Points Eng 106 Homework #2 Solution Winter 2013

10 3.2 A department store has offered you a credit card that charges interest at 1.65% per month, compounded monthly. What is the nominal interest (annual percentage) rate for this credit card?

Nominal Interest Rate, r = 1.65% * 12 = 19.80 %/yr

What is the effective annual interest rate?

$\iota_a = (1+.0165)^{12} - 1 = 21.70\%$/yr

5 3.4 A California bank, Berkeley Savings and Loan, advertised the following information: interest 7.55% and effective annual yield 7.842%. No mention is made of the interest period in the advertisement. Can you figure out the compounding scheme used by the bank?

ι_a = 7.842% and r =7.55%

The scheme is likely using a continuous compounding scheme.

In continuous compounding,

$\iota_a = e^{r} - 1 = 7.842 \approx$ e^(.0755)-1

15 3.42 Suppose that $1,500 is placed in a bank account at the end of each quarter over the next 20 years. What is the account's future worth at the end of 20 years when the interest rate is 8% compounded

Quarterly deposits over 20 years means N = 80.

(a) Semi-annually? (Need to skim deposits to end of each compounding period, so N=40.)

3000 (F/A, ι%, 40)

ι = 8%/2 = 4%/half year

3000(F/A, 4%, 40) = A[((1+ 4%)^{40}-1)/4% = 3000 * (95.0255) = $285,077.

(b) Monthly?

1500 (F/A, ι , 80)

$\iota_e = (1 + r/M)^C - 1$ = (1 + 8%/12 comp.periods/year)^(12 int. period/4 pay. periods) - 1

= (1 + .08/12)^(3) – 1 = 2.0133%

1500(F/A, 2.0133%, 80) = A[((1+ 2.0133%)^{80}-1)/ 2.0133% = 1500 * (195.031) = $292,547.

(c) Continuously?

1500 (F/A, ι , 80)

$\iota_e = e^{r/k} - 1$ = exp(8%/4) – 1 = 2.0201%

1500 (F/A, 2.020%, 80) = A[((1+ 2.020%)^{80}-1)/2.020% = 1500*(195.669) = $293,503.

15 3.57 Janie Curtis borrowed $22,000 from a bank at an interest rate of 9% compounded monthly. This loan is to be repaid in 36 equal monthly installments over three years. Immediately after her 20th payment, Janie desires to pay the remainder of the loan in a single payment. Compute the total amounts she must pay at that time.

P = $22,000, r = 9%, N = 36, and ι = 9%/12 months = 0.75%

A = P (A/P, 0.75%, 36) = 22,000 (0.0318) = $699.6 / month.

The owed amount is the equivalent value at end of period 20 of payments 21 through 36.

P = A(P/A, 0.75%, 16) = $699.6 (15.0243) = $10,524.

10 4.11 An annuity provides for 10 consecutive end-of-year payments of \$10,000. The average general inflation rate is estimated to be 5% annually, and the market interest rate is 9% annually. What is the annuity worth in terms of a single equivalent amount of today's dollars?

The annuity amounts are in current (rather than constant) dollars, so use market interest rate
$P = A\ (P/A, \iota\ , N) = 10{,}000\ (\ P/A, 9\ \%, 10\ ) = 10{,}000 * (6.4177) = \$64{,}177.$

10 4.14 The purchase of a car requires a \$12,000 loan to be repaid in monthly installments for four years at 9% interest compounded monthly. If the general inflation rate is 4% compounded monthly, find the actual and constant dollar value of the 20^{th} payment of this loan.

N = 12 months * 4 years = 48 periods. i = r/m = 0.75% monthly, f = 4%/12 = 1/3 % monthly. P = 12000
A = P (A/P, interest rate, N), where for actual dollars, $A = A_{20}$ because all amounts in the series are equal
$A_{20} = 12000 * (A/P, 0.75\%, 48) = 12000 * (.0249) = \$298.6$ in actual dollars.
Deflate to year zero dollars: $A_n*[(P/F, \text{inflation rate}, n)] = A'_{20}$
$A'_{20} = A_{20}*[P/F, 1/3\%\ , 20)] = \$298.6*0.9356 = \$279.4$ in constant (year-zero) dollars.

10 #7 Begin with an equal payment series in constant dollars of A' = \$1000 at the end of each of three years.
$\iota = 9\%/yr$ and $f = 3.8\%/year$.

$A'_1 = A'_2 = A'_3 = \$1000$
$\iota' = \left((1+\iota)/(1+f)\right) - 1 = 1.09/1.038 - 1 \approx 5\%$ a year

First, convert this to a single amount at the end of year 0.
$P' = A'(P/A, 5\%, 3) = 1000 * (2.7232) = \$2{,}723.2$

Then, convert to actual dollars series using i:
$A = P\ (A/P, 9, 3) = \$2{,}723.2\ (0.3951) = \$1{,}076.$

75 (Total points)