Problem 2.68 A 50-Ω lossless line is to be matched to an antenna with $Z_L = (75 - j20)\ \Omega$ using a shorted stub. Use the Smith chart to determine the stub length and distance between the antenna and stub.

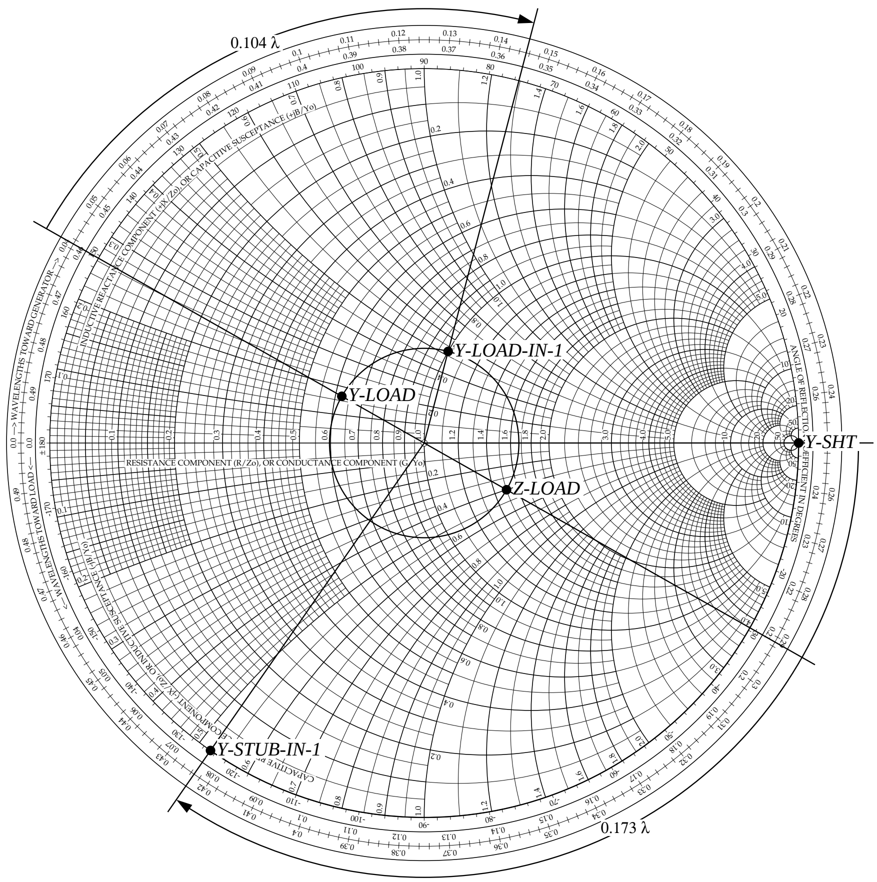


Figure P2.68: (a) First solution to Problem 2.68.

Solution: Refer to Fig. P2.68(a) and Fig. P2.68(b), which represent two different solutions.

$$z_L = \frac{Z_L}{Z_0} = \frac{(75 - j20)\ \Omega}{50\ \Omega} = 1.5 - j0.4$$

and is located at point *Z-LOAD* in both figures. Since it is advantageous to work in admittance coordinates, y_L is plotted as point *Y-LOAD* in both figures. *Y-LOAD* is at 0.041λ on the WTG scale.

For the first solution in Fig. P2.68(a), point *Y-LOAD-IN*-1 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-1 is at 0.145λ on the WTG scale, so the stub should be located at $0.145\lambda - 0.041\lambda = 0.104\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-1, $b = 0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 - j0.52$ is required. This point is *Y-STUB-IN*-1 and is at 0.423λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.423\lambda - 0.250\lambda = 0.173\lambda$ long (or some multiple of a half wavelength longer).

