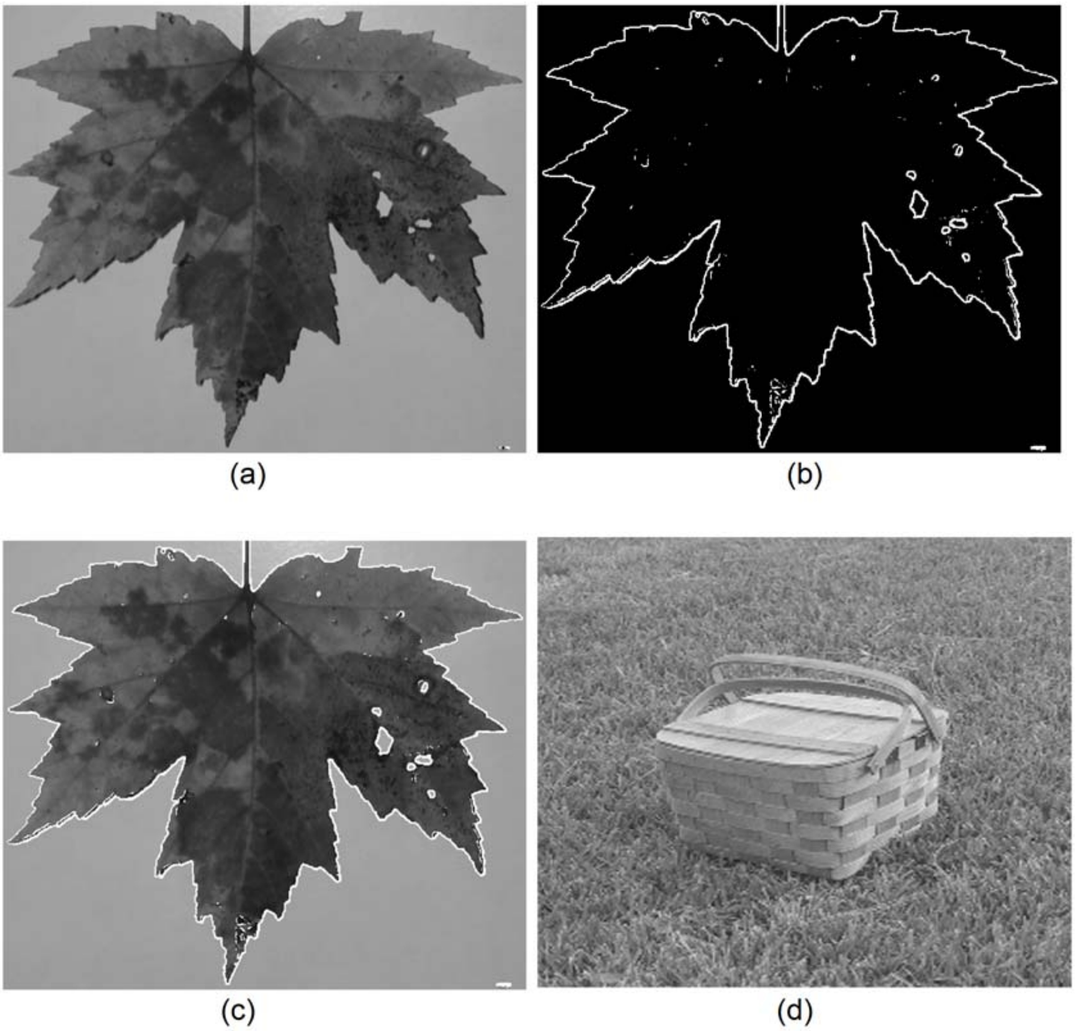
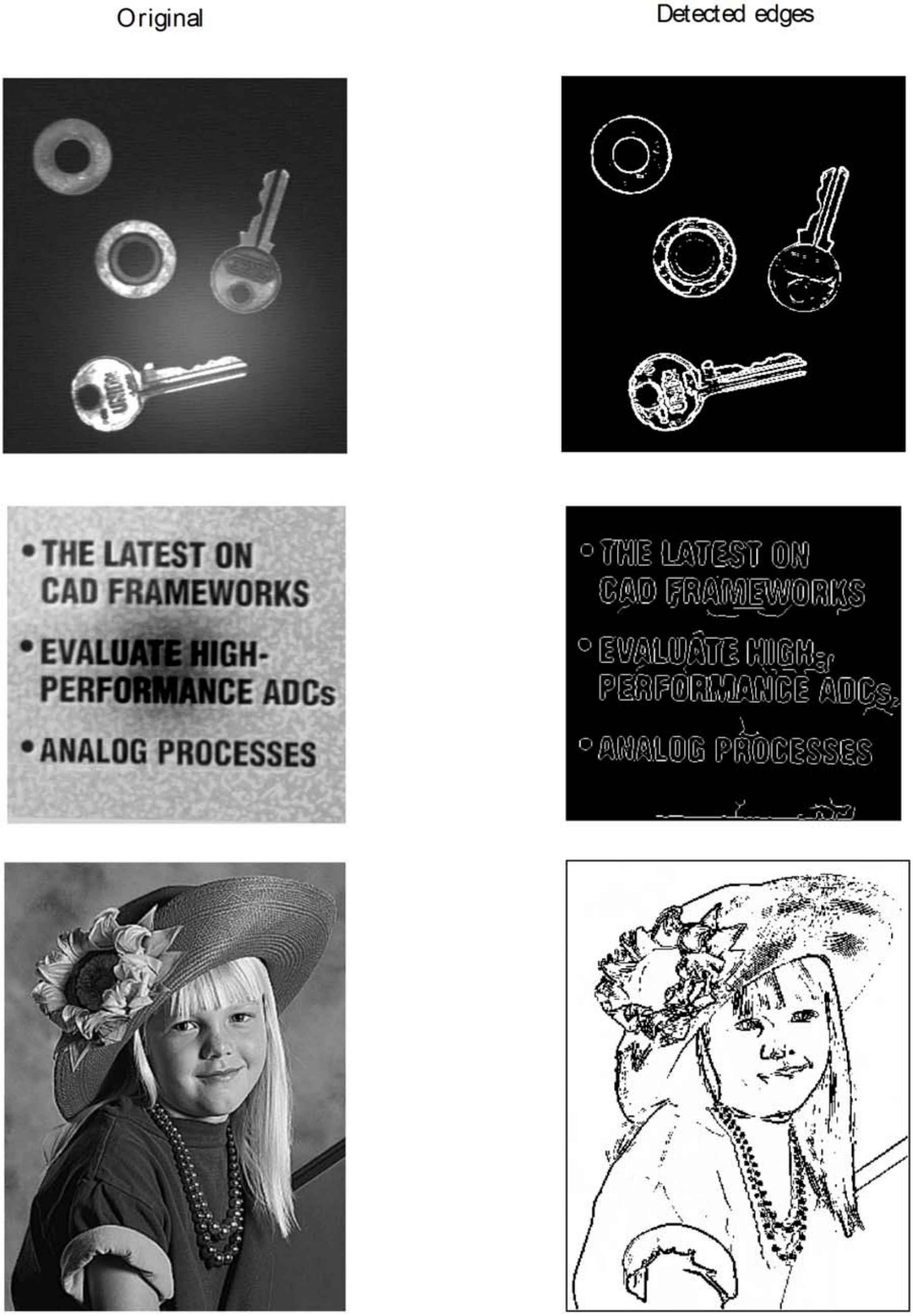
6 – EDGE DETECTION

Changes or discontinuities in an image attribute such as luminance or texture are important primitive features of an image since they often provide an indication of the physical extent of objects within the image. Edge detection contributes significantly to algorithms for feature detection, segmentation, and motion analysis.

