NUS National University of Singapore

Dean's Office, Faculty of Science (ANSWER SHEET)

2004 TEST AND ANSWERS

FORM CC1

USE 2B PENCILS ONLY

INSTRUCTIONS

Suggested answers to each question are given in the question paper. Choose an answer and shade the corresponding circle.

EXAMPLES OF SHADING

CORRECT ●

INCORRECT ⊗ ⊘ ⊖ ⊝

SECTION A STUDENT'S NAME

MODULE : MA 1505

YEAR/SEMESTER: 2004

DATE: ____________

SECTION B : MATRICULATION NUMBER

U								
⓪	⓪	⓪	⓪	⓪	⓪	Ⓐ	⓪	⓪
①	①	①	①	①	①	Ⓑ	①	①
②	②	②	②	②	②	Ⓔ	②	②
③	③	③	③	③	③	Ⓗ	③	③
④	④	④	④	④	④	Ⓙ	④	④
⑤	⑤	⑤	⑤	⑤	⑤	Ⓛ	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	Ⓜ	⑥	⑥
⑦	⑦	⑦	⑦	⑦	⑦	Ⓝ	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧	Ⓡ	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	Ⓤ	⑨	⑨
						Ⓦ		
						Ⓧ		
						Ⓨ		

1. Write your matriculation number here.
2. Now SHADE the corresponding circle in the grid for each digit or letter.

SECTION C : ANSWERS

1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
1 Ⓐ ● Ⓒ Ⓓ Ⓔ	11 Ⓐ Ⓑ ● Ⓓ Ⓔ	21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	31 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	41 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ ● Ⓔ	12 ● Ⓑ Ⓒ Ⓓ Ⓔ	22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	32 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	42 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 ● Ⓑ Ⓒ Ⓓ Ⓔ	13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	33 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	43 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ ● Ⓔ	14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	34 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	44 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ ●	15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	35 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	45 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ ● Ⓒ Ⓓ Ⓔ	16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	36 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	46 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ ● Ⓔ	17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	27 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	37 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	47 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ Ⓓ ●	18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	28 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	38 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	48 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9 Ⓐ Ⓑ Ⓒ Ⓓ ●	19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	29 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	39 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	49 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10 Ⓐ Ⓑ Ⓒ ● Ⓔ	20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	30 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	40 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	50 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
51 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	61 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	71 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	81 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	91 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
52 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	62 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	72 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	82 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	92 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
53 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	63 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	73 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	83 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	93 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
54 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	64 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	74 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	84 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	94 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
55 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	65 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	75 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	85 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	95 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
56 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	66 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	76 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	86 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	96 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
57 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	67 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	77 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	87 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	97 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
58 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	68 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	78 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	88 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	98 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
59 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	69 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	79 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	89 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	99 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
60 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	70 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	80 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	90 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	100 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

S/NO 27891

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1. $\ln y = x \ln x \Rightarrow y'(\frac{1}{y}) = 1 + \ln x \Rightarrow y' = x^{x}(1+\ln x)$

$y' = 0 \Leftrightarrow 1 + \ln x = 0$, i.e. $\ln x = -1$ or $x = e^{-1}$

2. $3x^2 + 3y^2 y' = 6(y + xy')$. at $(3,3)$, $3(9) + 27y' = 6(3+3y')$

$9y' = -9 \Rightarrow y' = -1$. tangent line: $\frac{y-3}{x-3} = -1 \Rightarrow y-3+x-3=0$

3. $\frac{d}{dx}(\sin^n x \cos nx) = n\sin^{n-1} x \cos x \cos nx + (\sin^n x)(-n \sin nx)$

$= n \sin^{n-1} x (\cos nx \cos x - \sin nx \sin x) = n \sin^{n-1} x \cos(n+1)x$

4. $f(x) = 1 + \frac{9}{x^2-9} \Rightarrow f'(x) = -18(x^2-9)^{-2}x$

$f''(x) = -18[(x^2-9)^{-2} + x(-2)(x^2-9)^{-3}(2x)]$

$f''(9) = -18(81-9)^{-3}[72 - 4(81)] = \frac{252(18)}{72^2(72)} = \frac{63}{72(72)} = \frac{7}{576}$

5. let $y^4 = 3+x$, $\lim_{y\to 3^{\frac14}} \frac{y-2y^4-3^{\frac14}+6}{y^4-3} = \lim_{y\to 3^{\frac14}} \frac{1-8y^3}{4y^3} = \lim_{y\to 3^{\frac14}} \frac{1}{4y^3} - 2$

