CHAPTER 5

Exercises

E5.1 (a) We are given $v(t) = 150\cos(200\pi t - 30^\circ)$. The angular frequency is the coefficient of t so we have $\omega = 200\pi$ radian/s. Then

$$f = \omega / 2\pi = 100 \text{ Hz} \qquad T = 1/f = 10 \text{ ms}$$

$$V_{rms} = V_m / \sqrt{2} = 150/\sqrt{2} = 106.1 \text{ V}$$

Furthermore, $v(t)$ attains a positive peak when the argument of the cosine function is zero. Thus keeping in mind that ωt has units of radians, the positive peak occurs when

$$\omega t_{\max} = 30 \times \frac{\pi}{180} \quad \Rightarrow \quad t_{\max} = 0.8333 \text{ ms}$$

(b) $P_{avg} = V_{rms}^2 / R = 225$ W

(c) A plot of $v(t)$ is shown in Figure 5.4 in the book.

E5.2 We use the trigonometric identity $\sin(z) = \cos(z - 90^\circ)$. Thus

$$100\sin(300\pi t + 60^\circ) = 100\cos(300\pi t - 30^\circ)$$

E5.3 $\omega = 2\pi f \cong 377$ radian/s $\qquad T = 1/f \cong 16.67$ ms $\qquad V_m = V_{rms}\sqrt{2} \cong 155.6$ V

The period corresponds to 360° therefore 5 ms corresponds to a phase angle of $(5/16.67)\times 360^\circ = 108^\circ$. Thus the voltage is

$$v(t) = 155.6\cos(377t - 108^\circ)$$

E5.4 (a) $\mathbf{V}_1 = 10\angle 0^\circ + 10\angle -90^\circ = 10 - j10 \cong 14.14\angle -45^\circ$

$$10\cos(\omega t) + 10\sin(\omega t) = 14.14\cos(\omega t - 45^\circ)$$

(b) $\mathbf{I}_1 = 10\angle 30^\circ + 5\angle -60^\circ \cong 8.660 + j5 + 2.5 - j4.330$

$$\cong 11.16 + j0.670 \cong 11.18\angle 3.44^\circ$$

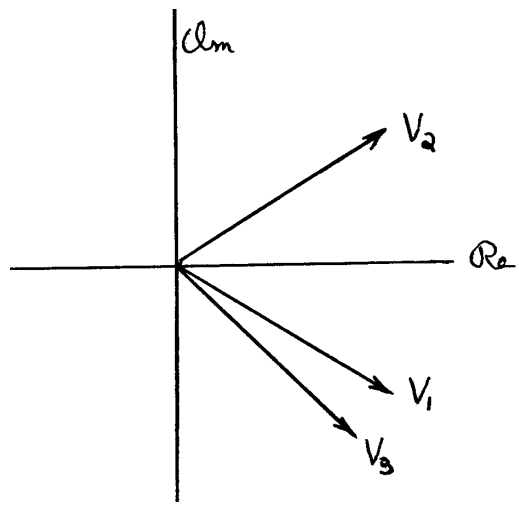
$$10\cos(\omega t + 30^\circ) + 5\sin(\omega t + 30^\circ) = 11.18\cos(\omega t + 3.44^\circ)$$

(c) $\mathbf{I}_2 = 20\angle 0^\circ + 15\angle -60^\circ \cong 20 + j0 + 7.5 - j12.99$

$$\cong 27.5 - j12.99 \cong 30.41\angle -25.28^\circ$$

$$20\sin(\omega t + 90^\circ) + 15\cos(\omega t - 60^\circ) = 30.41\cos(\omega t - 25.28^\circ)$$

E5.5 The phasors are $\mathbf{V}_1 = 10\angle -30^\circ$ $\mathbf{V}_2 = 10\angle +30^\circ$ and $\mathbf{V}_3 = 10\angle -45^\circ$



v_1 lags v_2 by 60° (or we could say v_2 leads v_1 by 60°)

v_1 leads v_3 by 15° (or we could say v_3 lags v_1 by 15°)

v_2 leads v_3 by 75° (or we could say v_3 lags v_2 by 75°)

E5.6 (a) $Z_L = j\omega L = j50 = 50\angle 90^\circ$ $\mathbf{V}_L = 100\angle 0^\circ$

$\mathbf{I}_L = \mathbf{V}_L / Z_L = 100 / j50 = 2\angle -90^\circ$

(b) The phasor diagram is shown in Figure 5.11a in the book.

