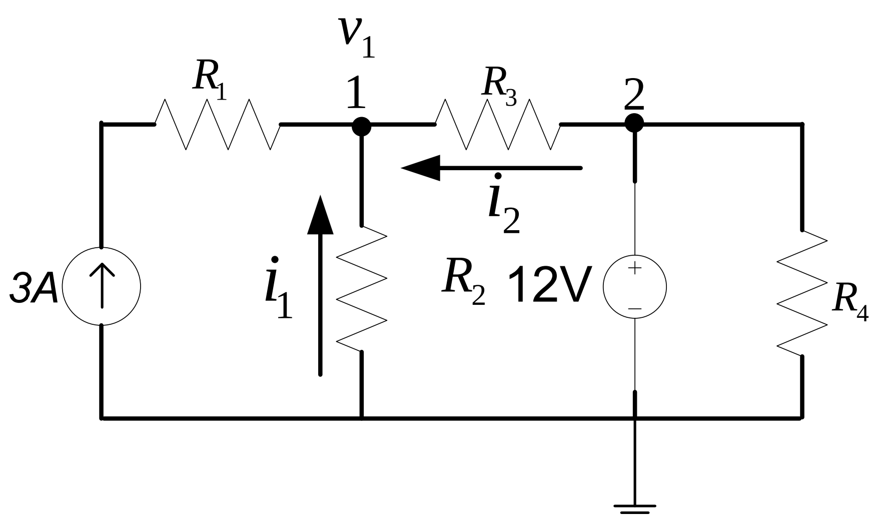
Tutorial 2

1. For the circuit shown in the figure find:
   a. The currents i1 and i2.
   b. The power delivered by the 3-A current source and by the 12-V voltage source.
   c. The total power dissipated by the circuit.

   Let R1=25 ohm, R2= 10ohm, R3=5 ohm, R4=7 ohm and express i1 an i2 as functions of v.



Solution:

Known quantities

A circuit with two known sources and a few resistances.

To find:

To find the current s in some branches.

Analysis:

We can find the currents if we know the voltage at node1.

Let the voltage at node 1 as v_1.

$$i_1 = -\frac{v_1}{R_2}$$

$$i_2 = \frac{12 - v_1}{R_3}$$

Ans:

a)

Applying the KCL at node 1, algebraic sum of currents entering =0.

$$3+\frac{0-v_1}{R_2}+\frac{12-v_1}{R_3}=0$$

$$\frac{3R_2R_3-v_1R_3+12R_2-v_1R_2}{R_2R_3}=0 \Rightarrow v_1=3\frac{(4+R_3)R_2}{R_2+R_3}$$

Putting the values of the resistances, $v_1 = 18V$.

