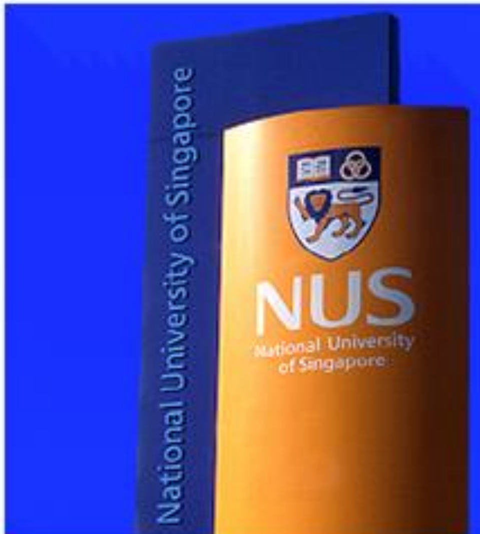


Singapore's Global University

CG1108
Electrical Engineering
Lecture 5

Dr. John-John Cabibihan

Dept. of Electrical and Computer Engineering

LÉON CHARLES THÉVENIN

(30 March 1857 - 21 September 1926)

was a French telegraph engineer who extended Ohm's law to the analysis of complex electrical circuits. As a result of these studies, he developed his famous Theorem, Thévenin's theorem.

Lecture Outline

- Revision
- Superposition theorem
- Thévenin’s theorem
- Norton’s theorem
- Maximum power transfer

Revision: Node Voltage Analysis

1. Objective: To find the unknown node voltages.
2. KCL is applied at the nodes to obtain the equations.
3. Branch currents are expressed in terms of the node voltages.
4. If the branch is a resistor then the branch current can be expressed as the difference of node voltages divided by the resistance.

Node Voltage Analysis

5. If the branch is an independent current source, then branch current is given.
6. If a branch is an ideal voltage source, then a super node enclosing the voltage source is used for applying the KCL.
7. For dependent sources, their values are written in terms of the node voltages.
8. Follow a convention, e.g. current leaving the nodes as positive.

Mesh Current Analysis

1. Objective: To find the unknown mesh currents.
2. KVL is applied at the meshes to obtain the equations.
3. Branch voltages are expressed in terms of the mesh currents.
4. If the branch is a resistor then the branch voltage can be expressed as the difference of mesh currents multiplied by the resistance.

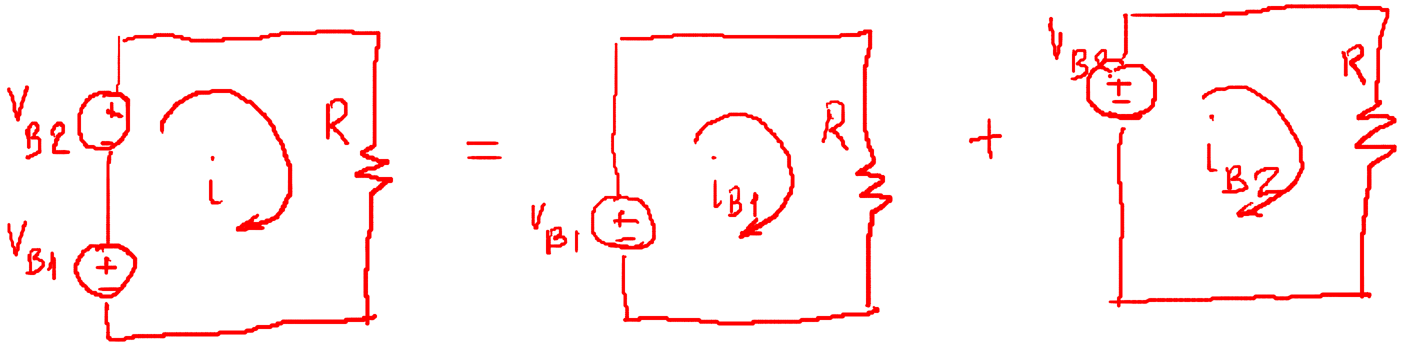
Mesh Current Analysis

5. If the branch is an independent voltage source, then branch voltage is given.
6. If a branch is an ideal current voltage, then a super mesh enclosing the current source is used for applying the KVL
7. For dependent sources, their values are written in terms of the mesh currents
8. Follow a convention, e.g. voltage fall as positive.

- **The principle of Superposition**
- **Linearity**
- **One-port networks and equivalent circuits**
  - Thevenin's equivalent circuit
  - Norton equivalent circuit
- **Maximum power transfer**

The principle of Superposition

- In a linear circuit containing *N* sources, each branch voltage and current is the sum of *N* voltages and currents, each of which may be computed by setting all but one source equal to zero and solving the circuit containing that single source.



$$i = \frac{V_{B1} + V_{B2}}{R} = \frac{V_{B1}}{R} + \frac{V_{B2}}{R} = i_{B1} + i_{B2}$$

To find the individual response…

- ‘Kill’ all the other independent sources but one.
- Use network analysis to find the required response.
- Do this one by one for all independent sources.
- Add up the responses to find the total response.

Killing a voltage source

- To kill a voltage source, we make its output voltage equal to zero.
- Replace the voltage source with a short circuit.

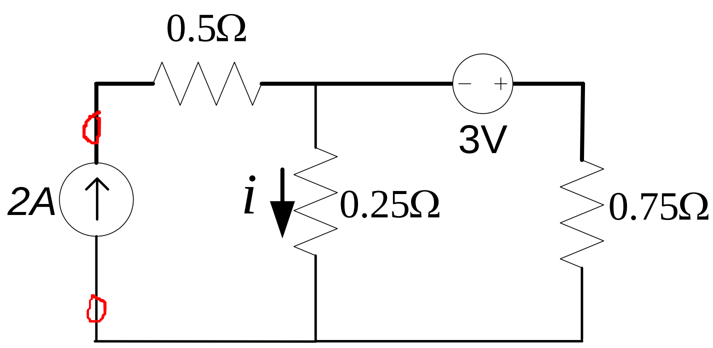
Killing a current source

- To kill a current source, we make its output current equal to zero.
- Replace the current source with an open circuit.

About the superposition principle…

- Superposition principle is not a precise analysis technique, but rather a conceptual aid to visualize the behavior of the linear circuits containing multiple sources.
- Sometimes, the analysis of a circuit is simplified by applying superposition principle i.e. by considering each independent source separately.

