CG1108 Sem 2 AY2010/11

Part2

Lecture 5

✓

DC
Steady-state

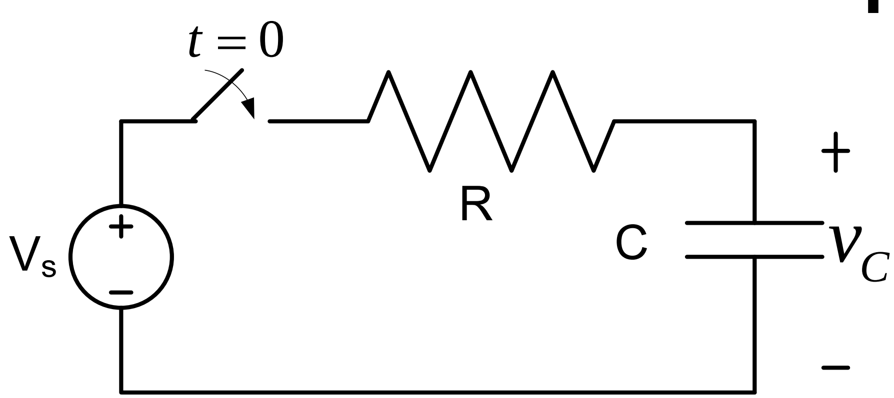
DC Transients

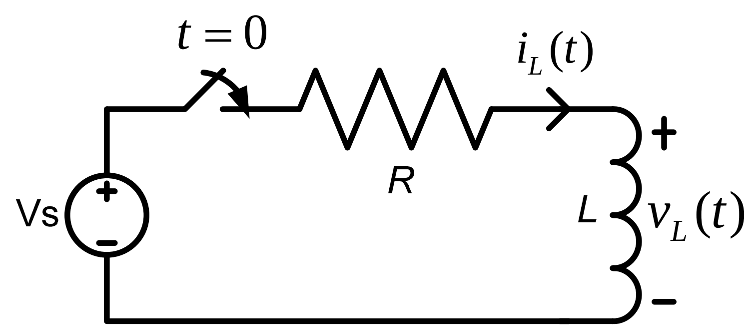
- Learning objectives:
  - Understand the meaning of transients.
  - Write differential equations for circuits containing inductors and capacitors.
  - Solving differential equations to find the time value of voltages and currents
  - Use of Oscilloscope and Signal generator

Transients

- The time-varying voltages and currents resulting from the adding or removing voltage and current source to circuits containing energy storage elements, are called **transients**.
- Voltage and current in such circuits are represented by **First-order differential equations.**

First-order circuits with DC supply





$$RC\frac{dv_c}{dt}+v_c=V_s$$ ✓

$$\tau\frac{dx}{dt}+x=K'$$

$$\frac{L}{R}\frac{di_L}{dt}+i_L=\frac{V_S}{R}$$

$x = v_c$

$\tau = RC$

$K' = V_s$

$v_c(ss) = V_s$

$\frac{dx}{dt} = 0$

$x = i_L$

$\tau = \frac{L}{R}$

$K' = \frac{V_s}{R}$

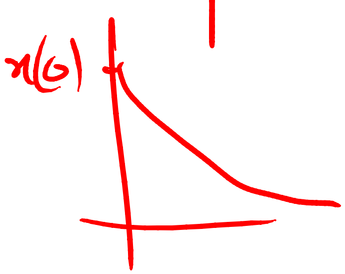
$i_L(ss) = \frac{V_s}{R}$

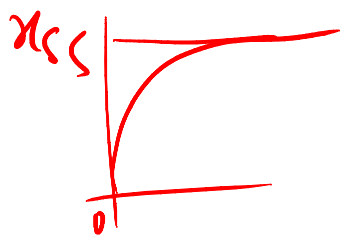
Solution of Differential eqn

$$\tau\frac{dx}{dt}+x(t)=K^{'}$$

For DC circuits, $$\frac{dx(t)}{dt}=0, x_{ss}=K^{'}$$

$$\boxed{x(t)=x(0)e^{-\frac{t}{\tau}}+x_{ss}(1-e^{-\frac{t}{\tau}})}$$



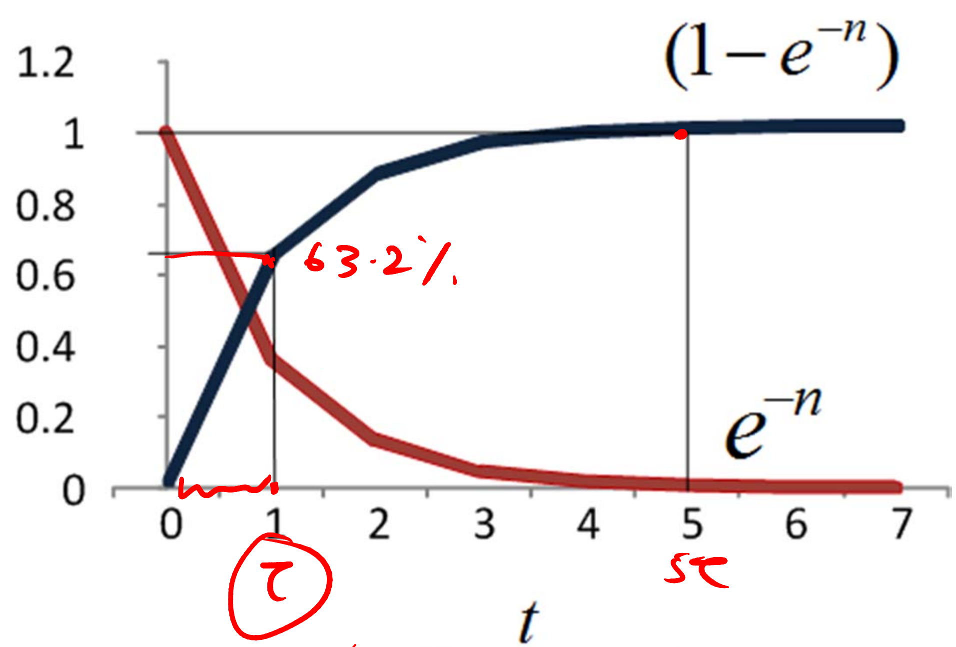


Nature of the solution

$$x(t) = x(0)e^{-\frac{t}{\tau}} + x_{ss}(1 - e^{-\frac{t}{\tau}})$$

τ = Time constant

n	e^{-n}	$(1-e^{-n})$
0	1	0
1	0.367879	0.632121
2	0.135335	0.864665
3	0.049787	0.950213
4	0.018316	0.981684
5	0.006738	0.993262
6	0.002479	0.997521
7	0.000912	0.999088



$n = \frac{t}{\tau}$ →

RC and RL comparing to the general form

$$\tau \frac{dx}{dt} + x = K' \text{ where } K' \text{ is the steady-state solution } (x_{ss})$$

