EE2023 TUTORIAL 2 (SOLUTIONS)

Solution to Q.1

Description of $x(t)$:

- $x(t)$ is a REAL & EVEN function of t $\because$ Spectrum is REAL and SYMMETRIC
- $x(t)$ has an average (or DC) value of 2 $\because$ Zero-frequency component has value 2
- $x(t)$ is APERIODIC $\because$ $\{\pi,\ \pi^2, \pi^3\}\ \cdots$ has no common factor
- $x(t)$ is a POWER SIGNAL $\because$ $\begin{cases}\text{Spectrum is defined only at discrete} \\ \text{frequency points (sum of sinusoids)}\end{cases}$

Since $x(t)$ is non-periodic, it does not have a Fourier series expansion.

Solution to Q.2

(a) The fundamental frequency of $x(t) = 6\sin(12\pi t) + 4\exp\left(j\left(8\pi t + \frac{\pi}{4}\right)\right) + 2$ is $\begin{cases} \boldsymbol{f_p = HCF\{6,4\} = 2} \\ \boldsymbol{T_p = 0.5} \end{cases}$.

Re-write $x(t)$ as a sum of complex exponentials:

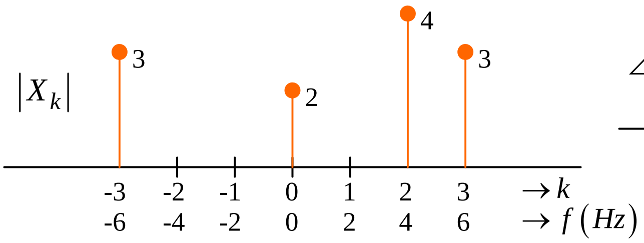
$$\begin{aligned} x(t) &= \frac{6}{j2}\left[\exp(j12\pi t) - \exp(-j12\pi t)\right] + 4\exp(j\pi/4)\exp(j8\pi t) + 2 \\ &= j3\exp(-j12\pi t) + 2 + 4\exp(j\pi/4)\exp(j8\pi t) - j3\exp(j12\pi t) \end{aligned} \tag{1}$$

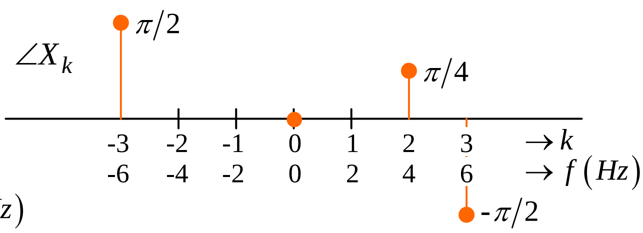
Express $x(t)$ as a complex exponential Fourier series:

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} X_k \exp\left(j2\pi\frac{k}{T_p}t\right) = \sum_{k=-\infty}^{\infty} X_k \exp(j4\pi kt) \\ &= \begin{pmatrix} \cdots + X_{-3}\exp(-j12\pi t) + X_{-2}\exp(-j8\pi t) + X_{-1}\exp(-j4\pi t) \\ +X_0 \\ +X_1\exp(j4\pi t) + X_2\exp(j8\pi t) + X_3\exp(j12\pi t) + \cdots\cdots\cdots \end{pmatrix} \end{aligned} \tag{2}$$

Comparing coefficients of complex exponential terms in (1) and (2), we conclude that:

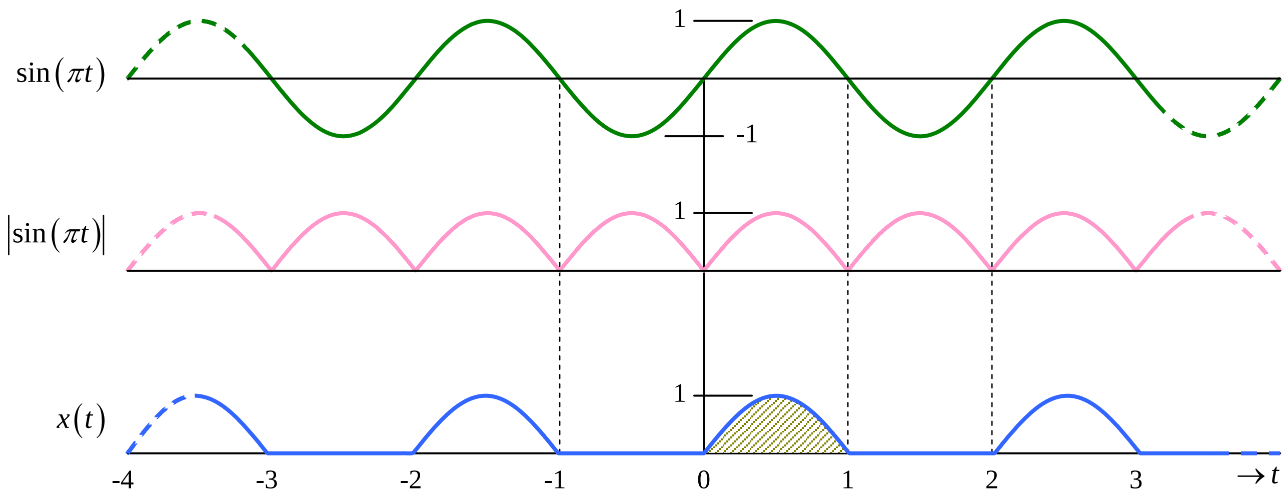
$$\boldsymbol{X_{-3} = j3,\quad X_0 = 2,\quad X_2 = 4\exp\left(j\frac{\pi}{4}\right),\quad X_3 = -j3} \text{ and } \boldsymbol{\left[X_k = 0;\ k \neq 0,\ 2,\ \pm 3\right]}.$$





Remarks: *If a periodic signal is given as a sum of sinusoids, then its Fourier series coefficients can be evaluated using the above method without the need to perform any integration.*

(b) $x(t)=\frac{1}{2}\left(\left|\sin(\pi t)\right|+\sin(\pi t)\right)$: Half-wave rectification of $\sin(\pi t)$.



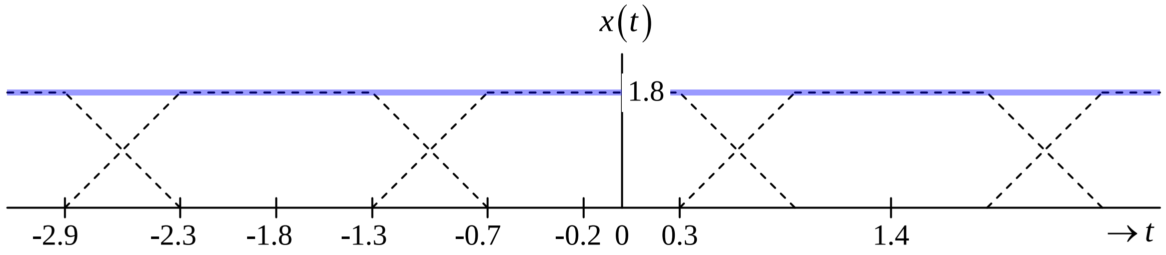
Period of $x(t)$: $T=2$

Coefficients of complex exponential Fourier series expansion of $x(t)$:

$$
\begin{aligned}
X_k &= \frac{1}{T}\int_0^T x(t)\exp(-j2\pi kt/T)\,dt = \frac{1}{2}\int_0^2 x(t)\exp(-j\pi kt)\,dt \\
&= \frac{1}{2}\int_0^1 \sin(\pi t)\exp(-j\pi kt)\,dt \\
&= \frac{1}{2}\int_0^1 \frac{1}{j2}\left[\exp(j\pi t)-\exp(-j\pi t)\right]\exp(-j\pi kt)\,dt \\
&= \frac{1}{j4}\int_0^1 \exp(-j\pi(k-1)t)-\exp(-j\pi(k+1)t)\,dt \\
&= \frac{1}{j4}\left[\frac{\exp(-j\pi(k-1)t)}{-j\pi(k-1)}-\frac{\exp(-j\pi(k+1)t)}{-j\pi(k+1)}\right]_0^1 \\
&= \frac{1}{j4}\left[\exp(-j\pi k)\left(\frac{-1}{-j\pi(k-1)}-\frac{-1}{-j\pi(k+1)}\right)-\left(\frac{1}{-j\pi(k-1)}-\frac{1}{-j\pi(k+1)}\right)\right] \\
&= \frac{\exp(-j\pi k)+1}{2\pi\left(1-k^2\right)} = \begin{cases} \dfrac{1+(-1)^k}{2\pi\left(1-k^2\right)}; & |k|\neq 1 \\ j/4; & k=-1 \\ -j/4; & k=1 \end{cases}
\end{aligned}
$$

