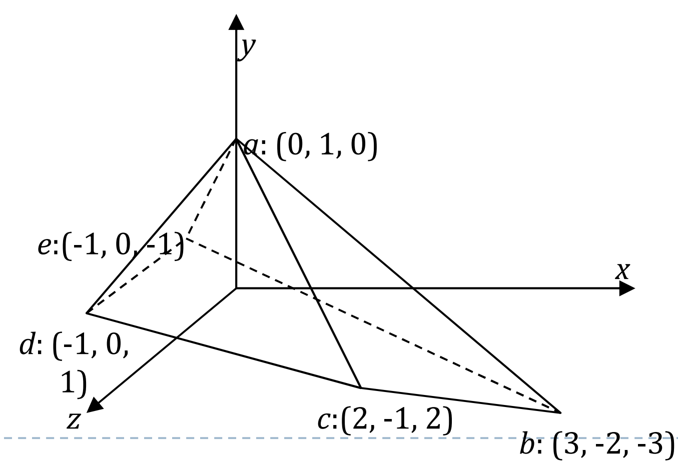
CS3241 Computer Graphics

Tutorial #5

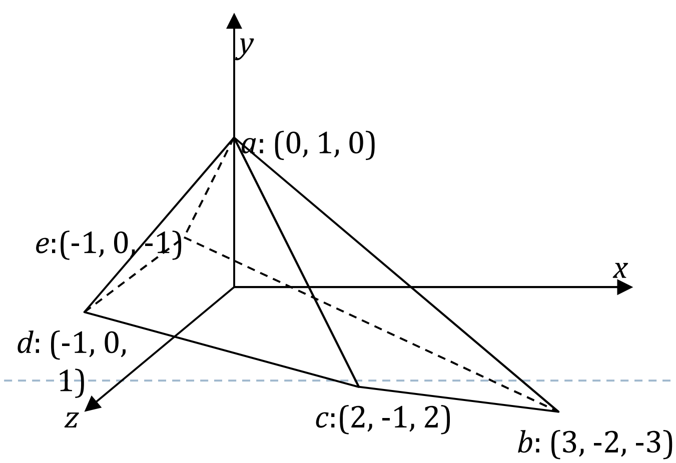
Question #1

- Here are 4 triangles (*abc*, *acd*, *ade* and *aeb*) on a mesh:
  1. Compute the normal vectors (which face "upwards", i.e. y > 0) for the 4 triangles.



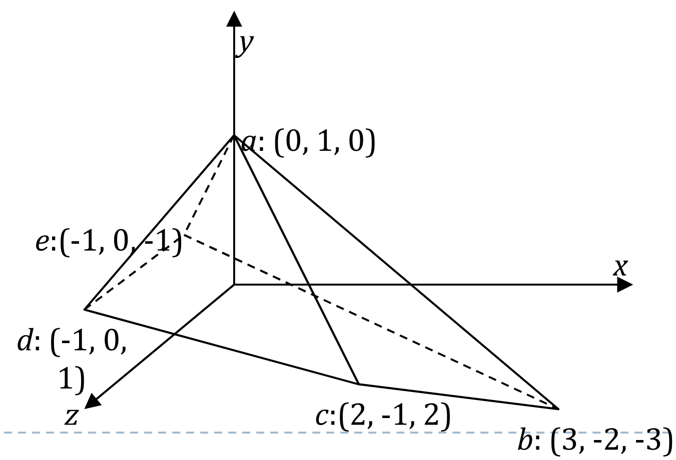
Question #1a

- Edge vectors:
  - ab = b – a = (3,-3,-3),
  - ac = c – a = (2,-2, 2),
  - ad = d – a = (-1,-1,1),
  - ae = e – a = (-1,-1,-1)



Question #1a

- Triangle Normals
  - abc : (1/√2) (1,1,0),
  - acd : (1/√2) (0,1,1),
  - ade : (1/√2) (-1,1,0),
  - aeb : (1/√2) (0,1,-1)



$$abc : ac \times ab = \begin{vmatrix} i & j & k \\ 2 & -2 & 2 \\ 3 & -3 & -3 \end{vmatrix} = (12,12,0)$$