$$x(t) = x(0)e^{-\frac{t}{\tau}} + x_{ss}(1 - e^{-\frac{t}{\tau}})$$

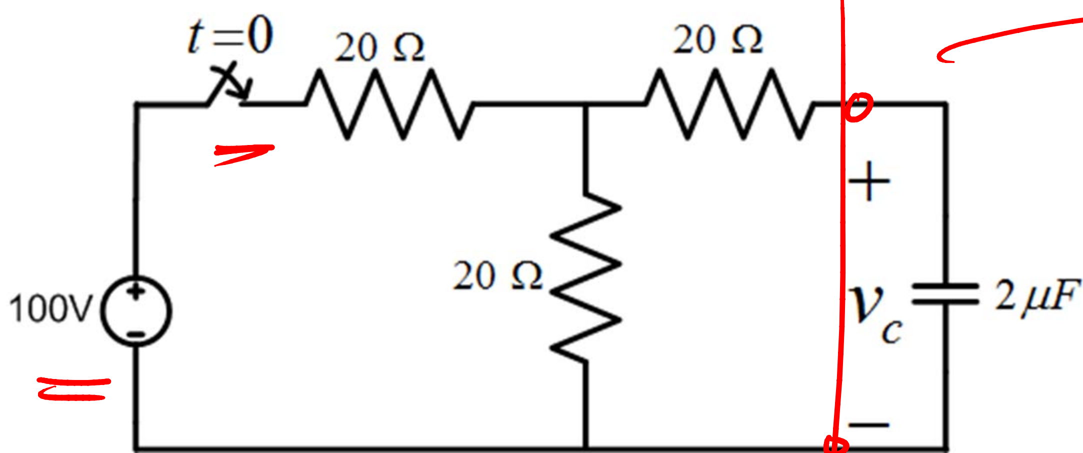
$$RC\frac{dv_c}{dt} + v_c = V_s$$

$$v_c(t) = v_c(0) \cdot e^{-t/RC} + V_S\left(1 - e^{-t/RC}\right)$$

$$\frac{L}{R}\frac{di_L}{dt} + i_L = \frac{V_S}{R}$$

$$i_L(t) = i_L(0) \cdot e^{-t/L/R} + \frac{V_S}{R}\left(1 - e^{-t/L/R}\right).$$

Example2



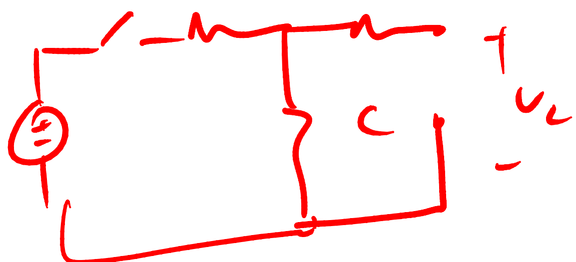
$$v_c(t) = 50\left(1 - e^{-t/60\times10^{-6}}\right)\,V$$

$v_c(t)$ for $t > 0$.

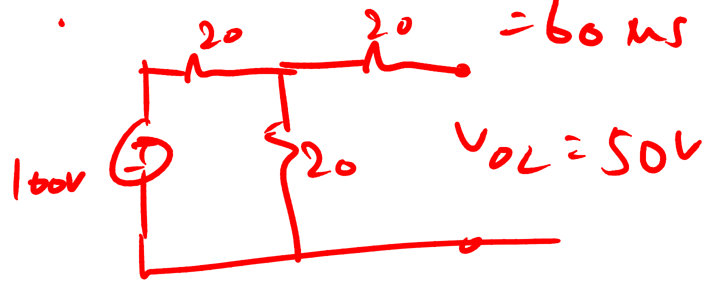
$v_c(0) = 0$

$v_{c\,ss} = 50$

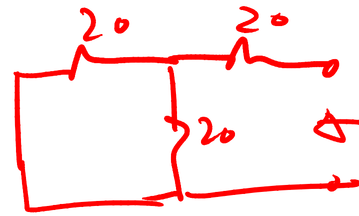
$\tau = R_{TH} \times C = 30 \times 2\times10^{-6} = 60\ \mu s$

