NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

ST2334 PROBABILITY AND STATISTICS

(Semester 2: AY 2005–2006)

April 2006 — Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer **ALL** the questions for TOTAL 50 marks.
3. Read the questions **CAREFULLY**, and label some important quantities clearly at your own convenience.
4. This is a **CLOSED** book examination. Candidates may bring in **TWO** A4 size (210×297mm) help sheets written on both sides.
5. **Decimal answers** should be given to at least **FOUR (4)** decimal places.

Matriculation No: ________________________________

Question	1	2	3	4	Total
Max. marks	5	12	15	18	50
Marks scored					

1. **[5 marks]** Let X be a binomial random variable with number of trials N and probability of success $p = 0.25$. Suppose a random sample of size n from X, denoted by $X_1, \ldots, X_n$, is given.

   (a) Give an unbiased estimator for N. **[3 marks]**

   (b) What is the standard error of your estimator in (a)? **[2 marks]**

2. **[12 marks]** Suppose Y_1 (in minutes) is the total time between a customer's arrival in a shop and departure from the checkout counter and Y_2(in minutes) is the time spent in queue together with check-out. The joint density of these variables is given by

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} e^{-y_1}, & 0 \le y_2 \le y_1 \le \infty \\ 0, & \text{elsewhere} \end{cases} .$$

(a) If a customer spends totally 5 minutes in the shop, find the probability that the customer spends more than 1 minute for "queuing". **[4 marks]**

(b) Suppose now we are interested in the "shopping time" of customers, the total time spent between a customer's arrival until starting to queue or until leaving the shop buying nothing.

(i) Find the probability that the "shopping time" of a customer is less than half of the total time s/he spends in the shop. [Hint: $\int_0^\infty x^a \, e^{-x} \, dx = a!$ for any positive integer a.] **[4 marks]**

(ii) Suppose we collected shopping information of 50 customers in this shop. **State clearly** the hypotheses, the test statistic, and the rejection region for testing whether the expected "shopping time" of each customer, μ, is less than 1 minute at 1% level. **[4 marks]**

3. **[15 marks]** A car park has two entrances along a road. Entrance I is only for private cars and motorbikes, while entrance II is for all other types of vehicles such as vans and lorries. Vehicles arrive at entrance I according to a Poisson distribution at an average of 5 per hour, while vehicles arrive independently at entrance II according to a Poisson distribution at an average of 2 per hour. There are entrance fees of \$3 and \$5 for vehicles using the two entrances, respectively. The parking fee is already included in the entrance fee and there is no limit for the parking time. Assume that the car park is so big that cars can come into the car park at any time upon their arrivals. Suppose the car park operates 24 hours a day and 7 days a week.

   (a) The expected total amount of money paid by customers arriving at the car park in every hour can be computed to be \$25. Argue why the variance of the total amount of money paid by the customers arriving at the car park in every hour is \$95. **[2 marks]**

   (b) Use normal approximation to compute the probability that the total weekly revenue of the car park is greater \$4,000. [Hint: revenue = the amount of money paid by all customers.] **[4 marks]**

(c) Show that the probability that there is less than 2 cars arriving at the car park in an hour is given by $8\,e^{-7}$. **[2 marks]**

(d) Compute the probability that in this week there are less than 3 hours during which less than 2 cars are arriving at the park

(i) exactly **[2 marks]**, and

(ii) by normal approximation. **[5 marks]**

4. **[18 marks]** A sample of 20-year conventional mortgage rates at 22 randomly chosen banks in country A yields a mean rate of 7.61% and a standard deviation of 0.39%. A similar sample taken at 7 randomly chosen banks in country B has a mean rate of 7.1%, with a standard deviation of 0.56%.

   (a) Construct a 95% confidence interval for μ_A, the mean mortgage rate of banks in country A. **State the assumption** that you need to make. **[3 marks]**

   (b) How do you interpret the confidence interval in (a)? **[2 marks]**

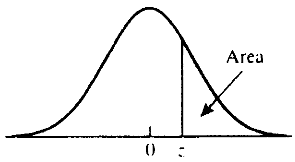
(c) Suppose one wants to improve the error of estimation in our estimation in (a) to be less than 0.05%. Give the least number of mortgage rates of additional banks that would have to be selected from country A. **[2 marks]**

(d) Construct a 98% confidence interval for the difference in the mean rates, $\mu_A - \mu_B$, where μ_B is the mean mortgage rate of banks in country B, to conclude whether the data provide sufficient evidence (at $\alpha = 2\%$) that the mean mortgage rates of the two countries are different. **State the assumptions** that you need to make. **[5 marks]**

(e) Suppose one wants to achieve higher precision by controlling the width of the confidence interval in (d) to be less than 0.5%. Assume that the total numbers of mortgage rates of banks from country A and B to be collected are identical. Give the least numbers of mortgage rates of additional banks that would have to be selected from countries A and B, respectively. **[3 marks]**

(f) Due to typographical errors, the sample sizes in the question are wrong; the data indeed refers to mortgage rates of 55 banks and 70 banks from countries A and B, respectively, while all other information is correct. Perform a 1%-level test to check whether the mean mortgage rate of banks in country A is greater than the mean mortgage rate of banks in country B. **[3 marks]**

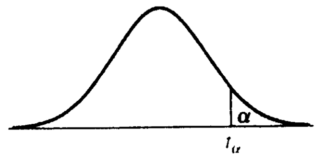
Normal curve areas
Standard normal probability in right-hand
tail (for negative values of z areas are found by symmetry)



	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

From R. E. Walpole. *Introduction to Statistics* (New York: Macmillan, 1968).

Percentage points of the t distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	d.f.
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

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