CG1108 Sem 2 AY2010/11

Part2

Lecture 5

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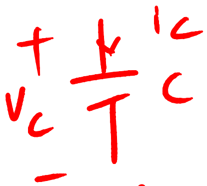
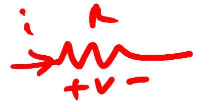
Mid-term Announcement

Date	:	5 Mar 2011 (Saturday)
Time	:	10am – 11am
Venue	:	LT6 and E3-06-01
Syllabus	:	Up to DC transients
		(Lecture 1 to Lecture 5)



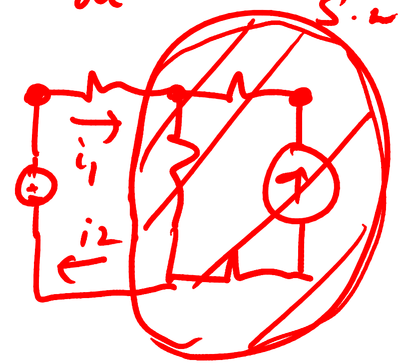


Topics learnt so far

$i_c = C\frac{dv_c}{dt}$ $v = iR$ $v_L = L\frac{di_L}{dt}$

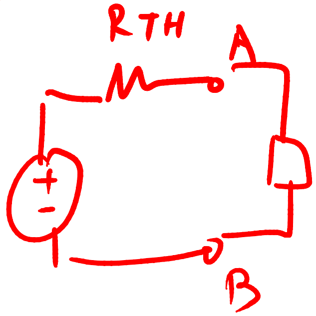
KCL to S.N.

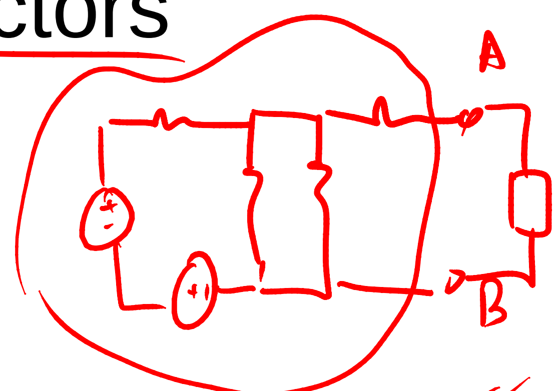
- Ohm's Law ✓ $I = \frac{V}{R}$, $V = IR$
- KVL, KCL Supernode
- Node analysis, Mesh analysis
- Thevenin equivalent, Norton equivalent
- Capacitors, Inductors



$i_1 - i_2 = 0$

$\Rightarrow i_1 = i_2$

$=$ 

$\leftarrow$ 

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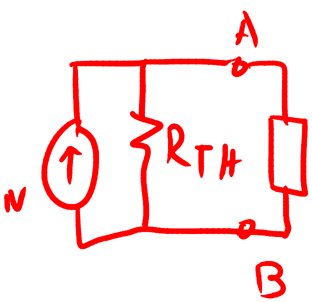
$\rightarrow$ 

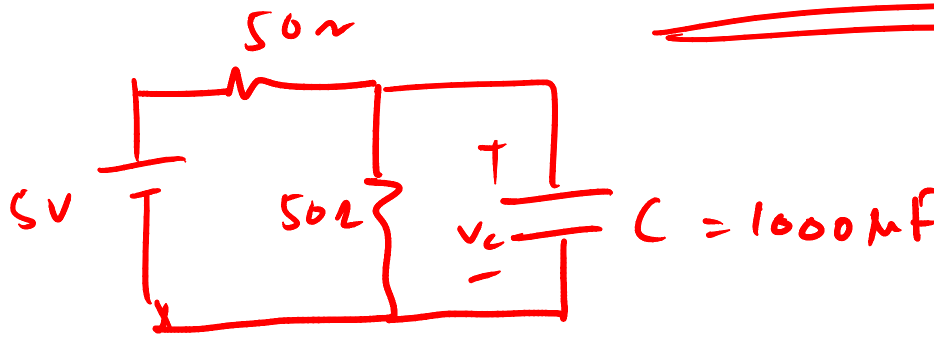
Remaining topics

- DC transients ✓ Lect 4 5 } Labs
- AC steady-state ✓ Lect 6
- Magnetic circuits and DC motor
- Diodes, BJTs and MOSFETs
- Digital Logic
- Instrumentation and Meas. Systems
- Autonomous Vehicle Project

