**NATIONAL UNIVERSITY OF SINGAPORE**

**EXAMINATION FOR**

**EE2011 — ENGINEERING ELECTROMAGNETICS**

(Semester I: 2011/2012)

November / December 2011 Time Allowed: 2 Hours

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**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **SIX (6)** questions in **TWO (2)** sections and comprises **SEVEN (7)** printed pages.

2. Attempt a total of **FOUR (4)** questions — any **TWO (2)** questions in Section A and any **TWO (2)** questions in Section B.

3. All questions carry equal marks.

4. You are allowed to bring into the examination hall a single A4-size information sheet (with material of your own choice on both sides of the paper).

5. Smith Charts are available on request. Remember to write your matriculation number on the Smith Chart(s) before tying to your examination script.

6. The following information may be useful**:**

   velocity of light in free space $c_o = 3 \text{ X } 10^{8}$ m $s^{-1}$
   
   free-space permittivity $\varepsilon_o = 8.85 \text{ X } 10^{-12}$ F $m^{-1}$
   
   free-space permeability $\mu_o = 4\pi \text{ X } 10^{-7}$ H $m^{-1}$
   
   free-space intrinsic impedance $\eta_o = 120\pi\ \Omega$

7. All symbols not specifically defined in the examination paper have their normally accepted meanings.

**SECTION A: Attempt any TWO (2) questions from this section (which consists of three questions).**

**Q1** A plane wave propagating in the +z direction in seawater, and its magnetic field intensity **H** at $z = 0$ is

$$\mathbf{H}(z=0,t) = \left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right)10\sin\left(10^7\pi t - \frac{\pi}{3}\right) \quad (\text{A/m})$$

The constitutive parameters of seawater are $\varepsilon_r = 78$, $\mu_r = 1$, and $\sigma = 6$ S/m.

(a) Determine the attenuation constant $\alpha$, intrinsic impedance $\eta$, phase velocity $u_p$, wavelength $\lambda$, and skin depth $\delta$.

(10 marks)

(b) Find the distance at which the amplitude of **H** is 0.1 A/m.

(3 marks)

(c) Write expressions for the phasor **H**($z$)

(6 marks)

(d) Write expressions for the phasor **E**($z$).

(6 marks)

**Q2**

**(a)** On a 150-Ω lossless transmission line, the following observations were noted: distance of the first voltage minimum from the load = 3 cm; distance of the first voltage maximum from the load = 9 cm; the voltage standing wave ratio (VSWR) S = 3. Find the load impedance $Z_L$.

(15 marks)