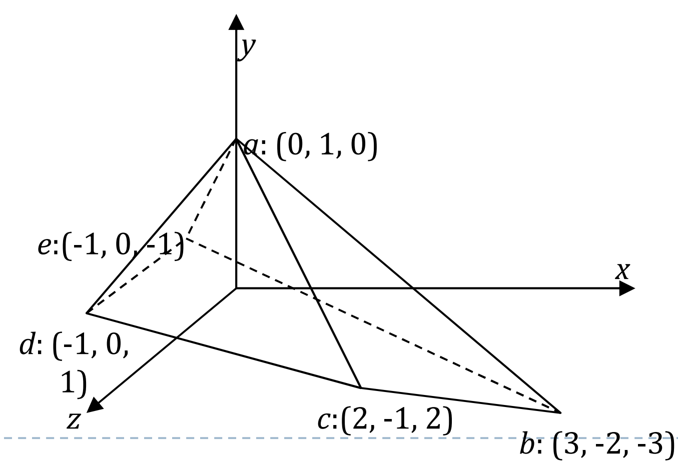
$$acd : ad \times ac = \begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ 2 & -2 & 2 \end{vmatrix} = (0,4,4)$$

$$ade : ae \times ad = \begin{vmatrix} i & j & k \\ -1 & -1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = (-2,2,0)$$

$$aeb : ab \times ae = \begin{vmatrix} i & j & k \\ 3 & -3 & -3 \\ -1 & -1 & -1 \end{vmatrix} = (0,6,-6)$$

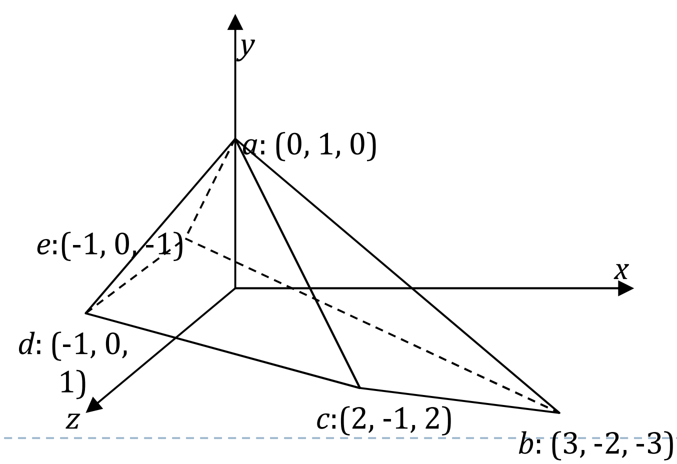
Question #1

- Here are 4 triangles (*abc*, *acd*, *ade* and *aeb*) on a mesh:
    1. Compute the normal vector at all the vertices for shading



Question #1b

- Calculate Vertex Normals
- Calculated from face normals:
  - abc = $(1/\sqrt{2})\ (1,1,0)$,
  - acd = $(1/\sqrt{2})\ (0,1,1)$,
  - ade = $(1/\sqrt{2})\ (-1,1,0)$,
  - aeb = $(1/\sqrt{2})\ (0,1,-1)$



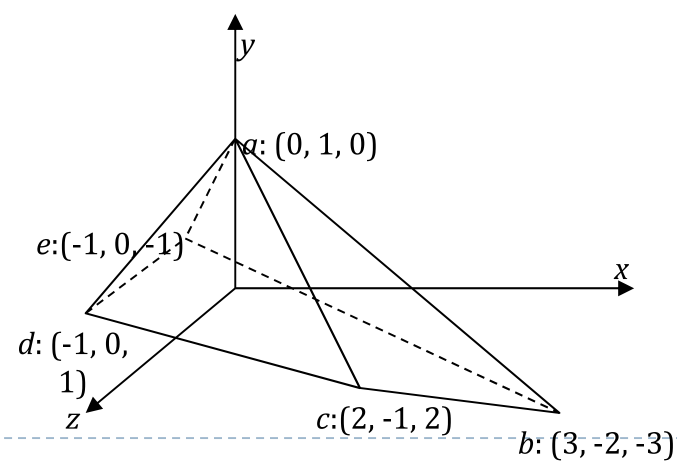
Question #1b

- Vertex *a*, shared by abc, ade,acd, aeb

$$n_a = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\1\\0\end{bmatrix}+\begin{bmatrix}-1\\1\\0\end{bmatrix}+\begin{bmatrix}0\\1\\1\end{bmatrix}+\begin{bmatrix}0\\1\\-1\end{bmatrix}\right)=\begin{bmatrix}0\\ 4/\sqrt{2}\\0\end{bmatrix}$$

$$\bar{n}_a = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

abc = (1/√2) (1,1,0),
acd = (1/√2) (0,1,1),
ade = (1/√2) (-1,1,0),
aeb = (1/√2) (0,1,-1)



