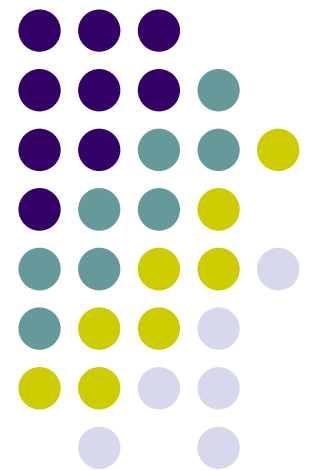


# Circuit analysis with Nonlinear Elements And Review of Part2

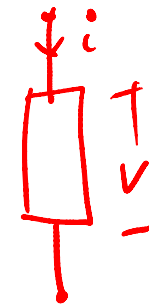
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# Linear elements

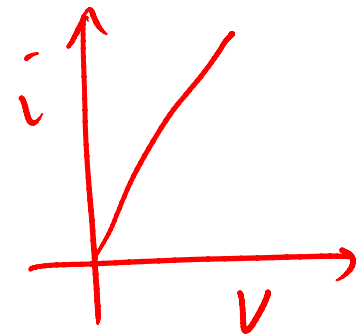
- Resistor

$$i = \frac{V}{R}$$



- Inductor

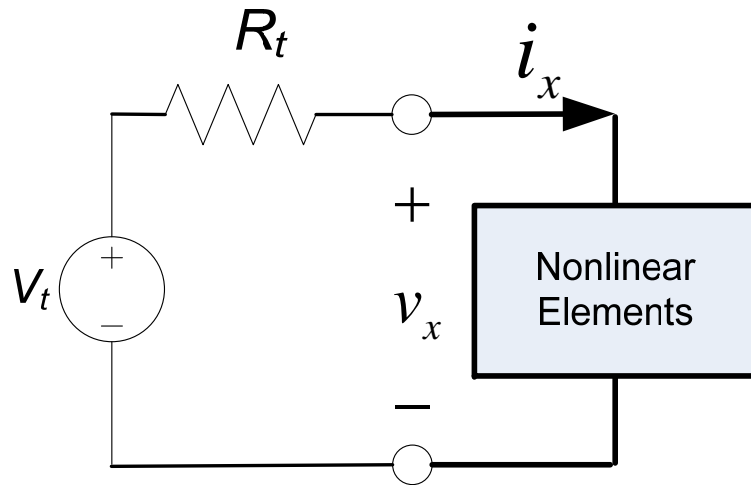
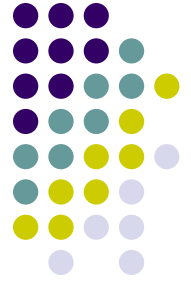
$$V = L \cdot \frac{di}{dt}$$



- Capacitor

$$i = C \cdot \frac{dV}{dt}$$

# Nonlinear elements

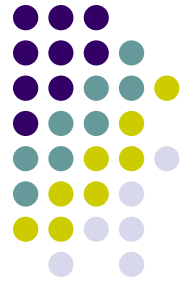


$$i_x = e^{v_x} : \text{Nonlinear} \quad \text{--- ①}$$

From KVL :

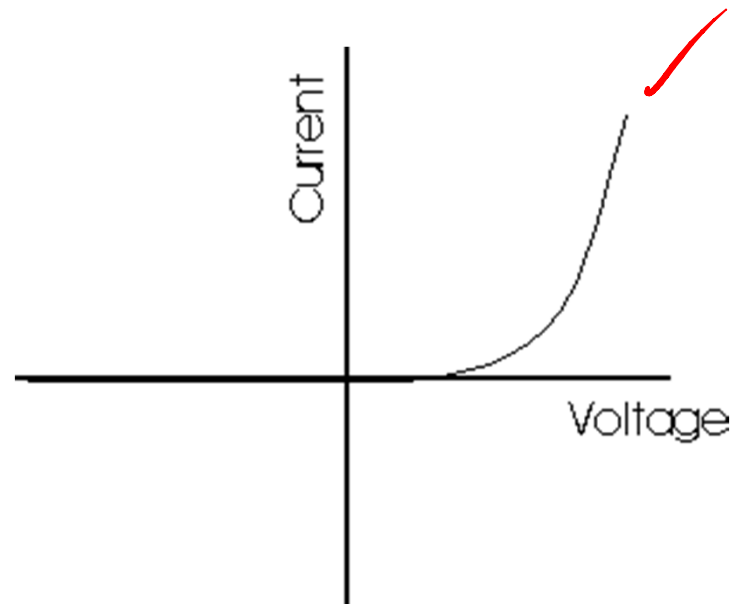
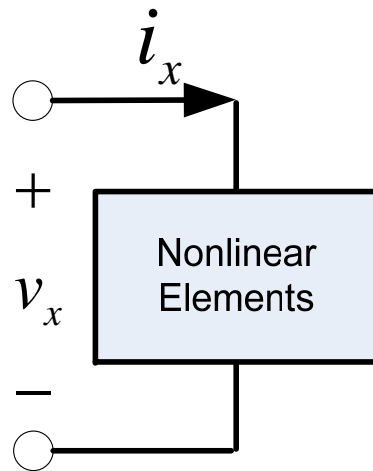
$$\text{Replace } i_x \text{ by } e^{v_x} : V_t - i_x \cdot R_t = v_x \quad \text{--- ②}$$
$$V_t - e^{v_x} \cdot R_t = v_x$$

