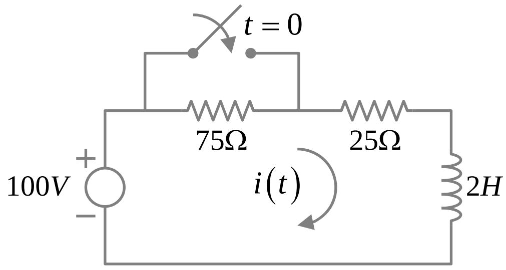
EE2023 TUTORIAL 5 (SOLUTIONS)

Solution to Q.1



Consider $t = 0^-$:

Assume that circuit has been in the same state for an extended period of time. Since the inductor acts as a short circuit in steady-state, we have

$$\left[i(t) = \frac{100}{75+25} = 1A \right] \rightarrow \boldsymbol{i(0^-) = 1A}$$

Consider $t \geq 0$:

At $t = 0$, the switch is closed, shorting out the 75Ω resistor. Applying Kirchoff voltage law:

$$2\frac{di(t)}{dt} + 25i(t) = 100 \quad \cdots\cdots \quad (\clubsuit)$$

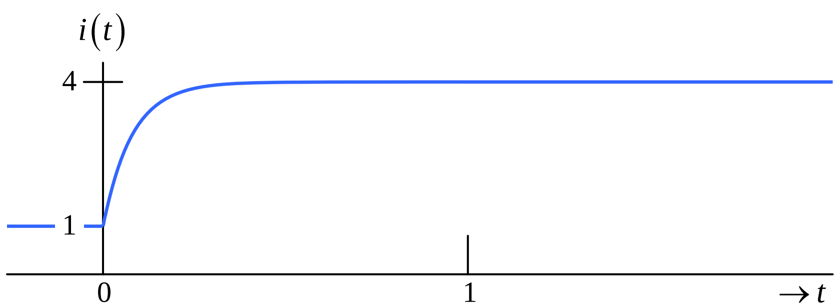
Using Laplace transform

Transforming $(\clubsuit)$ into the s-domain using Laplace transform:

$$\left(2\frac{di(t)}{dt} + 25i(t) = 100 \right) \leftrightarrow \left(2\left[sI(s) - i(0^-) \right] + 25I(s) = \frac{100}{s} \right)$$

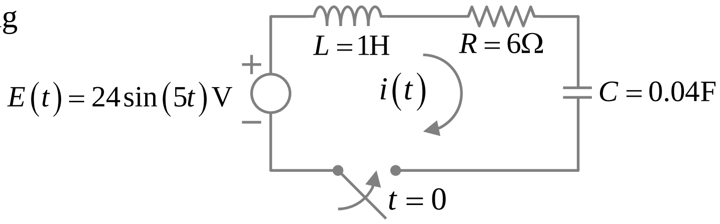
With $i(0^-) = 1$, we get

$$\left[I(s) = \frac{2s+100}{s(2s+25)} = \frac{4}{s} - \frac{3}{s+12.5} \right] \rightarrow \boldsymbol{i(t) = 4 - 3\exp(-12.5t)}.$$



Solution to Q.2

(a) Using KVL, the differential equation relating $i(t)$ to $E(t)$ is



$$L\frac{di(t)}{dt}+Ri(t)+\frac{1}{C}\int_{0^-}^{t} i(\tau)\,d\tau=E(t)$$

Differentiating both sides with respect to t:

$$L\frac{d^2i(t)}{dt^2}+R\frac{di(t)}{dt}+\frac{1}{C}i(t)=\frac{dE(t)}{dt} \quad \cdots \ (\clubsuit)$$

(b) Substituting $L=1$, $R=6$, $C=0.04$, and $E(t)=24\sin(5t)$ into $(\clubsuit)$, we have

$$\frac{d^2i(t)}{dt^2}+6\frac{di(t)}{dt}+25i(t)=120\cos(5t) \quad \cdots\cdots \ (\heartsuit)$$

