EE2011 Engineering Electromagnetics - Part CXD
Tutorial 1 - Solutions

Q1

(i) $\varepsilon_r = 2.9$

$Z_0 = 75\ \Omega$

$Z_L = 300\ \Omega$

$l = 38$ mm $= 0.038$ m

since $u_p = \dfrac{\omega}{k} = \dfrac{c}{\sqrt{\varepsilon_r}}$, we have

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r} = 71.33 \text{ rad/m}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(kl)}{Z_0 + jZ_L \tan(kl)} = 82.90 + j118.01\ \Omega.$$

(ii)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L - Z_0} = \frac{300-75}{300+75} = 0.6\angle 0° = |\Gamma_L| e^{j\theta_L}$$

$$\therefore \quad \theta_L = 0$$

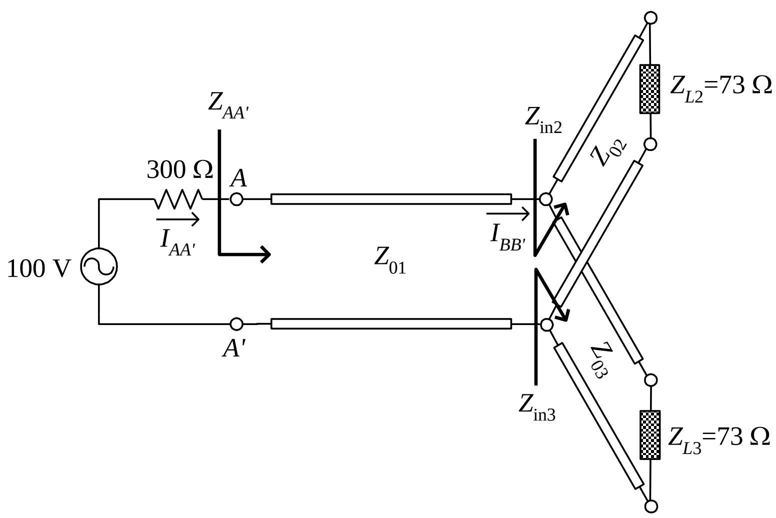
$$\lambda = \text{wavelength along the caoxial line} = \frac{2\pi}{k} = \frac{2\pi}{71.33} = 88 \text{ mm}$$

$$\ell_M = \text{maximum voltage locations} = \frac{\theta_L \lambda}{4\pi} + \frac{n\lambda}{2} = 44n \text{ (mm)}, \quad n = 0,1,2,\cdots$$

$$\begin{aligned} \ell_m &= \text{maximum current locations} \\ &= \text{minimum voltage locations} \\ &= \frac{\theta_L \lambda}{4\pi} + \frac{(2n+1)\lambda}{4} = (22 + 44n) \text{ (mm)}, \quad n = 0,1,2,\cdots \end{aligned}$$

Note: the current maximum locations are shifted $\lambda/4$ = 22 (mm) from the voltage maximum locations.

Q2



$$Z_{\text{in}2} = Z_{02}\,\frac{Z_{L2} + jZ_{02}\tan(2\pi\,3/8)}{Z_{02} + jZ_{L2}\tan(2\pi\,3/8)} = 118.042 - j92.553\,\Omega$$

$$Z_{\text{in}3} = Z_{03}\,\frac{Z_{L3} + jZ_{03}\tan(2\pi\,3/8)}{Z_{03} + jZ_{L3}\tan(2\pi\,3/8)} = 95.244 - j30.472\,\Omega$$

Total impedance as seen from *BB′*:

$$Z_{BB'} = Z_{\text{in}2} \,//\, Z_{\text{in}3} = 54.818 - j26.575\,\Omega$$

Total impedance as seen from *AA′*:

$$Z_{AA'} = Z_{01}\,\frac{Z_{BB'} + jZ_{01}\tan(2\pi\,3/8)}{Z_{01} + jZ_{BB'}\tan(2\pi\,3/8)} = 126.884 - j332.878\,\Omega$$

Total current at *AA′*: $I_{AA'} = \dfrac{100\angle 0^\circ}{300 + Z_{AA'}} = 0.1456 + j0.1136 = 0.1847\angle 0.6623\text{A}$

Average power supplied to transmission line at *AA′*:

$$P_{AA'} = \tfrac{1}{2}\left|I_{AA'}\right|^2 \operatorname{Re}\left[Z_{AA'}\right] = \tfrac{1}{2}(0.1847)^2(126.884) = 2.1643\text{ W}\,.$$

Since transmission line 1 is lossless, the average power supplied to the parallel transmission lines at *BB′* is $P_{BB'} = P_{AA'} = 2.1643\text{ W}$. The total average power supplied to the two antennas must therefore be equal to $P_{BB'}$, since transmission lines 2 and 3 are lossless.