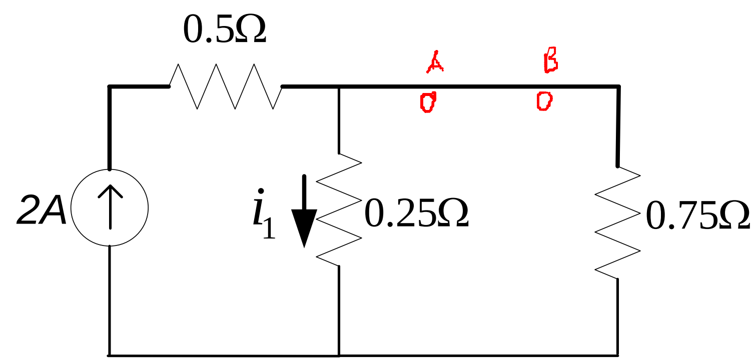
Example



Circuit with two independent sources

$i = i_1 + i_2$

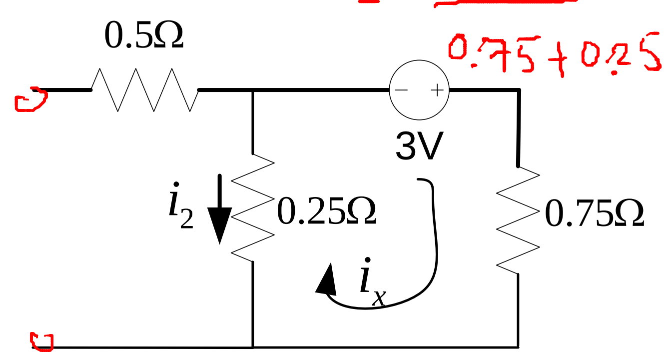
$= 1.5 - 3 = -1.5\,A$



Circuit with only the current source

$\underline{i_1 = 1.5\,A}$

$i_1 = \frac{0.75}{0.75+0.25} = 1.5\,A$



Circuit with only the voltage source

$i_2 = -i_x$

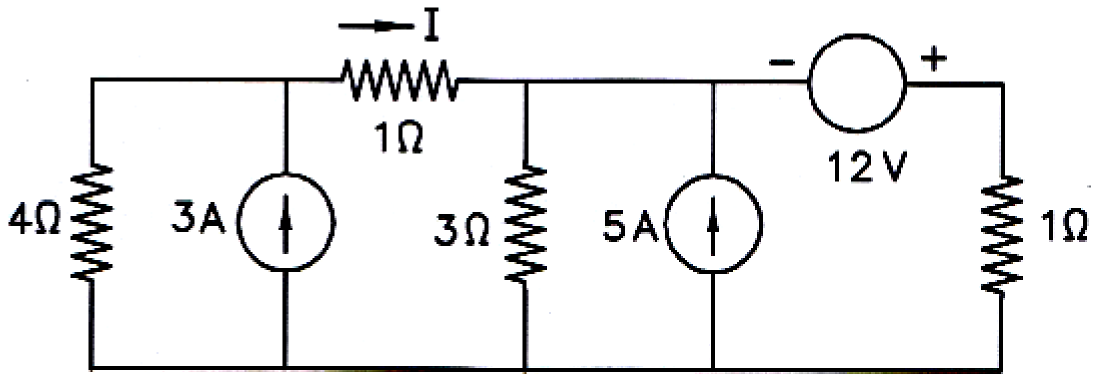
$-3 + 0.75\,i_x + 0.25\,i_x = 0$

$i_x = \frac{3}{1}\,A$

$i_2 = -3A$

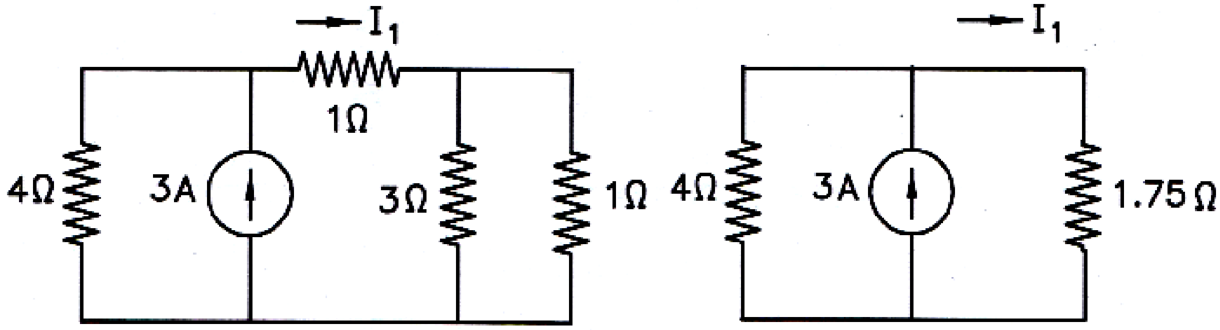
Superposition Theorem – Example

Determine *I* in the circuit shown below using superposition theorem.



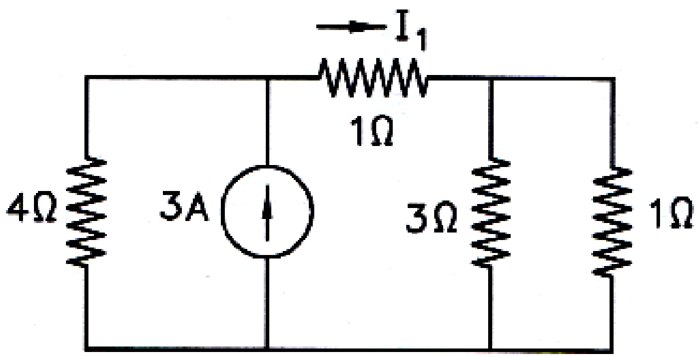
Solution:

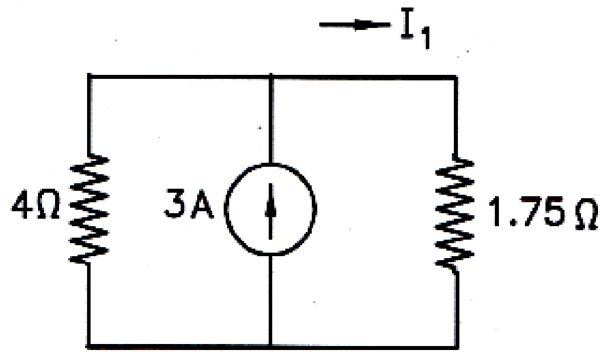
First, let us consider the circuit with 3 A current source.



Circuit with 3 A current source

Superposition Theorem – Example





Circuit with 3 A current source

The effective resistance of the resistor combination on the right hand side of the source is

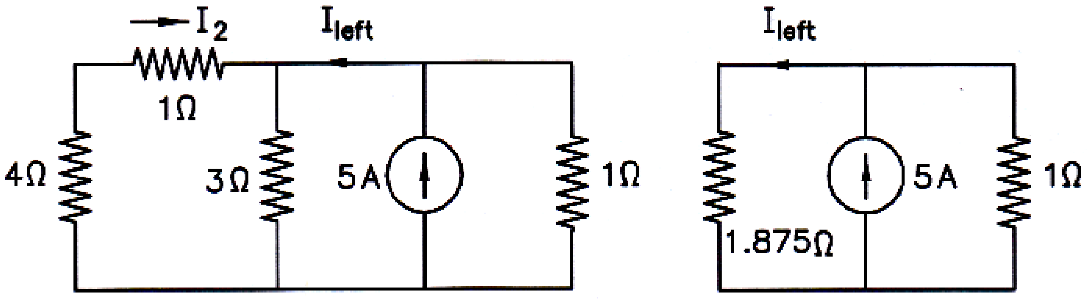
$$1 + (3||1) = 1.75 \;\; \Omega$$

As we have two resistances of 4 Ω and 1.75 Ω connected in parallel across the current source, we get

$$I_1 = \frac{4}{5.75} \times 3 = 2.087 \;\; \mathrm{A}$$

Superposition Theorem – Example

Next, let us consider the circuit with 5 A current source.



Circuit with 5 A current source

The effective resistance of the resistor combination on the left hand side of the source is

$$(4+1)||3 = 1.875 \ \ \Omega$$

As we have two resistances of 1.875 Ω and 1 Ω connected in parallel across the current source, we get current flow to the left side of the current source as

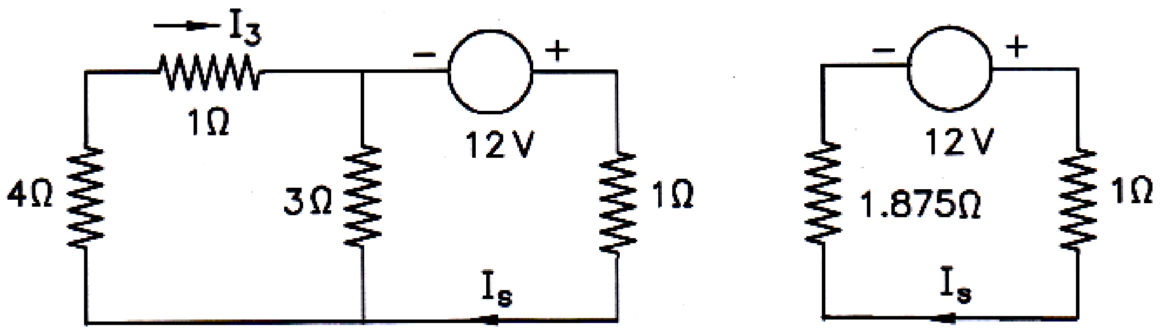
$$I_{left} = \frac{1}{2.875} \times 5 = 1.739 \ \ \mathrm{A}$$

The above current will flow through the two parallel paths on the left side of the source. Hence

$$I_2 = \frac{3}{8} \times (-I_{left}) = -\frac{3}{8} \times 1.739 = -0.652 \ \ \mathrm{A}$$

Superposition Theorem – Example

Finally, let us consider the circuit with 12 V voltage source.