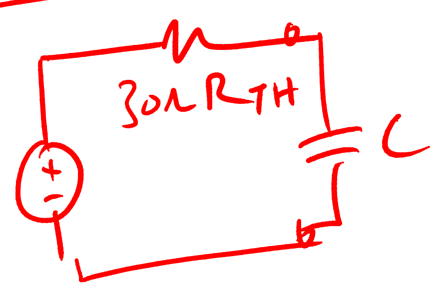


$V_{TH} =$

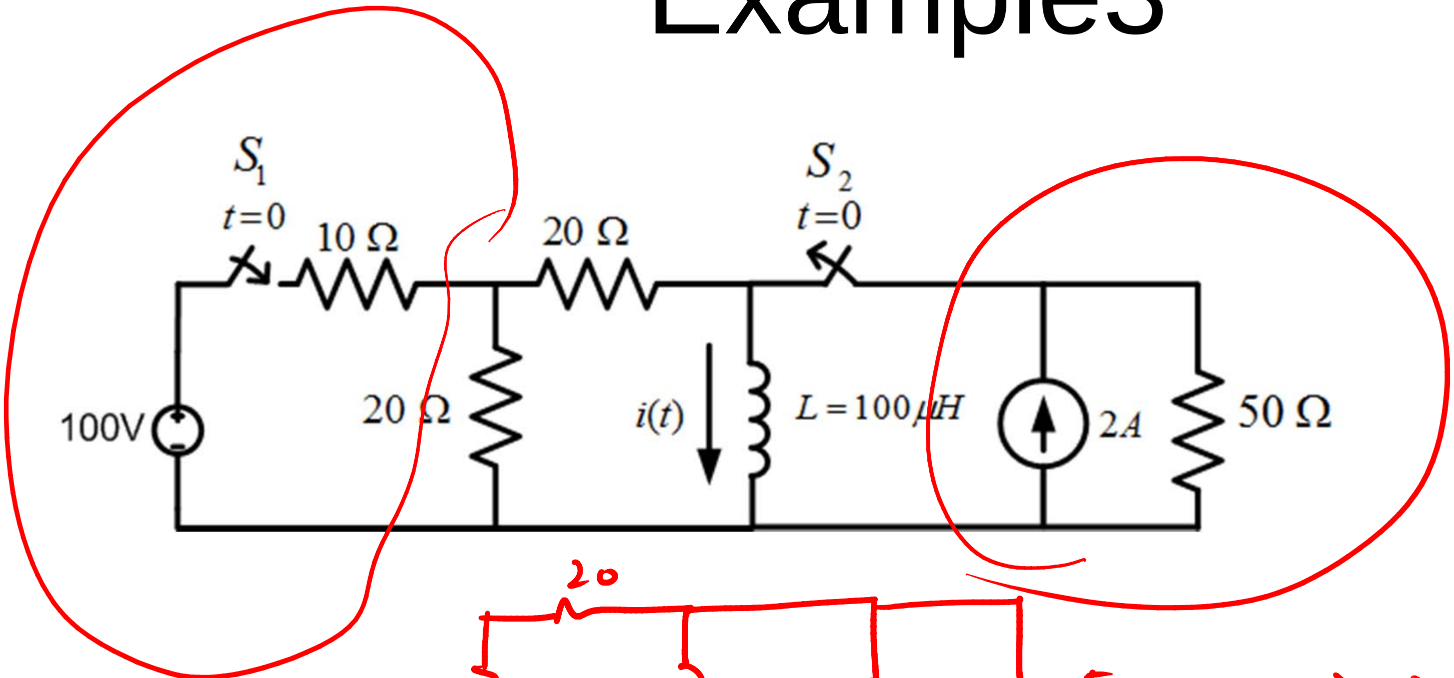


$R_{TH} =$

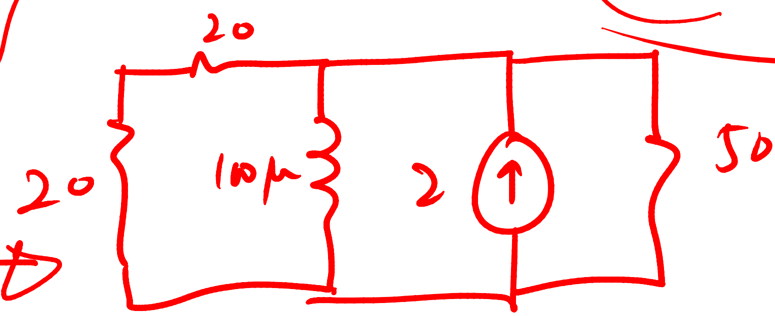




Example3

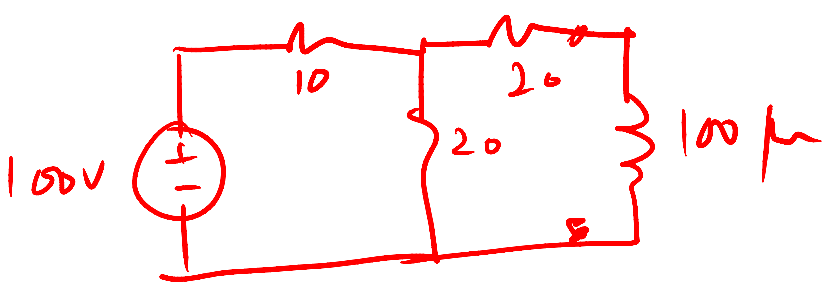


Before $t=0$



$\Rightarrow i(0)$ ✓

After $t=0$



$\Rightarrow i_{ss}$ ✓

τ

Summary of DC transients

1. Understand behaviour of Capacitor and Inductor in DC steady state.

   Cap → open ckt

   Ind → short ckt

2. Thevenin equivalent

3. Apply the transient formula

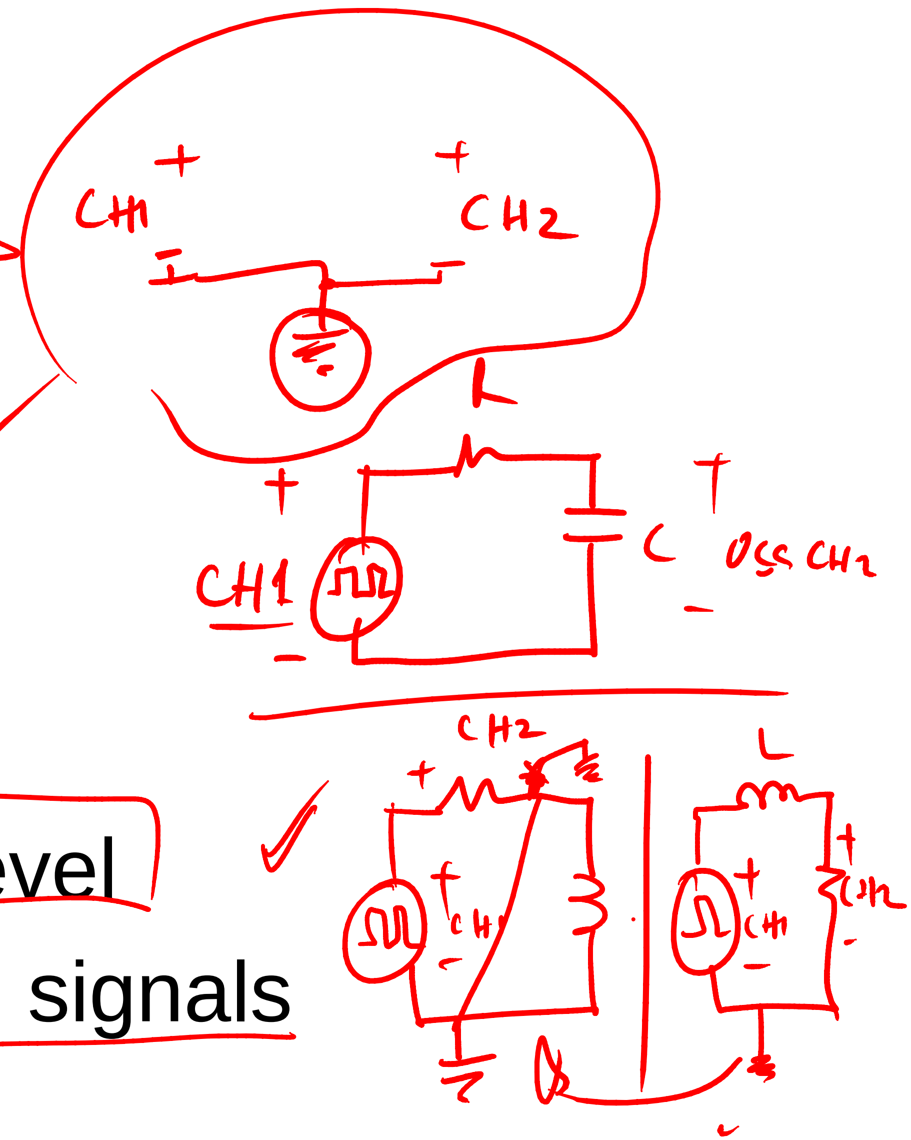
$$x(t) = x(0)\,e^{-t/\tau} + x_{ss}\left(1 - e^{-t/\tau}\right).$$

Lab4: Objectives

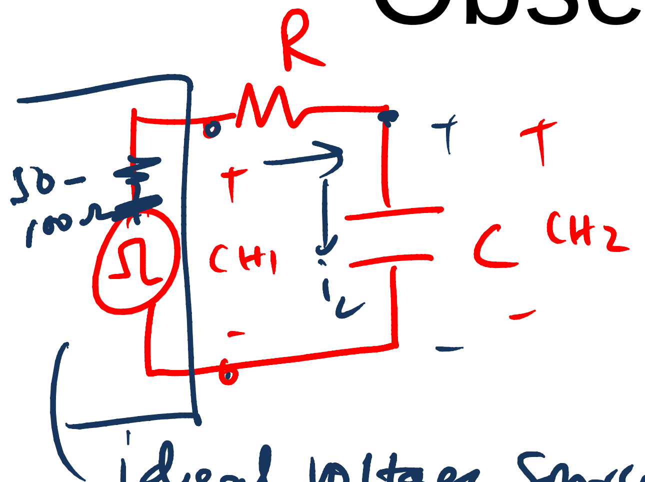
- To learn about the behavior of capacitors and inductors in DC circuits.
- To learn about the use of the Oscilloscope.
- To measure the time constants for RC, RL circuits using the oscilloscope.

Oscilloscope

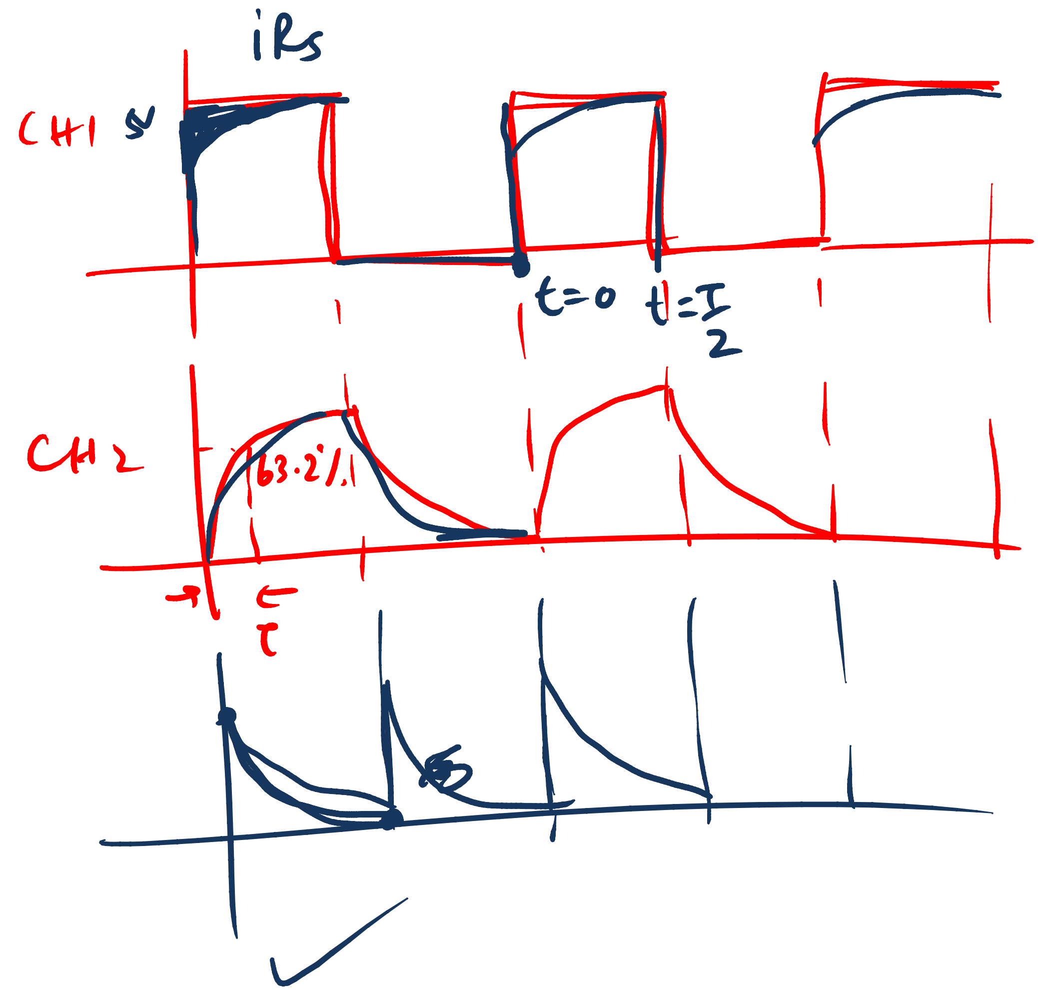
- Basic function
- The probe
- Internal square wave
- Auto-setup
- Vertical scale
- Horizontal scale
- Trigger source and level
- Cursor for measuring signals