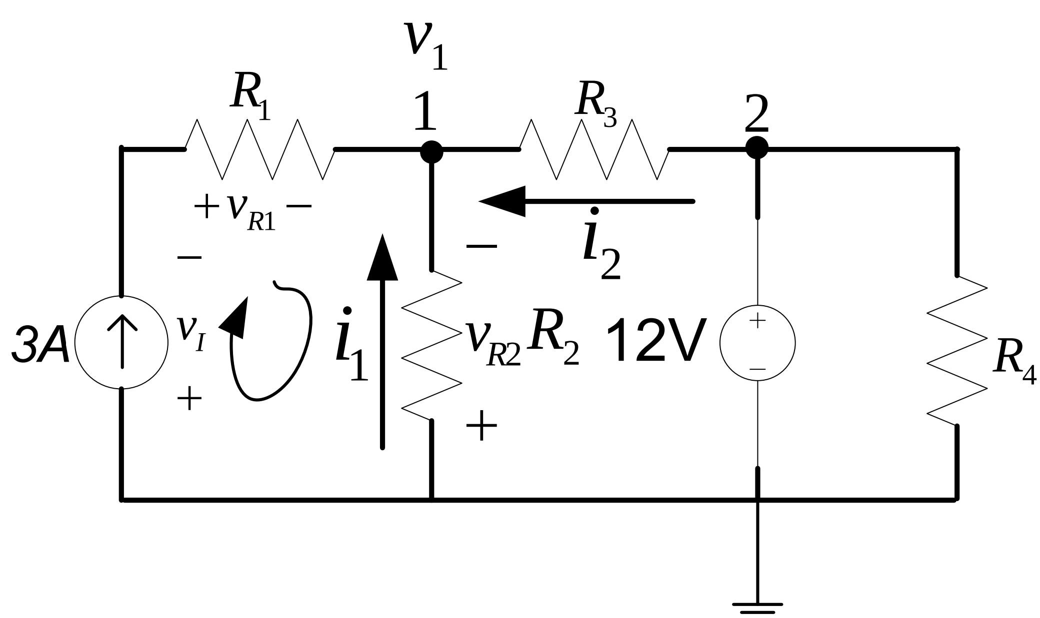
Therefore,

$$i_1 = -\frac{18}{10} = -1.8A$$

$$i_2 = \frac{12-v_1}{R_3} = \frac{12-18}{5} = -1.2A$$

b)

To find the power delivered by the current source, we need to know the votlage across it. We have labeled the voltages across the elements in the mesh shown in the figure. We can apply KVL around the loop to find this voltage.



Starting from node1 going clock-wise along the mesh, the algebraic sum of voltage rises around the mesh:

$$v_{R2}-v_I-v_{R1}=0$$

$$i_1R_2-v_I-3R_1=0$$

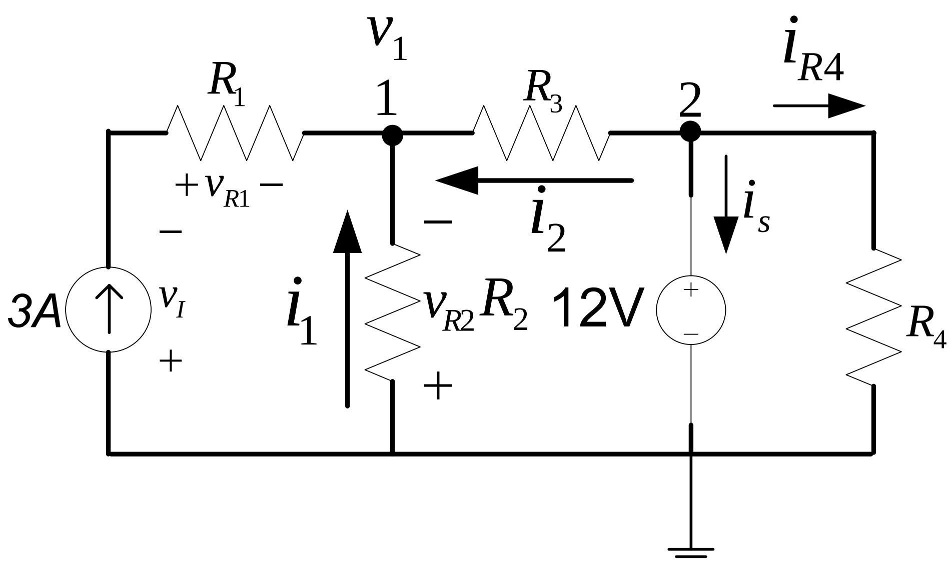
$$v_I=-1.8\times 10-3\times 25=-18-75=-93V$$

$$p=vi=-93\times 3=-279W$$

We have labeled the voltage and current reference directions according to the passive sign convention. Thus negative power implies that power is delivered by the current source.

c)

To find the power associated with the voltage source, we need to know the current through the voltage source. For this, we assign the current in resistance R4 and the source as shown.



Applying KCL at node2:

Sum of currents entering the node =0.