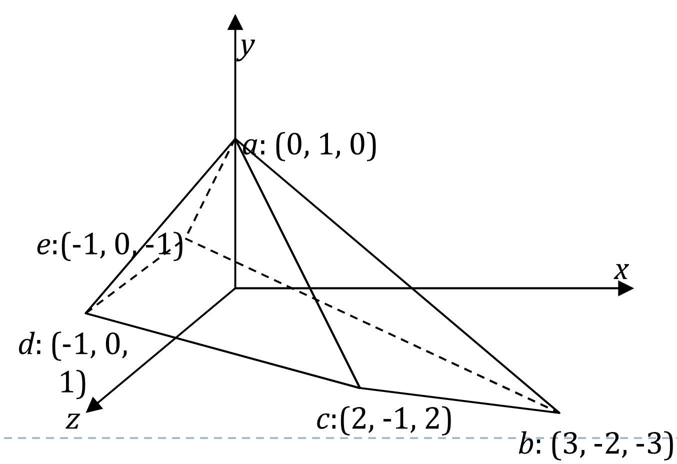
Question #1b

- Vertex *b*, shared by abc, aeb

$$n_b = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\1\\0\end{bmatrix}+\begin{bmatrix}0\\1\\-1\end{bmatrix}\right)=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

$$\bar{n}_b = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

abc = (1/√2) (1,1,0),
acd = (1/√2) (0,1,1),
ade = (1/√2) (-1,1,0),
aeb = (1/√2) (0,1,-1)



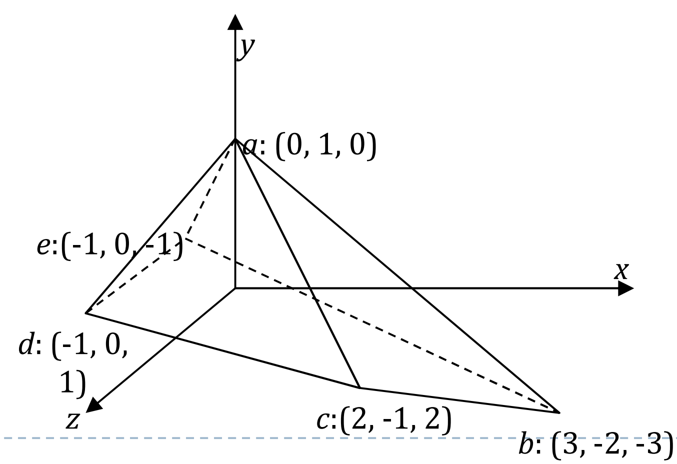
Question #1b

- Vertex *c*, shared by abc, acd

$$n_c = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\1\\0\end{bmatrix}+\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$\bar{n}_c = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

abc = (1/√2) (1,1,0),
acd = (1/√2) (0,1,1),
ade = (1/√2) (-1,1,0),
aeb = (1/√2) (0,1,-1)



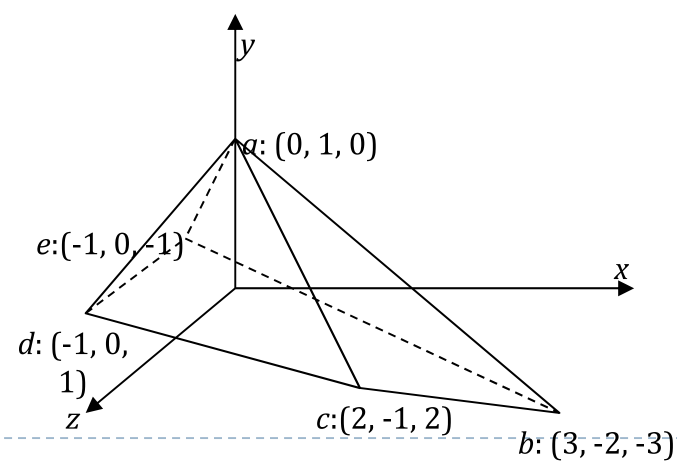
Question #1b

- Vertex *d*, shared by acd, ade

$$n_d = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}0\\1\\1\end{bmatrix}+\begin{bmatrix}-1\\1\\0\end{bmatrix}\right)=\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\2\\1\end{bmatrix}$$

