Singapore's Global University

CG1108
Electrical Engineering

Dr. John-John Cabibihan

Dept. of Electrical and Computer Engineering

EDWARD LAWRY NORTON
(28 July 1898 – 28 January 1983)

was an accomplished Bell Labs engineer famous for developing the concept of the Norton equivalent circuit.

Lecture Outline

- Pspice demonstration
- Revision
- Capacitance
- Inductance

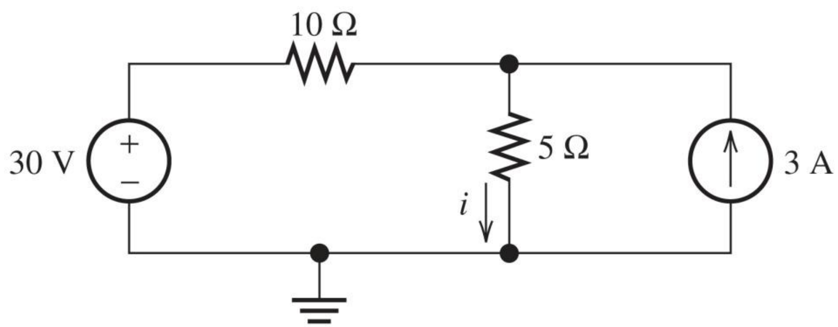
Superposition: killing a voltage/current source

- To kill a voltage source, we make its output voltage equal to zero.
- replace the voltage source with a short circuit

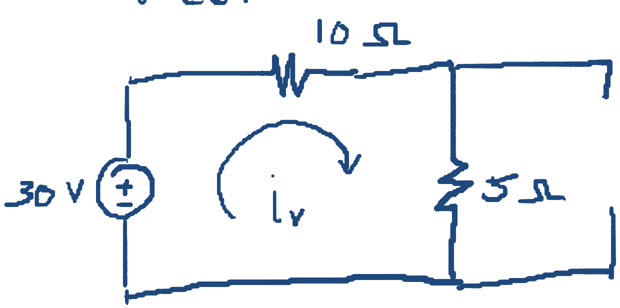
- To kill a current source, we make its output current equal to zero.
- replace the current source with an open circuit

Example

Use superposition to find the current *i*.

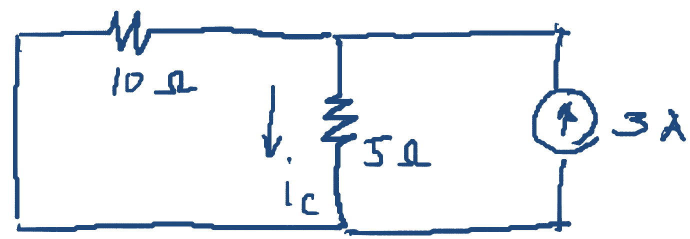


1. Zero the current source. Find the current due to the voltage source.



$i_v = 30/15 = 2A$

2. Zero the voltage source. Find the current due to the current source.

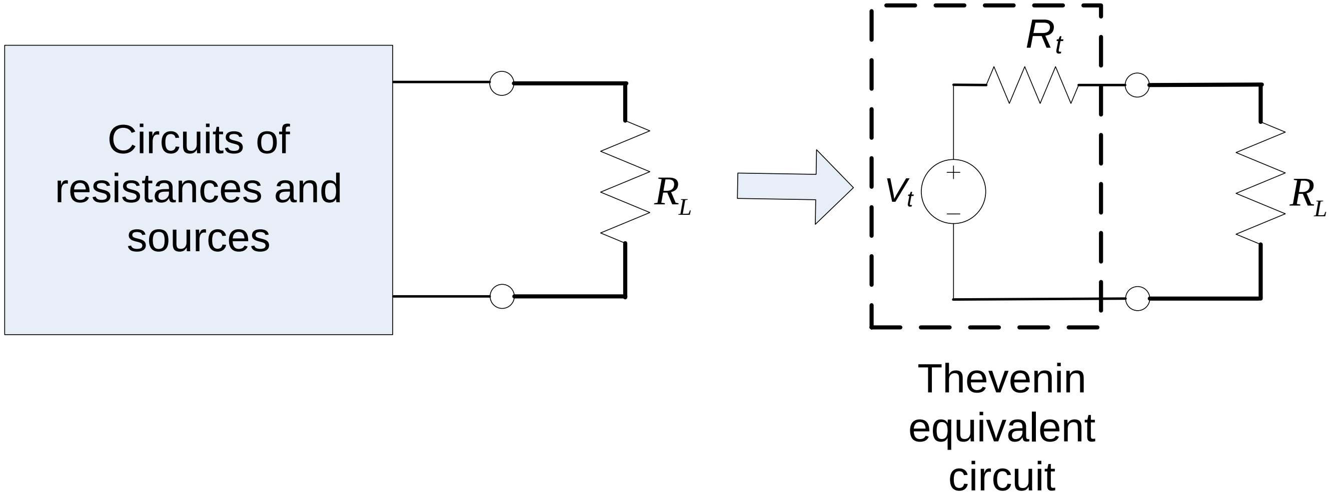


$$i_C = 3\left(\frac{10}{10+5}\right) = 2\,A$$

3. The total current is the sum of contributions from each source.

$$i = i_v + i_C = 2 + 2 = 4\,A$$

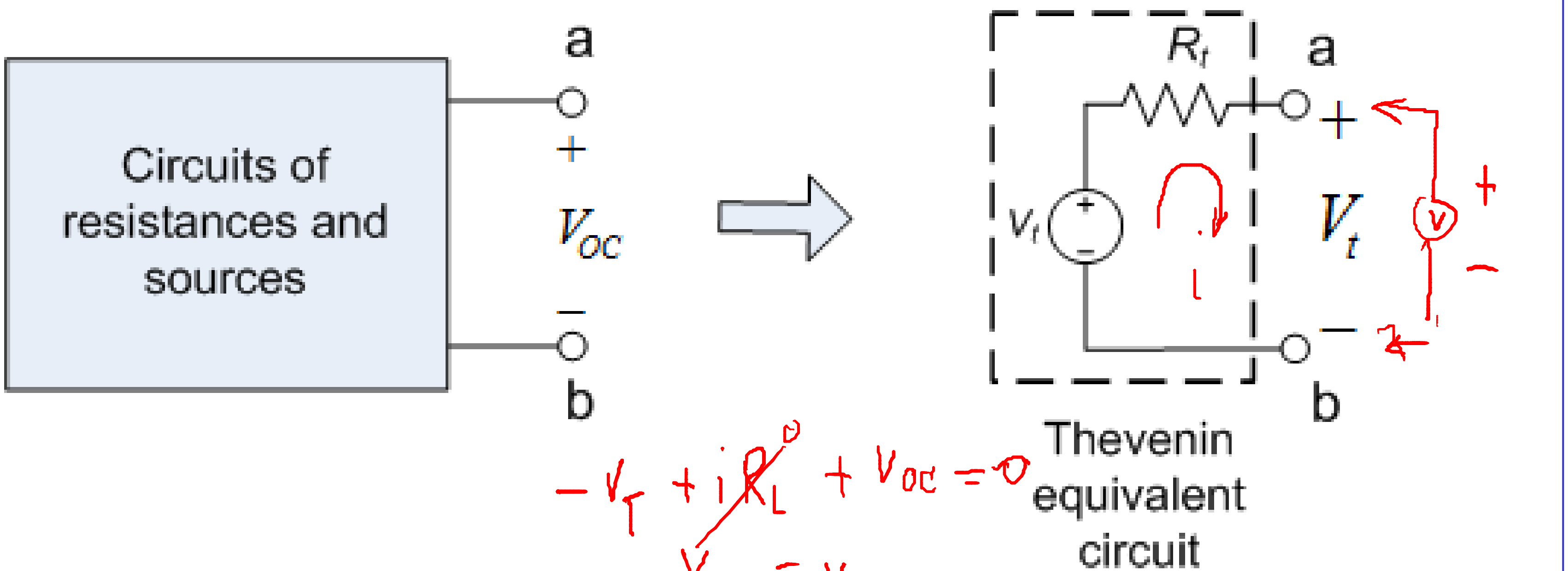
Thevenin equivalent



- A voltage source in series with a resistance
- The voltage source is called Thevenin’s voltage
- The series resistance is called Thevenin’s resistance

Thevenin Voltage

- The value of the voltage source is the open circuit voltage between the two terminals.
- This is called the Thevenin voltage.

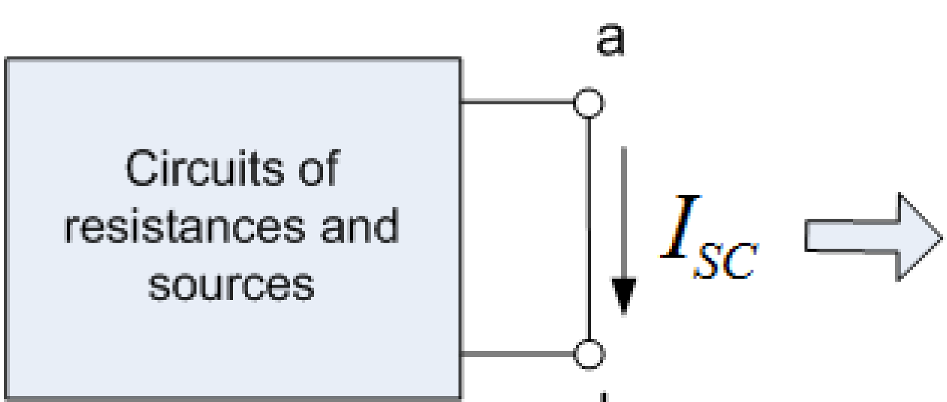


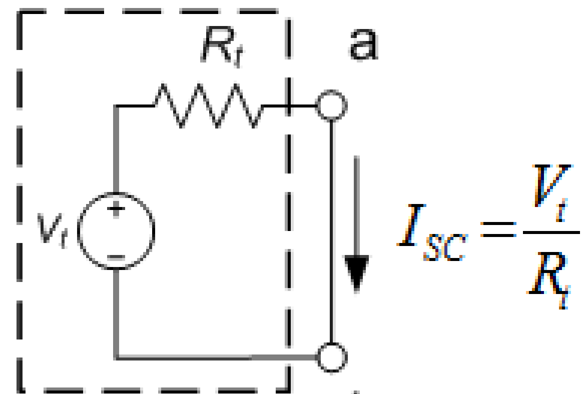
$-V_T + i\cancel{R_L}^{0} + V_{oc} = 0$

$V_{oc} = V_T$

Thevenin Resistance

- Find the short circuit current between the two terminals.
- Calculate the Thevenin resistance.



