Chapter 12 Instructor Notes

Chapter 12 introduces the subject of power electronics. The importance of power electronics cannot be overemphasized, considering the widespread industrial application of electric machines, and other high current loads in practical engineering applications. The chapter discusses the basic characteristics and limitations of power amplifiers, practical voltage regulators, inductive loads (such as electric motors), and SCRs. The aim is to give the student sufficient understanding of the device characteristics to be able to complete simple "order of magnitude" calculations to be able to size a device for a given application. This chapter is much more practically oriented than some of the others in the text, and may be used to accompany a course in electric power and machines based on Chapters 7, 16, 17 and 18.

After Sections 12.1 and 12.2 present a classification of power electronic devices (Figure 12.1, p. 578) and circuits (Table 12.1, p. 579), the discussion is divided into the topics of Voltage Regulators (Section 12.3), Power Amplifiers and Transistor Switches (Section 12.4), Rectifiers and Controlled Rectifiers (Section 12.5), and Electric Motor Drives (Section 12.6)

Homework problems are divided into three major sections. The first, on voltage regulators, includes problems two different voltage regulator circuits (12.2, 12.3) The second, on rectifiers and controlled , illustrates a battery charging circuit (12.7) and two simple motor speed control problems (12.9, 12.10). The last section, on drives, introduces choppers, and more advanced problems on DC motor supplies based on controlled rectifiers (12.23 and 12.24).

Learning Objectives

1. Learn the classification of power electronic devices and circuits. Sections 1 and 2.
2. Analyze the operation of practical voltage regulators. Section 3.
3. Understand the principal limitations of transistor power amplifiers. Section 4.
4. Analyze the operation of single- and three-phase controlled rectifier circuits. Section 5.
5. Understand the operation of power converters used in electric motor control, and perform simplified analysis on DC-DC converters. Section 6.

Section 12.3: Voltage Regulators

Problem 12.1

Solution:

Find:
Repeat Example 12.1 for a 7-V Zener diode.

Analysis:
Calculating the collector and base currents according to:

$$I_E = \frac{7-1.3}{10} = 0.57\,\text{A}$$

$$I_B = \frac{I_E}{11} = 51.8\,\text{mA}$$

We find:

$$I_R = \frac{20-7}{47} = 0.277\,\text{A}$$

$$I_Z = I_R - I_B = 0.225\,\text{A}$$

$$V_{CE} = 20 - V_L = 20 - 5.7 = 14.3\,\text{V} > 0.6\,\text{V}$$

Thus, the transistor is in the active region. The Zener power is: $I_Z \cdot V_Z = 1.576\,\text{W}$.

Problem 12.2

Solution:

Known quantities:
The current regulator circuit shown in Figure P12.2.

Find:
The expression for R_S.

Analysis:
Assuming that the Zener voltage is V_Z , that $V_{BE} = V_\gamma = 0.6\,\text{V}$, and that the required current is I, we have:

$$R_S = \frac{V_Z + V_{BE}}{I} = \frac{V_Z + 0.6}{I} .$$

Problem 12.3

Solution:

Known quantities:
The shunt-type voltage regulator shown in Figure P12.3.

Find:
The expression for the output voltage, V_{out} .

Analysis:
If the Zener diode is to be in the regulator mode, the CB junction must be forward biased; in this case, both the CB and the BE junctions are forward biased, since a substantial base current will be generated through

the Zener diode (depending on the value of the shunt resistor in the output circuit). Thus, the collector-emitter voltage is equal to: $V_{CEsat} \approx 0.2\,\mathrm{V}$, and the source current will be: $I_S = \frac{V_S - V_{CEsat}}{R_S} = I_C + I_Z$. The voltage across the shunt resistor will therefore be V_γ, and the output voltage is: $V_{out} = V_Z + V_\gamma$.

Section 12.5: Rectifiers and Controlled Rectifiers (AC-DC Converters)

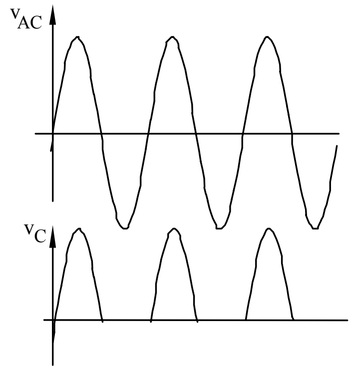
Problem 12.4

Solution:

Known quantities:
The circuit shown in Figure 12.17.

Find:
If the LR load is replaced by a capacitor, draw the output waveform and label the values.

Analysis:
When the sinusoidal source voltage is in the positive half cycle, the series diode conducts, and the shunt diode is an open circuit; thus, the positive half cycle appears directly across the capacitor (assuming ideal diodes). During the negative half cycle, the series diode is open, and therefore the voltage across the capacitor remains zero, as shown in the sketches below.



