EE2023 Signals & Systems Quiz

Semester 2 AY2011/12

Date : 8 March 2012 **Time Allowed : 1.5 hours**

Instructions :

1. Answer all 4 questions. Each question carries 10 marks.
2. This is a closed book quiz.
3. No programmable or graphic calculators allowed.
4. Write your answers in the spaces indicated in this question paper. No attachment is allowed.
5. Please enter your name and matric number in the spaces below.

Name : ______________________________

Matric # : ____________________________

Lecture Group # : ______________________

Question #	Marks
1	
2	
3	
4	
Total Marks	

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Group 3 : A/Prof Tan Woei Wan
Group 4 : Prof Lawrence Wong

Q1. Consider a periodic signal, $x(t)$, modelled by the following equation

$$x(t)=2je^{-j3t}+(2+3j)e^{-j2t}+5+(2-3j)e^{j2t}-2je^{j3t}$$

(a) What is the fundamental frequency of $x(t)$?

(b) By comparing $x(t)$ with the Fourier Series Expansion equation, $x(t)=\sum_{k=-\infty}^{\infty} X_k e^{jk\omega t}$, derive the magnitude, $|X_k|$, and phase, $\angle X_k$, of the Fourier Series coefficients when k = 0, 1, 2 and 3.

(c) An alternative method for evaluating the Fourier Series coefficients, X_k, of $x(t)$ is

$$X_k=\frac{1}{T}\int_0^T x(t)e^{-\frac{j2\pi kt}{T}}dt$$

What is the value of T?

(d) Suppose the Fourier Series coefficients for the signal $y(t)=4\sin(3t)$ is determined using the equation $X_k=\frac{1}{T}\int_0^T y(t)e^{-\frac{j2\pi kt}{T}}dt$, where T is the value determined in part (c). Can the resulting Fourier Series coefficients be used to correctly synthesize $y(t)$ via Fourier Series expansion ? Justify your answer.

Q1 ANSWER

a) Fundamental freq = 1 rad/s or $\frac{1}{2\pi}$ Hz

b) $|C_0| = 5$ $\quad\angle C_0 = 0°$

$|C_1| = 0$ $\quad\angle C_1 = 0°$

$|C_2| = \sqrt{13}$ $\quad\angle C_2 = -56.3°$

$|C_3| = 2$ $\quad\angle C_3 = -90°$

c) $T = 2\pi$

d) Fundamental period of $y(t) = \frac{2\pi}{3}$ sec

Since there are 3 complete cycles of $y(t)$ between 0 and 2π, $y(t)$ can be synthesized correctly.

Q2. Figure Q2 shows an exponentially decaying function $x(t)$ which is expressed as:

$$x(t) = \exp(-a|t|)$$

where $a > 0$.

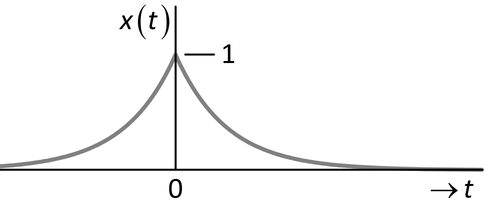


Figure Q2

(a) Determine the Fourier transform, $X(f)$, of the signal $x(t)$.

(b) Using the replication property of the Dirac-δ function, the periodic signal $x_p(t)$ can be obtained as:

$$x_p(t) = x(t) \ast \sum_{k=-\infty}^{\infty} \delta(t - kT_p)$$

where T_p is the period, and $\ast$ denotes convolution. Derive the Fourier transform, $X_p(f)$, of the periodic signal $x_p(t)$ based on this approach.

Q2 ANSWER

a) $x(t) = e^{\alpha t} u(-t) + e^{-\alpha t} u(t)$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{0} e^{\alpha t} e^{-j2\pi f t}\, dt + \int_{0}^{\infty} e^{-\alpha t} e^{-j2\pi f t}\, dt$$

$$= \left. \frac{e^{(\alpha - j2\pi f)t}}{\alpha - j2\pi f} \right|_{-\infty}^{0} + \left. \frac{e^{(-\alpha - j2\pi f)t}}{-\alpha - j2\pi f} \right|_{0}^{\infty}$$

$$= \frac{1}{\alpha - j2\pi f} + \frac{1}{\alpha + j2\pi f} = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

b) $$\sum_{K=-\infty}^{\infty} \rightleftharpoons \frac{1}{T_p} \sum_{K=-\infty}^{\infty} \delta\left(t - \frac{K}{T_p}\right)$$

$$X_p(f) = X(f) \cdot \frac{1}{T_p} \sum_{K=-\infty}^{\infty} \delta\left(t - \frac{K}{T_p}\right)$$

$$= \frac{1}{T_p} \sum_{K=-\infty}^{\infty} X\left(\frac{K}{T_p}\right)$$

$$= \frac{1}{T_p} \sum_{K=-\infty}^{\infty} \frac{2\alpha}{\alpha^2 + 4\pi^2 (K/T_p)^2}$$