$$v_x ?$$

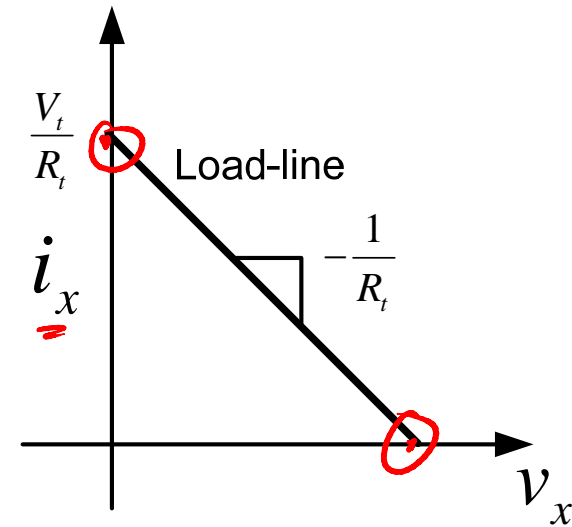
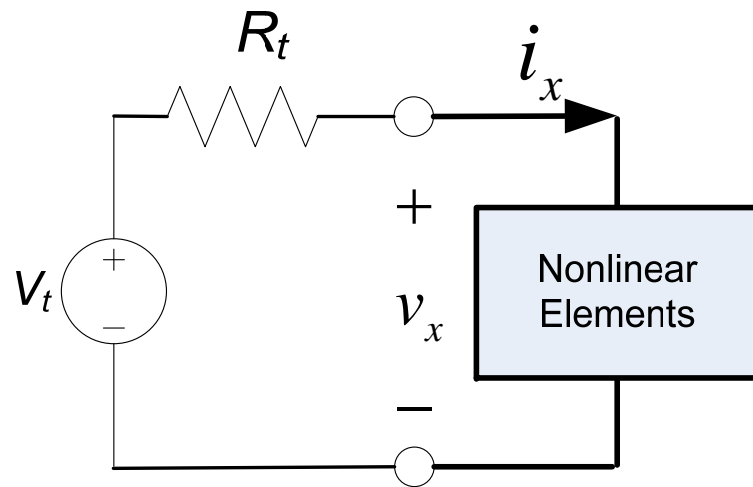
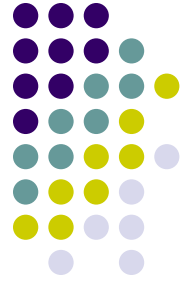


# Nonlinear elements

- Nonlinear elements may not have an analytical function



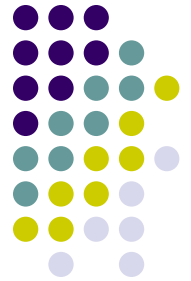
# Load line



$$\underline{\text{KVL}}: V_t - R_t \cdot i_x = v_x$$

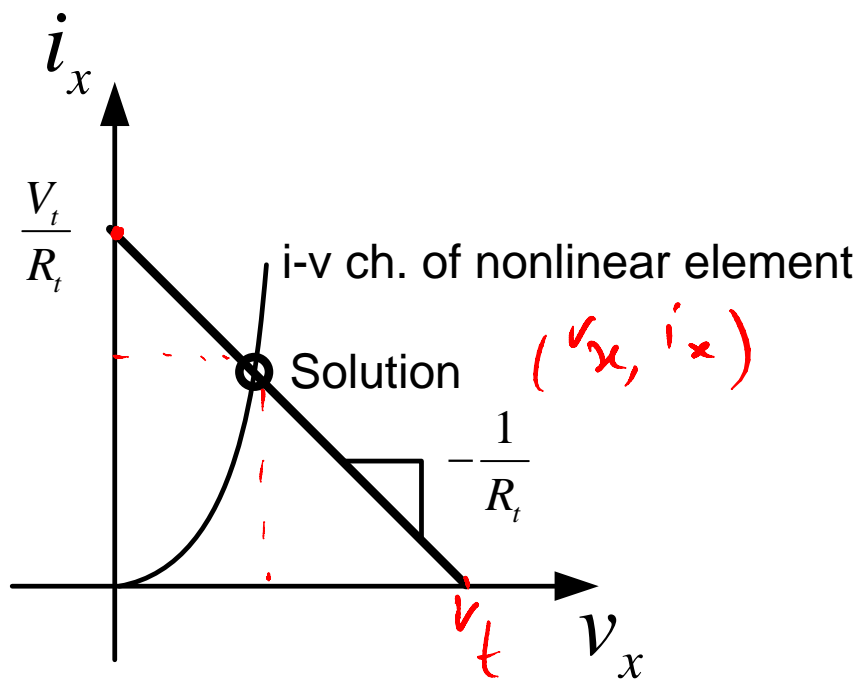
$$v_x = 0 \Rightarrow V_t - R_t \cdot i_x = 0 \Rightarrow i_x = \frac{V_t}{R_t}$$

$$i_x = 0 \Rightarrow V_t = v_x$$

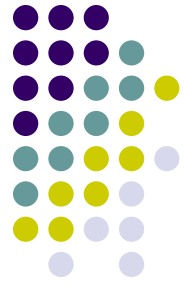


# Graphical (Load-line) analysis

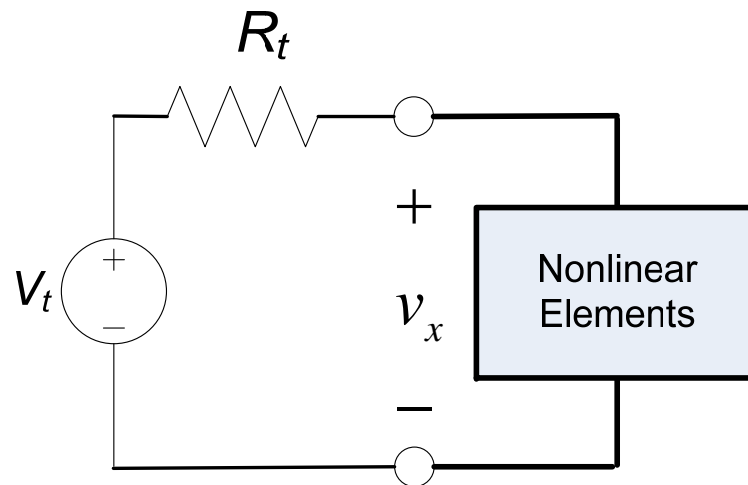
- Merge the load line onto the i-v curve of the nonlinear element
- The solution is at the intersection point



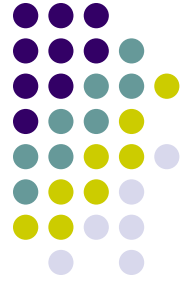
# Solving circuits with one Nonlinear element



- Replace the circuit by its Thevenin's equivalent considering the nonlinear element as the load
- Use graphical analysis technique



# Example

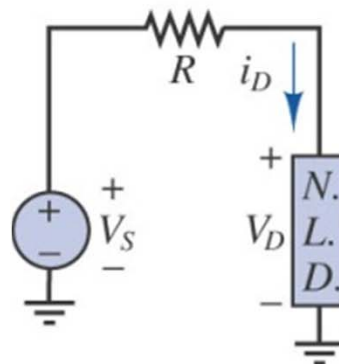


Find the current through the NLD.

