Chapter 5 Instructor Notes

Chapter 5 has been reorganized in response to a number of suggestions forwarded by users of the third edition of this book. The material is now divided into three background sections (overview of transient analysis, writing differential equations, and DC steady-state solution) and two major sections: first and second order transients. A few examples have been added, and all previous examples have been re-organized to follow the methodology outlined in the text.

In the section on first order transients, a *Focus on Methodology: First Order Transient Response* (p. 215) clearly outlines the methodology that is followed in the analysis of first order circuits; this methodology is then motivated and explained, and is applied to eight examples, including four examples focusing on engineering applications (5.8 - Charging a camera flash; 5.9 and 5.11 dc motor transients; and 5.12, transient response of supercapacitor bank). The analogy between electrical and thermal systems that was introduced in Chapter 3 is now extended to energy storage elements and transient response (*Make The Connection: Thermal Capacitanc*e, p. 204; *Make The Connection: Thermal System Dynamics*, p. 205; *Make The Connection: First-Order Thermal System*, p. 218-219;); similarly, the analogy between hydraulic and electrical circuits, begun in Chapter 2 and continued in Chapter 4, is continued here (*Make The Connection: Hydraulic Tank*, pp. 214-215). The box *Focus on Measurements: Coaxial Cable Pulse Response* (pp. 230-232) illustrates an important transient analysis computation (this problem was suggested many years ago by a Nuclear Engineering colleague).

The section on second order transients summarizes the analysis of second order circuits in the boxes *Focus on Methodology: Roots of Second-Order System* (p. 240) and *Focus on Methodology: Second Order Transient Response* (pp. 244-245). These boxes clearly outline the methodology that is followed in the analysis of second order circuits; the motivation and explanations in this section are accompanied by five very detailed examples in which the methodology is applied step by step. The last of these examples takes a look at an automotive ignition circuit (with many thanks to my friend John Auzins, formerly of Delco Electronics, for suggesting a simple but realistic circuit). The analogy between electrical and mechanical systems is explored in *Make The Connection: Automotive Suspension,* pp. 239-240 and pp. 245-246.

The homework problems are divided into four sections, and contain a variety of problems ranging from very basic to the fairly advanced. The focus is on mastering the solution methods illustrated in the chapter text and examples.

Learning Objectives

1. Write differential equations for circuits containing inductors and capacitors.
2. Determine the DC steady state solution of circuits containing inductors and capacitors.
3. Write the differential equation of first order circuits in standard form and determine the complete solution of first order circuits excited by switched DC sources.
4. Write the differential equation of second order circuits in standard form and determine the complete solution of second order circuits excited by switched DC sources.
5. Understand analogies between electrical circuits and hydraulic, thermal and mechanical systems.

Section 5.2: Writing Differential Equations for Circuits Containing Inductors and Capacitors

Problem 5.1

Solution:

Known quantities:

$L = 0.9\, mH, V_s = 12\, V, R_1 = 6\, k\Omega, R_2 = 6\, k\Omega, R_3 = 3\, k\Omega.$

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.21.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation. Note that the top node voltage is the inductor voltage, v_L.

$$\frac{v_L}{R_1 + R_2} + i_L + \frac{v_L}{R_3} = 0$$

Next, use the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$\frac{L}{R_1 + R_2}\frac{di_L}{dt} + i_L + \frac{L}{R_3}\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + \frac{(R_1 + R_2)R_3}{L(R_1 + R_2 + R_3)} i_L = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{di_L}{dt} + 2.67 \cdot 10^6 i_L = 0$$

Problem 5.2

Solution:

Known quantities:

$V_1 = 12\, V, C = 0.5\, \mu F, R_1 = 0.68\, k\Omega, R_2 = 1.8\, k\Omega.$

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.23.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation. Note that the top node voltage is the capacitor voltage, v_C.

$$i_C + \frac{v_C}{R_2} + \frac{v_C - V_1}{R_1} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$C\frac{dv_C}{dt} + \frac{R_1 + R_2}{R_1 R_2} v_C = \frac{V_1}{R_1} \quad \Rightarrow \quad \frac{dv_C}{dt} + \frac{R_1 + R_2}{C(R_1 R_2)} v_C = \frac{V_1}{CR_1}$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + (4052)v_C - 35292 = 0$$

Problem 5.3

Solution:

Known quantities:

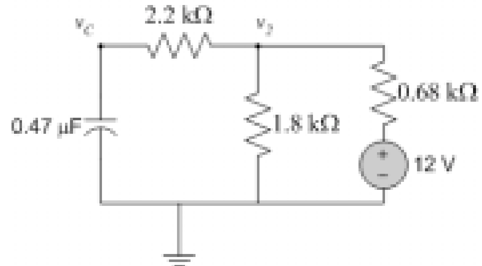
$V_1 = 12\,V, R_1 = 0.68\,k\Omega, R_2 = 2.2\,k\Omega, R_3 = 1.8\,k\Omega, C = 0.47\,\mu F.$

Find:
The differential equation for $t > 0$ (switch closed) for the circuit of P5.27.

Analysis:
Apply KCL at the two node (nodal analysis) to write the circuit equation. Note that the node #1 voltage is the capacitor voltage, v_C.

For node #1:

$$i_C + \frac{v_C - v_2}{R_2} = 0$$



For node #2:

$$\frac{v_2 - v_C}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - V_1}{R_1} = 0$$

Solving the system:

$$v_C = \frac{R_3}{R_1 + R_3}V_1 - \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_3}i_C$$

$$v_2 = \frac{R_3}{R_1 + R_3}V_1 - \frac{R_1R_3}{R_1 + R_3}i_C$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\frac{C(R_1R_2 + R_1R_3 + R_2R_3)}{R_1 + R_3}\frac{dv_C}{dt} + v_C = \frac{R_3}{R_1 + R_3}V_1$$

$$\frac{dv_C}{dt} + \frac{R_1 + R_3}{C(R_1R_2 + R_1R_3 + R_2R_3)}v_C = \frac{R_3}{C(R_1R_2 + R_1R_3 + R_2R_3)}V_1$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + (790)v_C - 6876 = 0$$

Problem 5.4

Solution:

Known quantities:

$V_{S2} = 13\,V, L = 170\,mH,\ R_2 = 4.3\,k\Omega, R_3 = 29\,k\Omega.$

Find:
The differential equation for $t > 0$ (switch open) for the circuit of P5.29.

Analysis:
Applying KVL we obtain:

$$(R_2 + R_3)i_L + v_L + V_{S2} = 0$$

Next, using the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$(R_2 + R_3)i_L + L\frac{di_L}{dt} + V_{S2} = 0$$

$$\frac{di_L}{dt} + \frac{(R_2 + R_3)}{L}i_L + \frac{V_{S2}}{L} = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{di_L}{dt} + 1.96 \cdot 10^5 i_L + 76.5 = 0$$

Problem 5.5

Solution:

Known quantities:

$I_0 = 17\,mA, C = 0.55\,\mu F, R_1 = 7\,k\Omega, R_2 = 3.3\,k\Omega.$

Find:
The differential equation for $t > 0$ for the circuit of P5.32.

Analysis:
Using the definition of capacitor current:

$$C\frac{dv_C}{dt} = I_0 \quad \Rightarrow \quad \frac{dv_C}{dt} = \frac{I_0}{C}$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} - 30909 = 0$$

Problem 5.6

Solution:

Known quantities:

$V_{S1} = V_{S2} = 11\,V$, $C = 70\,nF$, $R_1 = 14\,k\Omega$, $R_2 = 13\,k\Omega$, $R_3 = 14\,k\Omega$.

Find:
The differential equation for $t > 0$ (switch closed) for the circuit of P5.34.

Analysis:
Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_1 - V_{S2}}{R_1} + i_C + \frac{v_1}{R_3} = 0$$

Note that the node voltage v_1 is equal to:

$$v_1 = R_2 i_C + v_C$$

Substitute the node voltage v_1 in the first equation:

$$\left(\frac{1}{R_1}+\frac{1}{R_3}\right)v_C+\left(\frac{R_2}{R_1}+\frac{R_2}{R_3}+1\right)i_C-\frac{V_{S2}}{R_1}=0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\left(\frac{1}{R_1}+\frac{1}{R_3}\right)v_C+\left(\frac{R_2}{R_1}+\frac{R_2}{R_3}+1\right)C\frac{dv_C}{dt}-\frac{V_{S2}}{R_1}=0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt}+714.3v_C-3929=0$$

Problem 5.7

Solution:

Known quantities:

$V_S=20V, R_1=5\,\Omega, R_2=4\,\Omega, R_3=3\,\Omega, R_4=6\,\Omega, C_1=4\,F, C_2=4\,F, I_S=4\,A.$

Find:

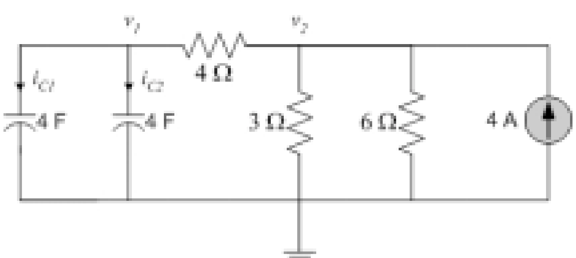
The differential equation for $t>0$ (switch closed) for the circuit of P5.41.

Analysis:

Apply KCL at the two node (nodal analysis) to write the circuit equation. Note that the node #1 voltage is equal to the two capacitor voltages, $v_{C1}=v_{C2}=v_C$.

For node #1:

$$i_{C1}+i_{C2}+\frac{v_C-v_2}{R_2}=0$$



For node #2:

$$\frac{v_2-v_C}{R_2}+\frac{v_2}{R_3}+\frac{v_2}{R_4}-I_S=0$$

Solving the system:

$$v_2=\frac{R_3R_4}{R_3+R_4}\left(i_{C1}+i_{C2}-I_S\right)$$

$$v_C=-\left(R_2+\frac{R_3R_4}{R_3+R_4}\right)\left(i_{C1}+i_{C2}\right)+\frac{R_3R_4}{R_3+R_4}I_S$$

Next, use the definition of capacitor current to eliminate the variables i_{Ci} from the nodal equation:

$$v_C+\left(R_2+\frac{R_3R_4}{R_3+R_4}\right)(C_1+C_2)\frac{dv_C}{dt}-\frac{R_3R_4}{R_3+R_4}I_S=0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt}+\frac{1}{48}v_C-\frac{1}{6}=0$$

Problem 5.8

Solution:

Known quantities:

$C = 1\mu F$, $R_S = 15\,k\Omega$, $R_3 = 30\,k\Omega$.

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.47.

Assume:

Assume that $V_S = 9\,\text{V}$, $R_1 = 10k\Omega$ and $R_2 = 20k\Omega$.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation.

1. Before the switch opens. Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_C - V_S}{R_S} + \frac{v_C}{R_1} + i_C + \frac{v_C}{R_2} + \frac{v_C}{R_3} = 0 \quad \Rightarrow \quad \left(\frac{1}{R_S} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v_C + i_C - \frac{V_S}{R_S} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\left(\frac{1}{R_S} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v_C + C\frac{dv_C}{dt} - \frac{V_S}{R_S} = 0$$

Substituting numerical value, we obtain the following differential equation:

$$\frac{dv_C}{dt} + 250v_C - 600 = 0$$

2. After the switch opens. Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_C - V_S}{R_S} + \frac{v_C}{R_1} + i_C = 0 \quad \Rightarrow \quad \left(\frac{1}{R_S} + \frac{1}{R_1}\right)v_C + i_C - \frac{V_S}{R_S} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\left(\frac{1}{R_S} + \frac{1}{R_1}\right)v_C + C\frac{dv_C}{dt} - \frac{V_S}{R_S} = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + \frac{500}{3}v_C - 600 = 0$$

Problem 5.9

Solution:

Known quantities:

Values of the voltage source, of the inductance and of the resistors.

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.49.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_1 - 100}{10} + \frac{v_1}{5} + i_L = 0 \quad \Rightarrow \quad 0.3v_1 + i_L - 10 = 0$$

Note that the node voltage v_1 is equal to:

$$v_1 = 2.5i_L + v_L$$

Substitute the node voltage v_1 in the first equation:

$$(1.75)i_L + (0.3)v_L - 10 = 0$$

Next, use the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$(0.3)(0.1)\frac{di_L}{dt} + (1.75)i_L - 10 = 0 \quad \Rightarrow \quad \frac{di_L}{dt} + (58.33)i_L - 333 = 0$$

Problem 5.10

Solution:

Known quantities:

$I_S = 5$ A, $L_1 = 1H$, $L_2 = 5H$, $R = 10k\Omega$.

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.52.

Analysis:

Applying KCL at the top node (nodal analysis) to write the circuit equation.

$$-I_S + \frac{v_1}{R} + i_L = 0$$

Note that the node voltage v_1 is equal to:

$$v_1 = v_{L1} + v_{L2}$$

Substitute the node voltage v_1 in the first equation:

$$\frac{v_{L1} + v_{L2}}{R} + i_L - I_S = 0$$

Next, use the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$\frac{(L_1 + L_2)}{R}\frac{di_L}{dt} + i_L - I_S = 0$$

Substituting numerical value, we obtain the following differential equation:

$$\frac{di_L}{dt} + \frac{5000}{3}i_L - \frac{25000}{3} = 0$$

Section 5.3: DC Steady-State Solution of Circuits Containing Inductors and Capacitors - Initial and Final Conditions

Problem 5.11

Solution:

Known quantities:

$L = 0.9\, mH, V_s = 12\,V, R_1 = 6k\Omega, R_2 = 6\,k\Omega, R_3 = 3\,k\Omega.$

Find:

The initial and final conditions for the circuit of P5.21.

Analysis:

Before opening, the switch has been closed for a long time. Thus we have a steady-state condition, and we treat the inductor as a short circuit. The voltages across the resistances R_1 and R_3 is equal to zero, since they are in parallel to the short circuit, so all the current flow through the resistor R_2:

$$i_L(0) = \frac{V_S}{R_2} = 2 \text{ mA.}$$

After the switch has been opened for a long time, we have again a steady-state condition, and we treat the inductor as a short circuit. When the switch is open, the voltage source is not connected to the circuit. Thus,

$$i_L(\infty) = 0 \text{ A.}$$

Problem 5.12

Solution:

Known quantities:

$V_1 = 12\,V, C = 0.5\,\mu F, R_1 = 0.68\,k\Omega, R_2 = 1.8\,k\Omega.$

Find:

The initial and final conditions for the circuit of P5.23.

Analysis:

Before closing, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. When the switch is open, the voltage source is not connected to the circuit. Thus,

$$v_C(0) = 0 \text{ V.}$$

After the switch has been closed for a long time, we have again a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistance R_2:

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} V_1 = \frac{1800}{1800 + 680} 12 = 8.71 \text{ V.}$$

Problem 5.13

Solution:

Known quantities:

$V_1 = 12\,V, R_1 = 0.68\,k\Omega, R_2 = 2.2\,k\Omega, R_3 = 1.8\,k\Omega, C = 0.47\,\mu F.$

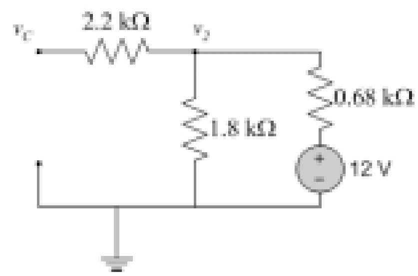
Find:
The initial and final conditions for the circuit of P5.27.

Analysis:
Before closing, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. When the switch is open, the voltage source is not connected to the circuit. Thus,

$$v_C(0) = 0 \text{ V}.$$

After the switch has been closed for a long time, we have again a steady-state condition, and we treat the capacitor as an open circuit. Since the current flowing through the resistance R_2 is equal to zero, the voltage across the capacitor is equal to the voltage across the resistance R_3 :

$$v_C(\infty) = \frac{R_3}{R_1 + R_3} V_1 = \frac{1800}{1800 + 680} 12 = 8.71 \text{ V}.$$



Problem 5.14

Solution:

Known quantities:

$V_{S1} = V_{S2} = 13\,V, L = 170\,mH,\ R_2 = 4.3\,k\Omega, R_3 = 29\,k\Omega.$

Find:
The initial and final conditions for the circuit of P5.29.

