Tutorial 6

5.2 If the probability density of a random variable is given by

$$f(x) = \begin{cases} kx^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find the value k and the probability that the random variable takes on a value

(a) between $\frac{1}{4}$ and $\frac{3}{4}$; (b) greater than $\frac{2}{3}$.

5.3 With reference to the preceding exercise, find the corresponding distribution function and use it to determine the probabilities that a random variable having this distribution function will take on a value

(a) greater than 0.8; (b) between 0.2 and 0.4.

5.4 If the probability density of a random variable is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

find the probabilities that a random variable having this probability density will take on a value

(a) between 0.2 and 0.8; (b) between 0.6 and 1.2.

5.5 With reference to the preceding exercise, find the corresponding distribution function, and use it to determine the probabilities that a random variable having the distribution function will take on a value

(a) greater than 1.8;

(b) between 0.4 and 1.6.

Tutorial 6

5.6 Given the probability density $f(x) = \dfrac{k}{1+x^2}$ for $-\infty < x < \infty$, find k.

5.13 Find μ and σ^2 for the probability density of Exercise 5.2.

5.14 Find μ and σ^2 for the probability density of Exercise 5.4.

5.21 Find z if the probability that a random variable having the standard normal distribution will take on a value

(a) less than z is 0.9911;

(b) greater than z is 0.1093;

(c) greater than z is 0.6443;

(d) less than z is 0.0217;

(e) between $-z$ and z is 0.9298.

5.24 Given a random variable having the normal distribution with $\mu = 16.2$ and $\sigma^2 = 1.5625$, find the probabilities that it will take on a value

(a) greater than 16.8;

(b) less than 14.9;

(c) between 13.6 and 18.8;

(d) between 16.5 and 16.7.

5.25 The time for a super glue to set can be treated as a random variable having a normal distribution with mean 30 seconds. Find its standard deviation if the probability is 0.20 that it will take on a value greater than 39.2 seconds.

Tutorial 6

5.31 Specifications for a certain job call for washers with an inside diameter of 0.300 ± 0.005 inch. If the inside diameters of the washers supplied by a given manufacturer may be looked upon as a random variable having the normal distribution with $\mu = 0.302$ inch and $\sigma = 0.003$ inch, what percentage of these washers will meet specifications?

5.32 With reference to the example on page 130, verify that if the variability of the filling machine is reduced to $\sigma = 0.025$ ounce, this will lower the required average amount of coffee to 4.05 ounces, yet keep 98% of the jars above 4 ounces.

5.33 A stamping machine produces can tops whose diameters are normally distributed with a standard deviation of 0.01 inch. At what "normal" (mean) diameter should the machine be set so that no more than 5% of the can tops produced have diameters exceeding 3 inches?

5.35 If a random variable has the binomial distribution with $n = 40$ and $p = 0.40$, use the normal approximation to determine the probabilities that it will take on

(a) the value 22;

(b) a value less than 8.

5.37 The probability that an electronic component will fail in less than 1,000 hours of continuous use is 0.25. Use the normal approximation to find the probability that among 200 such components fewer than 45 will fail in less than 1,000 hours of continuous use.

5.0 Verify that for standard normal distribution, $F(-z)=1-F(z)$

Tutorial 6

5.45 Find the distribution function of a random variable having a uniform distribution on (0, 1).

5.47 From experience Mr. Harris has found that the low bid on a construction job can be regarded as a random variable having the uniform density

$$f(x) = \begin{cases} \dfrac{3}{4C} & \text{for } \dfrac{2C}{3} < x < 2C \\ 0 & \text{elsewhere} \end{cases}$$

where C is his own estimate of the cost of the job. What percentage should Mr. Harris add to his cost estimate when submitting bids to maximize his expected profit?

5.200 For data: 0.09, 0.89, 0.07, 1.41, 1.16, 0.86, 0.04, 2.65, 0.26, 0.19, 0.75, 2.63, 1.29, 1.20, 1.45, 0.04, 0.21, 0.38, 0.05, 0.27, 0.08, 0.44, 0.14, 0.29, 0.07, 1.96, 0.69, 1.01, 0.40, 0.02, 4.57, 0.09, 0.67, 0.02, 0, 0.22, 0.76, 1.38, 1.33, 0.13, 0.22, 0.60, 0.75, 1.52, 1.17, 0.99, 2.29, 0.10, 0.01, 0.16,

(a) In R, draw the histogram and QQ-plot for the data.

(b) Whether the data has approximately a normal distribution?

(c) If not, what transformation can be taken to make the data closer to normal distribution? please check by QQ-plot.