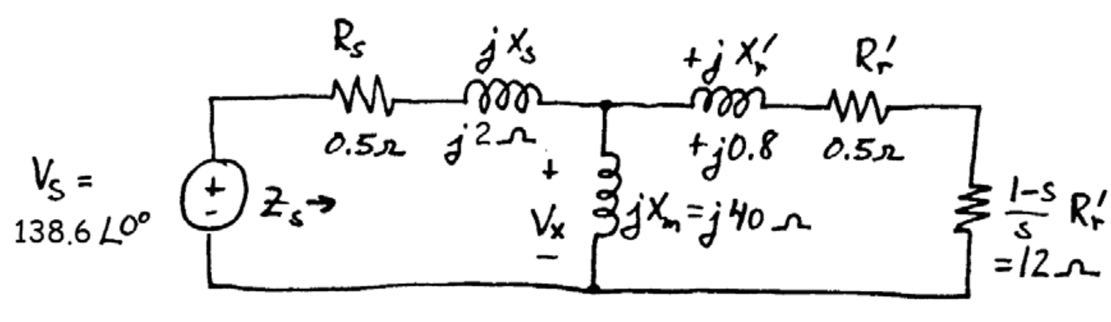
T17.2 Five of the most important characteristics for an induction motor are:

1. Nearly unity power factor.
2. High starting torque.
3. Close to 100% efficiency.
4. Low starting current.
5. High pull-out torque.

T17.3 An eight-pole 60-Hz machine has a synchronous speed of $n_s = 900$ rpm, and the slip is:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 864}{900} = 0.04$$

Because the machine is wye connected, the phase voltage is $240 / \sqrt{3} = 138.6$ V. (We assume zero phase for this voltage.)The per phase equivalent circuit is:



Then, we have

$$Z_s = 0.5 + j2 + \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8}$$
$$= 11.48 + j6.149 \quad \Omega$$
$$= 13.03\angle 28.17^\circ \quad \Omega$$

$\text{power factor} = \cos(28.17^\circ) = 88.16\%\ \text{lagging}$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{138.6\angle 0^\circ}{13.03\angle 28.17^\circ} = 10.64\angle -28.17^\circ \quad \text{A rms}$$

$$P_{\text{in}} = 3I_sV_s\cos\theta = 3.898\ \text{kW}$$

For a wye-connected motor, the phase current and line current are the same. Thus, the line current magnitude is 10.64 A rms.

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}'_r$.

$$\begin{aligned}\mathbf{V}_x &= \mathbf{I}_s \frac{j40(0.5 + 12 + j0.8)}{j40 + 0.5 + 12 + j0.8} \\ &= 123.83 - j16.244 \\ &= 124.9\angle -7.473^\circ\end{aligned}$$

$$\begin{aligned}\mathbf{I}'_r &= \frac{\mathbf{V}_x}{j0.8 + 0.5 + 12} \\ &= 9.971\angle -11.14^\circ\end{aligned}$$

The copper losses in the stator and rotor are:

$$\begin{aligned}P_s &= 3R_sI_s^2 \\ &= 3(0.5)(10.64)^2 \\ &= 169.7\ \text{W}\end{aligned}$$

and

$$\begin{aligned}P_r &= 3R'_r(I'_r)^2 \\ &= 3(0.5)(9.971)^2 \\ &= 149.1\ \text{W}\end{aligned}$$

Finally, the developed power is:

$$\begin{aligned}P_{\text{dev}} &= 3 \times \frac{1-s}{s} R'_r(I'_r)^2 \\ &= 3(12)(9.971)^2 \\ &= 3.579\ \text{kW}\end{aligned}$$

$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 3.429\ \text{kW}$$

The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 37.90\ \text{newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.97\%$$

T17.4 At 60 Hz, synchronous speed for an eight-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{8} = 900\ \text{rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{900 - 850}{900} = 5.56\%$$

The frequency of the rotor currents is the slip frequency. From Equation 17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

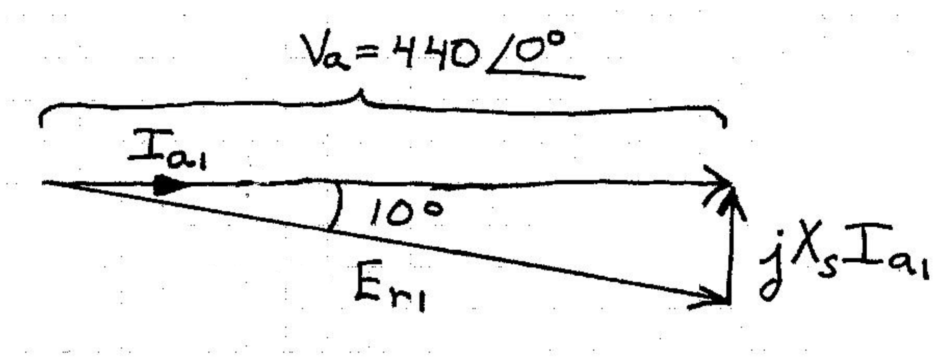
$$f_{\text{slip}} = sf = 0.05555 \times 60 = 3.333\ \text{Hz}$$

In the normal range of operation, slip is approximately proportional to output power and torque. Thus at 80% of full power, we estimate that $s = 0.8 \times 0.05555 = 0.04444$. This corresponds to a speed of 860 rpm.