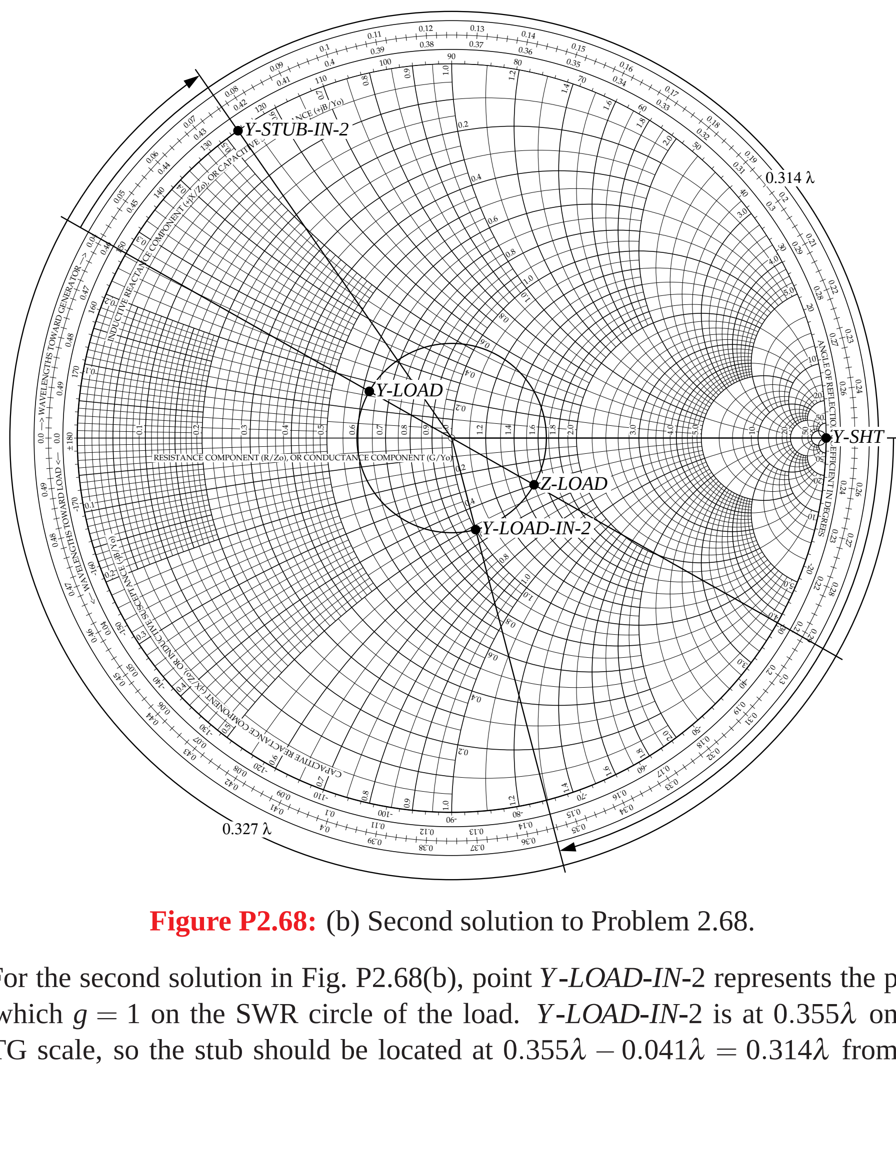


Figure P2.68: (b) Second solution to Problem 2.68.

For the second solution in Fig. P2.68(b), point *Y-LOAD-IN*-2 represents the point at which $g = 1$ on the SWR circle of the load. *Y-LOAD-IN*-2 is at 0.355λ on the WTG scale, so the stub should be located at $0.355\lambda - 0.041\lambda = 0.314\lambda$ from the load (or some multiple of a half wavelength further). At *Y-LOAD-IN*-2, $b = -0.52$, so a stub with an input admittance of $y_{\text{stub}} = 0 + j0.52$ is required. This point is *Y-STUB-IN*-2 and is at 0.077λ on the WTG scale. The short circuit admittance is denoted by point *Y-SHT*, located at 0.250λ. Therefore, the short stub must be $0.077\lambda - 0.250\lambda + 0.500\lambda = 0.327\lambda$ long (or some multiple of a half wavelength longer).