Circuit with 12 V voltage source

The effective resistance of the circuit is

$$1 + ((4 + 1)||3) = 2.875 \ \ \Omega$$

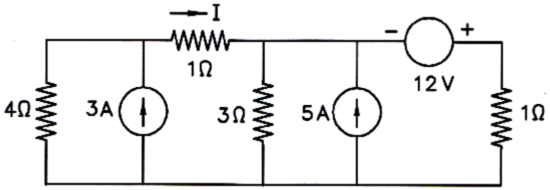
Hence the total current flow (I_S) from the 12 V source is

$$I_S = \frac{12}{2.875} = 4.174 \ \ \text{A}$$

So, we can find I_3 using current division method

$$I_3 = \frac{3}{8} \times 4.174 = 1.565 \ \ \text{A}$$

Superposition Theorem – Example



So the current flow I of the original circuit is

$$I = I_1 + I_2 + I_3 = 3 \text{ A}$$

Power absorbed by the 1 Ω resistor is

$$P_L = I^2 R = 9 \quad \text{W}$$

Due to currents I_1, I_2 and I_3 respectively, the power loss in the 1 Ω resistor will be

$$\begin{aligned} P_1 &= I_1^2 R = 4.356 \text{ W} \\ P_2 &= I_2^2 R = 0.425 \text{ W} \\ P_3 &= I_3^2 R = 2.449 \text{ W} \end{aligned}$$

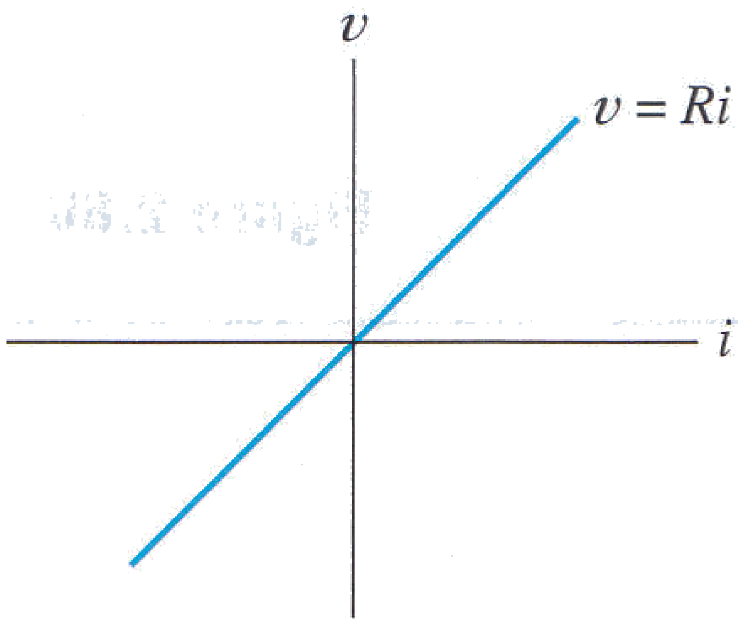
Clearly,

$$P_L \neq P_1 + P_2 + P_3$$

Note that superposition theorem cannot be applied to calculate the power.

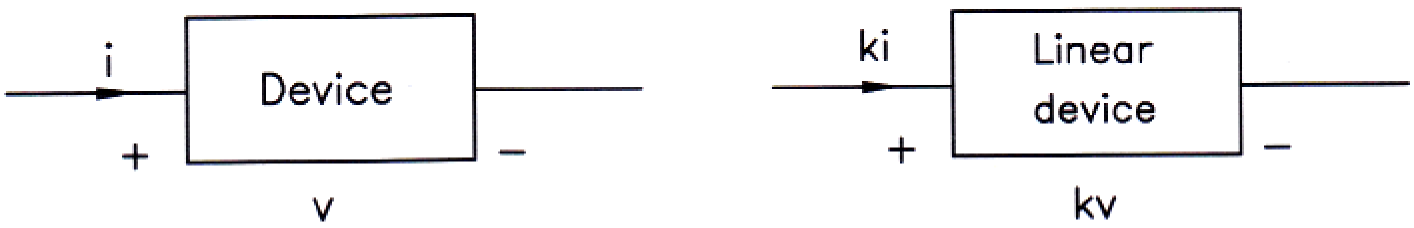
Linearity

Hambley, 4e



- If we plot voltage versus current for a resistance, we have a straight line.
- We say that Ohm's law is a linear equation.

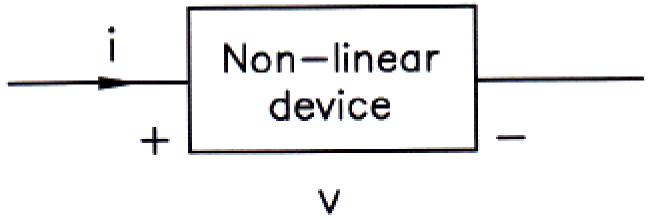
Linear Elements



Linear device

If the device current is given by $i = f(v)$, then the device is said to be linear if and only if $k\,i = f(k\,v)$ where k is a constant. The circuit elements R, L and C are linear elements. Any circuit with these elements are known as linear circuits.

Non-Linear Elements

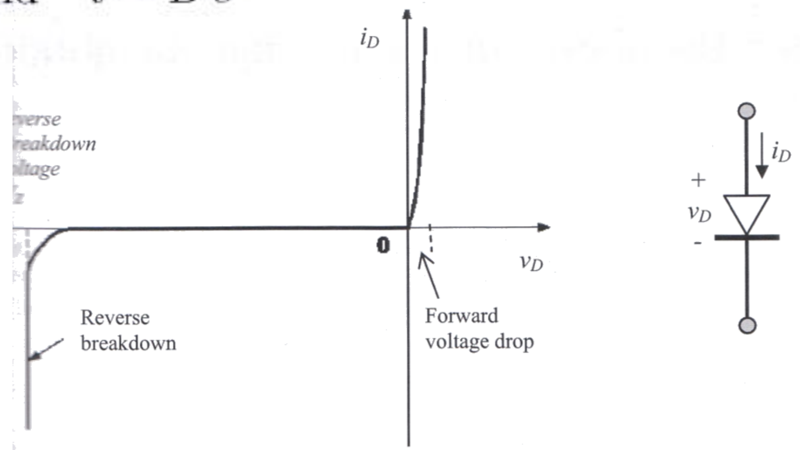


Non-linear device

In general, if the device current is given by $i = f(v)$, then the device is said to be non-linear if $i_{new} \neq k\, i$ when the device voltage is increased k times. For such devices, $f(k\, v) \neq k\, f(v)$. For example, the devices with the following $v - i$ characteristics are non-linear devices:

$$i = A\, v^{\alpha} \quad \text{and} \quad i = B\, e^{\beta v}$$

where k, A, B, α and β are constants.



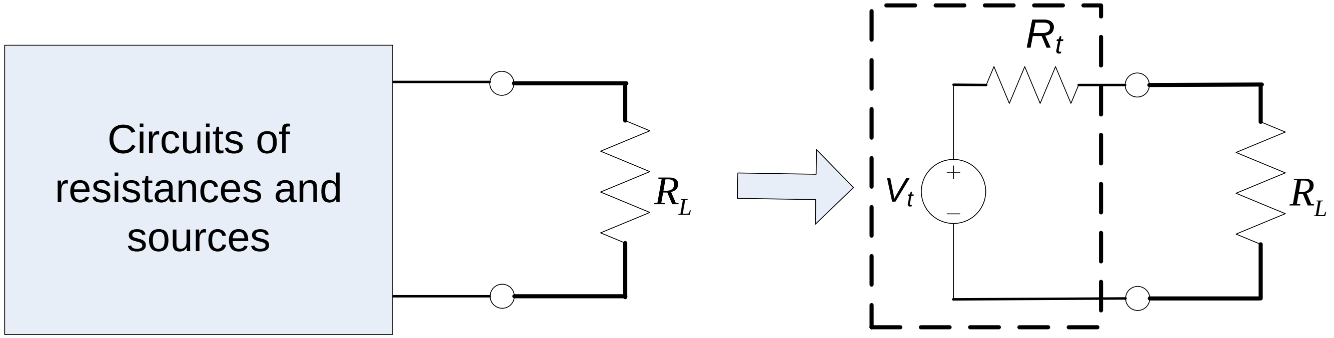
One-port networks and equivalent circuits

- Two-terminal circuits can be replaced by an equivalent circuit consisting of a source and a resistance.
- A voltage source with a series resistance

  (Thevenin equivalent circuit)
- A current source with a parallel resistance

  (Norton equivalent circuit)