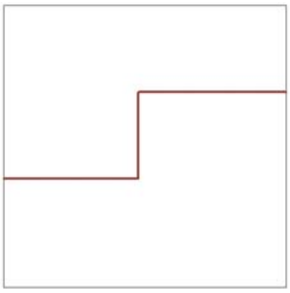
In this course, we consider edges to be local discontinuities in intensity.



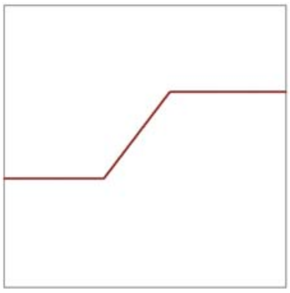
(a) Leaf (b) Detected edges (c) Edges superimposed on original (d) Texture example


Original
Detected edges
• THE LATEST ON
CAD FRAMEWORKS
• EVALUATE HIGH-
PERFORMANCE ADCs
• ANALOG PROCESSES

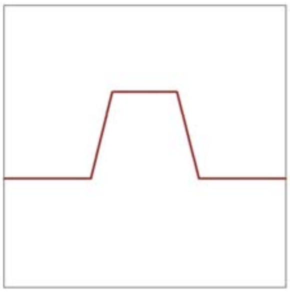
Example - Edge profiles



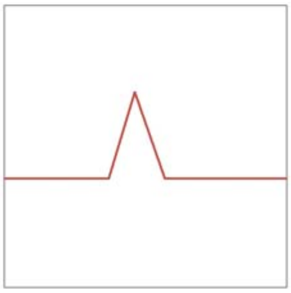
Step edge



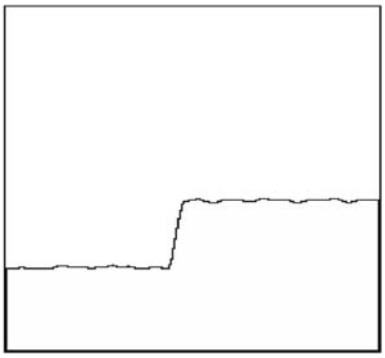
Ramp edge



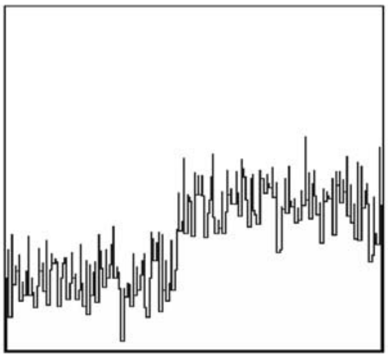
Line edge



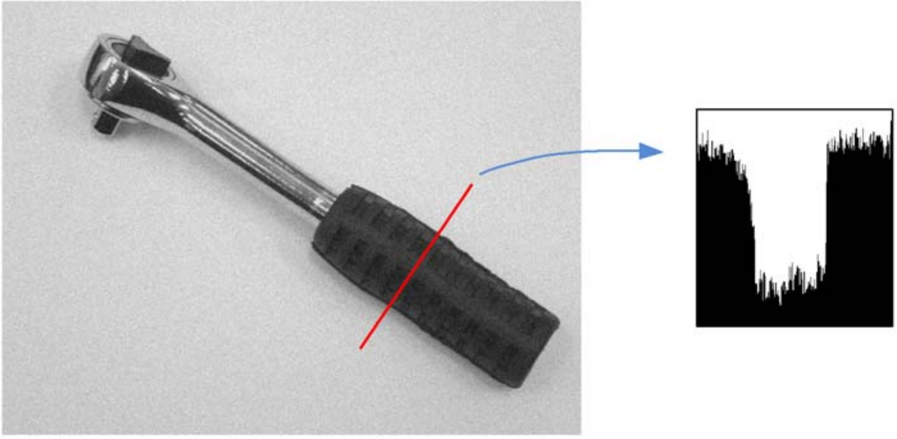
Roof edge



Example of a “real” edge



Noisy edge



Gradient Operators

For an image function $f(x,y)$, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are the rates of change in the x and y directions, respectively. The gradient at any point (x,y) is defined as

$$\mathbf{G}(x,y) = \begin{bmatrix} \partial f(x,y)/\partial x \\ \partial f(x,y)/\partial y \end{bmatrix} \equiv \begin{bmatrix} G_x \\ G_y \end{bmatrix} \tag{1}$$

The magnitude of the gradient is

$$|\mathbf{G}(x,y)| = [G_x^2 + G_y^2]^{1/2}. \tag{2}$$

An approximation is

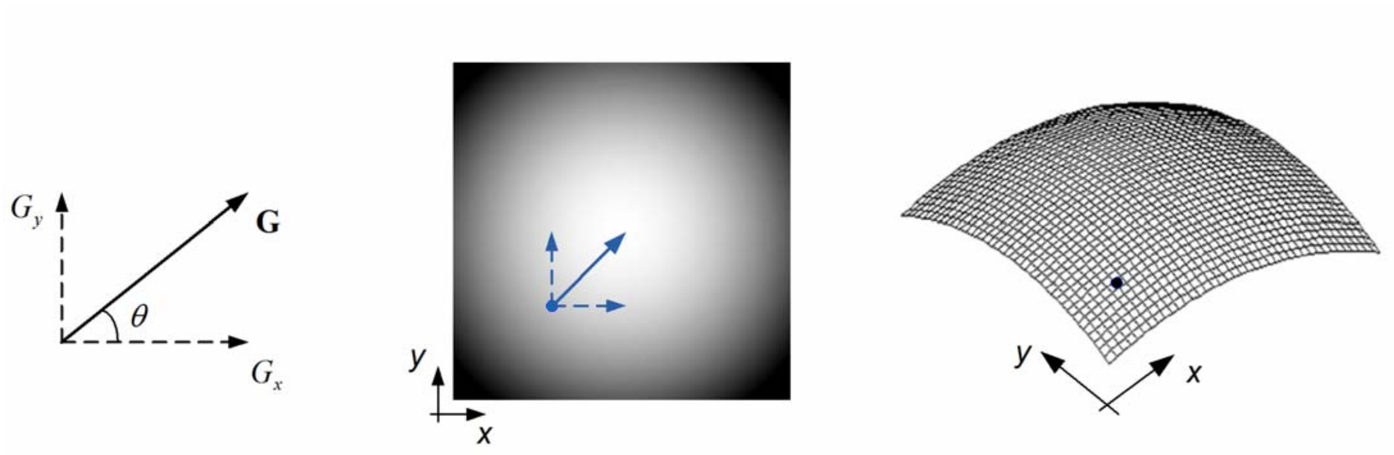
$$|\mathbf{G}(x,y)| \approx |G_x| + |G_y|. \tag{3}$$

