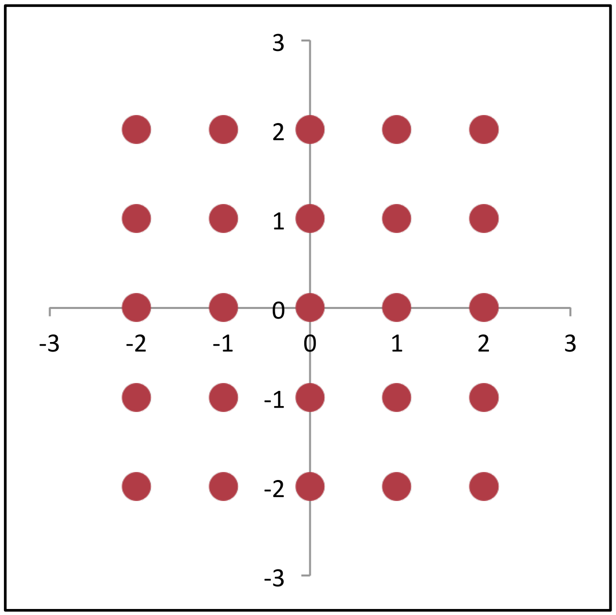
Spatial Transformation

- Simplest form: affine transformation

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

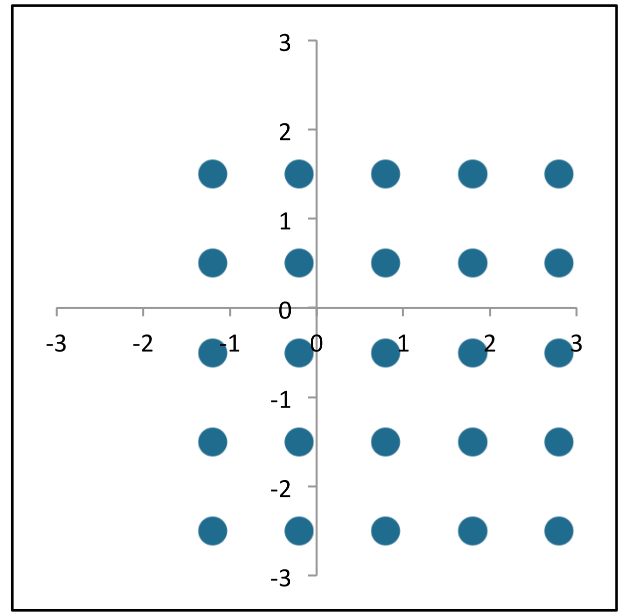
Affine Transformation

Source (x, y)



$$\begin{bmatrix} 1 & 0 & 0.8 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

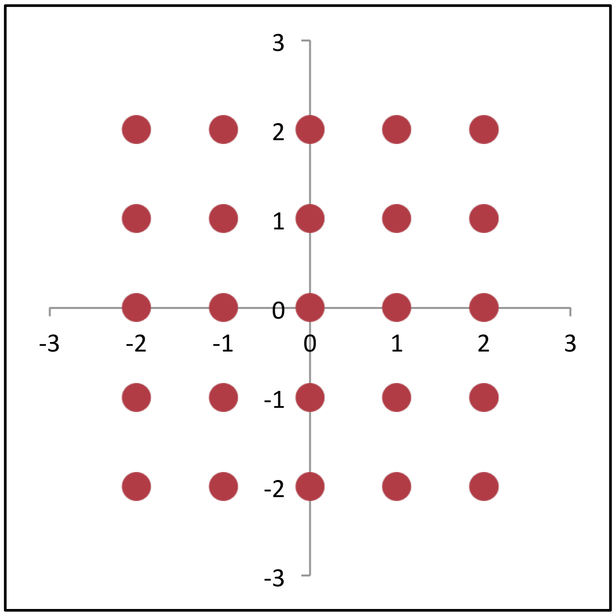
Target (u, v)