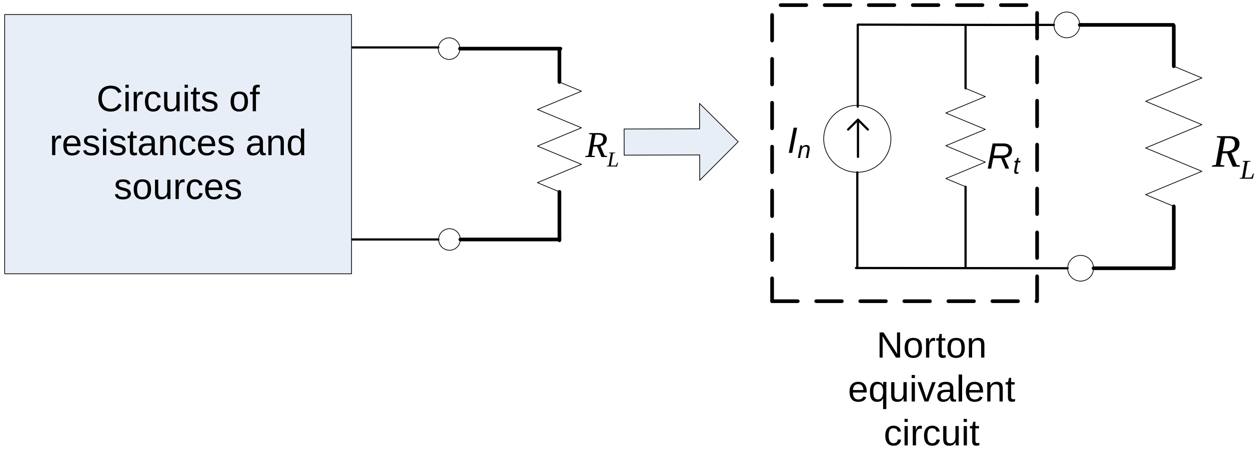


Thevenin equivalent circuit

$V_{oc} = V_T$

$I_{sc} = \frac{V_T}{R_T} \Rightarrow R_T = \frac{V_{oc}}{I_{sc}}$

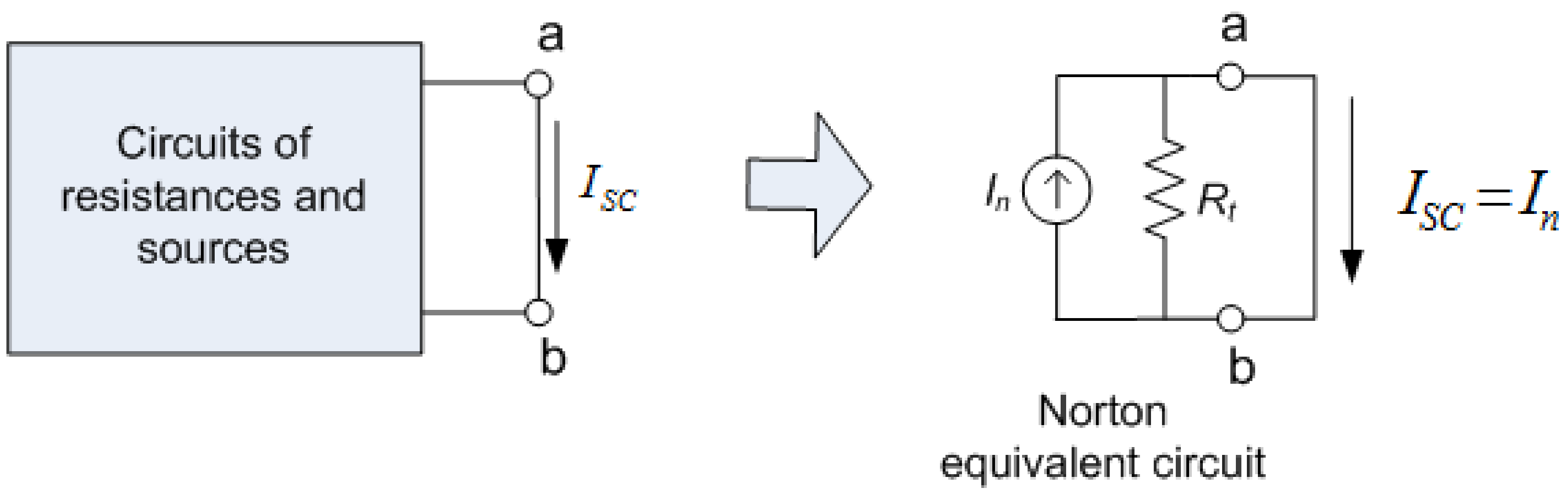
Norton Equivalent



- A current source in parallel with a resistance.
- The current source is called Norton's current.
- The series resistance is called Thevenin's resistance.

Norton's current

- The value of the current source is the short circuit current between the two terminals
- This is called the Norton's current



Steps to find the equivalent circuits

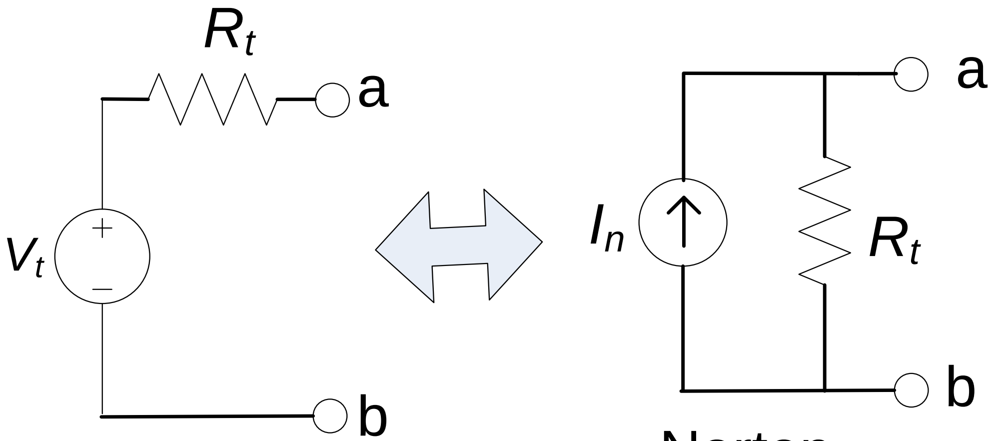
- Obtain the open circuit voltage between the two terminals – Thevenin's voltage:
- Obtain the short circuit current between the two terminals – Norton's current:
- Calculate the Thevenin's resistance as:

$$R_t = \frac{V_{OC}}{I_{SC}}$$

Source Conversion

- A voltage source with a series resistance is equivalent to a current source with the resistance in parallel.
- The values of the voltage and current source are given as

$$V_t = I_n R_t, \quad I_n = \frac{V_t}{R_t}$$

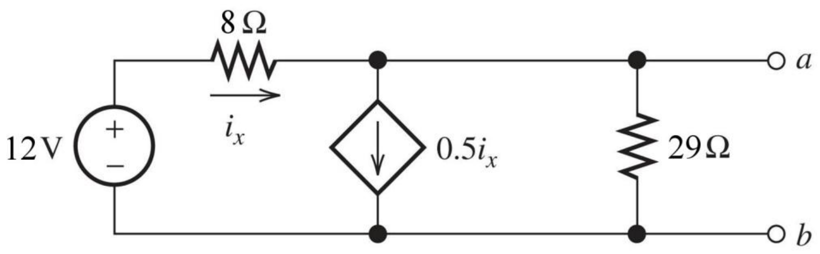


Thevenin equivalent circuit

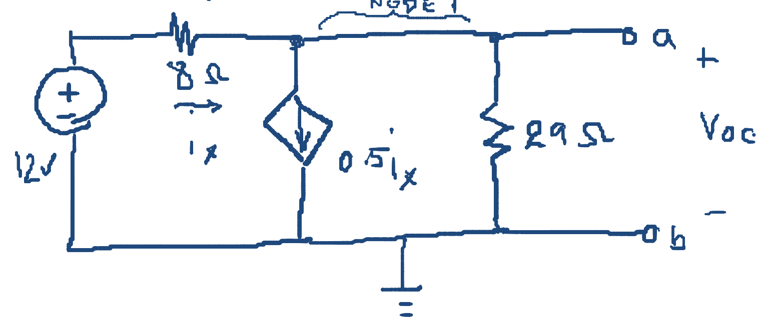
Norton equivalent circuit

Example

- Find the Thévenin and Norton equivalent circuits.



1. With open circuit conditions:



V_{oc} is the unknown node-voltage variable.