Observations in Lab4



ideal voltage source



$$i_c(0^+) = \frac{5 - V_c^{0}}{R_s + R}$$

$$i_c(T/2) = \frac{5 - V_c(T/2)}{R_s + R}$$

$$= 0 .$$

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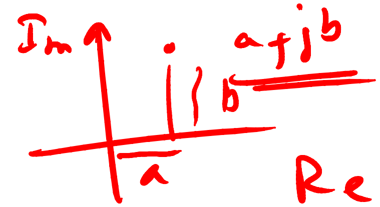
Lecture 6

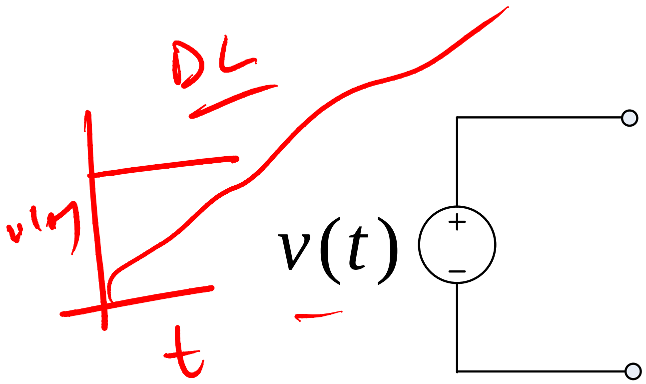
Learning objectives

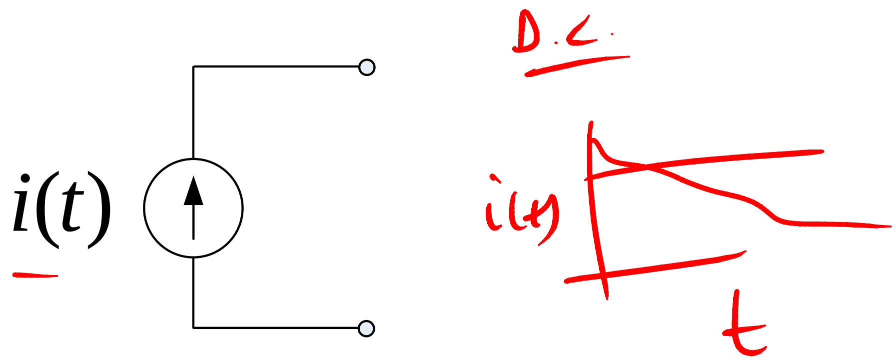
- Time-varying sources
- RMS (Root Mean Square) value for A.C.
- Sinusoids : $\sin(\omega t+\theta)$, $\cos(\omega t+\theta)$
- Complex algebra $1+j2$



- Phasor
- Impedances
- AC ckt analysis with phasors and impedances

Time dependent sources

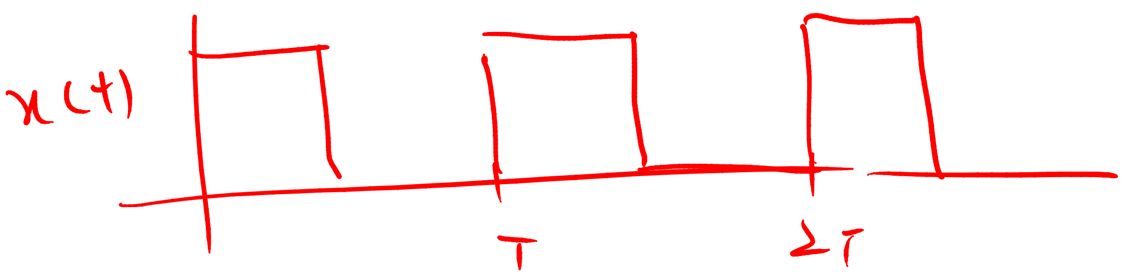




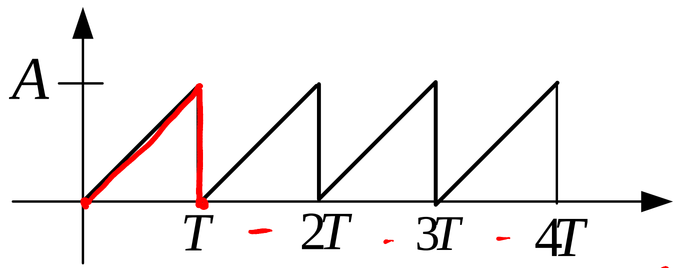
- Periodic functions

$$x(t) = x(t+nT), \quad n = 1,2,3,\ldots$$

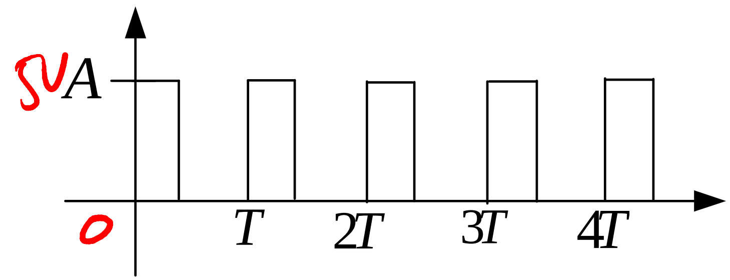
T = Time period



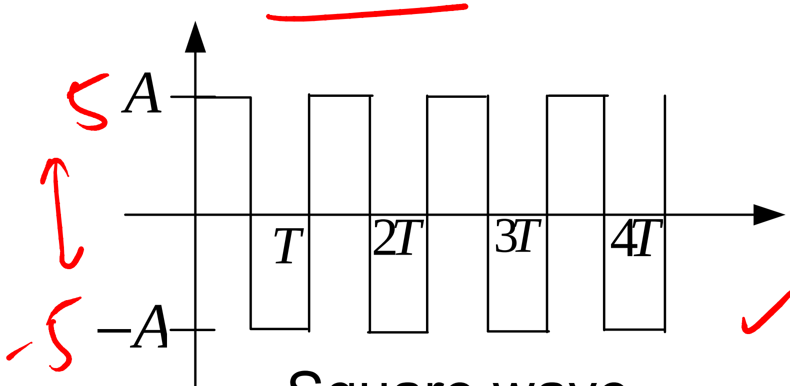
Common periodic signals



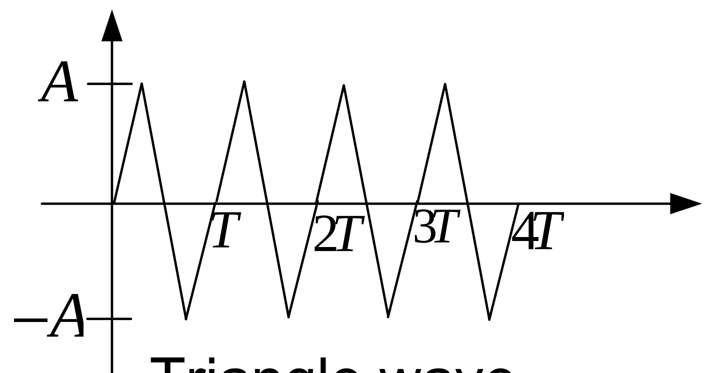
Sawtooth wave



Pulse train (TTL)



Square wave



Triangle wave

Periodic signals

- The **time period** of the signal is defined as the time taken to complete one cycle
- The **frequency** of the periodic signal is the number of cycles completed in one second. The units of frequency are hertz (Hz).
- **Angular frequency** (radians per second) , as one period in time corresponds to radian.