translation

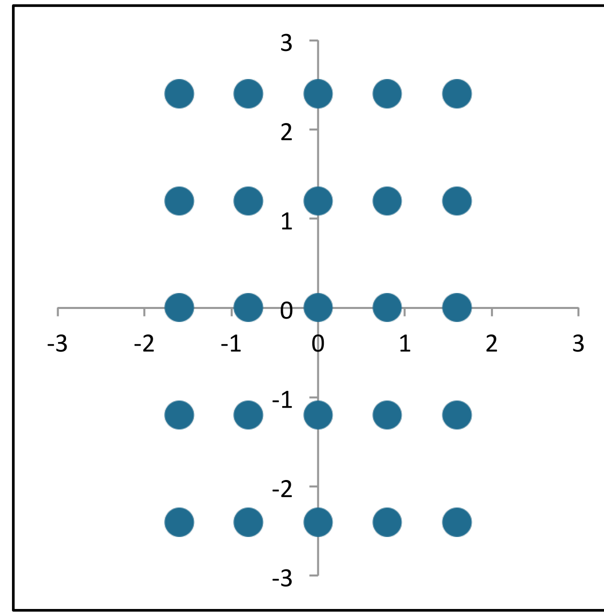
Affine Transformation

Source (x, y)



$$\begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

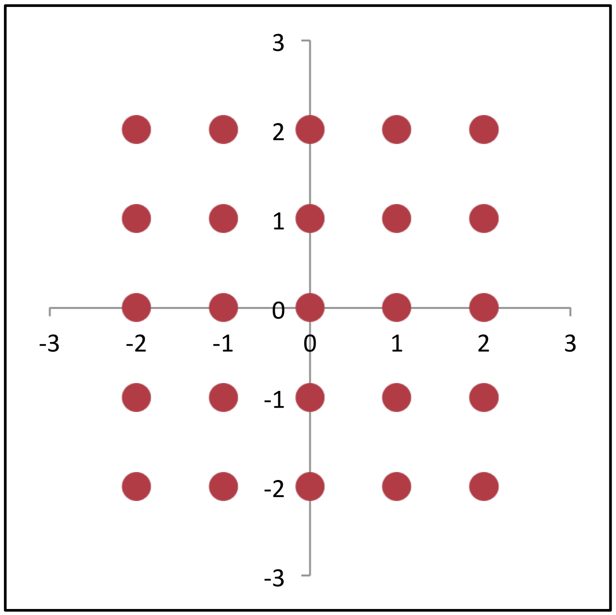
Target (u, v)



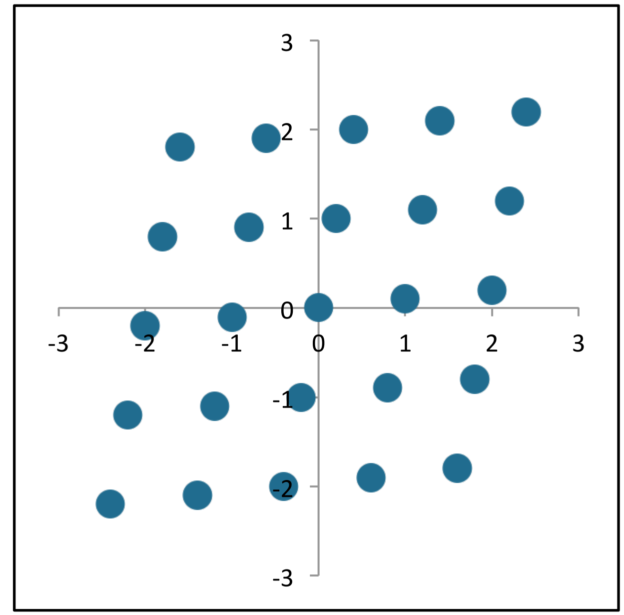
scaling

Affine Transformation

Source (x, y) | Target (u, v)



$$\begin{bmatrix} 1 & 0.2 & 0 \\ 0.1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