Analysis:
In a steady-state condition we can treat the inductor as a short circuit. Before the switch changes, applying the KVL we obtain:

$$V_{S1} - V_{S2} = (R_1 + R_2) i_L(0) \quad \Rightarrow \quad i_L(0) = \frac{V_{S1} - V_{S2}}{R_1 + R_2} = 0 \text{ mA}.$$

After the switch has changed for a long time, we have again a steady-state condition, and we treat the inductor as a short circuit. Thus, applying the KVL we have:

$$i_L(\infty) = -\frac{V_{S2}}{R_2 + R_3} = -0.39 \text{ mA}.$$

Problem 5.15

Solution:

Known quantities:

$I_0 = 17\,mA, C = 0.55\,\mu F, R_1 = 7\,k\Omega, R_2 = 3.3\,k\Omega.$

Find:
The initial and final conditions for the circuit of P5.32.

Analysis:
Before the switch changes, the capacitor is not connected to the circuit, so we don't have any information about its initial voltage.
After the switch has changed, the current source and the capacitor will be in series so the current to the capacitor will be constant at I_0. Therefore, the rate at which charge accumulates on the capacitor will also be constant and, consequently, the voltage across the capacitor will rise at a constant rate, without reaching any equilibrium state.

Problem 5.16

Solution:

Known quantities:

$V_{S1} = 17\,V, V_{S2} = 11\,V, C = 70\,nF, R_1 = 14\,k\Omega, R_2 = 13\,k\Omega, R_3 = 14\,k\Omega.$

Find:
The initial and final conditions for the circuit of P5.34.

Analysis:
In a steady-state condition we can treat the capacitor as an open circuit. Before the switch changes, applying the KVL we have that the voltage across the capacitor is equal to the source voltage V_{S1}:

$$v_C(0) = V_{S1} = 17 \text{ V.}$$

After the switch has changed for a long time, we have again a steady-state condition, and we treat the capacitor as an open circuit. Since the current flowing through the resistance R_2 is equal to zero, the voltage across the capacitor is equal to the voltage across the resistance R_3:

$$v_C(\infty) = \frac{R_3}{R_1 + R_3} V_{S2} = \frac{14000}{14000 + 14000} 11 = 5.5 \text{ V.}$$

Problem 5.17

Solution:

Known quantities:

$V_S = 20\,V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\,\Omega, R_4 = 6\,\Omega, C_1 = 4\,F, C_2 = 4\,F, I_S = 4\,A.$

Find:
The initial and final conditions for the circuit of P5.41.

Analysis:
The switch S_1 is always open and the switch S_2 closes at $t = 0$. Before closing, the switch S_2 has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitors as open circuits. When the switch is open, the current source is not connected to the circuit. Thus,

$$v_{C1}(0) = 0 \text{ V},$$
$$v_{C2}(0) = 0 \text{ V}.$$

After the switch S_2 has been closed for a long time, we have again a steady-state condition, and we treat the capacitors as open circuits. The voltages across the capacitors are both equal to the voltage across the resistance R_3:

$$v_{C1}(\infty) = v_{C2}(\infty) = (R_3 \parallel R_4) I_S = \frac{6 \cdot 3}{6+3} 4 = 8 \text{ V}.$$

Problem 5.18

Solution:

Known quantities:

$C = 1\mu F$, $R_S = 15k\Omega$, $R_3 = 30k\Omega$.

Find:
The initial and final conditions for the circuit of P5.47.

Assume:
Assume that $V_S = 9$ V, $R_1 = 10k\Omega$ and $R_2 = 20k\Omega$.

Analysis:
Before opening, the switch has been closed for a long time. Thus, we have a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistances R_1, R_2, and R_3. Thus,

$$v_C(0) = \frac{R_1 \parallel (R_2 \parallel R_3)}{R_S + R_1 \parallel (R_2 \parallel R_3)} V_S = \frac{10k \parallel 12k}{15k + 10k \parallel 12k} 9 = 2.4 \text{ V}.$$

After opening, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistance R_1. Thus,

$$v_C(\infty) = \frac{R_1}{R_1 + R_S} V_S = \frac{10000}{10000 + 15000} 9 = 3.6 \text{ V}.$$

Problem 5.19

Solution:

Known quantities:
Values of the voltage source, of the inductance and of the resistors.

Find:
The initial and final conditions for the circuit of P5.49.

Analysis:
Before the switch changes, apply KCL at the top node (nodal analysis) to write the following circuit equation.

$$\frac{v_1 - 100}{1000} + \frac{v_1}{5} + \frac{v_1}{2.5} = 0 \quad \Rightarrow \quad v_1 = \frac{100}{601} = 0.165 \text{ V}.$$

$$i_L(0) = \frac{v_1}{2.5} = \frac{40}{601} = 66 \text{ mA}.$$

After the switch has changed, apply KCL at the top node (nodal analysis) to write the following circuit equation.

$$\frac{v_1 - 100}{10} + \frac{v_1}{5} + \frac{v_1}{2.5} = 0 \quad \Rightarrow \quad v_1 = \frac{100}{7} = 14.285 \text{ V}.$$

$$i_L(\infty) = \frac{v_1}{2.5} = \frac{40}{7} = 5.714 \text{ A}.$$

Problem 5.20

Solution:

Known quantities:

$I_S = 5$ A, $L_1 = 1H$, $L_2 = 5H$, $R = 10k\Omega$.

Find:
The initial and final conditions for the circuit of P5.52.

Analysis:
Before closing, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the inductors as short circuits. The values of the two resistors are equal so the current flowing through the inductors is:

$$i_L(0) = \frac{I_S}{2} = 2.5 \text{ A}.$$

After the switch has been closed for a long time, we have again a steady-state condition, and we treat the inductors as short circuits. In this case the resistors are short-circuited and so all the current is flowing through the inductors.

$$i_L(\infty) = I_S = 5 \text{ A}.$$

Section 5.4: Transient Response of First-Order Circuits

Focus on Methodology

First-order transient response

1. Solve for the steady-state response of the circuit before the switch changes state ($t = 0^-$), and after the transient has died out ($t \to \infty$). We shall generally refer to these responses as $x(0^-)$ and $x(\infty)$.
2. Identify the initial condition for the circuit, $x(0^+)$, using continuity of capacitor voltages and inductor currents ($v_C(0^+) = v_C(0^-)$, $i_L(0^+) = i_L(0^-)$), as illustrated in Section 5.4.
3. Write the differential equation of the circuit for $t = 0^+$, that is, immediately after the switch has changed position. The variable $x(t)$ in the differential equation will be either a capacitor voltage, $v_C(t)$, or an inductor current, $i_L(t)$. It is helpful at this time to reduce the circuit to Thévenin or Norton equivalent form, with the energy storage element (capacitor or inductor) treated as the load for the Thévenin (Norton) equivalent circuit. Reduce this equation to standard form (Equation 5.8).
4. Solve for the time constant of the circuit: $\tau = R_T C$ for capacitive circuits, $\tau = L/R_T$ for inductive circuits.
5. Write the complete solution for the circuit in the form:

$$x(t) \quad x(\ ) \quad x(0) \quad x(\ )\ e^{\ t/}$$

Problem 5.21

Solution:

Known quantities:

Circuit shown in Figure P5.21, $L = 0.9\,mH, V_s = 12\,V, R_1 = 6\,k\Omega, R_2 = 6\,k\Omega, R_3 = 3\,k\Omega.$

Find:

If the steady-state conditions exist just before the switch was opened.

Assumptions:

$i_L = 1.70\,mA$ before the switch is opened at $t = 0$.

Analysis:

Determine the steady state current through the inductor at $t < 0$. If this current is equal to the current specified, steady-state conditions did exit; otherwise, opening the switch interrupted a transient in progress. To determine the steady-state current before the switch was opened, replace the inductor with an equivalent DC short-circuit and compute the steady-state current through the short circuit.

At steady state, the inductor is modeled as a short circuit:

$$V_{R3}\left(0^-\right) = 0 \qquad i_{R3}\left(0^-\right) = 0$$

Thus, the short-circuit current through the inductor is simply the current through R_2 which can be found by applying Kirchoff's Laws and Ohm's Law.

Apply KVL:

$$-V_s + i_{R3}\left(0^-\right)R_2 = 0 \qquad i_{R2}\left(0^-\right) = \frac{V_s}{R_2} = \frac{12}{6\times 10^3} = 2\,mH$$

Apply KCL:

$$-i_{R2}\left(0^-\right) + i_L\left(0^-\right) + i_{R3}\left(0^-\right) = 0,\ i_L\left(0^-\right) = i_{R2}\left(0^-\right) = 2\,mH$$

The actual steady state current through the inductor is larger than the current specified. Therefore, the circuit is not in a steady state condition just before the switch is opened.

Problem 5.22

Solution:

Known quantities:
Circuit shown in Figure P5.22,
$V_{S1} = 35V, V_{S2} = 130V, C = 11\mu F, R_1 = 17\,k\Omega, R_2 = 7\,k\Omega, R_3 = 23\,k\Omega.$

Find:
At $t = 0^+$ the initial current through R_3 just after the switch is changed.

Assumptions:
None.

Analysis:
To solve this problem, find the steady state voltage across the capacitor before the switch is thrown. Since the voltage across a capacitor cannot change instantaneously, this voltage will also be the capacitor voltage immediately after the switch is thrown. At that instant, the capacitor may be viewed as a DC voltage source.

At $t = 0^-$:
Determine the voltage across the capacitor. At steady state, the capacitor is modeled as an open circuit:

$$i_{R1}(0^-) = i_{R2}(0^-) = 0$$

Apply KVL:

$$V_{S1} + 0 - V_C(0^-) + 0 - V_{S2} = 0$$

$$V_C(0^-) = V_{S1} - V_{S2} = -95V$$

At $t = 0^+$:

$$V_C(0^+) = V_C(0^-) = -95V$$

$$i_{R2}(0^+) = i_{R3}(0^+)$$

Apply KVL:

$$V_{S2} - i_{R3}(0^+)R_2 + V_C(0^+) - i_{R3}(0^+)R_3 = 0$$

$$i_{R3}(0^+) = \frac{V_{S2} + V_C(0^+)}{R_2 + R_3} = \frac{130 - 95}{7\times10^3 + 23\times10^3} = 1.167\,mA$$

Problem 5.23

Solution:

Known quantities:
Circuit shown in Figure P5.23, $V_1 = 12\,V, C = 0.5\,\mu F, R_1 = 0.68\,k\Omega, R_2 = 1.8\,k\Omega.$

Find:
The current through the capacitor just before and just after the switch is closed.

Assumptions:
The circuit is in steady-state conditions for $t < 0$.

Analysis:
At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero. These are the steady state conditions, i.e.:

$$i_C(0^-) = 0 \qquad V_C(0^-) = 0$$

At $t = 0^+$, the switch is closed and the transient starts. Continuity requires:

$$V_C(0^+) = V_C(0^-) = 0$$

At this instant, treat the capacitor as a DC voltage source of strength zero i.e. a short-circuit. Therefore, all of the voltage V_1 is across the resistor R_1 and the resulting current through R_1 is the current into the capacitor.

Apply KCL: (Sum of the currents out of the top node)

$$i_C(0^+) + \frac{V_C(0^+) - 0}{R_2} + \frac{V_C(0^+) - V_1}{R_1} = 0$$

$$i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{0.68 \times 10^3} = 17.65\, mA$$

The CURRENT through the capacitor is NOT continuous but changes from 0 to $17.65\, mA$ when the switch is closed. The voltage across the capacitor is continuous because the stored energy CANNOT CHANGE INSTANTANEOUSLY.

Problem 5.24

Solution:

Known quantities:
Circuit shown in Figure P5.23, $V_1 = 12\, V, C = 150\, \mu F, R_1 = 400\, m\Omega, R_2 = 2.2\, k\Omega.$

Find:
The current through the capacitor just before and just after the switch is closed.

Assumptions:
The circuit is in steady-state conditions for $t < 0$.

Analysis:
At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero. These are the steady state conditions, i.e.:

$$i_C(0^-) = 0 \qquad V_C(0^-) = 0$$

At $t = 0^+$, the switch is closed and the transient starts. Continuity requires:

$$V_C(0^+) = V_C(0^-) = 0$$

At this instant, treat the capacitor as a DC voltage source of strength zero i.e. a short-circuit. Therefore, all of the voltage V_1 is across the resistor R_1 and the resulting current through R_1 is the current into the capacitor.

Apply KCL: (Sum of the currents out of the top node)

$$i_C(0^+) + \frac{V_C(0^+) - 0}{R_2} + \frac{V_C(0^+) - V_1}{R_1} = 0$$

$$i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{400 \times 10^{-3}} = 30\,A$$

The CURRENT through the capacitor is NOT continuous but changes from 0 to $30\,A$ when the switch is closed. The voltage across the capacitor is continuous because the stored energy CANNOT CHANGE INSTANTANEOUSLY.

Problem 5.25

Solution:

Known quantities:

Circuit shown in Figure P5.21, $V_s = 12\,V, L = 0.9\,mH, R_1 = 6\,k\Omega, R_2 = 6\,k\Omega, R_3 = 3\,k\Omega.$

Find:

The voltage across R_3 just after the switch is open.

Assumptions:

$i_L = 1.70\,mA$ before the switch is opened at $t = 0$.

Analysis:

When the switch is opened the voltage source is disconnected from the circuit and plays no role. Since the current through the inductor cannot change instantaneously the current through the inductor at t = 0^+ is also 1.70 mA. At this instant, treat the inductor as a DC current source and solve for the voltage across R_3 by current division or KCL and Ohm's Law.

Specify the polarity of the voltage across R_3.

$$i_L(0^+) = i_L(0^-) = 1.7\,mA$$

$$R_{eq} = R_1 + R_2 = 12\,k\Omega$$

Apply KCL: (Sum of the currents out of the top node)

$$\frac{V_{R3}(0^+)}{R_{eq}} + i_L(0^+) + \frac{V_{R3}(0^+)}{R_3} = 0$$

$$V_{R3}(0^+) = -\frac{i_L(0^+)}{\frac{1}{R_{eq}} + \frac{1}{R_3}} = \frac{1.7 \times 10^{-3}}{\frac{1}{12 \times 10^3} + \frac{1}{3 \times 10^3}} = -4.080V$$

Problem 5.26

Solution:

Known quantities:

Circuit shown in Figure P5.26, $V_1 = 12\,V, R_s = 0.7\,\Omega, R_1 = 22\,k\Omega, L = 100\,mH.$

Find:

The voltage through the inductor just before and just after the switch is changed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:
In steady-state the inductor acts like a short-circuit so it has no voltage across it for t < 0. However, its current is non-zero and is equal to the current out of the source V_S and through R_S. At the instant the switch is changed the current through the inductor is unchanged since the current through an inductor cannot change instantaneously. Also notice that after the switch is changed the current through R_1 is always equal to the inductor current and the voltage across R_1 is always equal to the inductor voltage. Thus, at t = 0+ the voltage across the inductor must be non-zero. That's fine since the voltage across an inductor can change instantaneously (or relatively so.)

Assume a polarity for the voltage across the inductor.

$t = 0^-$: Steady state conditions exist. The inductor can be modeled as a short circuit with:

$$V_L(0^-) = 0$$

Apply KVL;

$$-V_S + i_L(0^-)R_S + V_L(0^-) = 0$$

$$i_L(0^-) = \frac{V_S}{R_S} = \frac{12}{0.7} = 17.14\,A$$

At $t = 0^+$, the transient commences. Continuity requires:

$$i_L(0^+) = i_L(0^-)$$

Apply KVL:

$$i_L(0^+)R_1 + V_L(0^+) = 0$$

$$V_L(0^+) = -i_L(0^+)R_1 = -17.14 \times 22 \times 10^3 = -337.1\,kV$$

Problem 5.27

Solution:

Known quantities:
Circuit shown in Figure P5.27, $V_1 = 12\,V, R_1 = 0.68\,k\Omega, R_2 = 2.2\,k\Omega, R_3 = 1.8\,k\Omega, C = 0.47\,\mu F$.

Find:
The current through the capacitor at $t = 0^+$, just after the switch is closed.

Assumptions:
The circuit is in steady-state conditions for $t < 0$.

Analysis:
For t < 0, the switch is open and no power source is connected to the left half of the circuit. In steady state, by definition, the voltage across the capacitor and the current out of it must be constant. However, without a power source to replenish the energy dissipated by the resistors, that constant must be zero. Otherwise, current would flow out of the capacitor, its voltage would drop as it lost charge, and the energy of that charge would be dissipated by the resistors. This process would continue until no net charge remained on the capacitor and its voltage was zero. At steady state, then, the voltage across the capacitor is zero.