$$-i_s - i_2 - i_{R4} = 0$$

$$i_s = -i_2 - i_{R4} = 1.2 - \frac{12}{R_4} = -514.3mA$$

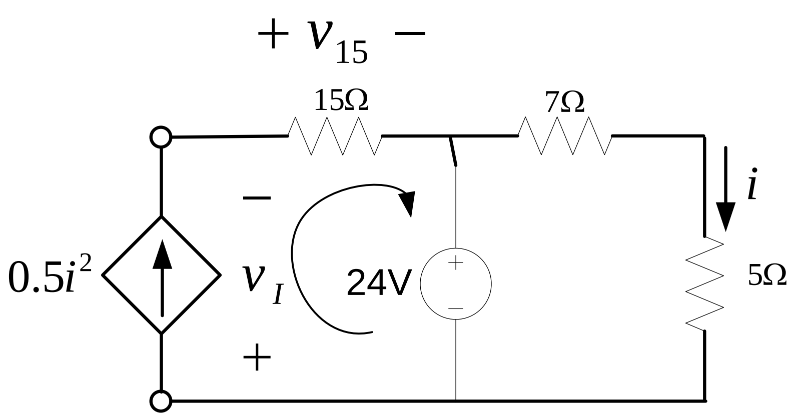
Power associated with the voltage source = $12 \times (-0.5143) = -6.17W$ i.e. power is delivered by the source.

c)

Total power dissipated by the circuit can be obtained from the conservation of energy principle, by equating with the total power delivered by the two sources.

Thus, total power dissipated = 279+6.17=285.7W.

2. Determine the power delivered by the dependent source in the circuit of the figure.



Known quantities:
A circuit with given voltage source and a dependent current source with known resistances.

Find:
To find the power delivered by the dependent current source.

Analysis:
To solve the voltage and current values for the dependent source.

We can find the current i from the circuit directly. Then we can find the value of the dependent current source. Then, we can apply KVL in the left mesh and find the voltage across the dependent current source.

Ans:

$$i = \frac{24}{7+5} = 2A$$

Value of the current source = $0.5 \times 4 = 2A$

Current through the 15 Ohm resistor = 2A

Applying KVL around the left mesh:
We have labeled the voltages according to passive sign convention.
Starting with the negative terminal of the voltage source, summing the voltage rises:

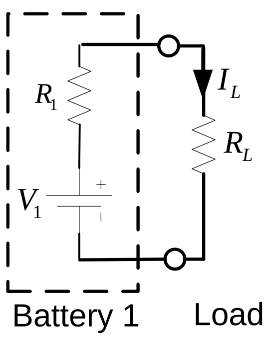
$$-v_I - v_{15} - 24 = 0$$

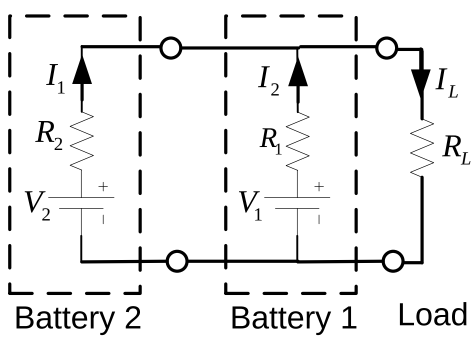
$$-v_I - 2 \times 15 - 24 = 0 \Rightarrow v_I = -54V$$

Power associated with the dependent current source = $v_I \times 2 = -54 \times 2 = -108W$ i.e power is delivered by the dependent current source.

Please note that there is current through the 24V source and hence no power delivered by the voltage source.

3. Consider NiMH hobbyist batteries shown in the circuit of the figure:
   a. If V1=12.0V, R1=0.15 ohm, RL=2.55 ohm, find the load current Il and the power dissipated by the load.
   b. If we connect a second battery in parallel with battery 1 that has voltage V2=12V and R2=0.28 ohm, will the load current IL increase or decrease? By how much?





Solution:
Known quantities:
The voltage and internal resistance of two batteries. The resistance of the load.

To find:
The change in load current if the second battery is connected in parallel to the first battery.

Analysis:

We can find the current with one battery easily.

When the second battery is connected, we can apply mesh current analysis to find the load current.

Ans:

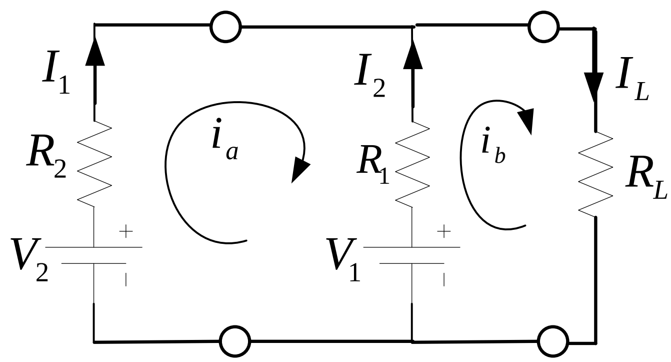
a)

With the first battery, the load current: $i_L = \dfrac{V_1}{R_1 + R_L} = \dfrac{12}{0.15 + 2.55} = \dfrac{12}{2.70} = 4.44A$.

