CHAPTER

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13.1 Color images

Gray level images use a single value per pixel that is called intensity or brightness, as in Chapter 2. The intensity represents the amount of light reflected or emitted by an object and is dependent on the object's material properties as well as on the sensitivity of the camera sensors. By using several sensors or filters, pixels can represent multiple values for light at different frequencies or colors. In this appendix, we describe how this multivalue characterization is represented and related to the human perception of color. In general, the processing of color images is an extensive subject of study, so this appendix aimed to introduce the fundamental ideas used to describe color in computer vision.

The representation of color is based on the relationships between colored light and perception. Light can be understood as an electromagnetic wave and when these waves hit an object, some light frequencies are absorbed while some others are reflected toward our eye and thus creating what we perceive as colors. Similarly, when the reflected light hits a camera's sensor, it obtains a measure of intensity by adding energy on a range of frequencies. In general, *multispectral* images maintain information about the absorption characteristics of particular materials by maintaining the energy measured over several frequencies. This can be achieved by using filters on the top of the sensors, by using prisms to disperse the light or by including several sensors sensitive to particular frequencies on the electromagnetic spectrum. In any case, color images are obtained by selecting different frequencies. Multispectral images which cover frequencies in the visible spectrum are called color images. Other multispectral images covering other part of the spectrum capture the energy with wavelengths that cannot be perceived by the human eye.

Since (color) cameras have several sensors per pixel over a specific frequency range, color images contain information about the luminance intensities over several frequencies. A color model gives meaning to this information by organizing colors in a way that can be related to the colors we perceive. In color image processing, colors are not described by a frequency signature, but they are described and organized according to our perception. The description of how light is perceived by the human eye is based on the *tristimulus theory*.

13.2 Tristimulus theory

Electromagnetic waves have an infinite range of frequencies, but the human eye can only perceive the range of frequencies in the visible spectrum which ranges from about 400 to 700 nm. Each frequency defines a different color as illustrated in Figure 13.1. Generally, we refer to light as the electromagnetic waves that transfer energy in this part of the spectrum. Electromagnetic waves beyond the visual spectrum have special names like X-rays, gamma rays, microwaves, or ultraviolet light.

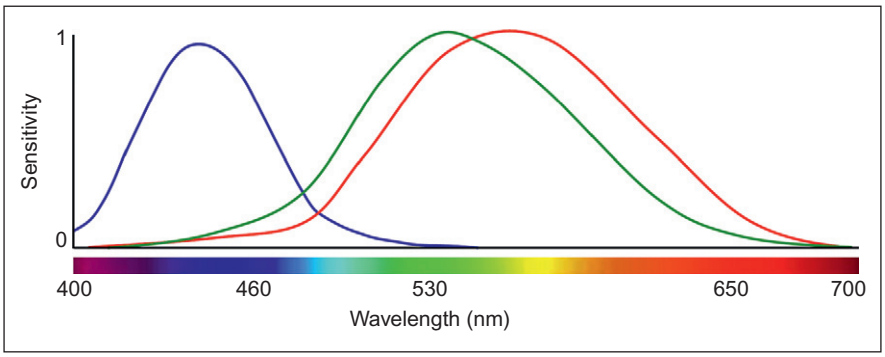


FIGURE 13.1

Visible spectrum and tristimulus response curves. This figure is also reproduced in color in the color plate section.

In the visible spectrum, each wavelength is perceived as a color; the extreme values are perceived as violet and red and between them there are greens and yellows. However, not all the colors that we perceive are in the visible spectrum, but many colors are created when light with different wavelength reaches our eye at the same time. For example, pink or white are perceived from a mix of light at different frequencies. In addition to new colors, mixtures of colors can produce colors that we cannot distinguish as new colors, but they may be perceived as a color in the visible spectrum. That is, the light created by mixing the colors of the spectrum does not produce a stimulus that we can identify as unique. This is why applications such as astronomy cannot identify materials from color images, but they rely on spectrograms to measure the actual spectral content of light. *Metamers* are colors that we perceive as the same but have different mix of light colors.

As explained in Section 1.3, our own representation of color is created by three types of cell receptors in our eyes that are sensitive to a range of frequencies near the blue, red, and green lights. Thus, instead of describing colors by frequency content or radiometric properties, colors can be represented by three stimuli according to the way we perceive them. This way of organizing colors is known as *trichromatic* or *tristimulus* representation. The tristimulus representation was widely used by artists in the eighteenth century and was experimentally developed by physicists. The theory was formally developed by Thomas Young and Hermann von Helmholtz (Sherman, 1981) with two main principles:

1. All the colors we perceive can be represented by a mixture of three primary colors.
2. The color space is linear. That is, the mixture is defined by summations, and the addition of two colors is achieved by adding its primary components.

In addition to these principles, the tristimulus representation establishes how the primaries are defined by considering the sensitivity of each cell receptor to each frequency in the visual spectrum. Each receptor defines a tristimulus

response curve as illustrated in Figure 13.1. That is, the blue receptor will generate a high response for energy around 430 nm, the green and the red around 550 and 560 nm, respectively. The receptors integrate the values in all frequencies and provide a single value, thus the same response can be obtained by different stimuli. For example, the blue receptor will provide the same response for a light with a high value at 400 nm and for a light with less intensity at 430 nm. That is, the response does not provide information about the frequencies that compose a color, but just about the intensity along a frequency range.

It is important to mention that color sensitivity is not the same for all people, so the curves only represent mean values for normal color vision. Also, it is known that color perception is more complex than the summation of three response curves, and the perception of a color is affected by other factors such as the surrounding regions (i.e., context), region sizes, light conditions, as well as more abstract concepts such as memory (temporal stimulus). In spite of this complexity, the tristimulus principles are the fundamental basis of our understanding of color. Furthermore, the tristimulus representation is not limited to the understanding of the perception of colors by the human eye, but the sensors in color cameras and color reproduction systems are based on the same principles. That is, according to the tristimulus theory, these systems only use three values to capture and re-create all the visible colors. This does not imply that the theory describes the nature of light composition or the true perception of the human eye, and it only provides a mechanism to represent the perception of colors.

13.3 Color models

13.3.1 The colorimetric equation

According to the tristimulus theory, all the possible colors we perceive can be defined in a 3D linear space. That is, if $[c_1\ c_2\ c_3]$ define *color components* (or weights) and $[A_1\ A_2\ A_3]$ some *base colors* (or *primaries*), then a color is defined by the colorimetric equation defined by

$$\mathbf{C} = c_1 A_1 + c_2 A_2 + c_3 A_3 \tag{13.1}$$

Here, superposition is expressed as an algebraic summation according to the Grassmann's law of linearity. This law was developed empirically and establishes that colors are combined linearly. Thus, a colorimetric relationship of our perception is written as a linear algebraic equation. It is important to note that the equality does not mean that the algebraic summation in the right side gives a numerical value **C** that can be used to represent or re-create the color. The symbol **C** is not a value or a color representation, but the equation expresses the idea that three stimuli combined by superposition of lights re-create the perception of the color **C**. The actual representation of the color is given by the triplet $[c_1\ c_2\ c_3]$.

The base colors in Eq. (13.1) can be defined according to the visual system by considering the response of the receptors in the human eye. That is, by

considering as primaries the colors that we perceive as red, green, and blue. However, there are other interpretations that give particular properties to the color space and that define different color models. For example, there are color models that consider how colors are created on reproduction systems like printers or models that rearrange colors such that special properties correspond to color properties. In any case, all the color models follow the tristimulus principles and they give a particular meaning to the values of $[c_1\ c_2\ c_3]$ and $[A_1\ A_2\ A_3]$ in Eq. (13.1).

A way to understand color models is to consider them as created by geometric transformations. If you can imagine that you can arrange all the colors that you can see in an enclosed space, then a color model will order those colors by picking up each color and give it the coordinates $[c_1\ c_2\ c_3]$ in a space delineated by the points $[A_1\ A_2\ A_3]$. Sometimes the transformation will be constrained to some colors, so not all the color models contain all the visible colors. Also, although the space is linear, the transformation can organize the colors using nonlinear mappings. Independent of the way the space is defined, since there are three components per color, a color space can be shown in a 3D graph. However, since the interpretation of 3D data is difficult, sometimes the data is shown using 2D graphs.

As such, each color model defines and represents colors that form a color order system. Geometric properties of the space are related to color properties making each model important for color understanding, synthesis, and processing. Therefore, many models have been developed. Historically, the first models were motivated by the scientific interest in color perception, the need of color representations in dye manufacture, as well as to provide practical guidance and color creation to painters. These models have created the fundamentals of color representation (Kuehni, 2003). Some of them, like the color sphere developed by Philipp Runge or the hexahedric model of Tobias Meyer, are close to the ideas of modern theory of color, but perhaps the first model with strong significance in modern theory of color is the CIE XYZ model. This model was developed from the CIE RGB model and it has been used as basis of other modern color representations. In order to explain these color models, it is important to have an understanding of the *luminosity function*.

13.3.2 Luminosity function

The expression in Eq. (13.1) provides a framework to develop color models by adding three components. However, this expression is related to the hue of a color, but not to its brightness. This can be seen by considering what happens to a color when its components are multiplied by the same constant. Since the intensity does not change the color wavelength and the equation is linear, we could expect to obtain a brighter (or darker) version of the color proportional to the constant. However, since the human eye does not have the same sensitivity to all frequencies, the color brightness actually depends on composition. For example, since the human eye is more sensitive to colors whose wavelength is close to green, colors having a large

green component will increase their intensity significantly when the components are increased. For the same increment in the components, blue colors will show less intensity. Colors composed of several frequencies can shift in hue according to the sensitivity to each frequency to the human eye.

The *luminosity* or *luminous efficiency function* is denoted as V_λ and it describes the average sensitivity of the human eye to a color's wavelength (Sharpe et al., 2005). This function was determined experimentally by the following procedure. First, the frequency of a light of constant intensity was changed until observers perceived the maximum brightness. The maximum was obtained with a wavelength of 555 nm. Secondly, a different light's wavelength was chosen and the power was adjusted until the perceived intensity of the new wavelength was the same as the 555 nm. Thus, the luminous efficiency for the light at the chosen wavelength was defined as the ratio between the power at the maximum and the power at the wavelength. The experiments for several wavelengths produce the general form illustrated in Figure 13.4. This figure represents the daytime efficiency (i.e., *photopic vision*). Under low light conditions (i.e., *scotopic vision*), the perception is mostly performed by the rods in the eye, so the curve is shifted to have a maximum efficiency of around 500 nm. In intermediate light conditions (i.e., *mesopic vision*), the efficiency can be expressed as a function of photopic and scotopic functions (Sagawa and Takeichi, 1986). The luminosity function in Figure 13.4 is normalized, thus it represents the relative intensity rather than the actual visible energy or power perceived by the human eye. The perceived power generally is expressed in *lumen* and it is proportional to this curve. Bear in mind that the perceived intensity is related to the luminous flux of a source while the actual physical power is related to the radiant flux and it is generally measured in *watts*.

In the description of color models, the luminous efficiency is used to provide a reference for the perceived brightness. This is achieved by relating the color components to the luminous efficiency via the *luminance coefficients* $[v_1\ v_2\ v_3]$. These coefficients define the contribution of each base color to the brightness as

$$V = v_1 c_1 + v_2 c_2 + v_3 c_3 \tag{13.2}$$

For example, the color coefficients [1 4 2] indicate that the second component contributes four times more to the brightness than the first one. Thus, an increase in the second component will create a color that is four times brighter than the color created by increasing the first one the same amount. It is important to emphasize that this function describes our perception of brightness and not the actual radiated power.

In general, the luminance coefficients of a color model can be computed by fitting the brightness to the luminosity function, i.e., the values that minimize the summation

$$\sum_{\lambda} |V_\lambda - (\alpha c_{1,\lambda} + \beta c_{2,\lambda} + \gamma c_{3,\lambda})| \tag{13.3}$$

where $\lfloor c_{1,\lambda}\ c_{2,\lambda}\ c_{3,\lambda} \rfloor$ are the components that generate the color with a single wavelength λ and $||$ defines a metric error. Colors formed by a single wavelength are referred to as *monochromatic*. Since the minimization is for all wavelengths, the best fit value only gives an approximation to our perception of brightness. However, in general, the approximation provides a good description of the perceived intensity, and luminance coefficients are commonly used to define and study the properties of color models.

13.3.3 Perception based color models: the CIE RGB and CIE XYZ

The CIE RGB and CIE XYZ color models were defined in 1931 by the Commission Internationale de L'Eclairage (CIE). Both models provide a description of the colors according to human perception and they characterize the same color's properties, nevertheless they use different base colors. While the CIE RGB uses visible physical colors, the XYZ uses imaginary or inexistent colors that only provide a theoretical basis. That is, the CIE RGB is the physical model developed based on perception experiments, while the CIE XYZ is theoretically derived from the CIE RGB. The motivation to develop the CIE XYZ is to have a color space with better descriptive properties. However, in order to achieve that description, the base colors are shifted out of the visible spectrum.

13.3.3.1 CIE RGB color model: Wright–Guild data

The base of the CIE RGB color space is denoted by the triplet $[R\ G\ B]$ and its components are denoted as $[r\ g\ b]$. Thus, the definition in Eq. (13.1) for this model is written as

$$\mathbf{C} = rR + gG + bB \tag{13.4}$$

This model considers how colors are perceived by the human eye and it was developed based on color matching experiments. The experiments were similar to previous experiments developed in the nineteenth century by Helmholtz and Maxwell that were used to organize colors according to its primary compositions (i.e., the Maxwell triangle). In the CIE RGB color model experiments, a person was presented with two colors: the first color defines a target color with a single known frequency wavelength and the second is produced by combining the light of three sources defined by the base colors. To determine the composition of the target color, the intensity of the base colors is changed until the color produced by the combination of lights matches the target color. The intensities of the composed sources define the color components of the target color.

The experiments that defined the CIE RGB model were published by Wright and Guild (Wright, 1929; Guild, 1932) and the results are known as the Wright–Guild data. Wright experiments used seven observers and light colors created by monochromatic lights at 650, 530, and 460 nm. The experiments matched monochromatic colors from 380 to 780 nm at 5 nm intervals. Guild used

ten observers and primaries composed of several wavelengths. In order to use both the Guild and Wright experimental data, the CIE RGB results are expressed in a common color base using color lights at 700, 546.1, and 435.8 nm. These lights were the standard basis used by the National Physical Laboratory in London and were defined since the last two are easily producible by a mercury vapor discharge and the 700 nm wavelength has the advantage of having a small perceptional change for different people. Therefore, small errors in the measure of the light intensity produce only small errors on the composed color.

An important result of the color matching experiments was the observation that many colors cannot be created by adding the primary lights, but they can only be produced by subtracting light values. In the experiments, subtraction does not mean using negative light intensities but to add a base color to the target color. This process **desaturates** the colors and since the mix of colors is linear, adding to the target is equal than subtracting from the light mixture that creates the second color in the experiments. For example, to generate violet requires adding a green light to the target, thus generating a negative green value. In practice, this means that the base colors are not saturated enough (far away from white) to generate those colors. In fact, there is no color basis that can generate all visible colors. However, it is possible to define theoretical basis that, although are too saturated to be visible, it can create all the colors. This is the base rationale for creating the CIE XYZ model that is presented later.

13.3.3.2 CIE RGB color matching functions

It is impractical to perform color matching experiments to obtain the components of all the visible colors, but the experiments should be limited to a finite set of colors. Thus, the color description should provide a rule that can be used to infer the components of any possible color according to the results obtained in the matching experiments. The mechanism that permits determination of the components of any color is based on the *color matching functions*.

The color matching functions are illustrated in Figure 13.2 and define the intensity values of the base colors that produce any monochromatic color with a normalized intensity. That is, for each color generated by a single wavelength and with unit intensity, the functions give three values that represent the components of that color. For example, to create the same color as a single light at 580 nm, we combine three base colors with intensities 0.24, 0.13, and −0.001.

It is important to mention that the color matching functions do not correspond to the actual intensities measured in the color matching experiments, but the values are manipulated to provide a normalized description that agrees with our color perception and such that they are referenced with respect to the white color. The definition of the color matching functions involves four steps (Broadbent, 2004). First, a different scale factor for each base color was defined such that the color mixture agrees with our perceptions of color. That is, yellow can be obtained by the same amount of red and green while the same amount of green and blue matched cyan (or the monochromatic light at 494 nm). Secondly, the

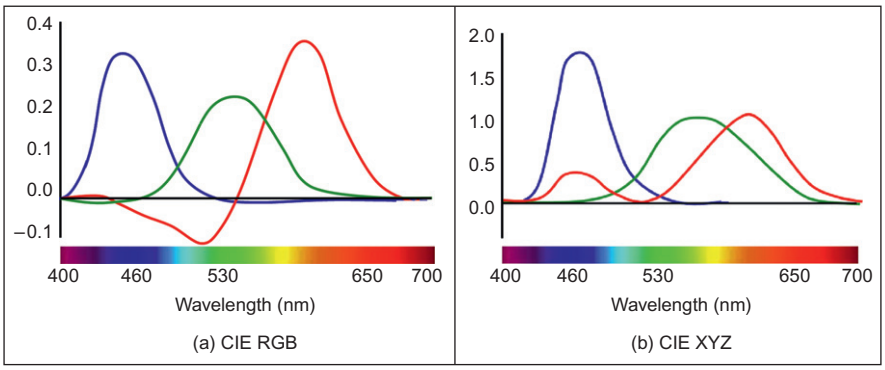


FIGURE 13.2

Color matching functions. This figure is also reproduced in color in the color plate section.

data is normalized such that the sum of the components for any given color is unity. That is, the color is made independent of the color luminous energy by dividing each measure by the total energy $[r + g + b]$. Thirdly, the color is centered using as a reference for white. Finally, the color is transformed to characterize color using colored lights at 700, 546.1, and 435.8 nm.