$$\bar{n}_d = \frac{1}{\sqrt{6}}\begin{bmatrix}-1\\2\\1\end{bmatrix}$$

abc = (1/√2) (1,1,0),
acd = (1/√2) (0,1,1),
ade = (1/√2) (-1,1,0),
aeb = (1/√2) (0,1,-1)



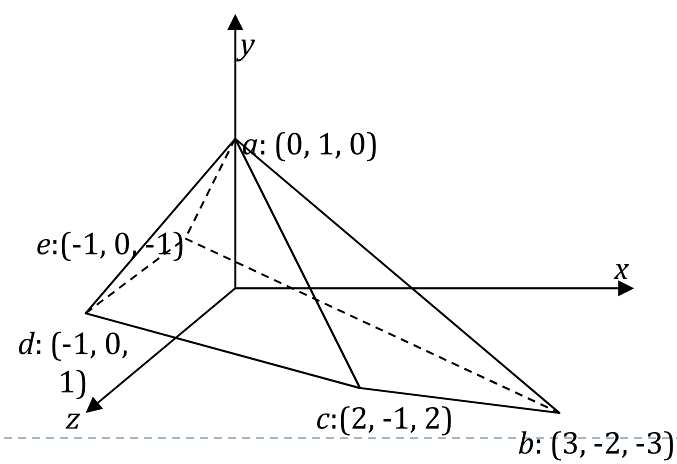
Question #1b

- Vertex e, shared by ade, aeb

$$n_e = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}-1\\1\\0\end{bmatrix}+\begin{bmatrix}0\\1\\-1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\2\\-1\end{bmatrix}$$

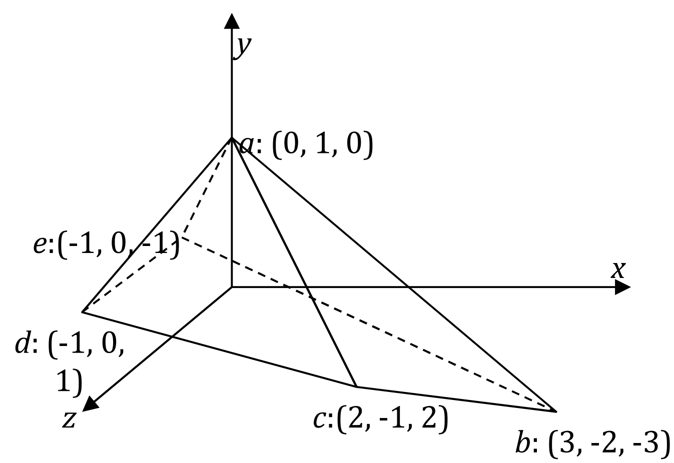
$$\bar{n}_e = \frac{1}{\sqrt{6}}\begin{bmatrix}-1\\2\\-1\end{bmatrix}$$

abc = (1/√2) (1,1,0),
acd = (1/√2) (0,1,1),
ade = (1/√2) (-1,1,0),
aeb = (1/√2) (0,1,-1)



Question #1c

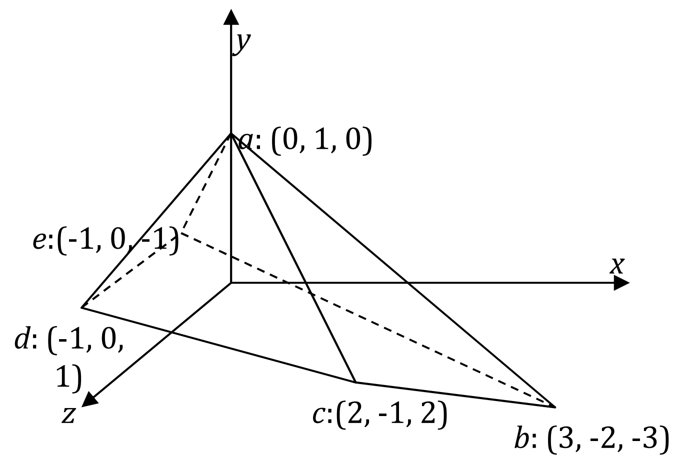
- Here are 4 triangles (*abc*, *acd*, *ade* and *aeb*) on a mesh:
  1. Compute the normal vector at the point p: (-0.5,0.5,0.5) for Phong Shading on triangle adc.