Thevenin equivalent

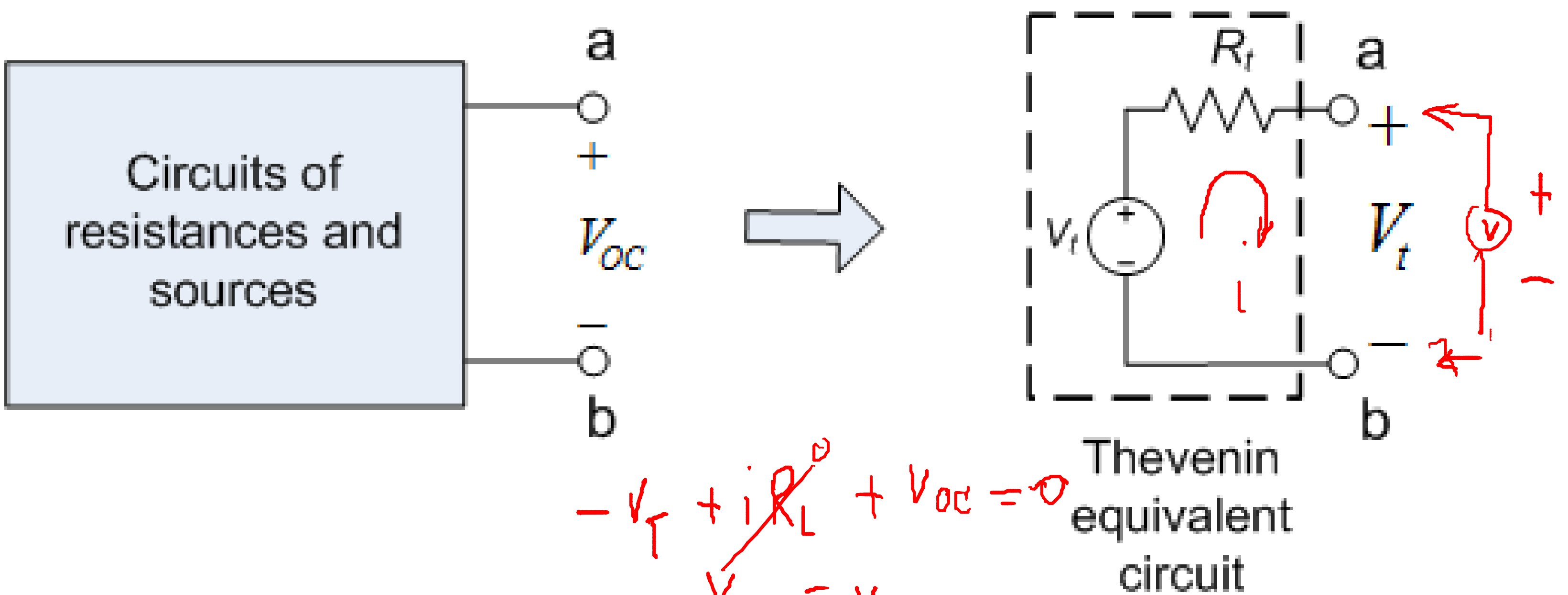


Thevenin equivalent circuit

- A voltage source in series with a resistance
- The voltage source is called Thevenin’s voltage
- The series resistance is called Thevenin’s resistance

Thevenin Voltage

- The value of the voltage source is the open circuit voltage between the two terminals.
- This is called the Thevenin voltage.

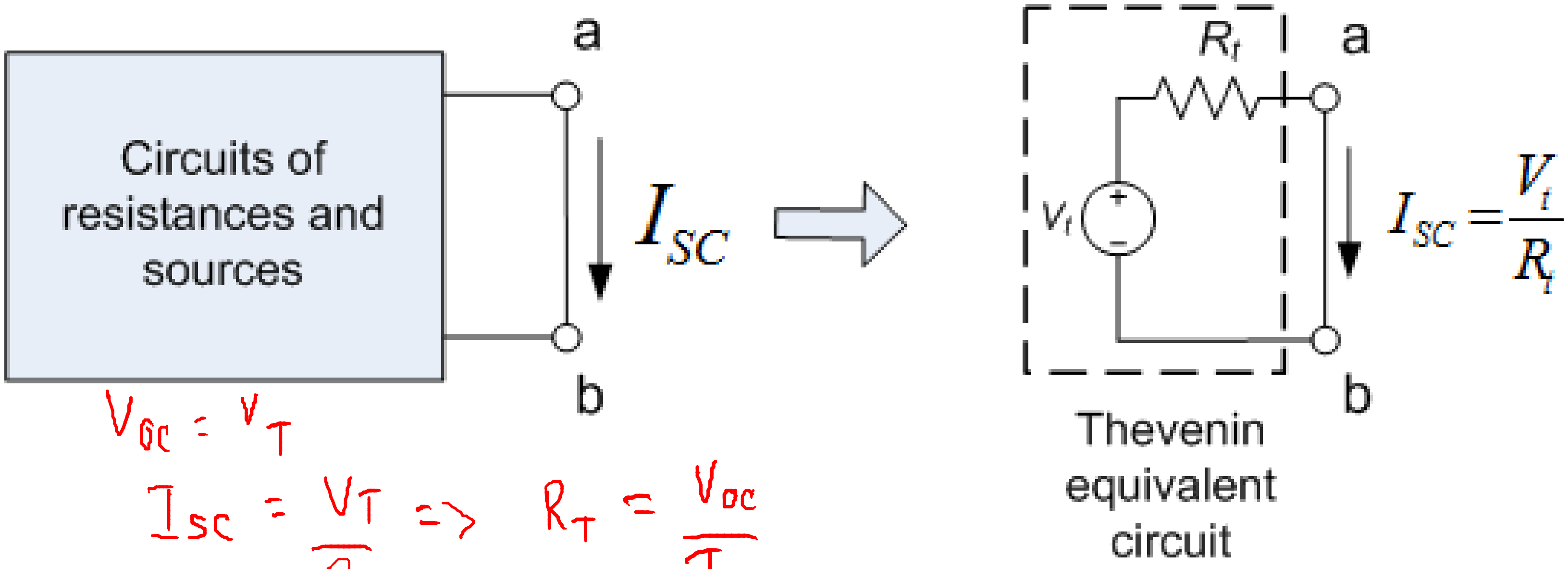


$-V_T + i\cancel{R_L}^{0} + V_{oc} = 0$

$V_{oc} = V_T$

Thevenin Resistance

- Find the short circuit current between the two terminals.
- Calculate the Thevenin resistance.

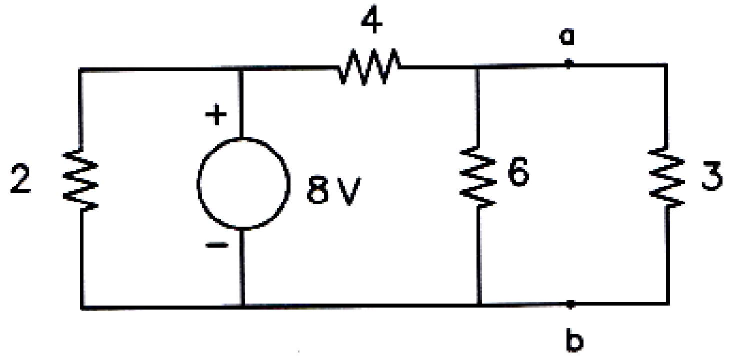


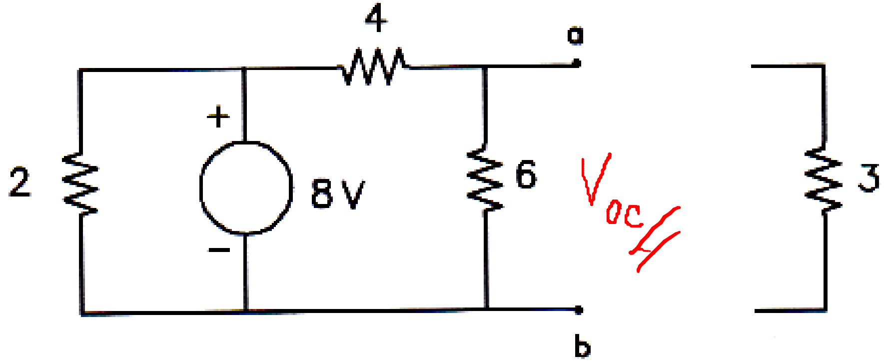
$V_{OC} = V_T$

$I_{SC} = \frac{V_T}{R_T} \Rightarrow R_T = \frac{V_{OC}}{I_{SC}}$

Thevenin’s Theorem – Example

In the circuit shown below, determine the current flow and voltage across the 3Ω resistor.





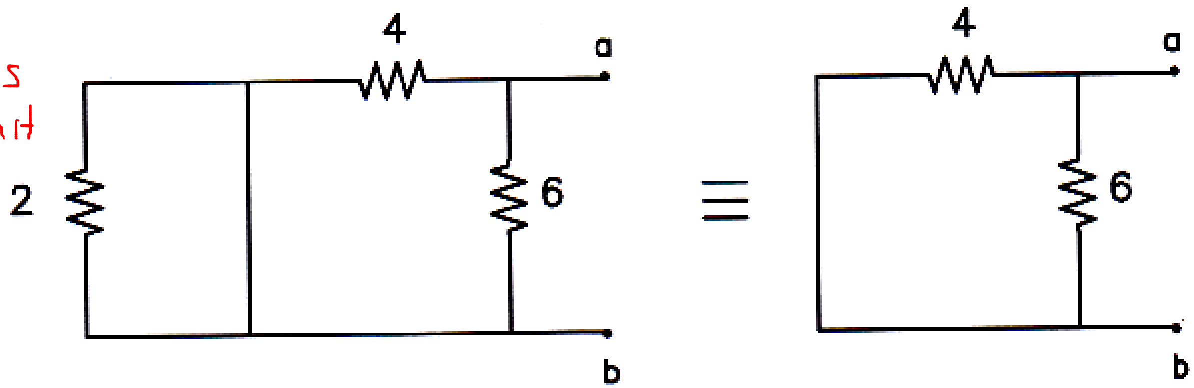
Then, the voltage across the terminals *a* and *b* is

$$V_{th} = V_{ab} = \frac{6}{4+6} \times 8 = 4.8 \text{ V}$$

Thevenin's Theorem – Example

Next, short the voltage source to determine R_{th} as shown in the figure below. Note that the 2Ω resistance is shorted when the voltage source is shorted. Hence it will not affect the calculation of R_{th}.