At $t = 0^+$, the voltage across the capacitor is still zero since the voltage across a capacitor cannot change instantaneously. At that instant, the capacitor can be treated as a voltage source of strength zero (i.e. a short-circuit.) However, the current through the capacitor can change instantaneously (or relatively so) from 0 to a new value. In this problem it will change as the switch is closed because the voltage source V1 will drive current through R1and the parallel combination of R_2 and R_3. The current through R_2 is the capacitor current.

$$V_C(0^+) = V_C(0^-) = 0$$

Apply KCL:

$$\frac{V_{R3}(0^+) - 0}{R_2} + \frac{V_{R3}(0^+)}{R_3} + \frac{V_{R3}(0^+) - V_1}{R_1} = 0$$

$$V_{R3}(0^+) = \frac{\frac{V_1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{V_1}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}} = \frac{12}{1 + \frac{0.68}{2.2} + \frac{0.68}{1.8}} = 7.114V$$

Recall that the voltage across the capacitor (Volts = Joules/Coulomb) represents the energy stored in the electric field between the plates of the capacitor. The electric field is due to the amount of charge stored in the capacitor and it is not possible to instantaneously remove charge from the capacitor's plates. Therefore, the voltage across the capacitor cannot change instantaneously when the circuit is switched.

However, the **rate** at which charge is removed from the plates of the capacitor (i.e. the capacitor current) **can** change instantaneously (or relatively so) when the circuit is switched.

Note also that these conditions hold only at the instant $t = 0^+$. For $t > 0^+$, the capacitor is gaining charge, all voltages and currents exponentially approach their final or steady state values.

Apply KVL:

$$-V_C(0^+) + i_C(0^+)R_2 + V_{R3}(0^+) = 0$$

$$i_C(0^+) = \frac{V_{R3}(0^+) - V_C(0^+)}{R_2} = \frac{7.114}{2.2 \times 10^3} = 3.234\,mA$$

Problem 5.28

Solution:

Known quantities:
NOTE: Typo in problem statement: should read "At t < 0, …"
Circuit shown in Figure P5.22,

$$V_{S1} = 35\,V, V_{S2} = 130\,V, C = 11\,\mu F, R_1 = 17\,k\Omega, R_2 = 7\,K\Omega, R_3 = 23\,k\Omega.$$

Find:

The time constant of the circuit for t > 0.

Assumptions:
The circuit is in steady-state conditions for t < 0.

Analysis:
For t > 0, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single capacitor system is $R_{eq}C$ where R_{eq} is the Thévenin equivalent resistance as seen by the capacitor, i.e., with respect to the port or terminals of the capacitor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question "What is the net equivalent resistance encountered in going from one terminal of the capacitor to the other through the network?" Then:

$$R_{eq} = R_2 + R_3 = 7\times10^3 + 23\times10^3 = 30\,k\Omega$$

$$\tau = R_{eq}C = 30\times10^3 \times 11\times10^{-6} = 330.0\,ms$$

Problem 5.29

Solution:

Known quantities:
Circuit shown in Figure P5.29,
$V_{S1} = 13\,V, V_{S2} = 13\,V, L = 170\,mH, R_1 = 2.7\,k\Omega, R_2 = 4.3\,k\Omega, R_3 = 29\,k\Omega.$

Find:
The time constant of the circuit for t > 0.

Assumptions:
The circuit is in steady-state conditions for t < 0.

Analysis:
For t > 0, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single inductor system is L/R_{eq} where R_{eq} is the Thévenin equivalent resistance as seen by the inductor, i.e., with respect to the port or terminals of the inductor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question "What is the net equivalent resistance encountered in going from one terminal of the inductor to the other through the network?" Then:

$$R_{eq} = R_2 + R_3 = 4.3\times10^3 + 29\times10^3 = 33.30\,k\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{170\times10^{-3}}{33.30\times10^3} = 5.105\,\mu s$$

Problem 5.30

Solution:

Known quantities:
Circuit shown in Figure P5.27, $V_1 = 12\,V, C = 0.47\,\mu F, R_1 = 680\,\Omega, R_2 = 2.2\,k\Omega, R_3 = 1.8\,k\Omega.$

Find:
The time constant of the circuit for t > 0.

Assumptions:
The circuit is in steady-state conditions for t < 0.

Analysis:
For t > 0, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single capacitor system is $R_{eq}C$ where R_{eq} is the Thévenin equivalent resistance as seen by the capacitor, i.e., with respect to the port or terminals of the capacitor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question "What is the net equivalent resistance encountered in going from one terminal of the capacitor to the other through the network?" Then:

$$R_{eq} = R_2 + \frac{R_3 R_1}{R_3 + R_1} = 2.2\times10^3 + \frac{1.8\times10^3 \times 0.68\times10^3}{1.8\times10^3 + 0.68\times10^3} = 2.694\,k\Omega$$

$$\tau = R_{eq}C = 2.694\times10^3 \times 0.47\times10^{-6} = 1.266\,ms$$

Problem 5.31

Solution:

Known quantities:

Circuit shown in Figure P5.21, $V_S = 12\,V, L = 0.9\,mH, R_1 = 6\,k\Omega, R_2 = 6\,k\Omega, R_3 = 3\,k\Omega.$

Find:

The time constant of the circuit for t > 0.

Assumptions:

The current through the inductor is $i_L = 1.70\,mA$ before the switch is opened at t = 0.

Analysis:

For t > 0, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single inductor system is L/R_{eq} where R_{eq} is the Thévenin equivalent resistance as seen by the inductor, i.e., with respect to the port or terminals of the inductor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question "What is the net equivalent resistance encountered in going from one terminal of the inductor to the other through the network?" Then:

$$R_{eq} = \frac{R_3(R_1 + R_2)}{R_3 + (R_1 + R_2)} = \frac{3\times 10^3(6\times 10^3 + 6\times 10^3)}{3\times 10^3 + (6\times 10^3 + 6\times 10^3)} = 2.400\,k\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.9\times 10^{-3}}{2.4\times 10^3} = 0.3750\,\mu s$$

Problem 5.32

Solution:

Known quantities:

Circuit shown in Figure P5.32, $V_c(0^-) = -7\,V, I_0 = 17\,mA, C = 0.55\,\mu F, R_1 = 7\,k\Omega, R_2 = 3.3\,k\Omega.$

Find:

The voltage $V_c(t)$ across the capacitor for t > 0.

Assumptions:

Before the switch is thrown the voltage across the capacitor is –7 V.

Analysis:

The current source and the capacitor will be in series so the current to the capacitor will be constant at I_0. Therefore, the rate at which charge accumulates on the capacitor will also be constant and, consequently, the voltage across the capacitor will rise at a constant rate. The integral form of the capacitor i-V relationship best expresses this accumulation process. The continuity of the voltage across the capacitor requires:

$$V_C(0^+) = V_C(0^-) = -7V$$
$$i_C(t) = I_0 = 17\,mA$$
$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)\,dt = \frac{1}{C}\left(\int_{-\infty}^{0} i_C(t)\,dt + \int_{0}^{t} I_0\,dt\right)$$
$$= V_C(0^+) + \frac{I_0}{C}\int_0^t dt = V_C(0^+) + \frac{I_0}{C}t\,|_0^t$$
$$= -7 + \frac{17\times10^{-3}}{0.55\times10^{-6}}t = -7 + 30.91\times10^3 t$$

Problem 5.33

Solution:

Known quantities:
Circuit shown in Figure P5.29,
$V_{S1} = 23\,V, V_{S2} = 20\,V, L = 23\,mH, R_1 = 0.7\,\Omega, R_2 = 13\,k\Omega, R_3 = 330\,k\Omega.$

Find:
The current $i_{R3}(t)$ through resistor R_3 for t > 0.

Assumptions:
The circuit is in steady-state conditions for $t < 0$. It is a resistive circuit with one storage element (e.g. inductor) so an assumed solution is of the form
$i_{R3}(t) = I_{SS} + (I_0 - I_{SS})e^{-t/\tau}$

Analysis:
The approach here is to first find the initial condition at t = 0^+ for the inductor and use it to determine the initial condition on the current through resistor R_3. Second, find the final steady-state condition of the circuit for t > 0. To do so, simply apply DC circuit analysis to solve for the current through resistor R_3 i.e. replace the inductor with a short-circuit. Finally, solve for the time constant of the circuit for t > 0. Each of these three results is needed to construct the complete transient solution.

At $t = 0^-$:
Assume steady state conditions exist. At steady state, the inductor is modeled as a short-circuit:
Apply KVL:
$$-V_{S2} + i_L(0^-)R_1 + i_L(0^-)R_2 + V_{S1} = 0$$
$$i_L(0^-) = \frac{V_{S2} - V_{S1}}{R_1 + R_2} = \frac{20 - 23}{0.7 + 13} = -0.22\,A$$
This current is flowing in the direction from the inductor to the switch.

Find I_0 at $t = 0^+$:
Continuity of the current through the inductor requires that:
$$i_L(0^+) = i_L(0^-) = -0.22\;A$$
$$I_0 = i_{R3}(0^+) = i_L(0^+) = -0.22\,A$$

Find I_{SS} at $t =$ infinity:
Assume that enough time has elapsed for steady state conditions to return. In steady state the inductor is modeled as a short circuit; therefore, the voltage across the inductor is zero. The result is a simple series

connection of resistors R_2 and R_3. The current across R_3 is found directly from Ohm's Law in this case.

$$I_{SS} = i_{R_3} = \frac{V_{S2}}{R_2 + R_3} = \frac{20\ V}{343\ \Omega} \cong 58\ mA$$

Find τ for $t > 0$**:**
To find the time constant t one first needs to determine the Thevenin equivalent resistance RTH across the terminals of the inductor. To do so, set all independent ideal sources to zero and determine the equivalent resistance "seen" by the inductor, i.e. with respect to the port or terminals of the inductor:

$$R_{eq} = R_2 + R_3 = 13 + 330 \times 10^3 = 330.0\,k\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{23 \times 10^{-3}}{330.0 \times 10^3} = 69.70\,ns$$

The complete response can now be written using the solution for transient voltages and currents in first-order circuits:

$$i_{R3}(t) = i_{R3}(\infty) + (i_{R3}(0^+) - i_{R3}(\infty))e^{-t/\tau}$$

$$= 0.058 + (-0.22 - 0.058)e^{-t/69.70\times10^{-9}}$$

$$= 0.058 - 0.28\ e^{-t/69.70\times10^{-9}}\ A$$

Problem 5.34

Solution:

Known quantities:
Circuit shown in Figure P5.34,
$V_{S1} = 17\,V, V_{S2} = 11\,V, R_1 = 14\,k\Omega, R_2 = 13\,k\Omega, R_3 = 14\,k\Omega, C = 70\,nF.$

Find:
a) $V(t)$ for $t > 0$.
b) The time for $V(t)$ to change by 98% of its total change in voltage after the switch is operated.

Assumptions:
The circuit is in steady-state conditions for $t < 0$. It is a resistive circuit with one storage element (e.g. capacitor) so an assumed solution is of the form

$$V_{R3}(t) = V_{SS} + (V_0 - V_{SS})e^{-t/\tau}$$

Analysis:
In general, the approach in problems such as this one is to first find the initial condition at $t = 0^+$ for the capacitor and use it to determine the initial condition on the voltage across resistor R_3. Second, find the final steady-state condition of the circuit for t > 0. To do so, simply apply DC circuit analysis to solve for the voltage across the resistor R_3 (i.e. replace the capacitor with an open-circuit and solve.) Finally, solve for the time constant of the circuit for t > 0 by finding the Thevenin equivalent resistance R_{TH} across the terminals of the capacitor. Each of these three results is needed to construct the complete transient solution.

a) **At** $t = 0^-$**:**

Steady state conditions are specified. At steady state, the capacitor is modeled as an open circuit:

$$i_C(0^-) = 0$$

Apply KVL:

$$V_{S1} = i_C(0^-)R_2 + V_C(0^-)$$
$$V_C(0^-) = V_{S1} = 17V$$

At $t = 0^+$:

The voltage across the capacitor remains the same:

$$V_C(0^+) = V_C(0^-) = 17V$$

Apply KCL:

$$\frac{V(0^+) - (V_{S2})}{R_1} + \frac{V(0^+) - V_C(0^+)}{R_2} + \frac{V(0^+) - 0}{R_3} = 0$$

$$V(0^+) = \frac{\frac{V_C(0^+)}{R_2} + \frac{V_{S2}}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{\frac{17}{13\times10^3} + \frac{11}{14\times10^3}}{\frac{1}{14\times10^3} + \frac{1}{13\times10^3} + \frac{1}{14\times10^3}} = 9.525V$$

At $t > 0$:

Determine the equivalent resistance as "seen" by the capacitor, ie, with respect to the port or terminals of the capacitor. Suppress the independent ideal voltage source:

$$R_{eq} = R_2 + \left(R_1 \| R_3\right) = R_2 + \frac{R_1R_3}{R_1 + R_3} = 13\times10^3 + \frac{14\times10^3 \times 14\times10^3}{14\times10^3 + 14\times10^3}$$
$$= 20.00\,k\Omega$$

$$\tau = R_{eq}C = 20\times10^3 \times 70\times10^{-9} = 1.400\,ms$$

At $t =$ **infinity:**

Steady state is again established. At steady state the capacitor is once again modeled as an open circuit:

$$i_C(\infty) = 0$$

Since the current through the capacitor branch is zero in steady state the voltage across R_3 may be found quickly by voltage division

$$V(\infty) = \frac{V_{S2}R_3}{R_3 + R_1}$$
$$= \frac{11\times14\times10^3}{14\times10^3 + 14\times10^3} = 5.500V$$

The complete response for $t > 0$ is then:

$$V(t) = V(\infty) + (V(0^+) - V(\infty))e^{-t/\tau}$$
$$= 5.5 + (9.525 - (5.5))e^{-t/1.4\times10^{-3}}$$
$$= 5.5 + 4.025e^{-t/1.4\times10^{-3}}\,V$$

b)

The time required for V(t) to reach 98% of its final value is found from

$$0.98 = \frac{V(t) - V(0^+)}{V(\infty) - V(0^+)}$$

or

$$\Delta V = V(\infty) - V(0^+) = 5.5 - (9.525)$$
$$= -4.025V$$
$$V(t_1) = V(0^+) + 0.98\Delta V$$
$$= 9.525 + 0.98\times(-4.025)$$
$$= 5.5805 = 5.5 + 4.025e^{-t_1/1.4\times10^{-3}}$$
$$t_1 = -1.4\times10^{-3}\ln\left(\frac{5.5805 - 5.5}{4.025}\right) = 5.477\,ms$$

Problem 5.35

Solution:

Known quantities:

Circuit shown in Figure P5.35, $V_G = 12\,V, R_G = 0.37\,\Omega, R = 1.7\,k\Omega.$

Find:

The value of L and R_1.

Assumptions:

The voltage across the spark plug gap V_R just after the switch is changed is $23\,kV$ and the voltage will change exponentially with a time constant $\tau = 13\,ms$.

Analysis:

At $t = 0^-$:

Assume steady state conditions exist. At steady state the inductor is modeled as a short circuit:

$$V_L(0^-) = 0$$

The current through the inductor at this point is given directly by Ohm's Law:

$$i_L(0^-) = \frac{V_G}{R_G + R_1}$$

At $t = 0^+$:

Continuity of the current through the inductor requires that:

$$i_L(0^+) = i_L(0^-) = \frac{V_G}{R_G + R_1}$$

$$V_R(0^+) = -i_L(0^+)R = -\frac{V_G R}{R_G + R_1}$$

$$R_1 = -\frac{V_G R}{V_R(0^+)} - R_G$$

$$= -\frac{12 \times 1.7 \times 10^3}{-23 \times 10^3} - 0.37 = 0.5170\,\Omega$$

Note that the voltage across the gap V_R was written as –23 kV since the current from the inductor flows opposite to the polarity shown for V_R; that is, the actual polarity of the voltage across R is opposite that shown.

At $t > 0$:

Determine the Thevenin equivalent resistance as "seen" by the inductor, ie, with respect to the port or terminals of the inductor:

$$R_{eq} = R_1 + R$$

$$\tau = \frac{L}{R_{eq}} = \frac{L}{R_1 + R}$$

$$L = \tau(R_1 + R) = 13 \times 10^{-3} \times (0.5170 + 1.7 \times 10^3) = 22.11\,H$$

Problem 5.36

Solution:

Known quantities:

Circuit shown in Figure P5.36, when $i_L \geq +2\,mA$, the relay functions.