The normalization of brightness and the center around the reference point of the transformation are very important factors related to *chromatic adaptation*. Chromatic adaptation is a property of the human visual system that provides constant perceived colors under different illumination conditions. For example, we perceive an object as white when we see it in direct sunlight or illuminated by an incandescent bulb. However, since the color of an object is actually produced by the light it reflects, the measure of the color is different when using different illumination. Therefore, the normalization and the use of a reference ensure that the measures are comparable and can be translated to different light conditions by observing the coordinates of the white color. As such, having white as reference can be used to describe color under different illumination.

In order to center the model based on white, observers were also presented with a standard white color to determine its components. There were large variations in each observer's measures, so the white color was defined by taking an average. The white color was defined by the values 0.243, 0.410, and 0.347. Thus, the results of the matching experiments were transformed such that the white color has its three components equal to 0.333. The values centered on white are finally transformed to the basis defined by 700, 546.1, and 435.8 nm.

Once the matching functions are defined, then the components for colors with a single wavelength can be obtained by interpolating the data. Moreover, the color matching functions can also be used to obtain the components of colors composed by mixtures of lights by considering the components of each wavelength in the mixture. To explain this, we consider that the components of a color $\hat{\mathbf{C}}_\lambda$ created

by a light with a normalized intensity value of one and a single frequency with wavelength λ is denoted as $[\hat{r}_\lambda \ \hat{g}_\lambda \ \hat{b}_\lambda]$. That is,

$$\hat{\mathbf{C}}_\lambda = \hat{r}_\lambda R + \hat{g}_\lambda G + \hat{b}_\lambda B \tag{13.5}$$

Since colors are linear, a color with an arbitrary intensity and same single frequency is

$$\mathbf{C}_\lambda = r_\lambda R + g_\lambda G + b_\lambda B = k(\hat{r}_\lambda R + \hat{g}_\lambda G + \hat{b}_\lambda B) \tag{13.6}$$

The value of the constant can be obtained by considering the difference between the intensities of the two target colors. That is, $k = |\mathbf{C}_\lambda|/|\hat{\mathbf{C}}_\lambda|$. Here, $|\mathbf{C}_\lambda|$ denotes the intensity of the color. Since the normalized values have an intensity of one, $k = |\mathbf{C}_\lambda|$. By using this value in Eq. (13.6), we have

$$\mathbf{C}_\lambda = |\mathbf{C}_\lambda|\hat{r}_\lambda R + |\mathbf{C}_\lambda|\hat{g}_\lambda G + |\mathbf{C}_\lambda|\hat{b}_\lambda B \tag{13.7}$$

According to this equation, the color components can be obtained by multiplying its intensity by the normalized components given by the color matching functions. That is,

$$r_\lambda = |\mathbf{C}_\lambda|\hat{r}_\lambda; \quad g_\lambda = |\mathbf{C}_\lambda|\hat{g}_\lambda; \quad b_\lambda = |\mathbf{C}_\lambda|\hat{b}_\lambda \tag{13.8}$$

This approach can be generalized to obtain the components of colors composed of several frequencies. For example, for two colors containing two frequency components λ_1 and λ_2, we have

$$\begin{aligned} \mathbf{C}_{\lambda_1} &= r_{\lambda_1} R + g_{\lambda_1} G + b_{\lambda_1} B \\ \mathbf{C}_{\lambda_2} &= r_{\lambda_2} R + g_{\lambda_2} G + b_{\lambda_2} B \end{aligned} \tag{13.9}$$

Since the color space is linear, the color containing both frequencies is given by

$$\mathbf{C}_{\lambda_1} + \mathbf{C}_{\lambda_2} = (r_{\lambda_1} + r_{\lambda_2})R + (g_{\lambda_1} + g_{\lambda_2})G + (b_{\lambda_1} + b_{\lambda_3})B \tag{13.10}$$

By using the definitions in Eq. (13.8), we have that the color components can be obtained by adding the color matching functions of each frequency. That is,

$$\mathbf{C}_{\lambda_1} + \mathbf{C}_{\lambda_2} = (|\mathbf{C}_{\lambda_1}|\hat{r}_{\lambda_1} + |\mathbf{C}_{\lambda_2}|\hat{r}_{\lambda_2})R + (|\mathbf{C}_{\lambda_1}|\hat{g}_{\lambda_1} + |\mathbf{C}_{\lambda_2}|\hat{g}_{\lambda_2})G + (|\mathbf{C}_{\lambda_1}|\hat{b}_{\lambda_1} + |\mathbf{C}_{\lambda_2}|\hat{b}_{\lambda_3})B \tag{13.11}$$

Therefore, the color components are the sum of the color matching functions multiplied by the intensity of each wavelength components. The summation can be generalized to include all the frequencies by considering infinite sums of all the wavelength components. That is,

$$\begin{aligned} r &= \int |C_\lambda|\hat{r}_\lambda \, \mathrm{d}\lambda \\ g &= \int |C_\lambda|\hat{g}_\lambda \, \mathrm{d}\lambda \\ b &= \int |C_\lambda|\hat{b}_\lambda \, \mathrm{d}\lambda \end{aligned} \tag{13.12}$$

As such, the color components of any color can be obtained by summing the color matching functions weighted by its spectral power distribution. Since the

color matching functions are represented in a tabular form, sometimes the integrals are expressed as a matrix multiplication of the form

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} \hat{r}_{\lambda_0} & \hat{r}_{\lambda_1} & \dots & \hat{r}_{\lambda_{n-1}} & \hat{r}_{\lambda_n} \\ \hat{g}_{\lambda_0} & \hat{g}_{\lambda_1} & \dots & \hat{g}_{\lambda_{n-1}} & \hat{g}_{\lambda_n} \\ \hat{b}_{\lambda_0} & \hat{b}_{\lambda_1} & \cdots & \hat{b}_{\lambda_{n-1}} & \hat{b}_{\lambda_n} \end{bmatrix} \begin{bmatrix} |C_{\lambda_0}| \\ |C_{\lambda_1}| \\ \vdots \\ |C_{\lambda_{n-1}}| \\ |C_{\lambda_n}| \end{bmatrix} \tag{13.13}$$

The first matrix in the right side of this equation is given by the CIE RGB color matching functions table and it is generally given by discrete values at 5 nm intervals from 380 to 480 nm. However, it is also common to use tables that have been interpolated at 1 nm intervals (Wyszecki and Stiles, 2000). The second matrix represents the power of the color in a wavelength interval.

13.3.3.3 CIE RGB chromaticity diagram and chromaticity coordinates

The CIE RGB model characterizes colors by three components, thus the graph of the full set of colors is a 3D volume. The general shape of this volume is illustrated on the top left in Figure 13.3. As colors increase in distance from the

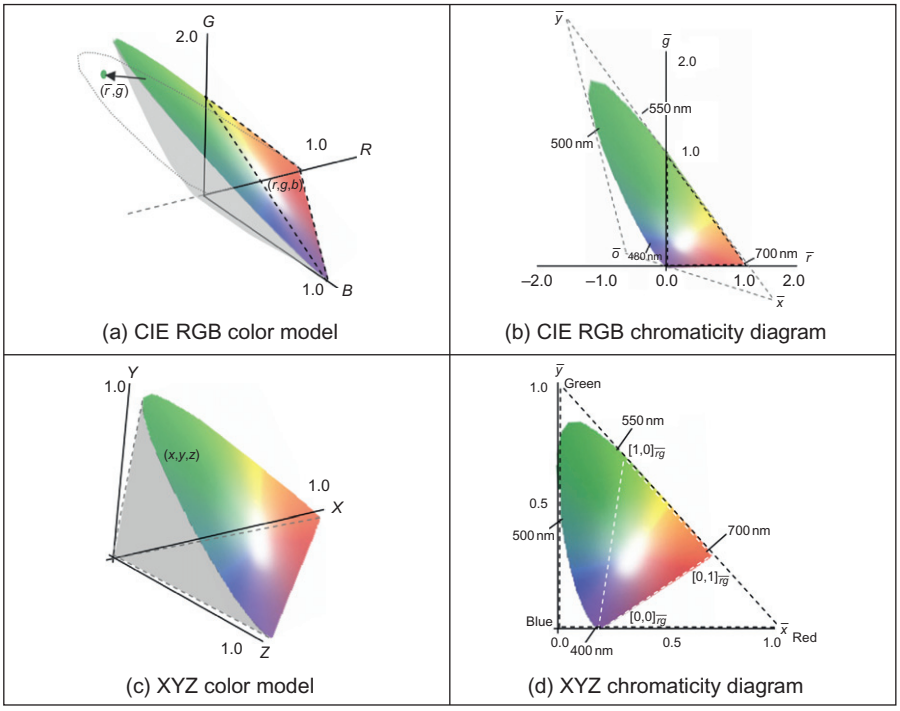


FIGURE 13.3

CIE RGB and XYZ color models. This figure is also reproduced in color in the color plate section.

origin, their brightness increases and more colors become visible forming a conical-shaped volume. In the figure, the base colors coincide with the corners of the triangle drawn with black dashed lines. Thus, the triangular pyramid defined by this triangle contains the colors that can be created by addition.

In general, the visualization and interpretation of colors using 3D representations is complicated, thus color properties can be better visualized using 2D graphs. The most common way to illustrate the CIE RGB color space is to only consider the color's *chromaticity*. That is, the *luminous energy* is eliminated by normalizing against the total energy. The *chromaticity coordinates* are defined as

$$\overline{r} = \frac{r}{r+g+b}; \quad \overline{g} = \frac{g}{r+g+b}; \quad \overline{b} = \frac{b}{r+g+b} \tag{13.14}$$

Only two of the three normalized colors are independent and one value can be determined from the other two. For example, we can compute blue as

$$\overline{b} = 1 - \overline{r} - \overline{g} \tag{13.15}$$

As such, only two colors can be used to characterize the chromaticity of the color model and the visible colors can be visualized using a 2D graph. The graph created by considering the color's chromaticity is called the *chromaticity diagram*.

The geometrical interpretation of the transformation in Eq. (13.14) is illustrated in Figure 13.3(a). Any point in the color space is mapped into the chromaticity diagram by two transformations. First, the central projection in Eq. (13.14) maps the colors into the plane that contains the colored shape in the figure. That is, by tracing radial lines from the origin to the plane. Secondly, the points are orthogonally projected into the plane *RG*. That is, the $\overline{b}$ coordinate is eliminated or set to zero. In the figure, the border of the area resulting from the projection is shown by the dotted curve in the *RG* plane.

Figure 13.3(b) shows the projected points into the *RG* plane and this corresponds to the chromaticity diagram for the CIE RGB model. Note from the transformation that any point in the same radial projection line will end up in the same point in the chromaticity diagram. That is, points in the chromaticity diagram characterize colors independent of their luminous energy. For example, the colors with chromaticity coordinates [0.5 0.5 0.5] and [1 1 1] are shown as the same point [1/3 1/3] in the diagram. This point represents both white and gray since they have the same chromaticity, but the first one is a less bright version of the second one. Since the chromaticity cannot show white and gray for the same point, it is colored by the normalized color $[\overline{r}, \overline{g}, \overline{b}]$.

It is not possible to use the inverse of Eq. (13.14) to obtain the color components from the chromaticity coordinates, but the inverse only defines a line passing through the origin and through colors with the same chromaticity. That is,

$$r = k\overline{r}; \quad g = k\overline{g}; \quad b = k\overline{b} \tag{13.16}$$

The value of k in this equation defines a normalization constant that according to Eq. (13.14) is given by $k = r + g + b$.

As illustrated in Figure 13.3(b), the visible spectrum of colors outlines a horseshoe region in the chromaticity diagram. The red and green components of each color are determined by the position of the colors in the axes in the graph while the amount of blue is determined according to Eq. (13.13). The top curved rim delineating the visible colors is formed by colors with a single frequency component. This line is called the spectral line and it represents lights from 400 to 700 nm. Single wavelength colors do not have a single component, but the diagram shows the amount of each component of the basis that is necessary to create the perception of the color. The spectral line defines the border of the horseshoe region since these colors are the limit of the human eye's perception. The straight line of the horseshoe region is called the purple line and is not formed by single wavelength colors, but each point in this line is formed by mixing the two monochromatic lights at 400 and 700 nm.

In addition to identifying colors, the chromaticity diagram can be used to develop a visual understanding of its properties and relationships. However, the interpretation of colors using chromaticity is generally performed in the XYZ color space, so we will consider the properties of the chromaticity diagram later.

13.3.3.4 CIE XYZ color model

The CIE RGB model has several undesirable properties. First, as illustrated in Figure 13.2, its color matching functions contain negative values. One of the graphs is negative at any wavelength. Negative colors do not fit well with the concept of producing colors by adding base colors and it introduces sign computations. This is important since at the time the XYZ model was developed, the computations were done manually. Secondly, the color components are not normalized, e.g., a color created by a light with a single frequency at 410 nm are [0.03 −0.007 0.22]. A better color description should have the components bounded to range from 0 to 1. Finally, all the base colors have a contribution to the brightness of a color. That is, the perceived brightness is changed by modifying any component. However, the distribution of cones and rods in the human eye has a different sensitivity for perception of brightness and color. Thus, a more useful description should concentrate the brightness on a single component such that the perception of a color can be related to the definition of chromaticity and brightness. The CIE XYZ model was developed to become a universal reference system that overcomes these unwanted properties.

The basis of the CIE XYZ model is denoted by the triplet $[X\ Y\ Z]$ and its components are denoted as $[x\ y\ z]$. Thus, the definition in Eq. (13.1) for this model is written as

$$\mathbf{C} = xX + yY + zZ \tag{13.17}$$

and the chromaticity coordinates are defined as

$$\overline{x} = \frac{x}{x+y+z}; \quad \overline{y} = \frac{y}{x+y+z}; \quad \overline{z} = \frac{z}{x+y+z} \tag{13.18}$$

Similar to Eq. (13.13), we have

$$\overline{z} = 1 - \overline{x} - \overline{y} \tag{13.19}$$

Thus, according to Eq. (13.16), colors with the same chromaticity are defined by the inverse of Eq. (13.18). That is,

$$x = k\overline{x}; \quad y = k\overline{y}; \quad z = k\overline{z} \tag{13.20}$$

At difference of the CIE RGB, the color components in the XYZ color model are not defined by matching color experiments, but they are obtained from the components of the CIE RGB model by a linear transformation. That is,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} r \\ g \\ b \end{bmatrix} \tag{13.21}$$

Here, M is a nonsingular 3×3 matrix. Thus, the mapping from the XYZ color model to the CIE RGB is given by

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = M^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{13.22}$$

The definition in Eq. (13.21) uses a linear transformation in order to define a one-to-one mapping that maintains collinearity. The one-to-one property ensures that the identity of colors is maintained, thus colors can be identified in both models without any ambiguity. Collinearity ensures that lines defined by colors with the same chromaticity are not changed. Thus, colors are not scrambled, but the transformation maps the colors without changing its chromaticity definition. Additionally, the transformation does not have any translation, so it actually rearranges the chromaticity lines defined from the origin by stretching the colors in the CIE RGB model. This produces a shift that translates the base colors into the invisible spectrum.

Equation (13.21) defines a system of three equations, thus the matrix can be determined by defining the mapping of three no coplanar points. That is, if we know the CIE RGB and XYZ components of three points, then we can substitute these values in Eq. (13.21) and solve for M. As such, in order to define the XYZ model, we just need to find three points. These points are defined by considering the criteria necessary to achieve desired properties in the chromaticity diagram (Fairman et al., 1997).

Since M is defined by three points, the development of the XYZ color model can be reasoned as the mapping of a triangle. This idea is illustrated in Figure 13.3. In this figure, the dashed triangle in the CIE RGB diagram shown in Figure 13.3(b) is transformed into the dark dashed triangle in the XYZ diagram shown in Figure 13.3(d). In Figure 13.3(d), the sides of the triangles coincide with the axis of the XYZ model and the visible colors are constrained to the triangle defined in the unit positive quadrant. By aligning the triangle to the *XYZ* axes,

we are ensuring that the transformation maps the color components to positive values. That is, since the triangle is at the right and top of the axis, $\overline{x} > 0$ and $\overline{y} > 0$. The definition of the diagonal side ensures that the remaining component is positive. This can be seen by considering that according to the definition of chromaticity in Eq. (13.18), we have

$$\overline{x} + \overline{y} = 1 - \overline{z} \tag{13.23}$$

Thus, in order for $\overline{z}$ to take values from 0 to 1, it is necessary that

$$\overline{x} + \overline{y} \leq 1 \tag{13.24}$$

That is, the colors should be under the diagonal line. Once the triangle in the XYZ chromaticity diagram has been defined, the problem of determining the transformation M in Eq. (13.21) consists on finding the corresponding triangle in the CIE RGB diagram. This can be achieved by considering the properties of the colors on the lines $\overline{ox}$, $\overline{oy}$, and $\overline{xy}$ that define the triangle. In other words, we can establish criteria to look for the corresponding lines in both diagrams. The first criterion to be considered is to give the contribution of brightness to a single component.

Since the human eye is more sensitive to colors whose wavelength is close to green, the contribution of brightness in the XYZ model is given by the Y component. That is, changes in the X and Z components of a color produce insignificant changes of intensity, but small changes along the Y axis will produce a strong intensity variation. For this reason, the Y component is called the *color intensity*. In the CIE RGB, all components have a contribution to the intensity of the color according to the luminance coefficients [1 4.59 0.06]. That is, the luminosity function in Eq. (13.2) for the CIE RGB model is given by

$$V = r + 4.59g + 0.06b \tag{13.25}$$

Since in the XYZ color model, the contribution to the intensity is only given by the Y component, the colors for which $\overline{y} = 0$ should have $V = 0$. That is, if $\overline{y} = 0$, then

$$r + 4.59g + 0.06b = 0 \tag{13.26}$$

This equation defines a plane that passes through the origin in the 3D CIE RGB color space. A projection into the chromaticity diagram is obtained by considering Eq. (13.13). That is,

$$0.17\overline{r} + 0.81\overline{g} + 0.01 = 0 \tag{13.27}$$

This line goes through the points $\overline{o}$ and $\overline{x}$ shown in Figure 13.3(b) and it corresponds to the line $\overline{ox}$ in Figure 13.3(d). The colors in this line are called *alychne* or colors with zero luminance, and these colors are formed by negative values of green or red. According to the definition of luminosity function, these colors do not produce any perceived intensity to the human eye and according to the locus of the line in the chromaticity diagram they are not visible. The closest sensation

we can have about a color that does not create any luminance is close to deep purple. In the XYZ chromaticity diagram, the $\overline{y}$ value defines a color from the alychne line.