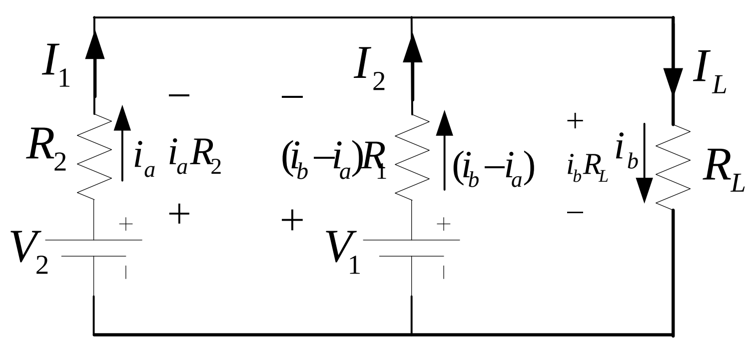
Power in the load = $i_L^2 R = 4.44 \times 4.44 \times 2.55 = 50.4W$

b)

We can consider two meshes as shown in Figure:



Then, we can find the branch currents and voltages in terms of the mesh currents:



Then, we apply the KVL around mesh a and mesh b to obtain the two independent equations.

$$V_2 - i_a R_2 + (i_b - i_a)R_1 - V_1 = 0$$
$$V_1 - (i_b - i_a)R_1 - i_b R_L = 0$$

Rearranging the two equations:

$$-i_a(R_1 + R_2) + i_b R_1 = V_1 - V_2 \qquad (1)$$

$$i_a R_1 - i_b(R_1 + R_L) = -V_1 \qquad (2)$$

Putting the values of the resistors and voltages we get:

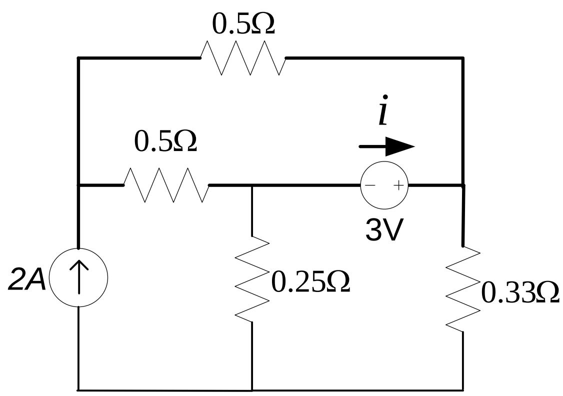
$$eqn(1): -0.43 i_a + 0.15 i_b = 0 \Rightarrow i_a = \frac{0.15}{0.43} i_b$$

$$eqn(2): 0.15 i_a - 2.70 i_b = -12 \Rightarrow i_b = 4.53A, i_a = 1.58A$$

The new load current = 4.53 A

The load current increases by 4.53-4.44=0.09A

4. Using node voltage analysis in the circuit of the figure, find the current i through the voltage source.



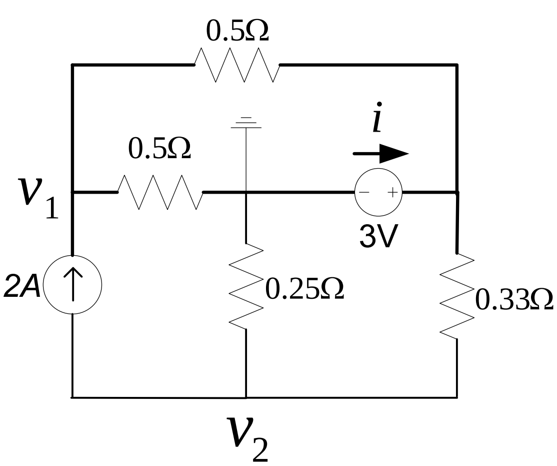
Known quantities:

Resistive circuit with one known current source and one known votlage source and four known resistances.