E5.7 (a) $Z_C = 1 / j\omega C = -j50 = 50\angle -90^\circ$ $\mathbf{V}_C = 100\angle 0^\circ$

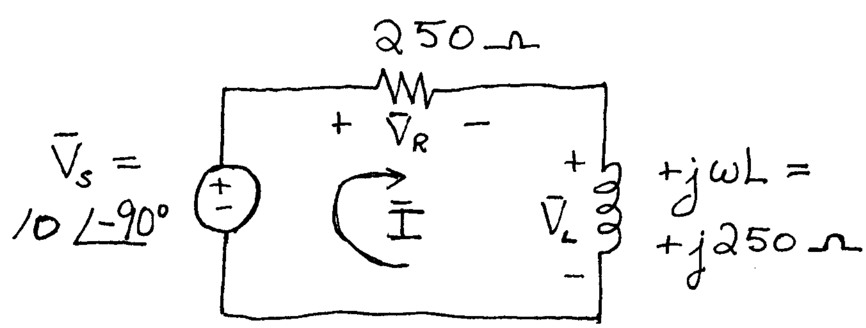
$\mathbf{I}_C = \mathbf{V}_C / Z_C = 100 / (-j50) = 2\angle 90^\circ$

(b) The phasor diagram is shown in Figure 5.11b in the book.

E5.8 (a) $Z_R = R = 50 = 50\angle 0^\circ$ $\mathbf{V}_R = 100\angle 0^\circ$ $\mathbf{I}_R = \mathbf{V}_R / R = 100 / (50) = 2\angle 0^\circ$

(b) The phasor diagram is shown in Figure 5.11c in the book.

E5.9 (a) The transformed network is:



$$\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{10\angle -90^\circ}{250 + j250} = 28.28\angle -135^\circ \text{ mA}$$

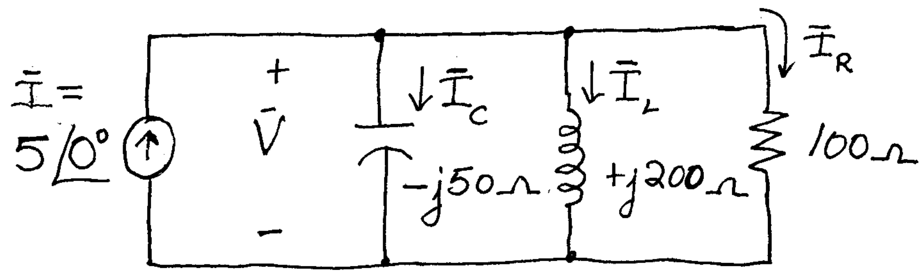
$i(t) = 28.28\cos(500t - 135^\circ)$ mA

$\mathbf{V}_R = R\mathbf{I} = 7.07\angle -135^\circ \qquad \mathbf{V}_L = j\omega L\mathbf{I} = 7.07\angle -45^\circ$

(b) The phasor diagram is shown in Figure 5.17b in the book.

(c) $i(t)$ lags $v_s(t)$ by 45°.

E5.10 The transformed network is:



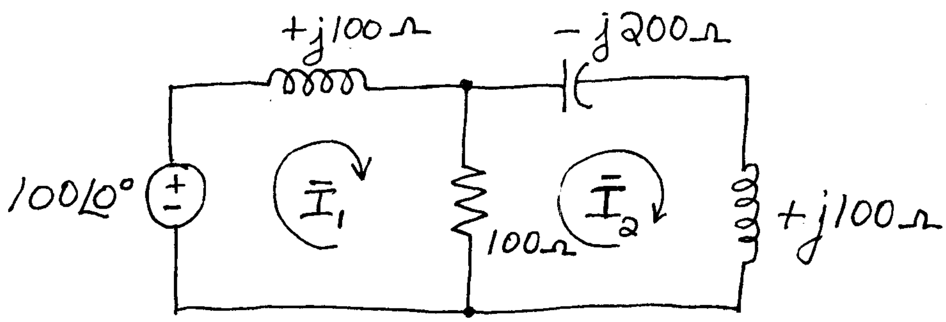
$$Z = \frac{1}{1/100 + 1/(-j50) + 1/(+j200)} = 55.47\angle -56.31^\circ \ \Omega$$

$\mathbf{V} = Z\mathbf{I} = 277.4\angle -56.31^\circ$ V $\qquad \mathbf{I}_C = \mathbf{V}/(-j50) = 5.547\angle 33.69^\circ$ A