But

$$P_{BB'} = \tfrac{1}{2}\operatorname{Re}\left[V_{BB'}\,I_{BB'}^*\right] = \tfrac{1}{2}\operatorname{Re}\left[V_{BB'}\,(V_{BB'}^*/Z_{BB'}^*)\right] = \tfrac{1}{2}\left|V_{BB'}\right|^2\operatorname{Re}\left[1/Z_{BB'}^*\right] = \tfrac{1}{2}\left|V_{BB'}\right|^2(0.01477)$$

$$\rightarrow\quad \left|V_{BB'}\right|^2 = 293.067$$

$$P_{L2} = \tfrac{1}{2}\left|V_{BB'}\right|^2\operatorname{Re}\left[1/Z_{\text{in}2}^*\right] = 0.7688\text{ W}$$

$$P_{L3} = \tfrac{1}{2}\left|V_{BB'}\right|^2\operatorname{Re}\left[1/Z_{\text{in}3}^*\right] = 1.3956\text{ W}$$

Note that $P_{L2} + P_{L3} = P_{BB'} = 2.1643\text{ W}$

Q3

Let $T = \tan(kl) = \tan(2\pi l/\lambda)$.

$$Z_{\text{in}} = Z_0\,\frac{(100 + j50) + jZ_0T}{Z_0 + j(100 + j50)T} = \frac{100Z_0 + j(50Z_0 + Z_0{}^2T)}{(Z_0 - 50T) + j100T} = 300\,\Omega$$

$$\rightarrow \quad 100Z_0 + j(50Z_0 + Z_0{}^2T) = (300Z_0 - 15000T) + j30000T \quad \text{----------} \ (*)$$

Considering the real part of the above equation:

$$100Z_0 = 300Z_0 - 15000T \quad \rightarrow \quad Z_0 = 75T$$

Putting this condition into the imaginary part of equation (*),

$$50Z_0 + Z_0{}^2T = 30000T \quad \rightarrow \quad 3750T + 5625T^3 = 30000T \quad \rightarrow \quad T = 2.1602$$

Thus $\tan(2\pi\, l/\lambda) = 2.1602$

$\Rightarrow 2\pi\, l/\lambda = \tan^{-1}(2.1602) + n\pi,\ n = 0,1,2,\cdots$

$\Rightarrow$ The minumum (l/λ) corrresponds to $n = 0$.

Hence,

$$l/\lambda = \frac{\tan^{-1}(2.1602)}{2\pi} = 0.181$$

and

$$Z_0 = 75T = 163.018\ \Omega$$

<u>Note:</u> the wavelength λ here is the effective wavelength in the transmission line and is not the free space wavelength λ_0.

Q4

(i)

Given:

$Z_{in} = 12.5 - j12.7\ \Omega$

$Z_L = 100 - j100\ \Omega$

$Z_g = 50\ \Omega$

$Z_0 = 50\ \Omega$

$f = 10^9$ Hz/s

To find the length l,

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0T}{Z_0 + jZ_LT} \text{ where } T = \tan(\beta l)$$

$$\therefore \quad T = \frac{-jZ_0(Z_L - Z_{\text{in}})}{Z_{\text{in}}Z_L - Z_0^2} = 1.7341$$

$\beta l = 1.0477$

$\lambda = c/f = 0.3$ m $\quad \beta = 2\pi/\lambda = 20.944$ rad/m

$\therefore \quad l = 0.05$ m $= 5$ cm

(The length of the transmission line is $0.05/0.3 = 0.167\lambda$)

(ii)

$v_g(t) = 5\cos(2\pi \times 10^9 t)$

$V_g = 5\angle 0°$

$$I_{\text{in}} = \frac{V_g}{Z_g + Z_{\text{in}}} = 0.0768 + j0.0156 = 0.0784\angle 0.2005 \quad \text{A}$$

But looking at the input end $(\ell = l)$ of the transmission line:

$$I_{in} = I(\ell = l) = \frac{I_L}{Z_0}\left[Z_0 \cos(\beta l) + jZ_L \sin(\beta l)\right] = I_L(2.2321 + j1.7326) \text{ A}$$

$$\therefore \quad I_L = 0.0249 - j0.0123 = 0.0277\angle -0.4597 \quad \text{A}$$

$$i_L(t) = \text{Re}\left[I_L \, e^{j2\pi\times 10^9 t}\right] = 0.0277\cos(2\pi\times 10^9 t - 0.4597) \quad \text{A}$$