CS3230 : Tutorial - 6

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The deadline is 1pm, 02-Oct-2012.

1. Given an array $a_1, \ldots, a_n$, we say that a pair (i, j) forms a significant inversion if $i < j$ and $a_i > 2a_j$. Give an $O(nlog(n))$-algorithm that computes the number of significant inversions.

2. You are given two databases each storing n numerical pairwise-distinct values. So, in total, there are $2n$ numerical values. You need to design a method that determines the median of these $2n$ values, which is the n-th smallest number out of the $2n$ numerical values.

   The only way you can access these numerical values is through queries. In each query you specify the database and a value k, and the chosen database returns the kth smallest value that it contains. Your method should compute the median using $O(logn(n))$ queries.

3. Design an algorithm that, given n numbers r_1, $\ldots$, r_n and a number x, detects if there are two numbers whose sum is x. The running time of your algorithm should be $O(log(n))$.

4. Let A be an array $a_1, \ldots, a_n$ that has been circularly shifted from a sorted array to k positions to the right. For instance, $25, 29, 31, 2, 4, 7, 9, 10, 15$ has been circularly shifted from $2, 4, 7, 9, 10, 15, 25, 29, 31$ to 3 positions to the right. Do the following:

   (a) Suppose you know the value of k. Give a $O(1)$ algorithm that finds the maximal number in A.

   (b) Suppose you do not know the value of k. Give $O(log(n))$-algorithm that finds the maximal number in the array A.