Remarks on T7

Q6

The force exerted by the rogue wave
in upward direction
over $t=T$ to $t=T+\tau$ is $-P/\tau$ See Appendix

Appendix

Force, momentum, impulsive force, impulse, Dirac delta function

Let P(t) and F(t) be a momentum and force at time t respectively

It is know that $$\frac{dP(t)}{dt} = F(t)$$

Hence $$P(t_2) - P(t_1) = \int_{t_1}^{t_2} F(t)dt$$

i.e., change of momentum from t_2 to t_1

$$= \int_{t_1}^{t_2} F(t)dt$$

Recall Dirac delta function

$$\delta(t) = 0 \text{ If } t\neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \Longrightarrow \int_{-\tau}^{\tau} \delta(t)dt = 1$$

for any small τ

δ(t) represents an impulsive force at t=0, i.e.,

a very large force (infinitely very large) exerted over [-τ,τ], where τ is infinitesimally small such that the change of momentum over [-τ,τ] is 1 δ(t) is also called impulse of one unit at t=0

Impulse of P units at t=0 means P δ(t), where P represents momentum over a very short time

$\delta(t - t_0)$ = impulsive force at t_0 with momentum 1

$P\delta(t - t_0)$ = impulsive force at t_0 with momentum P

$$L\{\delta(t)\} = 1 \qquad L\{\delta(t - t_0)\} = e^{-st_0}$$

Injection of drug

δ(t) can also be used to represent impulsive injection with one unit of drug over a very short time

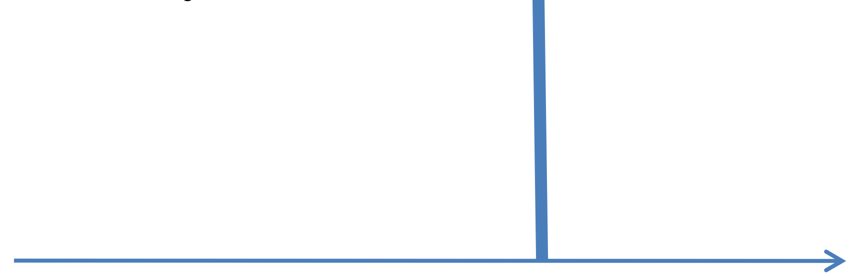
100 δ(t) represents impulsive injection of 100 units of drug over a very short time

How to imagine the graph of δ(t)

We can imagine the graph is infinitely tall , infinitesimally narrow and the area under the graph is 1

0

How to imagine $\delta(t-t_0)$



t_0