Problem 2

First normalize the load impedance to the system impedance.

$$z_L = \frac{Z_L}{Z_0} = \frac{1}{3}$$

Identify z_L on the Smith Chart.

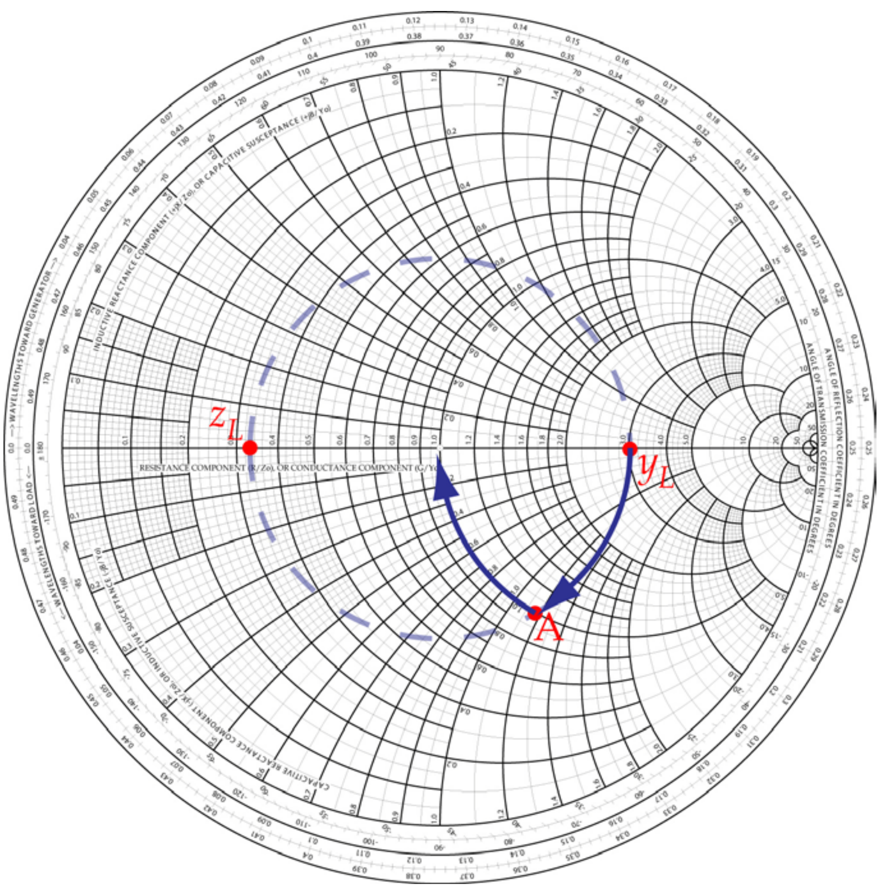
Find y_L on the Smith Chart

Move along the SWR circle until you intersect the *r*=1 circle at point A.

Find out the imaginary part of the admittance at point A. In this case, it's **-j1.15**. That means you an admittance of **y=j1.15** for matching.

Therefore, the required shunt impedance is

$$Z = Z_0 \frac{1}{y} = 75 \frac{1}{j1.15} = -j65.2\ (\Omega)$$



Problem 2.78 In response to a step voltage, the voltage waveform shown in Fig. P2.78 was observed at the sending end of a shorted line with $Z_0 = 50\ \Omega$ and $\varepsilon_r = 4$. Determine V_g, R_g, and the line length.

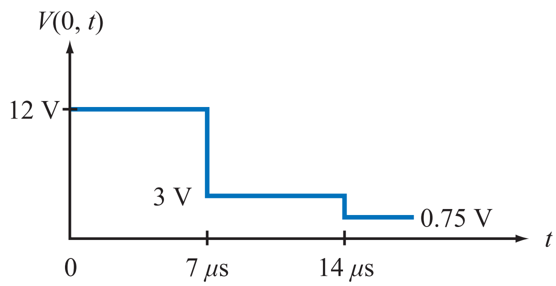


Figure P2.78: Voltage waveform of Problem 2.78.