Mode of learning

- Review past material
- Introduce the concepts
- Isolate and deal with the mathematics
- Solve examples
- Onto the hands-on part in the lab
- Tutorial

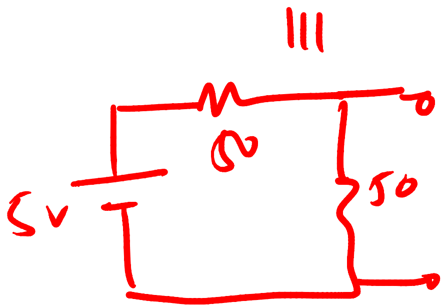
Review



$v_c = 2.5V$

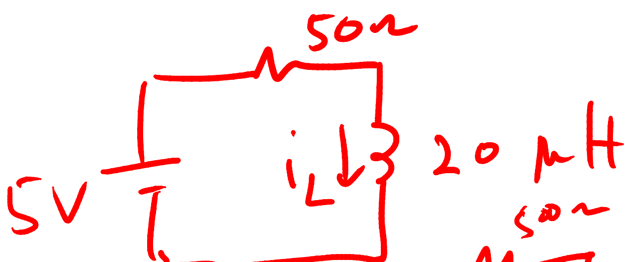
$i_c = C \dfrac{dv_c}{dt}$

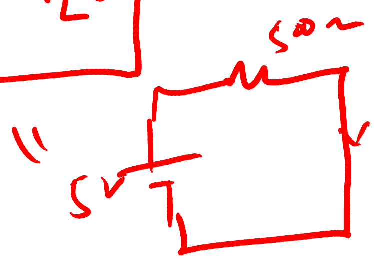
$v_L = L\, di/dt$



$v_c = 2.5V$ at steady-state.

$\dfrac{d}{dt}(x) = 0$





$I = \dfrac{5}{50} = 0.1A$

$i_L =$

open circuited → $i_c(s.s.) = 0$

short circuited $v_L(s.s.) = 0$

DC
Steady-State

DC Transients

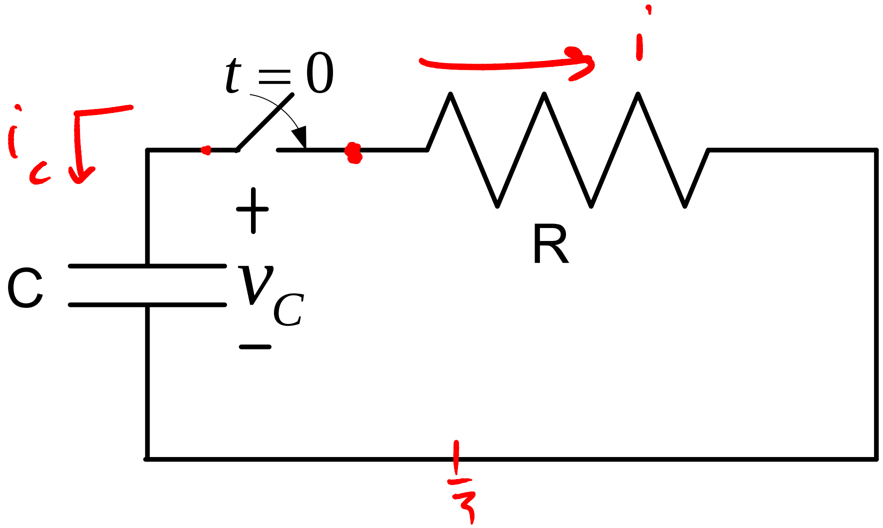
- Learning objectives:
  - Understand the meaning of transients.
  - Write differential equations for circuits containing inductors and capacitors.
  - Solving differential equations to find the time value of voltages and currents
  - Use of Oscilloscope and Signal generator

Transients

- The time-varying voltages and currents resulting from the adding or removing voltage and current source to circuits containing energy storage elements, are called **transients**.
- Voltage and current in such circuits are represented by **First-order differential equations.**