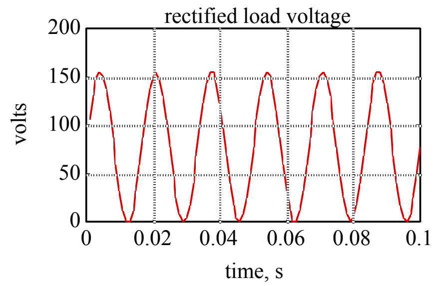
Problem 12.5

Solution:

Known quantities:
The circuit shown in Figure 12.17.

Find:
If the diode forward resistance is $50\ \Omega$, the forward bias voltage is $0.7\ \mathrm{V}$, and the load consists of a resistor $R = 10\Omega$ and an inductor $L = 2\,\mathrm{H}$, draw $v_L(t)$ and label the values for the given circuit.

Analysis:
To obtain exact numerical values, we assume a $111\ \mathrm{V_{rms}}$ source, $R = 10\Omega$, and $L = 2\,\mathrm{H}$, then:
$v_{AC}(t) = A\sin(\omega t) = 155.6 \cdot \sin(377t)$, and from Equation 12.6, the average load current is:
$I_L = \dfrac{155.6}{\pi R} = 4.95\,\mathrm{A}$. Using the approximation: $v_L(t) \approx \dfrac{A}{2} + \dfrac{A}{2}\sin(\omega t)$ we have:
$v_L(t) \approx \dfrac{A}{2} + \dfrac{A}{2}\sin(\omega t) = 77.8 + 77.8\sin(377t)$. The waveform is shown below:



Problem 12.6

Solution:

Known quantities:

For the circuit shown in Figure P12.6, v_{AC} is a sinusoid with 11 V peak amplitude, $R = 2\,\text{k}\Omega$ and the forward-conducting voltage of D is 0.7 V.

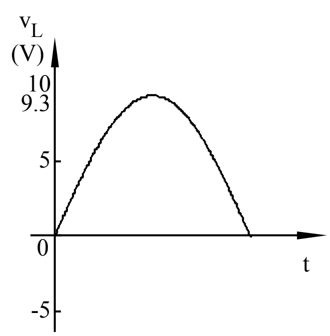
Find:

a. Sketch the waveform of $v_L(t)$.

b. Find the average value of $v_L(t)$.

Analysis:

a. Assume $v_{AC}(t) = 10\sin(\omega t)\text{V}$. The output voltage is: $v_L(t) = (10 - 0.7)\sin(\omega t)$. The waveform is shown below:



b. $\langle v_L \rangle = \dfrac{1}{2\pi}\int_0^{\pi} 9.3\sin(\omega t)\,d(\omega t) = \dfrac{9.3}{\pi} = 2.96\,\text{V}$.

Problem 12.7

Solution:

Known quantities:

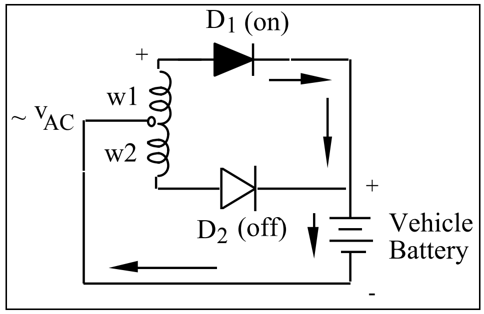
The vehicle battery charge circuit shown in Figure P12.7.

Find:

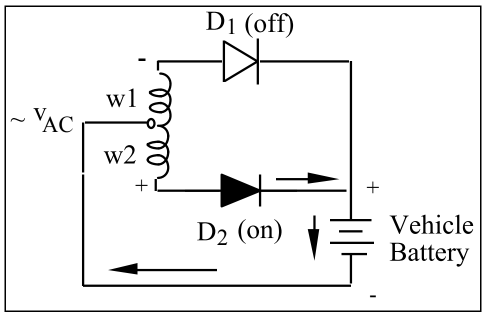
Describe the circuit, and draw the output waveform (L_1 and L_2 represent the inductances of the windings of the alternator).

Analysis:

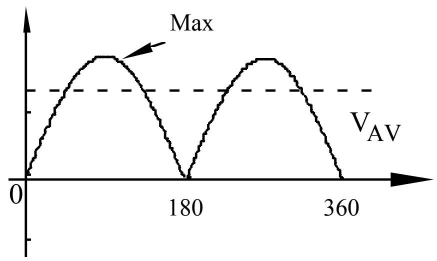
The positive half cycle from w1 is conducted by diode D_1. Diode D_2 does not conduct due to negative bias at w2. The first half cycle is passed through to the battery.



The second half cycle finds w2 positive and diode D_2 conducts current to the battery while diode D_1 is negatively biased and is off.



The full-wave rectified output waveform is shown below.



The average DC value V_{AV} is 63% of the peak value.

