Cyclic vs Angular Frequency

$$\left.\begin{array}{ll}\text{CYCLIC Frequency} & : f\ (\text{Hz}) \\ \text{ANGULAR Frequency} & : \omega\ (\text{rad/s})\end{array}\right\} \omega = 2\pi f$$

Fourier transforms:

	Forward Transform	**Inverse Transform**
In cyclic frequency $f:(\text{Hz})$	$G(f)=\int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)\,dt$	$g(t)=\int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)\,df$
In angular frequency $\omega:(\text{rad/s})$	$G(\omega)=\int_{-\infty}^{\infty} g(t)\exp(-j\omega t)\,dt$	$g(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\exp(j\omega t)\,d\omega$

Conversion between f and ω domain $\}$:

$$\underbrace{\begin{bmatrix} G(\omega)=G(f)\big|_{f=\frac{\omega}{2\pi}} \\ G(f)=G(\omega)\big|_{\omega=2\pi f}\end{bmatrix}}_{\text{In general}} \quad \cdots\cdots \quad \underbrace{\begin{bmatrix} \delta(\omega)=\delta(2\pi f)=\frac{1}{2\pi}\delta(f) \\ \delta(f)=\delta\left(\frac{\omega}{2\pi}\right)=2\pi\cdot\delta(\omega)\end{bmatrix}}_{\text{Conversion involving Dirac }\delta\text{-function}}$$

Examples of Fourier transforms of basic functions:

	$g(t)$	$G(f)$	$G(\omega)$
Constant	K	$K\delta(f)$	$2\pi K\delta(\omega)$
Unit impulse	$\delta(t)$	1	1
Unit step	$u(t)$	$\frac{1}{j2\pi f}+\frac{1}{2}\delta(f)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
Signum function	$\text{sgn}(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
Complex exponential	$\exp(j2\pi f_0 t)$ or $\exp(j\omega_0 t)$	$\delta(f-f_0)$	$2\pi\delta(\omega-\omega_0)$
Cosine	$\cos(2\pi f_0 t)$ or $\cos(\omega_0 t)$	$0.5\left[\delta(f-f_0)+\delta(f+f_0)\right]$	$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
Sine	$\sin(2\pi f_0 t)$ or $\sin(\omega_0 t)$	$-j0.5\left[\delta(f-f_0)-\delta(f+f_0)\right]$	$-j\pi\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$
Rectangular pulse	$\text{rect}\left(\frac{t}{T}\right)$	$T\ \text{sinc}(fT)$	$T\ \text{sinc}\left(\frac{\omega T}{2\pi}\right)$
Gaussian pulse	$\exp\left(-\alpha^2 t^2\right)$	$\frac{\pi^{0.5}}{\alpha}\exp\left(-\frac{\pi^2 f^2}{\alpha^2}\right)$	$\frac{\pi^{0.5}}{\alpha}\exp\left(-\frac{\omega^2}{4\alpha^2}\right)$
Comb function	$\sum_{m=-\infty}^{\infty}\delta(t-mT_0)$	$f_0\sum_{k=-\infty}^{\infty}\delta(f-kf_0)$; $\left[f_0=\frac{1}{T_0}\right]$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$; $\left[\omega_0=\frac{2\pi}{T_0}\right]$

Examples of Fourier transform properties:

	Time domain (t)	**Cyclic frequency domain** (f)	**Angular frequency Domain** (ω)
Linearity	$ag_1(t)+bg_2(t)$	$aG_1(f)+bG_2(f)$	$aG_1(\omega)+bG_2(\omega)$
Scaling	$g(at)$	$\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$	$\frac{1}{\lvert a\rvert}G\left(\frac{\omega}{a}\right)$
Duality	$G(t)$	$g(-f)$	$2\pi g(-\omega)$
Time Shifting	$g(t-t_o)$	$G(f)\exp(-j2\pi ft_o)$	$G(\omega)\exp(-j\omega t_o)$
Frequency Shifting (Modulation)	$g(t)\exp(j2\pi f_o t)$ or $g(t)\exp(j\omega_o t)$	$G(f-f_o)$	$G(\omega-\omega_o)$
Differentiation	$\frac{d^n}{dt^n}g(t)$	$(j2\pi f)^n G(f)$	$(j\omega)^n G(\omega)$
Integration	$\int_{-\infty}^{t} g(u)du$	$\frac{1}{j2\pi f}G(f)+\frac{1}{2}G(0)\delta(f)$	$\frac{1}{j\omega}G(\omega)+\pi G(0)\delta(\omega)$
Conjugate	$g^*(t)$	$G^*(-f)$	$G^*(-\omega)$
Multiplication	$g_1(t)g_2(t)$	$G_1(f)\ast G_2(f)$ $=\int_{-\infty}^{\infty} G_1(u)G_2(f-u)du$	$\frac{1}{2\pi}G_1(\omega)\ast G_2(\omega)$ $=\frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(u)G_2(\omega-u)du$
Convolution	$g_1(t)\ast g_2(t)$ $=\int_{-\infty}^{\infty} g_1(u)g_2(t-u)du$	$G_1(f)G_2(f)$	$G_1(\omega)G_2(\omega)$