

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA 1505 Mathematics I
Tutorial 7

1. Calculate the following iterated integrals:

(a) $\int_0^b \int_0^a (x^2 + y^2) dx dy$ (b) $\int_1^2 \int_0^1 \frac{xy}{\sqrt{4-x^2}} dx dy.$

Ans: (a) $\frac{1}{3}ab(a^2 + b^2)$ (b) $3 - 3\sqrt{3}/2$

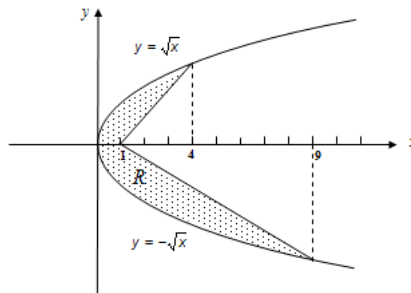
2. Evaluate the following double integrals:

(a) $\iint_R e^{x^2} dA$, R is the region bounded by $y = 0$, $y = x$, $x = 1$.

(b) $\iint_R (x + y) dA$, R is the region bounded by the two curves $y = \sqrt{x}$, $y = x^2$.

Ans: (a) $\frac{1}{2}(e - 1)$ (b) $\frac{3}{10}$

3. Evaluate the double integral $\iint_R x dA$ where R is the region as shown below.



Ans: 25

4. Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} dy dx$ by converting it to polar coordinates.

Ans: $\frac{1}{4}\pi(e - 1)$

5. Evaluate the following integrals by reversing the order of integration.

(a) $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy$ (b) $\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy$

Ans: (a) $\frac{1}{4}(e^{16} - 1)$ (b) 2

6. Find the volume of the solid whose base is the region in the xy -plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $x - z + 4 = 0$.

Ans: 625/12

7. Find the volume of the solid bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.

Ans: $\frac{16}{3}r^3$