Problem 12.8

Solution:

Find:
NOTE: Typo in problem statement, referring to the wrong example problem.
Repeat Example 12.6 for $\alpha = \pi/3$ and $\pi/6$.

Analysis:

a) For $\alpha = \pi/3$ we have: $v_L\left(\frac{\pi}{3}\right) = \frac{120\sqrt{2}}{2}\sqrt{1-\frac{1}{3}+\sin\frac{2\pi}{3}} = 105\,\text{V}$.

The power is: $P = \frac{{v_L}^2}{R} = 45.94\,\text{W}$.

b) For $\alpha = \pi/6$ we have: $v_L\left(\frac{\pi}{6}\right) = \frac{120\sqrt{2}}{2}\sqrt{1-\frac{1}{6}+\sin\frac{\pi}{3}} = 110.6\,\text{V}$.

The power is: $P = \frac{v_L^{\,2}}{R} = 50.97\,\text{W}$.

Problem 12.9

Solution:

Known quantities:

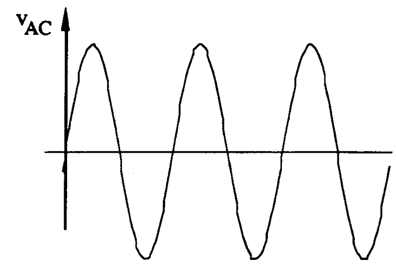
For the circuit shown in Figure P12.9, assume the thyristors are fired at $\alpha = 60^{o}$ and that the motor current is 20 A and is ripple free. The supply is $111\ V_{AC}$ (rms).

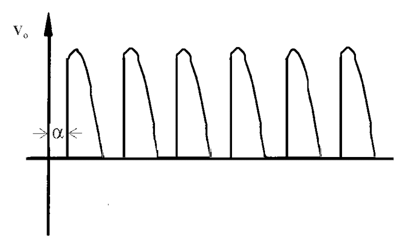
Find:

a) Sketch the output voltage waveform, v_0.
b) Compute the power absorbed by the motor.
c) Determine the volt-amperes generated by the supply.

Analysis:

a)





$$\alpha = 60 \times \frac{\pi}{180} = \frac{\pi}{3}, \quad R_a = 0.2 \quad .$$

b) $V_{o_{rms}} = \frac{120\sqrt{2}}{2}\left[1 - \frac{1}{3} + \sin 120^{\circ}\right]^{\frac{1}{2}} = 105\,\text{V} \rightarrow P_m = I_o V_{o_{rms}} = (20\,\text{A})(105\,\text{V}) = 2.1\,\text{kW}$.

c) $P_R = I_o^2 R_a = (20)^2 (0.2) = 80\,\text{W} \quad ; \quad P_S = P_m + P_R = 2180\,\text{W}$.

Problem 12.10

Solution:

Known quantities:

The circuit of Figure 12.2, replacing the resistive load with a DC motor. The motor operates at 110 V and absorbs 4 kW of power. The AC supply is 80 V, 60 Hz. Assume that the motor inductance is very large (i.e., the motor current is ripple free), and that the motor constant is 0.055 V/rev/min.

Find:
If the motor runs at $1{,}000$ rev/min at rated current:
a) Determine the firing angle of the converter.
b) Determine the rms value of the supply current.

Analysis:
Back emf $= k_T \times N = 0.055 \times 1000 = 55\,\text{V}$

$$I_{DC} = \frac{P}{V} = \frac{4000}{110} = 36.4\,\text{A}$$

$V = 110\,\text{V}$. Assume $R = 1$. $\quad I_{DC} = \frac{1}{\pi R}\left[\sqrt{2}V(\cos\alpha) - V_B(\pi - \alpha)\right]$

a) $36.4 = \frac{1}{\pi}\left[\sqrt{2}(110)\cos\alpha - 55(\pi - \alpha)\right]$. Solving yelds: $\alpha \approx \frac{7}{12}\pi$ rad.

b) With zero ripple, $I_{rms} = I_{DC} = 36.4\,\text{A}$.

Problem 12.11

Solution:

Find:
NOTE: Typo in problem statement, referring to the wrong example problem.
For the light dimmer circuit of Example 12.6, determine the load power at firing angles $\alpha = 0^0, 30^0, 60^0, 90^0, 120^0, 150^0, 180^0$, and plot the load power as a function of α.