$\mathbf{I}_L = \mathbf{V}/(j200) = 1.387\angle -146.31^\circ$ A

$\mathbf{I}_R = \mathbf{V}/(100) = 2.774\angle -56.31^\circ$ A

E5.11 The transformed network is:



We write KVL equations for each of the meshes:

$$j100\mathbf{I}_1 + 100(\mathbf{I}_1 - \mathbf{I}_2) = 100$$
$$-j200\mathbf{I}_2 + j100\mathbf{I}_2 + 100(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

Simplifying, we have

$$(100 + j100)\mathbf{I}_1 - 100\mathbf{I}_2 = 100$$
$$-100\mathbf{I}_1 + (100 - j100)\mathbf{I}_2 = 0$$

Solving we find $\mathbf{I}_1 = 1.414\angle -45^\circ$ A and $\mathbf{I}_2 = 1\angle 0^\circ$ A. Thus we have $i_1(t) = 1.414\cos(1000t - 45^\circ)$ A and $i_2(t) = \cos(1000t)$.

E5.12 (a) For a power factor of 100%, we have $\cos(\theta) = 1$, which implies that the current and voltage are in phase and $\theta = 0$. Thus, $Q = P\tan(\theta) = 0$. Also $I_{rms} = P/[V_{rms}\cos(\theta)] = 5000/[500\cos(0)] = 10$ A. Thus we have $I_m = I_{rms}\sqrt{2} = 14.14$ and $\mathbf{I} = 14.14\angle 40^\circ$.

(b) For a power factor of 20% lagging, we have $\cos(\theta) = 0.2$, which implies that the current lags the voltage by $\theta = \cos^{-1}(0.2) = 78.46^\circ$. Thus, $Q = P\tan(\theta) = 24.49$ kVAR. Also, we have $I_{rms} = P/[V_{rms}\cos(\theta)] = 50.0$ A. Thus we have $I_m = I_{rms}\sqrt{2} = 70.71$ A and $\mathbf{I} = 70.71\angle -38.46^\circ$.

(c) The current ratings would need to be five times higher for the load of part (b) than for that of part (a). Wiring costs would be lower for the load of part (a).

E5.13 The first load is a 10 μF capacitor for which we have

$Z_C = 1/(j\omega C) = 265.3\angle -90^\circ\ \Omega \quad \theta_C = -90^\circ \quad I_{Crms} = V_{rms}/|Z_C| = 3.770$ A

$P_C = V_{rms}I_{Crms}\cos(\theta_C) = 0 \quad Q_C = V_{rms}I_{Crms}\sin(\theta_C) = -3.770$ kVAR

The second load absorbs an apparent power of $V_{rms}I_{rms} = 10$ kVA with a power factor of 80% lagging from which we have $\theta_2 = \cos^{-1}(0.8) = 36.87^\circ$. Notice that we select a positive angle for θ_2 because the load has a lagging power factor. Thus we have $P_2 = V_{rms}I_{2rms}\cos(\theta_2) = 8.0$ kW and $Q_2 = V_{rms}I_{2rms}\sin(\theta) = 6$ kVAR.

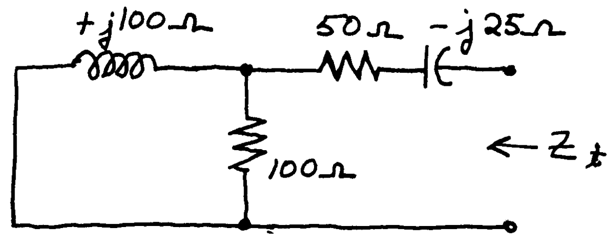
Now for the source we have:

$P_s = P_C + P_2 = 8$ kW $\quad Q_s = Q_C + Q_2 = 2.23$ kVAR

$V_{rms}I_{srms} = \sqrt{P_s^2 + Q_s^2} = 8.305$ kVA $\quad I_{srms} = V_{rms}I_{srms}/V_{rms} = 8.305$ A

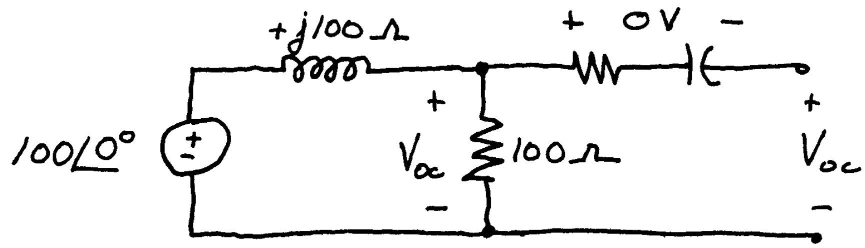
power factor $= P_s/(V_{rms}I_{srms}) \times 100\% = 96.33\%$