First order RC circuit

- Circuits with resistors and a single energy storage element (either inductor or capacitor) are said to be first-order circuit.



$$C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} = 0$$

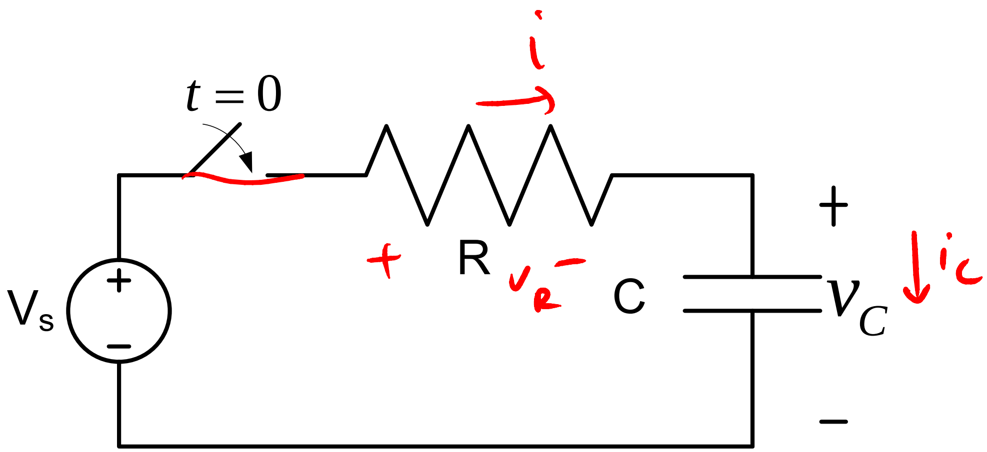
$$RC\frac{dv_c(t)}{dt} + v_c(t) = 0$$

$i_c = C\dfrac{dv_c}{dt}$

$i_c + i = 0$

$i = \dfrac{v_c}{R}$

RC Cricruit with a DC source



$$-V_s + v_R + v_C = 0$$ ✓ KVL

$$v_R = iR = RC\frac{dv_c}{dt}$$ ✓

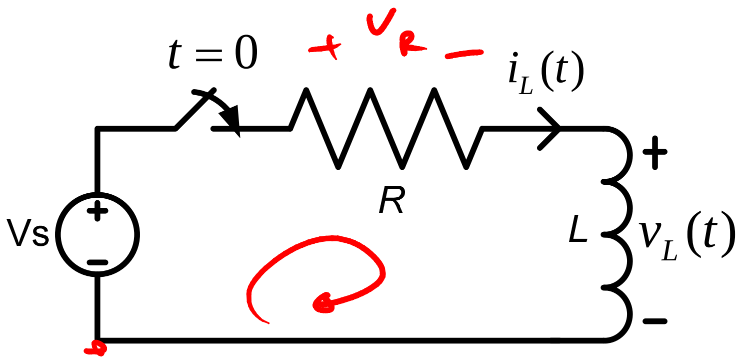
$$-V_s + RC\frac{dv_c}{dt} + v_c = 0$$ ✓

✓ $$RC\frac{dv_c}{dt} + v_c = V_s$$

v_c is the time-varying qty.

$v_{c\,ss} = V_S$

RL circuit with DC source



$-v_s + v_R + v_L = 0$ KVL

$$-V_s + iR + L\frac{di}{dt} = 0$$

$$\frac{L}{R}\frac{di}{dt} + i = \frac{V_s}{R}$$

$$i_{ss} = \frac{V_s}{R}$$

First order circuits with general sources

$$\tau \frac{dx(t)}{dt} + x(t) = f(t)$$

$f(t)$ is known as the forcing function

Steady-state is defined when time rate of the signal is zero.

$$\frac{dx(t)}{dt} = 0, x_{ss} = f(t)$$

Solution of Differential eqn

$$\tau \frac{dx(t)}{dt} + x(t) = f(t)$$

- Two parts of the general solution
  - Complementary solution (homogeneous eqn)
  - Particular solution (forced solution)

$$x(t) = x_c(t) + x_p(t)$$

Homogeneous equation

$$\tau \frac{dx_c(t)}{dt} + x_c(t) = 0$$ ← Homogeneous eqn.

