C. *Duality*

$$X(t) \rightleftarrows x(-f) \tag{2.6}$$

Example 2-4(C):

Consider the sinc function: $x(t) = \alpha \operatorname{sinc}(2Bt)$ *where* α *and* B *are positive constant. Find* $\Im\{x(t)\}$.

Start with the Fourier transform pair

$$\left[\tilde{x}(t) = A \operatorname{rect}\left(\frac{t}{T}\right)\right] \rightleftarrows \left[\tilde{X}(f) = AT \operatorname{sinc}(Tf)\right]$$

and substitute $T = 2B$ *and* $A = \dfrac{\alpha}{2B}$ *to get*

$$\left[\tilde{x}(t) = \frac{\alpha}{2B} \operatorname{rect}\left(\frac{t}{2B}\right)\right] \rightleftarrows \left[\tilde{X}(f) = \alpha \operatorname{sinc}(2Bf)\right].$$

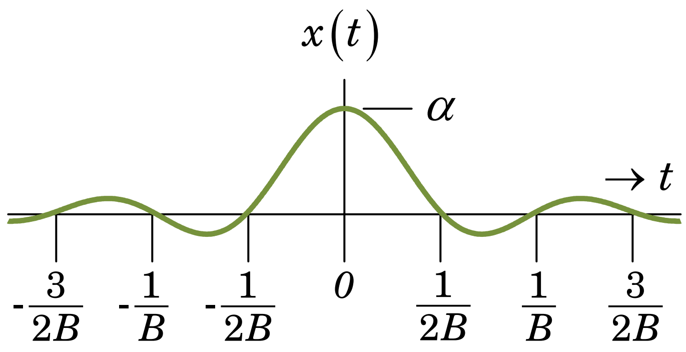
Applying the DUALITY property:

Errata

$$\left[\tilde{X}(t) = \alpha \operatorname{sinc}(2Bt)\right] \rightleftarrows \left[\tilde{x}(-f) = \frac{\alpha}{2B} \operatorname{rect}\left(-\frac{f}{2B}\right)\right].$$

Hence,

$$\Im\{\alpha \operatorname{sinc}(2Bt)\} = \frac{\alpha}{2B} \operatorname{rect}\left(-\frac{f}{2B}\right) = \frac{\alpha}{2B} \operatorname{rect}\left(\frac{f}{2B}\right).$$