Transforming $(\heartsuit)$ into the s-domain using Laplace transform:

$$\left(\frac{d^2i(t)}{dt^2}+6\frac{di(t)}{dt}+25i(t)=120\cos(5t)\right)$$

$$\updownarrow$$

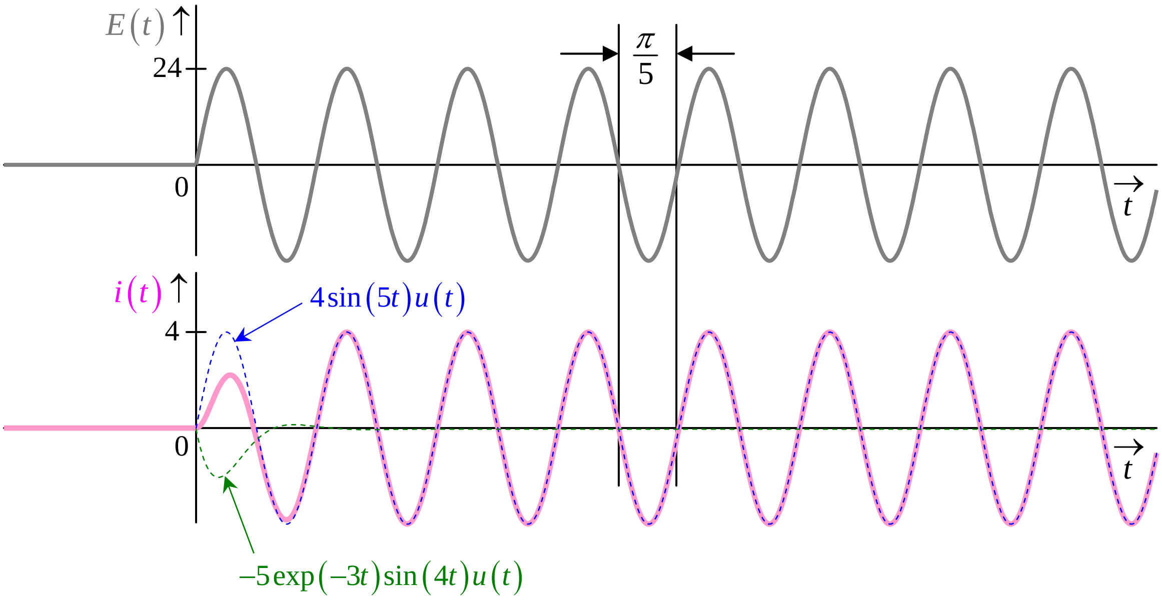
$$\left(\left[s^2I(s)-si(0^-)-i'(0^-)\right]+6\left[sI(s)-i(0^-)\right]+25I(s)=\frac{120s}{s^2+25}\right)$$

Since the initial conditions $i(0^-)$ and $i'(0^-)$ are zero, we have

$$I(s)\left(s^2+6s+25\right)=\frac{120s}{s^2+25}$$

$$I(s)=\frac{120s}{\left(s^2+6s+25\right)\left(s^2+25\right)}=-\frac{20}{s^2+6s+25}+\frac{20}{s^2+25}=-5\frac{4}{(s+3)^2+16}+4\frac{5}{s^2+25}$$

$$\therefore\ \boldsymbol{i(t)=-5\exp(-3t)\sin(4t)+4\sin(5t)}$$



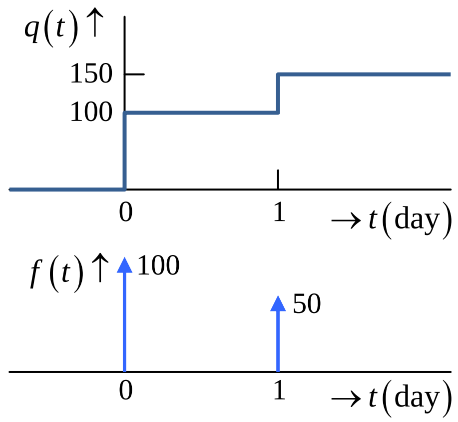
Solution to Q.3

(a) Assume that Ah Kow ingests that first tablet at $t=0$. The cumulative amount of drug administered up till time t should thus be