$V_S = 12\,V, L = 10.9\,mH, R_1 = 3.1\,k\Omega.$

Find:

R_2 so that the relay functions at $t = 2.3\,s$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

In this problem the current through the inductor is clearly zero before the switch is thrown. The task is determine the value of the resistance R_2 such that the current through the inductor will need 2.3 seconds to rise to 2 mA. Once again, we must find the complete transient solution, this time for the current through the inductor. Assume a solution of the form

$$i_L(t) = i_\infty + (i_0 - i_\infty)e^{-t/\tau}$$

At $t = 0^-$**:**

The current through the inductor is zero since no source is connected.

$$i_L(0^-) = 0$$

At $t = 0^+$**:**

$i_0 = i_L(0^+) = i_L(0^-) = 0$

At $t > 0$:

Determine the Thevenin equivalent resistance as "seen" by the inductor, i.e., with respect to the port or terminals of the inductor:

$$R_{TH} = (R_1 \| R_2) = \frac{R_1 R_2}{R_1 + R_2}$$

And so

$$\tau = \frac{L}{R_{TH}} = \frac{L(R_1 + R_2)}{R_1 R_2}$$

At $t =$ **infinity:**

Steady state is again established. At steady state the inductor is again modeled as a short circuit. Thus, the current through R_2 is zero and the current through the inductor is given by

$$i_\infty = i_L(\infty) = \frac{V_S}{R_1} = \frac{12}{3.1\times 10^3} = 3.87\, mA$$

Plug in the above quantities to the complete solution and set the current through the inductor equal to 2 mA and the time equal to 2.3 s.

$$2\times 10^{-3} = 3.87\times 10^{-3}\left(1 - e^{-2.3/\tau}\right)$$

or

$$\frac{-2.3}{\tau} = \ln\left[1 - \frac{2}{3.87}\right]$$

or

$$\tau \cong 3.16\,\text{seconds}$$

Solving for R_2:

$$3.16 = \frac{L(R_1 + R_2)}{R_1 R_2}$$

or

$$R_2 = \frac{LR_1}{3.16 R_1 - L} \cong 3.4\, m\Omega$$

Problem 5.37

Solution:

Known quantities:

Circuit shown in Figure P5.37, $V_1 = 12\, V, R_1 = 400\, m\Omega, R_2 = 2.2\, k\Omega, C = 150\, \mu F$.

Find:

The current through the capacitor just before and just after the switch is closed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:
At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero. Thus, in this circuit, and others like it that contain no connected sources prior to the switch being thrown, the steady state condition is zero currents and zero voltages.

$$i_C(0^-) = 0 \qquad V_C(0^-) = 0$$

At $t = 0^+$, the switch is closed; the transient starts. Continuity requires:

$$V_C(0^+) = V_C(0^-) = 0$$

Since the voltage across the capacitor is zero at this instant, the voltage across the resistor R2 is also zero at this instant which implies that no current passes through the resistor at t = 0^+. Therefore, at t = 0^+, the current out of the source must be the current into the capacitor.

$$i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{0.4} = 30A$$

The CURRENT through the capacitor is NOT continuous but changes from 0 to $30A$ when the switch is closed. The voltage across the capacitor is continuous because the stored energy CANNOT CHANGE INSTANTANEOUSLY.

Problem 5.38

Solution:

Known quantities:
Circuit shown in Figure P5.38, $V_S = 12\,V, R_S = 0.24\,\Omega, R_1 = 33\,k\Omega, L = 100\,mH$.

Find:
The voltage across the inductor before and just after the switch is changed.

Assumptions:
The circuit is in steady-state conditions for $t < 0$.

Analysis:

The solution to this problem can be visualized by noting that before the switch is thrown the circuit is presumed to be in a DC steady state. Therefore, before the switch is thrown the inductor may be modeled as a short circuit such that the voltage across it is zero. Moreover, the current through the inductor before the switch is thrown is simply V_S/R_S. Immediately after the switch is thrown the current through the inductor must be this same value since the current through an inductor cannot change instanteneously. However, after the switch is thrown this current must also be the current drawn through R_1 since that resistor and the inductor are then in series. Finally, the voltage across the inductor after the switch is thrown is always equal to the voltage across R_1, which is simply the product of i_L and R_1. Thus, the voltage across the inductor immediately after the switch is thrown must be $i_L(t=0^+)$ times R_1.

In quantitative terms, at t = 0^-,

$$v_L(0^-) = 0$$

$$i_L(0^-) = \frac{V_S}{R_S} = \frac{12}{0.24} = 50\,A$$

At t = 0^+,

$i_L(0^+) = i_L(0^-) = 50\,A$

and

$v_L(0^+) = i_L(0^+)R_1 = (50\,A)(33\,k\Omega) = 1.65\,MV$

This high side of this voltage is located at the ground terminal due to the direction of current flow through R_1.

ANSWER: $0V, -1.65\,MV$

Problem 5.39

Solution:

Known quantities:

Circuit shown in Figure P5.27, $V_1 = 12\,V, C = 150\,\mu F, R_1 = 4\,M\Omega, R_2 = 80\,M\Omega, R_3 = 6\,M\Omega$.

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

At $t = 0^+$, just after the switch is closed, a transient starts. Since this is a first order circuit (a single independent capacitance), the transient will be exponential with some time constant. That time constant is the product of the capacitance of the capacitor and the Thevenin equivalent resistance seen across the terminals of the capacitor.

To find the Thevenin equivalent resistance, suppress the independent, ideal voltage source which is equivalent to replacing it with a short circuit. Then with respect to the terminals of the capacitor:

$R_{TH} = R_2 + (R_1 \| R_3) = 80 + (4 \| 6) = 82.4\,M\Omega$

The time constant is simply

$\tau = R_{TH}C = (82.4\,M\Omega)(150\,\mu F) = 12360 \text{ sec} = 206 \text{ minutes} = 3.43 \text{ hours} +$

Notice that this value is quite independent of the magnitude of the voltage source.

ANSWER: $2.36\,ks = 206.0\,\text{min} = 3.433\,hr$

Problem 5.40

Solution:

Known quantities:

Circuit shown in Figure P5.21, $V_S = 12\,V, L = 100\,mH, R_1 = 400\,\Omega, R_2 = 400\,\Omega, R_3 = 600\,\Omega$.

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

$i_L = 1.70\,mA$ just before the switch is opened at $t = 0$.

Analysis:

At $t = 0^+$, just after the switch is opened, a transient starts. Since this is a first order circuit (a single independent capacitance), the transient will be exponential with some time constant. That time constant is the product of the inductance of the inductor and the Thevenin equivalent resistance seen across the terminals of the inductor.

In this problem, the Thevenin equivalent resistance is particularly easy to find since there are no sources connected. With respect to the terminals of the inductor:

$$R_{TH} = (R_1 + R_2) \| R_3 = 800 \| 600 \cong 343\ \Omega$$

The time constant is simply

$$\tau = \frac{L}{R_{TH}} = \frac{0.1}{343} \cong 292\ ns$$

ANSWER: $291.7\,ns$

Problem 5.41

Solution:

Known quantities:
Circuit shown in Figure P5.41,

$$V_S = 20V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\,\Omega, R_4 = 6\,\Omega, C_1 = 4\,F, C_2 = 4\,F, I_S = 4\,A.$$

Find:

a) The capacitor voltage $V_C(t)$ at $t = 0^+$.

b) The time constant τ for $t \geq 0$.

c) The expression for $V_C(t)$ and sketch the function.

d) Find $V_C(t)$ for each of the following values of t: $0, \tau, 2\tau, 5\tau, 10\tau$.

Assumptions:
Switch S_1 is always open and switch S_2 closes at $t = 0$.

Analysis:

a) Without any power sources connected the steady state voltages are zero due to relentless dissipation of energy in the resistors.

$$V_C(0^-) = V_C(0^+) = 0V$$

When the initial condition on a transient is zero, the general solution for the transient simplifies to

$$V_C(t) = V(\infty)\left(1 - e^{-t/\tau}\right)$$

b) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the current source (i.e. replacing it with an open circuit) and computing $R_2 + R_3 \| R_4$.

$$R_{TH} = 4 + (3 \| 6) = 6\,\Omega$$

$$\tau = R_{TH} C = (6)(8) = 48\,s$$

c) The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_3. This voltage is found readily by current division

$$V_C(\infty) = \frac{6\,\Omega}{3\,\Omega + 6\,\Omega}(4\,A)(3\,\Omega) = 8V$$

Plug in to the generalized solution given above to find

$$V_C(t) = 8\left(1 - e^{-t/48}\right) \quad , \quad t \geq 0$$
$$V_C(t) = 0 \quad , \quad t \leq 0$$

d)

$$V_C(0) = 0V; \qquad V_C(\tau) = 5.06V;$$
$$V_C(2\tau) = 6.9V; \qquad V_C(5\tau) = 7.95V;$$
$$V_C(10\tau) = 8.0V$$

Problem 5.42

Solution:

Known quantities:
Circuit shown in Figure P5.41,
$V_S = 20V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\Omega, R_4 = 6\Omega, C_1 = 4\,F, C_2 = 4\,F, I_S = 4\,A.$

Find:

a) The capacitor voltage $V_C(t)$ at $t = 0^+$.

b) The time constant τ for $t \geq 0$.

c) The expression for $V_C(t)$ and sketch the function.

d) Find $V_C(t)$ for each of the following values of t: $0, \tau, 2\tau, 5\tau, 10\tau$.

Assumptions:
Switch S_1 has been open for a long time and closes at $t = 0$; conversely, switch S_2 has been closed for a long time and opens at $t = 0$.

Analysis:

a) The capacitor voltage immediately after the switch S_1 is closed and the switch S_2 is opened is equal to that when the switches were the first opened and the second closed respectively. Since the second switch was closed for a long time one can assume that the capacitor voltage had reached a long-term steady state value. This value is found by replacing both capacitors with DC open circuits and solving for the voltage across R_3. This voltage is found readily by current division

$$V_C(0^+) = V_C(0^-) = \frac{6\,\Omega}{3\,\Omega + 6\,\Omega}(4\,A)(3\,\Omega) = 8V$$

b) The Thevenin equivalent resistance seen by the parallel capacitors is $R_1 \,||\, (R_2 + R_3)$.

$$\tau = R_{TH}C = \left(\frac{1}{5} + \frac{1}{4+3}\right)^{-1} \times (4+4) = \frac{70}{3}s$$

c) The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_3. This voltage is found readily by voltage division. Thus,

$$V_C(\infty) = \frac{R_2 + R_3}{R_1 + R_2 + R_3}V_S = \frac{4\Omega + 3\Omega}{5\Omega + 4\Omega + 3\Omega}(20V) = \frac{35}{3}V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + \left[V(0^+) - V(\infty)\right]e^{-t/\tau} = \frac{35}{3} + \left(8 - \frac{35}{3}\right)e^{-3t/70} = \frac{35}{3} - \frac{11}{3}e^{-3t/70} \quad , \quad t \ge 0$$

$$V_C(t) = 8 \quad , \quad t \le 0$$

d)

$V_C(0) = 8\ V;$ $V_C(\tau) = 10.318\ V;$

$V_C(2\tau) = 11.170\ V;$ $V_C(5\tau) = 11.642\ V;$

$V_C(10\tau) = 11.666 V$

Problem 5.43

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$V_S = 20 V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\,\Omega, R_4 = 6\,\Omega, C_1 = 4\,F, C_2 = 4\,F, I_S = 4\,A.$

Find:

a) The capacitor voltage $V_C(t)$ at $t = 0^+$.

b) The expression for $V_C(t)$ and sketch the function.

Assumptions:

Switch S_2 is always open; switch S_1 has been closed for a long time, and opens at $t = 0$. At $t = t_1 = 3\tau$, switch S_1 closes again.

Analysis:

The approach here is to find the transient solution in the interval 0 < t < 3 seconds and use that solution to determine the initial condition (the capacitor voltage) for the new transient after the switch S_1 closes again.

a) S_1 has been closed for a long time and in DC steady state the capacitors can be replaced with open circuits. Thus, by voltage division

$$V_C(0^+) = V_C(0^-) = \frac{7}{12} \times 20 = 11.67 V$$

b) As mentioned above, to find the complete transient solution for t > 0 it is necessary to find the capacitor voltage when switch S1 closes at t = 3 seconds. To do so it is first necessary to find the complete transient solution for when the switch is open (i.e. as if the switch never closed again.)

The long-term steady state capacitor voltage when the switch is held open is zero. The time constant is simply $R_{TH} C_{EQ}$ = 56 seconds. Thus, the complete transient solution for the first 3 seconds is

$$V_C(t) = V(0^+)e^{-t/56} = 11.67e^{-t/56} \quad , \quad 0 \le t \le 3$$

At t = 3 seconds, the capacitor voltage is

$$V_C(t = 3^-) = 11.67e^{-3/56} = 11.06$$

At t=3 seconds the switch S1 closes again. Continuity of voltage across the capacitors still holds so

$$V_C(t = 3^+) = V_C(t = 3^-) = 11.06$$

With the switch closed the long-term steady state capacitor voltage is the same as that found in part a.

$$V_C(\infty) = \frac{7}{12} \times 20 = 11.67V$$

The new time constant is found after suppressing the independent voltage source (i.e. replacing it with a short circuit) and finding the new Thevenin equivalent resistance seen by the capacitors.

$$R_{TH} = (4+3)\|5 = 2.92\ \Omega$$

and

$$\tau = R_{TH}C = (2.92)(4+4) = 23.3 \text{ seconds}$$

Finally, the transient solution for t > 3 is

$$V_C(t) = 11.67 + [11.06 - 11.67]e^{-t/23.3} = 11.67 - 0.61e^{-(t-3)/23.3} \quad , \quad t \geq 3$$

Notice the use of the shifted time scale (t-3) in the exponent.

Problem 5.44

Solution:

Known quantities:
Circuit shown in Figure P5.41,

$$V_S = 20V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\Omega, R_4 = 6\Omega, C_1 = 4F, C_2 = 4F, I_S = 4A.$$

Find:

a) The capacitor voltage $V_C(t)$ at $t = 0^+$.

b) The time constant τ for $t \geq 0$.

c) The expression for $V_C(t)$ and sketch the function.

d) Find $V_C(t)$ for each of the following values of t: $0, \tau, 2\tau, 5\tau, 10\tau$.

Assumptions:
Both switches S_1 and S_2 close at $t = 0$.

Analysis:

a) Without any power sources connected the steady state voltages are zero due to the complete dissipation of all circuit energy by the resistors.

$$V_C(0^-) = V_C(0^+) = 0V$$

When the initial condition on a transient is zero, the general solution for the transient simplifies to

$$V_C(t) = V(\infty)\left(1 - e^{-t/\tau}\right)$$

b) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the independent sources (i.e. by replacing the current source with an open circuit and the voltage source with a short circuit) and computing $R_1 \| (R_2 + R_3 \| R_4)$.

$$R_{TH} = [5\|(4 + (3\|6))] = [5\|6] = \frac{30}{11} \cong 2.73\Omega$$

$$\tau = R_{TH} C = \frac{30}{11} 8 = \frac{240}{11} \cong 21.8\, s$$

c) At this point only the long-term steady state capacitor voltage is needed to write down the complete transient solution. In DC steady state the capacitors can be modeled as open circuits. Furthermore, R3||R4 can be replaced with an equivalent resistance. This resistance is in parallel with the independent current source and the two can be replaced with a Thevenin source transformation of an appropriate voltage source in series with the same resistance. Once this replacement is made it is a simple matter of voltage division to determine the capacitor voltage.

$$R_3 \| R_4 = 3 \| 6 = 2\ \Omega$$

The source transformation results in an 8V voltage source in series with this resistance. Then, by voltage division,

$$V_C(\infty) = 8 + \frac{4+2}{4+2+5}(20-8) = \frac{160}{11} \cong 14.55\, V$$

Now plug in to the general form of the transient solution to find

$$V_C(t) \cong 14.5\left[1 - e^{-11t/240}\right] \quad , \quad t \geq 0$$

d)

$V_C(0) = 0V;$ $\qquad V_C(\tau) = 9.17V;$

$V_C(2\tau) = 12.5V;$ $\qquad V_C(5\tau) = 14.4V;$

$V_C(10\tau) = 14.5V$

Problem 5.45

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\,\Omega, R_4 = 6\,\Omega, C_1 = 4F, C_2 = 4F, I_S = 4A.$$

Find:

a) The capacitor voltage $V_C(t)$ at $t = 0^+$.

b) The time constant τ for $0 \leq t \leq 48s$.

c) The expression for $V_C(t)$ valid for $0 \leq t \leq 48s$.

d) The time constant τ for $t > 48s$.

e) The expression for $V_C(t)$ valid for $t > 48s$.

f) Plot $V_C(t)$ for all time.