Write a current eqn at node 1

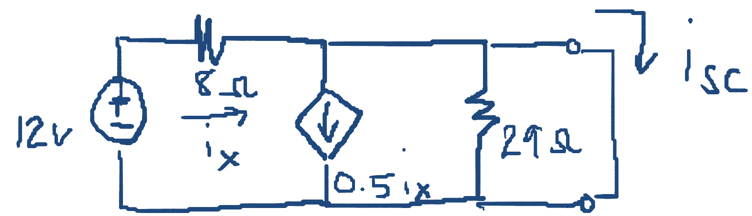
$$i_x - \frac{V_{oc}}{29} - 0.5 i_x = 0$$

2. Write an expression for the controlling variable i_x in terms of the node voltage

$$i_x = \frac{12 - V_{oc}}{8}$$

Solving: $V_t = V_{oc} = 7.733$ V

3. We consider short circuit conditions



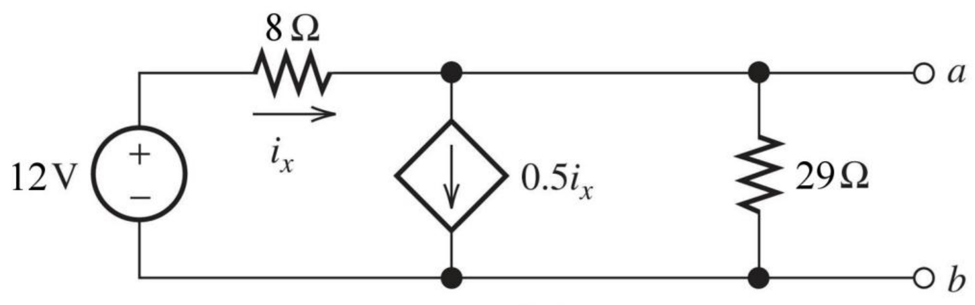
The current through the 29 Ω resistor is zero.

$$i_x = 12/8 = 1.5 \text{ A}$$

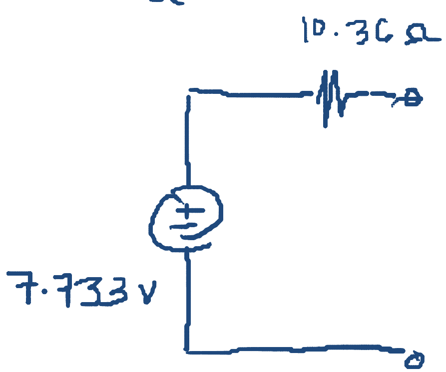
$$i_x - 0.5 i_x - i_{sc} = 0$$

$$i_{sc} = i_x - 0.5 i_x = 0.75 \text{ A}$$

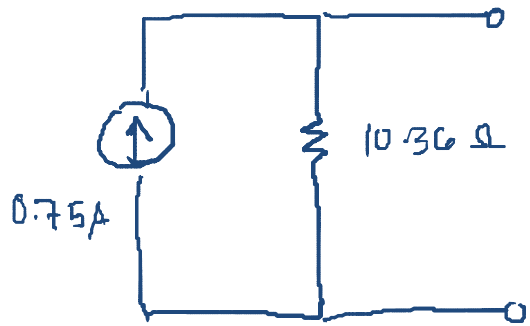
Example



$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{7.733}{0.75} = 10.3644\ \Omega$$



=



$$I_N = \frac{V_T}{R_T} = \frac{7.733}{10.36} = 0.75\ A$$

CG1108 AY2010/11

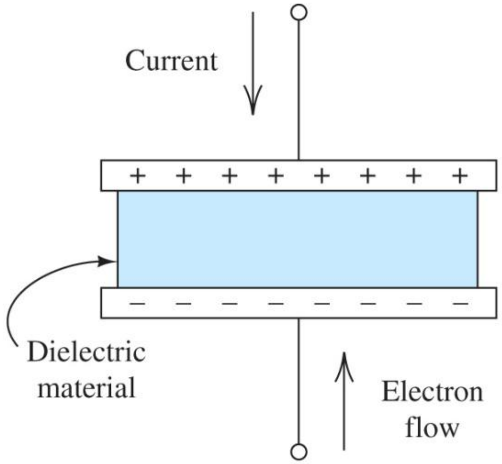
Lecture 4

Energy storage elements

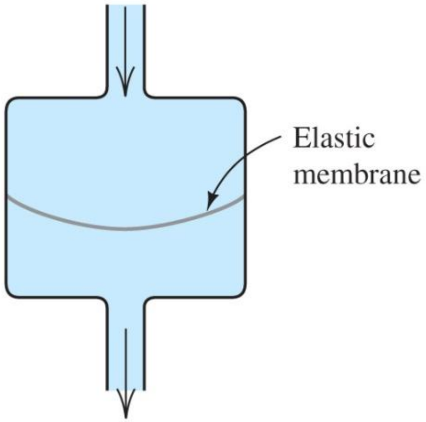
- **Learning Objectives**
  - **Capacitance**
  - **Inductance**

Capacitance

- Capacitors are constructed by separating two sheets of conductors by a thin layer of insulating material.
- The insulating material is called dielectric (air, paper, Mylar, polyester etc.)



(a) As current flows through a capacitor, charges of opposite signs collect on the respective plates

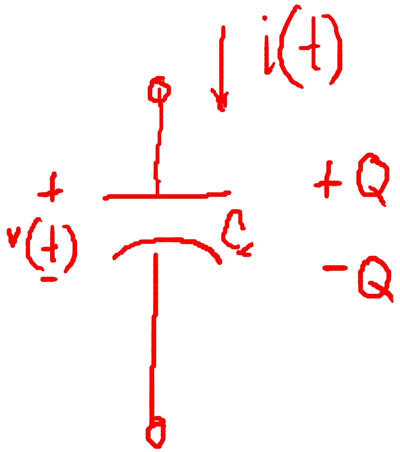


(b) Fluid-flow analogy for capacitance

Stored charge in terms of voltage

- In ideal capacitor, the charge developed in each plate (charge stored) is proportional to the voltage across it.
- the proportionality constant is called the capacitance. Its unit is farad (F).

$$Q = CV$$



Capacitance value

- One farad is equivalent to coulomb per volt.
- One farad is a large value.
- Practical capacitors have values only a few pico farad (10^{-12}) to a few micro farad (10^{-6}).

Current in terms of voltage

- Recall that current is the time rate of flow of charge.
- Taking the derivative of $q = Cv$ with respect to time:

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv)$$

- Ordinarily, capacitance is not a function of time. Thus, the relationship between current and voltage becomes:

$$i = C\frac{dv}{dt}$$

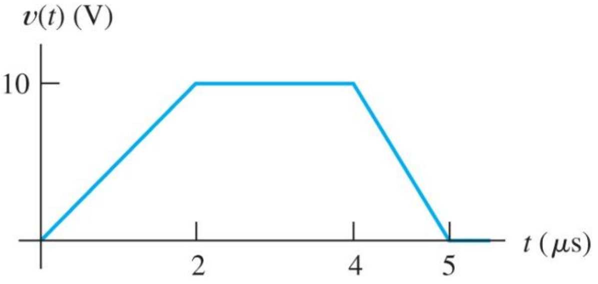
- As voltage increases, current flows through the capacitance and charge accumulates on each plate.
- If the voltage remains constant, the charge is constant and the current is zero. Thus, a capacitor appears to be an open circuit for a steady dc voltage.

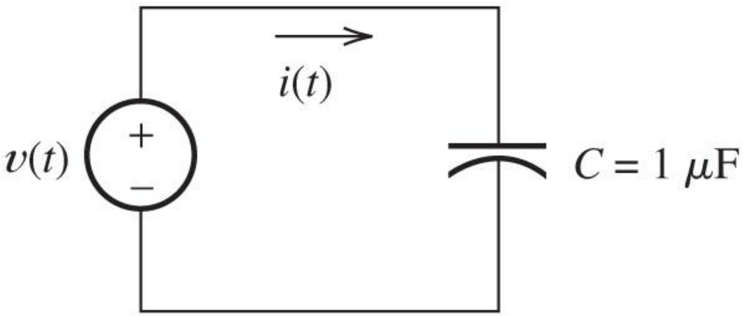
Capacitor current in terms of voltage

- Current is present only when charge is building up.
- At steady state, when the voltage is stable, the capacitor current will be zero.
- Hence, capacitors in DC circuits behave as an open circuit in steady state.

Determining current for a capacitance given voltage

Suppose that the voltage v(t) is applied to a 1 μF capacitance. Plot the stored charge and the current through the capacitance versus time.

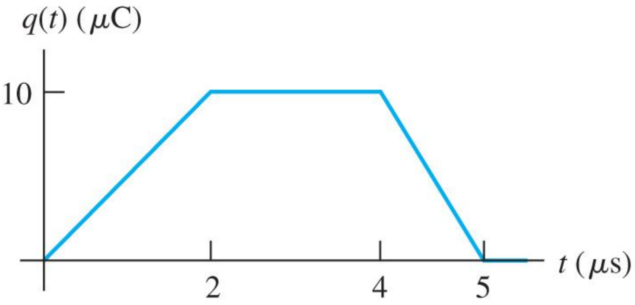




1. Plot the stored charge.

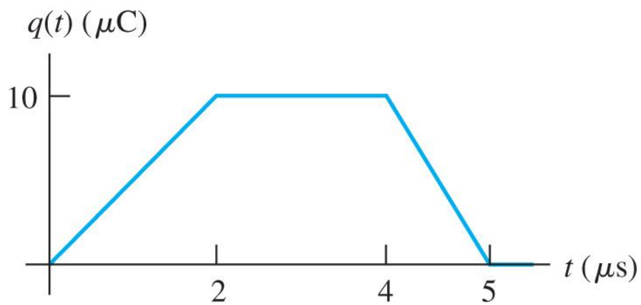
The stored charge is:

$$q(t) = C\,v(t)$$
$$= 10^{-6}\,v(t)$$



Determining current for a capacitance given voltage

2. Plot the current through the capacitance versus time.



The current is:

$$i(t) = C\frac{dv(t)}{dt} = 10^{-6}\frac{dv(t)}{dt}$$

The derivative of the voltage is the slope of the voltage vs time plot.
For t @ 0 to 2 μs

$$\frac{dv(t)}{dt} = \frac{10\ V}{2 \times 10^{-6} s} = 5 \times 10^{6}\ \frac{V}{s}$$

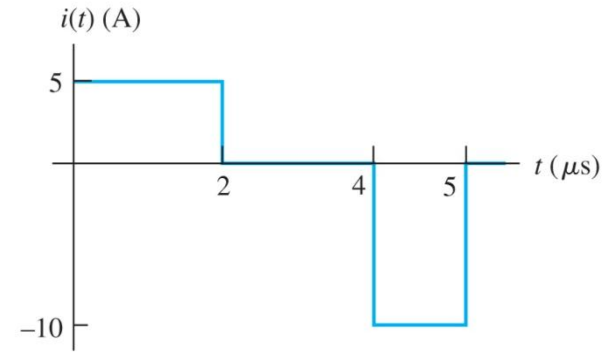
$$i(t) = C\frac{dv(t)}{dt} = 10^{-6} \times 5 \times 10^{6} = 5\ A$$

@ t = 2 and t = 4 μs, the voltage is constant (dv/dt = 0) and the current is zero.