$$\int \frac{dx_c(t)/dt}{x_c(t)} dt = \int \frac{-1}{\tau} \cdot dt$$

$$\ln\left[x_c(t)\right] = \frac{-t}{\tau} + c$$

$$x_c(t) = e^c e^{-t/\tau} = Ke^{-t/\tau}$$

- Determine the homogeneous solution by applying the initial condition to the complete solution

Particular solution

- The particular solution is obtained from the forcing function.
- It is normally of the same functional form as the forcing function and its derivatives.
- A table containing various forcing functions and their corresponding particular solutions are readily available.
- http://www.efunda.com/math/ode/linearode_undeterminedcoeff.cfm

When forcing function is DC

- The particular solution is a constant

$$\underline{\tau \frac{dx(t)}{dt} + x(t) = \boxed{f(t)}}$$

$$\tau \frac{dx_p}{dt} + x_p = \underline{K'}$$

Let $x_p = K'$.

Then $\underline{0} + \underline{K' = K'}$

i.e. $\underline{x_p = K'}$ is a solution

$f(t)$ is also the steady-state solution

Complete Solution

$$\tau\frac{dx}{dt}+x \;\; = K' \text{ where } K' \text{ is the steady-state solution } (x_{ss})$$

$$x(t) = Ke^{-\frac{t}{\tau}} + K'$$

Applying the initial condition, i.e. $x(t)$ at $t = 0$:

$$x(0) = Ke^{0} + K' \Rightarrow K = x(0) - K'$$

$\therefore$ final solution is:

$$x(t) = (x(0) - K')e^{-\frac{t}{\tau}} + K' = x(0)e^{-\frac{t}{\tau}} + K'(1 - e^{-\frac{t}{\tau}})$$

$$x(t) = x(0)e^{-\frac{t}{\tau}} + x_{ss}(1 - e^{-\frac{t}{\tau}})$$

Recap of the solution

$$\tau \frac{dx}{dt} + x = f(t)$$

$$x = x_c + x_p$$

S.S. soln : $\frac{dx}{dt} = 0 \Rightarrow x_{ss} = f(t)$

In our case $f(t)$ is a constant.

x_p is also a constant. $\underline{f(t) = k'}$

$\underline{x_p = k'} \rightarrow \tau \frac{d x_p}{dt} + x_p = k'$

$\tau \frac{dx_p}{dt} = 0$

$k' = k'$ ✓

$\underline{x_c}$

Homogeneous eqn : $f(t) = 0$

$$\tau \frac{dx}{dt} + x = 0 \Rightarrow \int \frac{dx}{x} dt = \int -1/\tau \cdot dt$$

$$\tau \frac{dx}{dt} = -x$$

$$\ln |x| = -\frac{t}{\tau} + C.$$

Recap of the solution

$$x_c = e^{\left(-t/\tau + c\right)} = \left(e^{c}\right) e^{-t/\tau} = K e^{-t/\tau}$$