Q3. Consider an energy signal $x(t)$. Let $X(f)$, E and B denote its *spectrum*, *energy* and *bandwidth*, respectively. With $x(t)$, we form another signal $y(t) = -0.5x(t-5)$.

(a) Express the spectrum of $y(t)$ in terms of $X(f)$.

(b) Express the energy of $y(t)$ in terms of E.

(c) Express the bandwidth of $y(t)$ in terms of B.

Q3 ANSWER

a) $Y(f) = -\frac{1}{2} X(f) e^{-j2\pi f(5)}$

b) $E = \int_{-\infty}^{\infty} |X(f)|^2 df$

Energy of $y(t) = \int_{-\infty}^{\infty} |Y(f)|^2 df$

$= \int_{-\infty}^{\infty} \frac{1}{4} | X(f) e^{-j10\pi f} | df$

$= \frac{1}{4} \int_{-\infty}^{\infty} |X(f)|^2 |e^{-j10\pi f}|^2 df$

$= \frac{1}{4} \int_{-\infty}^{\infty} |X(f)|^2 \cdot 1 \, df$

$= 0.25 E$

c) Bandwidth of $y(t) = B$

Q4. Consider a signal $x(t)$ (with Fourier Transform X(f)) whose amplitude spectrum is shown in Figure Q4 below.

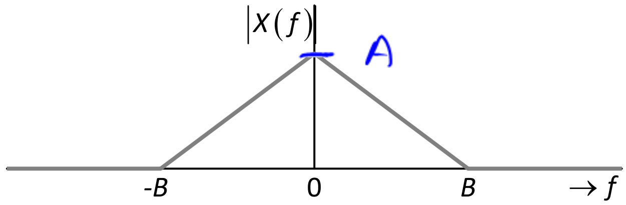


Figure Q4: Amplitude spectrum of $x(t)$

Consider also the signal $y(t) = x(t)\cos(10\pi t)$.

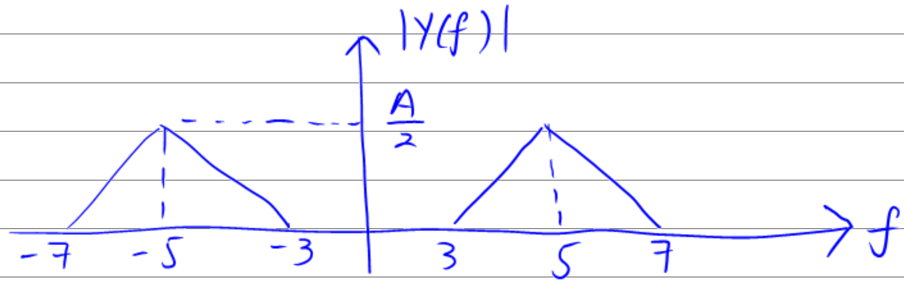
(a) If the bandwidth of $x(t)$ is $B = 2\ Hz$, sketch the amplitude spectrum of $y(t)$. Label clearly the frequency axis of the amplitude spectrum.

(b) If $y(t)$ is sampled at a sampling frequency of 15 *Hz*, write down the expression for the sampled signal of $y(t)$ in terms of the comb function.

(c) Sketch the amplitude spectrum of the sampled signal of $y(t)$.

Q4 ANSWER

a)



b) $y_s(t) = y(t) \cdot \sum_{K=-\infty}^{\infty} \delta\left(t - \frac{K}{15}\right)$

$= x(t)\cos(10\pi t) \cdot \sum_{K=-\infty}^{\infty} \delta\left(t - \frac{K}{15}\right)$

$= \sum_{K=-\infty}^{\infty} x\left(\frac{K}{15}\right) \cos\left(10\pi \frac{K}{15}\right) \delta\left(t - \frac{K}{15}\right)$

c)