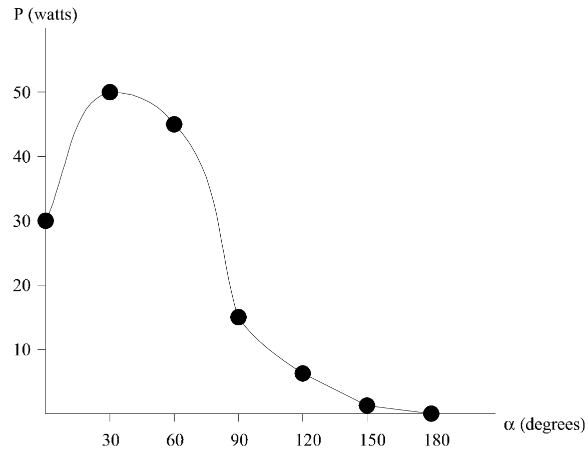
Analysis:

Note that $$V_{L_{rms}} = \frac{120}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \sin 2\alpha}\,, \quad \text{for } \alpha \le 90^0,$$

and $$V_{L_{rms}} = \frac{120}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}\,, \quad \text{for } a > 90^0$$

α	$V_{L_{rms}}$	$P = \frac{V_{L_{rms}}^2}{R_{BULB}}$
0°	84.85	30
30°	111.60	50.98
60°	115.00	45.98
90°	60.00	15.00
120°	37.53	5.87
150°	14.41	0.87
180°	0	0

A sketch of power vs. firing angle is shown below:



Problem 12.12

Solution:

Known quantities:

For the circuit shown in Figure P12.12:

$$V_L = 10\text{ V} \quad V_r = 10\% = 1\text{ V}$$

$$I_L = 650\text{ mA} \quad v_{line} = 170\cos\omega t\text{ V}$$

$$\omega = 2513\frac{\text{rad}}{\text{s}}$$

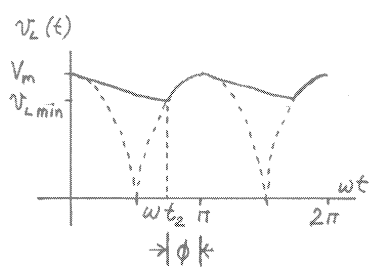
Find:

Determine the conduction angle of the diodes, if the diodes are fabricated form silicon.

Analysis:

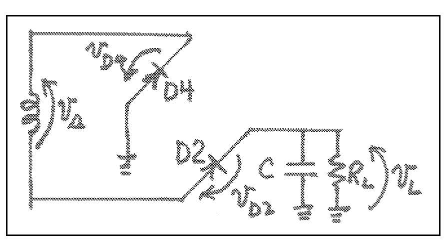
$$V_m = V_L + \frac{1}{2}V_r = 10\text{ V} + \frac{1}{2}[1\text{ V}] = 10.5\text{ V}$$

$$v_{L\text{-min}} = V_L - \frac{1}{2}V_r = 10\text{ V} - \frac{1}{2}[1\text{V}] = 9.5\text{ V}$$



D2 and D4 conduct during $\omega t_2 < \omega t < \pi$.

First, determine the amplitude of the source voltage. [The secondary of the transformer acts as a source.] Then use the same KVL to determine the angle at which the diodes start conducting.



$$KVL: \ +v_{D4}+v_s+v_{D2}+v_L = 0$$

$$At\ \omega t = \pi: \ v_{D2} = v_{D4} = v_{D\text{-}on} = 0.7\text{ V} \quad [Si]$$
$$v_s = -V_{so} \qquad v_L = V_m$$
$$V_{so} = v_{D\text{-}on}+v_{D\text{-}on}+V_m = 0.7\text{ V}+0.7\text{ V}+10.5\text{ V} = 11.9\text{ V}$$

$$At\ \omega t = \omega t_2: \ v_{D4} = v_{D2} = v_{D\text{-}on} = 0.7\text{ V} \qquad v_s = V_{so}\cos\omega t_2 \qquad v_L = v_{L\text{-min}}$$
$$\cos\omega t_2 = -\frac{v_{D\text{-}on}+v_{D\text{-}on}+v_{L\text{-min}}}{V_{so}} = -\frac{0.7\text{ V}+0.7\text{ V}+9.5\text{ V}}{11.9\text{ V}} = -0.91597$$
$$\omega t_2 = 156.3^\circ \qquad \therefore \phi = 180^\circ - 156.3^\circ = 23.66^\circ$$

Problem 12.13

Solution:

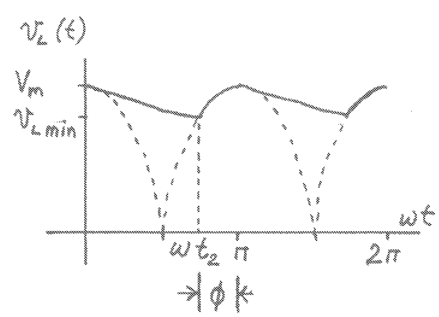
Known quantities:
For the circuit shown in Figure P12.13, assume that the conduction angle of the diodes shown (which are Silicon) is:

$$\Phi = 23^\circ$$
$$v_{s1}(t)=v_{s2}(t)=8\cos(\omega t)\text{V}$$
$$\omega = 377\frac{\text{rad}}{\text{s}} \qquad R_L = 20\text{ k}\Omega \qquad C = 0.5\,\mu\text{F}$$

Find:
The ripple voltage.