RECALL:
voltage across a short circuit is zero.

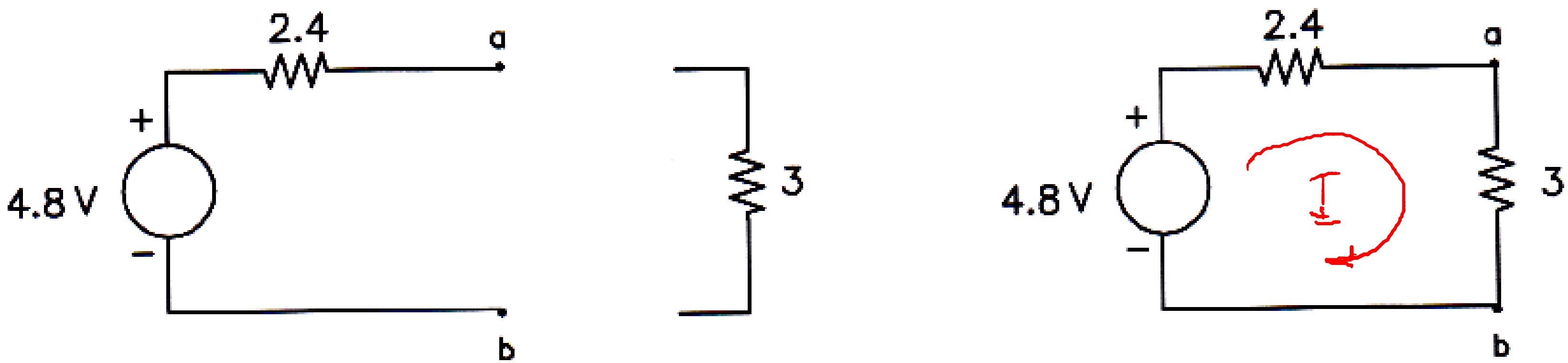


So, R_{th} is obtained as follows:

$$R_{th} = \frac{4 \times 6}{4 + 6} = 2.4 \ \Omega$$

Thevenin's Theorem – Example

The Thevenin equivalent circuit is shown in Figure below. So, the voltage across and current through the 3 Ω resistor are

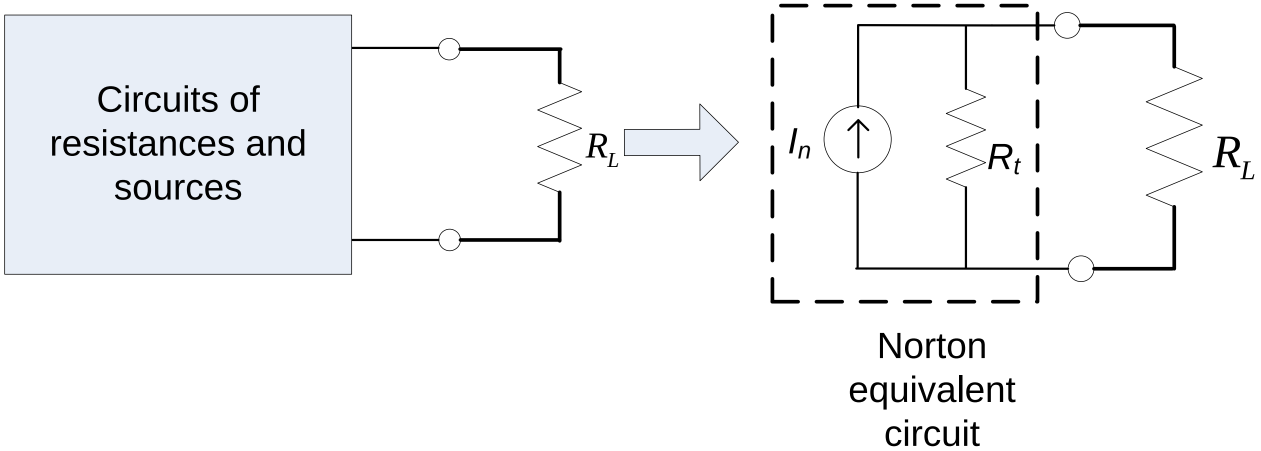


Thevenin's Equivalent Circuit

$$I = \frac{4.8}{2.4+3} = 0.889 \text{ A}$$

$$V_{ab} = \frac{4.8 \times 3}{2.4+3} = 2.667 \text{ V}$$

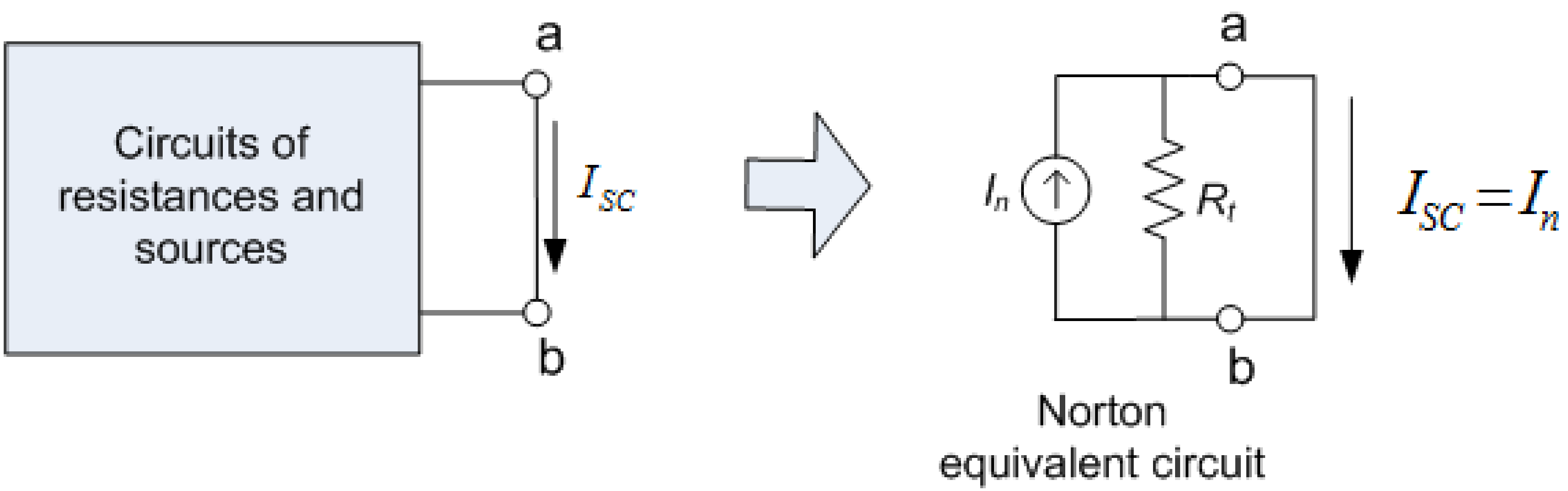
Norton Equivalent



- A current source in parallel with a resistance.
- The current source is called Norton's current.
- The series resistance is called Thevenin's resistance.

Norton's current

- The value of the current source is the short circuit current between the two terminals
- This is called the Norton's current



Steps to find the equivalent circuits

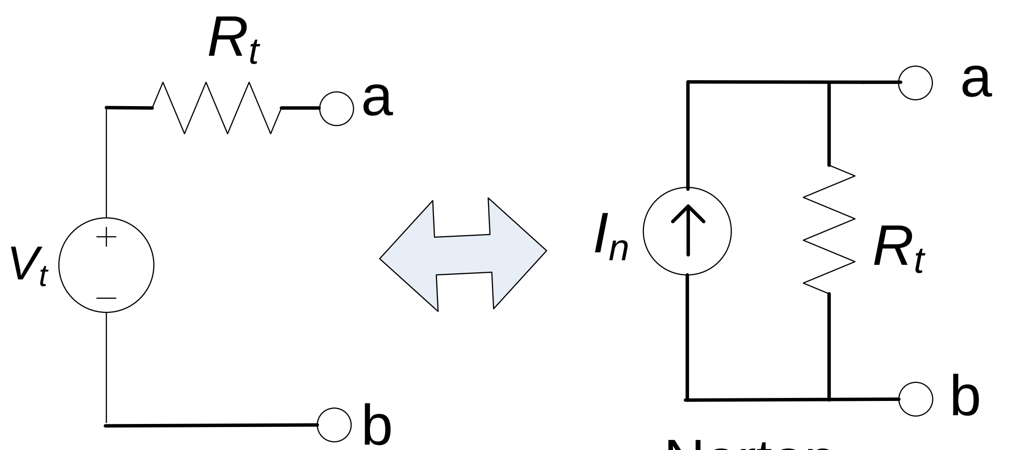
- Obtain the open circuit voltage between the two terminals – Thevenin's voltage:
- Obtain the short circuit current between the two terminals – Norton's current:
- Calculate the Thevenin's resistance as:

$$R_t = \frac{V_{OC}}{I_{SC}}$$

Source Conversion

- A voltage source with a series resistance is equivalent to a current source with the resistance in parallel.
- The values of the voltage and current source are given as

$$V_t = I_n R_t, \quad I_n = \frac{V_t}{R_t}$$

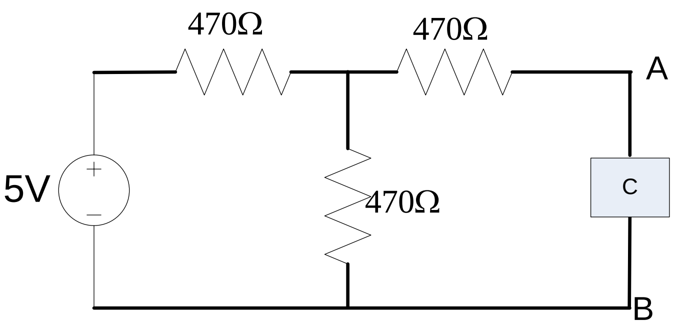


Thevenin equivalent circuit

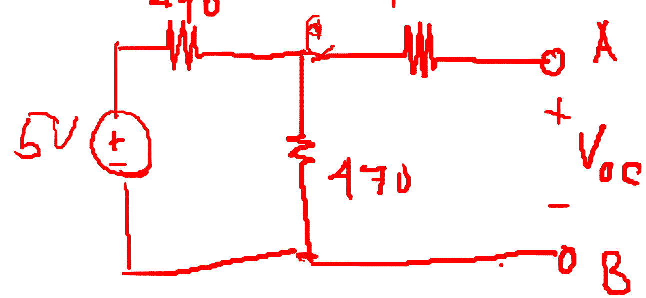
Norton equivalent circuit

Example

Find the open circuit voltage and short circuit current of the circuit:



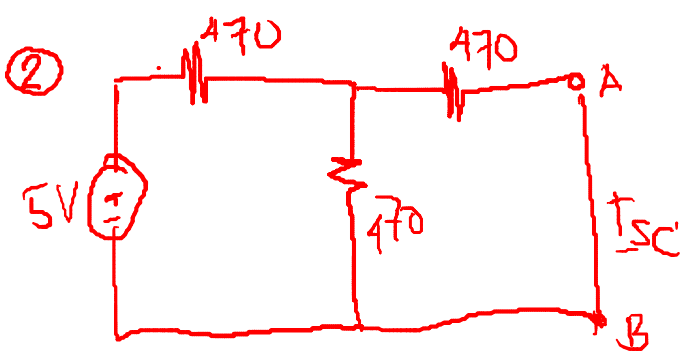
① Open circuit Voltage



$V_{oc} = V_{CB}$

$V_{CB} = \frac{5 \times 470}{470 + 470} = 2.5\,V$

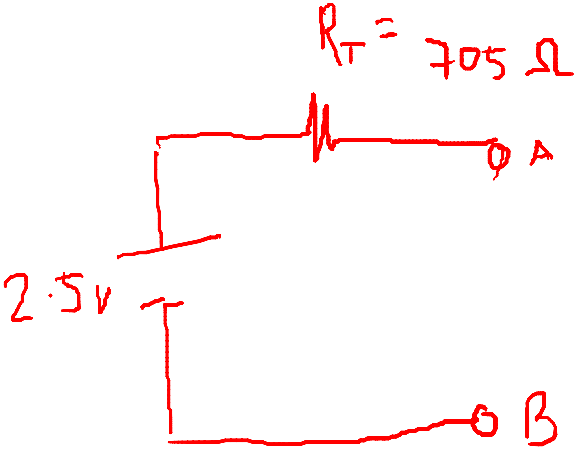
②

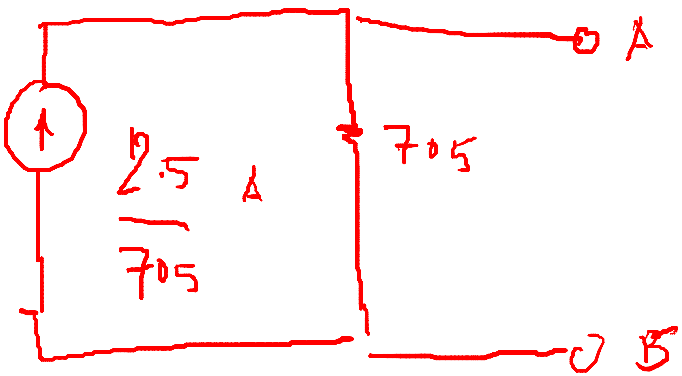


$I_{sc} = \frac{5}{\frac{470+470}{2}} * \frac{1}{2} = \frac{2.75}{705}$

$\boxed{R_T = 705\,\Omega}$

Example

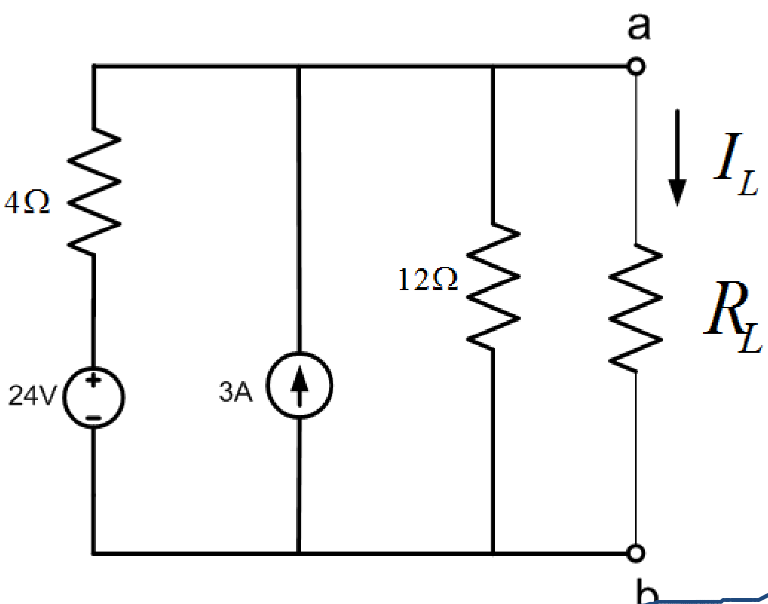


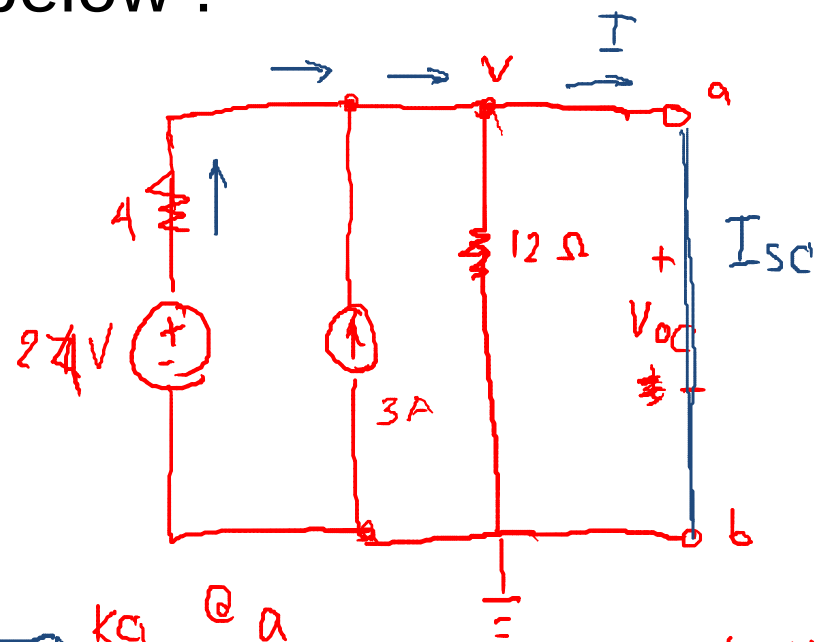


$$I_N = \frac{V_T}{R_T}$$

Example

Find the open circuit voltage and short circuit current of the circuit below :