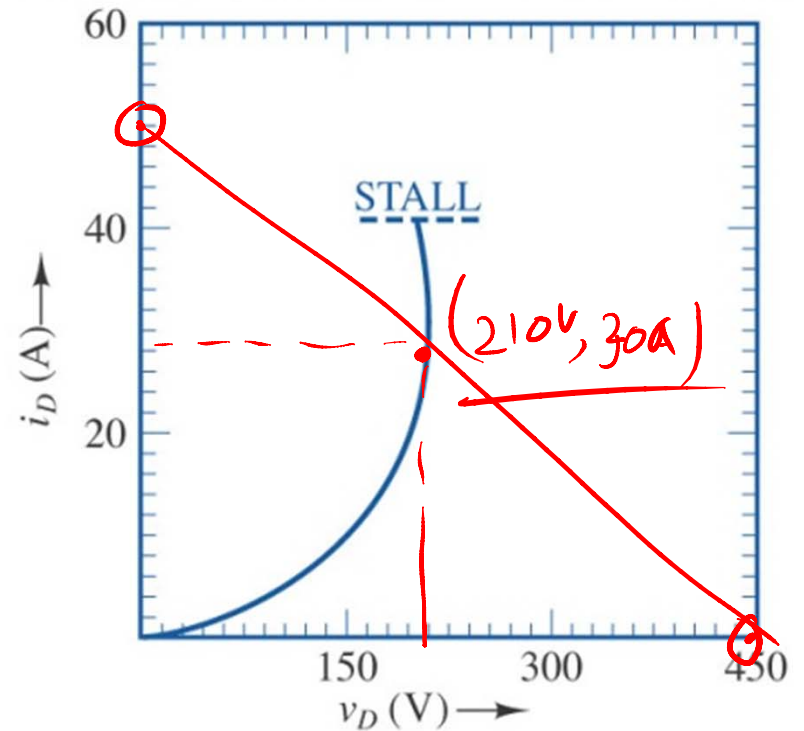
$$\frac{V_s}{R} = \frac{450}{9} = 50 \text{ A}$$

$$V_s = 450\text{V}$$

$$R = 9\Omega$$



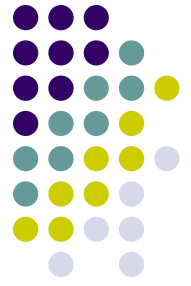
(a)



(b)

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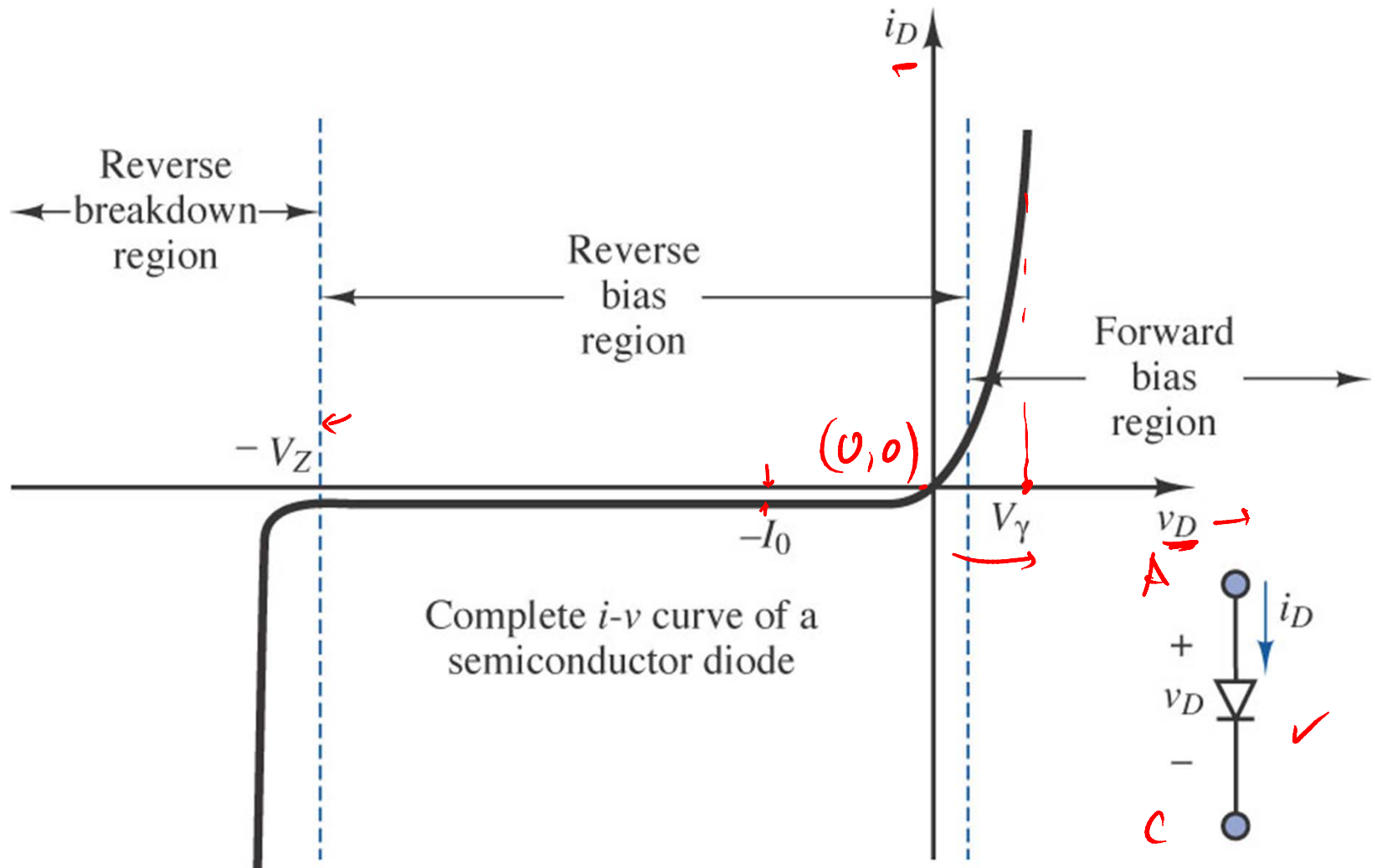
# Popular nonlinear devices