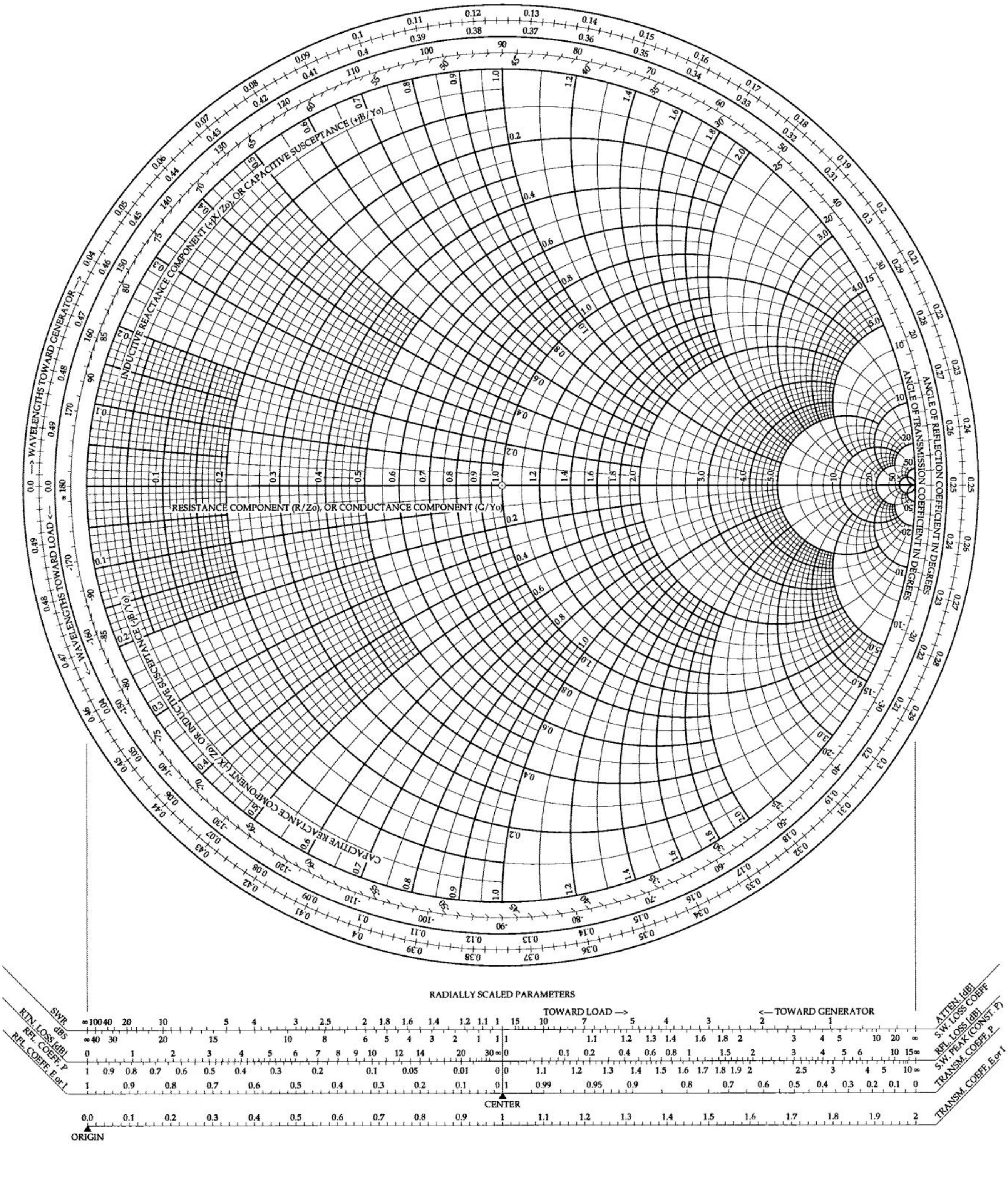
(b) A 50-Ω lossless transmission line has length $0.35\lambda$ and is terminated with a load $Z_L = 50 + j100\ \Omega$. Use Smith chart that is provide in the next page to find the input impedance $Z_{in}$ looking at the input of the transmission line.

(10 marks)

**Question 2 is continued in the next Page.**

# The Complete Smith Chart

Black Magic Design



**Q3** Consider the transmission line circuit shown in Fig. Q3. The voltage and impedance of the source are $V_g = 100$ V and $Z_g = 300\ \Omega$, respectively. Three lossless 150 Ω transmission lines, plotted as thick lines in Fig. Q3, all have lengths $3\lambda/8$. The impedance of the load of the second transmission line is known to be $Z_{L2} = 73\,\Omega$. The third line is short-circuited.

(a) Calculate the total load impedance $Z_{BB'}$, looking to the right at the port BB'.

(5 marks)

(b) What is the modulus $|V_{AA'}|$ of the voltage at the port AA'?

(5 marks)

(c) What is the modulus $|V_{BB'}|$ of the voltage at the port BB'? Is it equal to the $|V_{AA'}|$ obtained in part (ii)? Does the result contradict the Kirchhoff's Voltage Law? Briefly explain the result.

(7 marks)

(d) Calculate the power dissipated in the load $Z_{L2}$.

(5 marks)

(e) What is the voltage between points A' and B? Briefly explain the result.