E5.14 First, we zero the source and combine impedances in series and parallel to determine the Thévenin impedance.



$$Z_t = 50 - j25 + \frac{1}{1/100 + 1/j100} = 50 - j25 + 50 + j50$$
$$= 100 + j25 = 103.1\angle 14.04^\circ$$

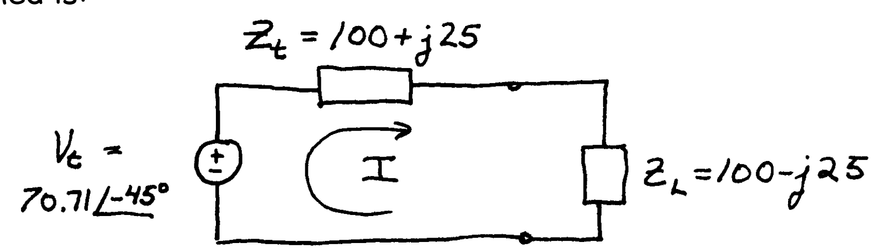
Then we analyze the circuit to determine the open-circuit voltage.



$$\mathbf{V}_t = \mathbf{V}_{oc} = 100 \times \frac{100}{100 + j100} = 70.71\angle -45^\circ$$

$$\mathbf{I}_n = V_t / Z_t = 0.6858\angle -59.04^\circ$$

E5.15 (a) For a complex load, maximum power is transferred for $Z_L = Z_t^* = 100 - j25 = R_L + jX_L$. The Thévenin equivalent with the load attached is:



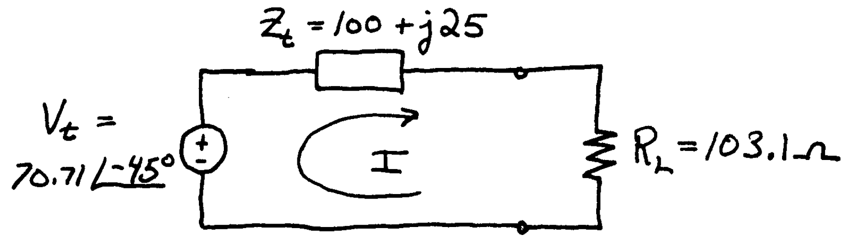
The current is given by

$$\mathbf{I} = \frac{70.71\angle -45^\circ}{100 + j25 + 100 - j25} = 0.3536\angle -45^\circ$$

The load power is

$$P_L = R_L I_{rms}^2 = 100(0.3536/\sqrt{2})^2 = 6.25\text{ W}$$

(b) For a purely resistive load, maximum power is transferred for $R_L = |Z_t| = \sqrt{100^2 + 25^2} = 103.1\ \Omega$. The Thévenin equivalent with the load attached is:



The current is given by

$$\mathbf{I} = \frac{70.71\angle -45^\circ}{103.1 + 100 - j25} = 0.3456\angle -37.98^\circ$$

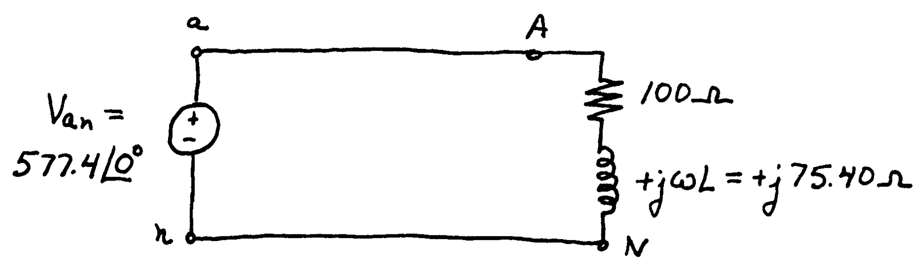
The load power is

$$P_L = R_L I_{rms}^2 = 103.1(0.3456/\sqrt{2})^2 = 6.157\ \text{W}$$

E5.16 The line-to-neutral voltage is $1000/\sqrt{3} = 577.4$ V. No phase angle was specified in the problem statement, so we will assume that the phase of $\mathbf{V}_{an}$ is zero. Then we have

$$\mathbf{V}_{an} = 577.4\angle 0^\circ \qquad \mathbf{V}_{bn} = 577.4\angle -120^\circ \qquad \mathbf{V}_{cn} = 577.4\angle 120^\circ$$

The circuit for the *a* phase is shown below. (We can consider a neutral connection to exist in a balanced Y-Y connection even if one is not physically present.)



