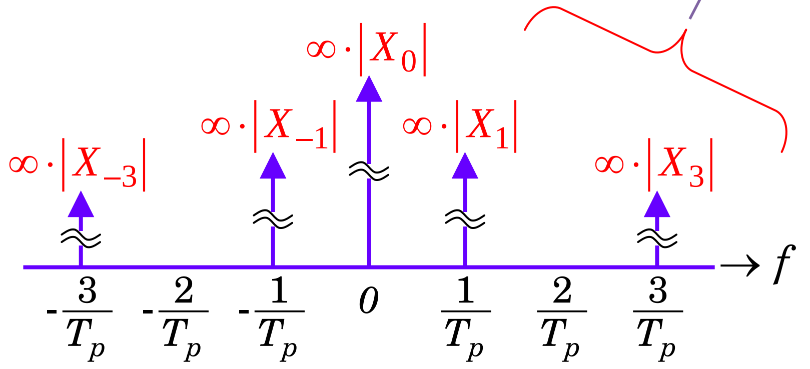
♠ ESD of $x_p(t)$:

$$E_x(f) = \underbrace{\sum_{k=-\infty}^{\infty} |X_k|^2 \delta^2\left(f - \frac{k}{T_p}\right) = \sum_{k=-\infty}^{\infty} \left(\infty \cdot |X_k|^2\right) \delta\left(f - \frac{k}{T_p}\right)}_{because\ \delta^2(\bullet)=\infty\cdot\delta(\bullet)} \Bigg\} \cdots \left[\begin{array}{l} \textit{with the understanding} \\ \textit{that } (\infty \cdot 0 = 0) \end{array}\right] \tag{3.7}$$

$\boldsymbol{E_x(f)}$

(Continuous-frequency)

Errata



♠ Total Energy of $x_p(t)$:

$$E = \underbrace{\int_{-\infty}^{\infty} |x_p(t)|^2 dt = \int_{-\infty}^{\infty} E_x(f)\, df}_{\textbf{Rayleigh Energy Theorem}} = \infty \tag{3.8}$$

***Proof* (optional):**

Errata

$$E=\int_{-\infty}^{\infty}\left|x_p(t)\right|^2 dt$$

$$=\int_{-\infty}^{\infty}\left[\mathfrak{I}^{-1}\left\{\sum_{k=-\infty}^{\infty}X_k\delta\left(f-\frac{k}{T_p}\right)\right\}\right]\cdot\left[\mathfrak{I}^{-1}\left\{\sum_{l=-\infty}^{\infty}X_l\delta\left(f-\frac{l}{T_p}\right)\right\}\right]^* dt$$

$$=\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}X_k\delta\left(f-\frac{k}{T_p}\right)\cdot e^{j2\pi ft}df\right]\cdot\left[\int_{-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_l^*\delta\left(\tilde{f}-\frac{l}{T_p}\right)\cdot e^{-j2\pi\tilde{f}t}d\tilde{f}\right]dt$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_kX_l^*\delta\left(f-\frac{k}{T_p}\right)\delta\left(\tilde{f}-\frac{l}{T_p}\right)\cdot\underbrace{\int_{-\infty}^{\infty}e^{j2\pi\left(f-\tilde{f}\right)t}dt}_{\delta\left(f-\tilde{f}\right)}\right]d\tilde{f}df$$

$$=\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_kX_l^*\delta\left(f-\frac{k}{T_p}\right)\delta\left(f-\frac{l}{T_p}\right)df$$

$$\overbrace{=\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\left|X_k\right|^2\delta^2\left(f-\frac{k}{T_p}\right)df=\int_{-\infty}^{\infty}\underbrace{\sum_{k=-\infty}^{\infty}\left(\infty\cdot\left|X_k\right|^2\right)\delta\left(f-\frac{k}{T_p}\right)}_{ESD:\ E_x(f)}}^{\textit{since}\ \ \delta^2(\bullet)=\infty\cdot\delta(\bullet)}df=\infty$$