$\omega = 2\pi \times f$

Root-mean-square (RMS) values

$$p(t)=\frac{v^2(t)}{R}$$

$$E_T=\int_0^T p(t)dt=\int_0^T \frac{v^2(t)}{R}dt$$

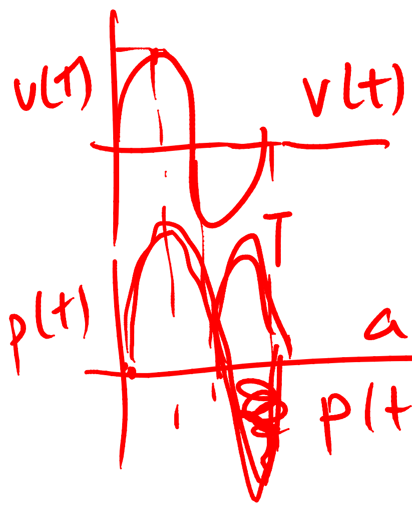
average power $= \frac{1}{T}\int_0^T p(t).dt$

$$P_{avg}=\frac{E_T}{T}=\frac{1}{T}\int_0^T p(t)dt=\frac{1}{T}\int_0^T\frac{v^2(t)}{R}dt=\frac{\frac{1}{T}\int_0^T v^2(t)dt}{R}$$

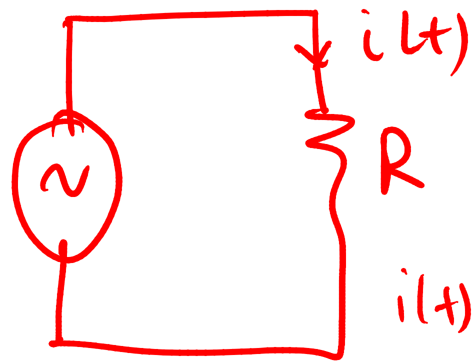
$$\frac{1}{T}\int_0^T v^2(t)dt=V_{rms}^2$$

$$\boxed{V_{rms}=\sqrt{\frac{1}{T}\int_0^T v^2(t)dt}}$$

$$P_{avg}=\frac{V_{rms}^2}{R}$$



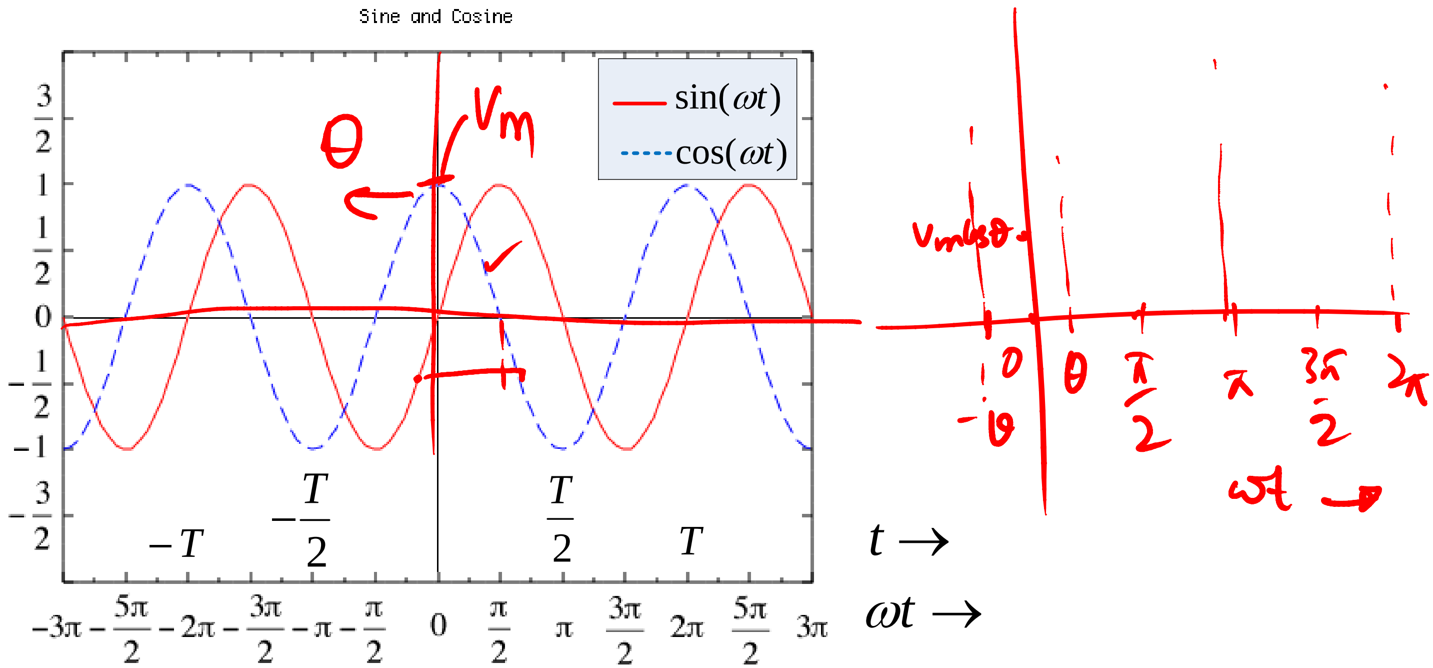
a.c.



$i(t)=\frac{v(t)}{R}$

$p(t)=v(t)\,i(t)=\frac{v^2(t)}{R}$

Sinusoidal signals



$v(t) = V_m \cos(\omega t + \theta)$ V_m is the peak value of the voltage

$V_m = V_m \cos(\omega t + \theta)$ θ is the phase angle

At $t = 0$

$\omega t + \theta = 0$

$v(t) = V_m \cos\theta$

$\omega t = -\theta$, ω = angular freq,

RMS for a sinusoid

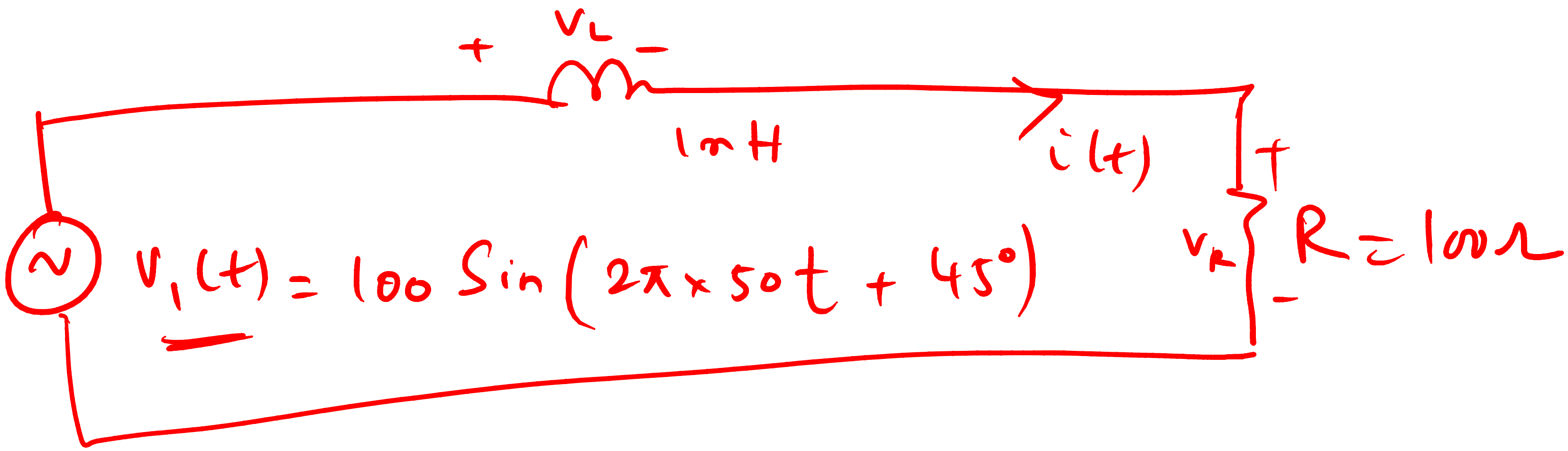
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta)dt}$$