Question 1c

- p is the midpoint of a and d
- The normal of p should be the average of the normals of a and d

$$\bar{n}_a = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \bar{n}_d = \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$



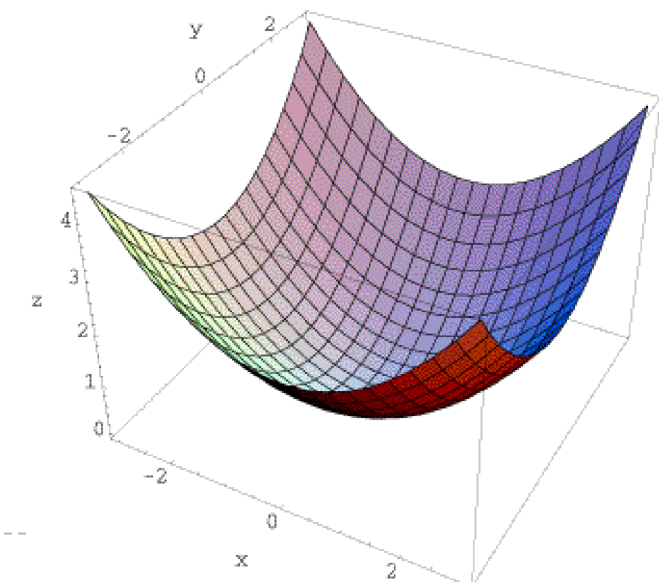
$$\bar{n}_p = \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right) / 2$$

Then normalize the normal

Question 2

- Other than computing a vertex normal vector for shading by averaging the normals of neighboring polygons, we can directly compute the normal vector of a vertex by other methods.
- For example, we would like to draw a paraboloid with the formula $z = x^2 + y^2$ by the following code:

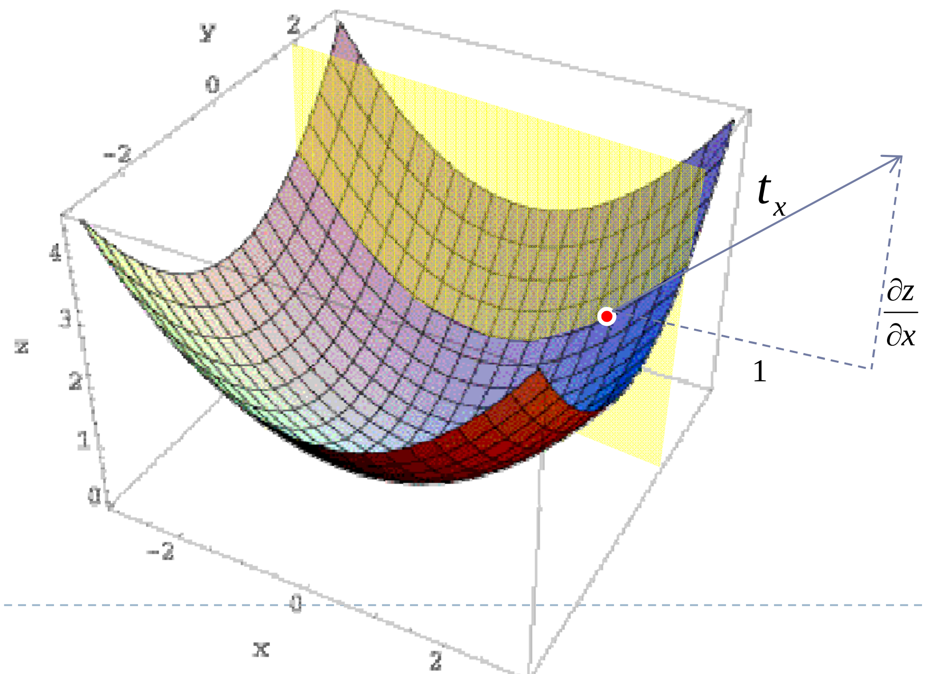
```
for (x = -2.5; x < 2.5; x+=0.25)
  for(y = -2.5; y < 2.5; y+=0.25)
  {
    x1 = x+0.25; y1 = y+0.25;
    glBegin(GL_POLYGON);
      glVertex3f(x,y, x*x + y*y);
      glVertex3f(x1,y, x1*x1 + y*y);
      glVertex3f(x1,y1, x1*x1 + y1*y1);
      glVertex3f(x,y1, x*x + y1*y1);
    glEnd();
  }
```