Assumptions:

Switch S_1 opens at $t = 0$; switch S_2 opens at $t = 48s$.

Analysis:

The approach here is to find the transient solution in the interval 0 < t < 48 seconds and use that solution to determine the initial condition (the capacitor voltage) for the new transient after the switch S_2 opens.

a) S_1 and S_2 have been closed for a long time and in DC steady state the capacitors can be replaced with open circuits. Thus, by node analysis and voltage division

$$\frac{V_S - V_C(0^-)}{R_1} = \frac{V_C(0^-)R_2}{R_2(R_2+R_3+R_4)} \quad \Rightarrow \quad V_C(0^-) = \frac{V_S(R_2+R_3+R_4)}{(R_1+R_2+R_3+R_4)}$$

$$V_C(0^+) = V_C(0^-) = \frac{20(4+3+6)}{(5+4+3+6)} = \frac{260}{18}V \cong 14.4V$$

b) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the independent sources (i.e. by replacing the current source with an open circuit) and computing (R_2 + R_3||R_4).

$$R_{TH} = R_2 + (R_3 \parallel R_4) = 4 + (3 \parallel 6) = 4 + 2 = 6\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 6 \cdot 8 = 48\,s$$

c) As mentioned above, to find the complete transient solution for t > 0 it is necessary to find the capacitor voltage when switch S_2 opens at t = 48 s. To do so it is first necessary to find the complete transient solution for when only the switch S_1 is open (i.e. as if the switch S_2 never opens.). The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_3. This voltage is found readily by current division. Thus,

$$V_C(\infty) = \frac{6\Omega}{3\Omega + 6\Omega}(4\,A)(3\,\Omega) = 8V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau} = 8 + (14.4 - 8)e^{-t/48} = 8 + 6.4e^{-t/48} \quad , \quad 0 \le t \le 48$$

At t = 48 s, the capacitor voltage is

$$V_C(t = 48^-) = 8 + 6.4e^{-1} = 10.35V$$

Continuity of voltage across the capacitors still holds so

$$V_C(48^+) = V_C(48^-) = 10.35V$$

d) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. When both the switches are opened, there are no independent sources connected to the circuit. Thus, the Thevenin equivalent resistance seen by the 8 F capacitance is found by computing (R_2 + R_3).

$$R_{TH} = R_2 + R_3 = 4 + 3 = 7\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 7 \cdot 8 = 56\,s$$

e) The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(t_0^{\,+}) - V(\infty)]e^{-(t-t_0)/\tau}$$

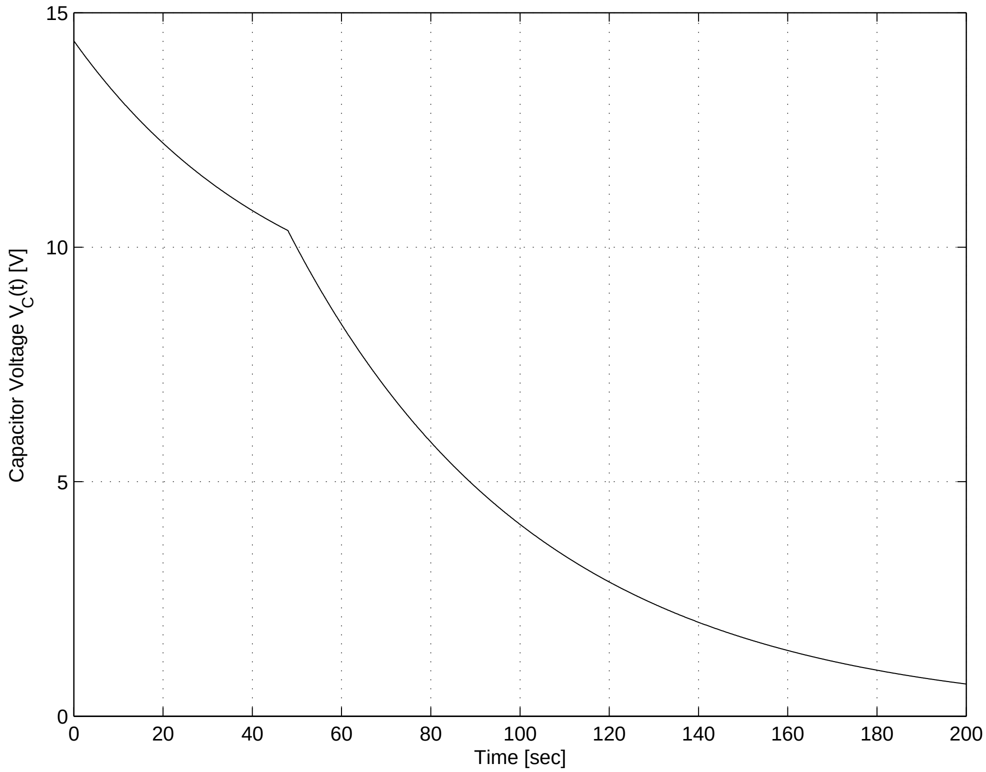
The long-term steady state capacitor voltage after the switch has been opened is zero since no independent sources are connected and all the initial energy in the circuit is eventually dissipated by the resistors. Thus,

$$V_C(\infty) = 0V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(48^+)e^{-t/\tau} = 10.35e^{-(t-48)/56} \quad , \quad t > 48$$

f) The plot of $V_C(t)$ for all time is shown in the following figure.



Problem 5.46

Solution:

Known quantities:
Circuit shown in Figure P5.41,

$V_S = 20V, R_1 = 5\,\Omega, R_2 = 4\,\Omega, R_3 = 3\,\Omega, R_4 = 6\,\Omega, C_1 = 4\,F, C_2 = 4\,F, I_S = 4\,A.$

Find:

a) The capacitor voltage $V_C(t)$ at $t = 0^+$.

b) The time constant τ for $0 \le t \le 96s$.

c) The expression for $V_C(t)$ valid for $0 \le t \le 96s$.

d) The time constant τ for $t > 96s$.

e) The expression for $V_C(t)$ valid for $t > 96s$.

f) Plot $V_C(t)$ for all time.

Assumptions:
Switch S_1 opens at $t = 96s$; switch S_2 opens at $t = 0$.

Analysis:

The approach here is to find the transient solution in the interval $0 < t < 96$ seconds and use that solution to determine the initial condition (the capacitor voltage) for the new transient after the switch S_1 opens.

a) S_1 and S_2 have been closed for a long time and in DC steady state the capacitors can be replaced with open circuits. Thus, by node analysis and voltage division

$$\frac{V_S - V_C(0^-)}{R_1} = \frac{V_C(0^-)R_2}{R_2(R_2+R_3+R_4)} \quad \Rightarrow \quad V_C(0^-) = \frac{V_S(R_2+R_3+R_4)}{(R_1+R_2+R_3+R_4)}$$

$$V_C(0^+) = V_C(0^-) = \frac{20(4+3+6)}{(5+4+3+6)} = \frac{260}{18}V \cong 14.4V$$

b) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the independent sources (i.e. by replacing the current source with an open circuit) and computing $R_1 \| (R_2 + R_3)$.

$$R_{TH} = R_1 \| (R_2 + R_3) = 5 \| (4+3) = 2.92\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 2.92 \cdot 8 = 23.3 s$$

c) As mentioned above, to find the complete transient solution for $t > 0$ it is necessary to find the capacitor voltage when switch S_1 opens at $t = 96$ s. To do so it is first necessary to find the complete transient solution for when only the switch S_2 is open (i.e. as if the switch S_1 never opens.). The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_2 and R_3. This voltage is found readily by voltage division. Thus,

$$V_C(\infty) = \frac{4\Omega + 3\Omega}{5\Omega + 4\Omega + 3\Omega}(20V) = 11.67V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau} = 11.67 + (14.4 - 11.67)e^{-t/23.3}$$

$$V_C(t) = 11.67 + 2.73e^{-t/23.3} \quad , \quad 0 \le t \le 96$$

At $t = 96$ s, the capacitor voltage is

$$V_C(t = 96^-) = 11.67 + 2.73e^{-96/23.3} = 11.71V$$

Continuity of voltage across the capacitors still holds so

$$V_C(96^+) = V_C(96^-) = 11.71V$$

d) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. When both the switches are opened, there are no independent sources connected to the circuit. Thus, the Thevenin equivalent resistance seen by the 8 F capacitance is found by computing ($R_2 + R_3$).

$$R_{TH} = R_2 + R_3 = 4 + 3 = 7\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 7 \cdot 8 = 56 s$$

e) The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(t_0^{\ +}) - V(\infty)]e^{-(t-t_0)/\tau}$$

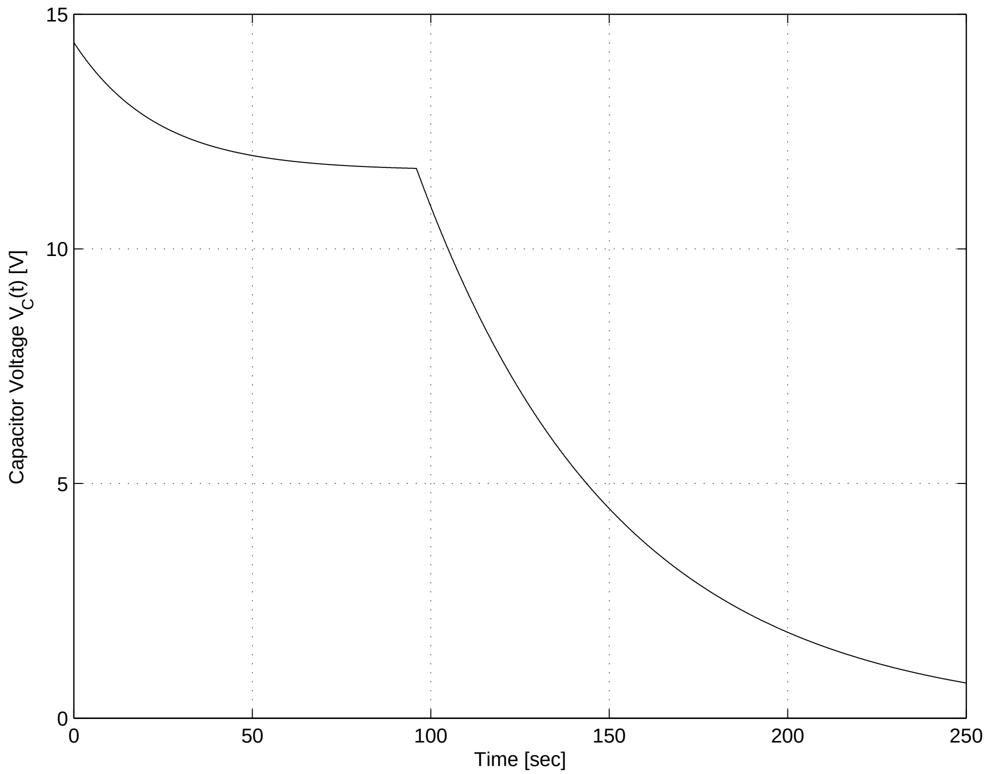
The long-term steady state capacitor voltage after the switch has been opened is zero since no independent sources are connected and all the initial energy in the circuit is eventually dissipated by the resistors. Thus,

$V_C(\infty) = 0V$

Plug in to the generalized solution given above to find

$V_C(t) = V(96^+)e^{-t/\tau} = 11.71e^{-(t-96)/56} \quad , \quad t > 96$

f) The plot of $V_C(t)$ for all time is shown in the following figure.



Problem 5.47

Solution:

Known quantities:

$R_S = 15\,k\Omega$, $\tau = 1.5$ ms, $\tau' = 10$ ms, $R_3 = 30\,k\Omega$, $C = 1\,\mu F$.

Find:

The value of resistors R_1 and R_2 .

Assumptions:

None.

Analysis:

Before the switch opens:

$$R_{eq} = R_S // R_1 // R_2 // R_3$$

$$\tau = R_{eq}C = 1.5\,ms$$

After the switch opens:

$$R_{eq}^{'} = R_S // R_1$$

$$\tau^{'} = R_{eq}^{'} C = 10\,ms$$

Solving the system of equations we have,

$$R_1 = \frac{R_S \tau'}{R_S C - \tau'} = \frac{15000 \cdot 0.01}{15000 \cdot 10^{-6} - 0.01} = 30k\Omega$$

$$R_2 = \left(\frac{C}{\tau} - \frac{1}{R_S} - \frac{1}{R_1} - \frac{1}{R_3} \right)^{-1} = \left(\frac{10^{-6}}{0.0015} - \frac{1}{15000} - \frac{1}{30000} - \frac{1}{30000} \right)^{-1} = 1875\Omega$$

Problem 5.48

Solution:

Known quantities:

Circuit shown in Figure P5.47, $V_S = 100V, R_S = 4k\Omega,\ R_1 = 2k\Omega, R_2 = R_3 = 6k\Omega, C = 1\mu F.$

Find:

The value of the voltage across the capacitor after $t = 2.666$ ms.

Assumptions:

None.

Analysis:

Before opening, the switch has been closed for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistance R_1. Thus,

$$V_C(t_0^-) = V_C(t_0^+) = \frac{R_S \parallel R_1 \parallel R_2 \parallel R_3}{R_S} V_S = \frac{300}{13} V \cong 23.077V$$

After the switch opens, the time constant of the circuit is

$$R_{eq} = R_S // R_1 = \frac{4000}{3}\Omega$$

$$\tau = R_{eq} C = \frac{1}{750} ms \cong 1.3ms$$

the generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(t_0^+) - V(\infty)]e^{-(t-t_0)/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_1. This voltage is found readily by voltage division. Thus,

$$V_C(\infty) = \frac{R_1}{R_S + R_1} V_S = \frac{2000\,\Omega}{4000\,\Omega + 2000\,\Omega} 20V = \frac{20}{3} V \cong 6.67V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(t_0^+) - V(\infty)]e^{-(t-t_0)/\tau} = \frac{20}{3} + \left(\frac{300}{13} - \frac{20}{3} \right) e^{-750(t-t_0)} = \frac{20}{3} + \frac{640}{39} e^{-750(t-t_0)}$$

Finally,

$$V_C(t_0 + 2.666ms) = \frac{20}{3} + \frac{640}{39} e^{-750(0.002666)} = 8.888V$$

Problem 5.49

Solution:

Known quantities:

As described in Figure P5.49.

Find:

The time at which the current through the inductor is equal to 5 A, and the expression for $i_L(t)$ for $t \geq 0$.

Assumptions:

None.

Analysis:

At $t < 0$:

Using the current divider rule:

$$i_L(0^-) = (\frac{100}{1000 + 5 // 2.5})(\frac{5}{5 + 2.5}) = 66.5\,mA$$

At $t > 0$:

Using the current divider rule:

$$i_L(\infty) = (\frac{100}{10 + 5 // 2.5})(\frac{5}{5 + 2.5}) = 5.71\,A$$

To find the time constant for the circuit we must find the Thevenin resistance seen by the inductor:

$$R_{eq} = 10 // 5 + 2.5 = 5.83\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.1}{5.83} = 17.1\,ms$$

Finally, we can write the solution:

$$i_L(t) = i_L(\infty) - (i_L(\infty) - i_L(0))e^{-t/\tau}$$

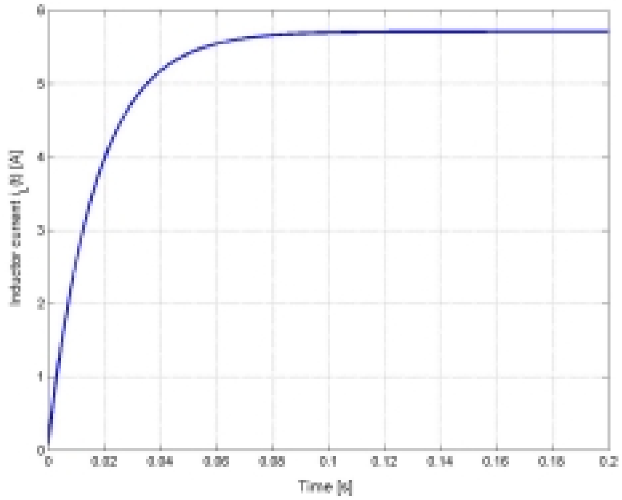
$$= 5.71 - (5.71 - 0.0665)e^{-t/17.1\times10^{-3}}$$

$$= 5.71 - 5.64e^{-t/17.1\times10^{-3}}\ A$$

Solving the equation we have,

$$i_L(\hat{t}) = 5.71 - 5.64e^{-\hat{t}/17.1\times10^{-3}} = 5 \quad \Rightarrow \quad \hat{t} = 35.437ms$$

The plot of $i_L(t)$ for all time is shown in the following figure.



