Question:

On Page 7-20 of Lecture Notes, how do we factorize $Y_{step}(s)=\dfrac{K\left(\sigma^2+\omega_d^2\right)}{s\left[(s+\sigma)^2+\omega_d^2\right]}$?

Answer:

Let

$$\frac{K\left(\sigma^2+\omega_d^2\right)}{s\left[(s+\sigma)^2+\omega_d^2\right]}=\frac{A}{s}+\frac{\beta(s)}{(s+\sigma)^2+\omega_d^2}. \qquad (1)$$

Multiply (1) throughout by s then set $s=0$ we get

$$\left[\frac{\cancel{s}K\left(\sigma^2+\omega_d^2\right)}{\cancel{s}\left[(s+\sigma)^2+\omega_d^2\right]}=\frac{\cancel{s}A}{\cancel{s}}+\frac{s\beta(s)}{(s+\sigma)^2+\omega_d^2}\right]_{s=0} \rightarrow \boldsymbol{A=K}. \qquad (2)$$

Substitute $\boldsymbol{A=K}$ into (1):

$$\frac{K\left(\sigma^2+\omega_d^2\right)}{s\left[(s+\sigma)^2+\omega_d^2\right]}=\frac{K}{s}+\frac{\beta(s)}{(s+\sigma)^2+\omega_d^2}=\frac{K\left[(s+\sigma)^2+\omega_d^2\right]+s\beta(s)}{s\left[(s+\sigma)^2+\omega_d^2\right]}. \qquad (3)$$

Equate the **numerators** in (3):

$$\left[\begin{aligned}K\left(\sigma^2+\omega_d^2\right)&=K\left[(s+\sigma)^2+\omega_d^2\right]+s\beta(s)\\&=Ks^2+2K\sigma s+K\sigma^2+K\omega_d^2+s\beta(s)\end{aligned}\right] \rightarrow \boldsymbol{\beta(s)=-K(s+2\sigma)} \qquad (4)$$

Substitute $\boldsymbol{\beta(s)=-K(s+2\sigma)}$ into (3):

$$\underline{\underline{\boldsymbol{\frac{K\left(\sigma^2+\omega_d^2\right)}{s\left[(s+\sigma)^2+\omega_d^2\right]}}}}=\frac{K}{s}-\frac{K(s+2\sigma)}{(s+\sigma)^2+\omega_d^2}=\underline{\underline{\boldsymbol{\frac{K}{s}-\frac{K(s+\sigma)}{(s+\sigma)^2+\omega_d^2}-\frac{K\sigma}{(s+\sigma)^2+\omega_d^2}}}}$$