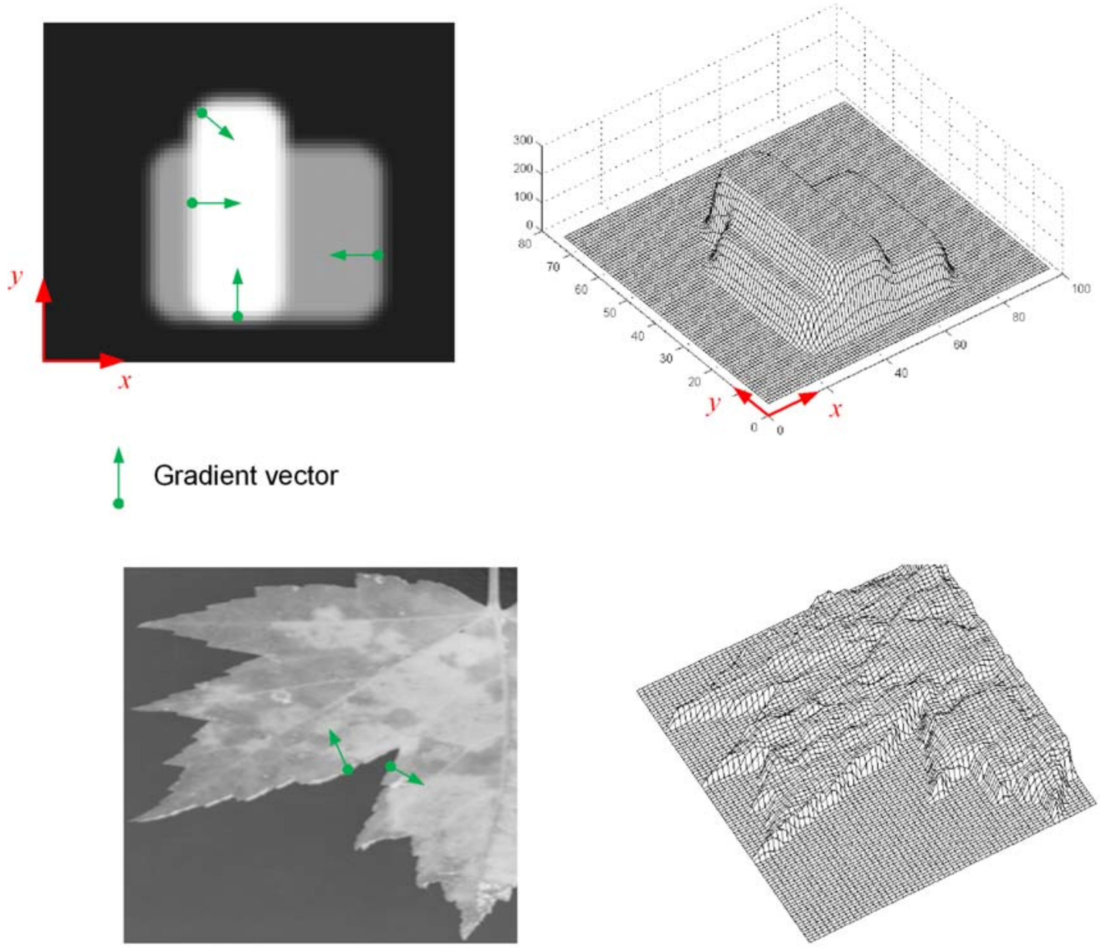
The direction of the gradient vector $\mathbf{G}$ at (x,y) measured with respect to the x axis is

$$\theta(x,y) = \tan^{-1}(G_y/G_x) \tag{4}$$

It can be shown that the gradient vector points in the direction of maximum rate of change of f at location (x,y).

An edge is a pixel whose gradient magnitude is "sufficiently large".





How do we obtain the gradient at any point in a digital image?

We can use the forward derivative derivative approximation, D_1, defined as

$$D_{1x} = \frac{f(x + \Delta x, y) - f(x)}{\Delta x} \tag{5}$$

to obtain

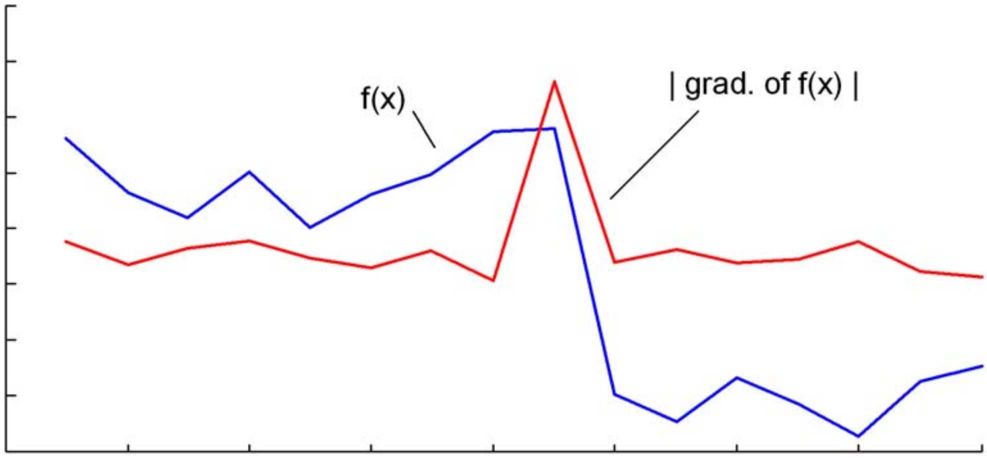
$$\frac{\partial f(x,y)}{\partial x} \approx D_{1x} = \frac{f(x + \Delta x, y) - f(x,y)}{\Delta x} \tag{6}$$

$$\frac{\partial f(x,y)}{\partial y} \approx D_{1y} = \frac{f(x, y + \Delta y) - f(x,y)}{\Delta y} \tag{7}$$

This corresponds to masking operations with

$$[-1 \quad 1] \qquad \text{and} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

(Δx and Δy may be set to 1, x axis points to the right, y axis points up.)

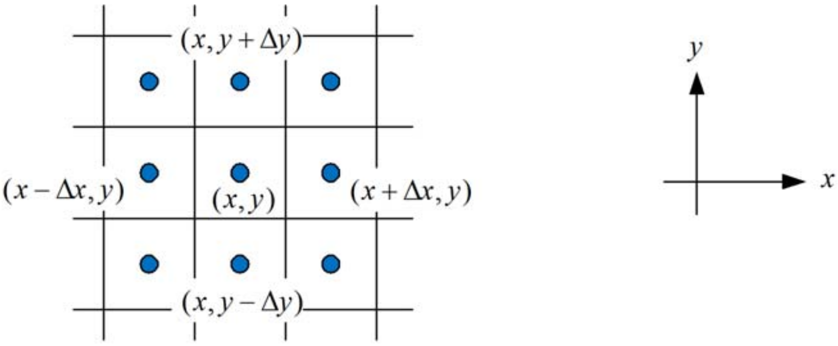


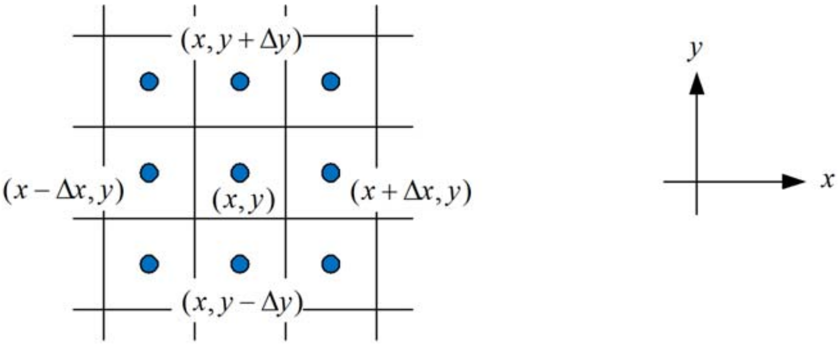
Better estimates are obtained using the *centred* difference approximation

$$D_{2x} = \frac{f(x+\Delta x, y) - f(x-\Delta x, y)}{2(\Delta x)} \tag{8}$$

equivalent to masking operations with

$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$





Variants on D_2 are the Prewitt masks:

$$D_{Px} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D_{Py} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

which form smoothed, centred difference operators.

The *Sobel* weighting masks emphasize the central pixel:

$$D_{Sx} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D_{Sy} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

The Sobel edge filter provides good edge detection and is somewhat insensitive to noise in the image.

