NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 8: SUGGESTED SOLUTIONS **SEMESTER II, AY 2010/2011**

1. $360 = 2^3 3^2 5 \implies \phi(360) = \phi(2^3)\phi(3^2)\phi(5) = 4 \times 6 \times 4$.

2.

3.

4. (a) $9!/3!/3!/3! - 3 \times 7!/3!/3! + 3 \times 5!/3! - 3!$;

   (b) Let P_a be the property that there are two consecutive a. Model $w(P_a)$ by considering permutations of $\boxed{aa}, a, b, b, b, c, c, c$. There are a total of $8!/3!/3! - 7!/3!/3!$ ways because of double counting whenever $\boxed{aa}$ and a are adjacent.

$$
\begin{aligned}
w(0) &= 9!/3!/3!/3! = 1680 \\
w(1) &= 3\,(8!/3!/3! - 7!/3!/3!) = 3 \times 980 \\
w(2) &= 3\,(7!/3! - 2(6!/3!) + 5!/3!) = 3 \times 620 \\
w(3) &= 6! - 3 \times 5! + 3 \times 4! - 3! = 426 \\
E(0) &= w(0) - w(1) + w(2) - w(3) = 174
\end{aligned}
$$

5.
- $w(0) = \binom{13}{5} = 1287$
- $w(1) = w(AB) + w(BC) + w(AD) + w(DC)$
- $\quad = \binom{5}{2}\binom{7}{3} + \binom{6}{2}\binom{6}{2} + \binom{5}{2}\binom{7}{2} + \binom{6}{3}\binom{6}{2} = 1085$
- $w(2) = w(AB, BC) + w(AD, DC)$
- $\quad = \binom{5}{2}\binom{6}{2} + \binom{5}{2}\binom{6}{2} = 300.$
- $w(3) = w(4) = 0$
- i) $E(0) = w(0) - w(1) + w(2) - w(3) + w(4) = 502.$
- ii) $E(1) = w(1) - 2w(2) + 3w(3) - 4w(4) = 485$
- iii) $E(2) = w(2) - \binom{3}{2}w(3) + \binom{4}{2}w(4) = 300.$