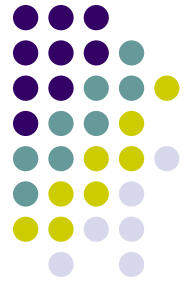
- Diode
- BJT(Bipolar Junction Transistor)
- MOSFET (Metal Oxide Semiconductor Field Effect Transistor)

We learn about their terminal characteristics without getting to know their physics

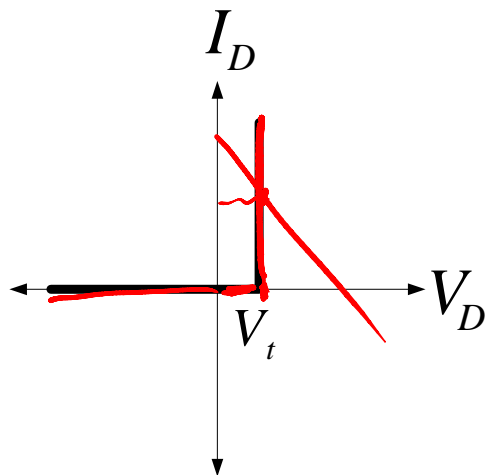
# Diode

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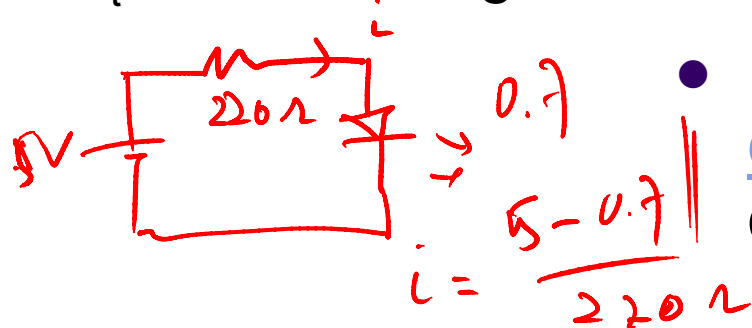




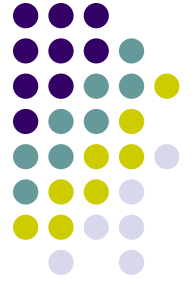
# Simplified diode model



$V_t$ : cut-in voltage

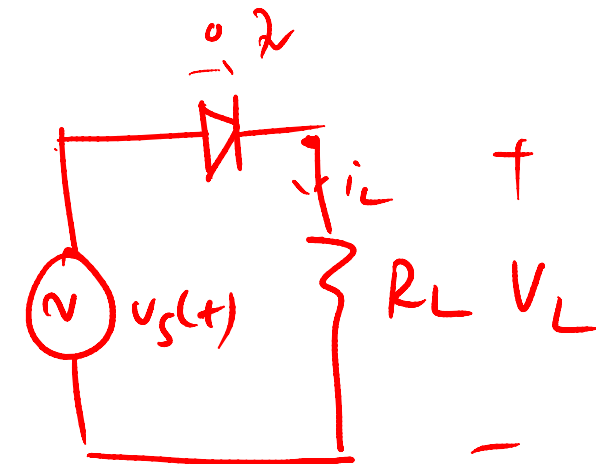
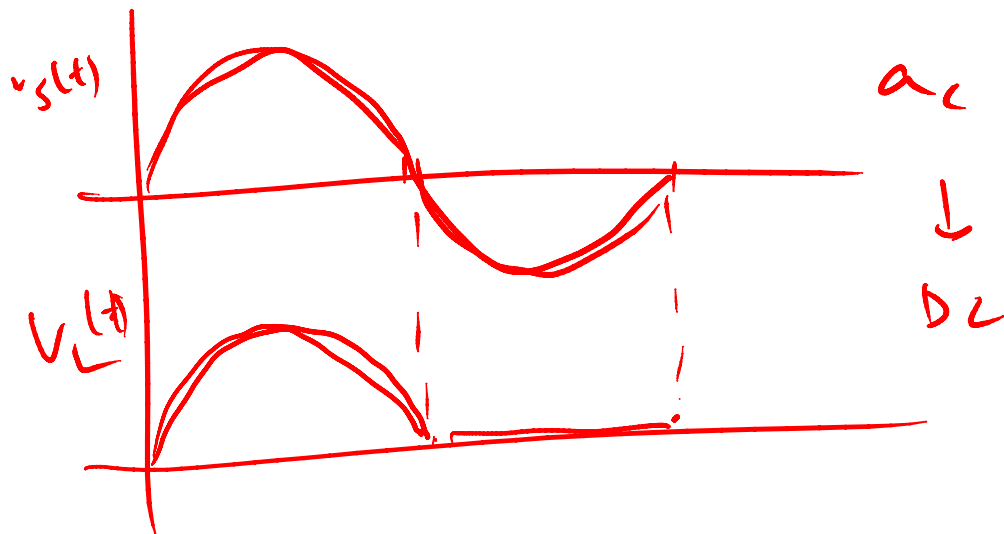


- In a normal silicon diode at rated currents, the arbitrary “cut-in” voltage is defined as 0.6 to 0.7 volts - used in rectification
- Schottky diodes can be rated as low as 0.2 V
- Red or blue light-emitting diodes (LEDs) can have values of 1.4 V and 4.0 V respectively.