6. $f'(x) = 6(x-2)(x+1) = 0$ at $x = -1, 2$, $f(-2) = 1$, $f(-1) = 12$

$-15 = f(2) \leqslant f(-2) \leqslant f(-1) \leqslant f(4) = 37$

7. $\frac{1}{6}\int \frac{6x}{\sqrt{3x^2+5}}dx = \frac{1}{6}\int (3x^2+5)^{-\frac12} d(3x^2+5) = \frac{1}{6}(2)(3x^2+5)^{\frac12} + C$

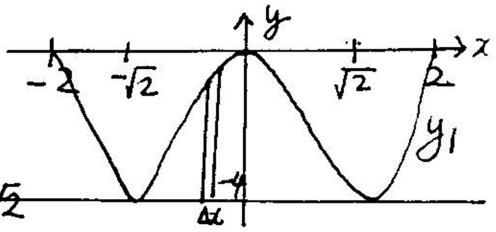
8. $\frac{d}{d(x^4)}\int_0^{x^4} \sin\sqrt{t}\, dt \,\frac{d(x^4)}{dx} = (\sin\sqrt{x^4})\, 4x^3 = (\sin|x^2|)4x^3$

9. Since $\sqrt{\sin x}\sqrt{1-\sin^2 x} = \sqrt{\sin x}\,|\cos x| = \sqrt{\sin(\pi - x)}\,|\cos(\pi-x)|$ on $[0,\pi$

$\int_0^{\pi} \sqrt{\sin x}\,|\cos x|\, dx = 2\int_0^{\frac{\pi}{2}} (\sin x)^{\frac12} d(\sin x) = \frac{4}{3}(\sin x)^{\frac32}\Big|_0^{\frac{\pi}{2}} = \frac{4}{3}$

10. $\int \ln(\cos x)\, d(\tan x) = \ln(\cos x)\tan x - \int \frac{\tan x}{\cos x}(-\sin x)\, dx$

$= \tan x \ln\cos x + \int \sec^2 x - 1\, dx = (\tan x)(\ln\cos x + 1) - x + C$

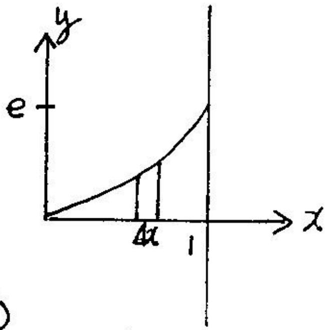


11. $y_1' = 4x(x^2-2) = 0$ at $x = 0, \pm\sqrt{2}$, $y_1(\pm\sqrt{2}) = -4$

Required Area $= 2\int_0^{\sqrt{2}} |y_1 - y_2|\, dx$

$= 2\int_0^{\sqrt{2}} x^4 - 4x^2 + 4\, dx = 2[\frac{x^5}{5} - \frac{4x^3}{3} + 4x]_0^{\sqrt{2}}$

$= 2\sqrt{2}[\frac{4}{5} - \frac{8}{3} + 4] = 2\sqrt{2}(\frac{32}{15})$



12. $y' = xe^x + e^x > 0$ for $x \geqslant 0$

volume of typical disk $= \pi(y-0)^2 \Delta x$

Required Volume $= \int_0^1 \pi x^2 e^{2x} dx = \frac{\pi}{2}\int_0^1 x^2 d(e^{2x})$

$= \frac{\pi}{2}[x^2 e^{2x}\Big|_0^1 - \int_0^1 e^{2x}(2x)\, dx] = \frac{\pi}{2}e^2 - \frac{\pi}{2}\int_0^1 x\, d(e^{2x})$

$= \frac{\pi}{2}e^2 - \frac{\pi}{2}[xe^{2x}\Big|_0^1 - \int_0^1 e^{2x} dx] = \frac{\pi}{2}e^2 - \frac{\pi}{2}e^2 + \frac{\pi}{2}(\frac12 e^{2x})\Big|_0^1 = \frac{\pi}{4}(e^2 - e$

1. Let $f(x) = x^x$, $x > 0$. Then $f'(x) = 0$ at

- **(A)** $x = e$
- **(B)** $x = e^{-1}$
- **(C)** $x = 1$
- **(D)** $x = e^{-2}$
- **(E)** $x = e^2$