$$=$$

$$x = x_c + x_p = \underbrace{K e^{-t/\tau} + K'}_{\text{unknown}}$$

Initial condition: $x(0) = x(t)\big|_{t=0}$

$$x(0) = K \cdot e^{-0/\tau} + K' = K + K'$$

$$\Rightarrow K = x(0) - K'$$

$$= x(0) - x_{ss}$$

$$x(t) = \left(x(0) - x_{ss}\right) \cdot e^{-t/\tau} + x_{ss}$$

$$= x(0) \cdot e^{-t/\tau} + x_{ss}\left(1 - e^{-t/\tau}\right)$$

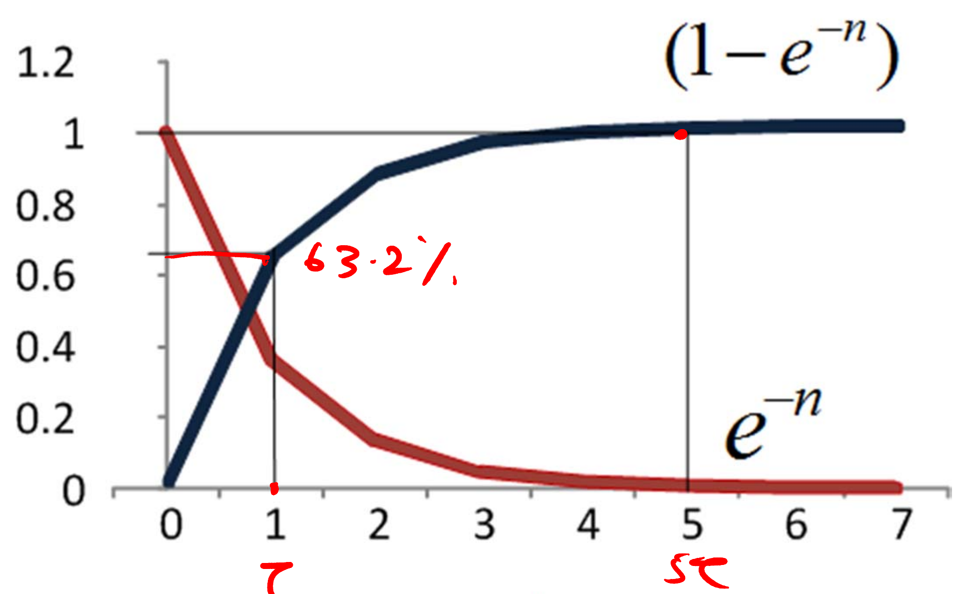
Nature of the solution

$$x(t) = x(0)e^{-\frac{t}{\tau}} + x_{ss}(1 - e^{-\frac{t}{\tau}})$$

τ = Time constant

n e^{-n} $(1-e^{-n})$

0	1	0
1	0.367879	0.632121
2	0.135335	0.864665
3	0.049787	0.950213
4	0.018316	0.981684
5	0.006738	0.993262
6	0.002479	0.997521
7	0.000912	0.999088



$$n = \frac{t}{\tau} \longrightarrow$$

RC and RL comparing to the general form

$$\tau \frac{dx}{dt} + x = K' \text{ where } K' \text{ is the steady-state solution } (x_{ss})$$

$$x(t) = x(0) e^{-\frac{t}{\tau}} + x_{ss}(1 - e^{-\frac{t}{\tau}})$$

$$RC \frac{dv_c}{dt} + v_c = V_s$$

$$\frac{L}{R} \frac{di_L}{dt} + i_L = \frac{V_S}{R}$$

$\tau_{RC} = RC$

$k' = V_S$

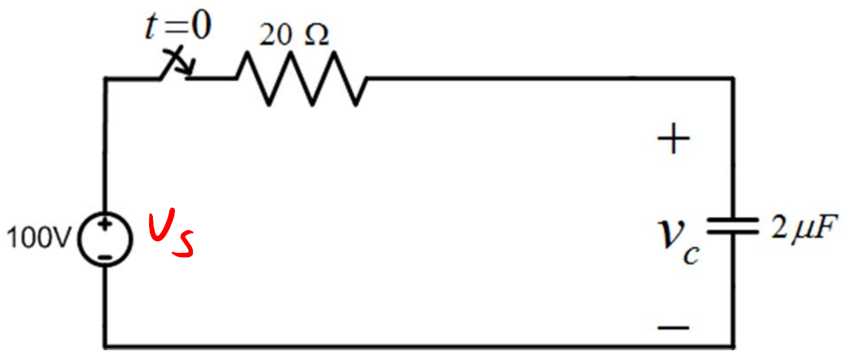
$v_c(t) = v_c(0) \cdot e^{-\frac{t}{RC}} + V_S(1 - e^{-t/RC})$

$\tau_{LR} = \frac{L}{R}$

$k' = \frac{V_S}{R}$

$i_L(t) = i_L(0) \cdot e^{-\frac{t}{4R}} + \frac{V_S}{R}(1 - e^{-t/4R})$

Example1



Find $v_c(t)$ after $t>0$.