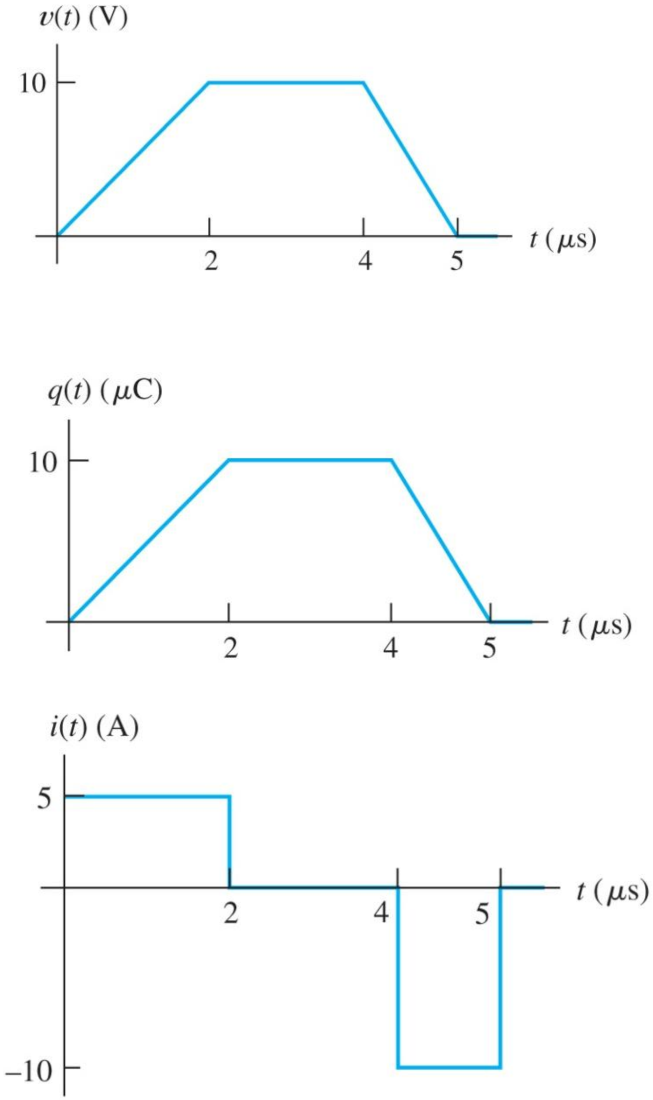
@ t = 4 and t = 5 μs

$$\frac{dv(t)}{dt} = \frac{-10\ V}{1 \times 10^{-6}\ s} = -10^{7}\ \frac{V}{s}$$

$$i(t) = C\frac{dv(t)}{dt} = 10^{-6} \times (-10^{7}) = -10\ A$$



Determining current for a capacitance given voltage



- As the voltage increases, current flows through the capacitor and charges accumulate on the plates.
- For constant voltage, the current is zero and the charge is constant.
- When the voltage decreases, the direction of the current reverses, and the stored charge is remove from the capacitor.

Capacitor voltage in terms of current

- Suppose we know the current *i(t)* flowing through a capacitance *C* and we want to compute the charge and voltage.
- Since current is the time rate of charge flow, we must integrate the current to compute charge.
- Often, action start at some initial time t_0, and the initial charge $q(t_0)$ is known.
- Charge as a function of time is given as:

$$q(t) = \int_{t_0}^{t} i(t)\,dt + q(t_0)$$

- Setting the right hand side of $q = Cv$ and the equation above equal to each other, and solving for the voltage *v(t)*:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + \frac{q(t_0)}{C}$$

Capacitor voltage in terms of current

- However, the initial voltage across the capacitance is given by:

$$v(t_0) = \frac{q(t_0)}{C}$$

- Substituting this, we have:

$$v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

- Usually, we take the initial time to be $t_0 = 0$.

Stored energy

- The power delivered to a circuit element is the product of the current and the voltage (provided that the references have the passive configuration).

$$p(t) = v(t)\, i(t)$$

$$i = C\frac{dv}{dt} \quad \Rightarrow \quad p(t) = C v \frac{dv}{dt}$$

- Suppose the capacitor initially has $v(t_0) = 0$. Then the initial stored electrical energy is zero. The capacitor is uncharged.
- Suppose that between time t_0 and some later time t the voltage changes from 0 to $v(t)$ volts.
- As the magnitude increases, energy is delivered to the capacitor, where it is stored in the electric field between the plates.
- If we integrate the power delivered from t_0 to t, we find the energy delivered.

$$w(t) = \int_{t_0}^{t} p(t)\,dt$$

Stored energy

$$w(t) = \int_{t_0}^{t} p(t)\,dt \Rightarrow p(t) = Cv\frac{dv}{dt} \Rightarrow w(t) = \int_{t_0}^{t} Cv\frac{dv}{\cancel{dt}}\,\cancel{dt}$$

$$w(t) = \int_{0}^{v(t)} Cv\,dv$$

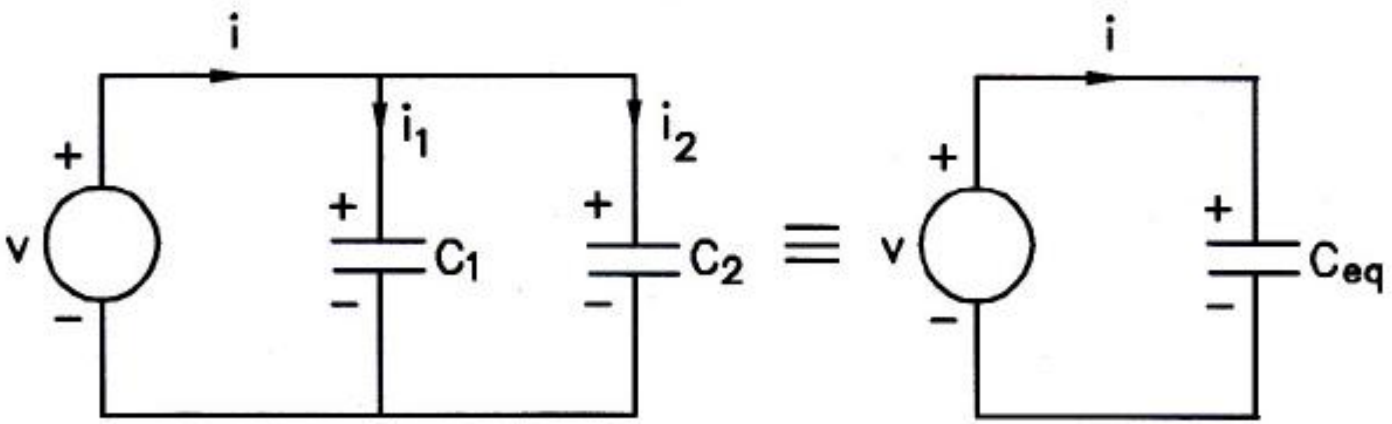
$$w(t) = \frac{1}{2}Cv^2(t)$$

With $q = Cv$, we can obtain two alternative expressions

$$w(t) = \frac{1}{2}v(t)q(t)$$

$$w(t) = \frac{q^2(t)}{2C}$$

Parallel Connection of Capacitors



Capacitors connected in parallel

Consider the circuit in Figure . Two capacitors of capacitance C_1 and C_2 are connected in parallel. The voltage applied across these two capacitors is the same (v). So we have

$$i_1 = C_1 \frac{dv}{dt}$$

$$i_2 = C_2 \frac{dv}{dt}$$

Parallel Connection of Capacitors

$$\begin{aligned} i &= i_1 + i_2 = (C_1 + C_2)\frac{dv}{dt} \\ &= C_{eq}\frac{dv}{dt} \end{aligned}$$

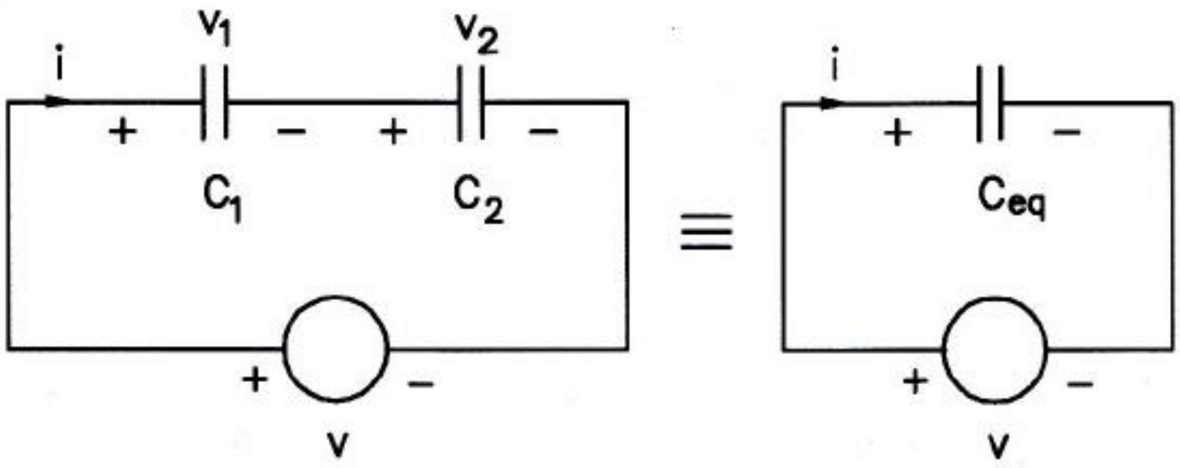
C_{eq} of two capacitors connected in parallel is obtained as follows:

$$C_{eq} = C_1 + C_2$$

So, if n capacitors are connected in parallel, the equivalent capacitance of the circuit will be given by

$$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Series Connection of Capacitors



