NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 10 **SEMESTER II, AY 2010/2011**

1. Find the number of ways to distribute n distinct objects to 5 distinct boxes, such that boxes 1, 3 and 5 must hold an odd number of objects and boxes 2 and 4 must hold an even number of objects.

2. Ten female workers and eight male workers are to be assigned to work in one of four different departments of a company. In how many ways can this be done if

   (a) each dept gets at least one worker;

   (b) each dept gets at least one female worker;

   (c) each dept gets at least one female worker and at least one male worker.

3. In how many ways can 5 letters of the name "NG ENG TENG" be rearranged?

4. Use generating functions to prove the solution to the recurrence relation $a_n = pa_{n-1} + q$ for $n \geq 1$ and $a_0 = r$ is given by

$$a_n = \begin{cases} qn + r & \text{if } p = 1; \\ rp^n + \dfrac{(1-p^n)q}{1-p} & \text{otherwise.} \end{cases}$$

   Hence solve the recurrence relation $a_n - a_{n-1} = 3$ for $n \geq 1$ and $a_0 = 5$ by

5. Solve the recurrence relation $a_n - 4a_{n-1} - 5a_{n-2} = 8n - 16$ for $n \geq 2$, $a_0 = 1$ and $a_1 = 9$.

6. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 10 \cdot 2^n$ for $n \geq 2$, $a_0 = 2$ and $a_1 = 1$.

7. Solve the recurrence relation $a_n - 7a_{n-1} + 12a_{n-2} = 5^n - 3^n$ for $n \geq 2$ and $a_0 = a_1 = 1$.

Answers

2. 67171367640; 53642526720 ; 33415260480

4. $3n + 5$