# Solutions to Tutorial 11

8.3 1. *Null hypothesis* $H_0 : \mu_1 - \mu_2 = 0$

*Alternative hypothesis* $H_1 : \mu_1 - \mu_2 \neq 0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* The null hypothesis specifies $\delta = \mu_1 - \mu_0 = 0$. Since the samples are large, we use the large sample statistic where we estimate each population variance by the sample variance.

$$Z = \frac{\overline{X} - \overline{Y} - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

The alternative is two-sided so we reject the null hypothesis for $Z > z_{.025}$ or $Z < -z_{.025}$

4. *Calculations:* Since $n_1 = 33$, $n_2 = 31$, $\overline{x} = 115.1$, $\overline{y} = 114.6$, $s_1 = 0.47$, and $s_2 = 0.38$

$$\sqrt{\frac{.47^2}{33} + \frac{0.38^2}{31}} = 0.10655$$

and

$$z = \frac{115.1 - 114.6}{0.10655} = 4.69 > 1.96,$$

5. *Decision:* Because $4.69 > 1.96$, we reject the null hypothesis at the .05 level of significance.

8.4 The sample sizes are large so we use the large samples confidence interval with $z_{.025} = 1.96$.

$$\overline{X} - \overline{Y} \pm z_{.025}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$115.1 - 114.6 \pm 1.96\sqrt{\frac{.47^2}{33} + \frac{0.38^2}{31}} = 115.1 - 114.6 \pm 1.96\,0.10655$$

or $0.29 < \mu_1 - \mu_2 < 0.71$ We are 95 % confident that the mean time to repair is 0.29 to 0.71 hour higher for the first kind of equipment.

8.5 (a) 1. *Null hypothesis* $H_0 : \mu_1 - \mu_2 = 0$

*Alternative hypothesis* $H_1 : \mu_1 - \mu_2 \neq 0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* The null hypothesis specifies $\delta = \mu_1 - \mu_0 = 0$. Since the samples are large, we use the large sample statistic where we estimate each population variance by the sample variance.

$$Z = \frac{\overline{X} - \overline{Y} - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

The alternative is two-sided so we reject the null hypothesis for $Z > z_{.025}$ or $Z < -z_{.025}$

4. *Calculations:* Since $n_1 = 75$, $n_2 = 75$, $\overline{x} = 83.2$, $\overline{y} = 90.8$, $s_1 = 19.3$, and $s_2 = 21.4$

$$\sqrt{\frac{19.3^2}{75} + \frac{21.4^2}{75}} = 3.3276$$

and

$$z = \frac{83.2 - 90.8}{3.3276} = -2.28 < -1.96,$$

5. *Decision:* Because $-2.28 < -1.96$, we reject the null hypothesis at the .05 level of significance. The P-value $.0226 = 2P[Z < -2.28]$ gives strong support for rejecting the null hypothesis.

8.6 (a) 1. *Null hypothesis* $H_0 : \mu_1 - \mu_2 = 0$
*Alternative hypothesis* $H_1 : \mu_1 - \mu_2 \neq 0$

2. *Level of significance:* $\alpha = 0.01$.

3. *Criterion:* The null hypothesis specifies $\mu_1 - \mu_0 = 0$. Since the samples are large, we use the large sample statistic where we estimate each population variance by the sample variance.

$$Z = \frac{\overline{X} - \overline{Y} - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

The alternative is two-sided so we reject the null hypothesis for $Z > z_{.005}$ or $Z < -z_{.005}$

4. *Calculations:* Since $n_1 = 40$, $n_2 = 30$, $\overline{x}_1 = 247.3$, $\overline{y} = 254.1$, $s_1 = 15.2$, and $s_2 = 18.7$

$$\sqrt{\frac{15.2^2}{40} + \frac{18.7^2}{30}} = 17.4323$$

and

$$z = \frac{247.3 - 254.1}{17.4323} = -1.629$$

5. *Decision:* Because $-1.629 > -2.58$, we fail to reject the null hypothesis at the .01 level of significance. The $P-$value is $P(Z < -1.629) = 0.052$

# Solutions to Tutorial 11

8.10 1. Let $\mu_1$ be the mean for Method A and $\mu_2$ be the mean for Method B. *Null hypothesis* $H_0 : \mu_1 - \mu_2 = 0$

*Alternative hypothesis* $H_1 : \mu_1 - \mu_2 < 0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* The null hypothesis specifies $\delta_0 = \mu_1 - \mu_0 = 0$. Since the samples are small, but we can assume that the populations are normal with the same variance, we use the two-sample $t$ statistic

$$t = \frac{(\bar{X} - \bar{Y}) - \delta_0}{\sqrt{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}} \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{n_1 + n_2}}$$

Since the alternative hypothesis is left-sided, $\delta < 0$, we reject the null hypothesis when $t < -t_{.05}$ or $t < -1.734$ since $t_{.05} = 1.734$ for 18 degrees of freedom.

