NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

EXAMINATION FOR
Semester 1 AY2009/2010

CS3230 – Design and Analysis of Algorithms

Nov/Dec 2009 Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **TEN (10)** questions and comprises **TWELVE (12)** printed pages, including this page. **BOTH** sides of the page are used. The total marks is 100.
2. Answer **ALL** questions within the space provided in this booklet.
3. This is an **Open Book** examination. You may cite any result in the textbook, lecture notes, tutorials, or labs.
4. Fill in your **Matriculation Number** here :

Examiners' Use Only		
Question	**Marks**	**Remarks (if any)**
Q1 (10)		
Q2 (10)		
Q3 (10)		
Q4 (5)		
Q5 (10)		
Q6 (10)		
Q7 (15)		
Q8 (10)		
Q9 (10)		
Q10 (10)		
Total (100)		

Q1 (Growth Rates of Functions) (total 10 marks)

Indicate whether the first function of each of the following pairs has a smaller, same or larger order of growth (to within a constant multiple) than the second function. Provide some explanation for each case (no formal proof required.)

a) 7^{n-1} and 7^n
b) $n(n+3)$ and $789n^2$
c) $(n-1)!$ and $n!$
d) $50n^3$ and $0.02n^4$
e) $\log_7 n$ and $\log_5 n$

(Answer 2 marks each)

Q2 (General) (total 10 marks)

Design an algorithm for the following problem: Given a set of n points $(2 \leq n)$ in the Cartesian plane, determine whether all of them lie on the same circumference (i.e., on the same circle). (a) Describe your algorithm clearly and in detail in your own words or in pseudo-code. (b) Describe any special cases. (c) What is the time complexity of your algorithm?

(Answer total 10 marks)

(total 10 marks)

Q3 (Proofs)

a) Prove that, if the polynomial $p(x) = ax^3 + bx^2 + cx + d$ has roots x_1, x_2, x_3, then $d = a \times x_1 \times x_2 \times x_3$.
(Answer 5 marks)

b) Find the second-degree polynomial $p(x) = ax^2 + bx + c$ such that $p(i) = 1/i$ for $i = 1, 2, 3$.
(Answer 5 marks)

Q4 (General) (total 5 marks)

What relationship do you see between Horner's rule as applied for binary exponentiation and the Russian Peasant's Multiplication method, if any? Provide a descriptive answer.

(Answer 5 marks)

Q5 (General) (total 10 marks)

Design an efficient algorithm for finding all sets of anagrams (of any length) in a large, given dictionary of English words. For example, 'eat', 'ate', and 'tea' belong to such a set (of length 3). (a) Describe your algorithm in your own words or pseudo-code. (b) What is the time complexity of your algorithm?

(Answer 10 marks)

Q6 (General) (total 10 marks)

There are n pancakes all of different sizes that are stacked on top of each other. You are allowed to slip a flipper under one of the pancakes and flip over the whole stack above the flipper. The purpose is to arrange the pancakes sorted according to their size with the biggest at the bottom. Design an algorithm that achieves this goal. (a) Describe your algorithm in your own words or pseudo-code. (b) What is the time complexity of your algorithm? (Hint: consider comparison of pancake sizes and flipping the flipper as basic operations.)

(Answer 10 marks)

Q7 (Dynamic Programming) (total 15 marks)

World Series odds. Consider two teams, A and B, playing a series of at most n games until one of the teams wins $\lceil \frac{n}{2} \rceil$ games. Assume that the probability of A winning a game is the same for each game and equal to p and the probability of A losing a game is $q = 1 - p$. (Hence, there are no ties.) Let $P(i, j)$ be the probability of A winning the series if A needs i more games to win the series and B needs j more games to win the series. E.g., $P(1, 1) = 0.4$ means that both A and B still need 1 game to win and the probability that A wins is 0.4.

(a) Set up a recurrence relation for $P(i, j)$ that can be used by a dynamic programming algorithm and state the initial conditions.

(b) Write a pseudo-code of the dynamic programming algorithm for solving this problem and determine its time efficiency.

(c) Find the probability of team A winning a seven-game series if the probability of it winning a game is 0.4, i.e., write down the filled dynamic programming table and find $P(4, 4)$.

(Answer 5 marks each; 15 total)

Q7 (continued)

Q8 (Greedy Technique) (total 10 marks)

Bridge/drain crossing revisited. Consider the generalization of the bridge crossing puzzle (Problem 4 in Tutorial 1) in which we have $n > 1$ people whose bridge/drain crossing times are $t_1 \leq t_2 \leq \ldots \leq t_n$. All the other conditions of the problem remain the same: at most two people at a time can cross the bridge (and they move with the speed of the slower of the two) and they must carry with them the only flashlight/key the group has.

Design a greedy algorithm for this problem. (a) Derive an expression (i.e., formula) of how long it will take to cross the bridge by using this algorithm in terms of $t_1, t_2, \ldots, t_n$. (b) Does your algorithm yield a minimum crossing time for every instance of the problem? If it does, prove it; if it does not, then find an instance with the smallest number of people for which it fails to yield the minimal crossing time.

(Answer total 10 marks)

Q9 (Divide-and-Conquer) (total 10 marks)

Consider the one-dimensional version of the closest-pair problem, i.e., the problem of finding the two closest numbers among a given set of n real numbers. (a) Design an algorithm that is directly based on the *divide-and-conquer* technique and describe its operation in your own words. (b) Provide the recurrence relation and solve it for the divide-and-conquer component of your algorithm, i.e., determine its efficiency class. Assume that the number of points is a power of two, i.e., $n = 2^k$. (c) What is the overall efficiency class of your algorithm?

(Answer total 10 marks)

Q10 (Dynamic Programming) (total 10 marks)

Consider the following recursive equation for computing a Binomial Coefficient: $C(n,k) = C(n-1,k-1) + C(n-1,k)$, $n > k > 0$, with $C(i,0) = C(i,i) = 1$, $0 \leq i \leq n$.

Is a memory function a good approach for computing a binomial coefficient? If not, why not? Explain your answer. (Hint: consider time and space efficiency.)

(Answer 10 marks)

End of Questions