The *Roberts* operator is defined by

$$D_+ = f(x+\Delta x, y+\Delta y) - f(x, y) \qquad (9)$$
$$D_- = f(x, y+\Delta y) - f(x+\Delta x, y) \qquad (10)$$

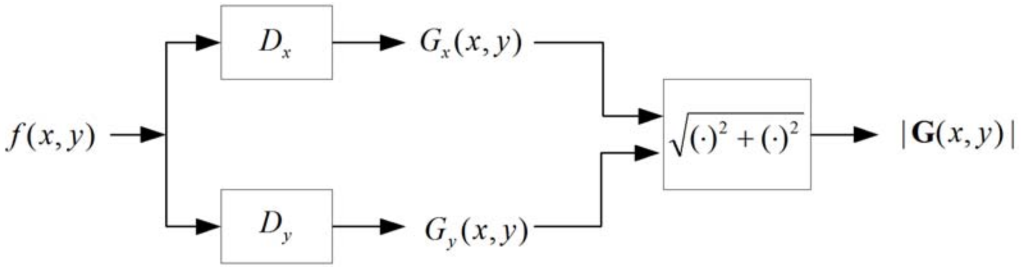
corresponding to filtering the image function with masks

$$D_+ = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \text{and} \qquad D_- = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The centre of each mask may be taken to be the lower left pixel.

The output image $G(x, y)$ can be set equal to the gradient of the input image $f(x, y)$ at that point:

$$G(x, y) = |\mathbf{G}[(x, y)]|. \tag{11}$$

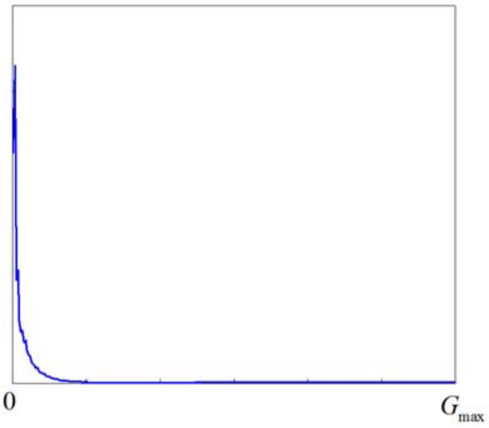


To obtain an edge map:

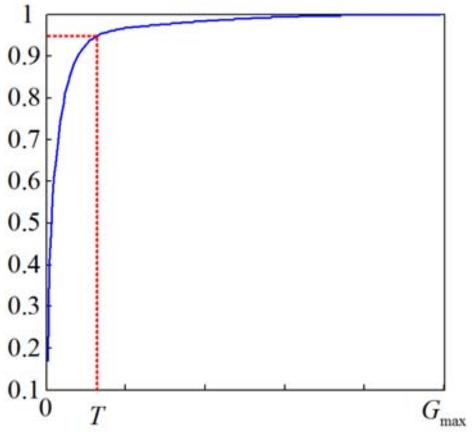
$$G(x, y) = \begin{cases} 1 & \text{if } |\mathbf{G}[(x, y)]| > T \\ 0 & \text{if } |\mathbf{G}[(x, y)]| \leq T. \end{cases} \tag{12}$$

The edge map is used for obtaining the object boundaries in an image. A suitable threshold T may be selected using the cumulative histogram of $G(x, y)$; typically, about 5 to 10% of pixels with the largest gradients are declared as edges.