The definition of the line $\overline{xy}$ considers that the line passing through the points [1 0] and [0 1] in the CIE RGB chromaticity diagram can be a good mapping for the diagonal line in the XYZ chromaticity diagram. It is a good mapping since it maximizes the coverage of the area defined by the visible colors and it delineates the contour of the color region that is tangential to the region defining the visible colors over a large wavelength range. However, this line does not encompass all the visible colors. This can be seen by considering that this line is defined by

$$\overline{r} + \overline{g} = 1 \tag{13.28}$$

Thus, the points on or below the line should satisfy the constraint given by

$$\overline{r} + \overline{g} \leq 1 \tag{13.29}$$

The blue color can be used in this equation by considering that according to Eq. (13.19)

$$\overline{r} + \overline{g} = 1 - \overline{b} \tag{13.30}$$

Thus, the constraint in Eq. (13.29) can be true only if $\overline{b}$ is positive. However, the color matching functions define small negative values between 546 and 600 nm. Consequently, some colors are above the line. To resolve this issue, the line that defines the XYZ model is obtained by slightly shifting the slope of the line in Eq. (13.28). The small change in the slope was calculated such that the line contains the color obtained by the minimum blue component. Thus, the second line that defines the XYZ model is given by

$$\overline{r} + 0.99\overline{g} = 1 \tag{13.31}$$

This line is illustrated in Figure 13.3(b) as the dotted line going through the points $\overline{x}$ and $\overline{y}$. The corresponding line in the XYZ chromaticity diagram can be seen in Figure 13.3(d).

The definition of the line $\overline{oy}$ in the CIE RGB chromaticity diagram was chosen to maximize the area covering the visible colors. This was achieved by defining the line tangential to the point defining the 500 nm color. The position of the line is illustrated by the points $\overline{o}$ and $\overline{y}$ in Figure 13.3(b). This line corresponds to the vertical axis of the XYZ diagram shown in the bottom right of the figure. The equation of the line in the CIE RGB chromaticity diagram is defined as

$$2.62\overline{r} + 0.99\overline{g} = -0.81 \tag{13.32}$$

Thus, the lines that define the triangle in the CIE RGB diagram are given by Eqs (13.27), (13.31), and (13.32). The vertices of the triangle are obtained by computing the intersection of these lines and they are given by the points [1.27 −0.27], [−1.74 2.76], and [−0.74 0.14]. In order to obtain the position of these points in the CIE RGB color space, it is necessary to include the $\overline{b}$

component. This is achieved by considering Eq. (13.13). Thus, the chromaticity coordinates of the points in the CIE RGB color model are [1.27 −0.27 0.002], [−1.74 2.76 −0.02], and [−0.74 0.14 1.6].

By using Eq. (13.16), we have that the color components defined by these coordinates are given by the three points

$$\begin{array}{lccc} \alpha[1.27 & -0.27 & 0.002] \\ \beta[-1.74 & 2.76 & -0.02] \\ \gamma[-1.74 & 2.76 & -0.02] \end{array} \tag{13.33}$$

The symbols α, β, and γ denote the normalization constants. In order to justify these constants, we should recall that points in the chromaticity diagram represent a line of points in the color space. That is, for any values of α, β, and γ, we obtain the same three points of the form $[\overline{r}\ \overline{g}]$. As such, for any value of the constants, the points in Eq. (13.33) have chromaticity coordinates that satisfy the criteria defined based on the chromaticity properties.

Since we define the triangle in the XYZ model to coincide with its axes, Eq. (13.21) transforms the points in Eq. (13.33) to the points [1 0 0], [0 1 0], and [0 0 1]. As such, the transformation M can be found by substitution of the three points defined in both spaces. However, this requires solving three systems of equations; each system gives a row of the matrix. A simpler approach consists on using Eq. (13.22) instead of Eq. (13.21). It is simpler to use Eq. (13.22) since the points in the XYZ system contain zeros in two of its elements. Thus, the three systems of equations are reduced to equalities. That is, by substitution of the three points in Eq. (13.22), we obtain the three equations

$$\alpha \begin{bmatrix} 1.27 \\ -0.27 \\ 0.002 \end{bmatrix} = M^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad \beta \begin{bmatrix} -1.74 \\ 2.76 \\ -0.02 \end{bmatrix} = M^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \quad \gamma \begin{bmatrix} -0.74 \\ 0.14 \\ 1.6 \end{bmatrix} = M^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{13.34}$$

The multiplication in the right side of the first equation gives the first column of M^{-1}, the second equation the second column, and the third the last column. That is,

$$M^{-1} = \begin{bmatrix} 1.27\alpha & -1.74\beta & -0.74\gamma \\ -0.27\alpha & 2.76\beta & 0.14\gamma \\ 0.002\alpha & -0.02\beta & 1.6\gamma \end{bmatrix} \tag{13.35}$$

We can rewrite this matrix as a product. That is,

$$M^{-1} = \begin{bmatrix} 1.27 & -1.74 & -0.74 \\ -0.27 & 2.76 & 0.14 \\ 0.002 & -0.02 & 1.6 \end{bmatrix} \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{13.36}$$

The normalization constants are determined by considering that the chromaticity of the reference point (i.e., white) is the same in both models. However,

instead of transforming the reference point by using Eq. (13.36), a simpler algebraic development can be obtained by considering the properties of the inverse of the matrix product. Thus, from Eq. (13.36),

$$M = \begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} 0.90 & 0.57 & 0.37 \\ 0.09 & 0.41 & 0.005 \\ -0.00002 & 0.006 & 0.62 \end{bmatrix} \tag{13.37}$$

In the CIE RGB, the coordinates of the reference white are [0.33 0.33 0.33]. By considering that this point is the same in the CIE RGB and in the XYZ color models, then according to Eq. (13.21),

$$\eta \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix} = \begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} 0.90 & 0.57 & 0.37 \\ 0.09 & 0.41 & 0.005 \\ -0.00002 & 0.006 & 0.62 \end{bmatrix} \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix} \tag{13.38}$$

This equation introduces the normalization constant η. This is because the constraint establishes that the chromaticity coordinates of the white point should be the same, but not their color components; by substitution in Eq. (13.14), it is easy to see that the colors [0.33 0.33 0.33] and η[0.33 0.33 0.33] have the same chromaticity values.

By developing Eq. (13.38),

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{\eta} \begin{bmatrix} 1/0.33 & 0 & 0 \\ 0 & 1/0.33 & 0 \\ 0 & 0 & 1/0.33 \end{bmatrix} \begin{bmatrix} 0.90 & 0.57 & 0.37 \\ 0.09 & 0.41 & 0.005 \\ -0.00002 & 0.006 & 0.62 \end{bmatrix} \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix} \tag{13.39}$$

Thus,

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{\eta} \begin{bmatrix} 1.84 \\ 0.52 \\ 0.62 \end{bmatrix} \tag{13.40}$$

By using these values in Eqs (13.35) and (13.37),

$$M = \frac{1}{\eta} \begin{bmatrix} 0.489 & 0.31 & 0.20 \\ 0.17 & 0.81 & 0.01 \\ 0.00 & 0.01 & 0.99 \end{bmatrix}; \quad M^{-1} = \eta \begin{bmatrix} 2.36 & -0.89 & -0.45 \\ -0.51 & 1.42 & -0.088 \\ -0.005 & -0.01 & 1.00 \end{bmatrix} \tag{13.41}$$

To determine η, we consider the second row of the transformation. That is,

$$y = (0.17r + 0.81g + 0.01b)/\eta \tag{13.42}$$

This value corresponds to the perceived intensity and it is given in Eq. (13.25), so

$$r + 4.59g + 0.06b = (0.17r + 0.81g + 0.01b)/\eta \tag{13.43}$$

Consequently,

$$\eta = \frac{0.17r + 0.81g + 0.01b}{r + 4.59g + 0.06b} = 0.17 \tag{13.44}$$

and

$$M = \begin{bmatrix} 2.76 & 1.75 & 1.13 \\ 1.0 & 4.59 & 0.06 \\ 0.00 & 0.05 & 5.59 \end{bmatrix}; \quad M^{-1} = \begin{bmatrix} 0.41 & -0.15 & -0.08 \\ -0.09 & 0.25 & -0.016 \\ 0.0 & 0.0 & 0.17 \end{bmatrix} \tag{13.45}$$

The second row in the first matrix defines γ by the luminance coefficients of the CIE RGB model. Thus the γ component actually gives the color's perceived brightness. Notice that since these equations were derived from the luminosity of the photopic vision, the maximum luminance is around 555 nm. However, alternative equations can be developed for considering other illuminations and other definitions of the white color.

13.3.3.5 CIE XYZ color matching functions

The transformation defined in Eq. (13.21) can be used to obtain the colors in the XYZ model from the components of the CIE RGB model. However, a definition of the XYZ color model cannot be given just by a transformation, but a practical definition of the color model should provide a mechanism that permits obtaining the representation of colors without reference to other color models. This mechanism is defined by the color matching functions.

Similar to the definition of the CIE RGB, the color components in the XYZ model can be defined by considering a sample of single colors. Subsequently, the components of any color can be obtained by considering its spectral composition. This process can be described in a way analogous to Eq. (13.12). That is,

$$x = \int |C_\lambda| \hat{x}_\lambda \, \mathrm{d}\lambda; \quad y = \int |C_\lambda| \hat{y}_\lambda \, \mathrm{d}\lambda; \quad z = \int |C_\lambda| \hat{z}_\lambda \, \mathrm{d}\lambda \tag{13.46}$$

Here, the components $[x\ y\ z]$ of a color are obtained from the intensity $|C_\lambda|$ at wavelength λ and the color matching functions $\hat{x}_\lambda$, $\hat{y}_\lambda$, and $\hat{z}_\lambda$. These functions are defined by the XYZ components of monochromatic lights. Thus, the definition of the XYZ system uses the transformation in Eq. (13.21) to determine the values for single colors that define the XYZ color matching functions. That is,

$$[\hat{x}_\lambda \quad \hat{y}_\lambda \quad \hat{z}_\lambda]^{\mathrm{T}} = M[\hat{r}_\lambda \quad \hat{g}_\lambda \quad \hat{b}_\lambda]^{\mathrm{T}} \tag{13.47}$$

The problem with this equation is that the $\hat{y}_\lambda$ values are related to the perceived intensity only in average terms. This can be seen by recalling that Eq. (13.25) only defines the perceived intensity that minimizes the error over all wavelengths. Thus, the value of $\hat{y}_\lambda$ will not be equal to the perceived intensity, but we can only expect that the average difference between these values for all wavelengths is small.

In order to make $\hat{y}_\lambda$ equal to V_λ, the definition of the XYZ model considered a different value for the constant η for every wavelength. To justify this definition, it should be noted that the selection of the value of the constant does not change the chromaticity properties of the model; the constant multiplies the three color components, thus it does not change the values obtained in Eq. (13.14). Accordingly, by changing the constant of the transformation for each wavelength, the criteria defined in the chromaticity diagram are maintained, and just the components (including the intensity) are rescaled. Thus, it is possible to define a scaling that satisfies the criteria and that makes the intensity equal to V_λ. As such, the scale that gives the value of the perceived intensity dependent on the wavelength λ is given by

$$\eta_\lambda = \frac{0.17\hat{r}_\lambda + 0.81\hat{g}_\lambda + 0.01\hat{b}_\lambda}{V_\lambda} \tag{13.48}$$

Equation (13.44) is a special form of this equation, but the constant is defined to obtain the best average intensity while this equation is defined per frequency. By considering the constant defined in Eq. (13.48) in Eq. (13.42), we have

$$y = \frac{0.17r + 0.81g + 0.01b}{0.17\hat{r}_\lambda + 0.81\hat{g}_\lambda + 0.01\hat{b}_\lambda} V_\lambda \tag{13.49}$$

Thus, if we are transforming the monochromatic color $\hat{x}_\lambda$, $\hat{y}_\lambda$, and $\hat{z}_\lambda$, the intensity y is equal to V_λ. This implies that there is a matrix for each wavelength. That is,

$$M_\lambda = \frac{1}{\eta_\lambda}\begin{bmatrix} 0.489 & 0.31 & 0.20 \\ 0.17 & 0.81 & 0.01 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \tag{13.50}$$

This matrix was obtained by considering Eq. (13.41) for the definition in Eq. (13.48). Thus, the transformation in Eq. (13.47) is replaced by

$$\begin{bmatrix} \hat{x}_\lambda & \hat{y}_\lambda & \hat{z}_\lambda \end{bmatrix}^{\mathrm{T}} = M_\lambda \begin{bmatrix} \hat{r}_\lambda & \hat{g}_\lambda & \hat{b}_\lambda \end{bmatrix}^{\mathrm{T}} \tag{13.51}$$

This transformation defines the color matching functions for the XYZ model. The general form of the curves is shown in Figure 13.2. The $\hat{y}_\lambda$ component illustrated as a green curve in the figure is equal to the intensity V_λ in Figure 13.4. Thus, a single component in the XYZ model gives the maximum perceivable brightness.

Evidently, since Eq. (13.51) defines the color matching functions, the calculations of the color components based on Eq. (13.21) are inaccurate. That is, the CIE RGB and the XYZ models are not related by a single matrix transformation; when computing a color by using the transformation and the color matching functions, we obtain different results since the color matching functions are obtained from several scaled matrices. Additionally, when considering colors composed of several frequencies, the transformation will include inaccuracies given the

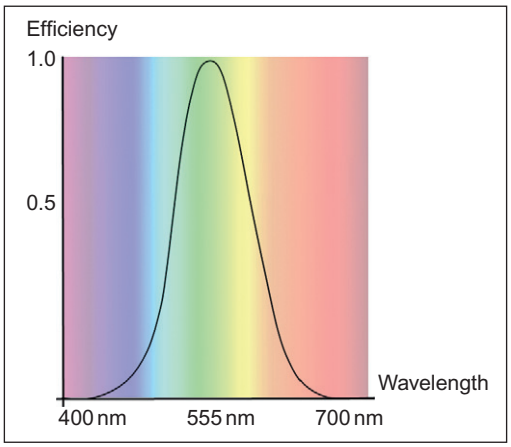


FIGURE 13.4

Luminous efficiency defined by the photopic luminosity function. This figure is also reproduced in color in the color plate section.

complexity of the actual intensity resulting from the mixture of wavelengths. Nevertheless, Eq. (13.21) is approximately correct on average and in practice can be used to transform colors. Alternatively, there are standard tables for the color matching functions of both models, so the representation of a color can be obtained by considering Eqs (13.12) and (13.46).

Actually there is little practical interest in transforming colors between the CIE RGB and XYZ models. The actual importance of their relationship is to understand the physical realization of color models and the theoretical criteria used to develop the XYZ model. The understanding of the physical realization of a color model describes perception or image capture. That is, how colors become numbers and what these numbers represent to our perception. The understanding of the XYZ criteria gives a justification to the creation of nonphysically realizable models to satisfy properties that are useful in understanding colors. In fact, there is always interest in using the properties of the XYZ model for other physical models; properties and color relationships in practical models are commonly explained by allusion to properties of the XYZ model. These properties are generally described using the XYZ chromaticity diagram.

13.3.3.6 XYZ chromaticity diagram

The visible colors of the XYZ model delineate the pyramid-like volume illustrated in Figure 13.3(c). Each line from the origin defines colors with the same chromaticity. The chromaticity coordinates are defined according to Eq. (13.18) and the chromaticity diagram shown in Figure 13.3(d) is obtained by considering the $\bar{x}$ and $\bar{y}$ values.

The chromaticity diagram provides a visual understanding of the properties of colors. The origin of the diagram is labeled as blue and the end of the axis as red and green. This indicates how colors change along each axis. The $\bar{y}$ value represents the perceived brightness. Similar to the CIE RGB chromaticity diagram, the visible colors define a horseshoe-shaped region. The colors along the curved rim of this region are colors with a single frequency component. This line is called the spectral line. The straight line of the horseshoe region is called the purple line. Colors in this line are created by mixing the monochromatic lights at the extremes of the visual spectrum, at 400 and 700 nm.

In addition to showing the palette of colors in the visual spectrum, the chromaticity diagram is also useful to visualize *hue* and *saturation*. These properties are defined by expressing colors relative to white using polar coordinates. By taking the white point [1/3 1/3] as reference, the hue of a color is defined as the angular component and its saturation as the radial length. The saturation is normalized such that the maximum value for a given hue is always one and it is given for the points in the border of the horseshoe region. As such, moving toward white on the same radial line produces colors with the same hue, but which are more desaturated. These define the shades of the color on the border of the horseshoe region. Any color with small saturation becomes white. Tracing curves such that their points keep the same distance to the border of the horseshoe region produces colors of different hue, but with constant saturation.

The chromaticity diagram is also useful to visualize relationships between mixtures of colors. The mix of colors that are generated from any two source colors is found by considering all the points in the straight line joining them. That is, the colors obtained by linearly combining the extreme points. Similarly, we can determine how a color can be obtained from another color by considering the line joining the points in the diagram. Any point in a line can be obtained by a linear combination of any other two points in the same line. Thus, the chromaticity diagram can be used to show how to mix colors to create the same perceived color (metamerism).