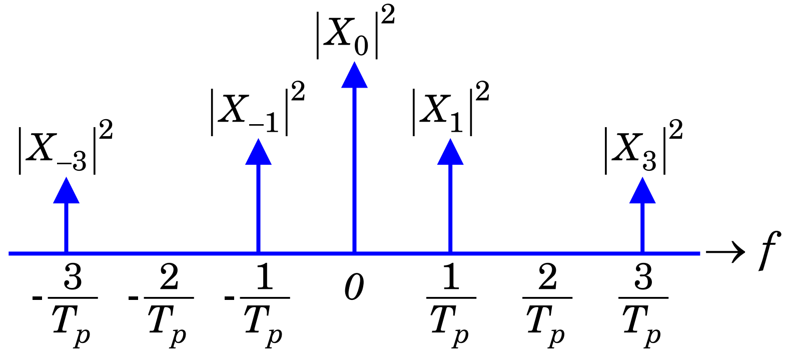
♠ PSD of $x_p(t)$:

Errata

$$P_x(f) = \sum_{k=-\infty}^{\infty} |X_k|^2 \delta\left(f - \frac{k}{T_p}\right) \tag{3.9}$$

$P_x(f)$

(Continuous-frequency)



♠ Average power of $x_p(t)$:

$$P = \underbrace{\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} P_x(f)\, df}_{\textbf{Parseval Power Theorem}} = \sum_{\tilde{k}=-\infty}^{\infty} |X_k|^2 \tag{3.10}$$

***Proof* (optional):**

Errata

$$
\begin{aligned}
P &= \frac{1}{T_p}\int_0^{T_p}\left|x_p(t)\right|^2 dt \\
&= \frac{1}{T_p}\int_{-0.5T_p}^{0.5T_p}\Im^{-1}\left\{\sum_{k=-\infty}^{\infty}X_k\delta\left(f-k/T_p\right)\right\}\left[\Im^{-1}\left\{\sum_{l=-\infty}^{\infty}X_l\delta\left(f-l/T_p\right)\right\}\right]^* dt \\
&= \frac{1}{T_p}\int_{-0.5T_p}^{0.5T_p}\left[\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}X_k\delta\left(f-k/T_p\right)\cdot e^{j2\pi ft}df\right]\left[\int_{-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_l^*\delta\left(\tilde{f}-l/T_p\right)\cdot e^{-j2\pi\tilde{f}t}d\tilde{f}\right]dt \\
&= \int_{-\infty}^{\infty}\left\{\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_kX_l^*\delta\left(f-k/T_p\right)\left[\frac{1}{T_p}\int_{-0.5T_p}^{0.5T_p}\left\{\int_{-\infty}^{\infty}\delta\left(\tilde{f}-l/T_p\right)e^{j2\pi\left(f-\tilde{f}\right)t}d\tilde{f}\right\}dt\right]\right\}df \\
&= \int_{-\infty}^{\infty}\left\{\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_kX_l^*\delta\left(f-k/T_p\right)\left[\frac{1}{T_p}\int_{-0.5T_p}^{0.5T_p}e^{j2\pi\left(f-l/T_p\right)t}dt\right]\right\}df \\
&= \int_{-\infty}^{\infty}\left\{\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}X_kX_l^*\delta\left(f-k/T_p\right)\mathrm{sinc}\left(k-l\right)\right\}df \\
&= \int_{-\infty}^{\infty}\underbrace{\left\{\sum_{k=-\infty}^{\infty}\left|X_k\right|^2\delta\left(f-k/T_p\right)\right\}}_{PSD:\ P_x(f)}df = \sum_{k=-\infty}^{\infty}\left|X_k\right|^2\int_{-\infty}^{\infty}\delta\left(f-k/T_p\right)df = \sum_{k=-\infty}^{\infty}\left|X_k\right|^2
\end{aligned}
$$