EEC 130A : Formula Sheet

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1 Waves and Phasors

The general expression for a one-dimensional wave

$$A_0 e^{-\alpha x} \cos\left(\frac{2\pi}{T} t \pm \frac{2\pi}{\lambda} x + \theta_0\right)$$

or

$$A_0 e^{-\alpha x} \cos\left(\omega t \pm \beta x + \theta_0\right)$$

In phasor form

$$A_0 e^{-\alpha x} e^{j(\pm \beta x + \theta_0)}$$

2 Transmission Lines

Telegrapher's Equations (phasor form)

$$-\frac{d\tilde{V}(z)}{dz} = \left(R' + j\omega L'\right) \tilde{I}(z)$$
$$-\frac{d\tilde{I}(z)}{dz} = \left(G' + j\omega C'\right) \tilde{V}(z)$$

Wave equation (expressed in voltage)

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0$$

Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{\left(R' + j\omega L'\right)\left(G' + jwC'\right)}$$

Characteristic Impedance

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + jwC'}}$$

For lossless transmission lines

$$\beta = \omega\sqrt{L'C'} = \frac{2\pi}{\lambda}$$

Phase velocity

$$u_p = \frac{\omega}{\beta} = f\lambda$$

For lossless transmission lines

$$u_p = \frac{1}{\sqrt{L'C'}}$$

For most TEM transmission lines

$$u_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

For quasi-TEM transmission lines on non-magnetic substrates, the above is approximated by

$$u_p = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_{eff}}} = \frac{c}{\sqrt{\epsilon_{eff}}}$$

Voltage expression on a lossless line for a wave propagating in the z direction (Solution to the wave equation)

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Current expression on a line for a wave propagating in the z direction

$$I(z) = I_0^+ e^{-j\beta z} - I_0^- e^{j\beta z}$$

Reflection coefficient from load Z_L

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Voltage standing wave ratio on a line with reflection coefficient Γ_L

$$SWR = \frac{|V|_{max}}{|V|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Position of voltage maximum

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2},$$

where $n = 0, 1, 2, \dots$ if $\theta_r \geq 0$, and $n = 1, 2, \dots$ if $\theta_r < 0$.

Position of voltage minimum

$$d_{min} = d_{max} \pm \frac{\lambda}{4}.$$

depending on whether d_{max} is greater or less than $\lambda/4$.

Input impedance of a transmission line seen at a distance l from a load Z_L

$$Z(l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

Input reflection coefficient of a lossless transmission line seen at a distance l from a load Z_L

$$\Gamma_{in} = \Gamma e^{-j2\beta l}$$

3 Electrostatics

Force on a point charge q inside a static electric field

$$\mathbf{F} = q\mathbf{E}$$

Gauss's law

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q \quad \text{or} \quad \nabla \cdot \mathbf{D} = \rho$$

Electrostatic fields are conservative

$$\nabla \times \mathbf{E} = 0 \quad \text{or} \quad \oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

Electric field produced by a point charge q in free space

$$\mathbf{E} = \frac{q\left(\mathbf{R} - \mathbf{R_i}\right)}{4\pi\epsilon_0 \left|\mathbf{R} - \mathbf{R_i}\right|^3}$$

Electric field produced by a volume charge distribution

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{V'} \hat{\mathbf{R}}' \frac{\rho_v \, dV'}{R'^2}$$

Electric field produced by a surface charge distribution

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{S'} \hat{\mathbf{R}}' \frac{\rho_s \, ds'}{R'^2}$$

Electric field produced by a line charge distribution

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_l \, dl'}{R'^2}$$

Electric field produced by an infinite sheet of charge

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_s}{2\epsilon}$$

Electric field produced by an infinite line of charge

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon} = \hat{\mathbf{r}} \frac{D_r}{\epsilon} = \hat{\mathbf{r}} \frac{\rho_l}{2\pi\epsilon r}$$

Electric field - scalar potential relationship

$$\mathbf{E} = -\nabla V \quad \text{or} \quad V_2 - V_1 = -\int_{P1}^{P2} \mathbf{E} \cdot d\mathbf{l}$$

Electrostatic potential due to a point charge (with infinity chosen as the reference)

$$V = \frac{q}{4\pi\epsilon_0 \left|\mathbf{R} - \mathbf{R_i}\right|}$$

Electrostatic potential due to charge distributions

$$V = \frac{1}{4\pi\epsilon}\int_S \frac{\rho_l}{R'}dl' \text{ (line charge)}$$

$$V = \frac{1}{4\pi\epsilon}\int_S \frac{\rho_s}{R'}ds' \text{ (surface charge)}$$

$$V = \frac{1}{4\pi\epsilon}\int_S \frac{\rho_v}{R'}dv' \text{ (volume charge)}$$