Problem 5.50

Solution:

Known quantities:

As described in Figure P5.49.

Find:

The expression for $i_L(t)$ for $0 \le t \le 5ms$. The maximum voltage between the contacts during the 5-ms duration of the switch.

Assumptions:

The mechanical switching action requires 5 ms.

Analysis:

a) At $t < 0$:

Using the current divider rule:

$$i_L(0^-) = (\frac{100}{1000 + 5 // 2.5})(\frac{5}{5 + 2.5}) = 66.5\, mA$$

For $0 \le t \le 5ms$:

The long term steady state inductor current after the switch has been opened is zero since no independent source is connected to the circuit and all the initial energy in the circuit is eventually dissipated by the resistors. Thus,

$$i_L(\infty) = 0\, A$$

To find the time constant for the circuit we must find the Thevenin resistance seen by the inductor:

$$R_{eq} = 5 + 2.5 = 7.5\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.1}{7.5} = 13.33\, ms$$

Finally, we can write the solution:

$$i_L(t) = i_L(0) e^{-t/\tau} = (0.0665) e^{-t/13.33\times 10^{-3}}\ A \qquad (0 \le t \le 5ms)$$

b) The voltage between the contacts during the 5-ms duration of the switching is equal to:

$$V_{cont}(t) = V_S - V_{5\Omega} = 100 - \left(V_L + 2.5 i_L\right)$$

where,

$$V_L(t) = L\frac{di_L(t)}{dt} = -\frac{(0.1)(0.0665)}{13.33\times10^{-3}} e^{-t/13.33\times10^{-3}} = (-0.5)e^{-t/13.33\times10^{-3}} V$$

Therefore,

$$V_{cont}(t) = 100 - [(2.5)(0.0665) - 0.5]e^{-t/13.33\times10^{-3}} = 100 + (0.33)e^{-t/13.33\times10^{-3}} V$$

Thus, the maximum voltage between the contacts during the 5-ms duration of the switch is:

$$V_{cont}^{MAX} = V_{cont}(t=0) = 100.33V$$

Problem 5.51

Solution:

Known quantities:

As described in Figure P5.51. The switch closes when the voltage across the capacitor voltage reaches v_M^C; The switch opens when the voltage across the capacitor voltage reaches $v_M^O = 1V$. The period of the capacitor voltage waveform is 200 ms.

Find:

The voltage v_M^C.

Assumptions:

The initial capacitor voltage is $1V$ and the switch has just opened.

Analysis:

With the switch open:

$$V_C(\infty) = 10V$$

$$\tau = RC = 0.15\,s$$

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(0)]e^{-t/\tau}$$

$$= 10 - (10-1)e^{-t/0.15}$$

$$= 10 - 9e^{-t/0.15} V$$

Now we must determine the time when $V_C(t) = v_M^C$.

Using the expression for the capacitor voltage:

$$v_M^C = 10 - 9e^{-t_0/0.15} \quad \Rightarrow \quad e^{-t_0/0.15} = \frac{10 - v_M^C}{9} \quad \Rightarrow \quad t_0 = -0.15\ln\left(\frac{10 - v_M^C}{9}\right)$$

With the switch closed, the capacitor sees the Thevenin equivalent defined by:

$$V_{eq} = V_C(\infty) = \frac{10}{10 + 10000} \times 10 \approx 1\times10^{-2} V \quad \text{(voltage division)}$$

$$R_{eq} = 10k\Omega \| 10\Omega \cong 10\Omega$$

$$\tau = R_{eq}C = 0.15\,ms$$

The initial value of this part of the transient is v_M^C at $t = t_0$. With these values we can write the expression for the capacitor voltage:

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(t_0)]e^{-(t-t_0)/\tau}$$
$$= 0.01 + \left(v_M^C - 0.01\right)e^{-(t-t_0)/0.15\times 10^{-3}} \; V$$

The end of one full cycle of the waveform across the 10Ω resistor occurs when the second transient reaches $v_M^O = 1V$. If we call the time at which this event occurs t_1, then:

$$V_C(t) = 1V \qquad at\, t = t_1 = 200ms$$

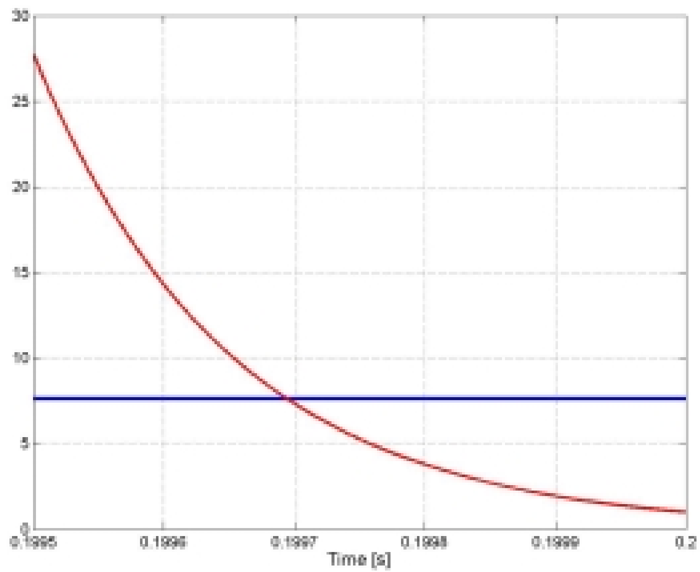
and so

$$1 = 0.01 + \left(v_M^C - 0.01\right)e^{-(t_1-t_0)/0.15\times 10^{-3}}$$

Graphically, the solution is the intersection between the following function:

$$v_M^C = 10 - 9e^{-t_0/0.15} \quad \text{(blue line)}$$

$$v_M^C = 0.01 + \frac{(1-0.01)}{e^{-(0.2-t_0)/0.15\times 10^{-3}}} \quad \text{(red line)}$$



which correspond to $v_M^C = 7.627$ V and $t_0 \cong 0.1997$ ms.

Problem 5.52

Solution:

Known quantities:

As describes in Figure P5.52. At $t = 0$, the switch closes.

Find:

a) $i_L(t)$ for $t \geq 0$.

b) $V_{L1}(t)$ for $t \geq 0$.

Assumptions:

$i_L(0) = 0\,A$.

Analysis:

a) In the long-term DC steady state after the switch is closed the inductors may be modeled as short circuits and so all of the current from the source will travel through the inductors.

$$i_L(\infty) = 5\,A$$

With the current source suppressed (treated as an open circuit) the Thevenin equivalent resistance seen by the inductors in series and the associated time constant are

$$R_{eq} = 10\,k\Omega$$

$$L_{eq} = L_1 + L_2 = 6\,H$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 0.6\,ms$$

$$i_L(t) = i_L(\infty)\left[1 - e^{-t/\tau}\right]$$

$$= 5\left[1 - e^{-t/0.6\times10^{-3}}\right]A$$

b) The voltage across either of the inductors is derived directly from the differential relationship between current and voltage for an inductor.

$$V_{L_1}(t) = L_1\frac{di_L(t)}{dt}$$

$$= (1)(5)\frac{d}{dt}(1 - e^{-t/0.6\times10^{-3}})$$

$$= 5\left(\frac{1}{0.6\times10^{-3}}e^{-t/0.6\times10^{-3}}\right)$$

$$= 8.333e^{-t/0.6\times10^{-3}}\,kV$$

Problem 5.53

Solution:

Known quantities:

As describes in Figure P5.52. At $t = 0$, the switch closes.

Find:

The voltage across the 10-$k\Omega$ resistor in parallel with the switch for $t \geq 0$.

Assumptions:

None.

Analysis:

When the switch closes at $t = 0$, the 10-$k\Omega$ resistor is in parallel with a short circuit, so its voltage is equal to zero for all time ($t \geq 0$).

Section 5.5: Transient Response of Second-Order Circuits

Focus on Methodology – roots of second order systems

Case 1: **Real and distinct roots**. This case occurs when ζ>1, since the term under the square root is positive in this case, and the roots are: $s_{1,2} \quad {}_n \quad {}_n\sqrt{\ \ ^2 \ \ 1}$. This leads to an **overdamped response**.

Case 2: **Real and repeated roots.** This case holds when ζ=1, since the term under the square root is zero in this case, and $s_{1,2} \quad {}_n \quad {}_n$. This leads to a **critically damped response**.

Case 3: **Complex conjugate roots.** This case holds when ζ<1, since the term under the square root is negative in this case, and $s_{1,2} \quad {}_n \quad j \ {}_n\sqrt{1 \quad ^2}$. This leads to an **underdamped response**.

Focus on Methodology

Second-order transient response

1. Solve for the steady-state response of the circuit before the switch changes state ($t = 0^-$), and after the transient has died out ($t \to \infty$). We shall generally refer to these responses as $x(0^-)$ and $x(\infty)$.
2. Identify the initial conditions for the circuit, $x(0^+)$, and $\dot{x}(0^+)$ using continuity of capacitor voltages and inductor currents ($v_C(0^+) = v_C(0^-)$, $i_L(0^+) = i_L(0^-)$), and circuit analysis. This will be illustrated by examples.
3. Write the differential equation of the circuit for $t = 0^+$, that is, immediately after the switch has changed position. The variable $x(t)$ in the differential equation will be either a capacitor voltage, $v_C(t)$, or an inductor current, $i_L(t)$. Reduce this equation to standard form (Equation 5.9, or 5.48).
4. Solve for the parameters of the second-order circuit: ω_n and ζ.
5. Write the complete solution for the circuit in one of the three forms given below, as appropriate:

Overdamped case (ζ> 1):

$$x(t) \quad x_N(t) \quad x_F(t) \qquad {}_1e^{\ {}_n \ \ {}_n\sqrt{\ ^2 \ 1}\ t} \qquad {}_2e^{\ {}_n \ \ {}_n\sqrt{\ ^2 \ 1}\ t} \qquad x(\ \ ) \quad t \quad 0$$

Critically damped case (ζ= 1):

$$x(t) \quad x_N(t) \quad x_F(t) \qquad {}_1e^{\ {}_n\ t} \qquad {}_2te^{\ {}_n\ t} \qquad x(\ \ ) \quad t \quad 0$$

Underdamped case (ζ= 1):

$$x(t) \quad x_N(t) \quad x_F(t) \qquad {}_1e^{\ {}_n \ j \ {}_n\sqrt{1 \ \ ^2}\ t} \qquad {}_2e^{\ {}_n \ j \ {}_n\sqrt{1 \ \ ^2}\ t} \qquad x(\ \ ) \quad t \quad 0$$

6. Apply the initial conditions to solve for the constants α_1 and α_2.

Problem 5.54

Solution:

Known quantities:
Circuit shown in Figure P5.54,

$V_{S1} = 15\,V, V_{S2} = 9\,V, R_{S1} = 130\,\Omega, R_{S2} = 290\,\Omega, R_1 = 1.1\,k\Omega, R_2 = 700\,\Omega, L = 17\,mH, C = 0.35\,\mu F.$

Find:
The voltage across the capacitor and the current through the inductor and R_{S2} as t approaches infinity.

Assumptions:
The circuit is in DC steady-state conditions for $t < 0$.

Analysis:
The conditions that exist at $t < 0$ have no effect on the long-term DC steady state conditions at $t \to \infty$. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. In this case, the inductor short circuits the R_1 branch and the R_2 C branch. Thus, the voltage across these branches and the current through them are zero. In other words all of the current produced by the 9V source travels through the inductor in this case.

$$i_L(\infty) = \frac{V_{S2}}{R_{S2}} = \frac{9}{290} = 31.03\,mA$$

Of course, this current is also the current traveling through the 290 Ω resistor.

$$i_{RS2}(\infty) = i_L(\infty) = 31.03\,mA$$

And since the voltage across the inductor in the long-term DC steady state is zero (short circuit)

$$0 + V_C(\infty) + i_{R2}(\infty)R_2 = 0$$

$$V_C(\infty) = 0$$

Problem 5.55

Solution:

Known quantities:
Circuit shown in Figure P5.54,

$V_{S1} = 12\,V, V_{S2} = 12\,V, R_{S1} = 50\,\Omega, R_{S2} = 50\,\Omega, R_1 = 2.2\,k\Omega, R_2 = 600\,\Omega, L = 7.8\,mH, C = 68\,\mu F.$

Find:
The voltage across the capacitor and the current through the inductor as t approaches infinity.

Assumptions:
The circuit is in DC steady-state conditions for $t < 0$.

Analysis:
The conditions that exist at $t < 0$ have no effect on the long-term DC steady state conditions at $t \to \infty$. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. In this case, the inductor short circuits the R_1 branch and the R_2 C branch. Thus, the voltage across these branches and the current through them are zero. In other words all of the current produced by the 12V source travels through the inductor in this case.

$$i_L(\infty) = \frac{V_{S2}}{R_{S2}} = \frac{12}{50} = 240\,mA$$

Of course, this current is also the current traveling through the 290 Ω resistor.

$$i_{RS2}(\infty) = i_L(\infty) = 240\,mA$$

And since the voltage across the inductor in the long-term DC steady state is zero (short circuit)

$$0 + V_C(\infty) + i_{R2}(\infty)R_2 = 0$$
$$V_C(\infty) = 0$$

Problem 5.56

Solution:

Known quantities:
Circuit shown in Figure P5.56,

$V_S = 170\,V, R_S = 7\,k\Omega, R_1 = 2.3\,k\Omega, R_2 = 7\,K\Omega, L = 30\,mH, C = 130\,\mu F.$

Find:

The current through the inductor and the voltage across the capacitor and R_1 at steady state.

Assumptions:
None.

Analysis:
As $t \rightarrow \infty$, the circuit will return to DC steady state conditions. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. Therefore, in this case, the current through R_2 is zero and thus the voltage across the capacitor must be equal to the voltage across R_1. Furthermore, the current through the inductor and R_1 is simply $V_S/(R_S + R_1)$.

$$i_C(\infty) = 0 \qquad i_L(\infty) = i_{R1}(\infty) = \frac{V_S}{R_S + R_1} = \frac{170}{7000 + 2300} \cong 18.3\,mA$$
$$V_{R1}(\infty) = i_L(\infty)R_1 = 18.28\times10^{-3}\times2.3\times10^{3} = 42.04V$$

and

$$V_C(\infty) = V_{R1}(\infty) = 42.04V$$

Problem 5.57

Solution:

Known quantities:
Circuit shown in Figure P5.57,

$V_S = 12\,V, C = 130\,\mu F, R_1 = 2.3\,k\Omega, R_2 = 7\,K\Omega, L = 30\,mH.$

Find:

The current through the inductor and the voltage across the capacitor and R_1 at steady state.

Assumptions:
None.

Analysis:
As $t \rightarrow \infty$, the circuit will return to DC steady state conditions (practically after about 5 time constants.) In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. Therefore, in this case, the voltage across the capacitor must be equal to the voltage across R_2. Furthermore, the current through the inductor and R_2 is simply $V_S/(R_1 + R_2)$.

$$i_C(\infty) = 0 \qquad V_L(\infty) = 0$$
$$i_L(\infty) = i_{R2}(\infty) = \frac{V_S}{R_1 + R_2} = \frac{12}{9.3\times10^{3}} = 1.29\,mA$$
$$V_{R1}(\infty) = i_S(\infty)R_1 = 1.29\times10^{-3}\times2.3\times10^{3} = 2.968V$$

And by observation

$$V_C(\infty) = V_{R_2}(\infty) = (1.29\times10^{-3})(7\times10^{3}) = 9.03V$$

All answers are positive indicating that the directions of the currents and polarities of the voltages assumed initially are correct. (You did do that, didn't you?)