Analysis:

$$KVL: \ -v_{s1}(t)+v_{D1}+v_L(t) = 0$$
$$\text{At } t=0:$$
$$v_L(0) = V_m = V_{so}-v_{D\text{-}on} = 8\text{ V}-0.7\text{ V} = 7.3\text{ V}$$
$$\omega t_2 = \pi-\Phi \quad \Rightarrow \quad t_2 = \frac{\pi - 23^\circ\dfrac{\pi}{180^\circ}}{377\dfrac{\text{rad}}{\text{s}}} = 7.268\text{ ms}$$



$$v_L(t) = v_L(\infty)+(v_L(0)-v_L(\infty))e^{-\frac{t}{T_C}} = 0+[\,V_m-0\,]\,e^{-\frac{t}{R_L C}}$$
$$v_L(t_2) = v_{L\text{-min}} = 7.3\cdot e^{-\frac{7.286\cdot10^{-3}}{[20\cdot10^3][0.5\cdot10^{-6}]}} = 3.529\text{ V}$$
$$V_r = V_m - v_{L\text{-min}} = 7.3-3.529 = 3.771\text{ V}$$

Note the ripple is quite large. This is primarily due to the very small (for this type circuit) value of the capacitance. Also the conduction angle assumed above is not correct for this circuit.

Problem 12.14

Solution:

Known quantities:
The diodes in the full-wave DC power supply shown are Silicon. If:

$$I_L = 85\text{ ma} \quad V_L = 5.3\text{ V} \quad V_r = 0.6\text{ V} \quad \omega = 377\frac{\text{rad}}{\text{s}} \quad v_{line} = 156\cos\omega t\text{ V}$$

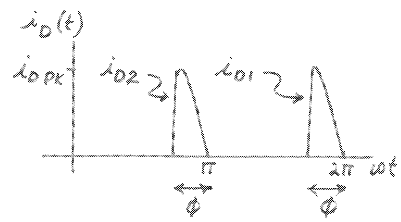
$$C = 1023\,\mu\text{F} \qquad \phi = Conduction\ Angle = 23.90^{\circ}$$

Find:
The value of the average and peak current through each diode.

Analysis:
Diodes D1 and D3 will conduct half of the load current and Diodes D2 and D4 will conduct the other half.

Therefore: $$i_{D\text{-}ave} = \frac{1}{2}I_L = \frac{1}{2}[\,85\text{ mA}\,] = 42.5\text{ mA}$$

The waveforms of the diode currents are complex but can be roughly approximated as triangular [recall area of triangle = bh/2]:



$$I_L = [\,i_{D1,3} + i_{D2,4}\,]_{ave} = \frac{1}{2\pi}\int_0^{2\pi}[\,i_{D1,3}(\omega t) + i_{D2,4}(\omega t)\,]\,\mathrm{d}[\omega t] = \frac{1}{2\pi}\Big[\frac{\phi\, i_{D\text{-}pk}}{2} + \frac{\phi\, i_{D\text{-}pk}}{2}\Big]$$

$$i_{D\text{-}pk} = \frac{2\pi I_L}{\frac{1}{2}\Phi + \frac{1}{2}\Phi} = \frac{2\pi I_L}{\phi} = \frac{[\,2\pi\text{ rad}\,][\,85\text{ ma}\,]}{[\,23.90^{\circ}\,][\frac{\pi\text{ rad}}{180^{\circ}}]} = 1.280\text{ A}.$$

Problem 12.15

Solution:

Known quantities:
The diodes in the full-wave DC power supply shown in Figure P12.13 are Silicon. If:

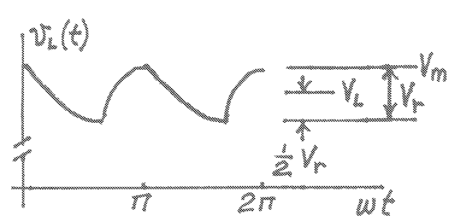
$$I_L = 600\text{ mA} \quad V_L = 50\text{ V} \quad V_r = 8\,\% = 4\text{ V} \quad v_{line} = 170\cos\omega t\text{ V} \quad \omega = 377\frac{\text{rad}}{\text{s}}$$

Find:
The value of the conduction angle for the diodes and the average and peak current through the diodes. The load voltage waveform is shown in Figure P12.15.