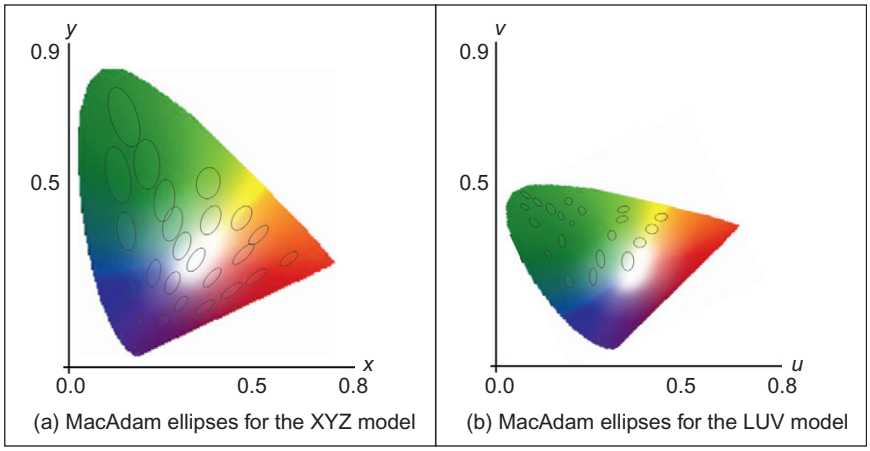
13.3.4 Uniform color spaces: CIE LUV and CIE LAB

The XYZ model is very useful to visualize the colors we can perceive and their relationships. However, it lacks uniformity or perceptual linearity. That is, the perceived difference between two colors is not directly related to the distance of the colors as represented in the chromaticity diagram. In other words, the perceived difference between points at the same distance in chromaticity can be significantly dissimilar. In practice, uniformity and linearity are important properties if we are using the measure of color differences as an indication of how similar are the colors for the visual system. For example, in image classification, if we measure a large difference between the color of two pixels, we may wrongly assume that they form part of a different class, but in fact these can be very similar to our eye. Another example of the importance of using uniform color systems

is when color measures are used to determine the accuracy of color reproduction systems. In this case, the quality of a system is given by how different the colors are actually perceived rather than how different are in the chromaticity diagram. Also linearity is desirable in reproduction systems since we do not want to spend resources storing different colors that look the same to the human eye.

The nonuniformity of the XYZ system is generally illustrated by using the MacAdam ellipses shown in Figure 13.5(a). These ellipses were obtained by experiments using matching colors (MacAdam, 1942). In the experiments, observers were asked to adjust the color components of one color until it matches a fixed color from the chromaticity diagram. The results showed that the accuracy of matching depends on the test color and that the matching colors obtained from different observers lie within ellipses with different orientations and sizes. The original experiments derived the 25 ellipses illustrated in Figure 13.5(a). The center of the ellipse is given by the fixed color and their area encompasses the matching colors by the observers.

The MacAdam experiments showed that our ability to distinguish between colors is not the same for all colors, thus distances in the chromaticity diagram are not a good measure of color differences. Ideally, observed differences should be delineated by circles with the same radius such that a given distance between colors has the same meaning independent of the position in the diagram. The study of the nonuniformity of the XYZ color model motivated several other models that look for better linearity. In 1976, the CIE provided two standards for these uniform spaces. They are known as the *CIE LUV* and the *CIE LAB* color models. The basic concept of these models is to transform the color components of the XYZ colors so that perceptual differences in the chromaticity diagram are more uniform.



(a) MacAdam ellipses for the XYZ model (b) MacAdam ellipses for the LUV model

FIGURE 13.5

CIE LUV uniformity. This figure is also reproduced in color in the color plate section.

The definition of the CIE LUV model is based on the following equations:

$$u = \frac{4\bar{x}}{\bar{x} + 15\bar{y} + 3}; \quad v = \frac{9\bar{y}}{\bar{x} + 15\bar{y} + 3} \tag{13.52}$$

Similar to Eq. (13.18), the overbar is used to indicate that the values represent chromaticity coordinates. This equation can also be expressed in terms of the color components by considering the definition in Eq. (13.18). That is,

$$u = \frac{4x}{x + 15y + 3z}; \quad v = \frac{9y}{x + 15y + 3z} \tag{13.53}$$

Both Eqs (13.52) and (13.53) are equivalent, but one is expressed using chromaticity coordinates and the other by using color components. In both cases, the transformation distorts the coordinates to form a color space with better perceptual linearity than the XYZ color model. The result is not a perfect uniform space, but the linearity between perceived differences is improved.

The transformation in Eq. (13.52) was originally used as a simple way to improve perceptual linearity in earlier color models (Judd, 1935). Later, the transformation was used in the LUV color model, but this model also includes the use of a reference point and it separates the normalization of brightness. As mentioned in Section 13.3.3.1, the white reference point is used to account for variations in illumination; the human eye adapts to the definition of white depending on the lighting conditions, thus having white as reference can be used to describe color under different lightings. The LUV color model uses as reference the standard indirect light white; however, it can be translated to represent other lights.

The LUV model defines a reference point denoted as $[u_n \; v_n]$. This point is obtained by transforming the chromaticity coordinates of the white color. That is, by considering the values $\bar{x}_n = 0.31$, $\bar{y}_n = 0.33$ in Eq. (13.52), we have

$$u_n = \frac{4\bar{x}_n}{-2\bar{x}_n + 12\bar{y}_n + 3}; \quad v_n = \frac{9\bar{y}_n}{-2\bar{x}_n + 12\bar{y}_n + 3} \tag{13.54}$$

This equation can also be expressed in terms of the color components of the white color by considering Eq. (13.53). In any case, the transformation of the white color for indirect daylight gives as a result a reference point close to [0.2 0.46]. This point is used to define the color components in the LUV model as

$$u^* = 13L^*(u - u_n); \quad v^* = 13L^*(v - v_n) \tag{13.55}$$

Here, $[u^* \; v^*]$ are the color components and the lightness L^* is given by

$$L^* = \begin{cases} \left(\frac{29}{3}\right)^3 \left(\frac{y}{y_n}\right), & \frac{y}{y_n} \leq \left(\frac{6}{29}\right)^3 \\ 116\left(\frac{y}{y_n}\right)^{1/3} - 16, & \text{otherwise} \end{cases} \tag{13.56}$$

Equations (13.55) and (13.56) transform color components. However, equivalent equations can be developed to map chromaticity coordinates by following Eq. (13.53) instead of Eq. (13.52).

In addition to centering the transformation on the reference point, Eq. (13.55) introduces a brightness scale value L^*. Remember that the Y axis gives the perception of brightness, thus by dividing y_n the color is made relative to the brightness of the white color and the linearization is made dependent on the vertical distance to the reference point. When using the white color as reference and since XYZ is normalized, $y_n = 1$. However, other values may be used when using a different reference point.

Equation (13.56) makes the perception of brightness more uniform and it has two parts that are defined by considering small and large intensity values. In most cases, the color is normalized by the part containing the cubic root, thus the normalization is exponentially decreased as y increases. That is, points closer to the $\overline{ox}$ axis in Figure 13.5(a) have a larger scale than points far away from this axis. However, for small values, the cube root function has a very large slope and as a consequence small differences in brightness produce very large values. Thus, the cubic root is replaced by a line that gives better scale values for small intensities. In addition to the cubic root, the normalization includes constant factors that made the value to be in a range from 0 to 100. This was arbitrarily chosen as an appropriate range for describing color brightness.

The constant values in Eq. (13.55) are chosen so that measured distances between systems can be compared. In particular, when the color differences are computed by using the Euclidean distance, a distance of 13 in the XYZ model corresponds to the distance of one in the LUV color model (Poynton, 2003). The constants produce a range of values between -134 to 220 for u^* and -140 to 122 for v^*. However, these values can be normalized as illustrated in Figure 13.5(b). This diagram is known as the uniform chromaticity scale diagram. The figure illustrates the shape of the MacAdam ellipses in the LUV color model with less eccentricity and more uniform size. However, they are not perfect circles. In practice, the approximation provides a useful model to measure perceived color differences.

The LAB color model is an alternative to the LUV model. It uses a similar transformation for the brightness, but it changes the way colors are normalized with respect to the reference point. The definition of LAB color model is given by

$$\begin{aligned} L^* &= 116 f\left(\frac{y}{y_n}\right) - 16; \quad a^* = 500\left(f\left(\frac{x}{x_n}\right) - f\left(\frac{y}{y_n}\right)\right); \\ b^* &= 200\left(f\left(\frac{y}{y_n}\right) - f\left(\frac{z}{z_n}\right)\right) \end{aligned} \tag{13.57}$$

for

$$f(s) = \begin{cases} \frac{1}{3}\left(\frac{29}{6}\right)^2 s + \frac{16}{116}, & s \leq (6/29)^3 \\ s^{1/3}, & \text{otherwise} \end{cases} \tag{13.58}$$

The definition of L^* is very similar to the LUV model. In fact, if we substitute Eq. (13.58) in the definition of L^* in Eq. (13.57), we obtain an equation that is almost identical to Eq. (13.56). The only difference is that the LUV model uses a line with zero intercept to replace the cubic root for small values while the LAB model uses a line with the same value and slope as the cubic part at the point $(6/29)^3$. In practice, the definition of L^* in both the LUV and LAB gives very similar values.

Although the definition of L^* is practically the same, the normalization by using the reference point in the LUV and LAB color models are different; the LUV color model uses subtraction while the LAB divides the color coordinates by the reference point. Additionally, in the LAB color model, the coordinates are obtained by subtracting opposite colors. The use of opposite colors is motivated by the observation that most of the colors we normally perceive are not created by mixing opposites (Nida-Rümelin and Suarez, 2009). That is, there is no reddish-green or yellowish-blue, but combinations of opposites have a tendency toward gray. Thus, the opposites provide natural axes for describing a color. As such, the a^* and b^* values are called the red/green and the yellow/blue *chrominances*, respectively, and they have positive and negative values. These values do not have limits and they extend to colors not visible by the human eye; however for digital representations, the range is limited by values between -127 and 127.

The a^*, b^* and the dark-bright luminosity define the axes of a 3D diagram referred to as the LAB chart and that is illustrated in Figure 13.6. In this figure,

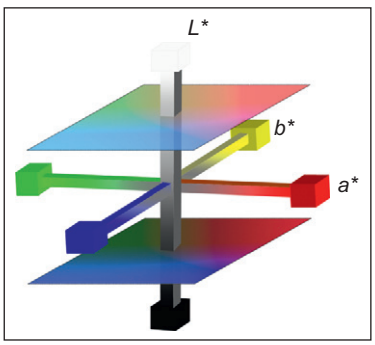


FIGURE 13.6

CIE LAB color space. This figure is also reproduced in color in the color plate section.

the top/bottom axis of this graph represents the lightness L^* and it ranges from black to white. The other two axes represent the red/green and yellow/blue values. Negative values in a^* indicate green while positive values indicate magenta. Similarly, negative and positive values of b^* indicate yellow and blue colors. Since visualizing 3D data is difficult, generally the colors in the LAB model are shown as slices parallel to the a^* and b^* axes. Two of these slices are illustrated in Figure 13.6.

In order to obtain an inverse mapping that obtains the components of a color in the XYZ color model from the LUV and LAB values, we can simply invert the equations defining the transformations. For example, the chromatic coordinates of a color can be obtained from the LUV coordinates by inverting Eqs (13.52) and (13.54). That is,

$$\bar{x} = \frac{9u}{6u + 16v + 12}; \quad \bar{y} = \frac{4v}{6u + 16v + 12} \tag{13.59}$$

and

$$u = \frac{u^*}{13L^*} + u_n; \quad v = \frac{u^*}{13L^*} + v_n \tag{13.60}$$

For the LAB color model, the coordinates in the XYZ space can be obtained by inverting Eqs (13.57) and (13.58). That is,

$$\begin{aligned} &y = y_n f^{-1}\left(\frac{L^* + 16}{116}\right); \quad x = x_n f^{-1}\left(\frac{L^* + 16}{116} + \frac{a^*}{500}\right); \\ &y = y_n f^{-1}\left(\frac{L^* + 16}{116} + \frac{b^*}{200}\right) \end{aligned} \tag{13.61}$$

for

$$f^{-1}(s) = \begin{cases} 3\left(\frac{6}{29}\right)^2\left(s - \frac{16}{116}\right), & s \leq 6/29 \\ s^3, & \text{otherwise} \end{cases} \tag{13.62}$$

It is important to understand that the colors in the LUV, LAB, and XYZ models are the same and they represent the colors we can perceive. These transformations just define mappings between coordinates. That is, they change the way we name or locate each color in a coordinate space. What is important is how coordinates of different colors are related to each other. That is, each color model arranges or positions the colors differently in a coordinate space, so the special relationships between colors have specific properties.

As explained before, the XYZ provides a good understanding of color properties and it is motivated by the way we match different colors using single frequency normalized components (i.e., color matching functions). The LUV and LAB color models provide an arrangement that approximates the way we perceive differences between colors. That is, they have better perceptual linearity,

chromatic adaptation, and they match better the human perception of lightness. This is important so, for example, to predict how observers will detect color differences in graphic displays. Additionally, there is some experimental works in the image processing literature that have shown that these models can also be useful for tasks such as color matching, detecting shadows, texture analysis, and edge classification. This may be related to their better perceptual linearity; however, it is important to remember that these models were not designed to provide the best information or correlation about colors, but to model and give a special arrangement of the human response to color data.

13.3.5 Additive and subtractive color models: RGB and CMY

13.3.5.1 RGB and CMY

The CIE RGB and XYZ models represent all the colors that can be perceived by the human eye by combining three monochromatic lights non-visible for the XYZ model. Thus, although it has important theoretical significance, they are not adequate for modeling practical color reproduction and capture systems such as photography, printers, scanners, cameras, and displays. In the case of reproduction systems, producing colors with a single frequency (e.g., lasers) with adequate intensity for generating visible colors with an adequate luminosity is very expensive. Similarly, sensors in cameras integrate the luminosity over a wide range of visible colors. Consequently, the base colors in capture and reproduction systems use visible colors composed of several electromagnetic frequencies. Thus, there is a need of device dependent color models that are finally determined by factors such as the amount of ink or video voltages. Fortunately, images rarely contain saturated colors, so a no monochromatic base provides a good reproduction for most colors without compromising intensity.

The RGB color models use base colors containing components close to the red, green, and blue wavelengths. These models are used, for example, by CRT (cathode ray tube) displays and photographic films. The base colors in these models are denoted as $[R\ G\ B]$ and their components as $[r\ g\ b]$. Other reproduction systems, such as inkjet and laser printers, use base colors close to the complementary of RGB, i.e., cyan, yellow, and magenta. These models are called CMY and their base colors and components are denoted as $[C\ M\ Y]$ and $[c\ m\ y]$, respectively. The CIE RGB is a particular RGB model; however, the term RGB models is generally only used to refer color models developed for practical reproduction systems. The motivation to have several RGB and CMY color models is to characterize the physical properties of different reproduction systems.

The RGB and CMY color models differ in the way in which the colors are created; RGB is an additive model while CMY is subtractive. The additive or subtractive nature of the models is determined by the physical mechanism used in the reproduction system. In the RGB, the base colors are generated by small light emitting components such as fluorescent phosphors, diodes, or semiconductors. These components are positioned very close to each other, so its light is combined

and perceived as a single color. Thus, the creation from colors stems from black and it adds the intensities of the base colors. In CMY, the base colors are colorants that are applied on a white surface. The colors act as filters between the white surface and the eye producing a change in our perception. That is, colors are subtracted from white. For example, to create green, we need to filter all the colors but green, thus we should apply the complementary or opposing color to green—magenta.

CMY has been extended to CMYK model by adding black to the base colors. The use of black has two practical motivations. First, in a reproduction system, it is cheaper to include a black than use CMY to generate black. Secondly, using three different colors produces less detail and shade than using a single color. This is particularly important if we consider that a great amount of printing material is in black and white.

Since RGB and CMY models are relevant for reproduction systems, in addition to the additive and subtractive properties, it is very important to describe the colors that are included in the model. This is called the *gamut* and it is generally described using a triangle in the chromaticity diagram as illustrated in Figure 13.7. In this figure, the triangles' vertices are defined by the base colors of typical RGB and CMY models. The triangle pointing upward illustrates a typical RGB color model while the upside down triangle illustrates a CMY model. Since colors are linearly combined, each triangle contains all the colors that can be obtained by the base colors, i.e., the gamut. This can be seen by considering that

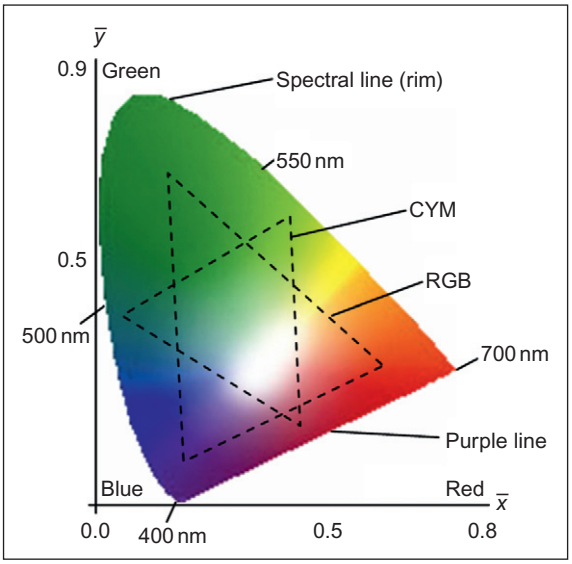


FIGURE 13.7

Chromaticity diagram. This figure is also reproduced in color in the color plate section.

any point between two of the base colors can be obtained by a linear combination between them. For example, any color that can be obtained by combining R and G is in the line joining those points. Thus, the full trace of lines between a point in this line and B fill in the triangle covering all the colors that can be created with the base.

In addition to visualizing the model using the chromaticity diagram, sometimes the colors in the RGB and CMY models are shown using a 3D cube where each axis defines one color of the base. This is called the RGB color cube and the range of possible values is generally normalized such that all colors are encompassed in a unit cube. The origin of the cube has coordinates [0 0 0] and it defines black, while the diagonal opposite corner [1 1 1] represents white. The vertices [1 0 0], [0 1 0], and [0 0 1] represent the base colors red, green, and blue, respectively, and the remaining three vertices represent the complementary colors yellow, cyan, and magenta. In practice, the chromaticity diagram is used to visualize the possible range of colors of a reproduction system while the cube representation is useful to visualize the possible color values.