Capacitors connected in series

Consider the circuit in the figure . Two capacitors of capacitance C_1 and C_2 are connected in series. The current flow through these two capacitors is the same (i). So we have

$$
\begin{aligned}
v_1 &= \frac{1}{C_1}\int i\,dt \\
v_2 &= \frac{1}{C_2}\int i\,dt
\end{aligned}
$$

Series Connection of Capacitors

$$
\begin{aligned}
v &= v_1 + v_2 = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt \\
&= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int i\,dt = \frac{1}{C_{eq}}\int i\,dt
\end{aligned}
$$

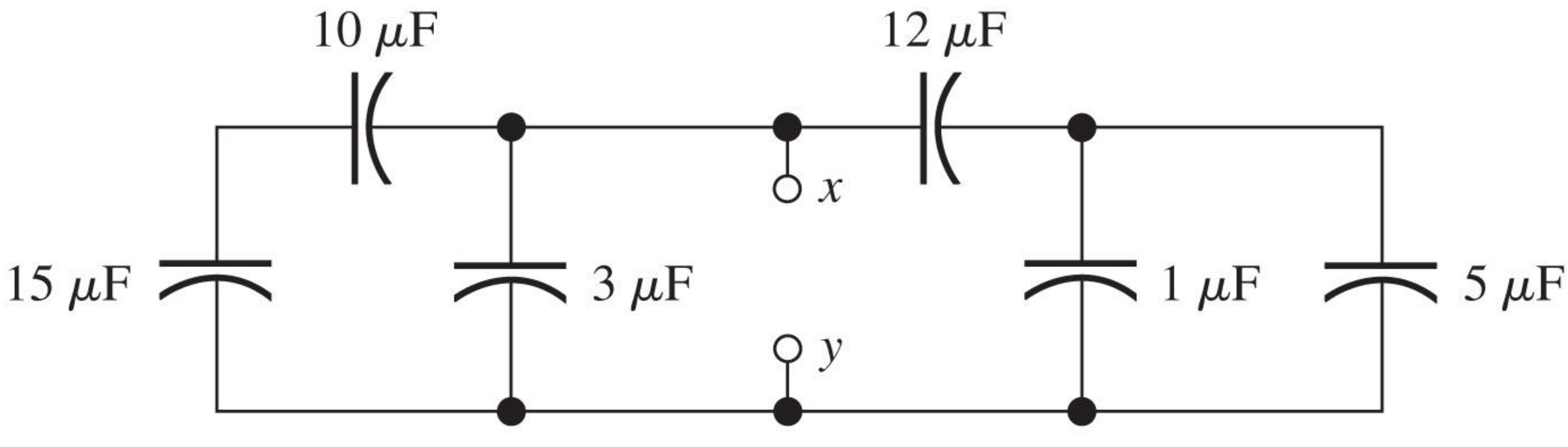
C_{eq} of two capacitors connected in series is obtained as follows:

$$
\begin{aligned}
\frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \\
C_{eq} &= \frac{C_1 C_2}{C_1 + C_2}
\end{aligned}
$$

So, if n capacitors are connected in series, the equivalent capacitance of the circuit will be

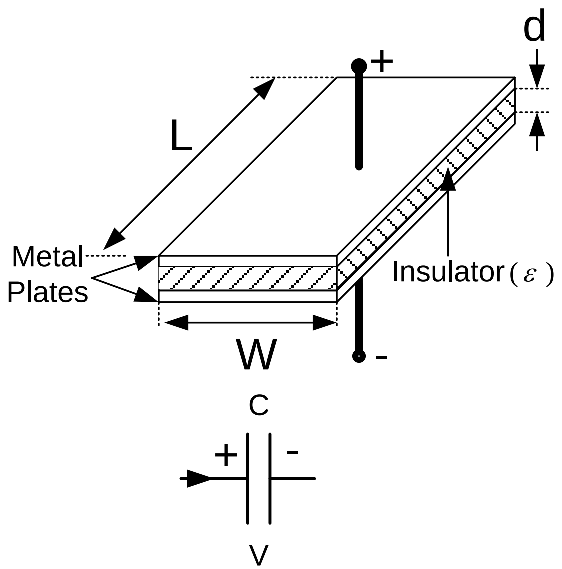
$$
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}
$$

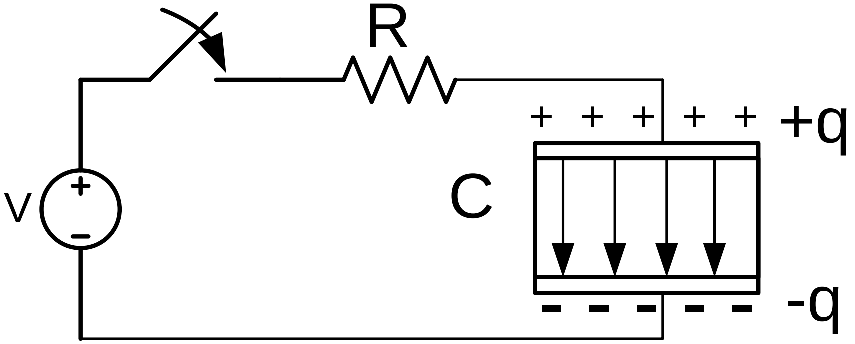
Find the capacitance between x and y:



$C_{eq} = 13\ \mu F$

Capacitance of the parallel-plate capacitor





$$C = \frac{\varepsilon A}{d} \qquad A = WxL$$

$$\varepsilon = \varepsilon_r \varepsilon_0$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \, F/m$$

Table 3.1 Relative permittivity of selected materials

Air	1.0
Diamond	5.5
Mica	7.0
Polyester	3.4
Quartz	4.3
Silicon dioxide	3.9

Example

- Compute the capacitance of a parallel-plate capacitor having rectangular plates 10 cm by 20 cm separated by a distance of 0.1 mm. The dielectric is air. Repeat if the dielectric is mica.

$C = \frac{\varepsilon A}{d} = ?$

$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

$\varepsilon = \varepsilon_r \varepsilon_0$

$A = L \times W = 0.02\ m^2$

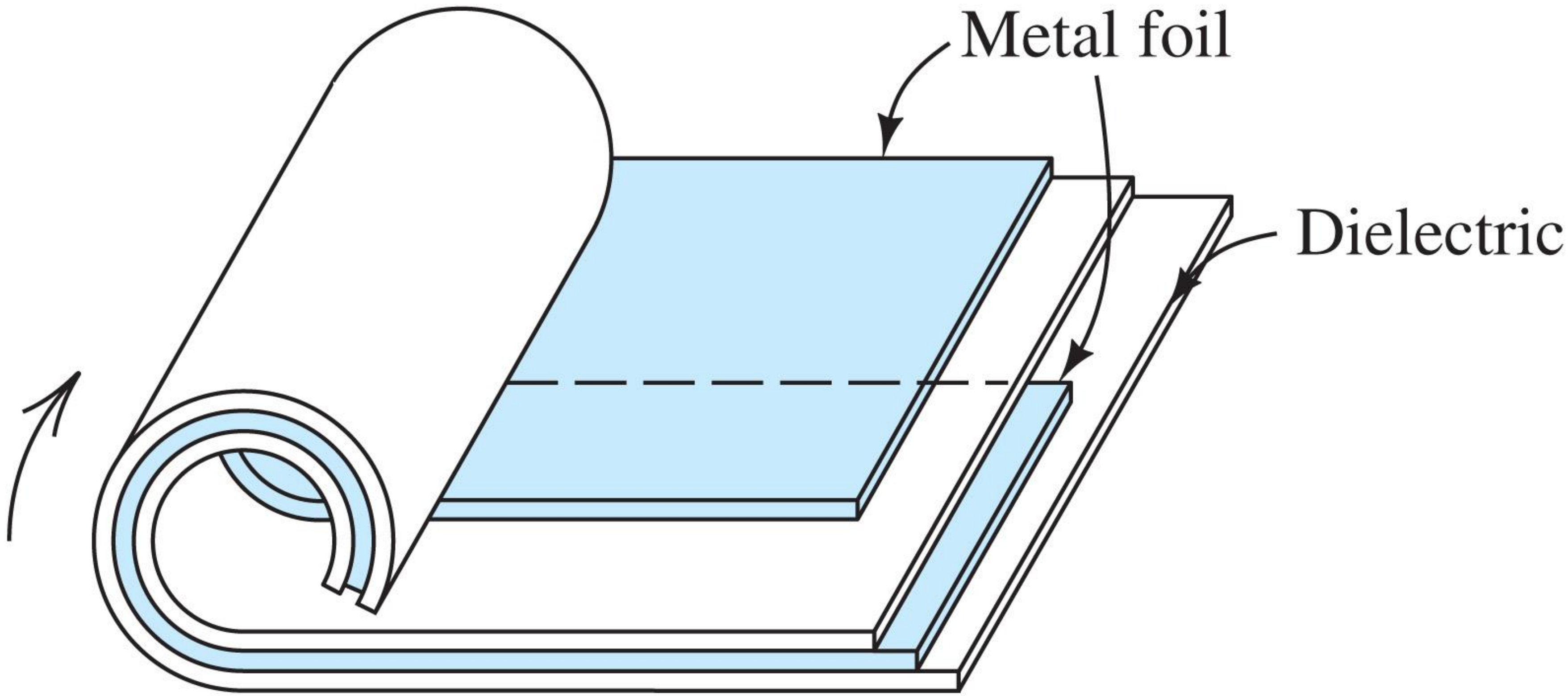
$\varepsilon = \varepsilon_r \varepsilon_0 = 1 \times 8.85 \times 10^{-12}\ F/m$

$C = \frac{\varepsilon A}{d} = 1770 \times 10^{-12}\ F$

$C = 12{,}390 \times 10^{-12}\ F$

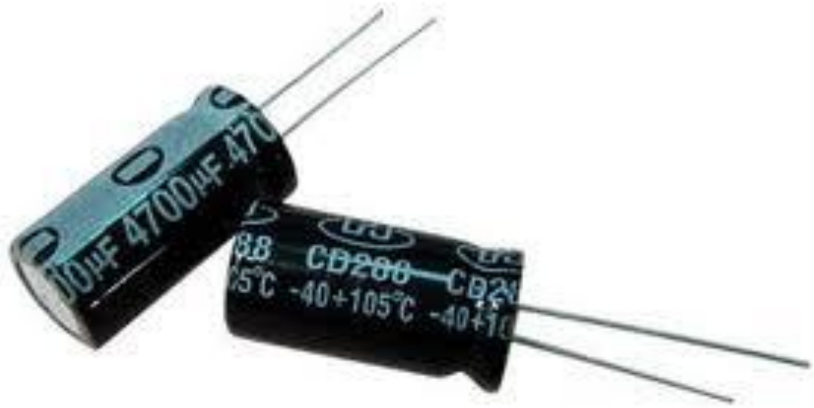
Air	1.0
Diamond	5.5
Mica	7.0
Polyester	3.4
Quartz	4.3
Silicon dioxide	3.9

Practical Capacitors



Practical capacitors can be constructed by interleaving the plates with two dielectric layers and rolling them up. By staggering the plates, connection can be made to one plate at each end of the roll.