Solution to Q.3

Graphically, we observe that $x(t) = \sum_{n=-\infty}^{\infty} 2p(t-1.6n) = 1.8$.



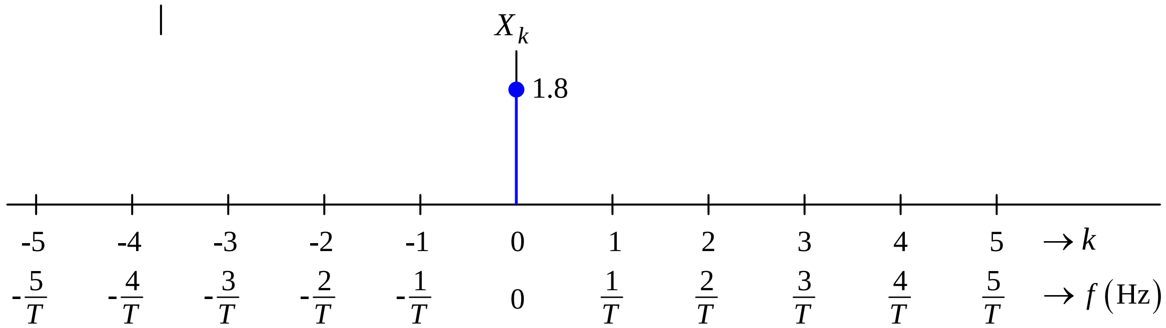
By Deduction:

- $x(t)$ has a *zero-frequency* component of value 1.8, which implies that $X_0 = 1.8$.
- $x(t)$ has no *non-zero frequency* components, which implies that $X_k = 0;\ \ k \neq 0$.

By Derivation:

Since $x(t)$ is a constant (or a DC signal), it may be treated as a periodic signal of arbitrary period *T*, where $0 < T < \infty$. Its Fourier series coefficients can thus be computed as

$$\begin{aligned}
X_k &= \frac{1}{T}\int_{-T/2}^{T/2} 1.8 \exp\left(-j2\pi \frac{k}{T} t\right) dt \\
&= \frac{1.8}{T}\left[\frac{\exp(-j2\pi kt/T)}{-j2\pi k/T}\right]_{-T/2}^{T/2} \\
&= \frac{1.8}{T}\left[\frac{\exp(-j\pi k)}{-j2\pi k/T} - \frac{\exp(j\pi k)}{-j2\pi k/T}\right] \\
&= 1.8\frac{\sin(\pi k)}{\pi k} \\
&= 1.8 \,\text{sinc}(k) \\
&= \begin{cases} 1.8; & k = 0 \\ 0; & k \neq 0 \end{cases}
\end{aligned}$$



Solution to Q.4

(a) The analysis subsystem assumes that the input $x(t)$ has a period of 1 and computes its Fourier series coefficients μ_k over the interval $[-0,5,0.5]$.

(b) The synthesis subsystem uses μ_k as Fourier series coefficients to synthesize a periodic signal of period equal to 1.

(c) The analysis subsystem uses an analysis interval of 1 (from -0.5 to 0.5). Thus, the segment $\left[x(t);\ |t| \le 0.5\right]$ is implicitly treated by the system as one period of the input signal although the actual period of $x(t)$ is 2/3. The output signal is simply obtained by replicating the segment $\left[x(t);\ |t| \le 0.5\right]$ at regular intervals of duration 1. With this notion we may sketch $y(t)$ without the need to compute $y(t) = \sum_{k=-\infty}^{\infty} \mu_k \exp(j2\pi kt)$.