Reproduction systems of the same type have similar base colors, but the exact spectral composition varies slightly. Thus, standards have been established to characterize different color reproduction systems. For example, the HDTV (high-definition television) uses points with chromaticity coordinates $R = [64\ \ 0.33]$, $G = [0.3\ \ 0.6]$ and $B = [0.15\ \ 0.06]$, while the NTSC (National Television System Committee) has the points $R = [0.67\ \ 0.33]$, $G = [0.21\ \ 0.71]$, and $B = [0.14\ \ 0.08]$. Other standards include the PAL (Phase Alternate Line) and the ROMM (Reference Output Medium Metric) developed by Kodak. In addition to standards, it is important to note that since color reproduction is generally done by using colors represented in digital form, often different color models are also strongly related to the way the components are digitally stored. For example, *true color* uses 8 bits per component while *high color* uses 5 bits for red and blue and 6 bits for green. However, independent of the type of model and storage format, the color representation uses the RGB and CMY color models.

13.3.5.2 Transformation between RGB color models

The transformation between RGB models is important to make data available to diverse reproduction and capture systems. Similar to Eq. (13.21), the transformation between RGB color models is defined by a linear transformation. That is,

$$\begin{bmatrix} r_1 \\ g_1 \\ b_1 \end{bmatrix} = M_{\text{RGB}} \begin{bmatrix} r_2 \\ g_2 \\ b_2 \end{bmatrix} \tag{13.63}$$

Here, $[r_1\ g_1\ b_1]$ and $[r_2\ g_2\ b_2]$ are the color components in two different RGB color models and M_{RGB} is a 3×3 nonsingular matrix. The matrix is generally derived by using the XYZ color model as a common reference. That is, M_{RGB} is

obtained by concatenating two transformations. First the component $[r_2\ g_2\ b_2]$ is mapped into the XYZ model and then it is mapped into $[r_1\ g_1\ b_1]$. That is,

$$\begin{bmatrix} r_1 \\ g_1 \\ b_1 \end{bmatrix} = M_{\text{RGB2,XYZ}} M_{\text{XYZ,RGB1}} \begin{bmatrix} r_2 \\ g_2 \\ b_2 \end{bmatrix} \tag{13.64}$$

The matrix $M_{\text{RGB2,XYZ}}$ denotes the transformation from $[r_2\ g_2\ b_2]$ to $[x\ y\ z]$ and $M_{\text{XYZ,RGB1}}$ denotes the transformation from $[x\ y\ z]$ to $[r_1\ g_1\ b_1]$.

In order to obtain $M_{\text{RGB,XYZ}}$, we can follow a similar development to the formulation presented in Section 13.3.3.4. However, in the RGB case, the coordinates of the points defining the color model are known from the color model standards. Thus, we only need to obtain the normalization constants. For example, the definition of the NTSC RGB color model gives the base colors with XYZ chromaticity coordinates [0.67 0.33], [0.21 0.71], and [0.14 0.08]. The definition also gives the white reference point [0.31 0.3161]. The position of these points in the color space is obtained by computing $\overline{z}$ according to Eq. (13.19) and by considering the mapping defined in Eq. (13.20). That is, the XYZ coordinates of the base colors for the NTSC model are given by

$$\begin{array}{l} \alpha[0.67 \quad 0.33 \quad 0.0] \\ \beta[0.21 \quad 0.71 \quad 0.08] \\ \gamma[0.14 \quad 0.08 \quad 0.78] \end{array} \tag{13.65}$$

This expression corresponds to Eq. (13.33) for the CIE RGB. However, in this case, the points are coordinates in the XYZ color space. Since these points are mapped into the points [1 0 0], [0 1 0], and [0 0 1] in the NTSC color space, we have

$$\begin{aligned} &\alpha \begin{bmatrix} 0.67 \\ 0.33 \\ 0.0 \end{bmatrix} = M_{\text{NTSC,XYZ}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad \beta \begin{bmatrix} 0.21 \\ 0.71 \\ 0.08 \end{bmatrix} = M_{\text{NTSC,XYZ}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \\ &\gamma \begin{bmatrix} 0.14 \\ 0.08 \\ 0.78 \end{bmatrix} = M_{\text{NTSC,XYZ}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{aligned} \tag{13.66}$$

That is,

$$M_{\text{NTSC,XYZ}} = \begin{bmatrix} 0.67\alpha & 0.21\beta & 0.14\gamma \\ 0.33\alpha & 0.71\beta & 0.08\gamma \\ 0.0\alpha & 0.08\beta & 0.78\gamma \end{bmatrix} \tag{13.67}$$

We can rewrite this matrix as

$$M_{\text{NTSC,XYZ}} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{13.68}$$

In order to compute the normalization constants, we invert this matrix. That is,

$$M_{\text{NTSC,XYZ}}^{-1} = \begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} 1.73 & -0.48 & -0.26 \\ -0.81 & 1.65 & -0.02 \\ 0.08 & -0.17 & 1.28 \end{bmatrix} \tag{13.69}$$

By using Eqs (13.19) and (13.20), we have that the XYZ coordinates of the NTSC reference point are $\eta[0.31\ 0.316\ 0.373]$, where η is a normalization constant. By considering this point in the transformation defined in Eq. (13.69), we have

$$\begin{bmatrix} 0.31 \\ 0.31 \\ 0.31 \end{bmatrix} = \eta \begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} 1.73 & -0.48 & -0.26 \\ -0.81 & 1.65 & -0.02 \\ 0.08 & -0.17 & 1.28 \end{bmatrix} \begin{bmatrix} 0.310 \\ 0.316 \\ 0.373 \end{bmatrix} \tag{13.70}$$

By rearranging the terms in this equation,

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \eta \begin{bmatrix} 1/0.31 & 0 & 0 \\ 0 & 1/0.31 & 0 \\ 0 & 0 & 1/0.31 \end{bmatrix} \begin{bmatrix} 1.73 & -0.48 & -0.26 \\ -0.81 & 1.65 & -0.02 \\ 0.08 & -0.17 & 1.28 \end{bmatrix} \begin{bmatrix} 0.310 \\ 0.316 \\ 0.373 \end{bmatrix} \tag{13.71}$$

Thus,

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \eta \begin{bmatrix} 0.92 \\ 0.84 \\ 1.45 \end{bmatrix} \tag{13.72}$$

By considering these values in Eq. (13.67),

$$M_{\text{NTSC,XYZ}} = \eta \begin{bmatrix} 0.62 & 0.18 & 0.20 \\ 0.30 & 0.59 & 0.11 \\ 0.00 & 0.07 & 1.13 \end{bmatrix} \tag{13.73}$$

The constant η is determined based on the perceived intensity. The brightest color in the NTSC model is given by the point [1 1 1]. According to Eq. (13.73), the intensity value is $0.3 + 0.59 + 0.11$. Since the maxima intensity in the XYZ color model is one, we have

$$\eta = \frac{1}{0.3 + 0.59 + 0.11} = 0.9805 \tag{13.74}$$

Thus,

$$M_{\text{NTSC,XYZ}}^{-1} = \begin{bmatrix} 0.60 & 0.17 & 0.02 \\ 0.29 & 0.58 & 0.11 \\ 0.00 & 0.06 & 1.11 \end{bmatrix} \tag{13.75}$$

By considering Eqs (13.74) and (13.72) in Eq. (13.68), we have

$$M_{\text{NTSC,XYZ}} = \begin{bmatrix} 1.91 & -0.53 & -0.28 \\ -0.98 & 1.99 & -0.02 \\ 0.05 & -0.11 & 0.89 \end{bmatrix} \tag{13.76}$$

The transformation matrices for other RGB color models can be obtained by following a similar procedure. For example, for the PAL RGB model, the chromaticity coordinates of the base points are [0.64 0.33], [0.29 0.60], and [0.15 0.06]. The definition also gives the white reference point [0.3127 0.3290]. Thus,

$$M_{\text{PAL,XYZ}} = \begin{bmatrix} 0.43 & 0.34 & 0.17 \\ 0.22 & 0.70 & 0.07 \\ 0.02 & 0.13 & 0.93 \end{bmatrix}; \quad M^{-1}_{\text{PAL,XYZ}} = \begin{bmatrix} 3.06 & -1.39 & -0.47 \\ -0.96 & 1.87 & 0.04 \\ 0.06 & -0.22 & 1.06 \end{bmatrix} \tag{13.77}$$

According to Eq. (13.64), the transformation between the NTSC and PAL model can be obtained by considering that

$$M_{\text{PAL,NTSC}} = M_{\text{PAL,XYZ}} M_{\text{XYZ,NTSC}} = M_{\text{PAL,XYZ}} M^{-1}_{\text{NTSC,XYZ}} \tag{13.78}$$

That is,

$$M_{\text{PAL,NTSC}} = \begin{bmatrix} 0.35 & 0.28 & 0.23 \\ 0.33 & 0.44 & 0.15 \\ 0.05 & 0.13 & 1.04 \end{bmatrix} \tag{13.79}$$

Thus, the transformation between different color models can be performed by considering the transformations using as reference the XYZ color model. The advantage of using transformations for the XYZ model is that transformations between any color model can be computed as a simple matrix multiplication. The transformations between different CMY models can be developed following a similar procedure, i.e., by computing the normalization constants according to three points and a reference white.

13.3.5.3 Transformation between RGB and CMY color models

A very simple approach to transform between RGB and CMY color models is to compute colors using the numerical complements of the coordinates. Thus, the transformation between RGB and CMY can be defined as

$$\begin{bmatrix} c \\ m \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} r \\ g \\ b \\ 1 \end{bmatrix} \tag{13.80}$$

The problem with this definition is that it does not actually transform the coordinates between models. That is, instead of looking for corresponding colors in

the XYZ model according to the RGB and CMY base colors, it assumes that the bases of the CMY are [0 1 1], [1 0 1], and [1 1 0] in RGB coordinates. However, the base of the CMY model certainly does not match the RGB model. Additionally, colors in the CMY that are out of the RGB gamut are not used. Consequently, these types of transformations generally produce very different colors in both models.

A better way to convert between RGB and CMY color models is to obtain a transformation by considering the base colors of the RGB and CMY in the XYZ reference. This approach is analogous to the way the transformation between models was developed as given in the previous section. However, this approach also has the problem of mapping colors out of the gamut. As shown in Figure 13.7, the triangles delineating the RGB and CMY models have large areas that do not overlap, thus some colors cannot be represented in both models. That is, a transformation based on the XYZ model will give coordinates of points outside the target gamut. Thus, for example, colors in a display will not be reproduced in a printed image. A solution to this problem is to replace colors mapped outside the target gamut by the closest color in the gamut. However, this loses the color gradients by saturating at the end of the gamut. Alternatively, the source colors can be scaled such that the gamut fits the target gamut. However, this reduces the color tones.

Since there is not a unique transformation between RGB and CMY models, the change between color models has been defined by using *color management systems*. These are software systems that use *color profiles* that describe the color transformation for particular hardware and viewing characteristics. The format of color profiles is standardized by the ICC (*International Color Consortium*) and they define the transformation from the source to the XYZ or CIE LAB. The transformation can be defined by parameters or by tables from where the intermediate colors can be interpolated. Since profiles use chromaticity coordinates, they also contain the coordinates of the white reference point.

Many capture systems such as cameras and scanners produce and use standard color models. Thus, the profile for these systems is commonly defined. However, since there is no best way to transform between models, every hardware device that captures or displays color data can have several profiles. They are generally provided by hardware manufacturers and they are obtained by carefully measuring and matching colors in their systems. Generally, there are profiles that provide the closest possible color matching as well as profiles that produce different colors but use most part of the target gamut. Other profiles manipulate colors to highlight particular parts of the gamut and saturate others. These profiles are denoted as profiles for different *rendering intent*. The best profile depends on factors such as the colors on the image, color relationships, desired lightness, and saturation as well as subjective perception.

As we have already explained, corresponding chromaticity coordinates to the XYZ model and a white reference point can be used to compute normalization constants that define the color model transformations. Thus, color management

systems use color profiles in a similar way to the color transformation defined in Eq. (13.64). That is, they use the transformation of the source to convert to the reference frame and then the inverse of the target to obtain the final transformed data. If necessary, it will also perform transformations between the XYZ and CIE LAB before transforming to the final color model. For example, a transformation from RGB to CMY can be performed by two transformations as

$$\begin{bmatrix} c \\ m \\ y \end{bmatrix} = M^{-1}_{\text{CMY,XYZ}} M_{\text{XYZ,RGB}} \begin{bmatrix} r \\ g \\ b \end{bmatrix} \tag{13.81}$$

Here, the transformations are represented as matrices, but generally they are defined by tables. Thus, the implementation performs lookups and interpolations. In a typical case, the first transformation will be defined by the profile of a camera or scan, while the second is given by an output device such as a printer.

13.3.6 Luminance and chrominance color models: YUV, YIQ, and YCbCr

The RGB color models define base colors according to practical physical properties of reproduction systems. Thus, the brightness of each color depends on all components. However, in some applications like video transmission, it is more convenient to have a separate single component to represent the perceived brightness. From a historical perspective, perhaps the most relevant models that use a component to represent brightness are the YUV and YIQ. It is important to mention that sometimes the term YUV is used to denote any color model that uses *luminance* and *chrominance* in different components; the *Y* component is called the *luma* and the remaining two components are referred to as the chrominance. However, YUV is actually a standard color model that, like YIQ, was specifically developed for analogue television transmission.

In the early development of television systems, it was important to have the brightness in a single component for two main reasons. First, the system was compatible with the old black and white televisions that contained a single luminance component; the added color data can be transmitted separately from the brightness. Secondly, the transmission bandwidth can be effectively reduced by dropping the bandwidth of the components having the chromaticity; since the human eye is more sensitive to luminance, the reduction in chromaticity produces less degradation in the images when using the RGB model. Thus, transmission errors are less noticeable by the human eye. Currently, the data reduction achieved with this color model is not only important for transmission and storing, but also for video processing. For processing, a separate luminance can be used to apply techniques based on gray level values as well of techniques that are independent of the luminosity.

The YUV and YIQ color models are specified by the NTSC and PAL television broadcasting standards. The difference between both color models is that the

YIQ has a rotation of 33° in the color components. The rotation defines the *I* axis to have colors between orange and blue and the *Q* axis to have colors between purple and green. Since the human eye is more sensitive to changes in the *I* axis than to the colors in the *Q* component, the signal transmission can use more bandwidth for *I* than for *Q* to create colors that are clearly distinguished. Unfortunately, the decoding of *I* and *Q* is very expensive and television sets did not achieved a full *I* and *Q* decoding. Nowadays, the NTSC and PAL standards are being replaced by digital standards such as the ATSC (Advanced Television Systems Committee).

Video signals can also be transmitted without combining them into a single channel, but by using three independent signals. This is called component video and it is commonly used for wire video transmission such as analogue video cameras and DVD players. The color model used in analogue component video is called YPbPr. The YCbCr is the corresponding standard for digital video. Since this standard separates luminance, it is adequate for data reduction and thus it has been used for digital compression encoding formats like MPEG (Moving Pictures Expert Group) and JPEG (Joint Photographic Expert Group). The data reduction in digital systems is implemented by having less samples of chrominance than luminance. Generally, the chrominance is only half or a quarter of the resolution of luma component. There are other color models such as YCC. This color model was developed for digital photography and it is commonly used in digital cameras.

There are applications that require converting between different luminance and chrominance models. For example, if processing increases the resolution of video images, then it will be necessary to change between the color model used in standard definition and the color model used in high definition. In these cases, the transformation can be developed in two steps by taking as reference RGB color models. More often, conversions between RGB and YUV color models are necessary when developing interfaces between transmission and reproduction systems, e.g., when printing a digital image from a television signal or when using an RGB display to present video data. Conversion to the YUV color model is also necessary when creating video data from data captured using RGB sensors and it can also be motivated by processing reasons. For example, applications based on color characterizations may benefit by using uniform spaces.

It is important to note that transformations between RGB color models and luminance and chrominance models do not change the color base, but they only rearrange the colors to give a different meaning to each component. Thus, the base colors of luminance and chrominance models are given by the RGB standards. For example, YIQ uses the NTSC RGB base colors. These are called the RGB base or primaries of the YIQ color model. That means that the luminance and chrominance models are defined from RGB base colors and this is the reason why sometimes luminance and chrominance are considered as a way of encoding RGB data rather than a color model per se.

13.3.6.1 Luminance and gamma correction

The transformation from RGB to YUV is defined by considering the y component as the perceived intensity of the color. The perceived intensity was defined by the luminosity function in Eq. (13.2). Certainly, this function depends on the composition of the base colors. For example, for the CIE RGB is defined by Eq. (13.25). Since the YUV and YIQ color models were developed for television transmission, perceived intensity was defined according to the properties of the CRT phosphorus used on early television sets. These are defined by the RGB NTSC base colors. If we consider the contribution that each component has to luminosity, then y will be approximately given by

$$y = 0.18r + 0.79g + 0.02b \tag{13.82}$$

This equation defines luminance. In YUV and YIQ, this equation is not directly used to represent brightness, but it is modified to incorporate a nonlinear transformation that minimizes the perceived changes in intensity. The transformation minimizes visible errors created by the necessary encoding of data using a limited bandwidth. Since the human eye distinguishes more clearly variations in intensity at low luminance than when the luminance is high, then an efficient coding of the brightness can be achieved if more bandwidth is used to represent dark values than bright values. Coding and decoding luminance is called *gamma encoding* or *gamma correction*.

The graph in Figure 13.8(a) illustrates the form of the transformations used in gamma correction. The horizontal axis of the graph represents the luminance y and the vertical axis represents the luma. The luma is generally denoted as y' and it is the value used to represent brightness in the YUV and YIQ color models. Accordingly, some texts use the notation Y′UV and YUV to distinguishing models using gamma corrected values. However, the transmission of analogue television always includes gamma corrected values. Curves representing gamma

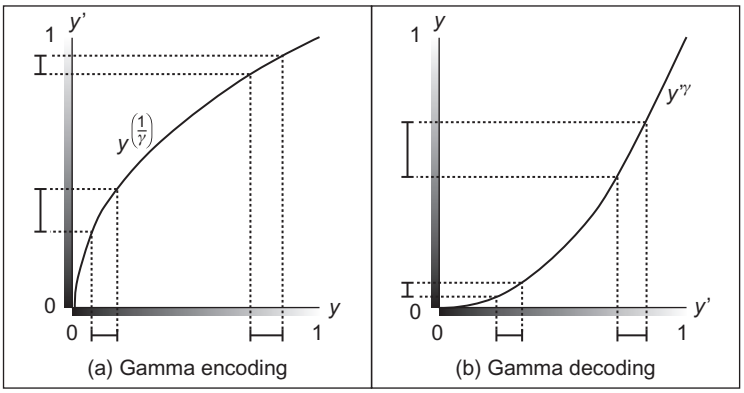


FIGURE 13.8

Gamma correction.

correction in Figure 13.8 only illustrate an approximation of the transformation used to obtain the luma. In practice, the transformation is defined by two parts: a power function is used for most of the curve and a linear function is used for the smallest values. The linear part is introduced to avoid generating insignificant values when the slope of the power exponential is close to zero.