Electrolytic Capacitors



- In such capacitors, one of the plates is metallic aluminum or tantalum, the dielectric is an oxide layer on the surface of the metal and the other ‘plate’ is an electrolytic solution.

Electrolytic Capacitors

- High capacitance per volume.
- These capacitors have polarity.
- If voltage of the opposite polarity is applied, capacitor may fail at high voltage.
- Commonly used in DC voltage systems as the bulk capacitor to filter out voltage ripples from the DC supply.
- These capacitors cannot be used where voltage polarity reverses.

Ceramic capacitor value

15×10^4 pF

47×10^4 pF

- There is a three digit code printed on a ceramic capacitor specifying its value. The first two digits are the two significant figures and the third digit is a base 10 multiplier. The value is given in picofarads (pF).

Ceramic capacitor value

- These capacitors do not have polarity
- Ceramic capacitors are suitable for moderately high-frequency work
- Often used as a decoupling capacitor (to supply small high frequency current at the point of demand).

Capacitive Sensors

The basic operation of a capacitive sensor can be seen from the familiar equation for a parallel-plate capacitor:

$$C = K\varepsilon_0(A/d)$$

where,

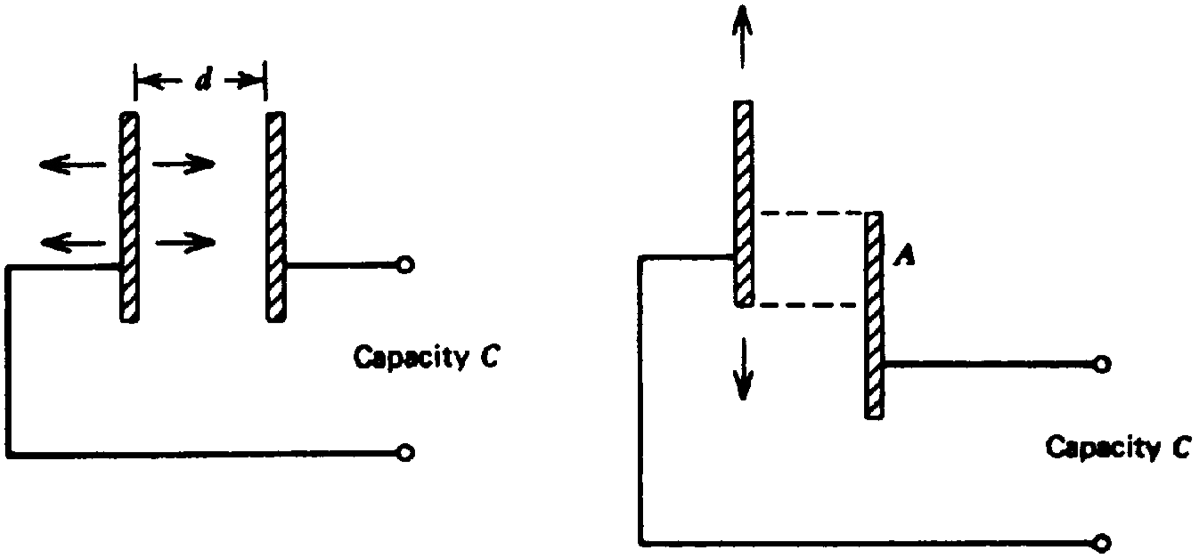
K = the dielectric constant (equals 1 for air);

ε_0 = permittivity (8.85 pF/m);

A = plate common area;

d = plate separation.

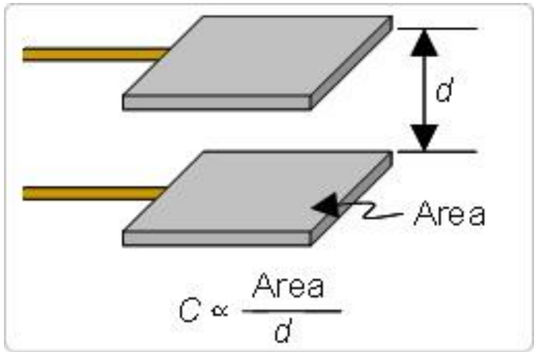
Capacitive Sensors

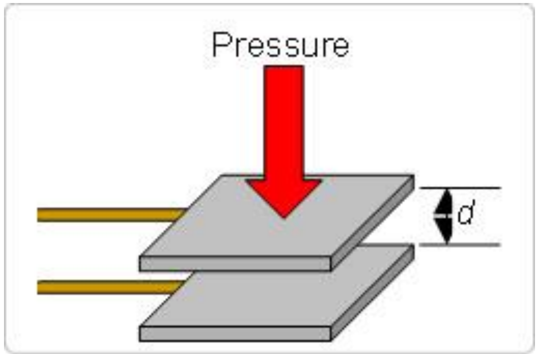


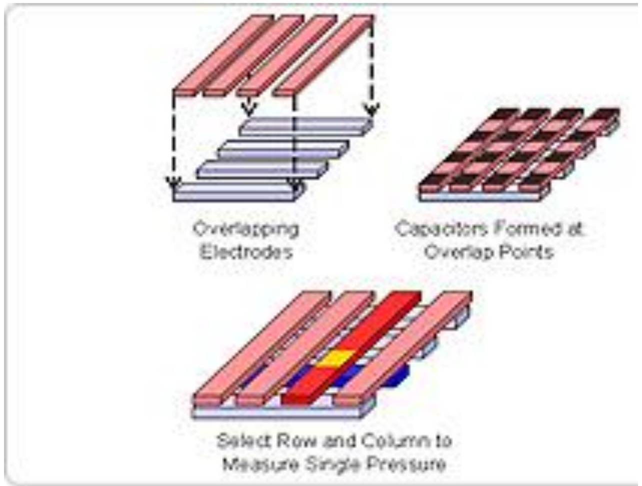
Capacity varies with the distance between the plates and the common area.
Both effects are used in sensors.

There are three ways to change the capacity: variation of the distance between the plates (d), variation of the shared area of the plates (A) and the variation of the dielectric constant (K).

Capacitive Sensors





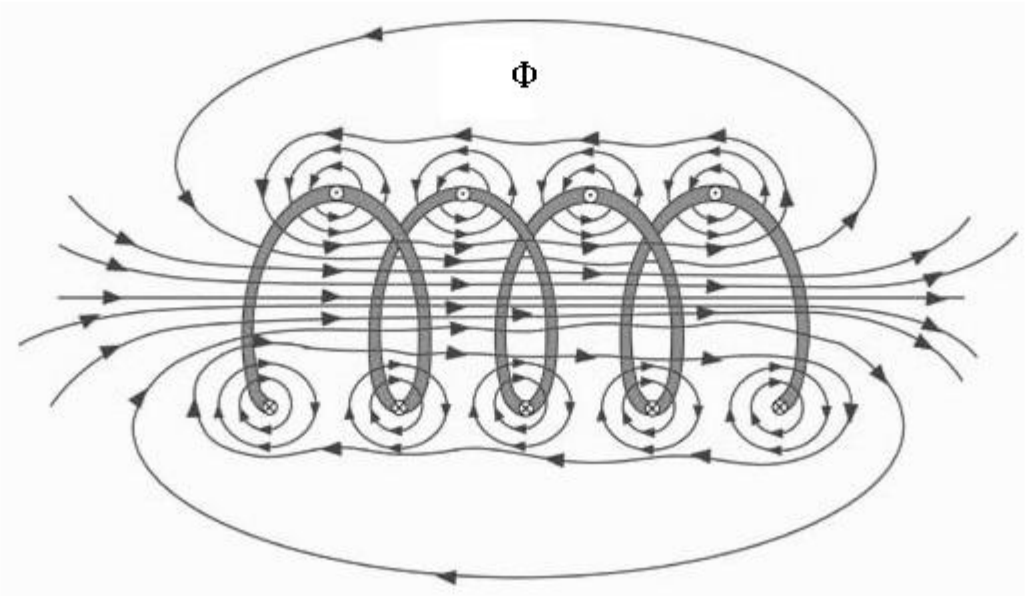


(Figures from Pressure Profile Systems)

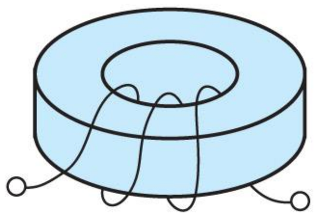
- Capacitance is a measure of the electrical charge stored between two electrodes separated by an air gap.
- As the electrodes are moved closer to or farther from one another, the air gap changes, and therefore so does the capacitance.
- The simplicity of a capacitor allows for a great deal of flexibility in design and construction, and results are more repeatable and less likely to degrade over time.

Inductance

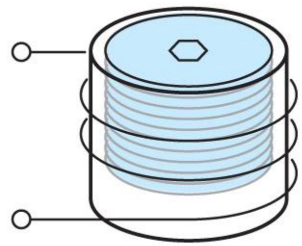
- An inductor is constructed by coiling a wire around some type of form. When current flows in the coil, a magnetic field is produced, with the magnetic flux linking the coil.