Grad. magnitude image

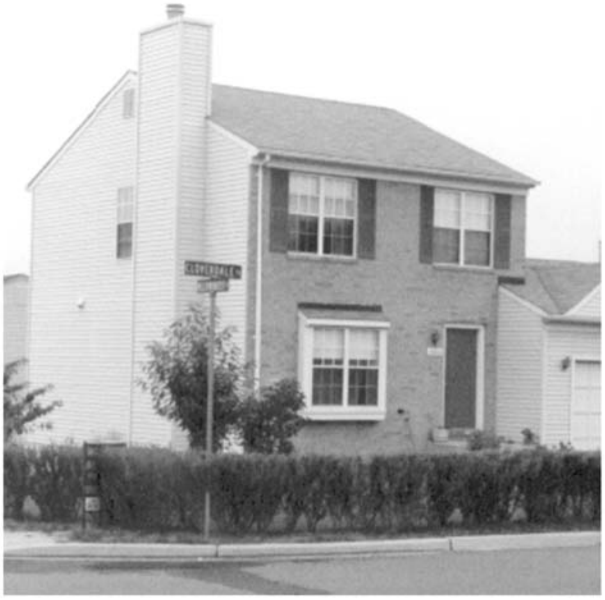


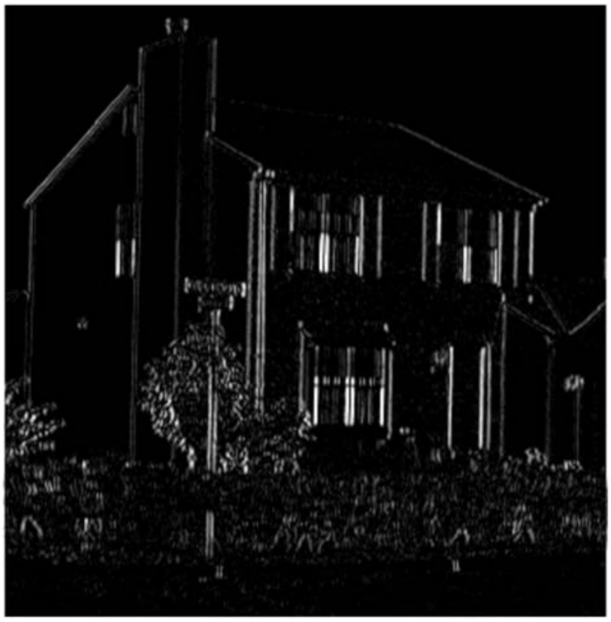
Histogram of grad. magnitude



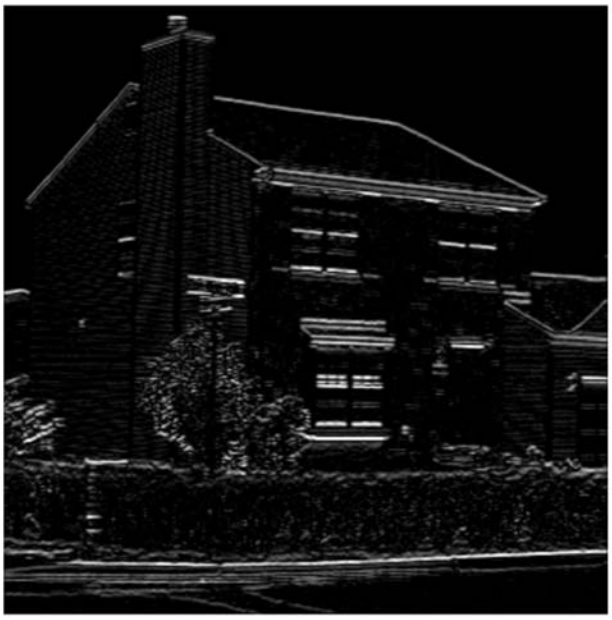
Cumulative histogram of grad. magnitude

Edge detection

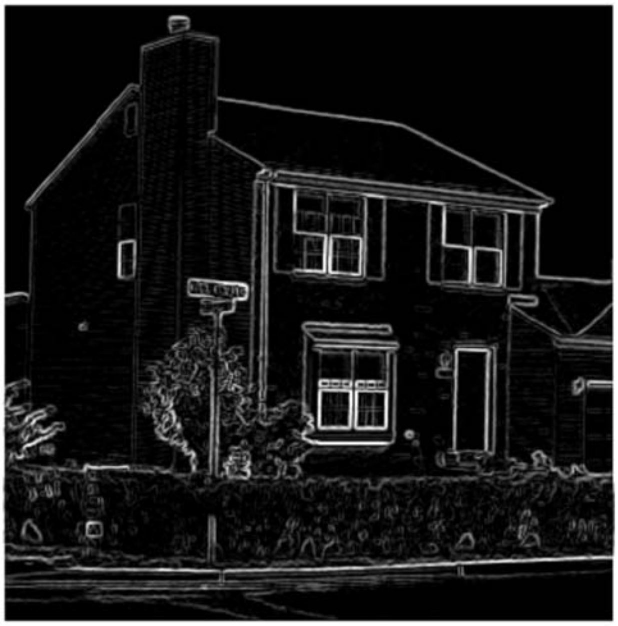




$|G_x|$

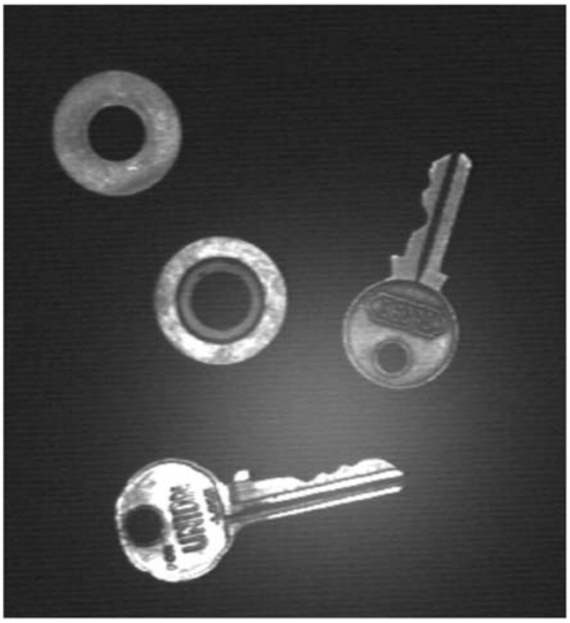


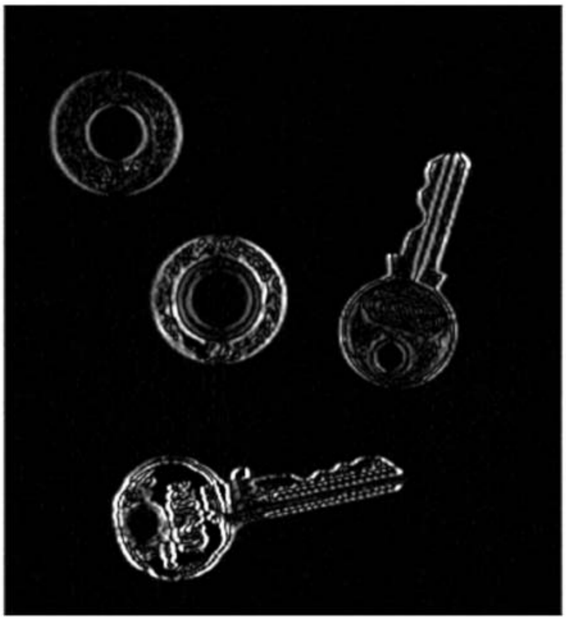
$|G_y|$



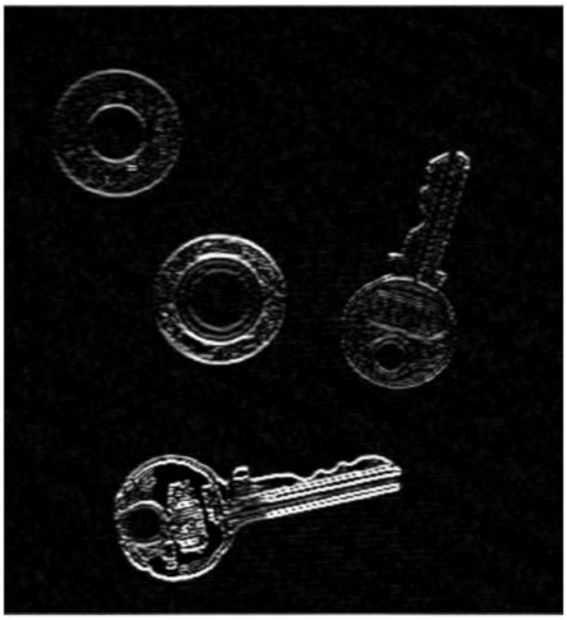
$\left(G_x^2 + G_y^2\right)^{1/2}$

Edge detection

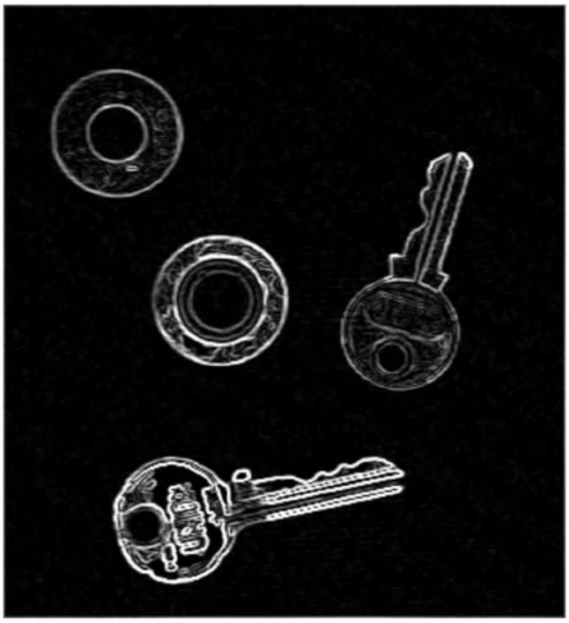




$|G_x|$

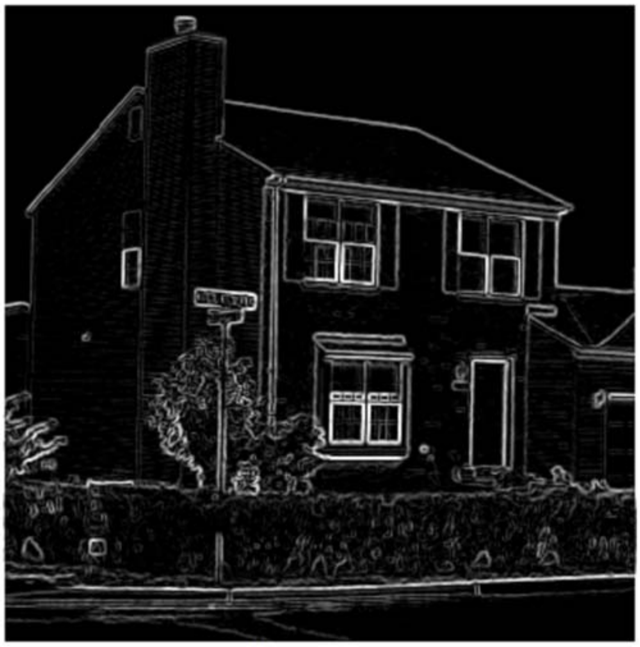