The *a*-phase line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_L} = \frac{577.4\angle 0^\circ}{100 + j75.40} = 4.610\angle -37.02^\circ$$

The currents for phases *b* and *c* are the same except for phase.

$$\mathbf{I}_{bB} = 4.610\angle -157.02^\circ \qquad \mathbf{I}_{cC} = 4.610\angle 82.98^\circ$$

$$P = 3\frac{V_Y I_L}{2}\cos(\theta) = 3\frac{577.4 \times 4.610}{2}\cos(37.02^\circ) = 3.188\ \text{kW}$$

$$Q = 3\frac{V_y I_L}{2}\sin(\theta) = 3\frac{577.4 \times 4.610}{2}\sin(37.02^\circ) = 2.404\ \text{kVAR}$$

E5.17 The a-phase line-to-neutral voltage is

$$\mathbf{V}_{an} = 1000 / \sqrt{3}\angle 0^\circ = 577.4\angle 0^\circ$$

The phase impedance of the equivalent Y is $Z_y = Z_\Delta / 3 = 50/3 = 16.67\ \Omega$.

Thus the line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_y} = \frac{577.4\angle 0^\circ}{16.67} = 34.63\angle 0^\circ\ A$$

Similarly, $\mathbf{I}_{bB} = 34.63\angle -120^\circ\ A$ and $\mathbf{I}_{cC} = 34.63\angle 120^\circ\ A$.

Finally, the power is

$$P = 3(I_{aA}/\sqrt{2})^2 R_y = 30.00\ \text{kW}$$

E5.18 Writing KCL equations at nodes 1 and 2 we obtain

$$\frac{\mathbf{V}_1}{100 + j30} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{50 - j80} = 1\angle 60^\circ$$

$$\frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{50 - j80} = 2\angle 30^\circ$$

In matrix form, these become

$$\begin{bmatrix} \left(\frac{1}{100 + j30} + \frac{1}{50 - j80}\right) & -\frac{1}{50 - j80} \\ -\frac{1}{50 - j80} & \left(\frac{1}{j50} + \frac{1}{50 - j80}\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1\angle 60^\circ \\ 2\angle 30^\circ \end{bmatrix}$$

The MATLAB commands are

```
Y = [(1/(100+j*30)+1/(50-j*80))  (-1/(50-j*80));...
     (-1/(50-j*80))  (1/(j*50)+1/(50-j*80))];
I = [pin(1,60); pin(2,30)];
V = inv(Y)*I;
pout(V(1))
pout(V(2))
```

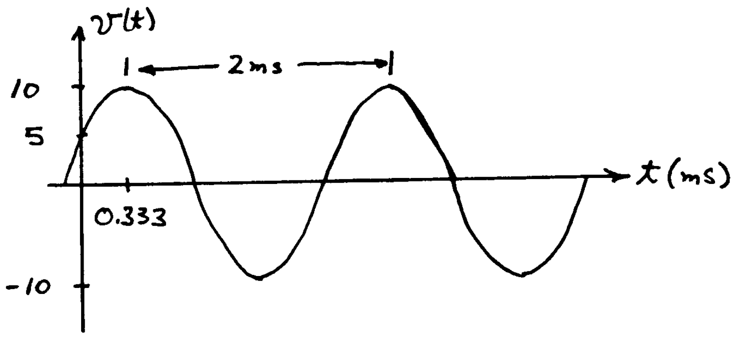
The results are

$$\mathbf{V}_1 = 79.98\angle 106.21^\circ \quad \text{and} \quad \mathbf{V}_2 = 124.13\angle 116.30^\circ$$

Answers for Selected Problems

P5.4*

$$\omega = 1000\pi \text{ rad/s}$$
$$f = 500 \text{ Hz}$$
$$\text{phase angle} = \theta = -60^\circ = -\pi/3 \text{ radians}$$
$$T = 2 \text{ ms}$$
$$V_{rms} = 7.071 \text{ V}$$
$$P = 1 \text{ W}$$
$$t_{peak} = 0.3333 \text{ ms}$$