$v_c(0) = 0$ initially uncharged

$v_c(ss) = 100V$

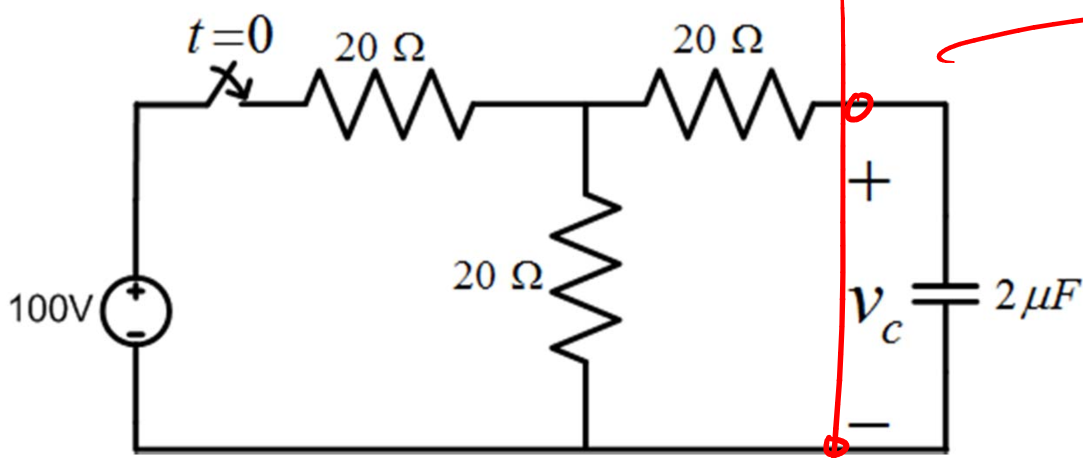
$\tau = RC = 20 \times 2\times10^{-6}$ S

$= 40\times10^{-6}$ S.

$$v_c(t) = 100\left(1 - e^{-\frac{t}{40\times10^{-6}}}\right) V$$

Example2

$v_c(t) = 50\left(1 - e^{-t/60\times10^{-6}}\right)\,V$

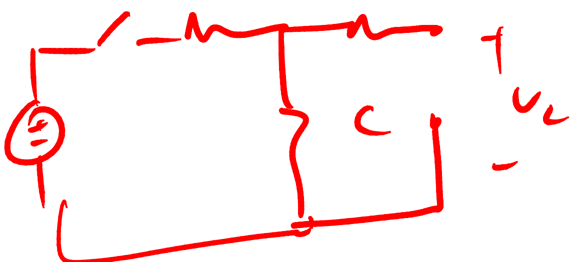


$v_c(t)$ for $t > 0$.