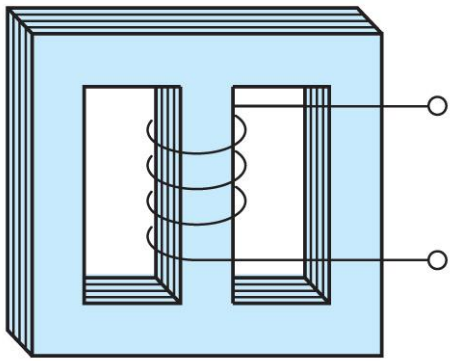
Inductor construction



(a) Toriodal inductor



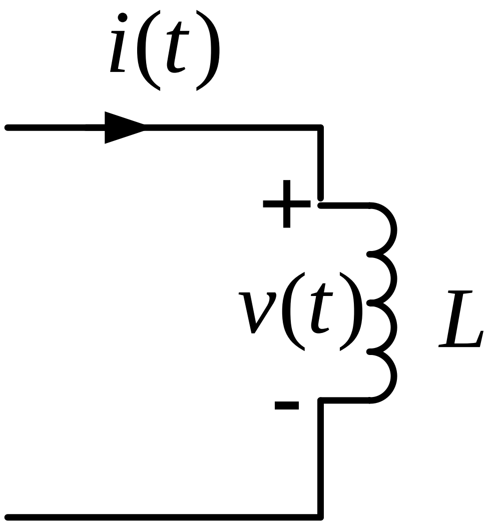
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

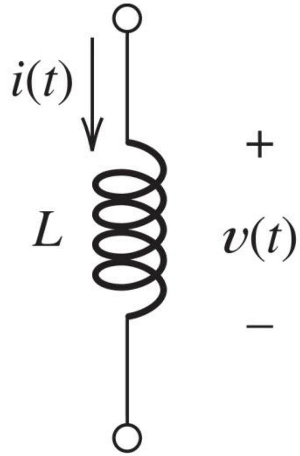


(c) Inductor with a laminated iron core

Faraday's law

- According to Faraday's law, a voltage is induced in a coil when the magnetic field linking it varies with time.
- The voltage is proportional to the time rate of change of current.





$$v(t) = L\frac{di}{dt}$$

Inductor current in term of voltage

- Suppose we know the initial current $i(t_0)$ and the voltage $v(t)$ across an inductance and we need to compute the current for $t > t_0$.

$$v(t) = L\frac{di}{dt} \Rightarrow di = \frac{1}{L} v(t)\,dt$$

- Integrating both sides:

$$\int_{i(t_0)}^{i(t)} di = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt$$

- Integrating, evaluating and re-arranging, we have:

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

- As long as $v(t)$ is finite, $i(t)$ can change only by an incremental amount in a time increment. Therefore, $i(t)$ must be continuous with no instantaneous jumps in value.

Stored energy

- The power delivered is the product of the current and voltage.

$$p(t) = v(t)\,i(t) \quad \Rightarrow \quad v(t) = L\frac{di}{dt} \quad \Rightarrow \quad p(t) = L\,i(t)\frac{di}{dt}$$

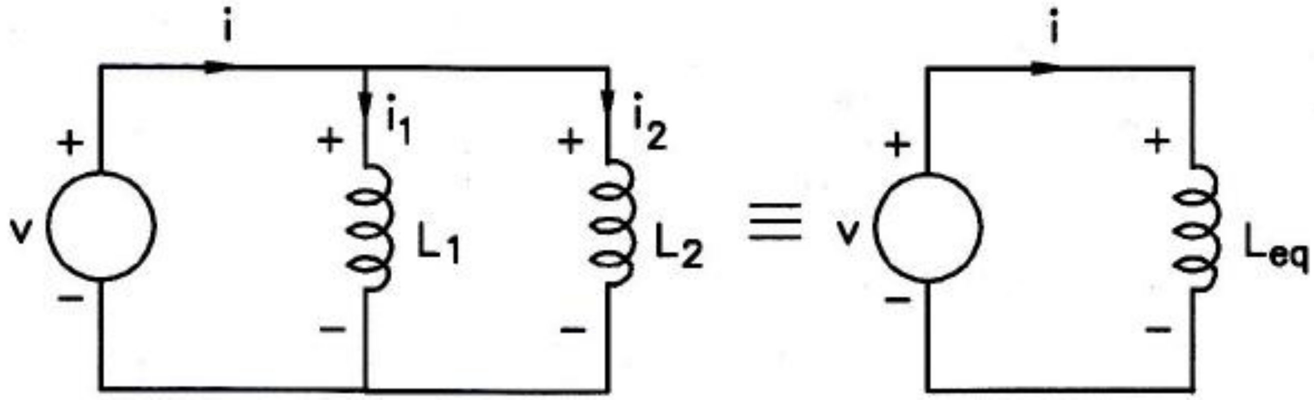
- Consider an inductor having an initial current *i(t₀)* =0. The initial electrical energy stored is zero.
- Assume that between time t_0 and some later time *t*, the current changes from 0 to *i(t)*. As the current magnitude increases, energy is delivered to the inductor, where it is stored in the magnetic field.
- Integrating the power from t_0 to *t*, the energy delivered is:

$$w(t) = \int_{t_0}^{t} p(t)\,dt \qquad w(t) = \int_{t_0}^{t} L\,i(t)\frac{di}{\cancel{dt}}\,\cancel{dt} \qquad w(t) = \int_{0}^{i(t)} L\,i\,di$$

- Integrating and evaluating:

$$w(t) = \frac{1}{2}L\,i^2(t)$$

Parallel Connection of Inductors



Inductors connected in parallel

Consider the circuit in Figure . Two inductors of inductance L_1 and L_2 are connected in parallel. The voltage applied across these two inductors is the same (v). So

$$i_1 = \frac{1}{L_1} \int v \, dt$$

$$i_2 = \frac{1}{L_2} \int v \, dt$$

Parallel Connection of Inductors

$$
\begin{aligned}
i &= i_1 + i_2 = (\frac{1}{L_1} + \frac{1}{L_2}) \int v\,dt \\
&= \frac{1}{L_{eq}} \int v\,dt
\end{aligned}
$$

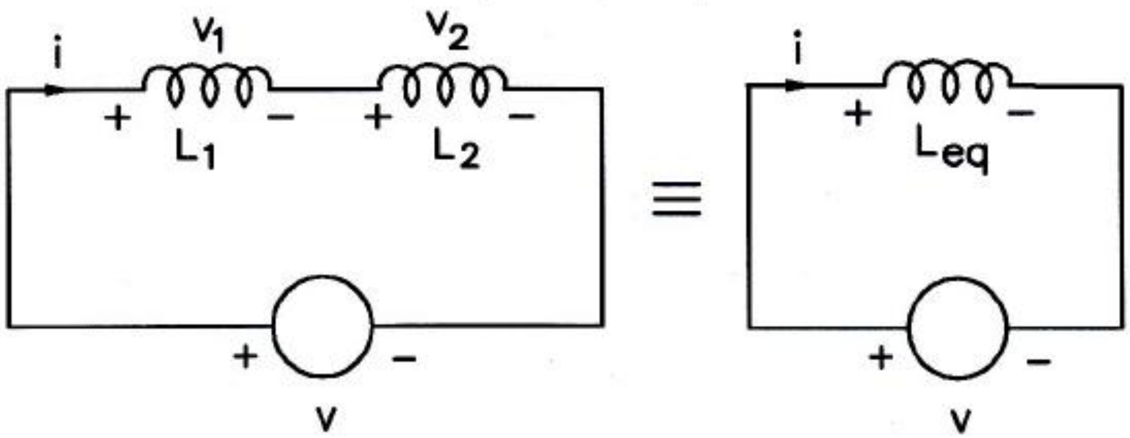
L_{eq} of two inductors connected in parallel is obtained as follows:

$$
\begin{aligned}
\frac{1}{L_{eq}} &= \frac{1}{L_1} + \frac{1}{L_2} = \frac{L_1 + L_2}{L_1\,L_2} \\
L_{eq} &= \frac{L_1\,L_2}{L_1 + L_2}
\end{aligned}
$$

So, if n inductors are connected in parallel, the equivalent inductance of the circuit will be given by

$$
\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}
$$

Series Connection of Inductors



Inductors connected in series

Consider the circuit in the Figure. Two inductors of inductance L_1 and L_2 are connected in series. The current flow through these two inductors is the same (i). So we have

$$
\begin{aligned}
v_1 &= L_1 \frac{di}{dt} \\
v_2 &= L_2 \frac{di}{dt}
\end{aligned}
$$

Series Connection of Inductors

$$\begin{aligned} v &= v_1 + v_2 = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \\ &= (L_1 + L_2) \frac{di}{dt} = L_{eq} \frac{di}{dt} \end{aligned}$$

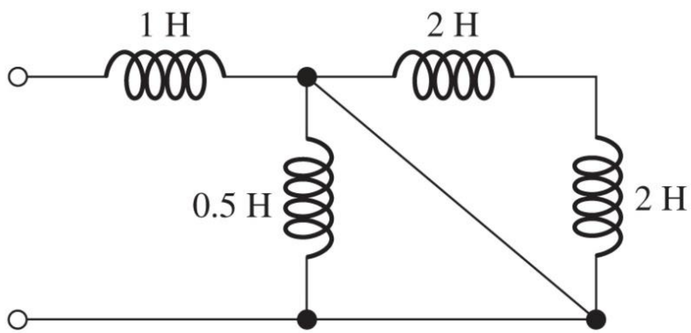
L_{eq} of two inductors connected in series is obtained as follows:

$$L_{eq} = L_1 + L_2$$

So, if n inductors are connected in series, the equivalent inductance of the circuit will be

$$L_{eq} = L_1 + L_2 + \ldots + L_n$$

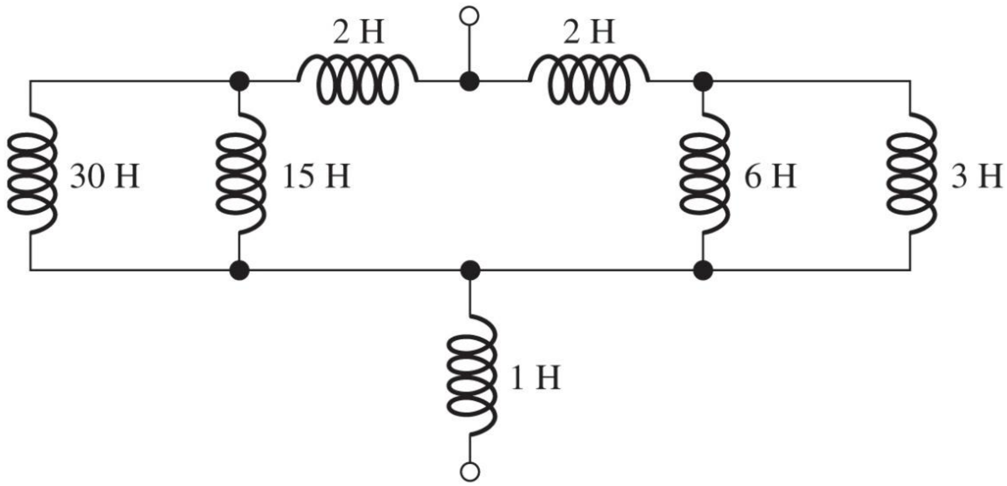
Examples



(a)

The 2 H, 2H and 0.5 H have no effect because they are in parallel with a short circuit.