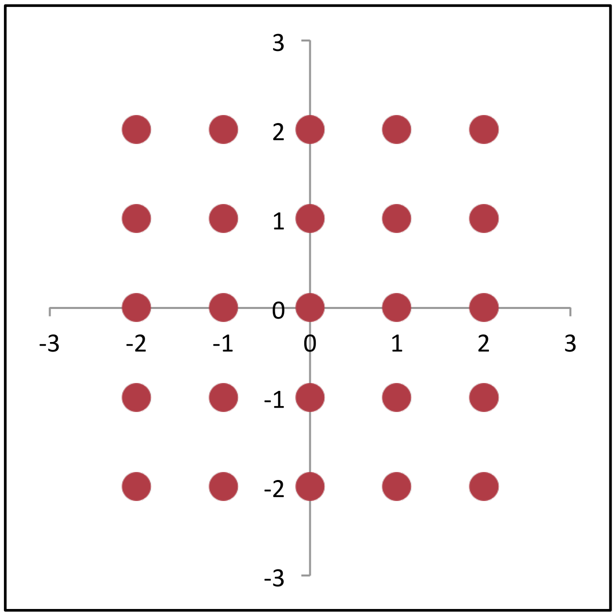


shearing

parallel lines remain parallel

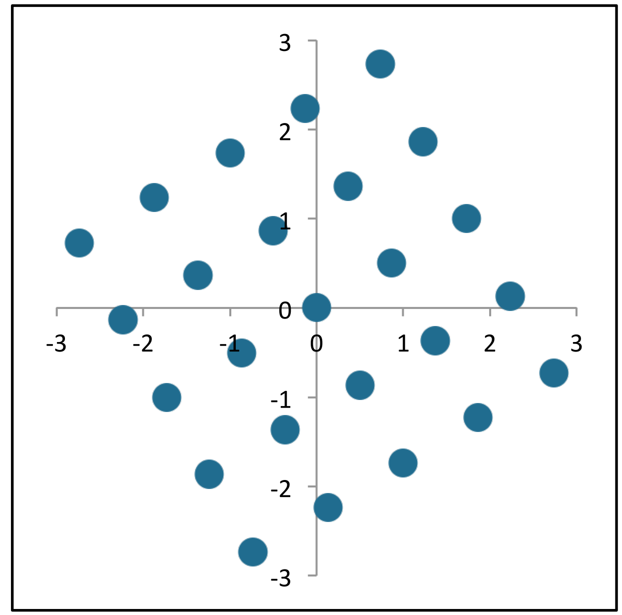
Affine Transformation

Source (x, y)



$$\begin{bmatrix} \cos & -\sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Target (u, v)



rotation

parallel lines remain parallel

Affine Transformation

- To solve for affine matrix:
  - For $i = 1, \ldots, n$, arrange into two matrix equations:

$$\begin{bmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

  - Then, solve each equation using linear least square.