4. *Calculations:* Here $n_1 = 10$ and $n_2 = 10$, and we first calculate $\overline{x} = 70$, $s_1 = 3.3665$, $\overline{y} = 74$, and $s_2 = 5.3955$. Then

$$t = \frac{70 - 74}{\sqrt{9(3.3665)^2 + 9(5.3955)^2}} \sqrt{\frac{10 \cdot 10 \cdot 18}{20}} = -1.989,$$

5. *Decision:* Since $-1.989 < -t_{.05} = -1.734$, we reject the null hypothesis at level of significance $\alpha = .05$. Thus, method B is more effective in terms of mean achievement score.

8.14 The sample size is small and we assume the difference has a normal distribution. There are $n = 5$ differences so $t_{.025} = 2.776$ for 4 degrees of freedom. Also, $\overline{d} = 1$ and $s_D = 1.414$. The 95 % confidence interval becomes

$$\overline{d} \pm t_{.025} \frac{s_D}{\sqrt{n}} = 1 \pm 2.776(\frac{1.414}{\sqrt{5}}) = 1 \pm 1.75$$

or $-0.75 < \mu_D < 2.75$. We are 95% confident that the mean difference in PCB's is between -0.75 and 2.75 ppb.

8.15 The sample size is small and we assume the difference has a normal distribution.

1. *Null hypothesis* $H_0 : \mu_D = 0$

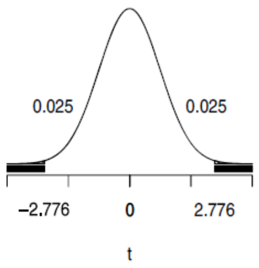
*Alternative hypothesis* $H_1 : \mu_D \neq 0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* We use the paired $t$ statistic

$$t = \frac{\overline{D} - \nu_{D\,0}}{S_D/\sqrt{n}}$$

Since $\alpha = .05$ and the alternative hypothesis is two-sided, we reject the null hypothesis if $t < -t_{.025}$ or if $t > t_{.025}$. There are 4 degrees of freedom so $t_{.025} = 2.776$.




4. *Calculations:* The sample mean of the differences is 1.0 and the variance is 2.0.

$$t = \frac{1.0 - 0}{\sqrt{2.0/5}} = 1.58$$

5. *Decision* We fail to reject the null hypothesis at level of significance .05.

10.23 1. *Null hypothesis* $H_0 : p = .06$

*Alternative hypothesis* $H_1 : p > .06$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* Using a normal approximation for the binomial distribution, we reject the null hypothesis when

$$Z = \frac{X - np_0}{\sqrt{np_0(1-p_0)}} \;>\; z_{.05}.$$

Since $\alpha = .05$ and $z_{.05} = 1.645$, the null hypothesis must be rejected if $Z > 1.645$.

4. *Calculations:* $p_0 = .06$, $X = 17$, and $n = 200$ so

$$Z = \frac{17 - 200(.06)}{\sqrt{200(.06)(.94)}} = 1.489.$$

5. *Decision:* Since the observed value $1.489 < z_{.05} = 1.645$, we cannot reject the null hypothesis at the 5% level.

# Solutions to Tutorial 11

10.32 1. *Null hypothesis* $H_0 : p_1 = p_2$

*Alternative hypothesis* $H_1 : p_1 > p_2$

2. *Level of significance:* $\alpha = 0.01$.

3. *Criterion:* We using the large sample statistic and reject the null hypothesis when

$$Z = \frac{X_1/n_1 - X_2/n_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \quad \text{with} \quad \hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

is greater than $z_{.01} = 2.33$.

4. *Calculations:* In this case, $x_1 = 26$, $n_1 = 200$, $x_2 = 12$, $n_2 = 200$, and

$$\hat{p} = \frac{26 + 12}{200 + 200} = .095$$

Hence

$$Z = \frac{26/200 - 12/200}{\sqrt{(.095)(.905)(2/200)}} = 2.41.$$

5. *Decision:* Since the observed value $2.41 > z_{.01} = 2.33$, we reject the null hypothesis of equal proportions at the 1 % level of significance.

10.33 Let $p_1$ and $p_2$ be proportions of reworking units before and after the training respectively. The 99% confidence interval for the true difference of the proportions, $p_1 - p_2$, is

$$\begin{aligned} & x_1/n_1 - x_2/n_2 \pm z_{\alpha/2} \sqrt{\frac{(x_1/n_1)(1 - x_1/n_1)}{n_1} + \frac{(x_2/n_2)(1 - x_2/n_2)}{n_2}} \\ &= 26/200 - 12/200 \pm 2.575 \sqrt{\frac{(26/200)(1 - 26/200)}{200} + \frac{(12/200)(1 - 12/200)}{200}} \\ &= .07 \pm .075 \end{aligned}$$

or $-.005 < p_1 - p_2 < .145$. We are 99 % confident that the proportion of units requiring reworking under the new method could be .145 lower to .005 higher than for the old method.