$$= \sqrt{\frac{1}{T}\frac{V_m^2}{2}\int_0^T (1+\cos 2(\omega t + \theta))dt} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

- For example when we say the PUB supply in Singapore is 230V, we mean that the RMS value of the PUB supply is 230V and its peak value would be

$$V_m = \sqrt{2}V_{rms} = 230 \times \sqrt{2} = 230 \times 1.414 = 325V$$

Circuits with sinusoidal sources



$$v_L = L\frac{di}{dt}$$

$$v_1(t) = v_L(t) + v_R(t)$$

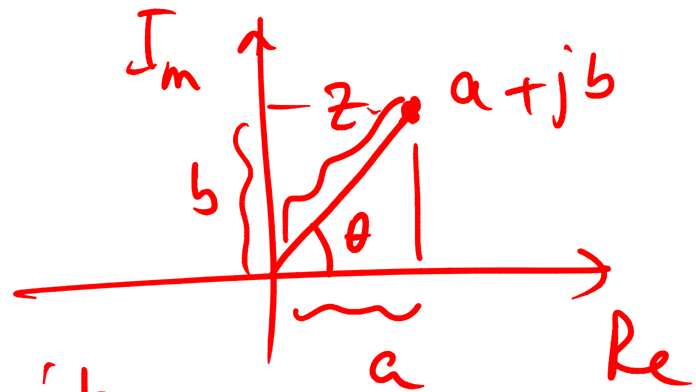
$$L\frac{di(t)}{dt} + iR = v_1(t)$$

Solve diff eqn with sinusoidal forcing fn?

$f(t)$ Forcing fn

Complex number

- Complex plane
  - Real axis
  - Imaginary axis



- Rectangular form : $a + jb$
- Polar form :
  - Magnitude
  - Angle

$$Z = \sqrt{a^2+b^2} \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

- Complex algebra

$A = 1 + j2$

$B = 0.5 + j1$

$A - B = (1-0.5) + j(2-1) = 0.5 + j(1)$

$A * B = (1+j2)(0.5+j1) =$

$A / B =$

Complex algebra

- Addition/Subtraction
  - Done in rectangular form
- Multiplication/Division
  - Done in polar form

$A = a_1 + jb_1$

$= Z_1 \angle \theta_1$

$B = a_2 + jb_2$

$= Z_2 \angle \theta_2$

$A * B = Z_1 \cdot Z_2 \angle \theta_1 + \theta_2$

$A \div B = (Z_1 \div Z_2) \angle \theta_1 - \theta_2$

Rect $\longrightarrow$ Polar

Phasor

- Sinusoidal voltages and currents can be represented as vectors in a complex plane.

Complex Number

- These are called Phasors and are very useful in steady-state analysis of sinusoidal voltages and currents.
- Phasor is just a definition. This gives rise to mathematical convenience. It has no physical significance.

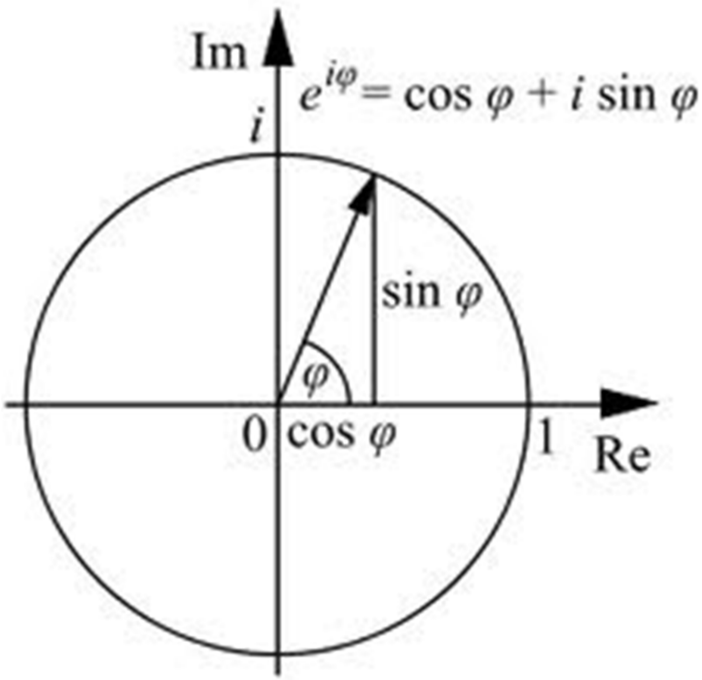
Phasor Definition

- For a sinusoidal voltage of

$$v_1(t) = V_1 \cos(\omega t + \theta)$$

- we define the phasor as: $\mathrm{V}_1 = V_1 \angle \theta_1$
- Thus, phasor of a sinusoid is a complex number having a magnitude equal to the peak value and having the same phase angle as the sinusoid.

Euler’s identity



$$e^{j\alpha} = \cos\alpha + j\sin\alpha$$

$$\cos\alpha = \mathrm{Re}(e^{j\alpha})$$

$$\cos(\omega t + \theta) = \mathrm{Re}(e^{j(\omega t + \theta)})$$

$$V_m \cos(\omega t + \theta) = \mathrm{Re}(V_m e^{j(\omega t + \theta)})$$

$$V_m e^{j(\omega t + \theta)} = V_m e^{j\theta} \times e^{j\omega t}$$

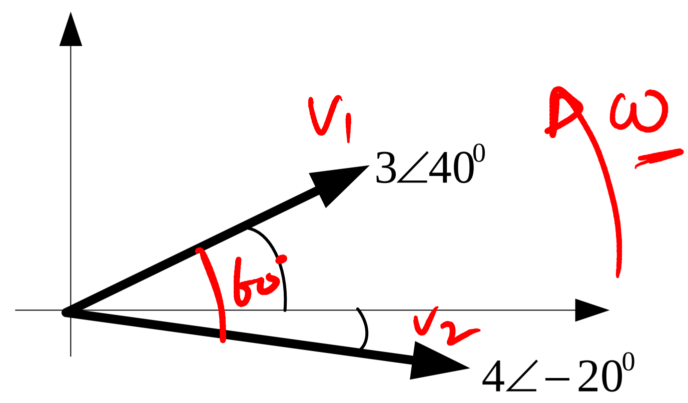
Phasor as a rotating vector

- represents a vector in the complex plane whose magnitude is and whose angle with the real axis is . The angle changes with time or in other words the vector keeps rotating at the angular speed of rad/sec. and its angle at t=0 was .
- The projection of the vector along the real axis will be the original sinusoidal function
- The phasor can be thought of as a snap shot of the rotating vector at t=0.