To find:
The current through the votlage soruce.

Analysis:
Use node votlage analysis.

Ans:



The negative terminal of the voltage source is marked as the reference.

Two nodes are marked as node voltage v_1, v_2.
The KCL at the two nodes will be:

$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} - 2 = 0 \quad (1)$$

$$\frac{v_2}{0.25} + \frac{v_2 - 3}{0.33} + 2 = 0 \quad (2)$$

Eqn(1)and (2) give:

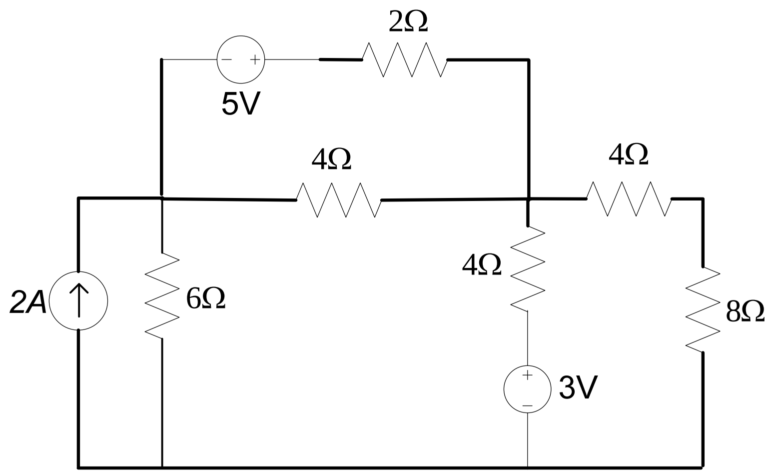
$$v_1 = 2V$$

$$v_2 = \frac{-2 * 0.25 * 0.33 + 3 * 0.25}{0.25 + 0.33} = 1.01V$$

The current through the voltage source, by apying KCL:

$$i = \frac{3 - v_1}{0.5} + \frac{3 - v_2}{0.33} = 2 + 6.03 = 8.03A$$

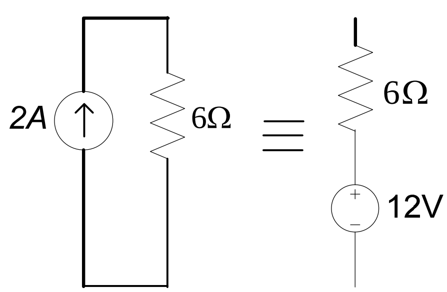
5. For the circuit in the figure, use mesh current analysis to find the matrices required to solve the circuit. DO NOT solve for the unknown currents. [Hint: you may find source transformation useful]



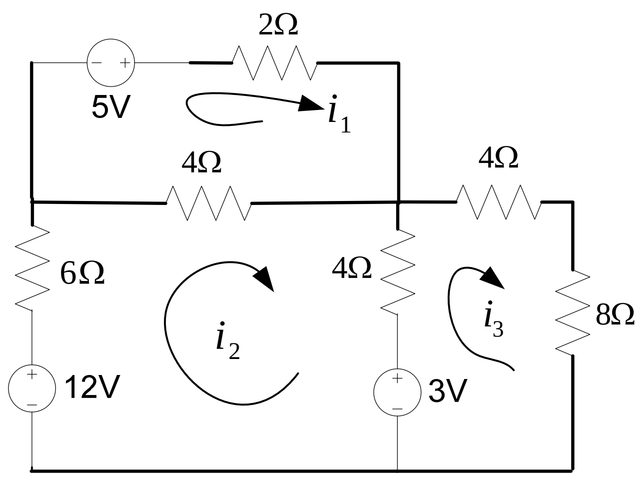
To find:
To solve the circuit using mesh analysis.

Analysis:

Current source with a parallel resistor can be transformed to a voltage source with a series resistor.



