Solutions to Tutorial 6

5.2 To find k, we must integrate $f(x)$ from $x = 0$ to $x = 1$ and set it equal to 1. Thus,

$$\int_0^1 kx^2 dx = 1 \quad \text{implies} \quad kx^3/3 \mid_0^1 = 1$$

which implies $k/3 = 1$. Thus, $k = 3$.

(a) $P(.25 \le X \le .75) = \int_{.25}^{.75} 3x^2 dx = x^3\big|_{.25}^{.75} = 13/32 = 0.4063$

(b) $P(X > 2/3) = \int_{2/3}^{1} 3x^2 = x^3\big|_{2/3}^{1} = 19/27 = 0.7037$

5.3 The distribution function is given by

$$F(x) = \int_{-\infty}^{x} f(s)ds = x^3$$

(a) $P(X > .8) = 1 - F(.8) = .488$

(b) $P(.2 < X < .4) = F(.4) - F(.2) = .056$

5.4 (a) Let X be a random variable with density $f(x)$. Then,

$$P(.2 < X < .8) = \int_{.2}^{.8} f(x)dx = \int_{.2}^{.8} xdx = x^2/2\big|_{.2}^{.8} = (.64 - .04)/2 = .30$$

(b)

$$\begin{aligned} P(.6 < X < 1.2) &= \int_{.6}^{1.2} f(x)dx = \int_{.6}^{1} xdx \; + \; \int_{1}^{1.2} (2-x)dx \\ &= x^2/2\big|_{.6}^{1} \; + \; (2x - x^2/2)\big|_{1}^{1.2} = .32 \; + \; .18 = .50 \end{aligned}$$

5.5

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(s)ds = \begin{cases} 0 & x < 0 \\ x^2/2 & 0 \le x \le 1 \\ 1/2 + [2s - s^2/2]\big|_1^x & 1 < x \le 2 \\ 1 & x > 2 \end{cases} \\ &= \begin{cases} 0 & x < 0 \\ x^2/2 & 0 \le x \le 1 \\ 2x - x^2/2 - 1 & 1 < x \le 2 \\ 1 & x > 2 \end{cases} \end{aligned}$$

(a) $P(X > 1.8) = 1 - F(1.8) = 1 - [2(1.8) - (1.8)^2/2 - 1] = 1 - .98 = .02$

(b) $P(.4 < X < 1.6) = F(1.6) - F(.4) = 2(1.6) - (1.6)^2/2 - 1 - (.4)^2/2 = .84$

5.6 We need to integrate $f(x)$ from $x = -\infty$ to $x = \infty$ and set it equal to 1.

$$\begin{aligned} \int_{-\infty}^{\infty} k/(1+x^2)dx &= k\int_{-\infty}^{\infty} 1/(1+x^2)dx = k \cdot \arctan x\big|_{-\infty}^{\infty} \\ &= k(\pi/2 \; + \; \pi/2) = k\pi = 1 \end{aligned}$$

Thus, $k = 1/\pi$.

Solutions to Tutorial 6

5.13 The density is

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Thus,

$$\mu = \int_0^1 3x^3 dx = 3x^4/4\Big|_0^1 = \frac{3}{4} = 0.75$$

$$\mu_2' = \int_0^1 3x^4 dx = 3x^5/5\Big|_0^1 = \frac{3}{5} = 0.6$$

and the variance is

$$\sigma^2 = \mu_2' - \mu^2 = 0.6 - (0.75)^2 = 0.0375$$

5.14 In this case,

$$\begin{aligned} \mu &= \int_0^2 x f(x) dx = \int_0^1 x^2 dx + \int_1^2 x(2-x) dx \\ &= x^3/3\Big|_0^1 + x^2\Big|_1^2 - x^3/3\Big|_1^2 = 1/3 + 4 - 1 - 8/3 + 1/3 = 1 \end{aligned}$$

and

$$\begin{aligned} \mu_2' &= \int_0^2 x^2 f(x) dx = \int_0^1 x^3 dx + \int_1^2 x^2(2-x) dx \\ &= x^4/4\Big|_0^1 + 2x^3/3\Big|_1^2 - x^4/4\Big|_1^2 \\ &= 1/4 + 16/3 - 2/3 - 16/4 + 1/4 = 7/6 \end{aligned}$$

Thus,

$$\sigma^2 = \mu_2' - \mu^2 = 7/6 - 1^2 = 1/6$$

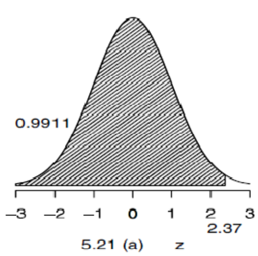
5.21 (a) $P(Z \le z) = F(z) = .9911$. Thus $z = 2.37$

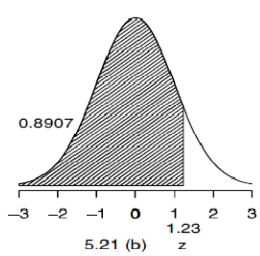
(b) $P(Z > z) = .1093$. That is, $P(Z \le z) = 1 - .1093$ or $F(z) = .8907$. Thus, $z = 1.23$

(c) $P(Z > z) = .6443$. That is, $F(z) = 1 - .6443 = .3557$. Using Table 3, $z = -.37$

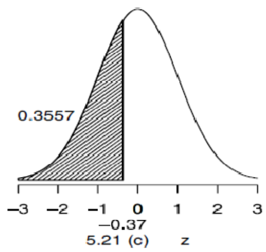
(d) $P(Z < z) = .0217$ so z is negative. From Table 3, $z = -2.02$.