(3 marks)

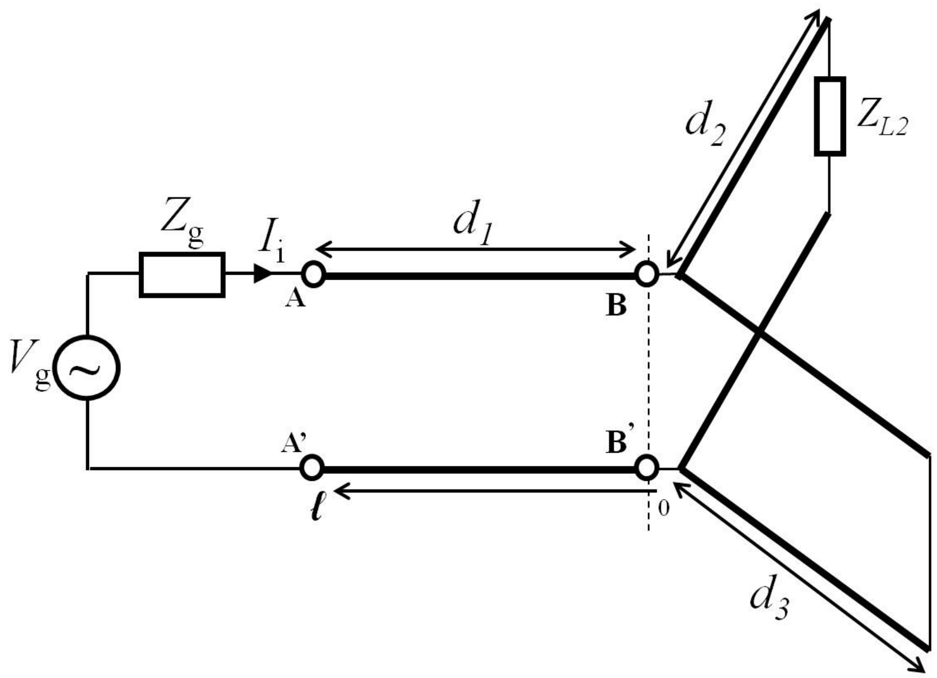



Fig. Q3

**SECTION B: Attempt any TWO (2) questions from this section (which consists of three questions).**

**Q4.** An engineer plans to design a special structure where the electric field at any point P (x, y, z) in the air space between the electrodes is given by**:**

$$\vec{E}\,(x, y, z) \;=\; -\,y\,\hat{u}_x - x\,\hat{u}_y + 0\,\hat{u}_z\ .$$

According to the design specifications, the potential at the point (2, 2, 0) has to be held constant at 4 V. All coordinates are given in meters.

(a) Derive an expression for the potential at any point P (x, y, z) in the air space between the electrodes.

(7 marks)

(b) For mechanical stability, his technician suggests that a thin metallic plate ought to be placed at the y = 0 plane. Deduce, with supporting reasons, whether the insertion of such a metallic plate will affect the electric-field pattern envisaged in the engineer's plan.

(6 marks)

(c) The engineer instructs the technician to check the surface charge distribution over the metallic plate to be placed at the y = 0 plane. Determine the surface charge density at the point (2, 0, 0) on the metallic plate.

(6 marks)

(d) The technician additionally suggests that it is important to perform the following test**:** choose a small test charge and measure the work done in moving it around a closed loop. Advise the engineer whether such a test is required after the special structure (with the metallic plate) has been constructed.

(6 marks)

**Q5** Reproduced in Figure Q5 is a diagram prepared by a lecturer for a class assignment. It depicts the cross-sectional view of a long cylindrical structure which has an inner conductor (in the shape of a square with dimension a) concentrically surrounded by an outer conductor (in the shape of a square with dimension 7a). The inner conductor is held at a potential of 10 V while the outer conductor has been earthed. The air region between the two conductors may be assumed to be free of charges. The origin O of the (x, y) coordinate system is sited at the bottom left corner of the outer conductor as shown in Figure Q5.