Figure 13.8(a) illustrates the encoding power function that maps each luminance value into a point in the vertical axis. This mapping shrinks intervals at high luminance and expands intervals at y values. Thus, when y is encoded, more bandwidth is given to the values where the human eye is more acute. The power function is

$$\Gamma(y) = y^{\left(\frac{1}{\gamma}\right)} \tag{13.83}$$

Here, $(1/\gamma)$ is called the gamma encoding value and it was chosen by practical considerations for television sets. Since the CRT on television sets had a nonlinear response that approximates the inverse of the transformation in Eq. (13.83), the gamma value was choosing to match the inverse response. As such, there is no need for decoder hardware, but the CRT nonlinearity acts as a decoder and the intensity reaching the eye is linear. Thus, by using gamma correction, the transmission not only encoded the luminance efficiently, but at the same time it corrects for nonlinearity of the CRT. It is important to emphasize that the main aim of gamma encoding is not to correct the nonlinearity of CRT displays but to improve the visual quality by efficiently encoding luminance. The gamma encoding of television transmission was carefully chosen such that the nonlinearity of the CRT was also corrected when the signal was displayed. However, gamma correction is important even when image data is not displayed on a CRT and video data is often gamma corrected. Consequently, to process the video data, it is often necessary to have gamma decoding. After processing, if the results ought to be displayed on a screen, then it should be gamma encoded to match the screen gamma.

Figure 13.8(b) illustrates the decoding gamma transformation. The function in this graph is a typical voltage/luminance response of a CRT and it corresponds to the inverse of Eq. (13.83). Thus, it expands intervals at high luminance and shrinks intervals at low luminance. Consequently, it will transform values that have been gamma encoded into linear luminance. Since the encoding occurs before the transmission of the signal, limited bandwidth of the transmission produces larger errors at low luminance values than at high luminance values. Accordingly, the encoding effectively improves the perceived quality of the images; image artifacts such as banding and roping produced by quantization are created at low intensities, so they are not evident to the human eye.

Evidently, the value of gamma varies depending on particular properties of the CRT, but for the YUV and YIQ standards, it defines a value of $\gamma = 2.2$. That is, the gamma encoding for YUV should transform the values in Eq. (13.82) by the power in Eq. (13.83) with encoding gamma of 0.45. Since the RGB

components in television sets were produced by three independent electron beans, the encoding cannot apply the transformation to combine luminance, but each component is separately gamma corrected. That is, the luma is defined as the sum of gamma corrected RGB components. Thus, by gamma correcting Eq. (13.82) for $\gamma = 2.2$, we have

$$y' = 0.299r' + 0.587g' + 0.114b' \tag{13.84}$$

The prime symbol in this equation is used to indicate gamma corrected values. That is, $r' = \Gamma(r)$, $g' = \Gamma(g)$, and $b' = \Gamma(b)$. These values have a range between zero and one.

There is an alternative definition of luma that was developed according to current displays used for HDTV technology. This definition is given by

$$y' = 0.212r' + 0.715g' + 0.072b' \tag{13.85}$$

In practice, Eq. (13.84) is defined for YUV and YIQ and it is used for standard television resolutions (i.e., SDTV), while Eq. (13.85) is part of the ATSC standards and it is used for HDTV.

13.3.6.2 Chrominance

The U and V components represent the chrominance and they are defined as the difference between the color and the white color at the same luminance. Given an RGB color, the white at the same luminance is defined by Eq. (13.84). Thus, the chrominance is given by

$$\begin{aligned} u &= K_u(b' - y') \\ v &= K_v(r' - y') \end{aligned} \tag{13.86}$$

Only two components are necessary since for chromaticity one component is redundant according to the definition in Eq. (13.13). This definition uses gamma encoded components and that a color is between black and white (i.e., gray level values), the components have the same value. Thus, $y' = b' = r'$ and the chrominance becomes zero.

The constants K_u and K_v in Eq. (13.86) can be defined such that the values of u and v are within a predefined range. In television transmission, the color components of YUV and YIQ are combined into a single composite signal that contains the luma plus a modulated chrominance. In this case, the composite transmission is constrained by the amplitude limits of the television signal. This requires that u be between ± 0.436, while the values of v must be between ± 0.613 (Poynton, 2003).

The desired television transmission ranges for u and v are obtained by considering the maximum and minimum of $b' - y'$ and $r' - y'$. The maximum of $b' - y'$ is obtained when $r = g = 0$ and $b = 1$, i.e., $1 - 0.114$. The minimum value is obtained when $r = g = 1$ and $b = 0$, i.e., $-(1 - 0.114)$. Similarly, for $r' - y'$, we have that the maximum is obtained when $b = g = 0$ and $r = 1$ and the minimum

when $b = g = 1$ and $r = 0$. That is, the extreme values are $\pm(1 - 0.299)$. Accordingly, the constants that bound the values to ± 0.436 and ± 0.613 are

$$\begin{aligned} K_u &= 0.436/(1 - 0.114) \\ K_v &= 0.615/(1 - 0.299) \end{aligned} \tag{13.87}$$

That is,

$$\begin{aligned} u &= 0.493(b' - y') \\ v &= 0.877(r' - y') \end{aligned} \tag{13.88}$$

These constants are not related to perception or properties of the colors but are defined such that signals are appropriate for composite transmission according to the NTSC and PAL standards. The same constants are used when the signal is transmitted over two channels (i.e., *S-Video*), but as we explain below they are different when the signal is transmitted over three channels.

13.3.6.3 Transformations between YUV, YIQ, and RGB color models

By considering the luma defined in Eq. (13.84) and by algebraically developing the chrominance defined in Eq. (13.88), we can express the mapping from RGB color model to YUV color by using a 3×3 transformation matrix. That is,

$$\begin{bmatrix} y' \\ u \\ v \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.288 & 0.436 \\ 0.615 & -0.514 & -0.100 \end{bmatrix} \begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} \tag{13.89}$$

A similar transformation for high-definition video can be obtained by replacing the first row of the matrix according to Eq. (13.85). The transformation from YUV to RGB is defined by computing the inverse of the matrix in Eq. (13.89), That is,

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 1.139 \\ 1.0 & -0.394 & -0.580 \\ 1.0 & 2.032 & 0.00 \end{bmatrix} \begin{bmatrix} y' \\ u \\ v \end{bmatrix} \tag{13.90}$$

In the case of the YIQ model, the luma and chrominance follow the same formulation, but the U and V components are rotated by 33°. That is,

$$\begin{bmatrix} i \\ q \end{bmatrix} = \begin{bmatrix} \cos(33) & -\sin(33) \\ \sin(33) & \cos(33) \end{bmatrix} \begin{bmatrix} 0.877(r' - y') \\ 0.499(b' - y') \end{bmatrix} \tag{13.91}$$

By developing this matrix and by considering Eq. (13.82), we have

$$\begin{bmatrix} y' \\ i \\ q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} \tag{13.92}$$

The transformation from YIQ to RGB is defined by taking the inverse of this matrix. That is,

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} y' \\ i \\ q \end{bmatrix} \tag{13.93}$$

13.3.6.4 Color model for component video: YPbPr

The YPbPr color model uses the definition of luma given in Eq. (13.84) and the chrominance is defined in an analogous way to Eq. (13.86). That is,

$$\begin{aligned} p_b &= K_b(b' - y') \\ p_r &= K_r(r' - y') \end{aligned} \tag{13.94}$$

The difference between this equation and Eq. (13.86) is that since YPbPr was developed for component video, it assumes that signals are transmitted independently and consequently there are different constraints about the range of the transmission signals. For component video, the luma is transmitted using a 1 V signal, but this signal also contains sync tips, thus the actual luma has a 0−700 mV amplitude range. In order to bring the chrominance to the same range, the normalization constants in Eq. (13.94) are defined such that the chrominance is limited to half the luma range (i.e., ± 0.5). Thus, signals are transmitted using a maximum amplitude of ± 0.350 mV that represent the same 700 mV range of the luma signal.

In a similar way to Eq. (13.87), in order to bound the chrominance values to ± 0.5, the normalization constants are defined by multiplying by the desired range:

$$\begin{aligned} K_b &= 0.5/(1 - 0.114) \\ K_b &= 0.5/(1 - 0.299) \end{aligned} \tag{13.95}$$

By using these constants in Eq. (13.94), we have

$$\begin{aligned} p_b &= 0.564(b' - y') \\ p_r &= 0.713(r' - y') \end{aligned} \tag{13.96}$$

As such, the transformation from the RGB color model to the YUV color model is given by

$$\begin{bmatrix} y' \\ p_b \\ p_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} \tag{13.97}$$

The inverse is then given by

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 1.402 \\ 1.0 & -0.344 & -0.714 \\ 1.0 & 1.772 & 0.0 \end{bmatrix} \begin{bmatrix} y' \\ p_b \\ p_r \end{bmatrix} \tag{13.98}$$

The transformations for HDTV can be obtained by replacing the first row in Eqs (13.97) and (13.98) according to the definition of luma in Eq. (13.85).

13.3.6.5 Color model for digital video: YCbCr

The YUV, YIQ, and YPbPr color models provide a representation of colors based on continuous values defined for the transmission of analogue signals. However, transmission and processing of data in digital technology require a color representation based on a finite set of values. The YCbCr color model defines a digital representation of color by digitally encoding the luma and chrominance components of the YPbPr model.

The YCbCr model encodes the values of YPbPr by using 8 bits per component, but there are extensions based on 10 bits. The luma byte represents an unsigned integer and its values range from 16 for black to 235 for white. Since chrominance values in the YPbPr model are positive and negative, the chrominance bytes in YCbCr represent two's complement signed integers centered at 128. Also, the YCbCr standard defines that the maximum chrominance values should be limited to 240. The ranges of the components in the YCbCr model are called *YCbCr video levels* and they do not cover the maximum range that can be represented using 8 bits. The range is clipped to avoid having YCbCr colors that when mapped to the RGB can create saturate colors out of the RGB gamut. That is, the range of the YCbCr components is chosen to be a subset of the RGB gamut. The xvYCC color model extends YCbCr representation by considering that modern displays and reproduction technologies can have a gamut that includes higher saturation values. Thus, the full 8 bit range is used. Also some applications, like JPEG encoding, have been considered more practical to use the full 8 bit range.

By considering the range of the components in the YCbCr model and by recalling that the luma in the YPbPr model ranges from 0 to 1 while the chrominance takes values between ± 0.5, then the transformation that defines the YCbCr color model is given by

$$y'_c = 16 + 219^* y'; \quad C_b = 128 + 224 p_b; \quad C_r = 128 + 224 p_r \tag{13.99}$$

Here we use y'_c to denote the luma component in the YCbCr color model. For applications using the full range represented by 8 bits, we have the alternative definition given by

$$y'_c = 255^* y'; \quad C_b = 128 + 256 p_b; \quad C_r = 128 + 256 p_r \tag{13.100}$$

By developing Eq. (13.99), according to the definitions in Eqs (13.84) and (13.96), we have that the transformation from RGB to YCbCr can be written as

$$\begin{bmatrix} y'_c \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 65.481 & 128.553 & 24.966 \\ -37.797 & -74.203 & 112.0 \\ 112.0 & -93.786 & -18.214 \end{bmatrix} \begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} + \begin{bmatrix} 16 \\ 128 \\ 128 \end{bmatrix} \tag{13.101}$$

By solving for r', g', and b', we have that the transformation from the YCbCr color model to the RGB color model is given by

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \begin{bmatrix} 0.00456 & 0.0 & 0.00625 \\ 0.00456 & -0.00153 & -0.00318 \\ 0.00456 & 0.00791 & 0.0 \end{bmatrix} \begin{bmatrix} y' - 16 \\ p_b - 128 \\ p_r - 128 \end{bmatrix} \tag{13.102}$$

For high-definition data, the definition should use Eq. (13.85) instead of Eq. (13.84). Also when converting data considering full range defined by 8 bits, the transformation equations are developed from Eq. (13.100) instead of using Eq. (13.99). Also, since this representation is aimed at digital data, there are formulae that approximate the transformation by using integers or bit manipulations.

Similar to color models used for analogue transmission, the YCbCr encodes colors efficiently by using more data for luma than for chrominance. This is achieved by using different samplings for the image data. The notation 4:2:2 is used to indicate that images have been codified by sampling the chrominance half the frequency than the luma. That is, each pair of pixels in an image's row has four bytes that represent two luminance values and two chrominance values; there is a luma for each pixel, but the chrominance is the same for both pixels. The notation 4:1:1 is used to indicate that 4 pixels share the same chrominance values. In addition to these representations, some standards like MPEG support vertical and horizontal sampling. In this case, four pixels in two consecutive rows and two consecutive columns are represented by six bytes; four for luminance and two for chrominance.

13.3.7 Perceptual color models: HSV and HLS

As mentioned in Section 13.3.5, RGB color models are aimed at representing colors created in reproduction systems. Thus, the combination of RGB components cannot be intuitive to human interpretation. That is, it is difficult to determine the precise values that should have color components that create a particular color. Even when using the visualization of the RGB color cube, the interpretation of colors is not simple since perceptual properties such as the color brightness vary indistinctly along the RGB axes. Of course the chromaticity diagram is very useful to visualize the relationships and properties of RGB colors. However, since this diagram is defined in the XYZ color space, it is difficult to relate color's properties to RGB component values. Other color models like YUV provide an intuitive representation of intensity, but chrominance only represents the difference to white at same luminance, thus the color ranges are not very intuitive. *Perceptual color models* are created by a transformation that rearranges the colors defined by the RGB color model such that their components are easy to interpret. This is achieved by relating components to colors' characteristics such as hue, brightness, or saturation. Thus, tasks such as color picking and color adjustments can be performed using color properties having an intuitive meaning.

There are many perceptual color models, but perhaps the most common are the HSV (hue, saturation, value) and the HLS (hue, lightness, saturation). The

HSV is also referred to as HSI (hue, saturation, intensity) or as the HSB (hue, saturation, brightness). HSV and HLS use two components to define the hue and saturation of a color but they use different concepts to define the component that represents the brightness. It is important to make clear that the definition of hue and saturation used by these color models does not correspond to the actual color's properties defined in Section 13.3.3.6 but are ad hoc measures based on intuitive observations of the RGB color cube. However, similar to the hue and saturation discussed in Section 13.3.3.6 and illustrated by using the chromaticity diagram shown in Figure 13.3, the hue and saturation in the HIS and HSV color models is defined by using polar coordinates relative to a reference gray or white point. The hue of a color that provides a meaning to the color family like, for example, red, yellow, or green is defined by the angular component and the saturation that provides an intuitive meaning of color sensation from white or gray is defined by the radial distance.

In order to compute hue and saturation according to the perception of the human eye, it is necessary to obtain the polar coordinates of the corresponding CIE RGB or XYZ color's chromaticity coordinates. However, the development of the HSV and HLS color models opts for a simpler method that omits the transformation between RGB and XYZ by computing the hue and saturation directly from the RGB coordinates (Smith, 1978). This simplicity in computation leads to three undesirable properties (Ford and Roberts, 1998): first, as we discussed in Section 13.3.5, the RGB coordinates are device dependent. Thus, the color description in these models will change depending on the reproduction or capture devices. That is, the same image used on television sets and on a digital camera will have different color's properties. Secondly, RGB coordinates are not based on human perception but are dependent on color reproduction technology. Thus, the computations are not based on reference values that match our perception. As such, the colors' properties in HSI and HSV color models give only rough approximations of perceived properties. Finally, since the color's luminance is not actually correlated to definitions like the luminosity functions and the computations use approximations, the brightness component does not correspond to the actual perceived brightness. Consequently, changes in hue or saturation can be perceived as changes in brightness and vice versa. However, in spite of these drawbacks, the intuitive definition provided by the HIS and HSV color models has demonstrated to be useful in developing tools for color selection. In image processing, these models are useful for operations that categorize range of colors and automatic color replacement since color rules and conditionals can be simply specified based on intuitive concepts.

Since HSV and HLS are defined by ad hoc practical notions rather than by formal concepts, there are several alternative transformations to compute the color components. All transformations are special developments of the original hexagon and triangle geometries (Smith, 1978). Both geometrics define brightness by using planes with normal along the line defining gray. In the hexagonal model, planes are defined as projections of subcubes in the RGB color cube while the triangle

model planes are defined by three points in the RGB axes. In general, the hexagon model should be preferred because the transformations are simple to compute (Smith, 1978). However, there are implementations of the HLS transformations suitable for real-time processing in current hardware. Thus, other factors such as the HIS model is more flexible about the definition of brightness, and the better distribution of the color makes the HIS color model more attractive for image processing applications.