$|G_y|$

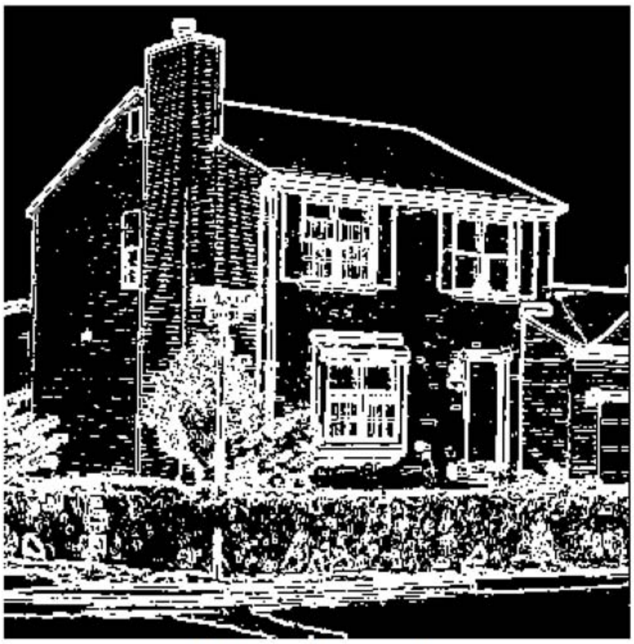


$\left(G_x^2 + G_y^2\right)^{1/2}$

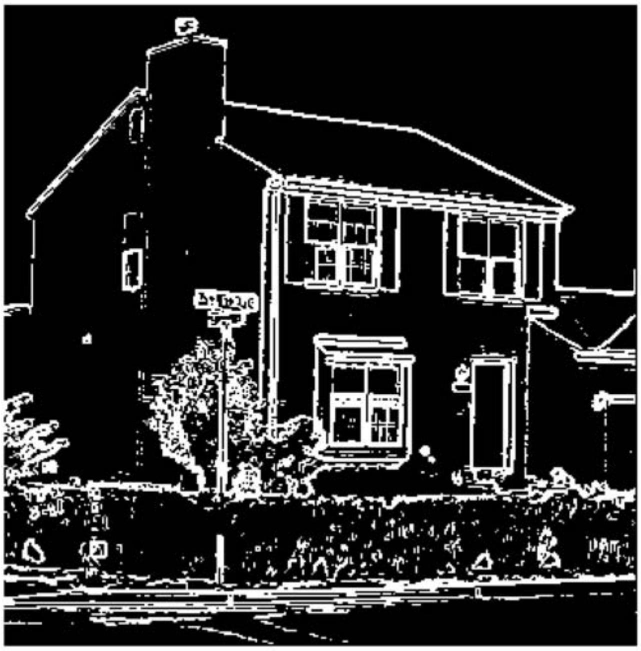
Edge map



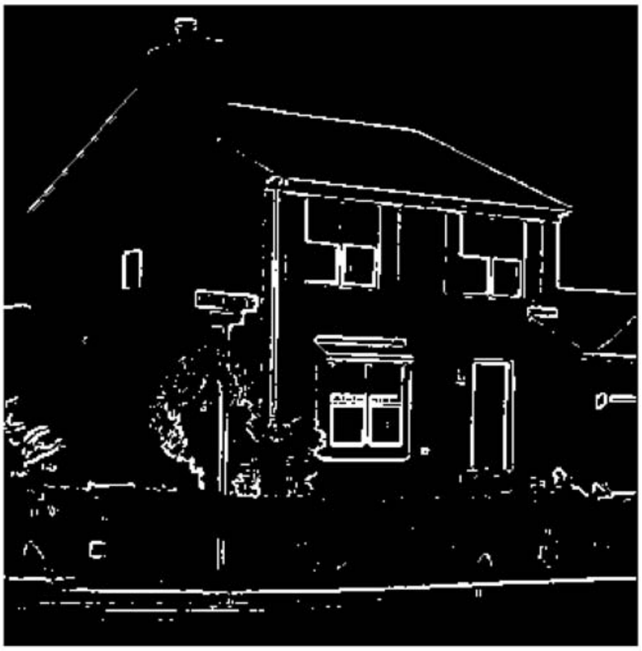
Gradient magnitude



With low threshold



With moderate threshold

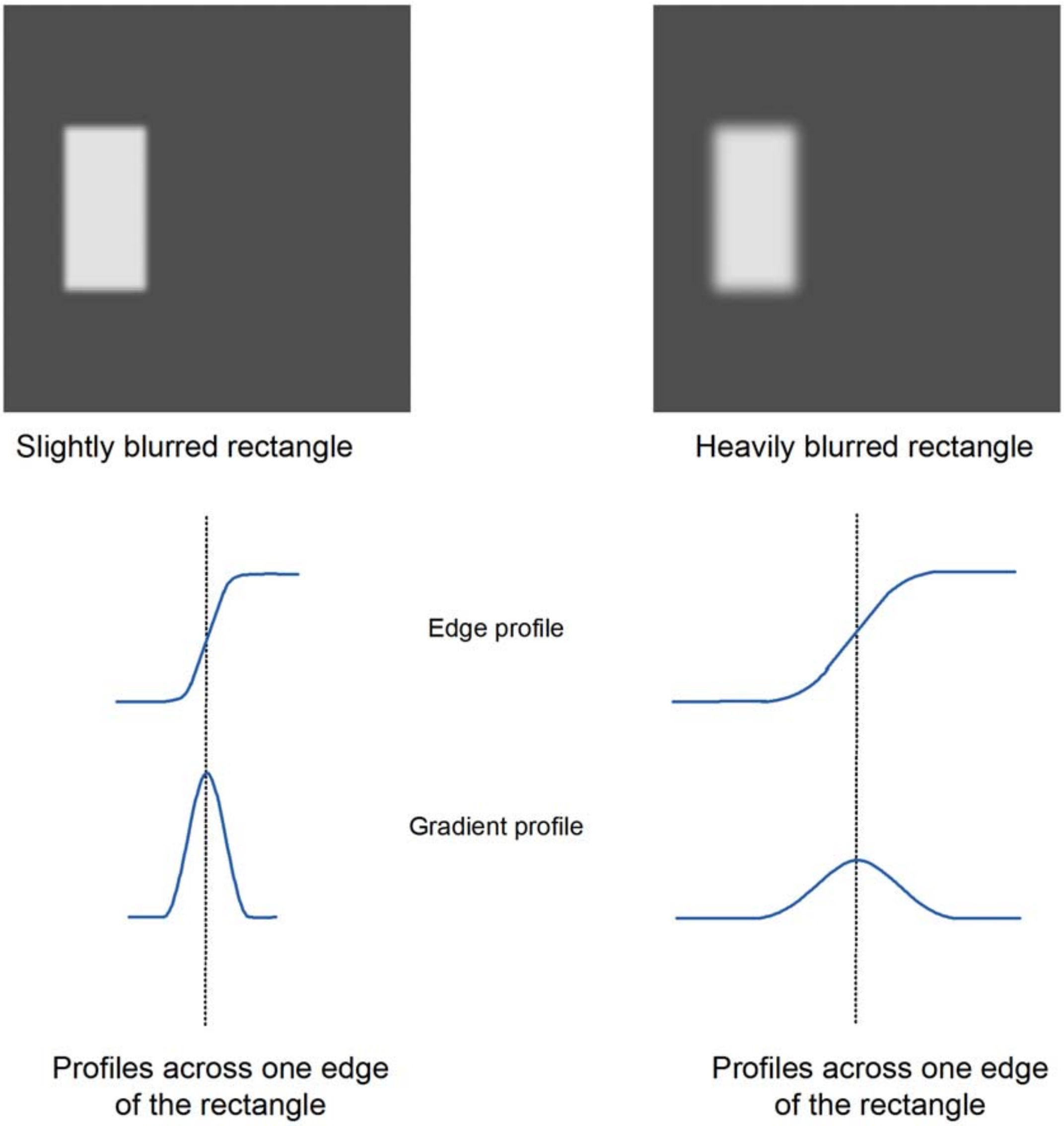


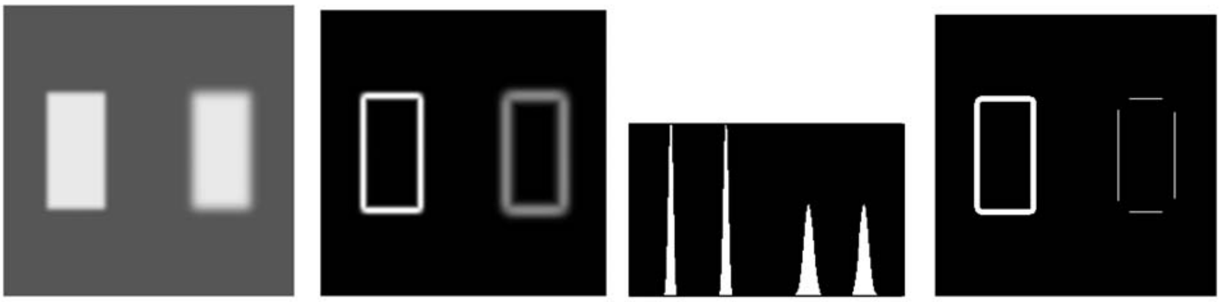
With high threshold

Problems

Problems may be encountered with the use of gradient operators:

1. Boundaries may be diffused instead of sharp; the former have lower gradients than the latter, thus complicating the threshold selection.