T17.5 The stator of a six-pole synchronous motor contains a set of windings (collectively known as the armature) that are energized by a three-phase ac source. These windings produce six magnetic poles spaced 60° from one another in alternating order (i.e., north-south-north-south-north-south). The field points from the stator toward the rotor under the north stator poles and in the opposite direction under the south stator poles. The poles rotate with time at synchronous speed (1200 rpm) around the axis of the motor.

The rotor contains windings that carry dc currents and set up six north and south magnetic poles evenly spaced around the rotor. When driving a load, the rotor spins at synchronous speed with the north poles of the rotor lagging slightly behind and attracted by the south poles of the stator. (In some cases, the rotor may be composed of permanent magnets.)

T17.6 Figure 17.22 in the book shows typical phasor diagrams with constant developed power and variable field current. The phasor diagram for the initial operating conditions is:



Notice that because the initial power factor is unity, we have $\theta_1 = 0°$ and $\mathbf{I}_{a1}$ is in phase with $\mathbf{V}_a$. Also, notice that $jX_s\mathbf{I}_{a1}$ is at right angles to $\mathbf{I}_{a1}$. Now, we can calculate the magnitudes of $\mathbf{E}_{r1}$ and of $X_s\mathbf{I}_{a1}$.

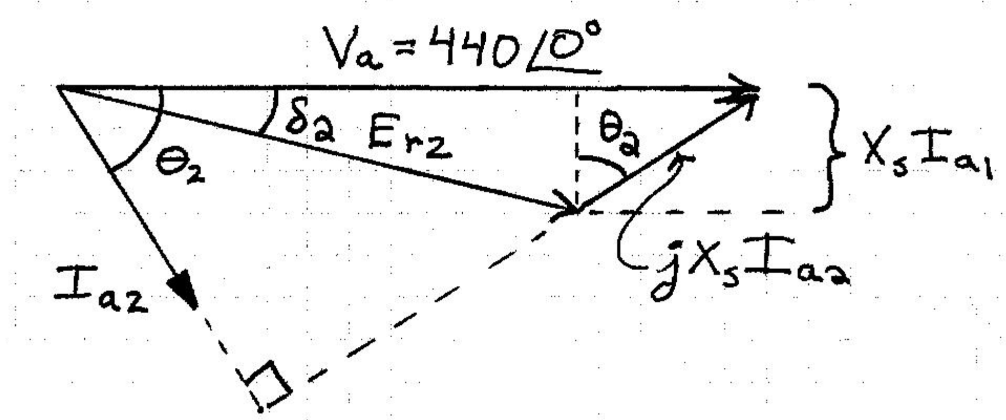
$$E_{r1} = \frac{V_a}{\cos\delta_1} = \frac{440}{\cos(10^\circ)} = 446.79\ \text{V}$$

$$X_s I_{a1} = E_{r1}\sin\delta_1 = 77.58\ \text{V}$$

Then, the field current is reduced until the magnitude of $\mathbf{E}_{r2}$ is 75% of its initial value.

$$E_{r2} = 0.75 \times E_{r1} = 335.09\ \text{V}$$

The phasor diagram for the second operating condition is:



Because the torque and power are constant, the vertical component of $jX_s\mathbf{I}_a$ is the same in both diagrams as illustrated in Figure 17.22 in the book. Thus, we have:

$$\sin\delta_2 = \frac{X_s I_{a1}}{E_{r2}} = \frac{77.58}{335.09}$$

which yields:

$$\delta_2 = 13.39^\circ$$

(Another solution to the equation is $\delta_2 = 166.61^\circ$, but this does not correspond to a stable operating point.)

Now, we can write:

$$(V_a - X_s I_{a1} \tan\theta_2)^2 + (X_s I_{a1})^2 = (E_{r2})^2$$

$$(440 - 77.58 \tan\theta_2)^2 + (77.58)^2 = (335.09)^2$$

Solving, we find $\theta_2 = 55.76°$, and the power factor is $\cos\theta_2 = 56.25\%$ lagging.