2. Find the equation of the tangent to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

- **(A)** $x - y = 0$
- **(B)** $x - 2y + 3 = 0$
- **(C)** $2x - y + 3 = 0$
- **(D)** $x + y - 6 = 0$
- **(E)** $x + 2y - 9 = 0$

3. $\frac{d}{dx}(\sin^n x \cos nx) =$

- **(A)** $n \sin^{n-1} x \cos(n+1) x$
- **(B)** $n \sin^{n-1} x \cos(n-1) x$
- **(C)** $n \sin^{n-1} x \sin(n+1) x$
- **(D)** $n \sin^{n-1} x \sin(n-1) x$
- **(E)** $n \sin^{n-1} x \sin nx$

4. Let $f(x)$ be a function defined by

$$f(x) = \frac{x^2}{x^2 - 9}$$

where $x \in [5, 10]$. Then $f''(9) =$

- **(A)** $\frac{3}{864}$
- **(B)** $\frac{7}{864}$
- **(C)** $\frac{3}{576}$
- **(D)** $\frac{7}{576}$
- **(E)** ∞

5. Evaluate $\lim_{x \to 0} \frac{(3+x)^{1/4} - 2(3+x) - (3)^{1/4} + 6}{x}$.

- **(A)** The limit does not exist
- **(B)** $\frac{1}{4(27)^{1/4}}$
- **(C)** $\frac{1}{4(3)^{1/4}} - 2$
- **(D)** $\frac{1}{4(3)^{1/4}}$
- **(E)** $\frac{1}{4(27)^{1/4}} - 2$

6. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$, $x \in [-2, 4]$. Let M and m denote the absolute maximum value and absolute minimum value of f respectively. Then

- **(A)** $M = 12$, $m = 1$
- **(B)** $M = 37$, $m = -15$
- **(C)** $M = 15$, $m = 0$
- **(D)** $M = 40$, $m = -12$
- **(E)** $M = 45$, $m = 5$

7. $\int \frac{x}{\sqrt{3x^2+5}} \mathrm{d}x =$

(A) $\frac{1}{9}\left(3x^2+5\right)^{3/2}+C$

(B) $\frac{1}{4}\left(3x^2+5\right)^{3/2}+C$

(C) $\frac{1}{12}\left(3x^2+5\right)^{1/2}+C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{1/2}+C$

(E) $\frac{3}{2}\left(3x^2+5\right)^{1/2}+C$

8. $\frac{d}{dx}\int_0^{x^4} \sin\sqrt{t}\mathrm{d}t =$

(A) $\sin x^4$

(B) $-\cos x^2$

(C) $x^4\sin x^2$

(D) $1-\cos x^2$

(E) $4x^3\sin x^2$

9. $\int_0^{\pi} \sqrt{\sin x - \sin^3 x} dx =$

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) 0

(D) $\frac{3}{4}$

(E) $\frac{4}{3}$

10. $\int \frac{\ln(\cos x)}{\cos^2 x}\, \mathrm{d}x =$

(A) $\tan x\,(1+\ln(\cos x)) + \sin x + C$

(B) $\tan x\,(1+\ln(\cos x)) - \sin x + C$

(C) $\tan x\,(1+\ln(\cos x)) + x + C$

(D) $\tan x\,(1+\ln(\cos x)) - x + C$

(E) $\tan x\,(1+\ln(\cos x)) + C$

11. Find the area of the region bounded by the curves $y = x^4 - 4x^2$ and $y = -4$.

(A) $\frac{8}{3}\sqrt{2}$

(B) $\frac{16}{5}\sqrt{2}$

(C) $\frac{64}{15}\sqrt{2}$

(D) $\frac{96}{25}\sqrt{2}$

(E) $\frac{128}{75}\sqrt{2}$

12. Let R denote the region in the first quadrant bounded by $y = xe^x$, $y = 0$ and $x = 1$. Find the volume generated by revolving the region R about the x-axis.

(A) $\frac{\pi}{4}\left(e^2 - 1\right)$

(B) $\frac{\pi}{4}e^2$

(C) $\frac{\pi}{4}\left(1 - \frac{1}{e^2}\right)$

(D) $\frac{\pi}{4}\left(e^2 - 2\right)$

(E) $\frac{\pi}{4}\left(e^2 - \frac{1}{2}\right)$

END OF PAPER