Question 2a

- Compute the two partial differentiations of z, namely $\partial z / \partial x$ and $\partial z / \partial y$. What is the meaning of these two numbers?

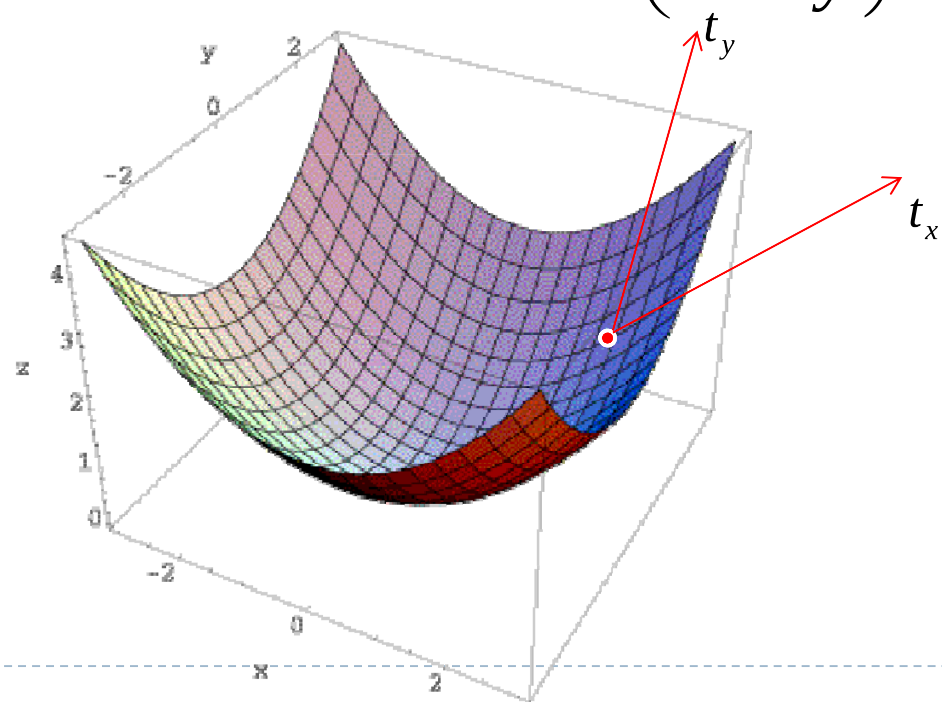
$$\frac{\partial z}{\partial x} = 2x \qquad \frac{\partial z}{\partial y} = 2y$$



Question 2b

- Compute the two tangent vectors of a point (*x*,*y*) along x and y directions

$$t_x = \left(1, 0, \frac{\partial z}{\partial x}\right) = (1, 0, 2x) \qquad t_y = \left(0, 1, \frac{\partial z}{\partial y}\right) = (0, 1, 2y)$$



Question 2c

- Compute the normal vector of (*x*,*y*). Hence, fill in the code for the normal vectors

$$n_{x,y} = t_x \times t_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix} = (-2x, -2y, 1)$$

```
for (x = -2.5; x < 2.5; x+=0.25)
  for(y = -2.5; y < 2.5; y+=0.25)
  {
    x1 = x+0.25; y1 = y+0.25;
    glBegin(GL_POLYGON);
      glNormal3f(-2*x, -2*y, 1);
      glVertex3f(x,y, x*x + y*y);
      glNormal3f(-2*x1, -2*y, 1);
      glVertex3f(x1,y, x1*x1 + y*y);
      glNormal3f(-2*x1, -2*y1, 1);
      glVertex3f(x1,y1, x1*x1 + y1*y1);
      glNormal3f(-2*x, -2*y1, 1);
      glVertex3f(x,y1, x*x + y1*y1);
    glEnd();
  }
```