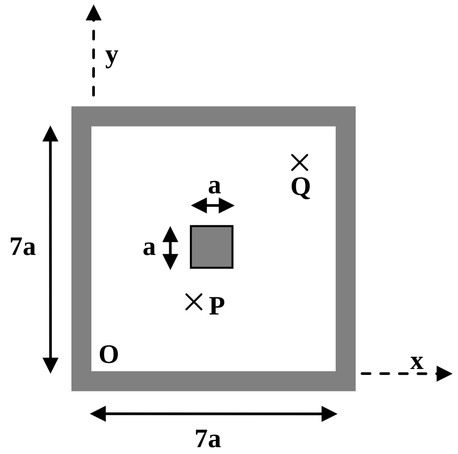



Figure Q5

(a) Student A asserts that his numerical computations involve only 5 unknown potential parameters. Deduce, with supporting reasons, whether you agree with this student's assertion.

(4 marks)

(b) The assignment requires students to estimate the potential at the point P with grid coordinates (3a, 2a). After having completed the assignment, the answers obtained by Student A and Student B are 5.2 V and 10.5 V respectively. Deduce, with supporting reasons, which of these two estimates is unlikely to be correct for this special coaxial cable.

(3 marks)

(c) Perform your own computations to obtain an estimate for the potential at the point Q with grid coordinates (6a, 6a).

(15 marks)

(d) Advise the students how they may improve the numerical accuracies that can be attained from such computations.

(3 marks)

**Q6** A research student has to design a long conducting rod for his project. Reproduced in Figure Q6(a) is the cross-sectional view (prepared by him for the workshop) showing that there is a cylindrical hole (with radius a) located concentrically in the cylindrical rod (with radius 3a).

His project requires the current carried by the rod-with-hole structure to be uniformly distributed over the cross-sectional surface. Another project requirement is that there must be a constant magnetic field intensity of $H_0 \hat{u}_\phi$ at the point P (2a, 0, 0) which is located on the x-axis (*i.e.* in the $\phi = 0$ direction).

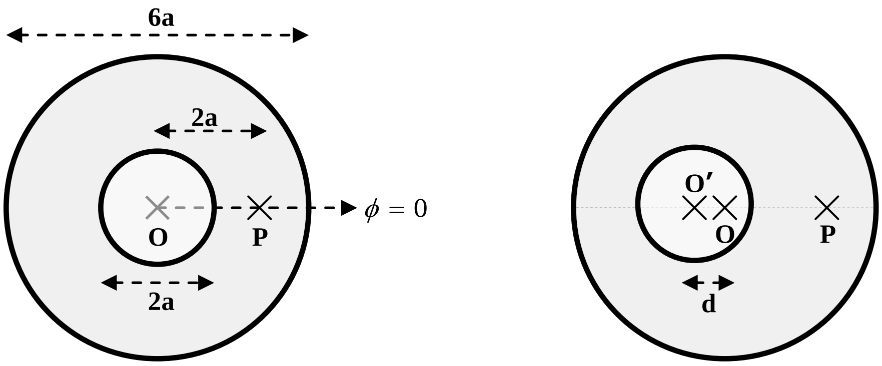



Figure Q6(a) Figure Q6(b)

(a) Show that, for compliance with the project requirements, the current density over the cross-sectional area must be given by:

$$J_0 = \frac{4H_0}{3a} .$$

(8 marks)

(b) Derive an expression (in terms of $H_0$, a and r) for the magnetic field intensity outside the cylindrical structure (where r is the distance from the origin O which is also the common center of the rod and hole).

(7 marks)

(c) The student proceeds with his experiment after collecting the rod-with-hole structure from the workshop. Unknown to him, however, the center O′ of the hole has been shifted during the machining process by a displacement d in the –x direction (*i.e.* in the $\phi = \pi$ direction) as sketched in Figure Q6(b).

(i) Deduce, with supporting reasons, whether the student will observe a smaller or larger magnetic field intensity at P while performing his experiment with the same current density $J_0$ specified in part (a).

(5 marks)

(ii) Deduce, with supporting reasons, whether the magnetic field intensity expression derived in part (b) remains valid for a distant point (*i.e.* when r > 100 a).

(5 marks)

**— END OF PAPER —**