Analysis:

$$v_{L\text{-min}} = V_L - \frac{1}{2}V_r = 50 - \frac{1}{2}4 = 48\text{ V}$$

$$V_m = V_L + \frac{1}{2}V_r = 50 + \frac{1}{2}4 = 52\text{V}$$



$$KVL:\ -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } t=0:\ -V_{so}\cos(0) + v_{D\text{-}on} + V_m = 0 \Rightarrow V_{so} = 0.7 + 52 = 52.7\text{ V}$$

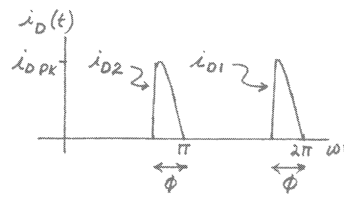
$$KVL: \ +v_{s2}(t) + v_{D2} + v_L(t) = 0$$

$$\text{At } t = t_2: \ +V_{so}\cos(\omega t_2) + v_{D\text{-}on} + v_{L\text{-min}} = 0$$

$$\omega t_2 = \cos^{-}[\ -\frac{v_{D\text{-}on} + V_{L\text{-min}}}{V_{so}}\ ] = \cos^{-}[\ -\frac{0.7\text{V} + 48\text{V}}{52.7\text{ V}}\ ] = 2.749\text{ rad}$$

$$\Phi = \pi - \omega t_2 = \pi - 2.749\text{ rad} = 392.1\text{ mrad}\frac{180^\circ}{\pi\text{ rad}} = 22.47^\circ$$

The waveforms of the diode currents are complex but can be roughly approximated as triangular [recall the area of triangle = bh/2].



$$i_{D1\text{-}ave} = i_{D2\text{-}ave} = \frac{1}{2}I_L = \frac{1}{2}600\text{ mA} = 300\text{ mA}$$

$$[\,i_{D1} + i_{D2}\,]_{ave} = I_L$$

$$\frac{1}{\omega T}\int_0^{2\pi}[\ i_{D1}(\omega t) + i_{D2}(\omega t)\ ]\ d[\omega t] = I_L$$

$$\frac{1}{\omega T}[\ \frac{1}{2}\Phi\, i_{D\text{-}pk} + \frac{1}{2}\Phi\, i_{D\text{-}pk}\ ] = I_L$$

$$i_{D\text{-}pk} = \frac{\omega T\, I_L}{\Phi} = \frac{[\,2\pi\text{ rad}\,][\,600\text{ mA}\,]}{392.1\text{ mrad}} = 9.615\text{ A}$$

Section 12.6: Electric Motor Drives

Problem 12.16

Solution:

Known quantities:

For the chopper of Figure P12.35, the supply voltage is 120 V, and the armature resistance of the motor is 0.15 Ω. The motor back emf constant is 0.05 V/rev/min and the chopper frequency is 250 Hz. Assume that the motor current is free of ripple and equal to 125 A at 120 rev/min.

Find:

a) The duty cycle of the chopper, δ, and the chopper-on time, t_1.
b) The power absorbed by the motor.
c) The power generated by the supply.

Analysis:

a)

$$v_o = i_o R_a + E_a = (125)(0.15) + 6 = 24.75\,\text{V}$$

$$24.75 = \delta(120) \Rightarrow \delta = \frac{24.75}{120} = 0.2063$$

$$\delta = \frac{t_1}{T} \Rightarrow t_1 = \delta T = (0.2063)\left(\frac{1}{250}\right) = 825\mu\text{s}$$

b)

$$P_m = E_a i_o = (6)(125) = 750\,\text{W}$$

$$P_R = R_a i_o^2 = (0.15)(125)^2 = 2.344\,\text{kW}$$

c)

$$P_S = P_m + P_R = 3.094\,\text{kW} \quad \text{or} \quad P_S = \delta \cdot V_S \cdot i_o = (0.2063)(120)(125) = 3.094\,\text{kW}.$$

Problem 12.17

Solution:

Known quantities:

For the circuit shown in Figure 12.39, the motor constant is 0.3 V/rev/min, the supply voltage is 600 V, and the armature resistance is $R_a = 0.2\Omega$.

Find:

If the motor speed is 800 rev/min and the motor current is 300 A, determine:

a) The duty cycle of the chopper, δ.
b) The power fed back to the supply.

Analysis:

a) $v_o = E_a + i_o R_a = 240 + (-300)(0.2) = 180\,\text{V} \quad \Rightarrow \quad \delta = \dfrac{180}{600} = 0.300$.

b)
$$\left.\begin{aligned} P_m &= E_a i_o = (240)(-300) = -72.0\,\text{kW} \\ P_R &= R_a i_o^2 = (0.2)(-300)^2 = 18\,\text{kW} \end{aligned}\right\} \Rightarrow \quad P_S = P_m + P_R = -54.0\,\text{kW}$$

or $P_S = \delta \cdot V_S \cdot i_o = (0.300)(600)(-300) = -54.0\,\text{kW}$.

Problem 12.18

Solution:

Known quantities:
For the two quadrant chopper of Figure 12.40, assume the thyristors S_1 and S_2 are turned on for time t_1 and off for time $T - t_1$ (T is the chopping period).

Find:
An expression for the average output voltage in terms of the supply voltage, V_S, and the duty cycle, δ.

Analysis:

$$\langle v_o \rangle = \frac{t_1}{T} V_S = \frac{t_1}{t \cdot t_1} V_S = \frac{1}{t} V_S = \delta \cdot V_S \,.$$

Problem 12.19

Solution:

Known quantities:
Supply voltage; chopper duty cycle.