$$q(t)=100u(t)+50u(t-1).$$

The system input $f(t)$, which is defined as the rate at which drug was administered, is therefore given by

$$f(t)=\frac{dq(t)}{dt}=100\delta(t)+50\delta(t-1).$$



(b) Given that there are no stress relief drug in Ah Kow's bloodstream when the first tablet was ingested. the initial conditions of the system are

$$y(0^-)=0 \text{ and } y'(0^-)=0.$$

(c) The DE describing the quantity of drug in Ah Kow's body given as

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=100\delta(t)+50\delta(t-1).$$

Applying Laplace transformation (with all initial conditions set to zero):

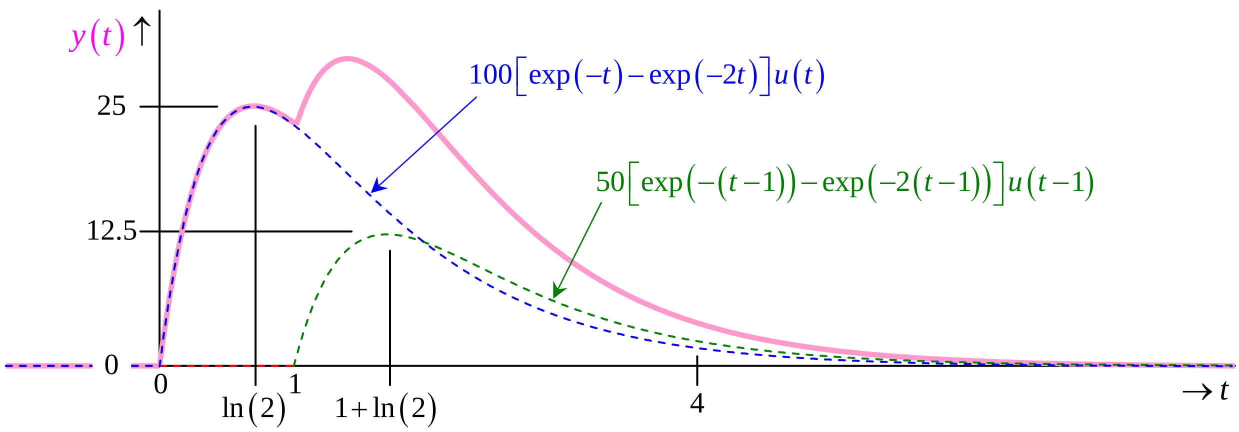
$$s^2Y(s)+3sY(s)+2Y(s)=100+50\exp(-s)$$

$$Y(s)=\frac{(2+\exp(-s))50}{s^2+3s+2}=(2+\exp(-s))\left[\frac{50}{s+1}-\frac{50}{s+2}\right]$$

Therefore,

$$\boldsymbol{y(t)=\mathcal{L}^{-1}\{Y(s)\}=\mathcal{L}^{-1}\left\{100\left[\frac{1}{s+1}-\frac{1}{s+2}\right]+50\left[\frac{1}{s+1}-\frac{1}{s+2}\right]\exp(-s)\right\}}$$

$$\boldsymbol{=100\left[\exp(-t)-\exp(-2t)\right]u(t)+50\left[\exp(-(t-1))-\exp(-2(t-1))\right]u(t-1)}$$



Amount of medicine in Ah Kow's body by the time of the exam is

$$y(4)=100\left[\exp(-4)-\exp(-8)\right]+50\left[\exp(-(3))-\exp(-6)\right]=4.1634\text{ mg}.$$