Solution:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{4}} = 1.5 \times 10^8 \text{ m/s},$$

$$7\ \mu\text{s} = 7 \times 10^{-6} \text{ s} = \frac{2l}{u_p} = \frac{2l}{1.5 \times 10^8}\,.$$

Hence, $l = 525$ m.

From the voltage waveform, $V_1^+ = 12$ V. At $t = 7\mu$s, the voltage at the sending end is

$$V(z=0, t=7\mu\text{s}) = V_1^+ + \Gamma_L V_1^+ + \Gamma_g \Gamma_L V_1^+ = -\Gamma_g V_1^+ \qquad \text{(because } \Gamma_L = -1\text{)}.$$

Hence, 3 V$= -\Gamma_g \times 12$ V, or $\Gamma_g = -0.25$. From Eq. (2.153),

$$R_g = Z_0 \left(\frac{1+\Gamma_g}{1-\Gamma_g}\right) = 50 \left(\frac{1-0.25}{1+0.25}\right) = 30\ \Omega.$$

Also,

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}\,, \qquad \text{or} \quad 12 = \frac{V_g \times 50}{30 + 50}\,,$$

which gives $V_g = 19.2$ V.

Problem 2.80 A generator circuit with $V_g = 200$ V and $R_g = 25\ \Omega$ was used to excite a 75-Ω lossless line with a rectangular pulse of duration $\tau = 0.4\ \mu$s. The line is 200 m long, its $u_p = 2 \times 10^8$ m/s, and it is terminated in a load $R_L = 125\ \Omega$.

(a) Synthesize the voltage pulse exciting the line as the sum of two step functions, $V_{g_1}(t)$ and $V_{g_2}(t)$.

(b) For each voltage step function, generate a bounce diagram for the voltage on the line.

(c) Use the bounce diagrams to plot the total voltage at the sending end of the line.

Solution:

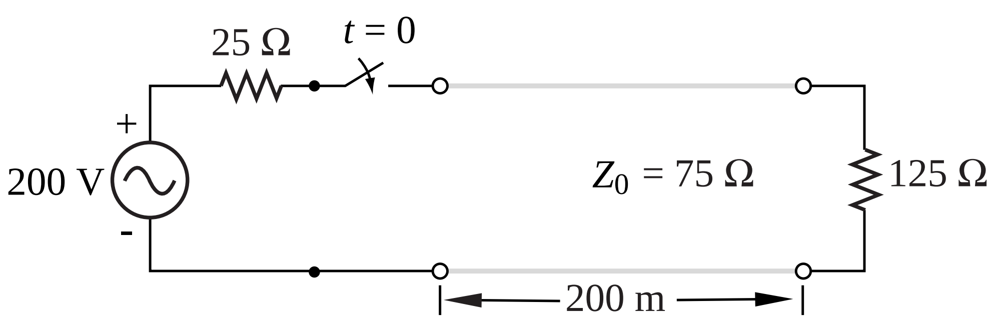


Figure P2.80: (a) Circuit for Problem 2.80.

(a) pulse length $= 0.4\ \mu$s.

$$V_g(t) = V_{g_1}(t) + V_{g_2}(t),$$

with

$$V_{g_1}(t) = 200\,U(t) \quad \text{(V)},$$
$$V_{g_2}(t) = -200\,U(t - 0.4\ \mu\text{s}) \quad \text{(V)}.$$

(b)

$$T = \frac{l}{u_p} = \frac{200}{2 \times 10^8} = 1\ \mu\text{s}.$$

We will divide the problem into two parts, one for $V_{g_1}(t)$ and another for $V_{g_2}(t)$ and then we will use superposition to determine the solution for the sum. The solution for $V_{g_2}(t)$ will mimic the solution for $V_{g_1}(t)$, except for a reversal in sign and a delay by 0.4 μs.