P5.6* $v(t) = 28.28\cos(2\pi 10^4 t - 72^\circ)$ V

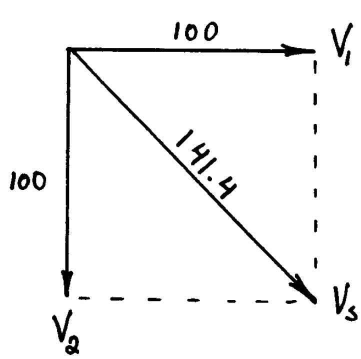
P5.12* $V_{rms} = 10.61$ V

P5.13* $V_{rms} = 3.808$ A

P5.23* $5\cos(\omega t + 75^\circ) - 3\cos(\omega t - 75^\circ) + 4\sin(\omega t) = 3.763\cos(\omega t + 82.09^\circ)$

P5.24*

$$v_s(t) = 141.4\cos(\omega t - 45^\circ)$$



$\mathbf{V}_2$ lags $\mathbf{V}_1$ by 90°
$\mathbf{V}_s$ lags $\mathbf{V}_1$ by 45°
$\mathbf{V}_s$ leads $\mathbf{V}_2$ by 45°

P5.25*

$$v_1(t) = 10\cos(400\pi t + 30^\circ)$$
$$v_2(t) = 5\cos(400\pi t + 150^\circ)$$
$$v_3(t) = 10\cos(400\pi t + 90^\circ)$$

$v_1(t)$ lags $v_2(t)$ by 120°

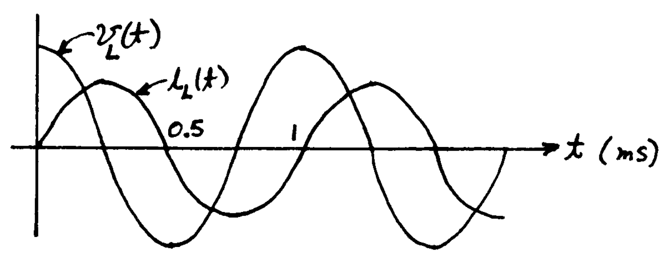
$v_1(t)$ lags $v_3(t)$ by 60°

$v_2(t)$ leads $v_3(t)$ by 60°

P5.35*

$$Z_L = 200\pi\angle 90^\circ$$
$$\mathbf{V}_L = 10\angle 0^\circ$$
$$\mathbf{I}_L = (1/20\pi)\angle -90^\circ$$
$$i_L(t) = (1/20\pi)\cos(2000\pi t - 90^\circ) = (1/20\pi)\sin(2000\pi t)$$

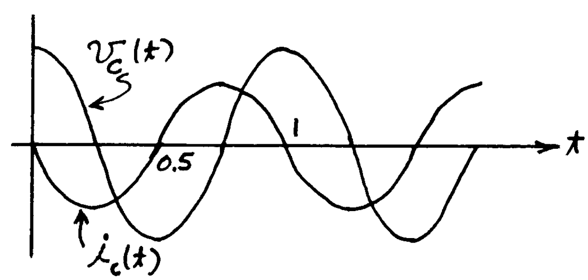
$i_L(t)$ lags $v_L(t)$ by 90°



P5.37*

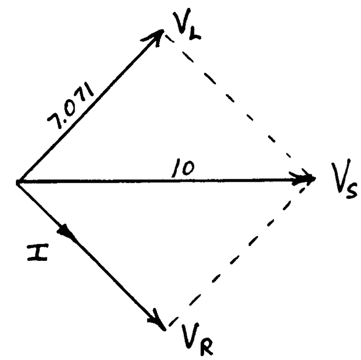
$$Z_C = 15.92\angle -90^\circ\ \Omega$$
$$\mathbf{V}_C = 10\angle 0^\circ$$
$$\mathbf{I}_C = \mathbf{V}_C / Z_C = 0.6283\angle 90^\circ$$
$$i_C(t) = 0.6283\cos(2000\pi t + 90^\circ) = -0.6283\sin(2000\pi t)$$

$i_C(t)$ leads $v_C(t)$ by 90°



P5.42*

$$\mathbf{I} = 70.71\angle -45^\circ \text{ mA}$$
$$\mathbf{V}_R = 7.071\angle -45^\circ \text{ V}$$
$$\mathbf{V}_L = 7.071\angle 45^\circ \text{ V}$$

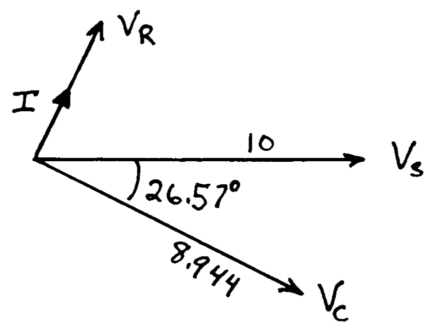


$\mathbf{I}$ lags $\mathbf{V}_s$ by 45°

P5.44*

$$\mathbf{I} = 4.472\angle 63.43^\circ \text{ mA}$$
$$\mathbf{V}_R = 4.472\angle 63.43^\circ \text{ V}$$
$$\mathbf{V}_C = 8.944\angle -26.57^\circ \text{V}$$