Slightly blurred rectangle
Heavily blurred rectangle
Edge profile
Gradient profile
Profiles across one edge of the rectangle
Profiles across one edge of the rectangle

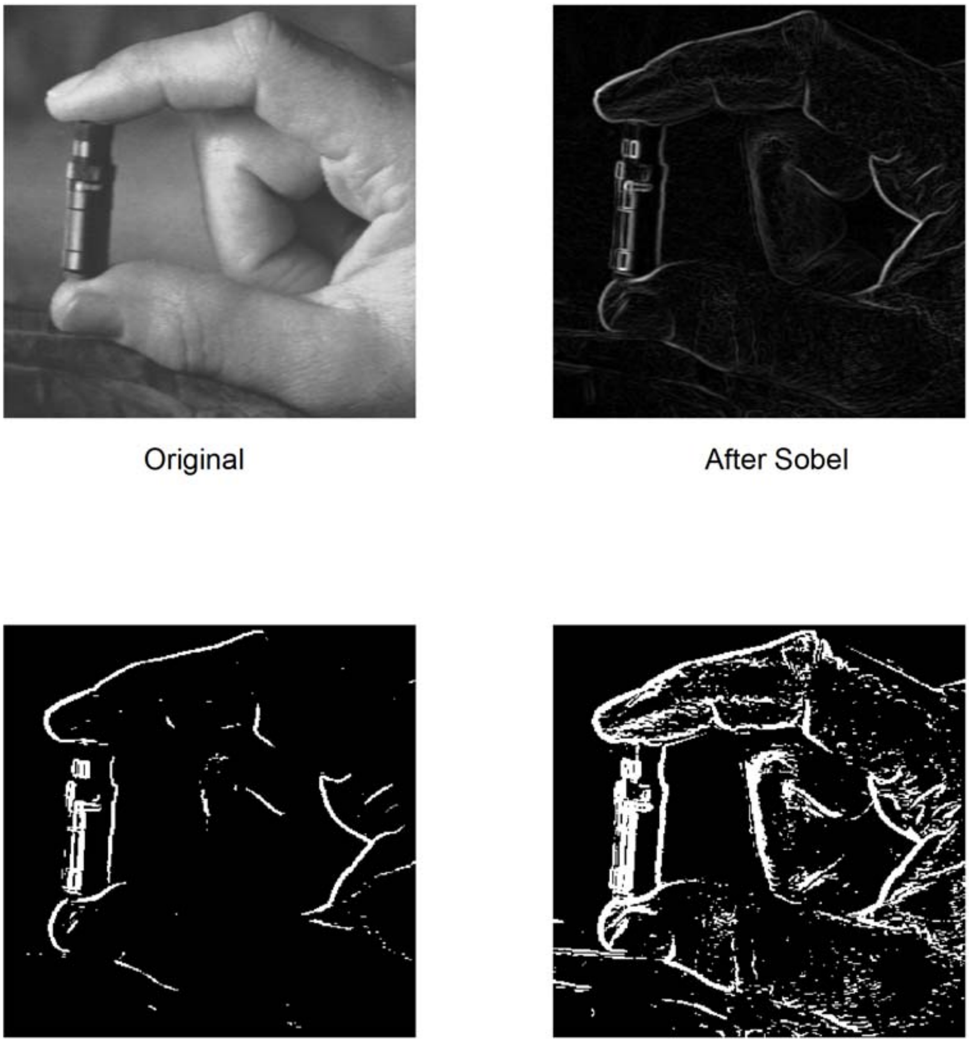


Original

After Sobel

Profile

After thresholding



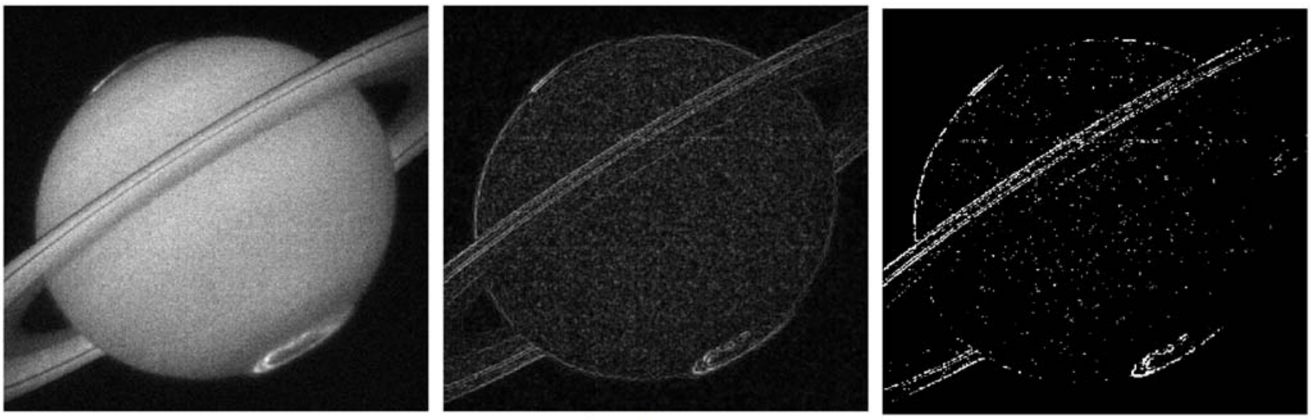
Original

After Sobel

High threshold

Low threshold

2. Noise can result in stronger gradients than meaningful edges.

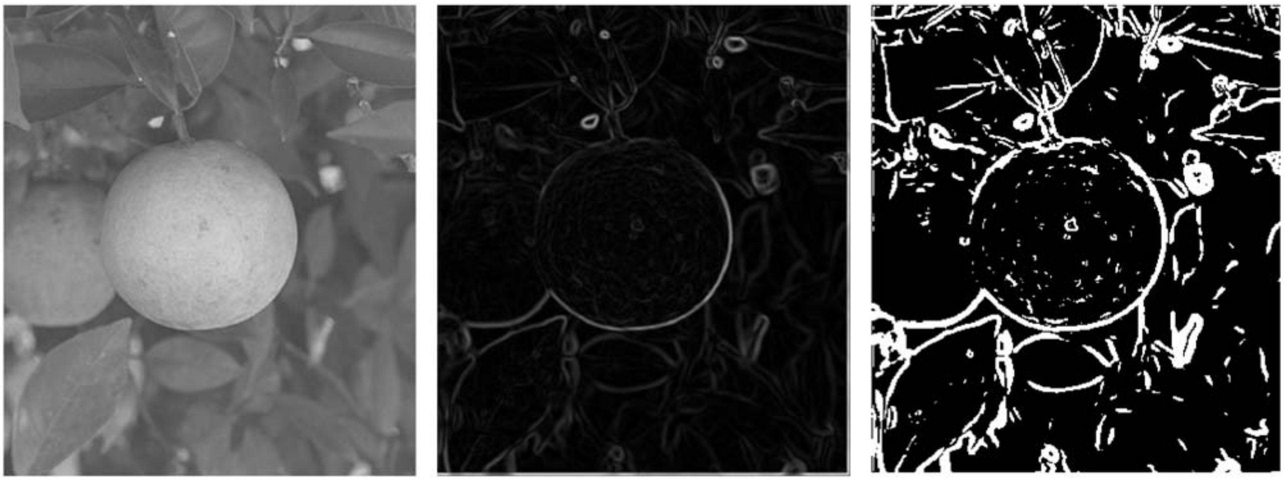


Original

Gradient magnitude

Edge map

3. Edges after thresholding are often thick.



Original

Gradient magnitude

Edge map

More sophisticated algorithms (e.g., Canny operator) can be used to give better results.