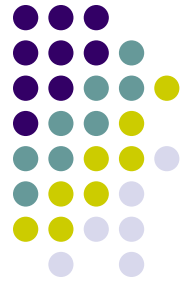
# Some applications of diode

- Rectification (ac to dc)
- LED ✓
- Over-voltage protection



$$V_s(t) = 100 \sin 100t$$

$$V_L(t) = \underline{i_L(t) \times R}$$



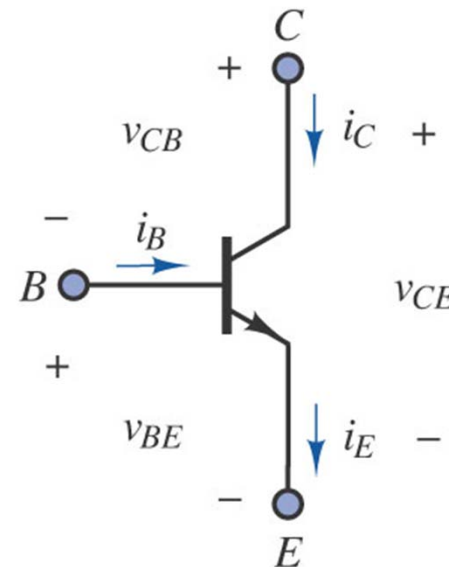
# BJT Symbol

(Bipolar Junction Transistor)

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- 3-terminal device
  - Base - B
  - Collector - C
  - Emitter - E
- Base current controls the behavior of the BJT

The operation of the BJT is defined in terms of two currents and two voltages:  $i_B$ ,  $i_C$ ,  $v_{CE}$ , and  $v_{BE}$ .



$$\text{KCL: } i_E = i_B + i_C$$

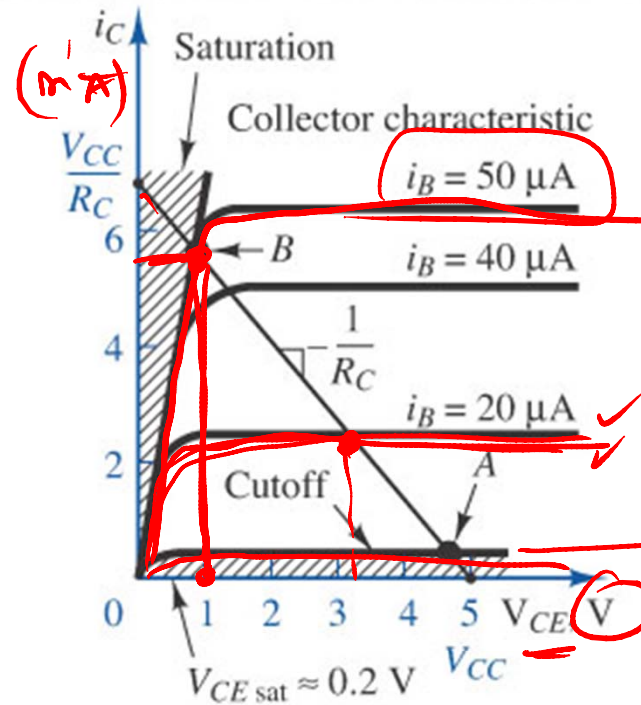
$$\text{KVL: } v_{CE} = v_{CB} + v_{BE}$$



# BJT in Circuit

$$V_{out} = \left( -100 \times \frac{R_C}{R_B} \right) \Delta V_{in}$$

- Separate i-v curve for each value of base current



$$i_B = 20 \mu A$$

$$i_C = 2 mA$$

$$i_C = \beta i_B$$

$\beta$  is large

- Two modes of operation:

- Amplifier
- Switch

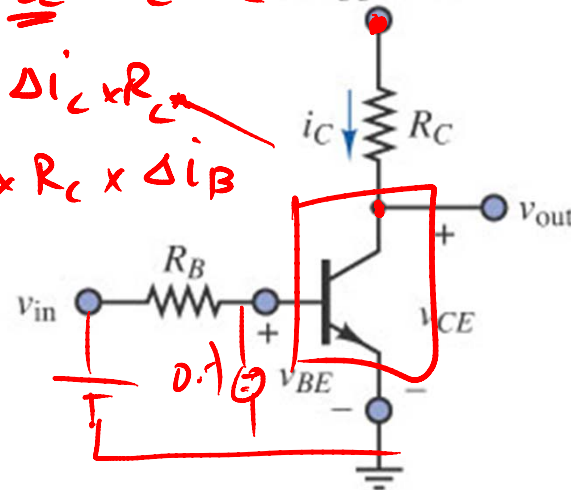
$$V_{out} = V_{CC} - i_C R_C \quad +V_{CC} = 5 V$$

