

## *EE2023 Midterm Quiz Information*

- *Date:* 4 October 2012 (Thursday) – Week 7.
- *Time:* 10 AM.
- *Duration:* 1.5 hours.
- *Venue:* Part of the cohort will be writing the quiz at LT5 or EA while the rest will write it at their original lecture venues. Your lecturer will inform you of your assigned venue in due course.
- *Coverage:* The SIGNALS part of the module.
- *Type:* Closed-book.
- *Calculator:* Programmable calculator is not allowed.
- *Formulas:* Tables of Fourier transforms and trigonometric identities shown in the next two pages will be provided during the quiz.

$$\text{Fourier Series: } \begin{cases} X_k = \frac{1}{T} \int_{\tilde{t}}^{\tilde{t}+T} x(t) \exp(-j2\pi k t/T) dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k t/T) \end{cases}$$

$$\text{Fourier Transform: } \begin{cases} X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi f t) dt \\ x(t) = \int_{-\infty}^{\infty} X(f) \exp(j2\pi f t) df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(f)$
Constant	$K$	$K\delta(f)$
Unit Impulse	$\delta(t)$	<b>1</b>
Unit Step	$u(t)$	$\frac{1}{2} \left[ \delta(f) + \frac{1}{j\pi f} \right]$
Sign (or Signum)	$\text{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(fT)$
Triangle	$\text{tri}\left(\frac{t}{T}\right)$	$T \text{sinc}^2(fT)$
Sine Cardinal	$\text{sinc}\left(\frac{t}{T}\right)$	$T \text{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f - f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2} [\delta(f - f_o) + \delta(f + f_o)]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2} [\delta(f - f_o) - \delta(f + f_o)]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha \pi^{0.5} \exp(-\alpha^2 \pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty} \delta(t - mT)$	$\frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{ \beta } X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t - t_o)$	$X(f) \exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t) \exp(j2\pi f_o t)$	$X(f - f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t) x_2(t)$	$\int_{-\infty}^{\infty} X_1(\zeta) X_2(f - \zeta) d\zeta$ or $X_1(f) * X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta) x_2(t - \zeta) d\zeta$ or $x_1(t) * x_2(t)$	$X_1(f) X_2(f)$
Integration in the time-domain	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j2\pi f} X(f) + \frac{1}{2} X(0) \delta(f)$
		$\frac{1}{j2\pi f} X(f) \quad \text{if } X(0) = 0$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j \sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = \frac{1}{j2}[\exp(j\theta) - \exp(-j\theta)]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$	$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$
$\cos^2(\theta) = \frac{1}{2}[1 + \cos(2\theta)]$	$\mathbf{C} \cos(\theta) - \mathbf{S} \sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2} \cos\left[\theta + \tan^{-1}\left(\frac{\mathbf{S}}{\mathbf{C}}\right)\right]$