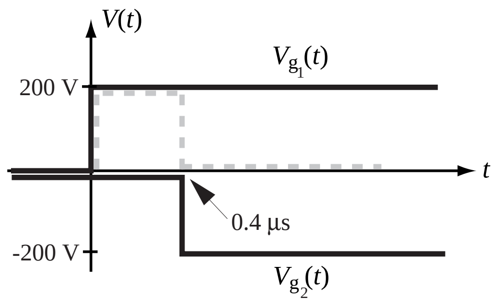


Figure P2.80: (b) Solution of part (a).

For $V_{g_1}(t) = 200U(t)$:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{25 - 75}{25 + 75} = -0.5,$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{125 - 75}{125 + 75} = 0.25,$$

$$V_1^+ = \frac{V_1 Z_0}{R_g + Z_0} = \frac{200 \times 75}{25 + 75} = 150 \text{ V},$$

$$V_\infty = \frac{V_g Z_L}{R_g + Z_L} = \frac{200 \times 125}{25 + 125} = 166.67 \text{ V}.$$

(i) $V_1(0,t)$ at sending end due to $V_{g_1}(t)$:

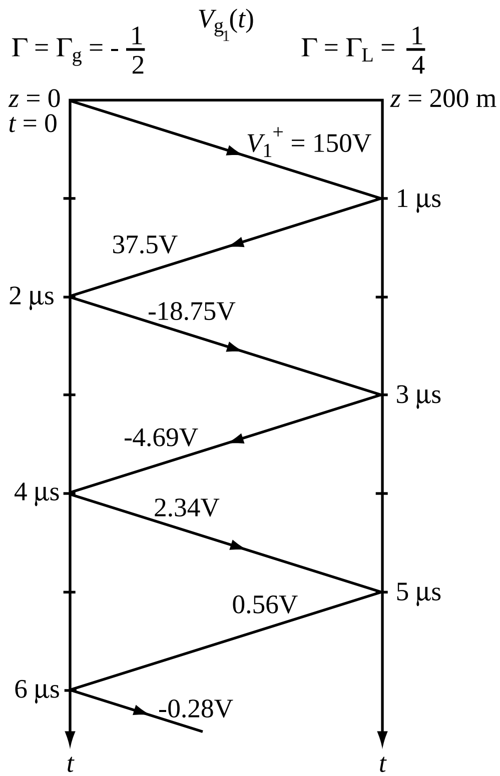


Figure P2.80: (c) Bounce diagram for voltage in reaction to $V_{g_1}(t)$.

(ii) $V_2(0,t)$ at sending end due to $V_{g_2}(t)$:

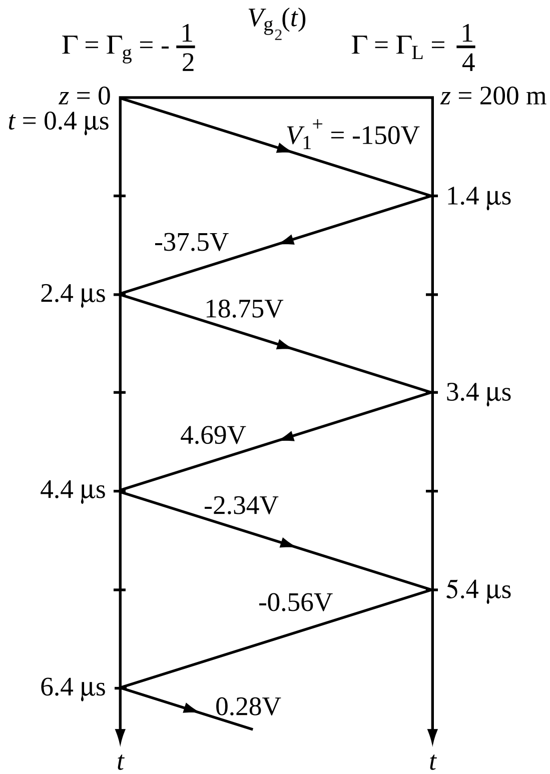


Figure P2.80: (d) Bounce diagram for voltage in reaction to $V_{g_2}(t)$.

(b)

(i) $V_1(0,t)$ at sending end due to $V_{g_1}(t)$: see Fig. P2.80(e).