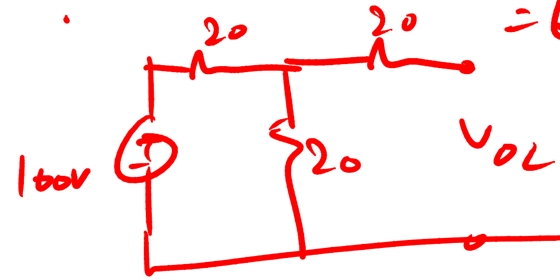
$v_c(0) = 0$

$v_{c\,ss} = 50$

$\tau = R_{TH} \times C = 30 \times 2\times10^{-6} = 60\ \mu s$

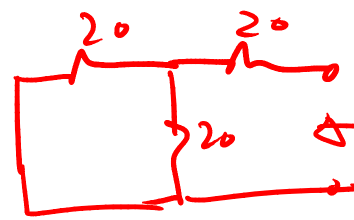


$V_{TH} =$



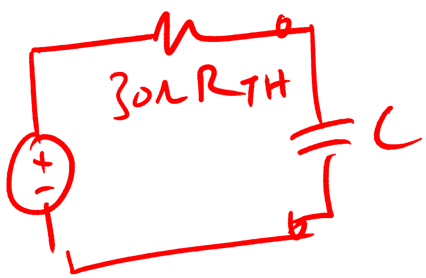
$V_{OC} = 50V$

$R_{TH} =$

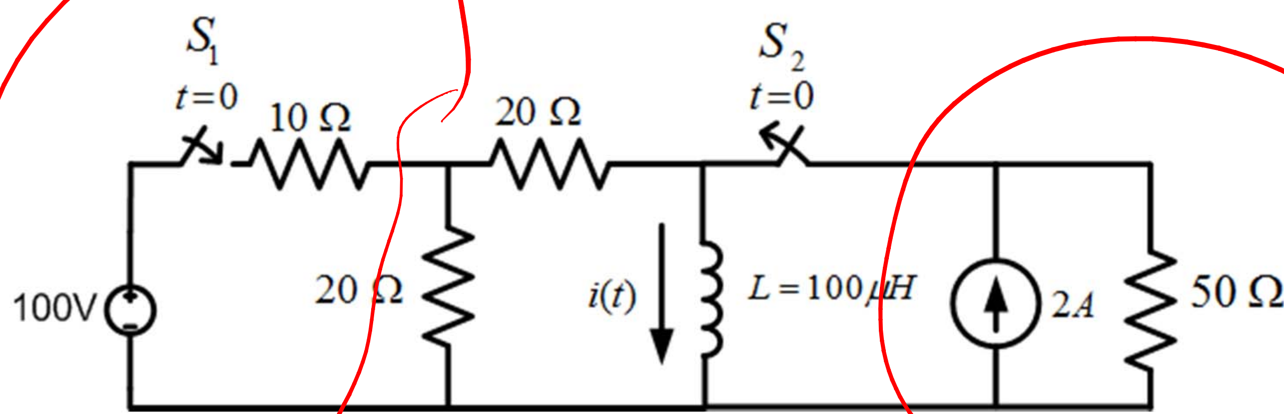


$R_{TH} = 30\Omega$

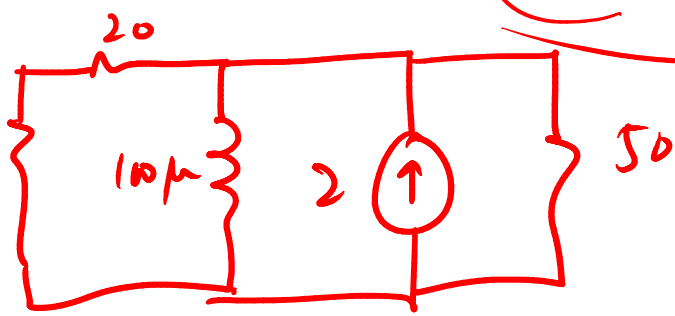
$50V\ V_{TH} = V_S$



Example3

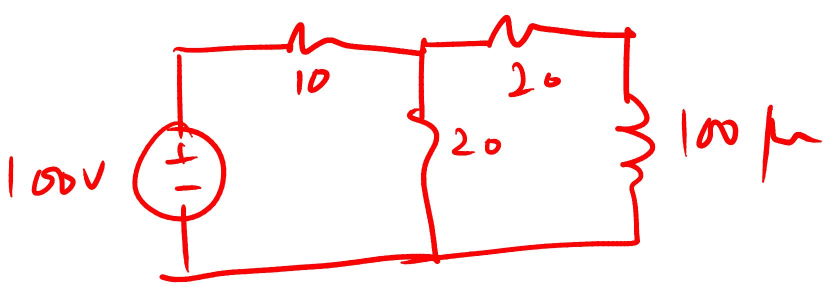


Before t=0



⇒ i(0)

After t=0



⇒ i_{ss}

τ

Lab4: Objectives

- To learn about the behavior of capacitors and inductors in DC circuits.
- To learn about the use of the Oscilloscope.
- To learn about the charging / discharging of capacitors.
- To measure the time constants for RC, RL circuits using the oscilloscope.

Lab4: Equipment to be used

- Lab DC power supply
- Digital multi-meter
- Breadboard
- Oscilloscope
- Signal Generator

Oscilloscope

- Basic function
- The probe
- Internal square wave
- Auto-exec
- Vertical scale
- Horizontal scale
- Trigger source and level
- Cursor for measuring signals

Oscilloscope Tutorial on Youtube

http://www.youtube.com/watch?v=qIfo_-d82Co&feature=channel

http://www.youtube.com/watch?v=hUIgAu3QQWQ&feature=channel

http://www.youtube.com/watch?v=g_KuGEh0PyA&feature=channel

Signal / Function Generator

- Functions
- Frequency setting
- Main and Aux/TTL output

http://www.youtube.com/watch?v=_pDz6e2ADew&feature=related