Image Warping Example

- With affine mapping

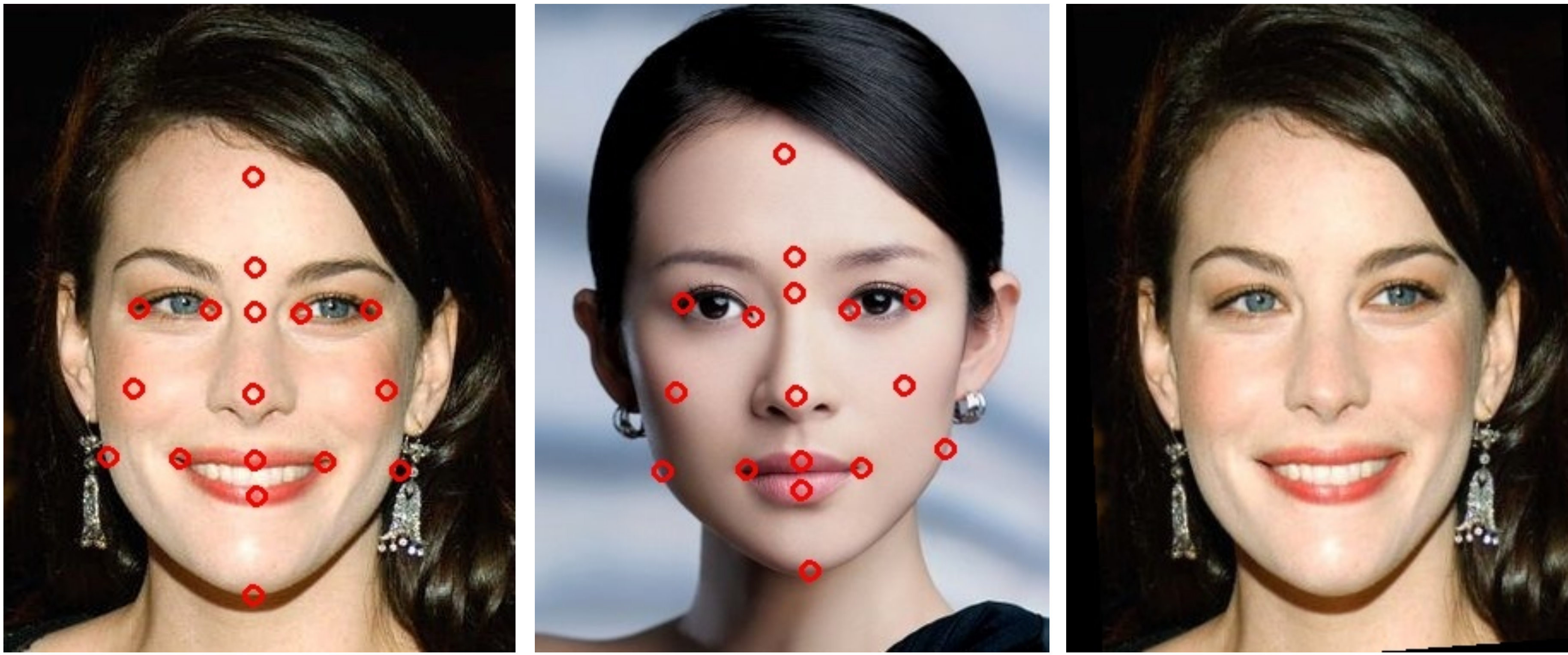


Image Warping Example

- With affine mapping