KCL @ a

$$\frac{V-24}{4} - 3 + \frac{V}{12} = 0$$

$V = V_{OC}$

$$\frac{24}{4} + 3 = I_{SC} = 9\ \text{A}$$

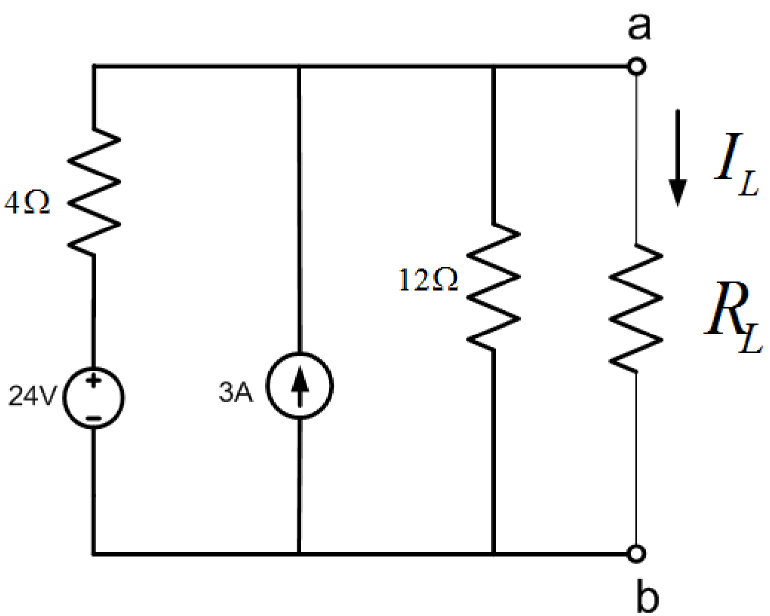
$R_T = ?$

Other ways to find Thevenin Resistance

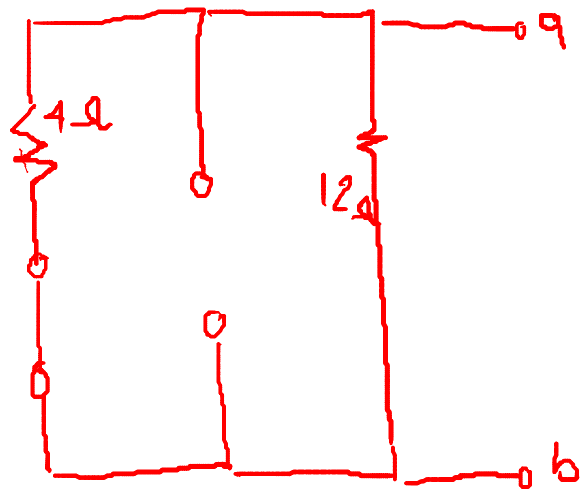
- If the circuit consists of independent sources only,
  - Kill all the sources. This would result in a purely resistive network
  - Find the equivalent resistance between the two terminals, by repeated application of series parallel rule
  - Thevenin resistance is equal to the equivalent resistance
- We cannot use this if there are dependent sources.

Example

Finding Thevenin resistance



1. Kill all the independent sources

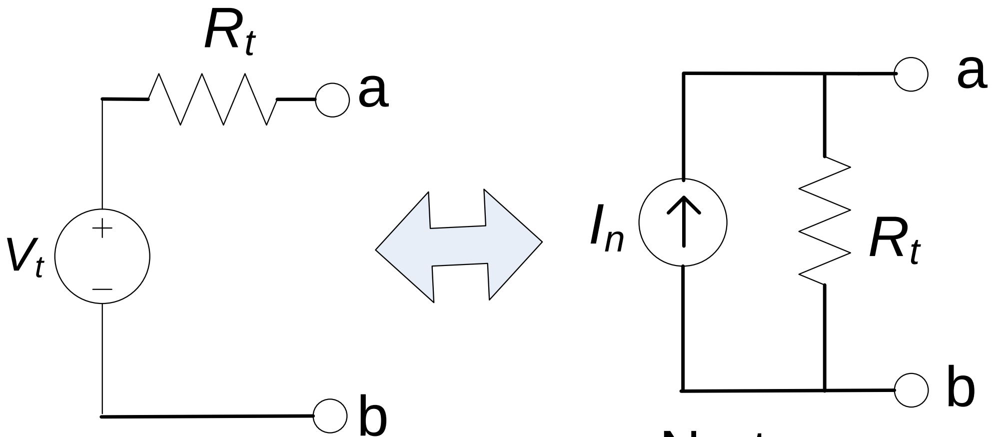


$$R_{eq} = \frac{4 \times 12}{4 + 12} = 3\ \Omega$$

Source Conversion

- A voltage source with a series resistance is equivalent to a current source with the resistance in parallel.
- The values of the voltage and current source are given as

$$V_t = I_n R_t, \quad I_n = \frac{V_t}{R_t}$$

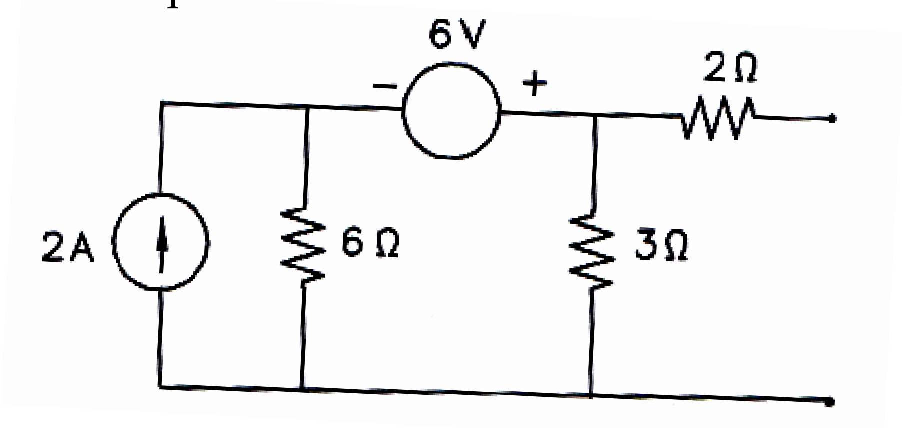


Thevenin equivalent circuit

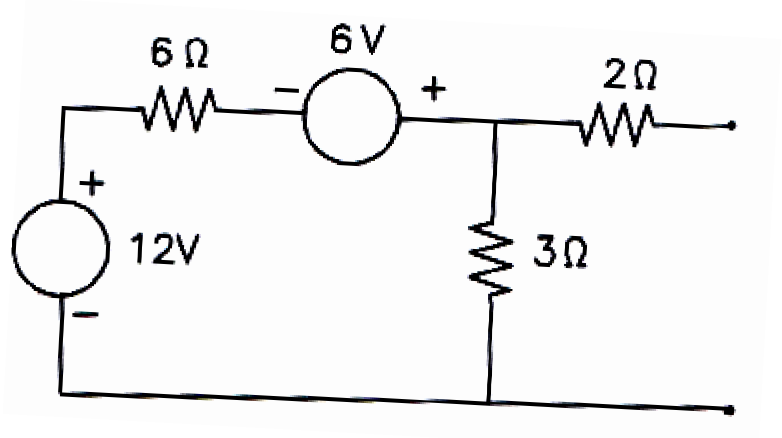
Norton equivalent circuit

Norton's Theorem - Example

Determine the Norton's Equivalent of the circuit shown in the figure below.

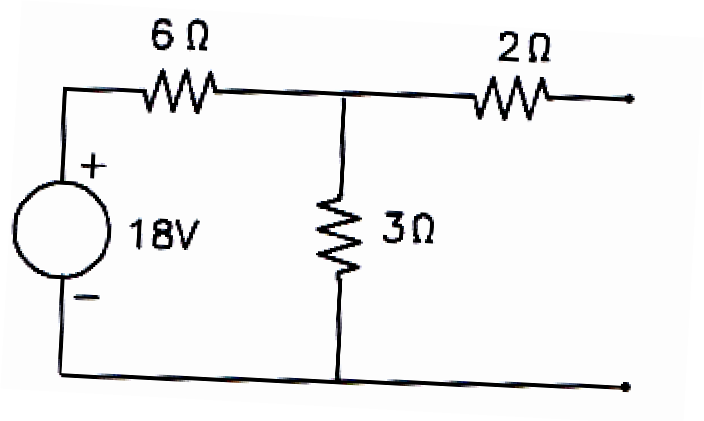


Solution: Converting the 2 A current source into an equivalent voltage source, we get the circuit shown below. The two voltage sources are connected in series here.

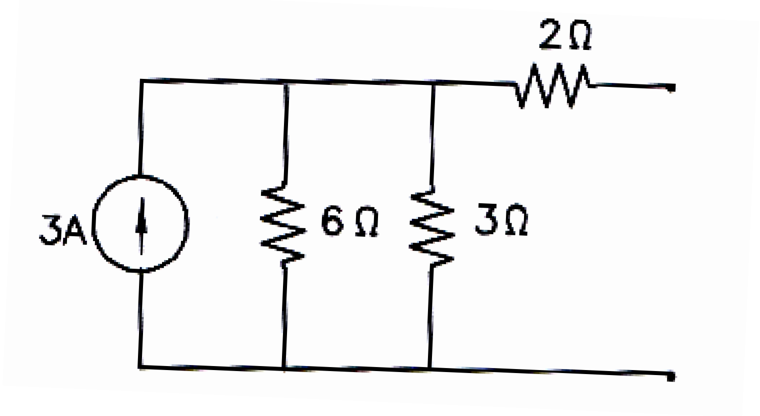


Norton's Theorem - Example

Replacing the voltage sources by a single equivalent voltage source, we obtain the circuit of figure shown below.