Problem 5.58

Solution:

Known quantities:
Circuit shown in Figure P5.58,

$V_S = 12\,V, C = 0.5\,\mu F,\ R_1 = 31\,k\Omega, R_2 = 22\,K\Omega, L = 0.9\,mH.$

Find:

The current through the inductor and the voltage across the capacitor at steady state.

Assumptions:
None.

Analysis:
As $t \to \infty$, the circuit will return to DC steady state conditions (practically after about 5 time constants). In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. Therefore, in this case, the voltage across the capacitor must be equal to the voltage across R_2. Furthermore, the current through the inductor and R_2 is simply $V_S/(R_1 + R_2)$.

$$i_C(\infty) = 0 \qquad V_L(\infty) = 0$$

$$i_L(\infty) = i_{R2}(\infty) = \frac{V_S}{R_1 + R_2} = \frac{12}{(31+22)\times10^{3}} \cong 226\ \mu A$$

$$V_{R_2}(\infty) = i_{R_2}(\infty)R_2 = (226\times10^{-3})(22\times10^{3}) \cong 4.98\ V$$

And by observation

$$V_C(\infty) = V_{R_2}(\infty) = 4.98\ V$$

Theoretically, when the switch is OPENED, the current through the inductor must continue to flow, at least momentarily. However, the inductor is in series with an OPEN switch through which current CANNOT flow. What the theory does not predict is that a very large voltage is developed across the gap and this causes an arc with a current (kind of like a teeny, weeny lightning bolt). The energy stored in the magnetic field of the inductor is rapidly dissipated in the arc. The same effect will be important later when discussing transistors as switches.

Problem 5.59

Solution:

Known quantities:
Circuit shown in Figure P5.59,

$V_S = 12\,V, C = 3300\,\mu F, R_1 = 9.1\,k\Omega, R_2 = 4.3\,k\Omega, R_1 = 4.3\,k\Omega, L = 16\,mH,.$

Find:

The initial voltage across R_2 just after the switch is changed.

Assumptions:
At $t < 0$ the circuit is at steady state and the voltage across the capacitor is $+7V$.

Analysis:
It is important to remember that only the values of the capacitor voltage and the inductor current are guaranteed continuity from immediately before the switch is thrown to immediately afterward. Therefore, to determine the initial voltage across R_2 it is necessary to first determine the initial voltage across the capacitor and the initial current through the inductor. Assume that before the switch was thrown DC steady state conditions existed. In DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. The initial voltage across the capacitor is given as +7V. The initial current through the inductor is equal to the current through R_3, which is given by Ohm's Law.

$$i_L(0^+) = i_L(0^-) = \frac{V_S}{R_3} = \frac{12}{4.3\times10^3} = 2.791\,mA$$

and

$$V_C(0^+) = V_C(0^-) = 7V$$

Apply KCL:

$$\frac{V_{R2}(0^+) - V_C(0^+)}{R_1} + \frac{V_{R2}(0^+)}{R_2} + i_L(0^+) = 0$$

$$V_{R2}(0^+) = \frac{\dfrac{V_C(0^+)}{R_1} - i_L(0^+)}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{(V_C(0^+) - i_L(0^+)R_1)R_2}{R_2 + R_1}$$

$$= \frac{(7 - 2.791\times10^{-3}\times9.1\times10^3)\times4.3\times10^3}{4.3\times10^3 + 9.1\times10^3} = -5.93V$$

One could also solve for V_{R2} by superposition.

$$V_{R2}(0^+) = \frac{R_2}{R_1 + R_2}(7V) - \frac{R_1R_2}{R_1 + R_2}(2.8mA) = -5.93\,V$$

Problem 5.60

Solution:

Known quantities:
Circuit shown in Figure P5.60,

$V_{S1} = 15\,V, V_{S2} = 9\,V, R_{S1} = 130\,\Omega, R_{S2} = 290\,\Omega, R_1 = 1.1\,k\Omega, R_2 = 700\,\Omega, L = 17\,mH, C = 0.35\,\mu F.$

Find:

The current through and the voltage across the inductor and the capacitor and the current through R_{S2} at $t = 0^+$.

Assumptions:

The circuit is in DC steady-state conditions for $t < 0$.

Analysis:

Since this was not done in the specifications above, you must note on the circuit the assumed polarities of voltages and directions of currents.

At $t = 0^-$:

Assume that steady state conditions exist. At steady state the inductor is modeled as a short circuit and the capacitor as an open circuit. Choose a ground. Note that because the inductor is modeled as a short circuit, there is no voltage drop from the top node to the bottom node and so before the switch there is no current through R_1.

$$V_L\left(0^-\right) = 0 \qquad i_C\left(0^-\right) = 0$$

Apply KCL:

$$\frac{0 - V_{S1}}{R_{S1}} + i_L(0^-) + \frac{0}{R_1} + 0 + \frac{0 - V_{S2}}{R_{S2}} = 0$$

$$i_L(0^-) = \frac{V_{S1}}{R_{S1}} + \frac{V_{S2}}{R_{S2}} =$$

$$= \frac{15}{130} + \frac{9}{290} = 146.4\,mA$$

Apply KVL:

$$V_C\left(0^-\right) + i_C\left(0^-\right)R_2 = 0$$

$$V_C\left(0^-\right) = 0$$

At $t = 0^+$:

$$i_L\left(0^+\right) = i_L\left(0^-\right) = 146.4\,mA$$

$$V_C\left(0^+\right) = V_C\left(0^-\right) = 0$$

Apply KVL:

$$-V_L(0^+) + 0 + i_C(0^+)R_2 = 0$$

$$i_C(0^+) = \frac{V_L(0^+)}{R_2}$$

Apply KCL:

$$i_L(0^+) + \frac{V_L(0^+)}{R_1} + \frac{V_L(0^+)}{R_2} + \frac{V_L(0^+) - V_{S2}}{R_{S2}} = 0$$

$$V_L(0^+) = \frac{\frac{V_{S2}}{R_{S2}} - i_L(0^+)}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{S2}}} = \frac{V_{S2} - i_L(0^+)R_{S2}}{\frac{R_{S2}}{R_1} + \frac{R_{S2}}{R_2} + 1}$$

$$= \frac{9 - 146.4\times10^{-3}\times0.29\times10^3}{\frac{0.29}{1.1} + \frac{0.29}{0.7} + 1} = -19.94V$$

$$i_C(0^+) = \frac{V_L(0^+)}{R_2} = \frac{-19.94}{0.7\times10^3} = -28.49\,mA$$

Apply KVL again:

$$-V_L(0^+) + i_{RS2}(0^+)R_{S2} + V_{S2} = 0$$

$$i_{RS2}(0^+) = \frac{V_L(0^+) - V_{S2}}{R_{S2}} = \frac{-19.94 - 9}{0.29\times10^3} = -99.79\,mA$$

Problem 5.61

Solution:

Known quantities:

Circuit shown in Figure P5.60,

$$V_{S1} = 12\,V, V_{S2} = 12\,V, R_{S1} = 50\,\Omega, R_{S2} = 50\,\Omega, R_1 = 2.2\,k\Omega, R_2 = 600\,\Omega, L = 7.8\,mH, C = 68\,\mu F.$$

Find:

The voltage across the capacitor and the current through the inductor as t approaches infinity.

Assumptions:

The circuit is in DC steady-state conditions for $t < 0$.

Analysis:

The conditions that exist at $t < 0$ have no effect on steady state conditions as $t \to \infty$. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. In this case, the inductor short circuits the R_1 branch and the R_2 C branch. Thus, the voltage across these branches and the current through them are zero. In other words all of the current produced by the 12V source travels through the inductor in this case.

Apply KCL;

$$i_L(\infty) + i_{R1}(\infty) + i_{R2}(\infty) + \frac{0 - V_{S2}}{R_{S2}} = 0$$

$$i_{R1}(\infty) = i_{R2}(\infty) = 0$$

$$i_L(\infty) = \frac{V_{S2}}{R_{S2}} = \frac{12}{50} = 240\,mA$$

Apply KVL:

$$0 + V_C(\infty) + i_{R2}(\infty)R_2 = 0$$
$$V_C(\infty) = 0$$

Problem 5.62

Solution:

Known quantities:

As described in Figure P5.62.

Find:

An expression for the inductor current for $t \geq 0$.

Assumptions:

The switch has been closed for a long time. It is suddenly opened at $t = 0$ and then reclosed at $t = 5s$

Analysis:

For $0 \leq t \leq 5$:

Define clockwise mesh currents, $t = 0$ in the lower loop, and $t = 0$ in the upper loop. Then the mesh equations are:

$$(5 + 5s)I_1 - 3I_2 = 0$$
$$-3I_1 + (3 + \frac{1}{4s})I_2 = 0$$

from which we determine that

$$(5 + 5s)(3 + \frac{1}{4s}) - 9 = 0$$
$$s = 0.242 \pm j0.158$$

Therefore, the inductor current is of the form:

$$i(t) = e^{-0.242\,t}[A\cos(0.158\,t) + B\sin(0.159\,t)]$$

From the initial conditions:

$$i(0) = \frac{6}{3} = 2 = A$$
$$L\frac{di}{dt}\bigg|_{t=0} = 5\frac{di}{dt}\bigg|_{t=0} = V_C(0^+) = -10$$
$$\Rightarrow \frac{di}{dt}\bigg|_{t=0} = -2 = -0.242A + 0.158B$$

Solving the above equations:

$$A = 2 \qquad B = -9.59$$
$$i(t) = e^{-0.242t}[2\cos(0.158t) - 9.59\sin(0.158t)]\ A \qquad for\, 0 \leq t \leq 5s$$

The solution for capacitor voltage will have the same form.

$$V_C(t) = e^{-0.242t}[A\cos(0.158t) + B\sin(0.158t)]V$$

From the initial conditions:

$$V_C(0) = 6 = A$$
$$\frac{dV_C}{dt}\bigg|_{t=0} = \frac{1}{C}i_C(0) = 0 \Rightarrow -0.242A + 0.158B = 0$$

Solving the above equations:

$$A = 6 \qquad B = 9.18$$

$$V_C(t) = e^{-0.242t}[6\cos(0.158t) + 9.18\sin(0.158t)]V \qquad for\, 0 \le t \le 5\,s$$

From the above results:

$$V_C(5) = 3.2807V$$

$$i(5) = -1.641\,A$$

These are the initial conditions for the solution after the switch recloses.

For $t \ge 5$:

The mesh equations are:

$$(3 + 5s)I_1 - 3I_2 = \frac{6}{s} + 5i(5)$$

$$-3I_1 + (3 + \frac{1}{4s})I_2 = -\frac{V_C(5)}{s}$$

from which we determine that

$$60s^2 + 5s + 3 = 0$$

$$s = 0.041 \pm j0.220$$

Therefore, the inductor current is of the form:

$$i(t) = 2 + e^{-0.041t}\{A\cos[0.220(t-5)] + B\sin[0.220(t-5)]$$

From the initial conditions:

$$2 + A = -1.641 \Rightarrow A = -3.641$$

$$\frac{di}{dt}\Big|_{t=5} = \frac{V_L(5)}{5} = \frac{6-238}{5} = 0.543$$

$$\Rightarrow -0.41A + 0.220B = 0.543$$

Solving the above equations:

$$A = -3.641 \qquad B = 1.77$$

$$i(t) = 2 + e^{-0.041t}\{-3.641\cos[0.220(t-5) + 1.77\sin[0.220(t-5)]\}\,A \quad for\, t \ge 5\,s$$

This, together with the previous result, gives the complete solution to the problem.

Problem 5.63

Solution:

Known quantities:

As described in Figure P5.63.

Find:

Determine if the circuit is underdamped or overdamped. The capacitor value that results in critical dumping.

Assumptions:

The circuit initially stores no energy. The switch is closed at $t = 0$.

Analysis:

a) For $t \ge 0$:

The characteristic polynomial is:

$$Ls^2 + Rs + \frac{1}{C} = 0$$

The damping ratio is:

$$\xi = \frac{RC}{2}\sqrt{\frac{1}{LC}} = \frac{400 \cdot 10^{-8}}{2}\sqrt{10^{10}} = 0.2 < 1$$

The system is underdamped, in fact we have the following complex conjugate roots:

$$s_{1,2} = -2\times10^4 \pm j(9.79\times10^4)$$

b) The capacitor value that results in critical damping is:

$$\xi = \frac{RC}{2}\sqrt{\frac{1}{LC}} = 1 \quad \Rightarrow \quad 200C\sqrt{\frac{100}{C}} = 1$$

$$200^2 C^2 \frac{100}{C} = 1 \quad \Rightarrow \quad C = \frac{1}{4\times10^6} F = 0.25\mu F$$

Problem 5.64

Solution:

Known quantities:

As described in Figure P5.63.

Find:

a) The capacitor voltage as t approaches infinity
b) The capacitor voltage after 20 μs
c) The maximum capacitor voltage.

Assumptions:

The circuit initially stores no energy. The switch is closed at $t = 0$.

Analysis:

For $t \ge 0$:
The characteristic polynomial is:

$$0.01s^2 + 400s + 10^8 = 0$$

$$s = -2\times10^4 \pm j(9.79\times10^4)$$

Therefore, the solution is of the form:

$$V_C(t) = 10 + e^{-2\times10^4 t}[A\cos(9.79\times10^4 t) + B\sin(9.79\times10^4 t)]$$

From the initial conditions:

$$10 + A = 0$$

$$-2\times10^4 A + 9.79\times10^4 B = 0$$

Solving the above equations:

$$A = -10 \qquad B = -2.04$$

$$V_C(t) = 10 + e^{-2\times10^4 t}[-10\cos(9.79\times10^4 t) - 2.04\sin(9.79\times10^4 t)]V$$

a) The capacitor voltage as *t* approaches infinity is:

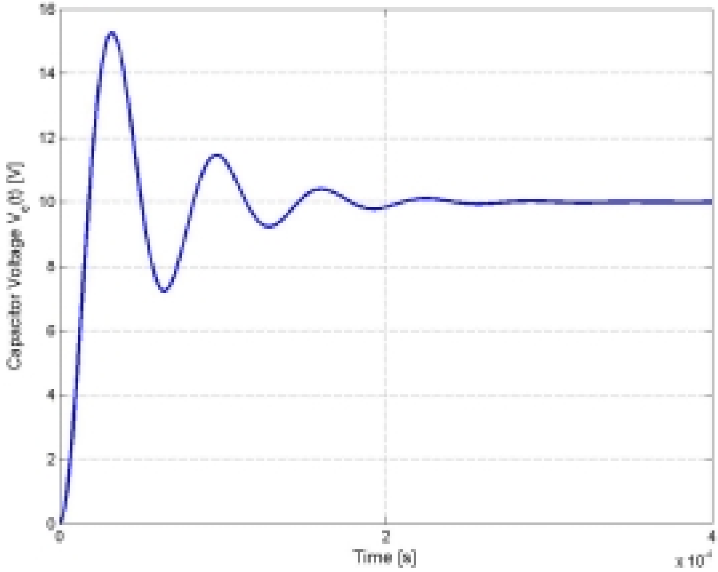
$$V_C(\infty) = 10V$$

b) The capacitor voltage after 20 μs is:

$$V_C(20\mu s) = 11.26V$$

c) Graphically, the maximum capacitor voltage is:

$$V_C^{\max} \cong 15V$$



Problem 5.65

Solution:

Known quantities:
As described in Figure P5.65.

Find:
An expression for the capacitor voltage for $t \geq 0$.

Assumptions:
The circuit initially stores no energy, the switch S_1 is open and the switch S_2 is closed. The switch S_1 is closed at $t = 0$ and the switch is opened S_2 at $t = 5\,s$.

Analysis:
This circuit has the same configuration during the interval $0 \leq t \leq 5\,s$ as the one for Problem 5.39 did for $t > 5\,s$. Therefore, the roots of the characteristic polynomial will be the same as those determined in that problem. They are:

$$s = -0.041 \pm j0.220$$

Ad the general form of the capacitor voltage is

$$V_C(t) = 6 + e^{-0.041t}[A\cos(0.220t) + B\sin(0.220t)]$$

For $0 \leq t \leq 5\,s$:
The initial conditions are:

$$V_C(0) = 0 \Rightarrow 6 + A = 0$$

$$\frac{dV_C}{dt}\Big|_{t=0} = \frac{1}{C} i_C(0) = 0$$

$$\Rightarrow -0.41A + 0.220B = 0$$

Solving the above equations:

$$A = -6 \qquad B = -0.904$$

$$V_C(t) = 6 + e^{-0.041t}[-6\cos(0.220t) - 0.904\sin(0.220t)]V \qquad for 0 \le t \le 5\,s$$

Note that $V_C(5) = 3.127V$.