$$\Delta V_o = -\Delta i_C \times R_C$$

$$= -100 \times R_C \times \Delta i_B$$

$$i_B = \frac{V_{in} - 0.7}{R_B}$$

$$\Delta i_B = \frac{\Delta V_{in}}{R_B}$$



Elementary BJT inverter

$$\beta = \frac{2 \times 10^{-3}}{20 \times 10^{-6}}$$

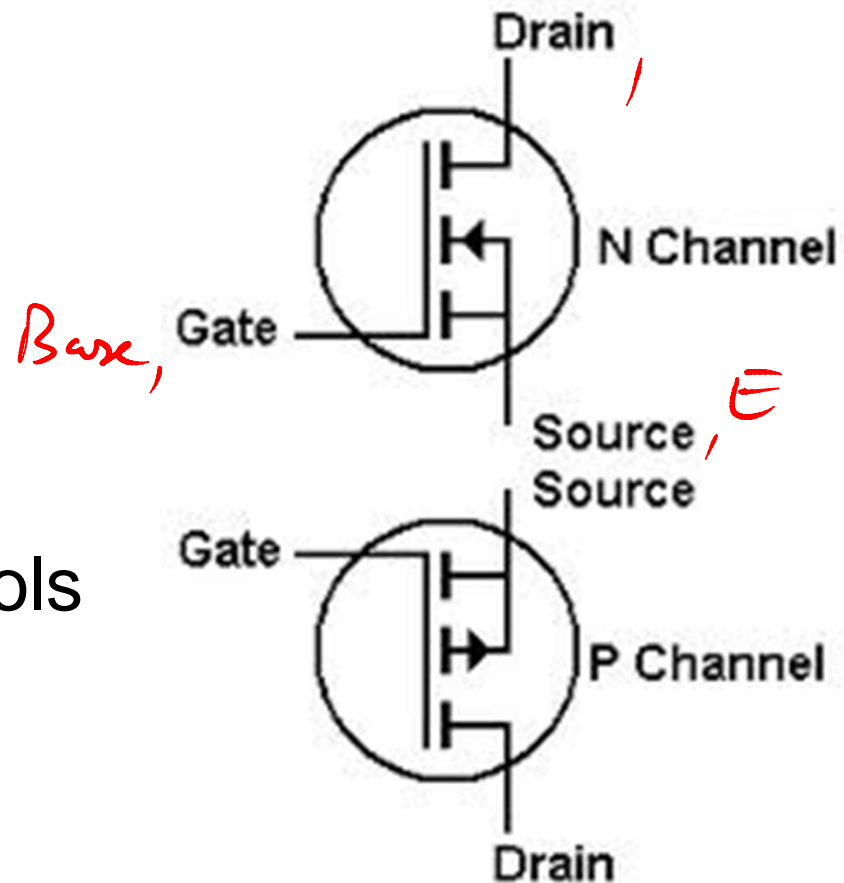
$$\approx 100$$

$$\Delta i_C = \beta \cdot \Delta i_B$$

$$= 100 \times \Delta i_B$$

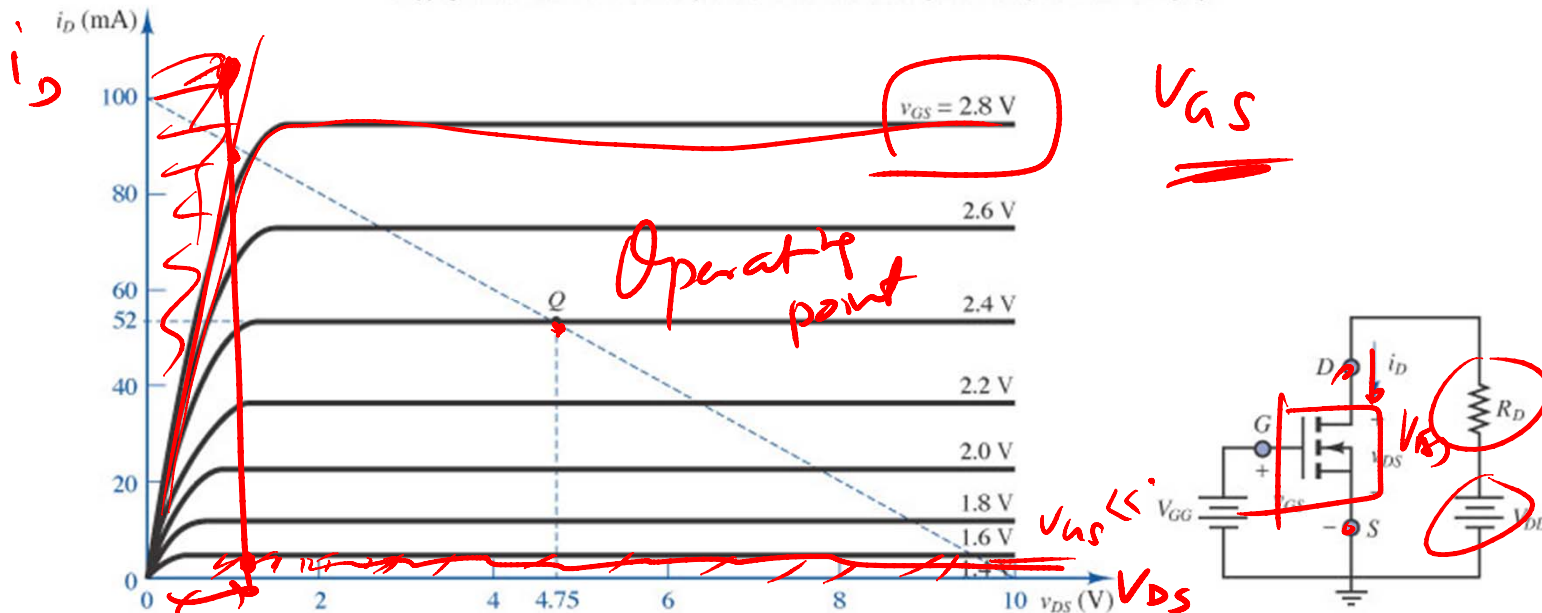
# MOSFET symbol

- 3-terminal device
  - Gate
  - Drain
  - Source
- Gate voltage controls the behavior of the MOSFET

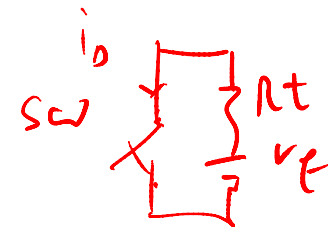


# 11\_07.jpg

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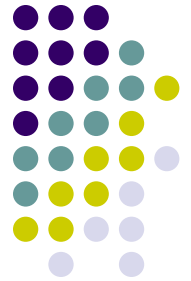


- Separate i-v curve for each value of base current
- Two modes of operation:
  - Amplifier
  - Switch



