EE3206/EE3206E INTRODUCTION TO COMPUTER VISION AND IMAGE PROCESSING

Semester 1, 2013/2014

Tutorial Set B

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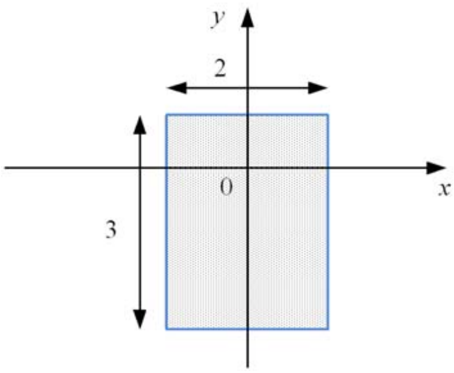
1. Verify the (continuous) Fourier transform pairs
   (a) $\delta(x, y) \leftrightarrow 1$
   (b) $1 \leftrightarrow \delta(u, v)$

2. A continuous image $f(x, y)$ consists of a light rectangle on a dark background. The sides of the rectangle are parallel to the coordinate axes, and the intensities of the rectangle and background are 50 and 10, respectively. The centroid of the rectangle is at $(0, -1)$. Obtain the Fourier spectrum of $f(x, y)$.



3. Let $f(x, y) = \cos 2\pi(ax + by)$. Show that the Fourier transform of $f(x, y)$ is given by
$$F(u, v) = \frac{1}{2}\delta(u - a, v - b) + \frac{1}{2}\delta(u + a, v + b)$$
   Obtain and sketch the Fourier spectrum of
   (a) $f_1(x, y) = \cos(20\pi x)$
   (b) $f_2(x, y) = \sin(40\pi y)$
   (c) $f_3(x, y) = \sin(30x + 40y)$
   (d) $f_4(x, y) = \sin(30x + 40y + 30)$

4. Consider the N-point sequence $f(x)$ and its DFT $F(u)$. Show that multiplying $f(x)$ by $(-1)^x$, i.e.,
$$f'(x) = f(x)\,(-1)^x, \qquad x = 0, 1, \ldots, N-1$$
   prior to taking the transform shifts the origin of the transform to the point $u = N/2$. (*Hint:* use the translation property of the DFT.)

5. You are given an 8-point sequence

$$f(x) = 1, 1, 1, 1, 0, 0, 0, 0$$

(a) Obtain $F(u)$, the DFT of $f(x)$.

(b) Sketch the magnitude and phase components for $-8 \leq u \leq 15$.

(c) Find the DFT of $g(x) = (-1)^x f(x)$.

6. Compute the DFTs of the following functions:

(a) $f_1(x) = 1, 1, 1, 1$

(b) $f_2(x) = 1, 0, 0, 0$

(c) $f_3(x) = 0, 1, 0, 0$

(d) $f_4(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(e) $f_5(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

This may be useful:

$$\sum_{x=0}^{N-1} \exp(-j2\pi ux/N) = \begin{cases} N & u = 0 \\ 0 & u \neq 0 \end{cases}$$

where u, x are integers and N is an even integer.