Phasor Diagram

$v_1(t) = 3\cos(\omega t + 40^0) \rightarrow 3\angle 40^0$

$v_2(t) = 4\cos(\omega t - 20^0) \rightarrow 4\angle -20^0$



phasor diagram

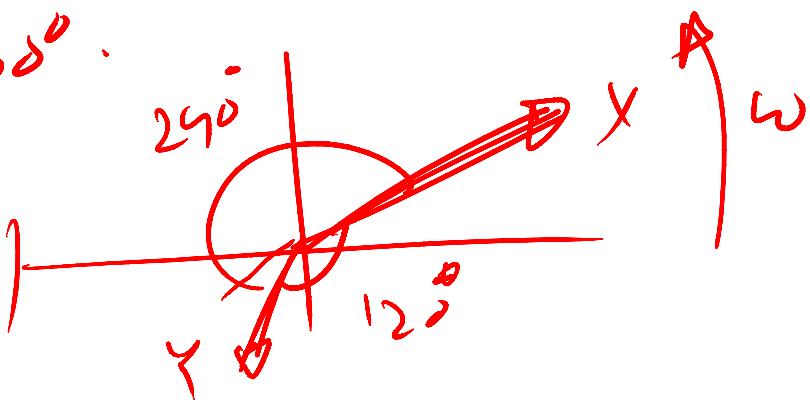
- **Leading / lagging**

v_1 leads v_2 by $(40-(-20)) = 60^\circ$

V_2 lags V_1 by 60°.

Y is lagging X by 120°

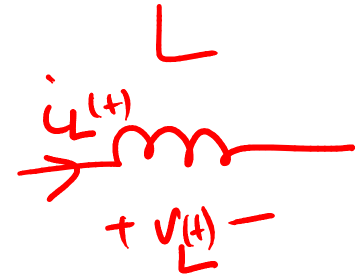
Y is leading X by 240°



Impedances

- Also known as complex resistance, frequency-dependent resistance.
- With understanding of impedances for the circuit elements like resistance, inductance and capacitance, the sinusoidal steady-state analysis will be same as analysis of purely resistive circuits under DC supply.

Inductance



$$i_L(t) = I_m \sin(\omega t + \theta) = I_m \cos(\omega t + \theta - 90^0)$$

$$v_L(t) = L\frac{di_L(t)}{dt} = L\omega I_m \cos(\omega t + \theta)$$

$$\mathrm{I_L} = I_m \angle \theta - 90^0$$

$I_L = I_m \angle \theta - 90^\circ$, $V_L = L\omega I_m \cdot \angle \theta$

$$\mathrm{V_L} = \omega L I_m \angle \theta = \omega L \angle 90^0 \times I_m \angle \theta - 90^0 = \mathrm{Z_L} \times \mathrm{I_L}$$

$$Z_L = \omega L \angle 90^0 = j\omega L$$

: Impedance of an inductor.

Ohm's law: $V = R \cdot I$

complex

resistance

Freq. dependent

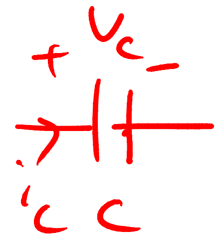
range

Capacitance

$$V_c = Z_c I_c$$

$$Z_c = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^0$$

$$i_c = C\frac{dV_c}{dt}$$



$$V_c(t) = V_m \cos(\omega t + \theta)$$

$$i_c = C \cdot V_m \cdot \omega\,(-1) \cdot \sin(\omega t + \theta)$$

$$-\sin\theta = \cos\left(\theta + \frac{\pi}{2}\right)$$

$$= C V_m \omega \cos\left(\omega t + \theta + \frac{\pi}{2}\right)$$

$$v_c(t) \rightarrow V_C = V_m \angle\theta$$

$$i_c(t) \rightarrow I_c = C\omega V_m \angle\theta + \frac{\pi}{2} = C\omega\angle\pi/2 \cdot V_m\angle\theta = \frac{V_c}{Z_c}$$

$$\frac{1}{Z_c} = C\omega\angle\pi/2$$

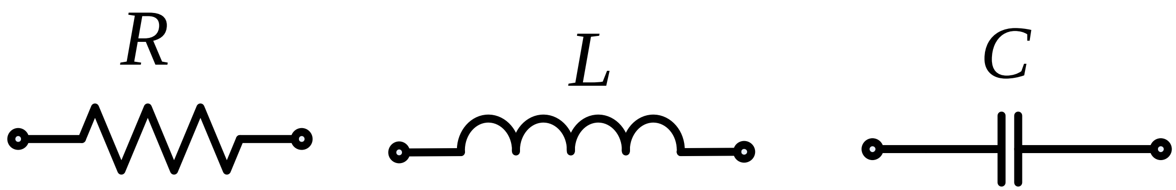
$$Z_c = \frac{1}{C\omega}\cdot\angle -\pi/2$$