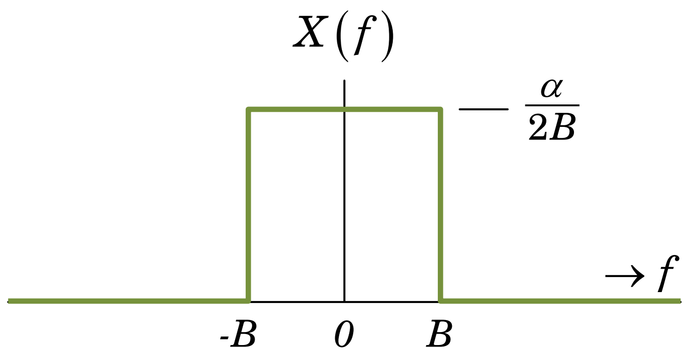


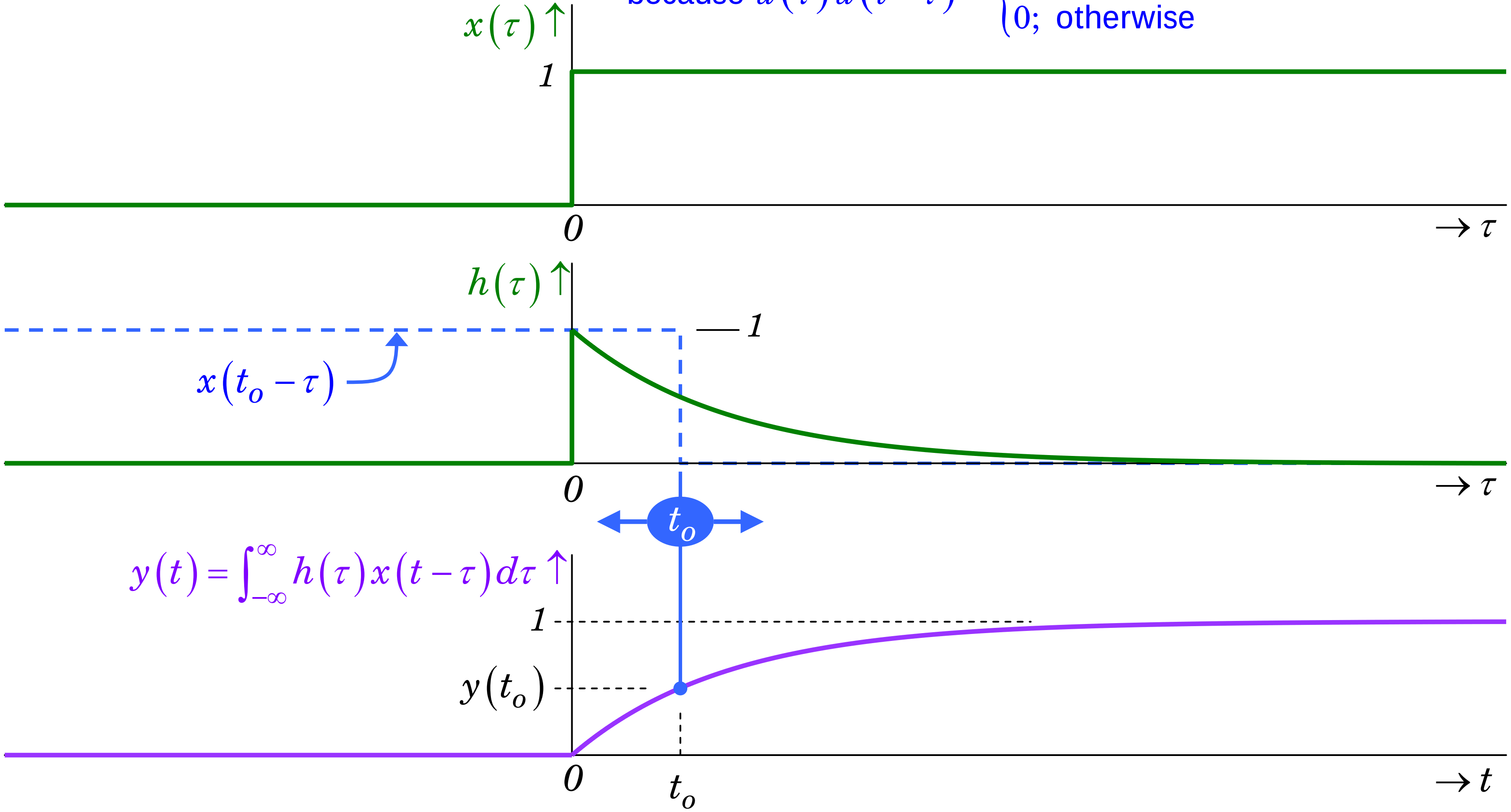
Illustration (Convolution): www.jhu.edu/~signals/index.html

Suppose $x(t)=u(t)$ *and* $h(t)=\exp(-t)u(t)$. *How do we evaluate* $y(t)=h(t)*x(t)$?

$$y(t)=\int_{-\infty}^{\infty}h(\tau)x(t-\tau)d\tau=\underbrace{\int_{-\infty}^{\infty}\exp(-\tau)u(\tau)u(t-\tau)d\tau=\int_{0}^{t}\exp(-\tau)d\tau}=1-\exp(-t)$$

because $u(\tau)u(t-\tau)=\begin{cases}1; & 0\le\tau\le t\\ 0; & \text{otherwise}\end{cases}$

Errata

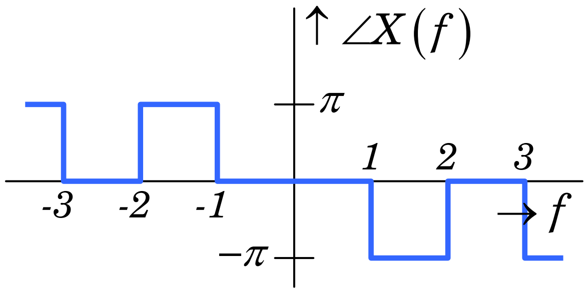


- $\boldsymbol{x(t)}$ ***is REAL and EVEN:*** $\left[\boldsymbol{x^*(t) = x(t)} \text{ and } \boldsymbol{x(t) = x(-t)}\right]$