13.3.7.1 The hexagonal model: HSV

Figure 13.9 illustrates the derivation of the HSV color model according to the hexagonal model. In this model, the RGB color cube is organized by considering a collection of subcubes formed by changing the coordinates of the components from zero to the maxima possible coordinate value. The quantity defining the size of the subcubes is called the *value* which generally ranges from 0 to 1. A value of 0 defines a subcube enclosing a single color (i.e., black) and a value of 1 encompasses the whole RGB cube. The subcubes do not contain all the colors they can enclose but only include the colors in the three faces that are visible from the point defining the white corner of the RGB color cube and looking toward the origin. In Figure 13.9(a), these are the shaded faces of the smaller subcube. As such, each color in the RGB color cube is uniquely included in a subcube and the value that defines the subcube for any chosen color can be determined by computing

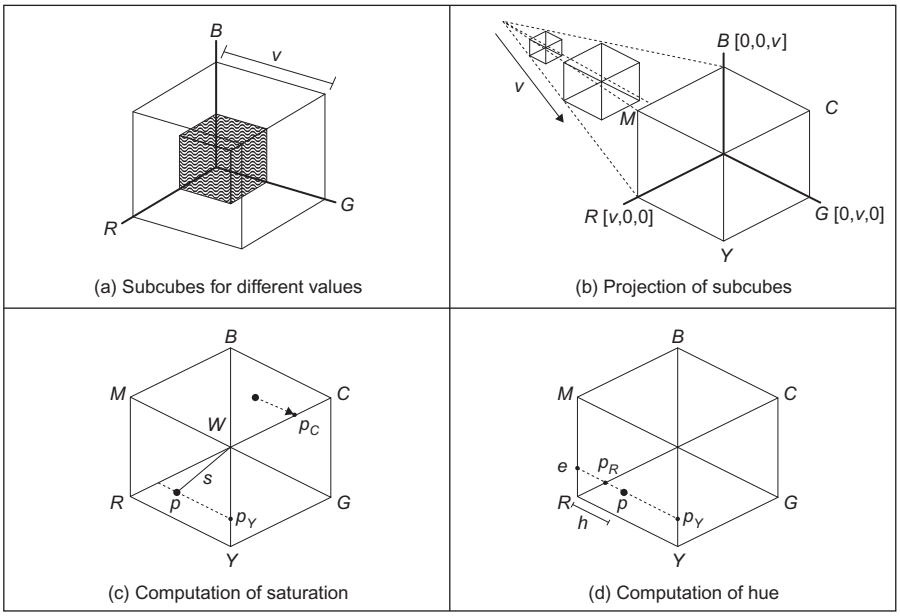


FIGURE 13.9

HSV color model.

the maxima of its coordinates. That is, a color $[r\ g\ b]$ is included in the cube defined by a value given by

$$v = \max(r, g, b) \tag{13.103}$$

According to this definition, the value in the HSV color model is related to the distance from black. Fully saturated colors like red, green, and yellow are in the same plane in the HSV color space. Evidently, this is not in accordance with the perceived intensity as defined by the luminosity function in Figure 13.4. However, this definition of blackness is useful to create user interfaces that permit the selection of colors given that hue is independent of brightness. In this method, the user can choose a desired hue or color base and then add blackness to change its *shade*. Change in *tint* or whiteness is given by the saturation, while tint and shade define the *tone* of the color. In the HSV color model, saturation is sometimes referred to as *chroma*.

The definition of value in Eq. (13.103) can be interpreted as a projection that takes all the colors in the faces of a subcube and maps them into a single plane as illustrated in Figure 13.9(b). Here, the view is aligned with the points defining black and white such that the projection defines a hexagon. Three of the vertices of the hexagon are defined by the RGB axis and they have coordinates $[v\ 0\ 0]$, $[0\ v\ 0]$, and $[0\ 0\ v]$. The other three vertices define yellow, cyan, and magenta, given by $[v\ v\ 0]$, $[0\ v\ v]$, and $[v\ 0\ v]$. The color is then defined as a position on a hexagonal plane around the lightness axis. The size of the hexagon is given by v and consequently the set of hexagons for all the subcubes form a hexahedron with the peak in the location of black. The value that defines brightness is determined by the color's vertical position in the axis of the hexahedron; at the peak of the hexahedron there is no brightness, so all colors are black while the brightest colors are at the other end.

Since in the projection the center of the hexagon defines gray levels, the saturation can be intuitively interpreted as the normalized distance from the color to the hexagon's center; when s is zero the color is gray, so it is desaturated. When the color is saturated then s is unity and the color lies in the border of the hexagon. Thus, the computation of saturation can be based on the geometry illustrated in Figure 13.9(c). Here, the center of the hexagon is indicated by the point w and the saturation for a point p is the distance s. Figure 13.9(c) illustrates an example for a color lying in the region between the axes R and G. In this case, the distance from w to p can be computed by considering a point p_Y on the Y axis. The subindex on the point indicates that the point lies on a particular axis. Thus, p_M and p_C are the points on the M and C axes that are used for colors in the GB and BR regions, respectively. By considering the geometry in Figure 13.9(c), the saturation is defined by three equations that are applicable depending on the region where the point p lies. That is,

$$s = \frac{|wp_Y|}{|wy|}; \quad s = \frac{|wp_C|}{|wc|}; \quad s = \frac{|wp_M|}{|wm|} \tag{13.104}$$

The first equation defines saturation when p is in the RG regions and the two remaining equations when it is in the GB and BR regions. In these equations, the notations $|wy|$, $|wc|$, and $|wm|$ indicate the distances from the point w to the maxima point along the Y, C, and M axis. Thus, the divisor normalizes the distance to be between zero and one. In Figure 13.9(c), these distances correspond to the length of the subcube defining the hexagon given in Eq. (13.103). By considering the geometry in Figure 13.9(c), the distance for each point can be computed as

$$|wp_Y| = |wy| - |yp_Y|; \quad |wp_C| = |wc| - |wp_C|; \quad |wp_M| = |wm| - |wp_M| \tag{13.105}$$

Thus, by considering Eqs (13.103) and (13.105) in Eq. (13.104),

$$s = \frac{v - |yp_Y|}{v}; \quad s = \frac{v - |yp_C|}{v}; \quad s = \frac{v - |yp_M|}{v} \tag{13.106}$$

We can also see in Figure 13.9(c) that the distances in these equations correspond to the color component in the direction of the axis where the point lies. That is,

$$s = \frac{v - b}{v}; \quad s = \frac{v - g}{v}; \quad s = \frac{v - r}{v} \tag{13.107}$$

In order to combine these three equations into a single relationship, it is necessary to observe how the $[r\ g\ b]$ coordinates of a color determine its region in the hexagon. By observing the projection of the cube illustrated in Figure 13.9, it can be seen that a color is in the region RG only if the b component of the color is lower than r and g. Similarly, the color is in the GB region only if r is the smallest component and it is in the region BR only if g is the smallest component. Accordingly,

$$s = \frac{v - \min(r, g, b)}{v} \tag{13.108}$$

Similar to saturation, the hue of a color is intuitively interpreted by considering the geometry of the hexagon obtained by the subcube's projection. As such, the hue is considered as the angular value taking as reference the center of the hexagon; by changing the angle, we change the color from red, yellow, green, cyan blue, and magenta. Naturally, the computation of the hue is also dependent on the part of the hexagon where the color lies.

Figure 13.9(d) illustrates the geometry used to compute the angle for a point between the R and Y lines. The angular position of the point p is measured as a distance from the R line as

$$h = \frac{1}{6}\frac{|p_R p|}{|p_R p_Y|} \tag{13.109}$$

The divisor in the second factor normalizes the distance, thus the hue is independent of the saturation. According to this equation, the hue value is zero when the point is on the R line and it is 1/6 when it is on the Y line. This factor is

included since we are measuring the distance in one sextant of the hexagon, thus the distance around all the hexagon is one.

By considering the geometry in Figure 13.9(d), Eq. (13.109) can be rewritten as

$$h = \frac{|ep| - |ep_R|}{6 \cdot |wp_Y|} \tag{13.110}$$

The distance $|ep|$ is equal to the value given by the g component and by the similarity of the triangles in the figure, we have that $|ep_R|$ is equal to $|yp_Y|$. That is,

$$h = \frac{g - |yp_Y|}{6 \cdot |wp_Y|} \tag{13.111}$$

By considering Eq. (13.105), Eq. (13.111) can be rewritten as

$$h = \frac{g - |yp_Y|}{6 \cdot (|wy| - |yp_Y|)} \tag{13.112}$$

$|wy|$ corresponds to the length of the subcube defining the hexagon given in Eq. (13.103). Thus,

$$h = \frac{g - |yp_Y|}{6 \cdot (v - |yp_Y|)} \tag{13.113}$$

According to Eqs (13.106) and (13.107), the distance $|yp_Y|$ can be computed by the minimum value of the RGB components of the color. Thus,

$$h = \frac{g - \min(r, g, b)}{6 \cdot (v - \min(r, g, b))} \tag{13.114}$$

This equation is generally algebraically manipulated to be expressed as

$$h = \frac{v - \min(r, g, b) - (v - g)}{6 \cdot (v - \min(r, g, b))} \tag{13.115}$$

As such, the hue is defined by

$$h = \frac{(1 - h_G)}{6} \tag{13.116}$$

where

$$h_G = \frac{(v - g)}{v - \min(r, g, b)} \tag{13.117}$$

In order to obtain the hue for any color, it is necessary to consider all the regions in the hexagon. This leads to the following equations for each region:

$$\begin{array}{llll} h = (1 - h_G)/6 & \text{for } RY; & h = (1 + h_R)/6 & \text{for } YG \\ h = (3 - h_B)/6 & \text{for } GC; & h = (3 + h_G)/6 & \text{for } CB \\ h = (5 - h_R)/6 & \text{for } BM; & h = (5 - h_B)/6 & \text{for } MR \end{array} \tag{13.118}$$

In this notation, *RY* means when the color is between the line *R* and *Y* in the hexagon and

$$h_R = \frac{(v-r)}{v-\min(r,g,b)}; \quad h_B = \frac{(v-b)}{v-\min(r,g,b)} \tag{13.119}$$

The definitions in Eq. (13.118) add the angular displacements of each sextant, such that that 0 is obtained for the red color, 1/6 for yellow, 2/6 for green, etc. That is, the value of h ranges from 0 to 1. In practical implementations, the h value is generally multiplied by 360 to represent degrees or by 255 so it can be stored in a single byte. Also, h is not defined when $r = g = b$, i.e, for desaturated colors. In these cases, implementations generally use the colors of neighboring pixels to obtain a value for h or just use an arbitrary value.

The implementation of Eq. (13.118) requires determining in which sextant is a given color. This is done by considering the maximum and minimum values of RGB. The color will be in the regions *RG*, *GB*, or *GR* when blue, red, or green is the smallest value, respectively. Similarly, we can see in Figure 13.9 that a color will be in the regions *MY*, *YC*, or *CM* when r, g, or b are the maxima, respectively. Thus, by combining these conditions, we have that a color will be in a particular sextant according to the following relationships:

$$\begin{array}{lllll} RY & \text{if} & r = \max \text{RGB} & \text{and} & b = \min \text{RGB} \\ YG & \text{if} & g = \max \text{RGB} & \text{and} & b = \min \text{RGB} \\ GC & \text{if} & g = \max \text{RGB} & \text{and} & r = \min \text{RGB} \\ CB & \text{if} & b = \max \text{RGB} & \text{and} & r = \min \text{RGB} \\ BM & \text{if} & b = \max \text{RGB} & \text{and} & g = \min \text{RGB} \\ MR & \text{if} & r = \max \text{RGB} & \text{and} & g = \min \text{RGB} \end{array} \tag{13.120}$$

The maxima here can be substituted by v defined in Eq. (13.102).

The transformation from RGB to HSV color models is defined by solving for r, g, and b in Eqs (13.103), (13.108), and (13.118). Since the transformations are defined for each sextant, the inverse is also defined for each sextant. For the case of colors in the *RY* region, we can observe that according to Eq. (13.120), r is greater than the other two components, thus

$$r = v \tag{13.121}$$

Since in this sextant the minimum is b, the saturation is given by the first relationship in Eq. (13.107). By using this relationship and Eq. (13.121), we have

$$b = v(1-s) \tag{13.122}$$

The green component can be obtained by considering Eq. (13.114). That is,

$$h = \frac{g-b}{6 \cdot (v-b)} \tag{13.123}$$

This equation was developed for the RY region wherein b is the minimum of the RGB components. The value of g expressed in terms of h, s, and v can be obtained by substitution of Eq. (13.122) in Eq. (13.123). That is,

$$g = v(1 - s(1 - 6h)) \tag{13.124}$$

By performing similar developments for the six triangular regions on the hexahedron, the transformation from the HSV color model to the RGB color model is defined as

$$\begin{array}{llll} RY & r = v; & g = k; & b = m \\ YG & r = n; & g = v; & b = m \\ GC & r = m; & g = v; & b = k \\ CB & r = m; & g = n; & b = v \\ BM & r = k; & g = m; & b = v \\ MR & r = v; & g = m; & b = n \end{array} \tag{13.125}$$

for

$$\begin{aligned} m &= v(1 - s) \\ n &= v(1 - s^*F) \\ k &= v(1 - s(1 - F)) \end{aligned} \tag{13.126}$$

The value of F in these equations is introduced since the equations use the displacement from the start of the interval defined by the region. That is,

$$F = 6h - \text{floor}(6h) \tag{13.127}$$

Thus, for the region RY, the displacement is measured from the R axis; for the region YG, is measured from the Y axis, etc. The development in Eqs (13.122) and (13.124) uses $6h$ instead of F since both values are the same for the interval RY. In implementation of Eq. (13.125), the region of the color can be simply determined by considering the angle defined by h. The index of the region starting from zero for RY and ending with five for MR is floor($6h$).

13.3.7.2 The triangular model: HSI

Figure 13.10 illustrates the definition of the triangular model. In this model, the colors in the RGB cube are organized by a set of triangles formed by three points in the RGB axes. Each triangle defines a plane that contains colors with the same lightness value. As the lightness increases, the triangle moves further away from the origin, thus it contains brighter colors. The lightness in this model is defined by the value given by

$$l = w_R r + w_G g + w_B b \tag{13.128}$$

The weights w_R, w_G, and w_B are parameters of the color model and they scale each of the axes. When the axes are scaled, the triangles' center is biased toward a particular point. For example, if $w_R = 0.2$, $w_G = 0.4$, and $w_B = 0.4$, then the triangle will intersect the R axis at the middle of the distance of the other axes, thus

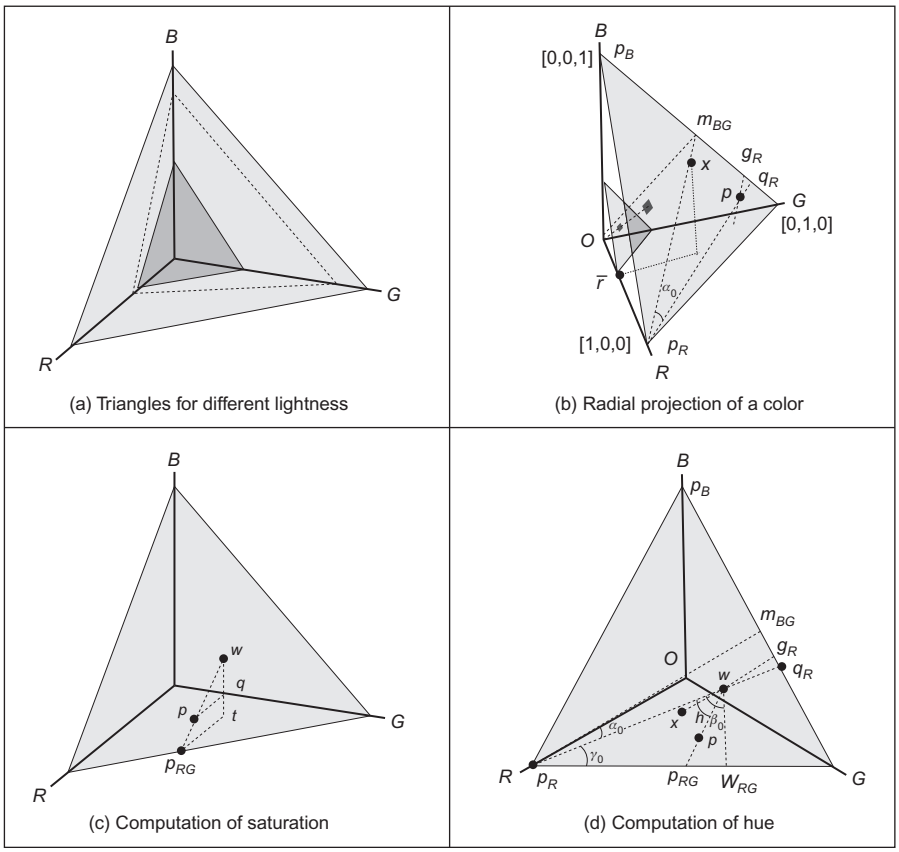


FIGURE 13.10

HSI color model.

its center will be biased toward the green and blue. This type of shift is illustrated by the dotted triangle in the diagram shown in Figure 13.10(a).

In the triangle model, a color is normalized to be independent of brightness by division by l.

$$\overline{r} = w_r \frac{r}{l}; \quad \overline{g} = w_g \frac{g}{l}; \quad \overline{b} = w_b \frac{b}{l} \tag{13.129}$$

As such, a color can be characterized by the lightness l and by its hue and saturation computed from normalized coordinates. The definition in Eq. (13.129) is similar to Eq. (13.14). This type of equation defines a central projection that maps the colors by tracing radial lines from the origin of the coordinate system. In the case of Eq. (13.129), the projection uses radial lines to map the colors into the normalized triangle defined by the points [1 0 0], [0 1 0], and [0 0 1].

Figure 13.10(b) illustrates this mapping. In this figure, the square in the small triangle is mapped into the larger triangle. The dotted line in the figure corresponds to the radial axis of the projection. The hue and saturation of any triangle is computed by using normalized coordinates. That is, the hue and saturation of any color are independent of its lightness and they are computed by considering the geometric measures in the normalized triangle.