(ii) $V_2(0,t)$ at sending end: see Fig. P2.80(f).

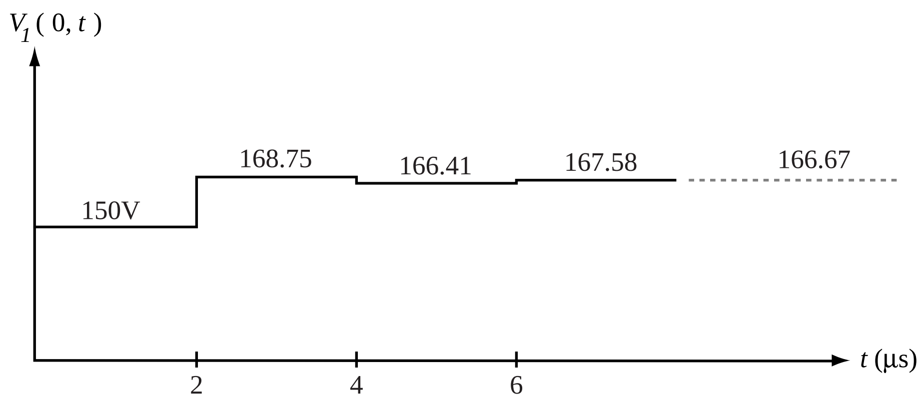


Figure P2.80: (e) $V_1(0,t)$.

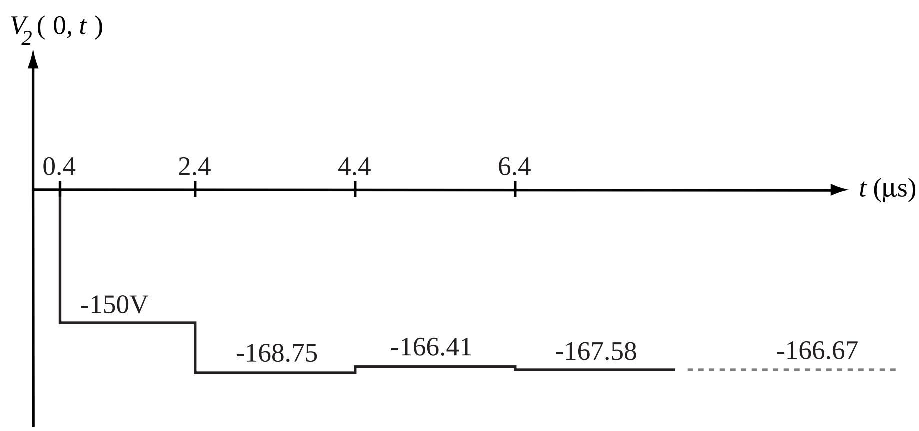


Figure P2.80: (f) $V_2(0,t)$.

(iii) Net voltage $V(0,t) = V_1(0,t) + V_2(0,t)$: see Fig. P2.80(g).

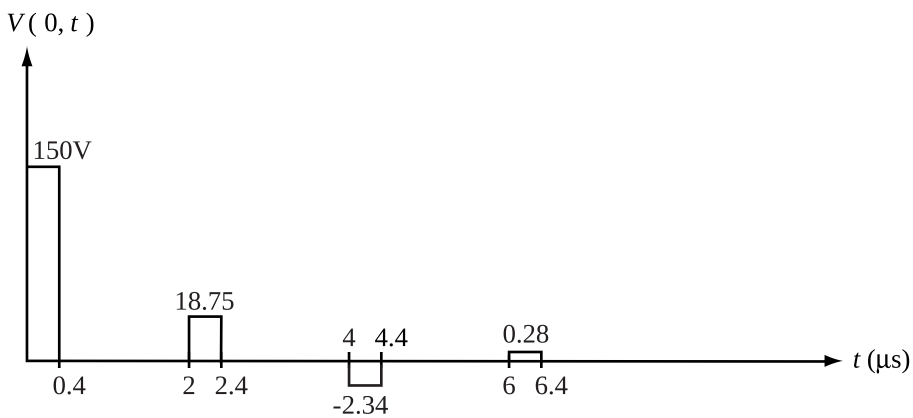


Figure P2.80: (g) Net voltage $V(0,t)$.

