NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 3: SUGGESTED SOLUTIONS **SEMESTER II, AY 2010/2011**

1.

2.

3.

4. Note that at most two unique digits will appear, giving three possible cases:

   (a) 1 digit appearing 7 times = 9 ways from $\{1,\ldots,9\}$

   (b) first digit appears 3 times
   - 9 ways to choose the first digit X from $\{1,\ldots,9\}$
   - $\binom{6}{2}$ ways to choose two positions to place the other two X
   - 9 ways to choose the other digit Y from $\{0,\ldots,9\}$ but different from X. The positions of Y are fixed.
   - Subtotal = $9\times 9\times 15 = 1215$.

   (c) first digit appears 4 times
   - 9 ways to choose the first digit X
   - $\binom{6}{3}$ ways to choose the positions of the other X
   - 9 ways to choose the other digit Y. The positions of Y are fixed
   - Subtotal = $9\times 9\times 20 = 1620$.

   By the addition principle, the total number is

$$9+1215+1620=2844.$$

5.

6.

7. Hint: Write P_n as a product of two matrices using the Chu-Vandermonde identity.