Poisson's equation

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

Constitutive relationship in dielectric materials

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$$

where $\mathbf{P}$ is the polarization.

$$\mathbf{P} = \epsilon_0\chi_e\mathbf{E}$$

Electrostatic energy density

$$w_e = \frac{1}{2}\epsilon E^2$$

Boundary conditions

$$E_{1t} = E_{2t} \quad \text{or} \quad \hat{\mathbf{n}} \times (\mathbf{E_1} - \mathbf{E_2}) = 0$$

$$D_{1n} - D_{2n} = \rho_s \quad \text{or} \quad \hat{\mathbf{n}} \cdot (\mathbf{D_1} - \mathbf{D_2}) = \rho_s$$

Ohm's law

$$\mathbf{J} = \sigma\mathbf{E}$$

Conductivity

$$\sigma = \rho_v\mu$$

where μ stands for charge mobility.

Joule's law

$$P = \int \mathbf{E} \cdot \mathbf{J}\ dv$$

4 Magnetostatics

Force on a moving charge q inside a magnetic field

$$\mathbf{F} = q\mathbf{u} \times \mathbf{B}$$

Force on an infinitesimally small current element $Id\mathbf{l}$ inside a magnetic field

$$d\mathbf{F}_m = Id\mathbf{l} \times \mathbf{B}$$

Torque on a N-turn loop carrying current I inside a uniform magnetic field

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

where $\mathbf{m} = \hat{\mathbf{n}}NIA$.

Gauss's law for magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \text{or} \quad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

Ampere's law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \text{or} \quad \oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

Magnetic flux density — magnetic vector potential relationship

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Magnetic potential produced by a current distribution

$$\mathbf{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\mathbf{J}}{R'}dV'$$

Vector Poisson's Equation

$$\nabla^2\mathbf{A} = -\mu\mathbf{J}$$

Magnetic field intensity produced by an infinitesimally small current element (Biot-Savart law)

$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2}$$

Magnetic field produced by an infinitely long wire of current in the z-direction

$$\mathbf{H} = \hat{\boldsymbol{\phi}}\frac{I}{2\pi r}$$

Magnetic field produced by a circular loop of current in the ϕ-direction

$$\mathbf{H} = \hat{\mathbf{z}}\frac{Ia^2}{2(a^2 + z^2)^{3/2}}$$

Constitutive relationship in magnetic materials

$$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$$

Magnetization

$$\mathbf{M} = \chi_m\mathbf{H}$$

Boundary conditions

$$B_{1n} = B_{2n} \quad \text{or} \quad \hat{\mathbf{n}} \cdot (\mathbf{B_1} - \mathbf{B_2}) = 0$$

$$H_{1t} - H_{2t} = J_s \quad \text{or} \quad \hat{\mathbf{n}} \times (\mathbf{H_1} - \mathbf{H_2}) = \mathbf{J_s}$$

Magnetostatic energy density

$$w_m = \frac{1}{2}\mu H^2$$

5 Maxwell's Equations

Integral form

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$$

Faraday's Law

$$V_{emf} = -N\frac{\partial \Phi}{\partial t}$$

Motional EMF

$$V_{emf} = \int (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

6 Useful Integrals

$$\int \frac{dx}{\sqrt{x^2 + c^2}} = \ln(x + \sqrt{x^2 + c^2})$$

$$\int \frac{dx}{x^2 + c^2} = \frac{1}{c}\tan^{-1}\frac{x}{c}$$

$$\int \frac{dx}{\left(x^2 + c^2\right)^{3/2}} = \frac{1}{c^2}\frac{x}{\sqrt{x^2 + c^2}}$$

$$\int \frac{x\ dx}{\sqrt{x^2 + c^2}} = \sqrt{x^2 + c^2}$$

$$\int \frac{x\ dx}{x^2 + c^2} = \frac{1}{2}\ln\left(x^2 + c^2\right)$$

$$\int \frac{x\ dx}{(x^2 + c^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + c^2}}$$

$$\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$$

7 Constants

Free space permittivity

$$\epsilon_0 = 8.85 \times 10^{-12} \qquad \mathrm{F/m}$$

Free space permeability

$$\mu_0 = 4\pi \times 10^{-7} \qquad \mathrm{H/m}$$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi$ $+ \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi$ $+ \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi$ $+ A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi$ $+ A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi$ $+ \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi$ $+ \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi$ $+ A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi$ $+ A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$