Problem 2.81 For the circuit of Problem 2.80, generate a bounce diagram for the current and plot its time history at the middle of the line.

Solution: Using the values for Γ_g and Γ_L calculated in Problem 2.80, we reverse their signs when using them to construct a bounce diagram for the current.

$$I_1^+ = \frac{V_1^+}{Z_0} = \frac{150}{75} = 2 \text{ A},$$
$$I_2^+ = \frac{V_2^+}{Z_0} = \frac{-150}{75} = -2 \text{ A},$$
$$I_\infty^+ = \frac{V_\infty}{Z_L} = 1.33 \text{ A}.$$

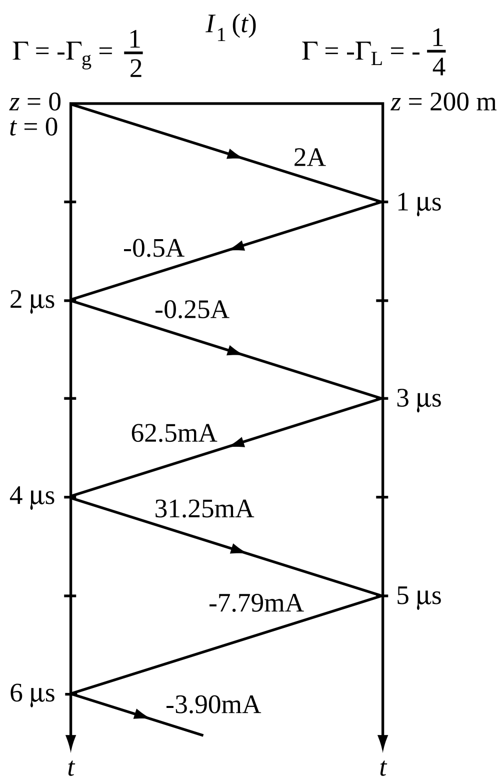


Figure P2.81: (a) Bounce diagram for $I_1(t)$ in reaction to $V_{g_1}(t)$.

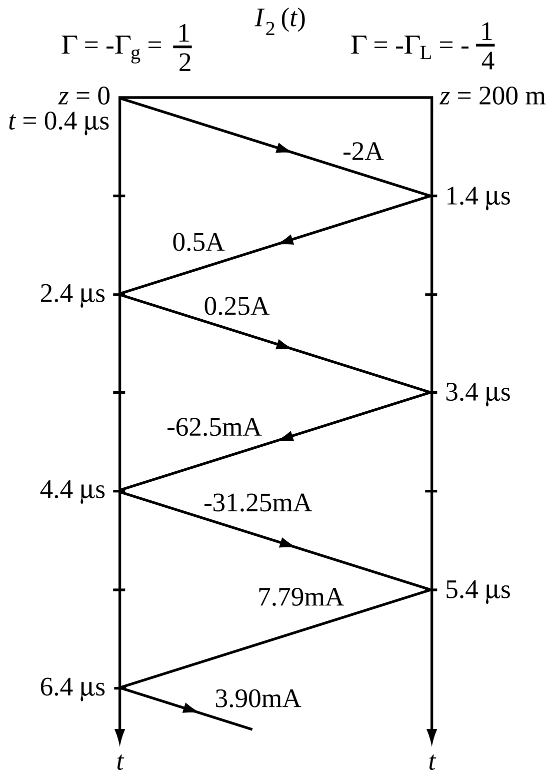


Figure P2.81: (b) Bounce diagram for current $I_2(t)$ in reaction to $V_{g_2}(t)$.