$\mathbf{I}$ leads $\mathbf{V}_s$ by 63.43°



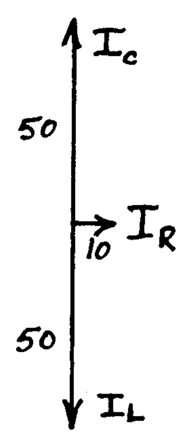
P5.46* $\omega = 500: \quad Z = 158.1\angle -71.57^\circ$

$\omega = 1000: \quad Z = 50\angle 0^\circ$

$\omega = 2000: \quad Z = 158.1\angle 71.57^\circ$

P5.49*

$$\mathbf{I}_R = 10\angle 0^\circ \text{ mA}$$
$$\mathbf{I}_L = 50\angle -90^\circ \text{ mA}$$
$$\mathbf{I}_C = 50\angle 90^\circ \text{ mA}$$



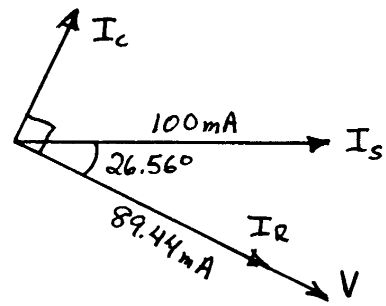
The peak value of $i_L(t)$ is five times larger than the source current!

P5.52* $\mathbf{V} = 8.944\angle -26.56^\circ$ V

$\mathbf{I}_R = 89.44\angle -26.56^\circ$ mA

$\mathbf{I}_C = 44.72\angle 63.44^\circ$ mA

$\mathbf{V}$ lags $\mathbf{I}_s$ by 26.56°



P5.67* $\mathbf{I} = 15.11\angle 20.66^\circ$

$P = 10$ kW

$Q = -3.770$ kVAR

Apparent power $= 10.68$ kVA

Power factor $= 93.57\%$ leading

P5.69* This is a capacitive load.

$P = 22.5$ kW

$Q = -11.25$ kVAR

power factor $= 89.44\%$

apparent power $= \sqrt{P^2 + Q^2} = 25.16$ KVA

P5.78* $P_s = 22$ kW

$Q_s = 13.84$ kVAR

Apparent power $= 26$ kVA

Power factor $= 84.62\%$ lagging

P5.83* (a) $\mathbf{I} = 400\sqrt{2}\angle -75.52^\circ$

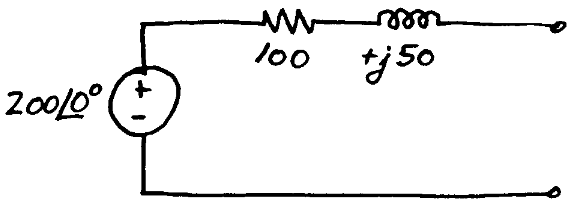
(b) $C = 1027\ \mu\text{F}$

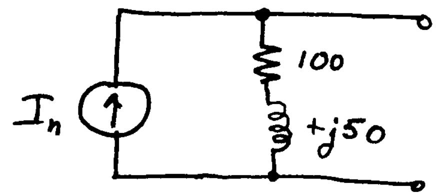
The capacitor must be rated for at least 387.3 kVAR.

$\mathbf{I} = 100\angle 0^\circ$

(c) The line current is smaller by a factor of 4 with the capacitor in place, reducing I^2R losses in the line by a factor of 16.

P5.87* (a) $\mathbf{I}_n = 1.789\angle -26.57^\circ$





(b) $P_{load} = 50\ \text{W}$

(c) $P_{load} = 47.21\ \text{W}$

P5.91* $R_{load} = 12.5\ \Omega$
$C_{load} = 106.1\ \mu\text{F}$

P5.95* $Z_\Delta = 70.29\angle -62.05^\circ\ \Omega$

P5.96* $V_L = 762.1\ \text{V rms}$
$I_L = 14.67\ \text{A rms}$
$P = 19.36\ \text{kW}$

P5.99* $\mathbf{I}_{aA} = 59.87\angle 0^\circ$
$\mathbf{V}_{An} = 322.44\angle -21.80^\circ$
$\mathbf{V}_{AB} = 558\angle 8.20^\circ$
$\mathbf{I}_{AB} = 34.56\angle 30^\circ$
$P_{load} = 26.89\ \text{kW}$
$P_{line} = 5.38\ \text{kW}$

