NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

$\mathbb{MA}$ 1505 Mathematics I
Tutorial 8

1. Find the area of the surface consisting of the part of the sphere of radius 2 centered at origin that lies above the horizontal plane $z = 1$. (Equation of this sphere is given by $x^2 + y^2 + z^2 = 2^2$.)

   Ans: 4π

2. Find the centre of mass of the lamina of density $\rho(x, y) = x^2$ that occupies the region R bounded by the parabola $y = 2 - x^2$ and the line $y = x$.

   Ans: $(-8/7, -20/49)$

3. Evaluate the following triple integral:
   $\iiint_D (x^2 + 2z)\, dV$, $\quad D$ is the solid cube $\{-\frac{1}{2} \le x \le \frac{1}{2}, -\frac{1}{2} \le y \le \frac{1}{2}, -\frac{1}{2} \le z \le \frac{1}{2}\}$.

   Ans: $\frac{1}{12}$

4. Let $\mathbf{F}(x, y, z) = 2xy\mathbf{i} + (x^2 + 2yz)\mathbf{j} + y^2\mathbf{k}$. Show that $\mathbf{F}$ is a conservative vector field. Find a function f such that $\nabla f = \mathbf{F}$.

   Ans: $f(x, y, z) = x^2y + y^2z + K$

5. Evaluate $\int_C g(x, y, z)\, ds$, where $g(x, y, z) = x^2 - yz + z^2$ and C is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$.

   Ans: $4\sqrt{14}/3$

6. Compute the work done by the force $\mathbf{F}(x, y, z) = yz\mathbf{i} + 2y\mathbf{j} - x^2\mathbf{k}$ on a particle that moves along the curve C given by the vector function $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$, for $0 \le t \le 1$.

   Ans: $17/30$

7. Evaluate $\int_C 2xy\, dx + (x^2 + z)\, dy + y\, dz$, where C consists of two line segments: C_1 from $(0, 0, 0)$ to $(1, 0, 2)$, and C_2 from $(1, 0, 2)$ to $(3, 4, 1)$.

   Ans: 40