(i) $I_1(l/2,t)$ due to $V_{g_1}(t)$:

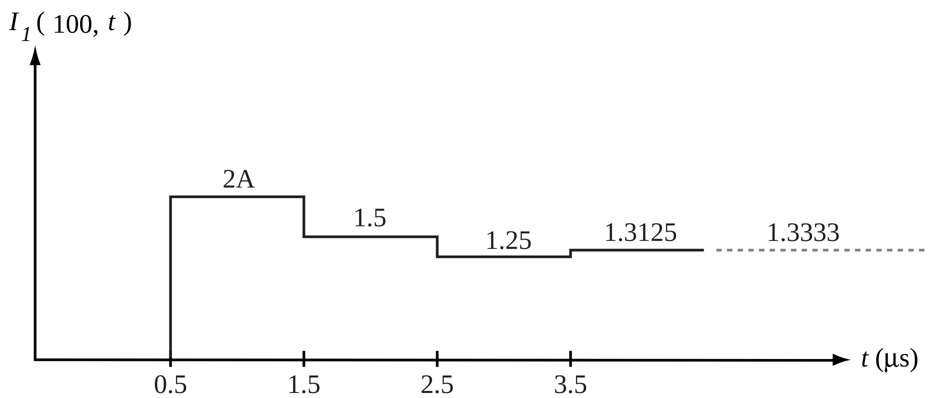


Figure P2.81: (c) $I_1(l/2,t)$.

(ii) $I_2(l/2,t)$ due to $V_{g_2}(t)$:

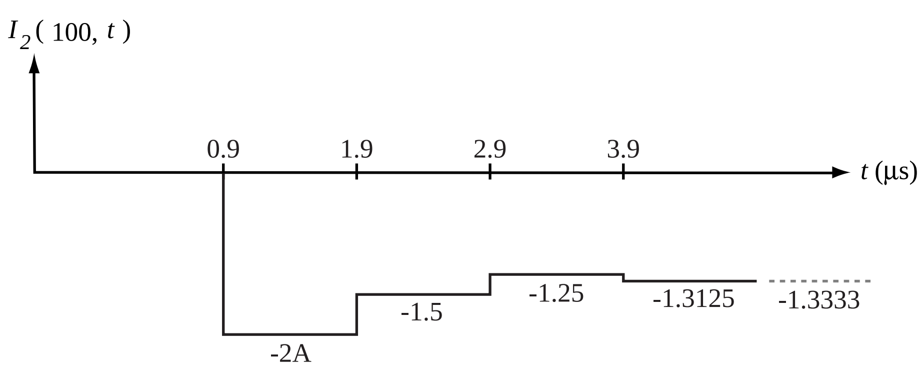


Figure P2.81: (d) $I_2(l/2,t)$.

(iii) Net current $I(l/2,t) = I_1(l/2,t) + I_2(l/2,t)$:

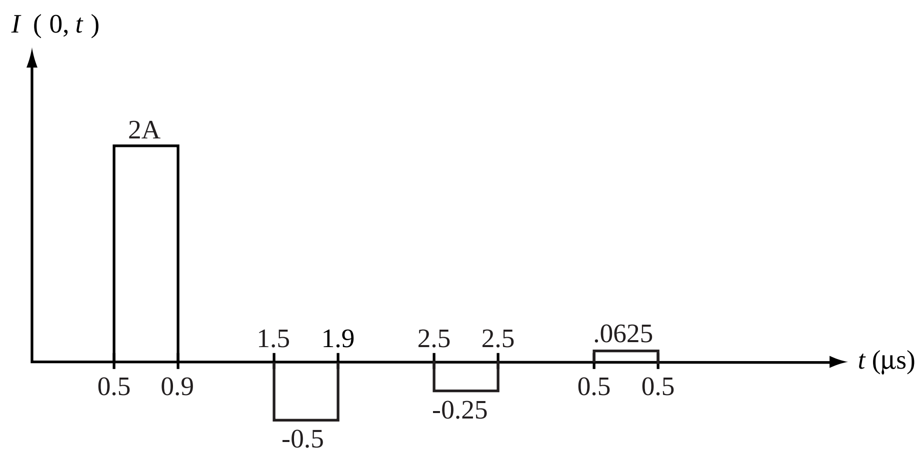


Figure P2.81: (e) Total $I(l/2,t)$.