CS2010 Semester 1 2012/2013

Data Structures and Algorithms II

Tutorial 03 - Heaps

For Week 05 (10 September - 14 September 2012)

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1 Introduction and Objective

The purpose of this tutorial is to reinforce the concepts of Binary Heap data structure which can be used as Priority Queue. We will also discuss PS2 Subtask 1 during this tutorial.

You can use `http://www.comp.nus.edu.sg/~stevenha/visualization/heap.html` to *verify* the answers of some questions in this tutorial. However during written tests, you have to be able to do this by yourself though.

2 Tutorial 03 Questions

Heaps, Heaps and more Heaps !

Q1. Is the tree shown in Figure 1 below a valid max heap?

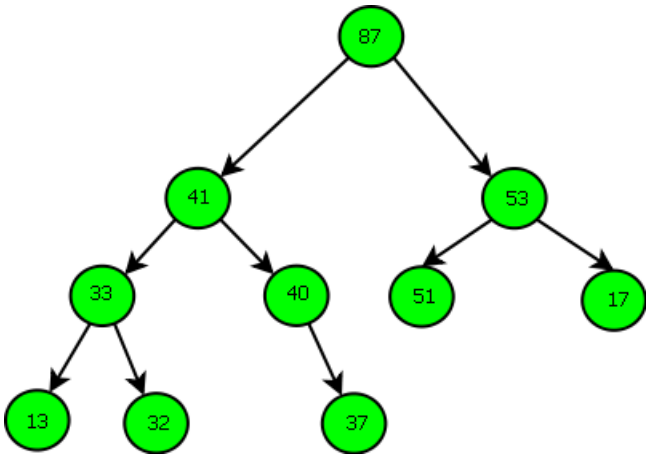

Figure 1: Is this a heap?

Q2a. Show the result of inserting 10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13, 2 one by one into an initially empty max heap (in another word, execute `BuildHeapSlow(array)` as shown in the lecture).

Q2b.) Show the result of using $O(n)$ `BuildHeap(array)` to build a max heap using the same input.

Q2c.) Show the result of 3 `ExtractMax()` operations on the max heap built in a.) and the one in b.)

Q3. What is the minimum and maximum number of comparisons between heap elements required to construct a max heap of 8 elements using the $O(n)$ `BuildHeap(array)`?

Q4. What modifications are required so that *both* `ExtractMax()` and `ExtractMin()` can be done in $O(\log n)$ time and every other heap operation retains the same running time?

Q5. Give an algorithm to find all nodes bigger than some value x in a max heap that runs in $O(k)$ time where k is the number of nodes in the output.

Q6. The *second* largest element in a max heap with more than two unique elements is always one of the children of the root. Is this true? If yes, show a simple proof. Otherwise, show a counter example.

Problem Set 2

Q7. Discussion of PS2 subtask 1.