Find:
Average and rms value of ideal switched supply voltage.

Analysis:
The duty cycle is $\delta \quad \frac{t_1}{T} \quad \frac{1}{2.5} \quad 0.4$ The average value is therefore $<V_{supply}> = \delta \times V_{supply} = 40$ V.

To compute the rms value we use the definition of eq. 4.24:

$$\tilde{V}_{\text{supply}} \quad \sqrt{\int_0^{\delta} V_{\text{supply}}{}^2 dt} \quad \sqrt{\delta \, 100^2} \quad 63.25 \text{ V}$$

Problem 12.20

Solution:

Known quantities:
Load resistance and inductance; ideal supply voltage; duty cycle.

Find:
Average values of current and voltage; power drawn from battery supply.

Analysis:
From the data given, $\delta = 0.333$. Since the period of the waveform is 3 ms, we can calculate the switching frequency to be:

$$f \quad \frac{1}{T} \quad \frac{1}{3 \quad 10^{\quad 3}} \quad 333.33 \text{ Hz}$$

$$\omega = 2{,}094.4 \text{ rad/s}$$

The time constant of the load impedance is $\tau = \frac{L}{R} = \frac{10^{-3} \text{ H}}{0.5} = 2 \text{ ms}$.

The average load voltage is: $<V_L> = \delta \times V_{supply} = 33.33$ V.

The average load current is $\quad I_L \quad \delta \frac{V_{\text{supply}}}{R} \quad 0.33 \frac{100 \text{ V}}{0.5} \quad 66.67 \text{ A}$.

To compute the power drawn from the battery supply (which is assumed equal to the load power if switching losses are held negligible), we really need to compute the rms load current, since $P_L \quad \tilde{I}_L{}^2 R$; however, this calculation cannot be completed without knowing exactly the shape of the load current.

Without further analysis we can only state that the power drawn from the battery is greater than $<P_L> = <V_L><I_L> = 2.22$ kW.

Problem 12.21

Solution:

Known quantities:
The converter of Problem 12.20 with a DC motor as load.

$$R_a = 0.2\Omega,\ L_a = 1 \text{ mH},\ E_a = 10 \text{ V},\ T = 3 \text{ ms},\ \delta = \frac{1}{3}$$

Find:
The average load current and voltage.

Analysis:
The average load voltage is: $<V_L> = \delta \times V_{supply} = 33.33$ V.

The average current is: $< I_L >= \dfrac{< V_L > - E_a}{R_a} = \dfrac{33.33 - 10}{0.2} = 116.5 \text{ A}$

Problem 12.22

Solution:

Known quantities:
Load resistance and inductance; load current; motor armature constant; DC supply and desired rpm range.

Find:
Range of duty cycles required.

Analysis:
Assume steady-state operation, so that the effects of the load inductance may be neglected. When the rpm is zero, the back emf is also 0, so

$V_L \quad I_a R_a \quad 25 \quad 0.3 \quad 7.5$ V .

The motor emf constant is $k_a\phi =$ 0.00167 V-s/rev, or 60*0.00167 = 0.1004 V-min/rev

At 2000 rpm, the back emf is:

$E_a = k_a \phi n = 200.4$ V.

Thus, the total load voltage is

$V_L \quad I_a R_a \quad E_a \quad 7.5 \quad 200.4 \quad 207.9$ V

From the range of voltages required by the motor for its proper operation, we conclude that the range of required duty cycles is:

$$\delta_{min} \quad \frac{V_{min}}{V_{supply}} \quad \frac{7.5}{220} \quad 0.0341$$

$$\delta_{max} \quad \frac{V_{max}}{V_{supply}} \quad \frac{207.9}{220} \quad 0.943$$

Problem 12.23

Solution:

Known quantities:
Motor ratings; motor armature resistance and armature constant; power supply ratings.

Find:
Motor speed, power factor and efficiency for $\alpha = 0°$ and $\alpha = 20°$.

Analysis:
The nominal torque from the rated data is

$$T_m = \frac{P}{\omega_m} = \frac{10000}{\frac{2\pi 1000}{60}} = 95.49\,\text{Nm}$$

It follows that the DC current is

$$I_a = \frac{T_m}{K_a} = \frac{95.49}{2} = 47.75\text{ A}$$

The average load voltage for firing angle of 0° is

$$< V_{L,0^\circ} >= \frac{2\sqrt{2}}{\pi} V_S = \frac{2\sqrt{2}}{\pi} 240 = 216\text{ V}$$

The speed at zero degree of firing angle is given by

$$< V_{L,0^\circ} >= E_{a,0^\circ} + R_a I_a \Rightarrow E_{a,0^\circ} = 216 - 0.42 \cdot 47.75 = 196\text{ V}$$

$$\omega_{m,0^\circ} = \frac{E_{a,0^\circ}}{K_a} = \frac{196}{2} = 97.97\ \text{rad}/\text{s}$$

