NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

SEMESTER I: 2010–2011

EXAMINATION FOR

CS3230 – DESIGN AND ANALYSIS OF ALGORITHMS

November 2010 – Time Allowed 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper consists of Eight (8) questions and comprises Sixteen (16) printed pages (including this page).
2. Answer **ALL** questions. Maximum Marks is 60.
3. This is an **Open Book** examination.
4. DO NOT use answer books. All answers must be written in the space provided in this question paper.
5. Write your matriculation number on each page.
6. Matriculation Number ____________________

Question	Internal Examiner	External Examiner	Average
Q1			
Q2			
Q3			
Q4a			
Q4b			
Q5			
Q6			
Q7			
Q8			
Total			

Matric No. ________________

Question 1 (7 marks)

Express the following recurrence relation T in terms of $O(\ )$ notation. Your grade will depend on how tight your bound is.

$$T(0) = T(1) = T(2) = 1.$$

$$\text{For } n \geq 3,\ T(n) = T(\lfloor n/2 \rfloor) + T(\lfloor n/3 \rfloor) + \lfloor n \log_2 n \rfloor.$$

Give a proof for your answer.

Matric No. ____________

Question 2. (8 marks)

Suppose you are visiting a carnival and there are n activities in which you can participate. Activity number i has a starting time $S[i]$ and an ending time $E[i]$.

Suppose you want to maximize the number of activities that you participate in (here if you start an activity, then you must stay with it until it finishes; you cannot participate in two activities simultaneously.)

(a) Give an algorithm to determine which activities you should participate in.

(b) Give time complexity of your algorithm in $O(\ )$ notation.

(c) Prove that your algorithm given in (a) gives an optimal answer. Also prove the time complexity analysis of your algorithm as done in part (b).

Your grading will depend on how good/complex your algorithm is.

Matric No. ____________________

(Blank page)

Matric No. ____________________

(Blank page)

Matric No. ____________________

Question 3 (7 marks)

Suppose we are given coin denominations as $d_1 > d_2 > d_3 \ldots > d_n = 1$.

Prove or give a counterexample to the following claim:

If for all i such that $1 \leq i \leq n-2$,

$$d_{i+2} \text{ divides } d_i - d_{i+1}$$

then the greedy algorithm for coin change problem is optimal.

Matric No. ________________

Question 4 (a) (4 marks)

Suppose we are given a minimal spanning tree T of a graph $G = (V, E)$. Suppose e is an edge in T. Suppose we get sub-trees T_1 and T_2 when we remove e from T. Then prove or disprove that e must be an edge of minimal weight which connects T_1 and T_2.

Matric No. ____________________

Question 4 (b) (4 marks)

Suppose we are given the following frequencies for the characters a, b, c, d, e, f:

$freq(a) = 5$, $freq(b) = 3$, $freq(c) = 12$, $freq(d) = 3$, $freq(e) = 8$, $freq(f) = 8$

Give an optimal Huffman coding tree for the above characters. Based on the Huffman tree constructed by you, give the codes for the different characters.

Matric No. ____________________

Question 5. (7 marks)

Build a table to show how the dynamic programming algorithm done in class will work for finding the optimal algorithm for the following matrix multiplication.

$M_1 \times M_2 \times M_3 \times M_4$, where

M_1 is a matrix of size 5×3

M_2 is a matrix of size 3×1

M_3 is a matrix of size 1×3

M_4 is a matrix of size 3×7

Matric No. ____________________

Question 6 (7 marks)

Suppose X and Y are two sequences of the same length, and both X and Y have a as their third character. Then can we claim that the longest common subsequence of X and Y also has a in it?

Either prove the above claim or give a counterexample.

Matric No. ________________

Question 7 (8 marks)

A search tree is a binary tree, with nodes labeled by numbers (keys). A search tree has the property that numbers in the left subtree of any node are strictly less than the number at the node, and the numbers in the right subtree of any node are strictly greater than the number at the node.

The time taken to search a number (present in the tree) is equal to the number of comparisons needed to find the node containing the number using the standard binary search (thus the time taken is 1+depth of the node containing the number, where depth of root is taken as 0).

Suppose we are given a set of numbers $a_1, a_2, \ldots, a_n$, for which we need to make a search tree. Furthermore, suppose we are also given the number of times f_i that a query regarding a_i is going to be asked.

Then, the cost of a binary search tree T for $a_1, a_2, \ldots, a_n$ is:

$$\sum_{i=1}^{n} f_i * (1 + d_i)$$

where d_i is the depth of node containing a_i in the tree T.

(a) Give a dynamic programming algorithm to find the cost of the optimal search tree (with respect to cost as defined above).

(b) What is the time complexity of your algorithm (in O notation)?

Matric No. ____________

(Blank page)

Matric No. ____________________

(Blank page)

Matric No. ____________________

Question 8 (8 marks)

The distance between two vertices u and v in a graph (denoted by $distance(u, v)$) is the number of edges in the shortest path between u, v.

Consider the following problem:

Input: An undirected graph $G = (V, E)$ and two positive integers f and d.

Question: Is there a subset V' of V such that (i) V' has at most f vertices, and (ii) for each vertex $u \in V$, there exists a vertex $v \in V'$, such that $distance(u, v) \leq d$.

(Intuitively, above asks whether there is a way to select f vertices of G, on which to locate "firehouses", so that no vertex in the graph is at a distance more than d from a firehouse.)

Show that the above problem is NP-complete.

Hint: Reduce 3SAT to the above problem.

Matric No. ____________

(Blank page)

Matric No. ____________________

(Blank page)

END of QUESTIONS