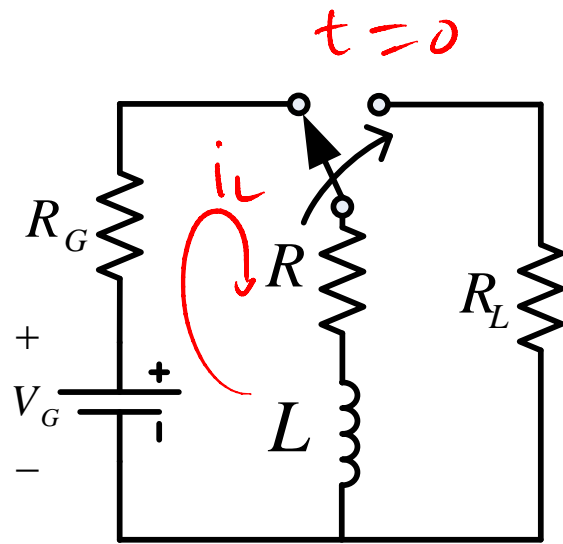
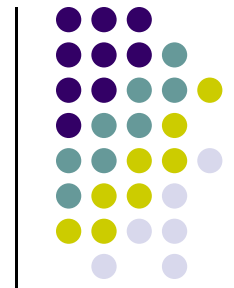


# Examinable Syllabus for part 2



1. DC transients analysis ✓
2. AC steady-state analysis ✓
3. Digital Logic ✓
4. Circuits with Nonlinear elements ✓

# Mid-term question Q4

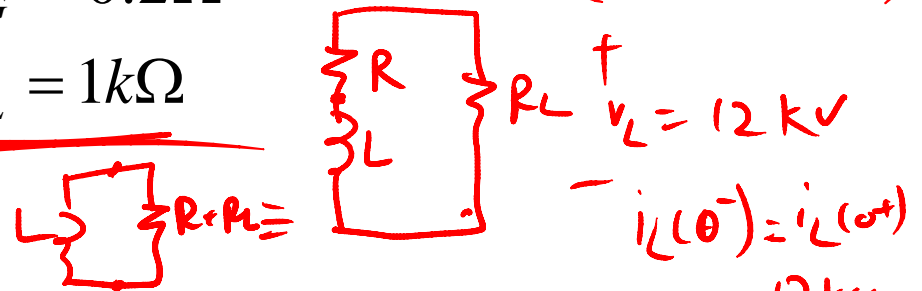


$$\frac{12}{0.2 + R} = 12 \Rightarrow R = 0.8 \Omega$$

$$i_L(\infty) = \frac{V_G}{R_G + R} = 12$$

(L is short)

$$\begin{aligned} V_G &= 12V \\ R_G &= 0.2\Omega \\ R_L &= 1k\Omega \end{aligned}$$



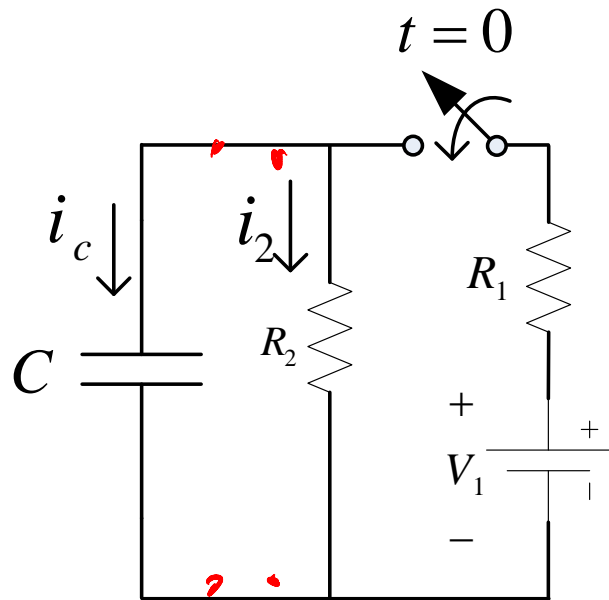
- The load requires 12KV to start with, and it decays at time constant  $\tau = 10\mu s$

$$\tau = \frac{L}{R + R_L} \Rightarrow L = \tau \times (R + R_L)$$

- Find the value of L and R in the circuit.

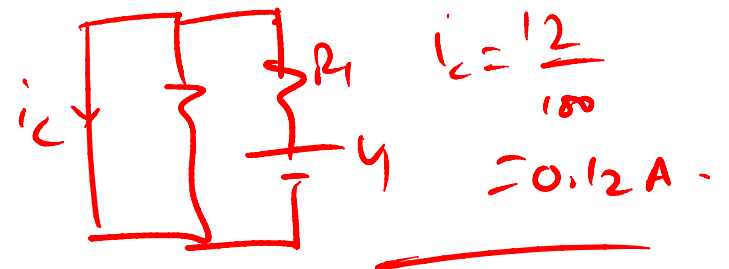
$$\begin{aligned} &= \frac{12kV}{1k\Omega} \\ &= 12A \end{aligned}$$

# Mid-term question Q4 ~~5~~



$$\begin{aligned} V_1 &= 12 \text{ V} \\ R_1 &= 100 \, \Omega \\ R_2 &= 100 \, \Omega \\ C &= 10 \, \mu\text{F} \end{aligned}$$

At  $t=0$



$$i_2(t) = \frac{v_c(t)}{R_2}$$

So  $R_T = R_1$

$$i_2(t) = \frac{v_c(t)}{R_2} = \frac{1}{R_2} \left( 6 \left( 1 - e^{-t/R_T} \right) \right)$$

- $i_c$  immediately after the switch is closed
- $i_2(t)$  after switch is closed