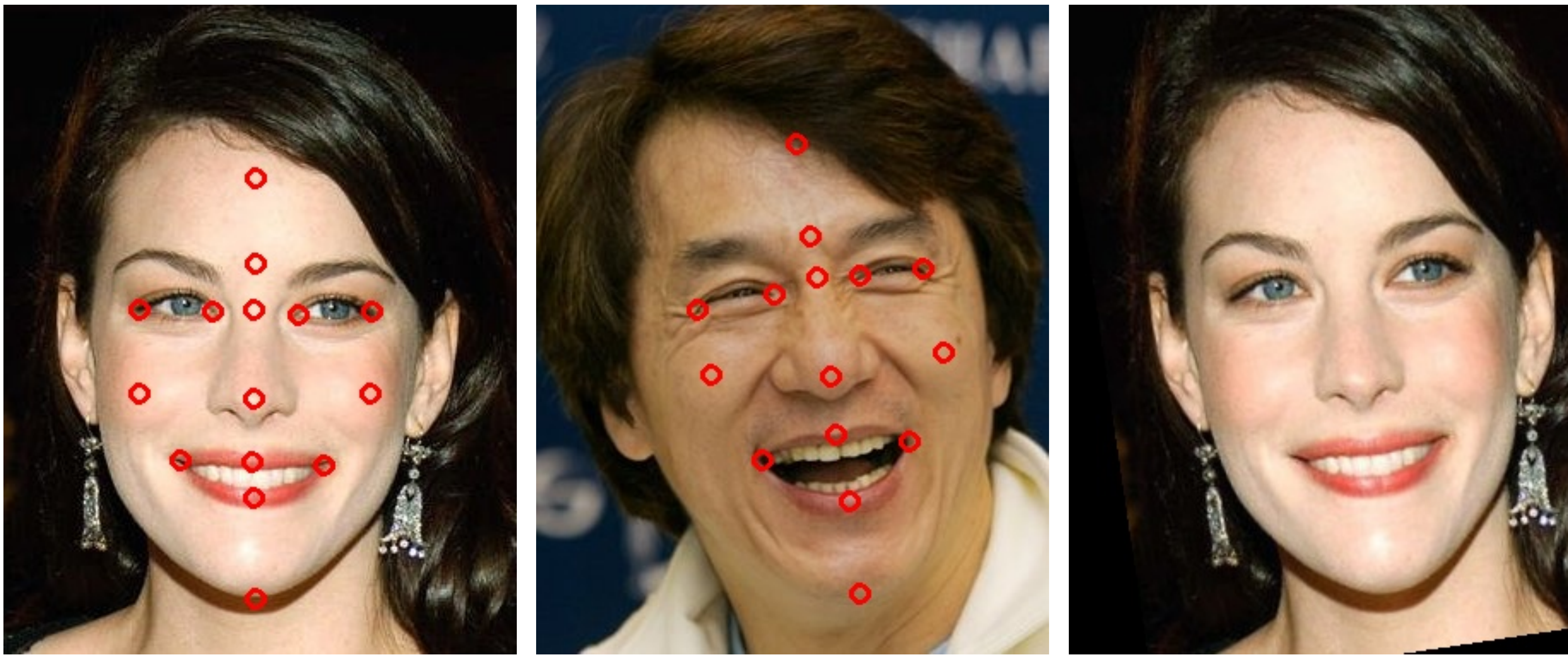


Image Warping Example

- With affine mapping

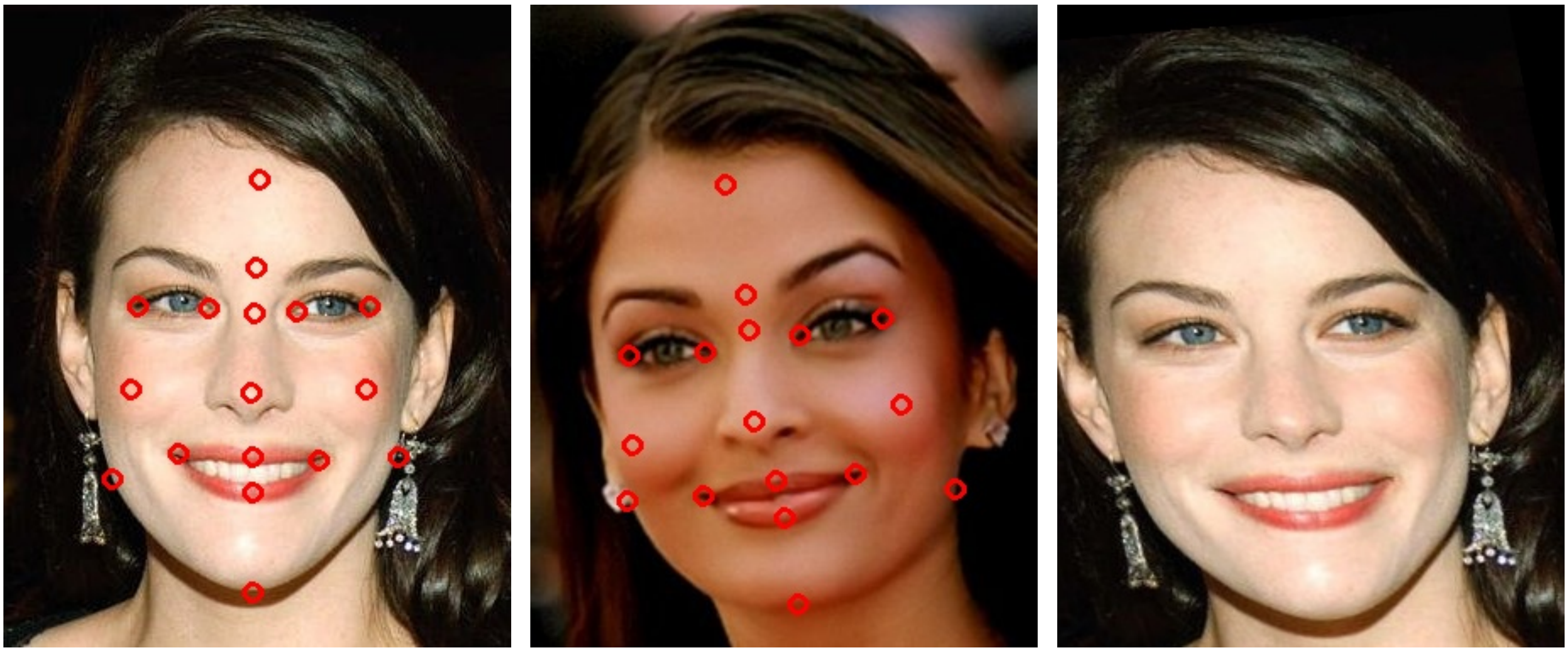
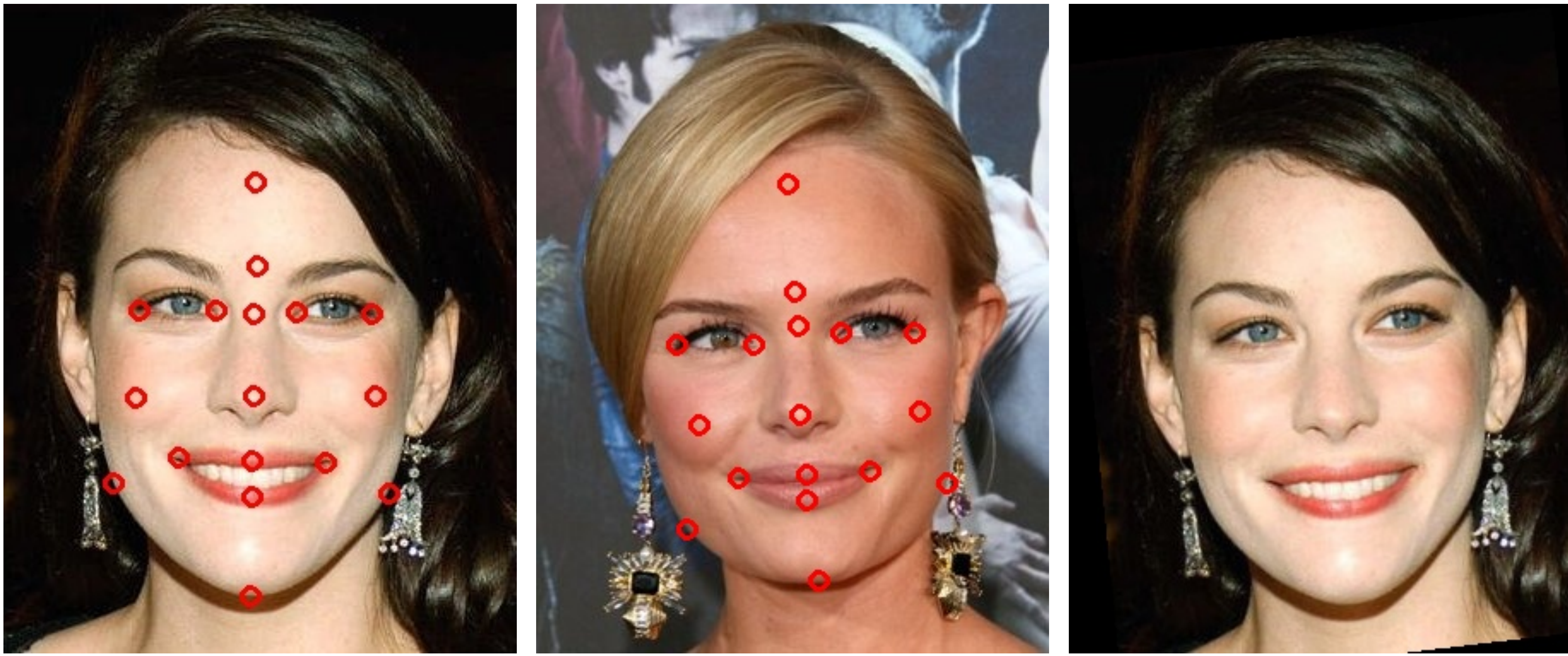