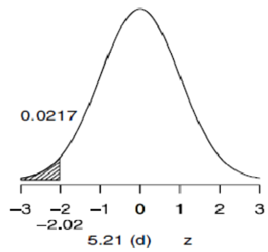
(e) $P(-z \le Z \le z) = .9298$. That is, $F(z) - F(-z) = .9298$, which implies that $F(z) - (1 - F(z)) = .9298$ or $F(z) = (1 + .9298)/2 = .9649$. By Table 3, $z = 1.81$.

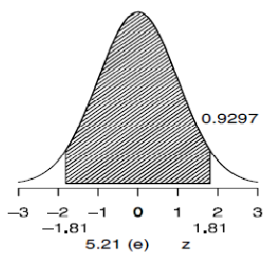




Solutions to Tutorial 6







5.24 Let X have distribution $N(16.2, 1.5625)$.

(a) $P(X > 16.8) = 1 - F((16.8 - 16.2)/1.25) = 1 - F(.48) = 1 - .6844 = .3156$

(b) $P(X < 14.9) = F((14.9 - 16.2)/1.25) = F(-1.04) = .1492$

(c) $P(13.6 < X < 18.8) = F((18.8 - 16.2)/1.25) - F((13.6 - 16.2)/1.25)$
$= F(2.08) - F(-2.08) = .9812 - .0188 = .9624$

(d) $P(16.5 < X < 16.7) = F((16.7 - 16.2)/1.25) - F((16.5 - 16.2)/1.25)$
$= F(.4) - F(.24) = .6554 - .5948 = .0606$

5.25

$$P[X > 39.2] = .20 \quad \text{so} \quad P[\frac{X - 30}{\sigma} > \frac{9.2}{\sigma}] = .20$$

That is, $1 - F(9.2/\sigma) = .20$, and $F(9.2/\sigma) = .80$. But $F(.842) = .80$. Thus $9.2/\sigma = .842$, so $\sigma = 10.93$.

5.31 $P(.295 \le X \le .305) = F((.305 - .302)/.003) - F((.295 - .302)/.003)$
$= F(1) - F(-2.333) = .8413 - .0098 = .8315$
Thus, 83.15 percent will meet specifications.

5.32 We must find μ such that $F((4 - \mu)/.025) = .02$ or $F((\mu - 4)/.025) = .98$. But, $F(2.05) = .98$. Thus, $(\mu - 4)/.025 = 2.05$ or $\mu = 4.05$.

5.33 We need to find μ such that $F((3 - \mu)/.01) = .95$. Thus, from Table 3,
$(3 - \mu)/.01 = 1.645$ or $\mu = 2.98355$.

5.35 If $n = 40$ and $p = .40$ then $\mu = 40(.40) = 16$ and $\sigma^2 = 40(.4)(.6) = 9.6$ or $\sigma = 3.0984$.

(a) $P(22) = F((22.5 - 16)/3.0984) - F((21.5 - 16)/3.0984)$
$= F(2.098) - F(1.775) = .9820 - .9621 = .0199$

(b) $P(\text{less than } 8) = F((7.5 - 16)/3.0984) = F(-2.743) = .0030$

Solutions to Tutorial 6

5.37 In this case, $n = 200$, $p = .25$, $\mu = np = 50$, $\sigma^2 = np(1-p) = 37.5$, $\sigma = 6.1237$. Thus,

$$\begin{aligned} P(\text{fewer than 45 fail}) &= F((44.5 - 50)/6.1237) \\ &= F(-.90) \ = \ .1841 \end{aligned}$$

5.0 It is obvious by the symmetry of the density function.

5.45 The uniform density is:

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Thus, the distribution function is

$$F(x) = \begin{cases} 1 & x \geq 1 \\ x & 0 < x < 1 \\ 0 & x \leq 0 \end{cases}$$

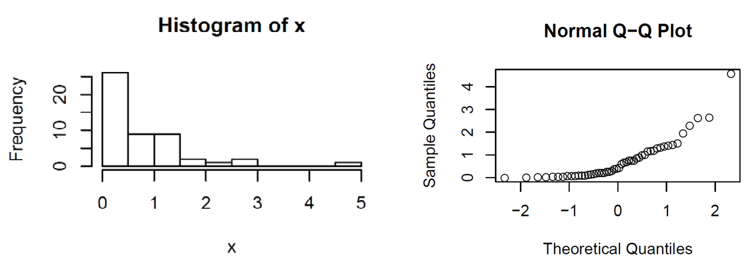
5.47 Suppose Mr. Harris bids $(1+x)c$. Then his expected profit is:

$$\begin{aligned} & 0P(\text{low bid } < (1+x)c) + xcP(\text{low bid } \geq (1+x)c) \\ = \ & xc \int_{(1+x)c}^{2c} \frac{3}{4c} ds \ = \ 3xc[2c - (1+x)c]/4c \ = \ 3c(x - x^2)/4 \end{aligned}$$

Thus, his profit is maximum when $x = 1/2$. So his bid is $3/2$ times his cost. Thus, he adds 50 percent to his cost estimate.

5.200

(a) and (b) Both histogram and QQ-plot show the data is not normally distributed



(c) By taking $x^{1/4}$, the data looks more like normally distributed.