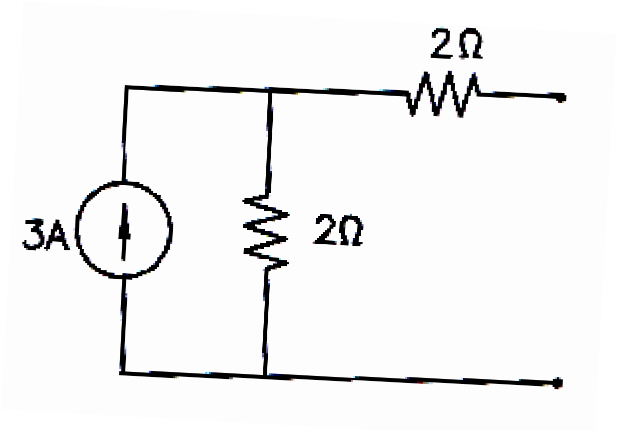
Converting this voltage source into an equivalent current source, the circuit becomes as shown in the figure shown below.



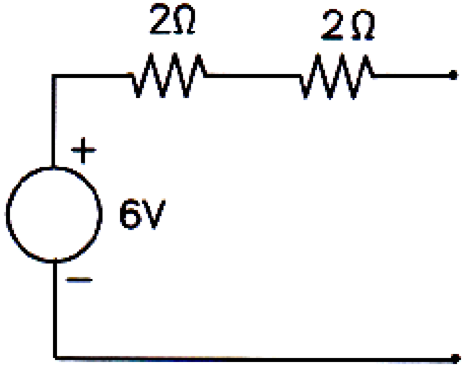
Norton's Theorem - Example

The 6Ω and 3Ω resistors in parallel in the circuit, may be replaced by an equivalent resistor of

$$(6||3) = 2\ \Omega$$

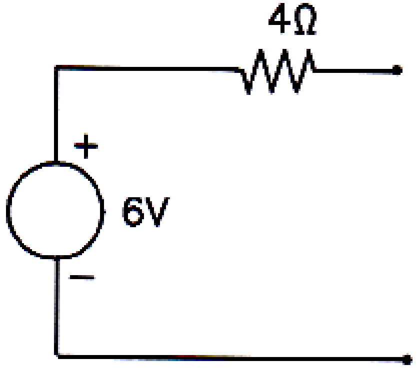


Converting the 3A current source of the circuit above into an equivalent voltage source, we get the circuit shown below.

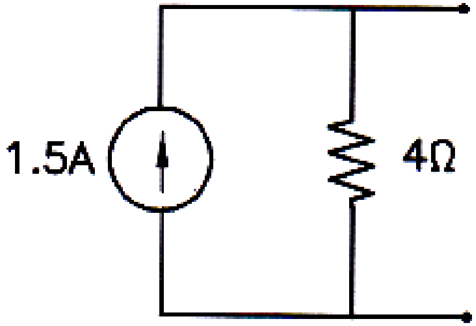


Norton's Theorem - Example

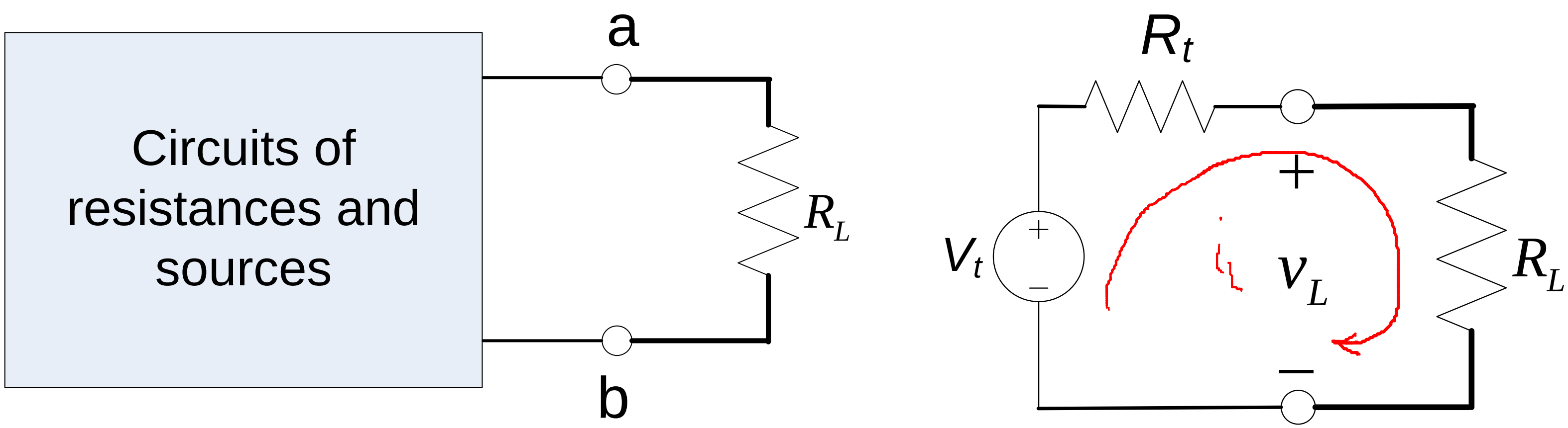
This circuit can be redrawn as shown in



Converting the voltage source of the above circuit into an equivalent current source, we get the required Norton's equivalent circuit as shown in circuit below.



Maximum power transfer



- The power absorbed by the load is given by:

$$P_L = i_L^2 R_L$$

- The load current is given by:

$$i_L = \frac{V_T}{R_L + R_T}$$

- Combining the two expressions, we can compute for the load power as:

$$P_L = \frac{V_T^2}{(R_L + R_T)^2} R_L$$

Maximum power transfer

- To find the value of R_L that maximizes the expression for P_L (assuming V_T and R_T are fixed, solve for the simple maximization problem.

$$\frac{dP_L}{dR_L} = 0$$

- Computing the derivative, we obtain the following expression:

$$\frac{dP_L}{dR_L} = \frac{v_T^2 (R_L + R_T)^2 - 2 v_T^2 R_L (R_L + R_T)}{(R_L + R_T)^4}$$

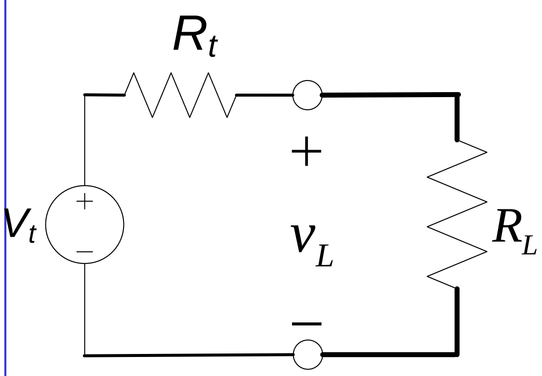
- Which leads to the expression:

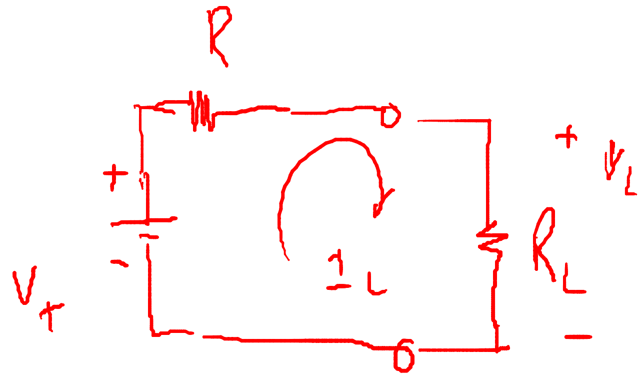
$$(R_L + R_T)^2 = 2 R_L (R_L + R_T) = 0$$

- It is easy to verify that the solution is

$$R_L = R_T$$

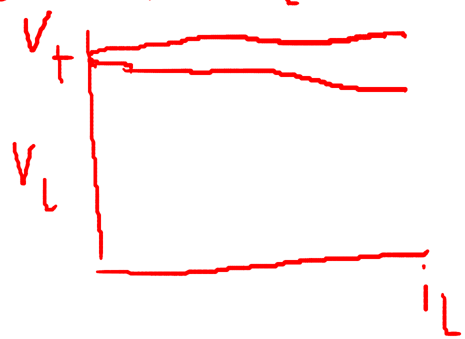
Source loading





$V_L - V_T + I_L R_T$

$V_L = V_T - i_L R_T$



Real voltage source