$L_{eq} = 1\ H$



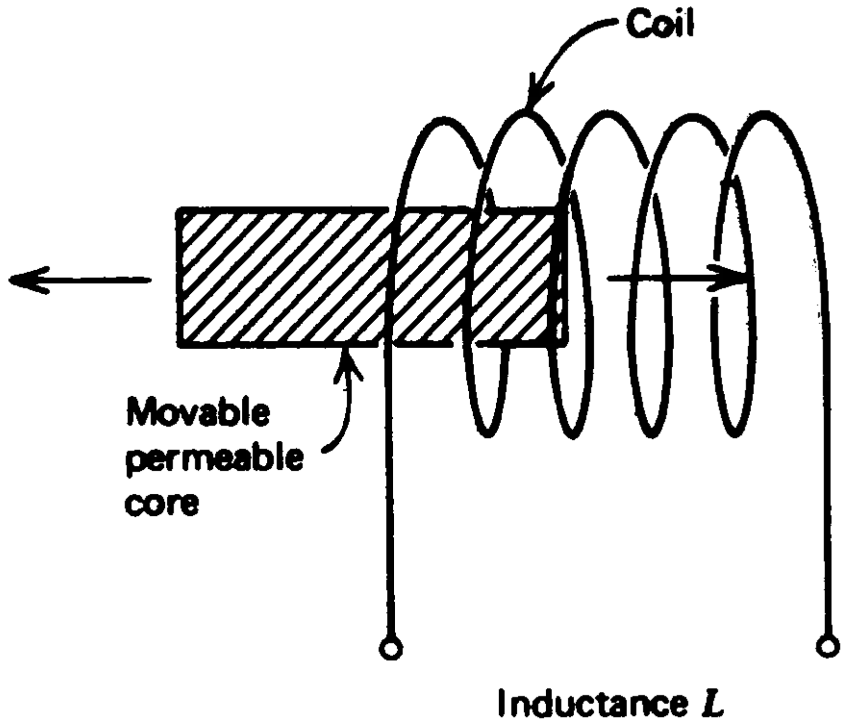
(b)

$L_{eq} = 4\ H$

Inductive Sensing

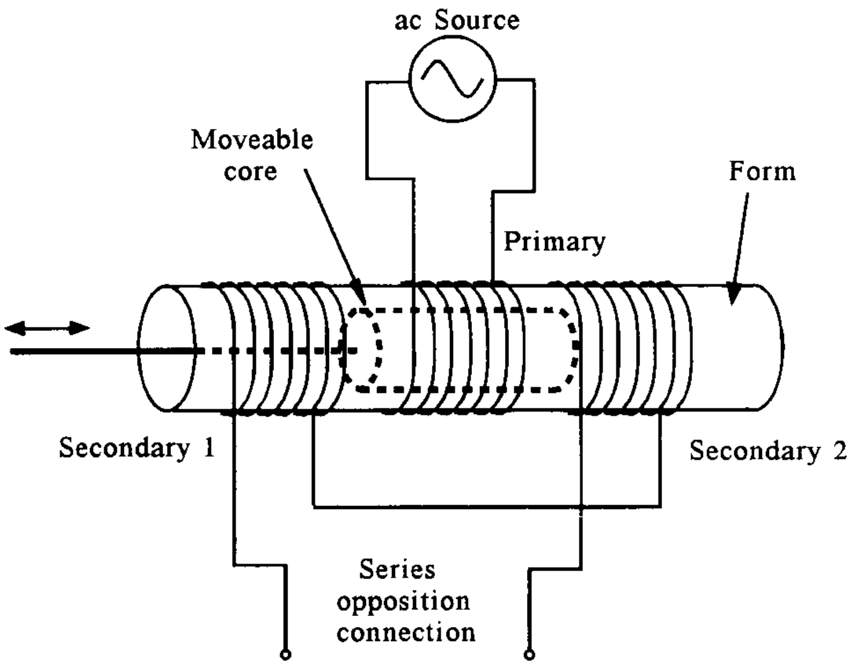
- Inductive sensing – If a permeable core is inserted into an inductor as shown, the net inductance is increased. Every new position of the core produces a different inductance.

- The inductor and movable core assembly may be used as a displacement sensor.



This variable-reluctance displacement sensor changes the inductance in a coil in response to core motion.

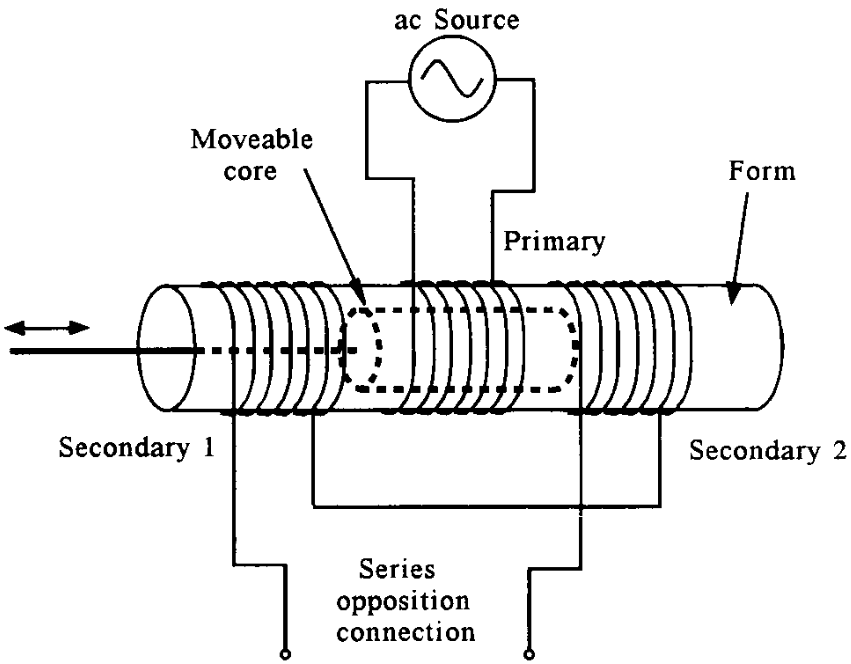
Variable-Reluctance Sensors



The LVDT has a movable core with the three coils as shown.

- A core of permeable material can slide freely through the center of the form.
- The inner core is the primary, which is excited by some ac source.
- Flux formed by the primary is linked to the secondary coils, inducing an ac voltage in each coil.

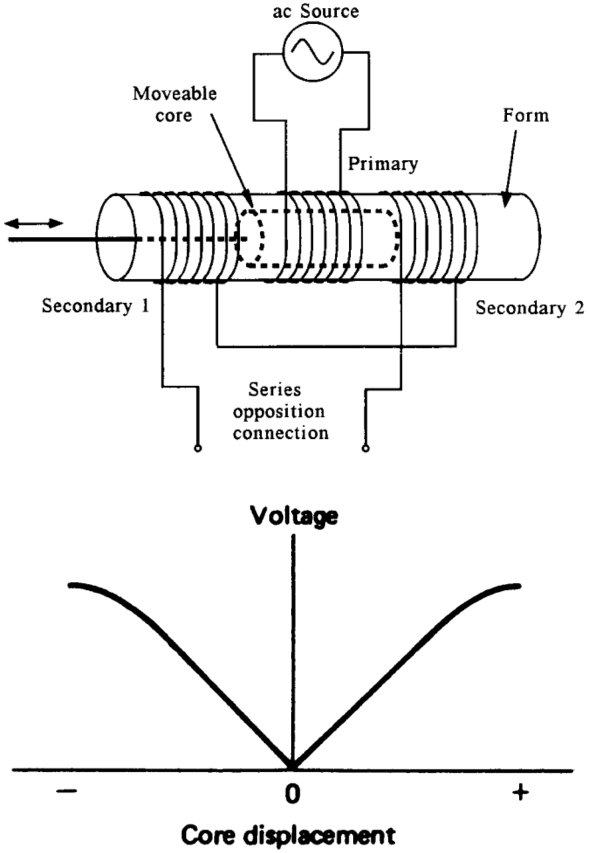
Variable-Reluctance Sensors



The LVDT has a movable core with the three coils as shown.

- When the core is centrally located in the assembly, the voltage induced in each primary is equal.
- If the core moves to one side or the other, a larger ac voltage in one coil and a smaller ac voltage in the other because of changes in the flux linkage associated with the core.

Variable-Reluctance Sensors



The LVDT secondary voltage amplitude for a series-opposition connection varies linearly with displacement.

- If the two secondary coils are wired in series opposition, then the two voltages will subtract; i.e. the differential is formed.
- When the core is moved to one side, the net voltage amplitude will increase.
- The differential amplitude is found to increase linearly as the core is moved to one side or the other.

Animation: http://www.rdpe.com/displacement/lvdt/lvdt-principles.htm