

Remarks of T9

Q1 By the hint given, we have
the following 6x6 transition matrix

$$A = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \end{pmatrix} \end{matrix}$$

Then use the matrix calculator at

<http://wims.unice.fr/wims>

to find A^5

To find A^n , we use $A^n = PD^nP^{-1}$ $A^\infty = \lim_{n \rightarrow \infty} A^n$

You may use matrix calculator or MATLAB to
find eigenvalues and eigenvectors of A

Q3 eigen-engine at

<http://www.aw-bc.com/ide/idefiles/media/JavaTools/eignengn.html>

can only be applied to matrices with
entries $-3, -2.5, -2, \dots, 0, 0.5, 1, 1.5, \dots, 3$

Complex eigenvectors can't be found there

You may use the matrix calculator at

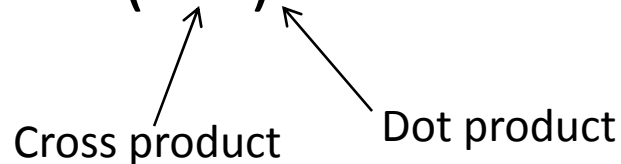
<http://wims.unice.fr/wims> or MATLAB
to find eigenvalues and eigenvectors

Q6

(A) How to check that the vector $[1,2,3]$ is on the plane, vector $[1,2,4]$ is not on the plane ?

HINT:

Recall if the plane is generated by two vectors u and v , then $(u \times v) \cdot w = 0$ iff vector w is on the plane


Cross product Dot product

If $u = [u_1, u_2, u_3]$ $v = [v_1, v_2, v_3]$ $w = [w_1, w_2, w_3]$

then

$$(u \times v) \cdot w = \det \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$$

You may use wims website to find det

<http://wims.unice.fr/wims>

Q6 (cont.) (B) Why the image of the following transformation A is a plane

HINT: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Let u, v, z be the eigenvectors of A

These three eigenvectors are linearly indep. (any pair of them are not parallel).

Hence any vector in 3-dim space can be written as

$$\alpha u + \beta v + \gamma z \quad \text{Then}$$

$$A(\alpha u + \beta v + \gamma z)$$

$$= \dots = \alpha \lambda_1 u + \beta \lambda_2 v + \gamma \lambda_3 z$$

where $\lambda_1, \lambda_2, \lambda_3$

are corresponding eigenvalues

Q6 (cont.) 2nd method

First det of A is zero. So the image is either a plane or a st line. Then look at

$$Ai = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \quad Aj = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} \quad Ak = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

They are not parallel. Hence the image is a plane

(C) The plane is generated by

any two nonparallel vectors in the image of A

For example, we may choose any two from Ai, Aj, Ak and eigenvectors