Laplacian Operator

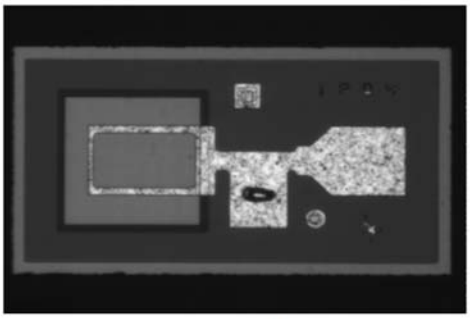
The Laplacian is a second-order derivative operator defined as

$$L[f(x,y)] = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{13}$$

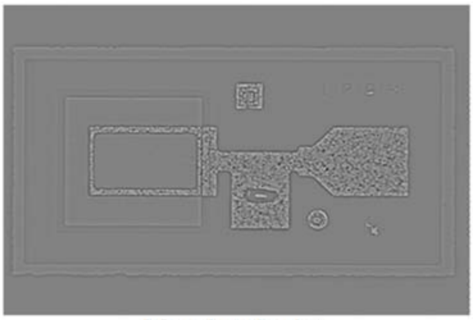
A Laplacian mask is

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The Laplacian operator is seldom used by itself for edge detection because it is unacceptably sensitive to noise. It is used in conjunction with a Gaussian filter for edge detection. This is known as a Laplacian of Gaussian (LOG) filter.

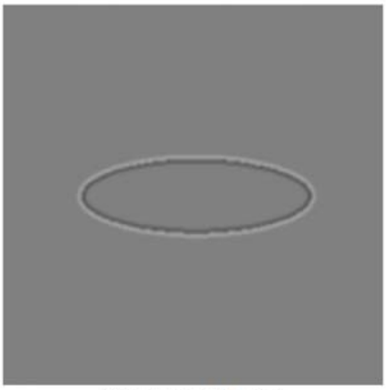


Original

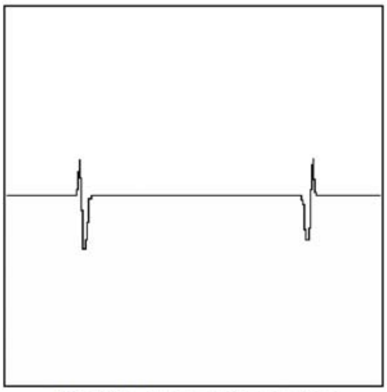


After Laplacian
(gray levels + 128)

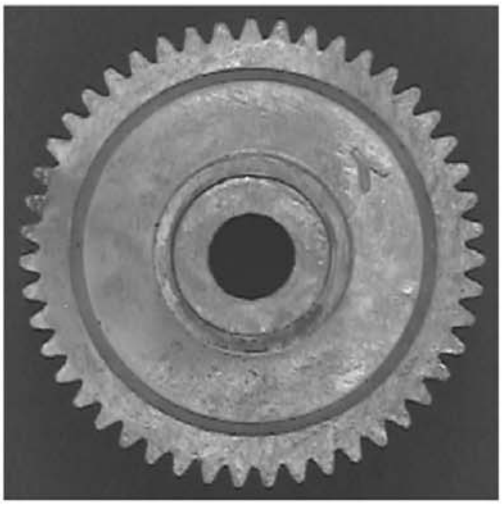
Original



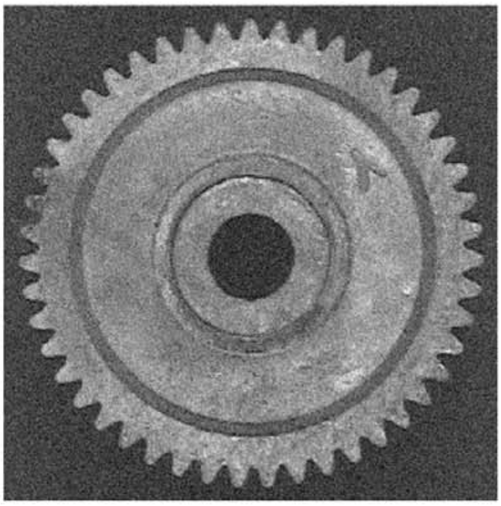
After Laplacian
(gray levels + 128)



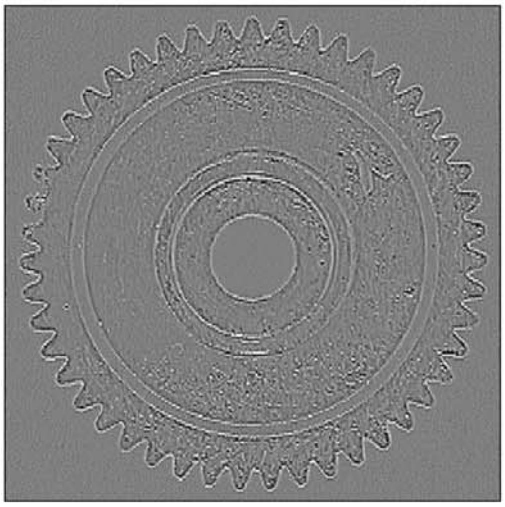
Profile through center



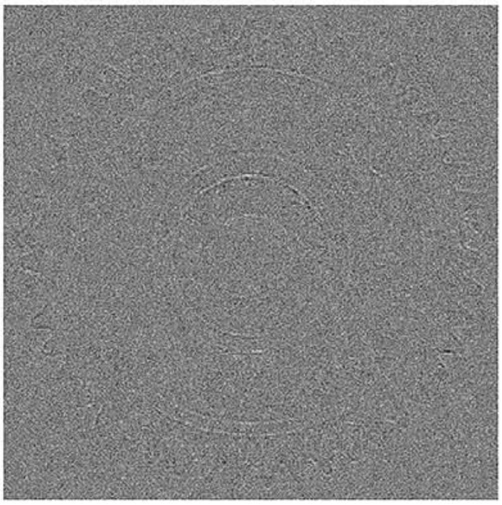
Original



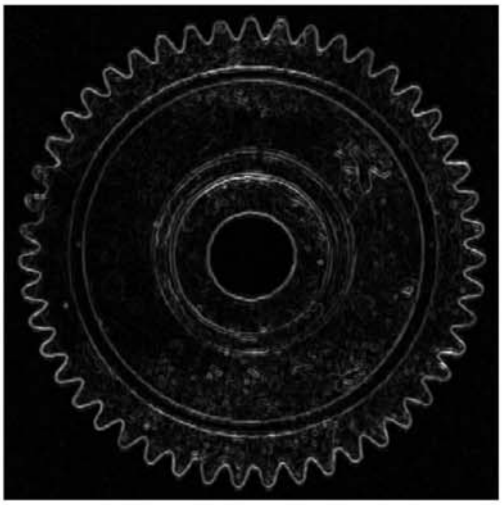
With Gaussian noise



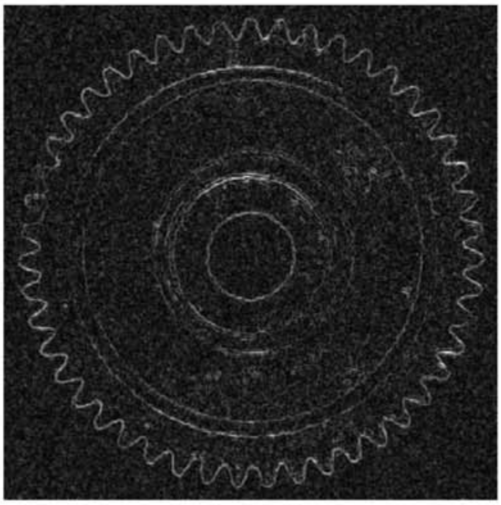
Laplacian



Laplacian



Sobel



Sobel