The efficiency is

$$\eta = \frac{E_{a,0^\circ} I_a}{E_{a,0^\circ} I_a + R_a I_a^2} = \frac{E_{a,0^\circ}}{E_{a,0^\circ} + R_a I_a} = \frac{196}{196 + 0.42 * 47.75} = 91\,\%$$

The rms voltage is

$$V_{rms,L} = V_S = 240$$

The power factor can be calculate as follows

$$\text{pf}\,(0^\circ) \cong \frac{E_{a,0^\circ} I_a + R_a I_a^2}{V_{rms,L} I_a} = \frac{E_{a,0^\circ} + R_a I_a}{V_{rms,L}} = \frac{196 + 0.42 * 47.75}{240} = 0.9$$

In the case of firing angle of 20°, we have

$$< V_{L,20^\circ} >= \frac{2\sqrt{2}}{\pi} V_S \cos\, 20^\circ = 203\ \text{V}$$

$$E_{a,20^\circ} =< V_{L,20^\circ} > - R_a I_a = 203 - 0.42 * 47.75 = 182.95\ \text{V}$$

$$\omega_{m,20^\circ} = \frac{E_{a,20^\circ}}{K_a} = \frac{182.95}{2} = 91.47\ \text{rad}/s$$

$$\eta = \frac{E_{a,20^\circ}}{E_{a,20^\circ} + R_a I_a} = \frac{182.95}{182.95 + 0.42 \cdot 47.75} = 90\%$$

$$\text{pf}\,(20^\circ) \cong \frac{E_{a,20^\circ} + R_a I_a}{V_{rms,L}} = \frac{182.95 + 0.42 * 47.75}{240} = 0.846$$

Problem 12.24

Solution:

Known quantities:
Separately excited DC motor:

$P = 10\,\text{kW}, V = 300\,\text{V}, \omega = 1000\,\text{rev/min}, R_a = 0.2\Omega, K_a = 1.38\,\text{V}\cdot\text{s/rad}$

Power supply: $V_S = 220\text{ (rms)V}, f = 60\text{ Hz}$

Three phase controlled bridge rectifier.

Find:
Speed, power factor, and efficiency for a firing angle of 30 deg.

Assumptions:
Load torque is constant, and the DC motor deliver the power *P* at 0 deg of firing angle.
Additional inductance is present to ensure continuous conduction.

Analysis:
The average voltage supplied by the rectifier at 0 deg of firing angle is

$$< V_{L,0^\circ} >= \frac{3\sqrt{3}}{\pi}\sqrt{2}V_S = 514.6\text{ V}$$

It follows

$$< V_{L,0^\circ} >= R_a I_a + E_{a,0^\circ} = R_a \frac{P}{E_{a,0^\circ}} + E_{a,0^\circ} \Rightarrow E_{a,0^\circ}^2 - < V_{L,0^\circ} > E_{a,0^\circ} + R_a P = 0 \Rightarrow E_{a,0^\circ}^2 - 514.6 E_{a,0^\circ} + 2000 = 0$$

$$E_{a,0^\circ} = 510.68\text{V} \Rightarrow I_a = I_{a,0^\circ} = I_{a,30^\circ} = \frac{P}{E_{a,0^\circ}} = \frac{10000}{510.68} = 19.58\text{A}$$

The average voltage supplied by the rectifier at 30° of firing angle is

$$< V_{L,30^\circ} >=< V_{L,0^\circ} > \cdot \cos 30^\circ = \frac{3\sqrt{3}}{\pi}\frac{\sqrt{3}}{2}\sqrt{2}V_S = 445.65\text{V}$$

The emf of the DC motor in this condition is given by

$$E_{a,30^\circ} =< V_{L,30^\circ} > - R_a I_a = 445.65 - 0.2 \cdot 19.58 = 441.74\text{V}$$

Finally, the speed is given by

$$\omega_m = \frac{E_{a,30^\circ}}{K_a} = 320.1\ \frac{\text{rad}}{\text{s}}$$

If we assume that the inductance is big enough, then the current ripple is negligible.
Under this conditions

$$\eta = \frac{E_{a,30^\circ} I_a}{E_{a,30^\circ} I_a + R_a I_a^2} = \frac{8649.27}{8725.94} = 99.1\%$$

The rms voltage supplied for a firing angle of 30° is

$$V_{L,rms} = 453.03\text{ V}$$

The power factor is

$$\text{pf} = \frac{P_{out}}{V_{L,rms} I_{L,rms}} \cong \frac{E_{a,30^\circ} I_a + R_a I_a^2}{V_{L,rms} I_a} = \frac{E_{a,30^\circ} + R_a I_a}{V_{L,rms}} = \frac{441.74 + 3.92}{453.03} = 0.984$$