P5.105* $\mathbf{V}_1 = 9.402\angle 29.58^\circ$
$\mathbf{V}_2 = 4.986\angle 111.45^\circ$

P5.107* $\mathbf{I}_1 = 1.372\angle 120.96^\circ$
$\mathbf{I}_2 = 1.955\angle 136.22^\circ$

Practice Test

T5.1 $$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{3}\int_0^2 (3t)^2 dt} = \sqrt{t^3\Big|_0^2} = \sqrt{8} = 2.828\ A$$

$$P = I_{rms}^2 R = 8(50) = 400\ \ \text{W}$$

T5.2 $$\mathbf{V} = 5\angle -45^\circ + 5\angle -30^\circ = 3.5355 - j3.5355 + 4.3301 - j2.5000$$

$$\mathbf{V} = 7.8657 - j6.0355 = 9.9144\angle -37.50^\circ$$

$$v(t) = 9.914\cos(\omega t - 37.50^\circ)$$

T5.3 (a) $V_{1rms} = \dfrac{15}{\sqrt{2}} = 10.61\ \text{V}$

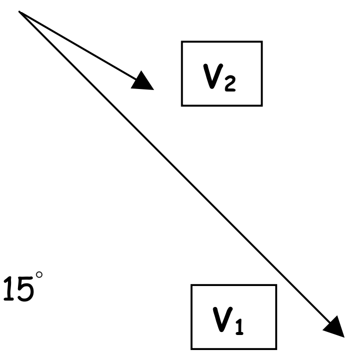
(b) $f = 200\ \ \text{Hz}$

(c) $\omega = 400\pi$ radians/s

(d) $T = 1/f = 5\ \ \text{ms}$

(e) $\mathbf{V}_1 = 15\angle -45^\circ$ and $\mathbf{V}_2 = 5\angle -30^\circ$

V$_1$ lags **V**$_2$ by 15° or **V**$_2$ leads **V**$_1$ by 15°



T5.4 $$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L - j/\omega C} = \frac{10\angle 0^\circ}{10 + j15 - j5} = \frac{10\angle 0^\circ}{14.14\angle 45^\circ} = 0.7071\angle -45^\circ\ \ A$$

$$\mathbf{V}_R = 10\mathbf{I} = 7.071\angle -45^\circ\ \text{V} \qquad \mathbf{V}_L = j15\mathbf{I} = 10.606\angle 45^\circ\ \ \text{V}$$

$$\mathbf{V}_C = -j5\mathbf{I} = 5.303\angle -135^\circ\ \ \text{V}$$

T5.5 $$\mathbf{S} = \tfrac{1}{2}\mathbf{VI}^* = \tfrac{1}{2}(440\angle 30^\circ)(25\angle 10^\circ) = 5500\angle 40^\circ = 4213 + j3535\ \ \text{VA}$$

$$P = \text{Re}(\mathbf{S}) = 4213\ \text{W}$$

$$Q = \text{Im}(\mathbf{S}) = 3535\ \text{VAR}$$

$$\text{Apparent power} = |\mathbf{S}| = 5500\ \text{VA}$$

$$\text{Power factor} = \cos(\theta_V - \theta_I) = \cos(40^\circ) = 76.6\%\ \ \text{lagging}$$

T5.6 We convert the delta to a wye and connect the neutral points with an ideal conductor.

$$Z_y = Z_\Delta / 3 = 2 + j8/3$$

$$Z_{total} = Z_{line} + Z_y = 0.3 + j0.4 + 2 + j2.667 = 2.3 + j3.067$$

$$Z_{total} = 3.833\angle 53.13°$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z_{total}} = \frac{208\angle 30°}{3.833\angle 53.13°} = 54.26\angle -23.13° \text{ A}$$

T5.7 The mesh equations are:

$$j10\mathbf{I}_1 + 15(\mathbf{I}_1 - \mathbf{I}_2) = 10\angle 45°$$

$$-j5\mathbf{I}_2 + 15(\mathbf{I}_2 - \mathbf{I}_1) = -15$$

In matrix form these become

$$\begin{bmatrix} (15+j10) & -15 \\ -15 & (15-j5) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45° \\ -15 \end{bmatrix}$$

The commands are:

```
Z = [(15+j*10)  -15; -15  (15-j*5)]
V = [pin(10,45); -15]
I = inv(Z)*V
pout(I(1))
pout(I(2))
```