TUTORIAL 7: SUGGESTED SOLUTIONS   SEMESTER II, AY 2010/2011

1.

2.

3. First construct an example with 8 people that shows $R(3,4) > 8$. Next we show that for any set of 9 people, there are either 3 who are mutually acquainted or 4 who are mutually not acquainted. Proof: Suppose not. Consider person A who knows k other persons. What is the maximum or minimum value for k?

   k cannot be bigger than 3. If A knows four persons, since there are no 3 who are mutual acquaintances, these four must be mutually unacquainted giving us a contradiction. Similarly, k cannot be less than 3. If A only knows 2 persons, there are six people remaining who do not know A. Since $R(3,3) = 6$, among these 6 there are either 3 mutual acquaintances (contradiction again) or 3 who are mutually unacquainted. But these 3 together with A gives us a set of 4 who are mutually unacquainted.

   So we conclude A knows exactly 3 persons, and this is true for all 9 people. If we consider people as points and acquaintances are joined by lines, there will be exactly $\dfrac{9 \times 3}{2} = \dfrac{27}{2}$ lines which is a contradiction.

4.

5. Using inclusion-exclusion, you should arrive at the answer $\# \le 150 - 103 - 102 - 94 + 84 + 65 + 75 = 75$. But this bound is not meaningful since we know that at most 65 read both algebra and combinatorics, so students reading all three modules should not be more than 65.

6.

7.

8.