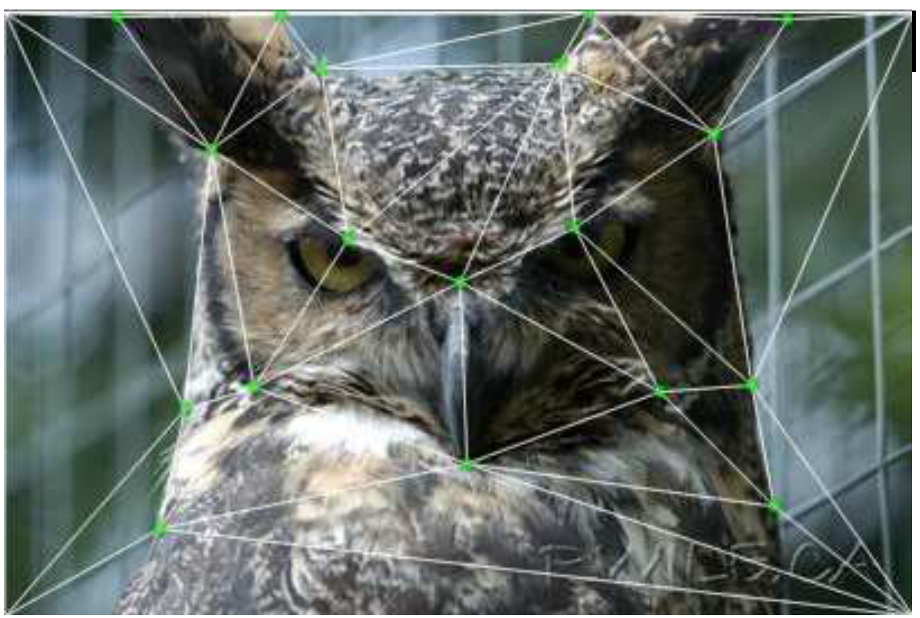
Image Warping Example

- With affine mapping



Local Transformation

- Divide image into regions.
- Warp each region by a different transform.
  - Ensure changes over boundaries are smooth.



Summary

- Need to mark good corresponding points.
- Lower-order function may not warp enough.
- Higher-order function can lead to distortions.
- Local transformations give finer control.