

**NATIONAL UNIVERSITY OF SINGAPORE**  
**DEPARTMENT OF MATHEMATICS**  
**MA2214 COMBINATORIAL ANALYSIS**

**TUTORIAL 6**

**SEMESTER II, AY 2010/2011**

1. Let  $A$  be the set of partitions of  $n - 4$  into exactly 4 parts, each part not exceeding 4. Let  $B$  be the set of partitions of  $n - 5$  into exactly 3 parts, each part not exceeding 5. Find a bijection from  $A$  to  $B$ .
2. (a) Find all partitions of 10 into distinct parts.  
(b) Find all partitions of 10 into odd parts.
3. Find the least  $n$  such that given any  $n$  distinct integers, there are at least four of them whose sum is divisible by 4.
4. A computer program generated 175 positive integers at random, none of which had a prime divisor larger than 10. Prove that we can always find three numbers among them whose product is the cube of an integer.
5. Five friends run a race every day during the last four months of the year. There are never any ties. Prove that there are two races which end the same way.
6. Prove that the infinite sequence of integers 1, 11, 111, 1111, ..., contains an element that is divisible by 2011.
7. (Challenging question: Euler's result)  
Find a bijection between the partitions of  $n$  into distinct parts and the partitions of  $n$  into odd parts.

**Answers**