The modified circuit will be:



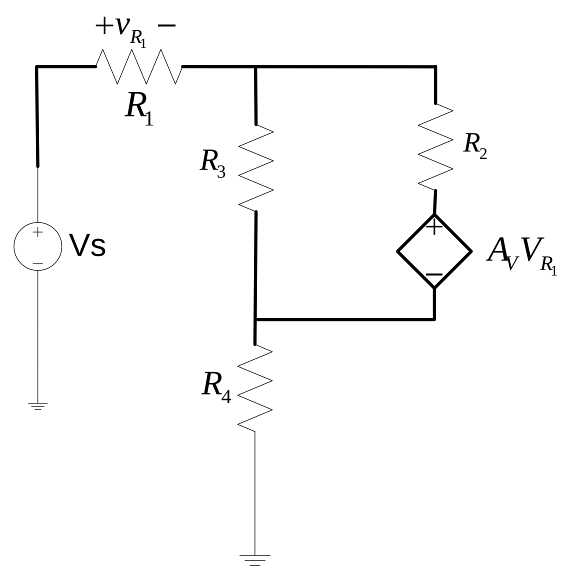
The matrix equation R I = V

$$\begin{bmatrix} 2+4 & -4 & 0 \\ -4 & 6+4+4 & -4 \\ 0 & -4 & 4+4+8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 12-3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 14 & -4 \\ 0 & -4 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 3 \end{bmatrix}$$

We can solve the matrix in various ways.

6. Using KCL, perform node analysis n the circuit shown in figure and determine voltage across R4. Note that one source is a controlled voltage source! Let Vs=5V, Av=70, R1=2.2kohm, R2=1.8kohm, R3=6.8kohm, R4=220ohm.



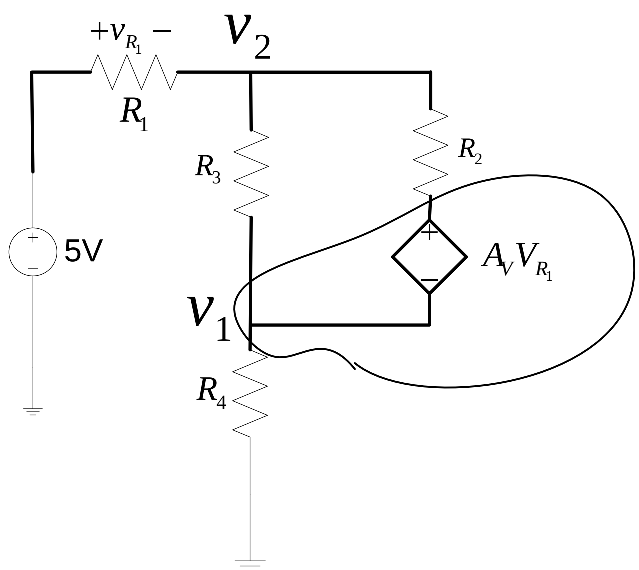
Solution:

Known quantities:
One independent source, one dependent source

To find:

Votlage across a resistor in the network.

Ans:
There are two unknown node voltages as shwon in the figure:



The dependent source voltage can be written in terms of the node voltages:

$$A_v V_{R1} = A_v(5 - v_2)$$

Writing the KCL at the two nodes:

At the super node:

$$\frac{v_1}{R_4} + \frac{v_1 - v_2}{R_3} + \frac{v_1 + A_v(5 - v_2) - v_2}{R_2} = 0$$

$$v_1\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{R_2}\right) + v_2\left(-\frac{1}{R_3} + \frac{-A_v - 1}{R_2}\right) = \frac{-5A_v}{R_2}$$

At node2:

$$\frac{v_2 - 5}{R_1} + \frac{v_2 - v_1}{R_3} + \frac{v_2 - (v_1 + A_v(5 - v_2))}{R_2} = 0$$

$$v_1\left(-\frac{1}{R_3} - \frac{1}{R_2}\right) + v_2\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1+A_v}{R_2}\right) = \frac{5A_v}{R_2} + \frac{5}{R_1}$$

Putting the values (factoring our 10^{-3}), we get:

$$5.2481v_1 - 39.592v_2 = -194.44$$

$$-0.7026v_1 + 40.046v_2 = 196.717$$

Solving these equaiton swe get:

$$v_1 = 8.7572mV$$

$$v_2 = 4.9124V$$