$$\left.\begin{array}{l} \left(\underbrace{x^*(t) = x(t) \;\Rightarrow\; X^*(f) = X(-f)}_{x(t) \text{ is REAL, see (2.13)}}\right) \\ \left(\begin{array}{l} x(-t) \rightleftarrows X(-f) \;\cdots\; \text{Scaling Property} \\ \underbrace{x(t) = x(-t)}_{x(t) \text{ is EVEN}} \;\Rightarrow\; X(f) = X(-f) \end{array}\right) \end{array}\right\} \Rightarrow \underbrace{\overbrace{X^*(f) = X(f)}^{\text{Real}} \text{ and } \overbrace{X(f) = X(-f)}^{\text{Even}}}_{\boldsymbol{X(f)} \textit{ is REAL and EVEN}} \tag{2.14}$$

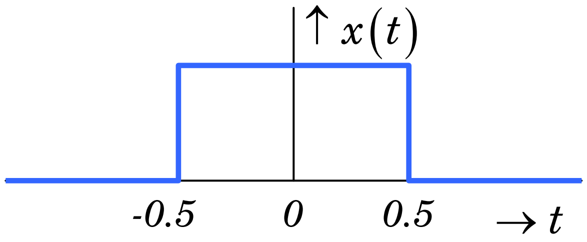
$$\angle X(f) = \begin{cases} 0; & X(f) \geq 0 \\ \pm\pi; & X(f) < 0 \end{cases}$$

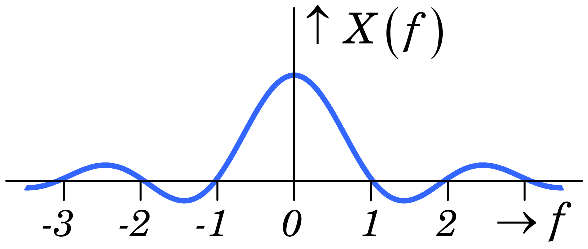
Errata



Example 2-9:

$$\left[\, x(t) = \text{rect}(t) \,\right]$$
$$\downarrow\uparrow$$
$$\left[X(f) = \text{sinc}(f)\right]$$



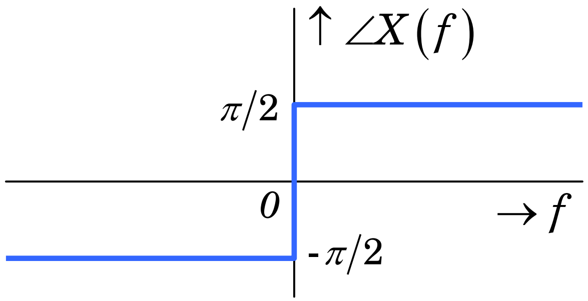


- $\boldsymbol{x(t)}$ ***is REAL and ODD:*** $\left[\boldsymbol{x^*(t) = x(t)} \text{ } \textbf{\textit{and}} \text{ } \boldsymbol{x(-t) = -x(t)}\right]$

$$\left.\begin{array}{l}\left(\underbrace{x^*(t) = x(t) \quad \rightarrow \quad X^*(f) = X(-f)}_{x(t)\text{ is REAL, see (2.13)}}\right. \\ \left(\begin{array}{l} x(-t) \rightleftarrows X(-f) \quad \cdots \text{ Scaling Property} \\ \underbrace{x(t) = -x(-t)}_{x(t)\text{ is ODD}} \quad \rightarrow \quad X(f) = -X(-f) \end{array}\right. \end{array}\right\} \rightarrow \underbrace{\overbrace{X^*(f) = -X(f)}^{\text{Imaginary}} \text{ and } \overbrace{X(f) = -X(-f)}^{\text{Odd}}}_{\boldsymbol{X(f)}\ \textbf{\textit{is IMAGINARY and ODD}}} \qquad (2.15)$$

Errata

$$\angle X(f) = \begin{cases} \pi/2; & \mathrm{Im}\left[X(f)\right] \geq 0 \\ -\pi/2; & \mathrm{Im}\left[X(f)\right] < 0 \end{cases} = \frac{\pi}{2}\mathrm{sgn}\left(\mathrm{Im}\left[X(f)\right]\right)$$



Example 2-10:

$$\left[x(t) = -(2\pi)^{-0.5}\, t \exp\left(-t^2/2\right)\right]$$

$$\downarrow\uparrow$$

$$\left[X(f) = j2\pi f \exp\left(-2\pi^2 f^2\right)\right]$$

see Example 2-4(F)