NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

(Semester I: 2003–2004)

ST2334 PROBABILITY AND STATISTICS

November 2003 — Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **NINE (9)** printed pages.
2. Answer **ALL** the questions. The number in [ ] indicates the number of marks allocated for that part. The total number of marks for this paper is 60.
3. Write your answers in the spaces provided.
4. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
5. Candidates may bring in **ONE (1)** handwritten A4-size (210 × 297 mm) help sheet.
6. Statistical tables are provided in the Appendices.

Matriculation No.: ____________________

Question	1	2	3	4	5	**Total**
Marks	4	15	15	10	16	60
Scores						

Consider two independent tosses of a fair coin. Let A be the event that the first toss lands heads, let B be the event that the second toss lands heads, and let C be the event that both tosses land on the same side. Show that the events A, B and C are pairwise independent but not (mutually) independent. [4 marks]

Suppose Mary takes only two meals, brunch and dinner, every day. Her calorie intakes at brunch and dinner are normally distributed with means 900 and 1300, and standard deviations 100 and 200, respectively. If Mary's total calorie intake for a day falls between 2000 and 2500, it is considered a "healthy" day for her. Assume that her calorie intakes at different meals and on different days are independent of one another.

(a) Find the probability that Mary's calorie intake at dinner will exceed her calorie intake at brunch tomorrow. (Hint: Sum of normal random variables is also a normal random variable.) [5 marks]

(b) For any given day, what is Mary's chance of having a "healthy" day? Using this result and the Central Limit Theorem, find the probability that the proportion of "healthy" days for Mary over the next (365-day) year exceeds 80%? [10 marks]

Stores *GioC* and *GioJ*, which belong to George, are located in two different towns, Clementi and Jurong, respectively. Suppose the probability density function of the weekly profit (X) of *GioC*, in thousands of dollars, is given by

$$f_X(x) = \begin{cases} \frac{1}{4}x & \text{if } 1 < x < 3, \\ 0 & \text{otherwise.} \end{cases}$$

and the probability density function of the weekly profit (Y) of *GioJ*, in thousands of dollars, is given by

$$f_Y(y) = \begin{cases} \frac{2}{15}y & \text{if } 1 < y < 4, \\ 0 & \text{otherwise.} \end{cases}$$

If the profit of one store is independent of the other, find the probability that for next week:

(a) the total profit of both *GioC* and *GioJ* exceeds \$5000. [5 marks]

(b) one store makes at least $1000 more than the other store? [10 marks]

Let $X_1, X_2, \ldots, X_{10}$ be a random sample from an exponential distribution with mean $\mu = 1/\lambda$. If $W = \sum_{i=1}^{10} X_i$, then $2\lambda W$ has a chi-squared distribution with 20 degrees of freedom.

(a) If $(0.05324W, 0.24213W)$ is a $(1-\alpha)100\%$ confidence interval for μ, determine α. [4 marks]

(b) Using the same α found in (a), one can always find another $(1-\alpha)100\%$ confidence interval of the form $(0, kW)$ for μ. Determine k. Which interval is better? Explain. [6 marks]

David wanted to investigate the relationship between distress and delight in males' and females' reactions to frightening films. In his study, he measured the emotional responses of 10 men and 20 women after viewing a segment from a horror film. A summary of data obtained in the study is given below.

	Distress Index		Delight Index	
Gender	Mean	Std Dev	Mean	Std Dev
Men	31.2	10.0	12.02	3.65
Women	40.4	9.1	9.19	5.55

After analyzing the data, David made two claims:

Claim 1: Females were more likely to express distress than males.

Claim 2: Males were more likely to express delight than females.

By setting up appropriate hypotheses and carrying out appropriate tests, determine whether you want to support or refute David's claims. Assume equal variances for distress index and unequal variances for delight index. Further assume normal populations and use $\alpha = 0.05$.

(a) Do you support or refute Claim 1? Justify. [8 marks]

(b) Do you support or refute Claim 2? Justify. [8 marks]

END OF PAPER