Resistance

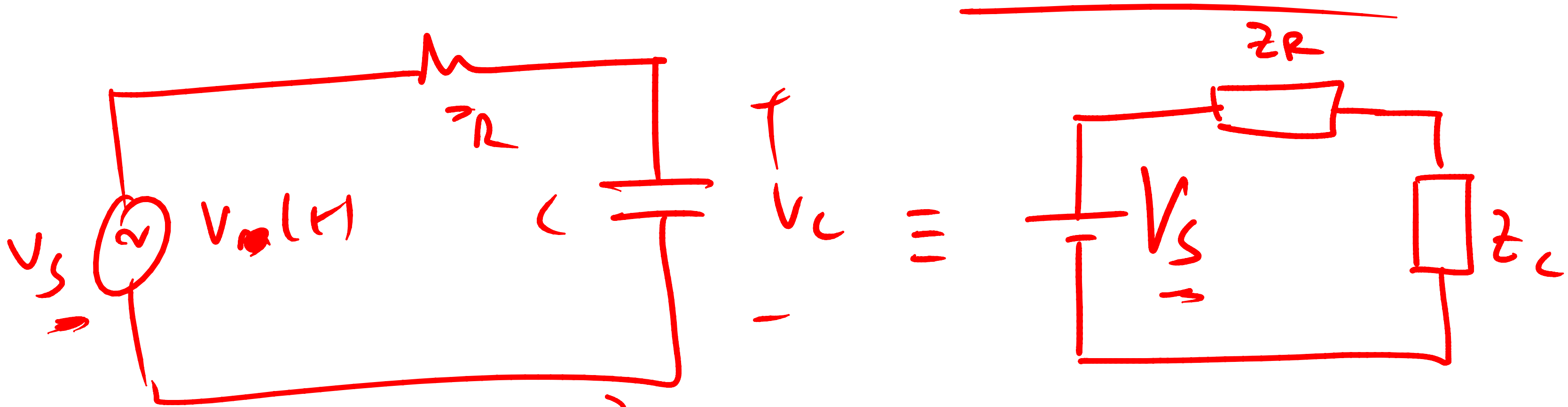
$$V_R = RI_R$$

$$Z_R = R$$

Impedances for R, L and C

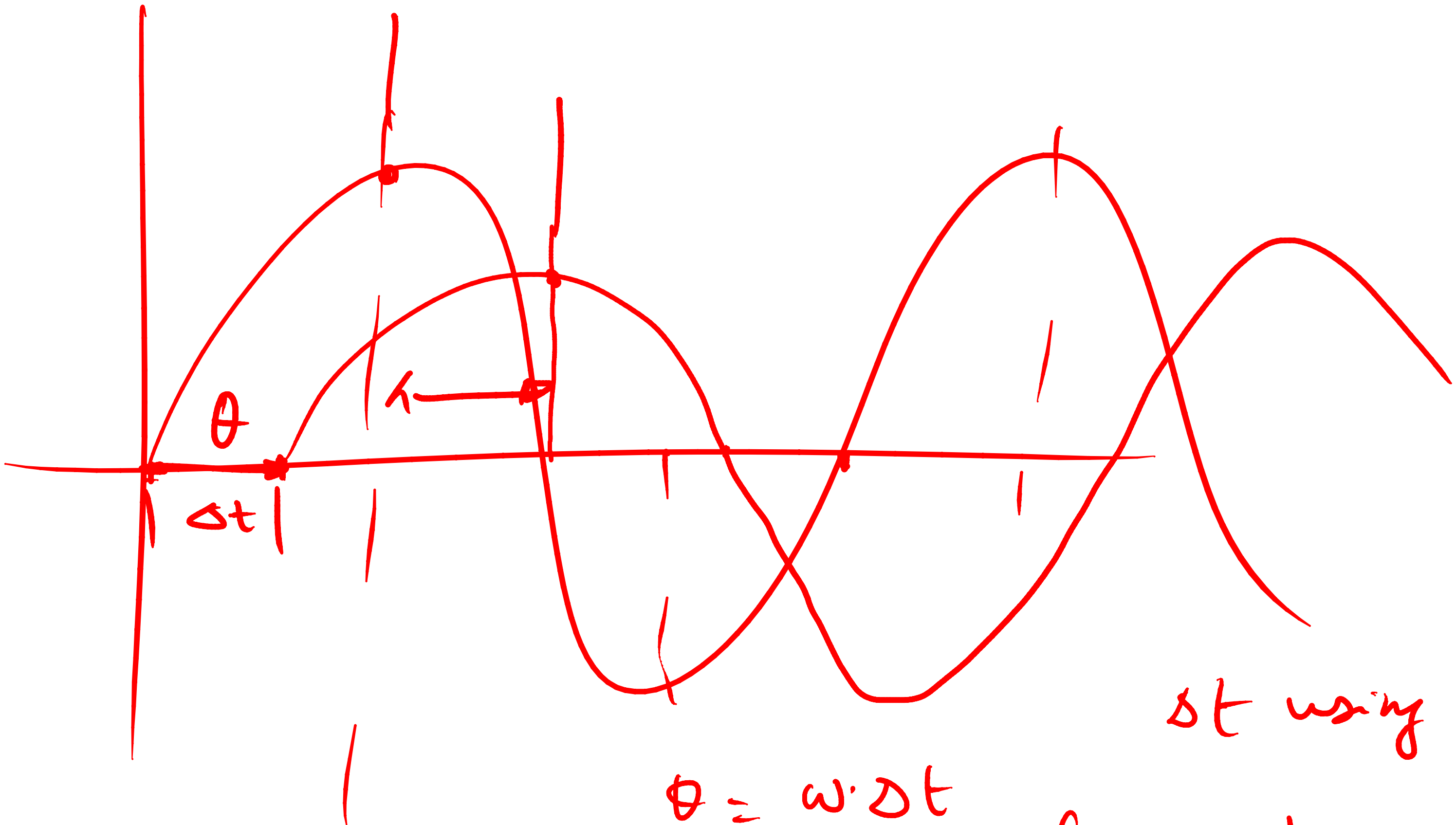


$Z_R = R$ $\qquad$ $Z_L = j\omega L$ $\qquad$ $Z_C = \dfrac{1}{j\omega C}$



$$\underline{V}_C = \underline{V}_s \cdot \frac{Z_c}{Z_R + Z_c}$$

Circuit Analysis with phasors and Complex Impedances



Δt using cursors

$\theta = \omega \cdot \Delta t$

$= 2\pi \times f \times \Delta t$

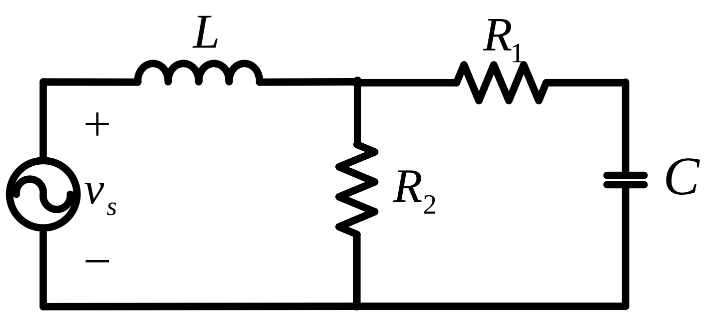
Step-by-step procedures for steady-state analysis of circuits with sinusoidal sources:

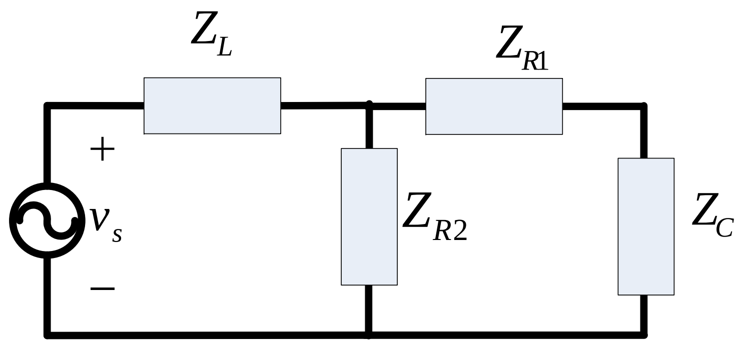
- All sources must have the same frequency.
- Replace the time descriptions of the voltage and current sources with the corresponding phasors.
- Replace the inductance with its impedance and capacitance with its impedance of . Resistances have the same impedance as their resistance.

Step-by-step procedures for steady-state analysis of circuits with sinusoidal sources:

- Analyze the circuits as before with DC sources and resistances only.
- Convert the final results in phasor to time-domain form

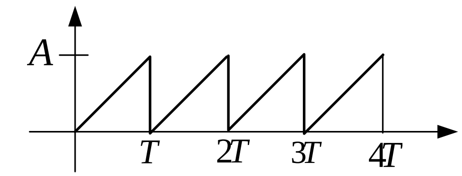
Example



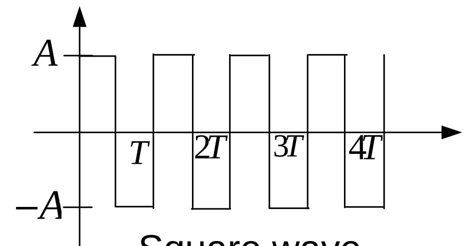


Lab5 Studying AC signals

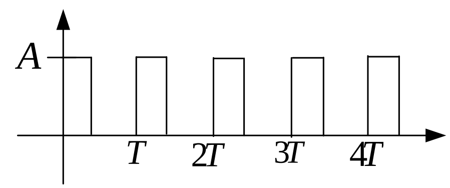
- Observing various types of ac signals using signal generator and oscilloscope.
- Measuring RMS of the signals with same peak-to-peak.



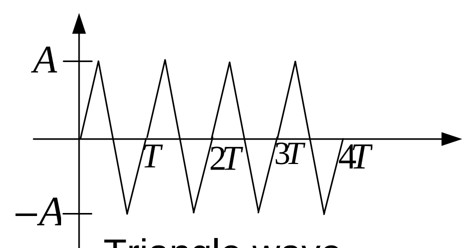
Sawtooth wave



Square wave

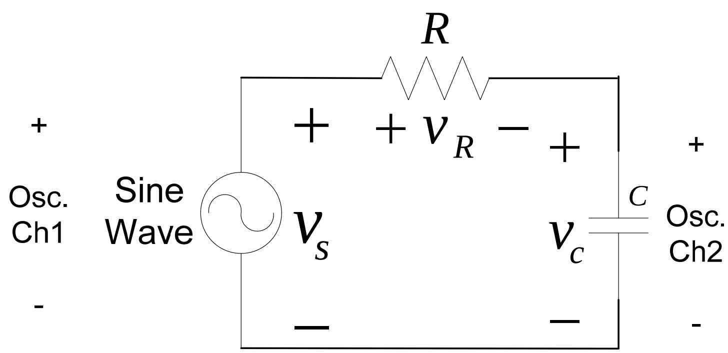


Pulse train



Triangle wave

Measuring Phase difference



Measuring Phase difference