There are two cases of interest for the scale settings. The first case is called the **unbiased case** and the second is called the **biased NTSC case**. The first considers that $w_R = w_G = w_B = 1/3$. That is, the gray points defined at the centers of the triangles are $[l/3\ \ l/3\ \ l/3]$. The white point is obtained for maxima lightness, [1/3 1/3 1/3]. According to Eq. (13.129), the lightness in the unbiased case is given by

$$l_{\text{unbiased}} = \frac{(r + g + b)}{3} \tag{13.130}$$

The problem with this definition is that the combination of luminance is poorly matched to the brightness perceived by the human eye. As shown in Figure 13.4, the perceived brightness in the human eye is stronger for green colors than for red and blue. The *biased NTSC case* is aimed at giving a better correlation between lightness and the brightness perceived by the human eye by using the weights given by $w_R = 0.3$, $w_G = 0.59$, and $w_B = 0.11$. These weights shift the gray points to be at $[0.3l\ \ 0.59l\ \ 0.11l]$ and the white point is located at [0.3 0.59 0.11]. According to Eq. (13.129), the lightness in the biased NTSC case is given by

$$l_{\text{NTSC}} = 0.3r + 0.59g + 0.11b \tag{13.131}$$

This equation is the same as the definition in Eq. (13.84), thus it corresponds to the luma in the YUV and YIQ color models. Accordingly, the lightness in this case should be well correlated to the human perception of luminance and it is compatible with analogue television. However, in order to be accurate, it is important that the RGB components to be gamma corrected. Another issue is that the weight values move the center point to colors that do not match perceived gray colors. The gray values in the RGB color models are generally defined for equal coordinate values, thus the hue and saturation are biased. It is also important to note that although the triangle model uses the mapping in Eq. (13.14), it does not use the chrominance diagram to define the coordinates, but the mapping is only used to obtain a radial projection. The chromaticity diagram is only defined for the CIE RGB and XYZ color models since they are based on perception experiments.

The geometry used to define the saturation in the triangle color model is illustrated in Figure 13.10(c). A color is indicated by the point p and w denotes the white point. Both points are normalized according to Eq. (13.129), thus they lie on the plane defined by the normalized triangle. The location of the point w changes for biased and unbiased cases. In the figure, t is the projection of w on the plane $b = 0$ and q is on the line defined by the points w and t.

Saturation is defined as the difference of a color from gray. That is, it can be intuitively interpreted as the normalized distance from p to w. When the distance is zero, the point represents a gray color and when it is one it represents one of the colors in the perimeter of the triangle. In order to formalize this concept, it is necessary to consider three different regions in the color space. The regions are illustrated in Figure 13.10(d) which shows the normalized triangle with the observer looking at the center of the RGB color cube. The three gray triangles in this figure define the regions *RG*, *GB*, and *BR*. The geometry in Figure 13.10(c) corresponds to a color in the *RG* region. In this case, the distance is normalized by dividing it by the distance to the point in the line border between the axes *R* and *G*. That is,

$$s_{RG} = \frac{|wp|}{|wp_{RG}|} \tag{13.132}$$

The subindex on s indicates that this equation is valid only for colors in the region *RG*. If α is the angle formed by the lines $p_{RG}w$ and $p_{RG}t$, then according to the dotted triangles in Figure 13.10(c), we have the following two trigonometric identities:

$$\sin(\alpha) = \frac{|wq|}{|wp|}; \quad \sin(\alpha) = \frac{|wt|}{|wp_{RG}|} \tag{13.133}$$

By substituting the values of $|wp|$ and $|wp_{RG}|$ from this equation into Eq. (13.132), we have

$$s_{RG} = \frac{|wq|}{|wt|} \tag{13.134}$$

By considering the definition of $|wq|$,

$$s_{RG} = \frac{|wt| - |qt|}{|wt|} = 1 - |qt| \tag{13.135}$$

The distance $|qt|$ corresponds to the blue component of the point p. This point is the projection of the color according to Eq. (13.129). Thus,

$$s_{RG} = 1 - \frac{b}{l} \tag{13.136}$$

Similar developments can be performed for colors in the regions *GB* and *BR*. In these cases, the point t is the projection of w into the planes $r = 0$ and $g = 0$, respectively. This leads to the following equations that define the saturation on each region:

$$s_{GB} = 1 - \frac{r}{l}; \quad s_{BR} = 1 - \frac{g}{l} \tag{13.137}$$

It is possible to combine Eqs (13.136) and (13.137) into a single equation that defines the saturation for any color by considering the way in which the $[r\ g\ b]$

components determine the region of the color. By observing the projection of the color in Figure 13.10(d), it can be seen that a color is in the region *RG* only if *b* is the smallest component. It is in the *GB* region if *r* is the smallest component, and it is in the region *BR* if *g* is the smallest component. Since the smallest component coincides with the color component used to define the saturation in Eqs (13.136) and (13.137),

$$s = 1 - \frac{\min(r, g, b)}{l} \tag{13.138}$$

The hue of a color is intuitively interpreted by considering the angular value in the normalized triangle by taking as reference the line joining the white point and the red color. This is illustrated in Figure 13.10(d). Here, the hue for the color represented by the point p corresponds to the angle defined between the lines wp_R and wp. In the example in this figure, the white point does not coincide with the center of the coordinates; however, the same definitions and formulations are applicable for the unbiased case. In both cases, an angle of zero corresponds with the red color.

By considering that the white point has the coordinates $\lfloor w_r\, w_g\, w_b \rfloor$ and the point p_R has the coordinates [1 0 0], then the vector from w to p_R is given by

$$wp_R = [1 - w_r \quad -w_g \quad -w_b] \tag{13.139}$$

Since the coordinates of the point p are defined by Eq. (13.129), the vector from w to p is given by

$$wp = \left[\frac{r}{l} - w_r \quad \frac{g}{l} - w_g \quad \frac{b}{l} - w_b\right] \tag{13.140}$$

The angle between the vectors in Eqs (13.139) and (13.140) can be obtained by considering the dot product. That is,

$$wp_R \cdot wp = |wp_R||wp|\cos(h) \tag{13.141}$$

By solving for h, we have

$$h = \cos^{-1}\left(\frac{wp_R \cdot wp}{|wp_R||wp|}\right) \tag{13.142}$$

The dot product and the two modules can be computed for Eqs (13.139) and (13.140). Thus,

$$h = \cos^{-1}(k) \tag{13.143}$$

For

$$k = \frac{(1 - w_r)(r - w_r) - w_g(g - w_g) - w_b(b - w_b)}{\sqrt{(1-w_r)^2 + w_g^2 + w_b^2 + (r-w_r)^2 + (g-w_g)^2 + (b-w_b)^2}} \tag{13.144}$$

The transformation in Eq. (13.143) is generally implemented by using an alternative expression that uses the arctangent function. That is, by using trigonometric identities, Eq. (13.143) becomes

$$h = \frac{\pi}{2} - \tan^{-1}\left(\frac{k}{\sqrt{1-k^2}}\right) \tag{13.145}$$

This equation will give the correct values only for angles between 0 and π corresponding to colors for which $b < g$. When the angle exceeds π, it is necessary to consider that the angle is negative (or measured clockwise). That is,

$$h = \begin{cases} \dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{k}{\sqrt{1-k^2}}\right), & \text{for } b < g \\ 2\pi - \dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{k}{\sqrt{1-k^2}}\right), & \text{otherwise} \end{cases} \tag{13.146}$$

This equation gives a range of values from 0 to 2π. In an implementation, the value obtained is generally expressed on degrees so it can be represented by an integer number. Alternatively, the range can be quantized to be represented by a single byte.

The transformation from RGB to HSL is defined by Eqs (13.130), (13.131), (13.138), and (13.146). Thus, the inverse transformation is obtained by solving for r, g, and b in these equations. Naturally, the inverse depends on which region is the color. This region can be determined by comparing the angle h against the angles formed between the red and green and between the green and blue axes. These angles are denoted by a_0 and a_1. For the unbiased case, the w point is in the middle of the triangle, thus $a_0 = a_1 = 120°$. In the biased case, these angles are $a_0 = 156.8°$ and $a_1 = 115.68°$. As such, the region of a color is determined by

$$\begin{array}{ll} RG, & \text{if } h < a_0 \\ GB, & \text{if } a_0 \leq h < a_0 + a_1 \\ BR, & \text{otherwise} \end{array} \tag{13.147}$$

Once the region of a color has been determined, a color component can be obtained by considering Eq. (13.136) or (13.137). For example, when the color is in the RG region, we have

$$b = l(1 - s) \tag{13.148}$$

Similarly, Eq. (13.137) can be used to find the red and green colors when the color is in the GB and RB regions, respectively.

The computation of the remaining two color components is based on the geometrical property of the normalized triangle (Figure 13.10(b)). Consider the triangle in 3D space that is formed by the points O, p_R, and m_{BG}. Here, the point m_{BG} is the midpoint of the BG line, so the angle between $p_R m_{BG}$ and the line between

the G and B axes is 90°. By following a similar development to the triangle relationships in Eq. (13.134), it is possible to relate the ratios between the distances along the OR axis and distances along the $p_R m_{BG}$ line. Thus, the distance for any color represented by the point x on the line $p_R q_R$ can be related to distances along the OR axis by the following expression:

$$\frac{|O\overline{r}|}{|Op_R|} = \frac{|m_{BG}x|}{|m_{BG}p_R|} \tag{13.149}$$

However, the distance $|Op_R|$ is one and $|O\overline{r}|$ is the red coordinate of the point x. Thus, this equation can be simply written as

$$\overline{r} = \frac{|m_{BG}x|}{|m_{BG}p_R|} \tag{13.150}$$

That is the red component of a color is defined as a ratio in the diagonal line. Here we denote the red component as $\overline{r}$. This is because the point x is on the normalized triangle, thus the red component actually corresponds to the normalized value given in Eq. (13.129). Similar expressions can be obtained for other color components. For example, for the blue component, we have

$$\overline{b} = \frac{|m_{GR}x|}{|m_{GR}p_B|} \tag{13.151}$$

Here m_{GR} is the middle point in the GR line and x is a color on the line $p_R m_{GR}$.

The relationship in Eq. (13.150) can be extended to lines that do not intersect BG at its middle point. For the point p on the line $p_R q_R$ in Figure 13.10(b), we have that the red value is given by

$$\overline{r} = \frac{|pg_R|}{|q_R p_R|\cos(\alpha_0)} \tag{13.152}$$

where

$$|pg_R| = |q_R p|\cos(\alpha_0) \tag{13.153}$$

Here, α_0 is the angle between the lines $p_R m_{BG}$ and $q_R p_R$. The cosine is introduced such that distances are measured in the same direction as that of the midline. That is, the substitution of Eq. (13.153) in Eq. (13.152) leads to Eq. (13.150).

Figure 13.10(d) illustrates how the definition in Eq. (13.152) can be used to obtain the red component for the color represented by a point p. In the figure, the point x is the orthogonal projection of p on the line wg_R. Similar to Figure 13.10 (b), this line has the same direction as the middle line $p_R m_{BG}$. The angle α_0 in the figure is defined by the location of the point w; for the unbiased case, w is in the middle of the triangle, thus $\alpha_0 = 0$, and for the biased case the angle is $\alpha_0 = 21.60°$. From Figure 13.10(d),

$$|xg_R| = |g_R w| + |wx| \tag{13.154}$$

That is,

$$|xg_R| = |q_R w|\cos(\alpha_0) + |wp|\cos(h - \alpha_0) \tag{13.155}$$

Here, the subtraction of the angles α_0 and h define the angle between the lines wx and wp. The subtraction is sometimes expressed as a summation by considering that α_0 is negative. By substitution of Eq. (15.55) in Eq. (13.152), we have

$$\overline{r} = \frac{|q_R w|\cos(\alpha_0) + |wp|\cos(\alpha_0 - h)}{|q_R p_R|\cos(\alpha_0)} \tag{13.156}$$

The first term in the right side of this equation defines the distance ratio for the point w. That is,

$$w_R = \frac{|q_R w|\cos(\alpha_0)}{|q_R p_R|\cos(\alpha_0)} \tag{13.157}$$

Thus,

$$\overline{r} = w_R + \frac{|wp|\cos(\alpha_0 - h)}{|q_R p_R|\cos(\alpha_0)} \tag{13.158}$$

By considering Eq. (13.132), this equation can be rewritten as

$$\overline{r} = w_R + \frac{s|wp_{RG}|\cos(\alpha_0 - h)}{|q_R p_R|\cos(\alpha_0)} \tag{13.159}$$

The distance $|wp_{RG}|$ can be obtained by considering the angle β_0 defined between the lines wp_{RG} and ww_{RG}. We can observe from Figure 13.10(d) that

$$\cos(\beta_0 - h) = \frac{|ww_{RG}|}{|wp_{RG}|} \tag{13.160}$$

Thus,

$$|wp_{RG}| = \frac{|ww_{RG}|}{\cos(\beta_0 - h)} \tag{13.161}$$

The angle β_0 in this equation can be expressed in terms of α_0 by observing the triangles in the figure that $\alpha_0 + \gamma_0 = 30$ and $\alpha_0 + \beta_0 = 90$. That is,

$$\beta_0 = \alpha_0 + 60 \tag{13.162}$$

By substitution of Eqs (13.161) and (13.162) in Eq. (13.159), we have

$$\overline{r} = w_R + s\frac{|ww_{RG}|\cos(\alpha_0 - h)}{|q_R p_R|\cos(\alpha_0)\cos(60 + \alpha_0 - h)} \tag{13.163}$$

By observing that the sides of the normalized triangle have the same length, we have that the middle distances are related by

$$|q_R p_R|\cos(\alpha_0) = |p_R m_{BG}| = |p_B m_{GR}| \tag{13.164}$$

That is,

$$\frac{|ww_{RG}|}{|q_R p_R|\cos(\alpha_0)} = \frac{|ww_{RG}|}{|p_B m_{GR}|} \tag{13.165}$$

The distances are measured in the same direction to the midline $|m_{GR}p_B|$. Thus, according to Eq. (13.151), the ratio in the left side in Eq. (13.165) defines the blue coordinate of the point. That is,

$$\frac{|ww_{RG}|}{|p_B m_{GR}|} = w_B \tag{13.166}$$

Thus, the equation for the red component is obtained by substitution of this relationship in Eq. (13.163). That is,

$$\overline{r} = w_R + sw_B \frac{\cos(\alpha_0 - h)}{\cos(60 + \alpha_0 - h)} \tag{13.167}$$

This equation represents the red normalized component of a color. The actual red component can be obtained by considering Eq. (13.129). That is,

$$r = l + sl \frac{w_B \cos(\alpha_0 - h)}{w_R \cos(60 + \alpha_0 - h)} \tag{13.168}$$

As such, the r and b components for a color can be computed using Eqs (13.148) and (13.168). The remaining component color can be computed using Eq. (13.128). That is,

$$g = \frac{(l - w_R r - w_B b)}{w_G} \tag{13.169}$$

Similar developments can be performed for obtaining the RGB components of colors in the regions GB and BR. Therefore, the complete transformation from HSL to RGB according to the definitions in Eq. (13.147) is given by

$$\begin{aligned}
&\text{if } h < a_0\text{:}\\
&\quad b = l(1-s); \quad r = l + s\frac{w_B \cos(A_0)}{w_R \cos(60 + A_0)}; \quad g = \frac{(l - w_B b - w_R r)}{w_G}\\
&\text{if } a_0 \le h < a_0 + a_1\text{:}\\
&\quad r = l(1-s); \quad g = l + s\frac{w_R \cos(A_1)}{w_G \cos(60 + A_1)}; \quad b = \frac{(l - w_R r - w_G g)}{w_B}\\
&\text{otherwise:}\\
&\quad g = l(1-s); \quad b = l + s\frac{w_G \cos(A_2)}{w_B \cos(60 + A_2)}; \quad r = \frac{(l - w_G g - w_B b)}{w_R}
\end{aligned} \tag{13.170}$$

For

$$\begin{aligned}
A_0 &= \alpha_0 - h\\
A_1 &= \alpha_1 - h - a_0\\
A_2 &= \alpha_2 - h - a_0 - a_1
\end{aligned} \tag{13.171}$$

These equations introduce the subtraction of the angles a_0 and a_1, so the computations are made relative to the first axis defining the region. In the unbiased model, the white point is at the center of the triangle, so the constants are defined by

$$\begin{aligned} &w_R = 0.33; \quad w_G = 0.33; \quad w_B = 0.33 \\ &a_0 = 120^\circ; \quad a_1 = 120^\circ \\ &\alpha_0 = 0; \quad \alpha_1 = 0; \quad \alpha_2 = 0 \end{aligned} \tag{13.172}$$

For the biased model, they are

$$\begin{aligned} &w_R = 0.30; \quad w_G = 0.59; \quad w_B = 0.11 \\ &a_0 = 156.58^\circ; \quad a_1 = 115.68^\circ \\ &\alpha_0 = 21.60^\circ; \quad \alpha_1 = -14.98^\circ; \quad \alpha_2 = -10.65^\circ \end{aligned} \tag{13.173}$$

The alpha sign is negative for the angles used in the *GB* and *BR* regions. This is because the lines defining those angles are in opposite direction to the direction of the angle α_0 used in the presented development.

13.3.8 More color models

This appendix has discussed different types of color spaces that have been created according to different motivations; there are color spaces aimed to formalize and standardize our perception of color, while other models are developed for a more practical nature according to the way reproduction systems work or how data should be organized for particular process such as video signal transmission. In any case, the color models are based on the tristimulus theory that formalized the sensations created by wavelengths in space. Thus, these models do not describe the physical spectral nature of color, but they provide a way to specify, re-create, and process our visual sensation of color using a 3D space.

It should be noted that this appendix has considered the most common color spaces; however, there are other important spaces that have similar motivations and properties, but they change the way colors are described. For example, the CIE LCH color model uses the same transformations as the LAB, but it uses cylindrical coordinates instead of rectangular. This gives a uniform space with polar coordinates, so it can be related to hue and saturation. The saturation in this space is generally referred to as *chroma* and it has the advantage of be more perceptually linear. Another example of an important color description is the Munsell color model. This color model also uses cylindrical coordinates, it uses perceptual uniform saturation, and it is based on measures of human perception. It is also important to mention that there exist other color models that have focused on achieving a practical color description. For example, the PANTONE color system consists of a large catalog of standardized colors.

In addition to many color spaces, the literature has alternative transformations for the same color space. Thus, in order to effectively use color information in image processing, it is important to understand the exact meaning of the color components in each color model. As such, the importance of the transformations

between models is not to define a recipe to convert colors but to formalize the relationships defined by the particular concepts that define each color space. Accordingly, the transformations presented in this appendix have been aimed at illustrating particular properties of the color spaces to understand their strengths and weaknesses rather than to prescribe how color spaces should be manipulated.

13.4 References

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