NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 5 **SEMESTER II, AY 2010/2011**

1. Prove that for positive integers $k \leq n$

   (a) $\left\{ \begin{matrix} n+1 \\ k \end{matrix} \right\} = \sum_{i=0}^{n} \binom{n}{i} \left\{ \begin{matrix} i \\ k-1 \end{matrix} \right\}$;

   (b) $\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \sum_{m=1}^{n} k^{n-m} \left\{ \begin{matrix} m-1 \\ k-1 \end{matrix} \right\}$.

2. We want to divide 9 children into any number of playgroups. However, we want Kate and William to be in different playgroups. How many ways are there to do this? (Playgroups with one child are allowed.)

3. Define $P_k(n)$ as the # of partitions of n into at most k parts. Prove the following

   (a) $P_k(n) = \sum_{j=1}^{k} p(n, j)$

   (b) $P_k(n) = P_{k-1}(n) + P_k(n-k)$

4. What is the # of partitions of 15 into at most 4 parts?

5. A partition is called self-conjugate if taking the conjugate of its Ferrers diagram produces the same diagram. Construct a self-conjugate partition for every positive integer $n \neq 2$.

6. (a) Find all self-conjugate partitions of 16.

   (b) Find all partitions of 16 into distinct odd parts.

7. (Challenging question: Sylvester's result)

   Prove that the total # of self-conjugate partitions of n is equal to the # of partitions of n into distinct odd parts.

Answers

4. 54