# Review of AC steady-state

- Review of Complex numbers ✓

$+$ ,  $-$ ,  $/$ ,  $*$  Rect  $\leftrightarrow$  Polar

- Phasors sinusoidal voltage and current

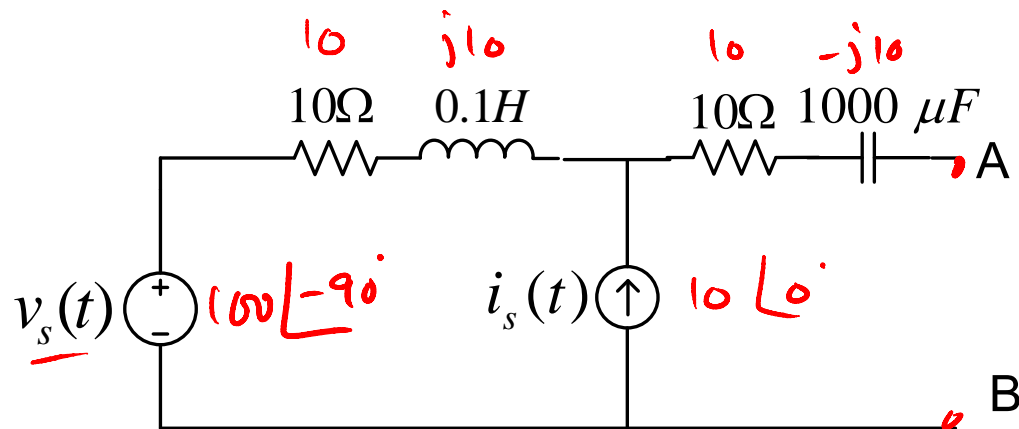
- Impedance for the R, L and C

$$Z_R = R, Z_L = j\omega L = \omega L \angle 90^\circ$$

- Solve circuits using all the techniques for DC circuit analysis

$$Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} \\ = \frac{1}{\omega C} \angle -90^\circ$$

# AC steady-state review Qn

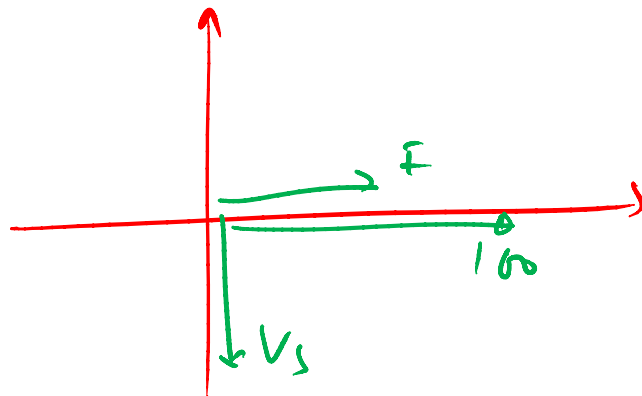
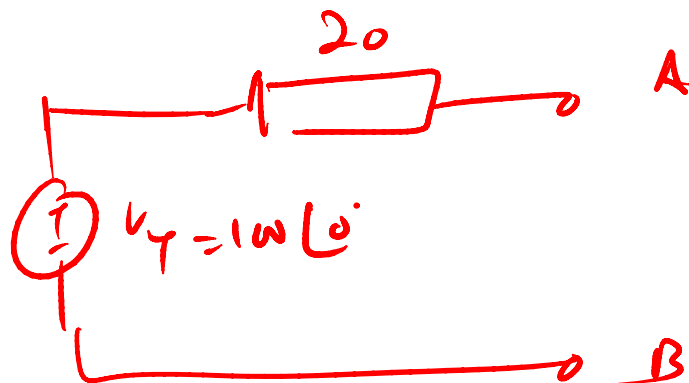
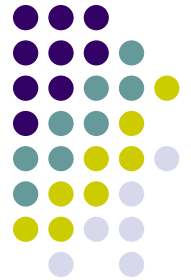
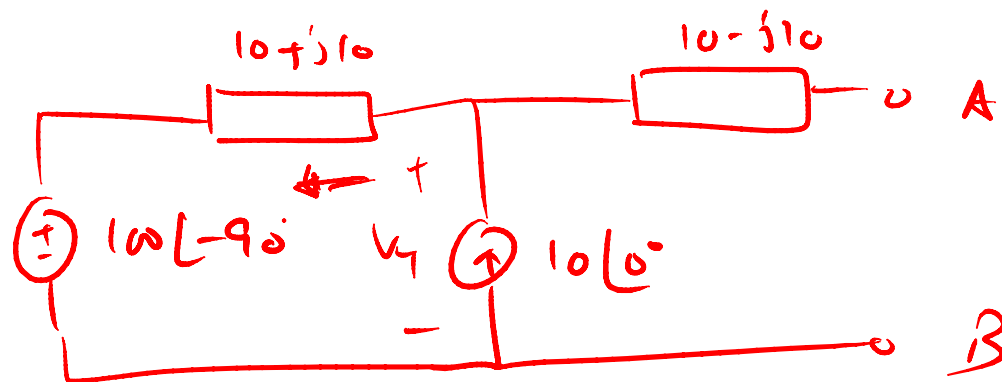


$$v_s(t) = 100 \sin(100t) \text{ V}$$

$$i_s(t) = 10 \cos(100t) \text{ A.}$$

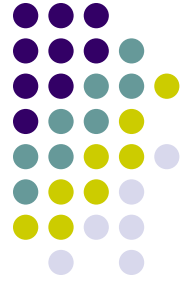
- Find the Thevenin equivalent between A and B
- Draw the Phasor diagram showing the voltage source, current source and the Thevenin voltage

$$0.1 \text{ H} \quad j \times 100 \times 0.1 = j10 \quad -j \frac{1}{\omega C} = -j \cdot \frac{1}{100 \times 10^{-3}} = -j10$$



$$\begin{aligned}
 V_T &= 100\angle -90^\circ + I_x(10 + j10) \\
 &= -j100 + 10\angle 0^\circ \times (10 + j10) \\
 &= -j100 + 10(10 + j10) \\
 &= -j100 + 100 + j100 = 100 \\
 V_T &= 100\angle 0^\circ
 \end{aligned}$$

# Digital Design



- Basic Boolean operation ✓
- Logic gates – AND, OR, NOT, NAND, NOR ✓
- Truth table for a digital system
- SOP and POS expression from Truth table
- Minimization of logic expression using K-map
- Realization of logic expression using NAND/NOR gates only with 'alternate gate representation'