For $t > 5\,s$:
We have a simple RC decay:

$$V_C(t) = 3.127e^{-\frac{t-5}{12}}\,V$$

Problem 5.66

Solution:

Known quantities:
Circuit shown in Figure P5.66, $C = 1.6\,nF$; After the switch is closed at $t = 0$, the capacitor voltage reaches an initial peak value of $70V$ when $t = {5\pi}/{3}\,\mu s$, a second peak value of $53.2V$ when $t = 5\pi\,\mu s$, and eventually approaches a steady-state of $50V$.

Find:
The values of R and L .

Assumptions:
The circuit is underdamped and the circuit initially stores no energy.

Analysis:
Using the given characteristics of the circuit step response and the assumption that the circuit is underdamped, the damping ratio and natural frequency can be determined as follows:

$$\zeta = \sqrt{\frac{1}{\left(\pi / \ln(a/A)\right)^2 + 1}}$$

where *a* is the overshoot distance and *A* is the steady-state value

$$\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}}$$

where *T* is the period of oscillation
In our case,

$$a = 70 - 50 = 20 \text{ and } A = 50$$

$$\zeta = \sqrt{\frac{1}{\left(\pi / \ln(20/50)\right)^2 + 1}} = 0.28$$

The period of the waveform is

$$T = \left(5\pi - \frac{5\pi}{3}\right)\times 10^{-6} = \frac{10\pi}{3}\times 10^{-6}$$

$$\omega_n = \frac{2\pi}{\frac{10\pi}{3}\times 10^{-6}\sqrt{1-0.28^2}} = 6.25\times 10^5$$

Implying the characteristic polynomial for the circuit is

$$s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}$$

Compare this with the standard form of the characteristic polynomial for a series RLC circuit:

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Matching terms yields $L = 1.6\,\mu H, R = 0.56\,\Omega$.

Problem 5.67

Solution:

Known quantities:

Same as P5.66, but the first two peaks occur at $5\pi\ \mu s$ and $15\pi\ \mu s$

Find:
Explain how to modify the circuit to meet the requirements.

Assumptions:
The capacitor value C cannot be changed.

Analysis:

Assuming we wish to retain the same peak amplitudes, we proceed as follows:
The new period is

$$T = 15\pi \times 10^{-6} - 5\pi \times 10^{-6} = 10\pi \times 10^{-6}$$

Using the given characteristics of the circuit step response and the assumption that the circuit is underdamped, the damping ratio and natural frequency can be determined as follows:

$$\zeta = \sqrt{\frac{1}{(\pi / \ln(a/A))^2 + 1}}$$

where *a* is the overshoot distance and *A* is the steady-state value

$$\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}}$$

where *T* is the period of oscillation

In our case,

$a = 70 - 50 = 20$ and $A = 50$

$$\zeta = \sqrt{\frac{1}{(\pi / \ln(20/50))^2 + 1}} = 0.28$$

$$\omega_n = \frac{2\pi}{10\pi \times 10^{-6}\sqrt{1-0.28^2}} = 2.17 \times 10^5$$

Implying the characteristic polynomial for the circuit is

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Compare this with the standard form of the characteristic polynomial for a series RLC circuit:

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Matching terms yields $L = 13.3\,\mu H, R = 1.61\,\Omega$.

Note that the frequency for this problem is one-third that of Problem 5.66, the inductance is 3^2 times that of Problem 5.66, and the resistance is 3 times that of Problem 5.66.

Problem 5.68

Solution:

Known quantities:
Circuit shown in Figure P5.68, $i(0) = 0\,A, V(0) = 10V$.

Find:
$i(t)$ for $t > 0$.

Assumptions:
None.

Analysis:
The initial condition for the capacitor voltage is $V(0^-) = 10V$. Applying KCL,

$$i_R + i_C + i = 0$$

where

$$i_R = \frac{V}{1\Omega}, \qquad i_C = 0.5\frac{dV}{dt}$$

Therefore,

$$i + V + 0.5\frac{dV}{dt} = 0$$

where

$$V = 2\frac{di}{dt} + 4i$$

Thus,

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 5i = 0$$

Solving the differential equation:

$$i(t) = k_1 e^{(-2+j)t} + k_2 e^{(-2-j)t} \qquad t > 0$$

$$i(0) = k_1 + k_2 = 0$$

$$V(0) = 2\frac{di(0)}{dt} + 4i(0) = -(4 - j2)k_1 - (4 + j2)k_2 = 10$$

Solving for k_1 and k_2 and substituting, we have

$$i(t) = -j\frac{5}{2}e^{(-2+j)t} + j\frac{5}{2}e^{(-2-j)t}\ A \qquad for\, t > 0$$

Problem 5.69

Solution:

Known quantities:

As described in Figure P5.69.

Find:

The maximum value of V.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

$$V(0^-) = V(0^+) = 0$$

Applying KVL:

$$\frac{d^2V}{dt^2} + 4\frac{dV}{dt} + 4V = 48$$

Solving the differential equation:

$$V = k_1 e^{-2t} + k_2 te^{-2t} + 12$$

From the initial condition:

$$V(0) = 0 \Rightarrow k_1 = -12$$

$$i_L(0) = C\frac{dV(0)}{dt} \Rightarrow 6 + \frac{k_2}{4} = 3 \Rightarrow k_2 = -12$$

$$V(t) = -12e^{-2t} - 12te^{-2t} + 12V \qquad for\, t > 0$$

The maximum value of V is:

$$V_{\max} = V(\infty) = 12V$$

Problem 5.70

Solution:

Known quantities:

As described in Figure P5.70.

Find:

The value of t such that $i = 2.5\,A$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

In steady state, the inductors behave as short circuits. Using mesh analysis, we can find the initial conditions.

$$i(0^-) = i(0^+) = 5A$$

$$V(0^-) = 0V$$

After the switch is closed, the circuit is modified.
Applying nodal analysis:

$$V(0^+) = 0V$$

$$\frac{d^2 i}{dt^2} + 7\frac{di}{dt} + 6i = 0$$

Solving the differential equation:

$$i(t) = k_1 e^{-t} + k_2 e^{-6t} \qquad for\, t > 0$$

$$i(0) = k_1 + k_2 = 5$$

$$V(0) = -k_1 - 6k_2 = 0$$

Solving for the unknown constants, using the initial conditions, we have:

$$i(t) = 6e^{-t} - e^{-6t}\ A \qquad for\, t > 0$$

Therefore,

$$i(\bar{t}) = 6e^{-\bar{t}} - e^{-6\bar{t}} = 2.5 \quad \Rightarrow \quad \bar{t}_1 = 873 ms$$

Problem 5.71

Solution:

Known quantities:

As described in Figure P5.71.

Find:

The value of t such that $i = 6A$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

In steady state, the inductors behave as short circuits. Using mesh analysis, we can find the initial conditions.

$$i(0^-) = i(0^+) = 12.5A$$

$$V(0^-) = 0V$$

After the switch is closed, the circuit is modified.
Applying nodal analysis:

$$V(0^+) = -15V$$

$$\frac{d^2 i}{dt^2} + 7\frac{di}{dt} + 6i = 0$$

Solving the differential equation:

$$i(t) = k_1 e^{-t} + k_2 e^{-6t} \qquad for\, t > 0$$

$$i(0) = k_1 + k_2 = 12.5$$

$$V(0) = -k_1 - 6k_2 = -15$$

Solving for the unknown constants, using the initial conditions, we have:

$$i(t) = 12e^{-t} + 0.5e^{-6t}\ A \qquad for\, t > 0$$

Therefore,

$$i(\bar{t}) = 12e^{-\bar{t}} + 0.5e^{-6\bar{t}} = 6 \quad \Rightarrow \quad \bar{t}_1 = 694 ms$$

Problem 5.72

Solution:

Known quantities:

As described in Figure P5.72.

Find:

The value of t such that $V = 7.5V$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

The circuit at $t = 0^-$ has the capacitors replaced by open circuits.
By current division:

$$i_{2\Omega} = \frac{3}{3+5} \times 20 = 7.5\,A$$

$$V(0^-) = V(0^+) = 2i_2 = 15V$$

$$i(0^-) = 0A$$

After the switch opens, apply KCL:

$$i(0^+) = 0A$$

$$i + i_1 + i_2 = 0$$

$$i = C\frac{dV}{dt} = \frac{1}{6}\frac{dV}{dt}, \qquad i_2 = \frac{V}{2}$$

$$i_1 = -\frac{1}{6}\frac{dV}{dt} - \frac{V}{2}$$

Applying KVL:

$$V = 3i_1 + V_1 = 3(-\frac{1}{6}\frac{dV}{dt} - \frac{V}{2}) + V_1$$

$$V_1 = V - 3(-\frac{1}{6}\frac{dV}{dt} - \frac{V}{2})$$

$$i_1 = \frac{1}{6}\frac{dV_1}{dt} = \frac{1}{12}\frac{d^2V}{dt^2} + \frac{5}{12}\frac{dV}{dt}$$

Applying KCL:

$$\frac{1}{12}\frac{d^2V}{dt^2} + \frac{5}{12}\frac{dV}{dt} + \frac{V}{2} + \frac{1}{6}\frac{dV}{dt} = 0$$

$$\Rightarrow \frac{d^2V}{dt^2} + 7\frac{dV}{dt} + 6V = 0$$

Solving the differential equation,

$$V(t) = k_1 e^{-t} + k_2 e^{-6t} \qquad for\, t > 0$$

$$V(0) = k_1 + k_2 = 15$$

$$i(0) = \frac{1}{6}\left(-k_1 - 6k_2\right) = 0$$

Solving for the unknown constants, using the initial conditions, we have:

$$V(t) = 18e^{-t} - 3e^{-6t}\,V \quad for\, t > 0$$

Therefore,

$$V(\bar{t}) = 18e^{-\bar{t}} - 3e^{-6\bar{t}} = 7.5 \quad \Rightarrow \quad \bar{t}_1 = 873ms$$

Problem 5.73

Solution:

Known quantities:

As described in Figure P5.73.

Find:

The maximum value of V and the maximum voltage between the contacts of the switches.

Assumptions:

The circuit is in steady state at $t = 0^-$. $L = 3$ H.

Analysis:

At $t = 0^-$:

$$i(0^-) = i(0^+) = \frac{10}{5} = 2\,A$$

$$V(0^-) = V(0^+) = 0V$$

After the switch is closed:
Applying KVL:

$$\frac{1}{L}\int_{-\infty}^{t} Vdt + \frac{1}{12}\frac{dV}{dt} + \frac{V}{3} = 0$$

$$\Rightarrow \frac{d^2V}{dt^2} + 4\frac{dV}{dt} + \frac{12}{L}V = 0$$

The particular response is zero for $t > 0$ because the circuit is source-free.

$$L = 3\,H \Rightarrow s^2 + 4s + 4 = 0$$

$$s_1 = s_2 = -2$$

$$V(t) = e^{-2t}(A + Bt) \qquad for\, t > 0$$

From the initial condition:

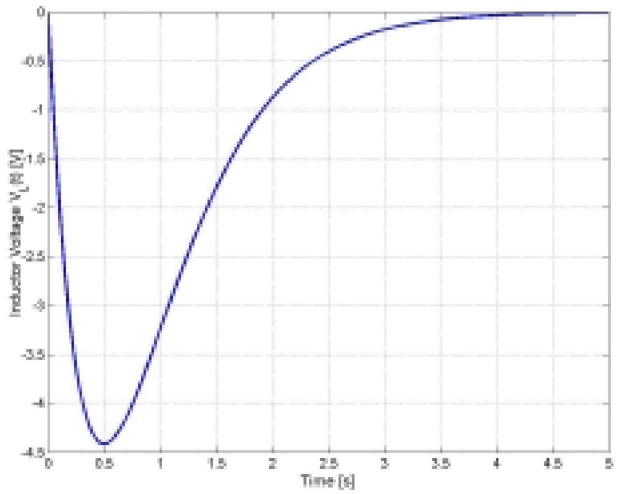
$$V(0^+) = 0 = e^0(A + B(0)) \Rightarrow A = 0$$

Substitute the solution into the original KCL equation and evaluate at $t = 0^+$:

$$\frac{1}{3}(0) + \frac{1}{12}\frac{d}{dt}[e^{-2t}(A + Bt)]|_{t=0} + 2 = 0$$

$$0 + \frac{1}{12}[Be^{-2t} - 2Bte^{-2t})]|_{t=0} + 2 = 0 \Rightarrow B = -24$$

$$V(t) = -24te^{-2t}\ \ V \qquad for\, t > 0$$



The maximum absolute value of V is:

$$V_{\max} = V(t = 0.5s) = \left| -\frac{12}{e} \right| V \cong 4.414V$$

The maximum voltage between the contacts of the switches is:

$$V_{switch}^{MAX} = V_S = 10V$$

since the voltage between the contacts of the switches is a constant.

Problem 5.74

Solution:

Known quantities:

As described in Figure P5.74.

Find:

V at $t > 0$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

At $t = 0^-$:

$$V(0^-) = 12V$$

$$i_L(0^-) = i_L(0^+) = 6\,A$$

$$i_C(0^-) = 0\,A$$

$$V_C(0^-) = V_C(0^+) = 4V$$

$$V_L(0^-) = 0V$$

For $t > 0$:

$$i_C(0^+) = -2A$$

$$V_L(0^+) = 4V$$

$$V(0^+) = 8V$$

$$\frac{dV}{dt}\left(0^+\right) = -\frac{dV_C}{dt}\left(0^+\right) = -\frac{i_C}{C} = -\frac{-2}{1/4} = 8$$

Using KVL and KCL, we can find the differential equation for the voltage in the resistor:

$$12 - 2i - 0.8\frac{di_L}{dt} = 0$$

$$i = i_L + i_C \Rightarrow i_L = i - i_C$$

$$i_C = C\frac{dV_C}{dt} = \frac{1}{4}\frac{d}{dt}\left(12 - V\right) = -\frac{1}{4}\frac{dV}{dt}$$

$$12 - 2i - 0.8\frac{d}{dt}\left(i - i_c\right) = 0 \Rightarrow 12 - 2i - 0.8\frac{di}{dt} + 0.8\frac{d}{dt}\left(-\frac{1}{4}\frac{dV}{dt}\right) = 0$$

$$i = V/2 \Rightarrow 12 - V - 0.4\frac{dV}{dt} - 0.2\frac{d^2V}{dt^2} = 0$$

$$0.2\frac{d^2V_{2\Omega}}{dt^2} + 0.4\frac{dV_{2\Omega}}{dt} + V_{2\Omega} = 12$$

$$\Rightarrow \frac{d^2V_{2\Omega}}{dt^2} + 2\frac{dV_{2\Omega}}{dt} + 5V_{2\Omega} = 60$$

Solving the differential equation:

Homogeneous Solution:

$$V_{2\Omega,h} = K_1e^{(-1+2j)t} + K_2e^{(-1-2j)t} \quad t > 0$$

$$V(0) = 8V, \frac{dV}{dt}\left(0\right) = 8$$

$$K_1 + K_2 = 8$$

$$\left(-1 + 2j\right)K_1 - \left(1 + 2j\right)K_2 = 8$$

$$K_1 = 4 - 4j$$

$$K_2 = 4 + 4j$$

$$V_{2\Omega,h} = \left(4 - 4j\right)e^{(-1+2j)t} + \left(4 + 4j\right)e^{(-1-2j)t} \qquad t > 0$$

Particular Solution:

$$V_{2\Omega,p} = \left(-6 + 3j\right)e^{(-1+2j)t} + \left(-6 - 3j\right)e^{(-1-2j)t} + 12 \qquad t > 0$$

The Total Solution:

$$V_{2\Omega} = V_{2\Omega,h} + V_{2\Omega,p}$$

$$V_{2\Omega} = \left(4 - 4j\right)e^{(-1+2j)t} + \left(4 + 4j\right)e^{(-1-2j)t}\left(-6 + 3j\right)e^{(-1+2j)t} + \left(-6 - 3j\right)e^{(-1-2j)t} + 12$$

$$V_{2\Omega} = \left(-2 - j\right)e^{(-1+2j)t} + \left(-2 + j\right)e^{(-1-2j)t} + 12 \quad t > 0$$
