NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

FINAL EXAMINATION FOR

Semester 1, AY2009/2010

CS4243 COMPUTER VISION & PATTERN RECOGNITION

November 2009 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains two parts: Part I contains **TWENTY (20)** multiple choice questions; Part II contains **TWO (2)** structured problems. There are **SIXTEEN (16)** printed pages, including this page.
2. Answer **ALL** questions. The maximum mark is 60.
3. For Part I, write your answers in the OCR form by completely filling the appropriate circle. Do not write your answers in this booklet, as they will be ignored.
4. For Part II, write your answers in the space provided in this booklet.
5. Write legibly.
6. This is an OPEN BOOK examination.
7. Please write your Matriculation Number below, and also in the OCR form.

Matriculation No.: ______________________________

This portion is for examiner's use only.

Question	Marks	Remarks
Part I		
P1		
P2		
Total		

Part II: Structured Problems

Write your answers to Part II in this booklet.

Problem 1: KNN Classification [10 marks]

Consider a 2-class pattern recognition problem using a two-dimensional feature vector $\mathbf{x}$. The class-conditional pdfs, $P(\mathbf{x} \mid \omega_i)$, are uniform over unit circles centered at $(0,0)$ and $(5,0)$, respectively, as shown in Figure 3. More precisely, they are defined as

$$P(\mathbf{x} \mid \omega_1) = \begin{cases} \frac{1}{\pi}, & \text{if } ||\mathbf{x}|| \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad P(\mathbf{x} \mid \omega_2) = \begin{cases} \frac{1}{\pi}, & \text{if } ||\mathbf{x} - \mathbf{c}|| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

where $\mathbf{c} = [5 \quad 0]^\top$. The class priors are equal: $P(\omega_1) = P(\omega_2) = \frac{1}{2}$.

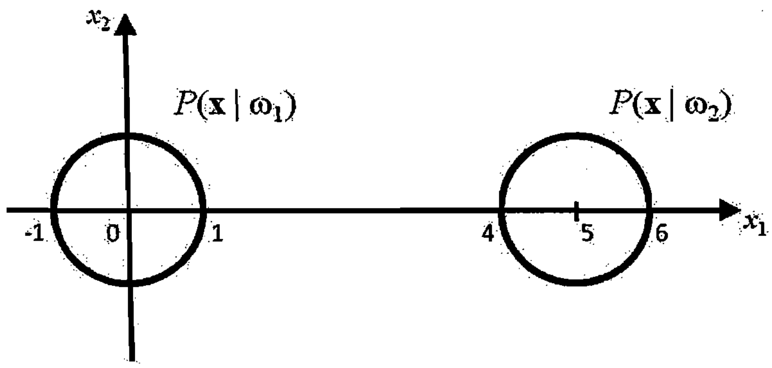

Figure 3: Class-conditional pdfs.

P1(a). Calculate the probability of error for the Bayes' classifier that uses these pdfs. [1 mark]

Let $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ be a set of n independent labeled training samples, and let $\mathcal{D}_k(\mathbf{x}) = \{\mathbf{x}'_1, \ldots, \mathbf{x}'_n\}$ denote the k nearest neighbors of $\mathbf{x}$. Recall that the $k-$nearest-neighbor rule for classifying $\mathbf{x}$ is to give $\mathbf{x}$ the label most frequently represented in $\mathcal{D}_k(\mathbf{x})$.

P1(b). Show that if k is odd, the probability of error of the KNN classifier is

$$P(error) = \frac{1}{2^n} \sum_{j=0}^{(k-1)/2} \binom{n}{j}$$

[7 marks]

(*Hint:* Write $P(error) = P(error \mid \omega_1)P(\omega_1) + P(error \mid \omega_2)P(\omega_2)$. To calculate $P(error \mid \omega_1)$, consider how a query point $\mathbf{q}$, whose true label is ω_1, can be misclassified.)

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P1(c). Show that for this case the single-nearest-neighbor classifier has a smaller error than the k-nearest-neighbor classifier for $k > 1$. [2 marks]

Optional Humor

Problem 2: Interactive Presentation [10 marks]

You have been hired to implement a projector-camera system that allows a user to draw on the projector screen with a laser pointer, and the system responds by drawing the same thing in the image being projected. That is, your system is to track a laser dot as it moves on the projector screen, and then draw the same curve in the image in real-time.

Figure 4(a) shows the system setup, comprising a projector, screen, camera, and a computer. The projector may be thought of as the inverse of a camera: it forms the desired image on its internal LCD panel (image plane), and this image is then projected onto the screen in the real world (Figure 4(b)).

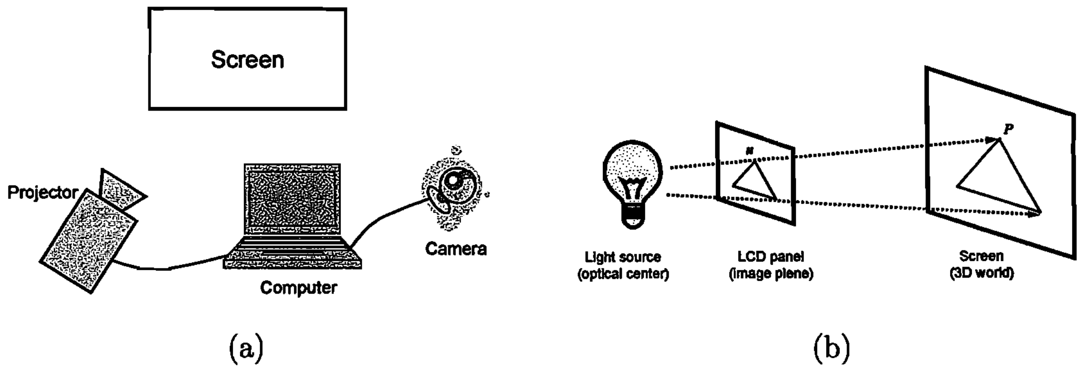

Figure 4: (a) The projector-camera system. (b) Projecting an image.

Denote the image to be projected by $\mathcal{I}$ (which is also the projector's image plane), and the image captured by the camera by $\mathcal{R}$, as shown in Figure 5. The transformation between these two planes is a homography $\mathbf{H}$. Knowing this will allow you to map a point in $\mathcal{R}$ to its corresponding point in $\mathcal{I}$.

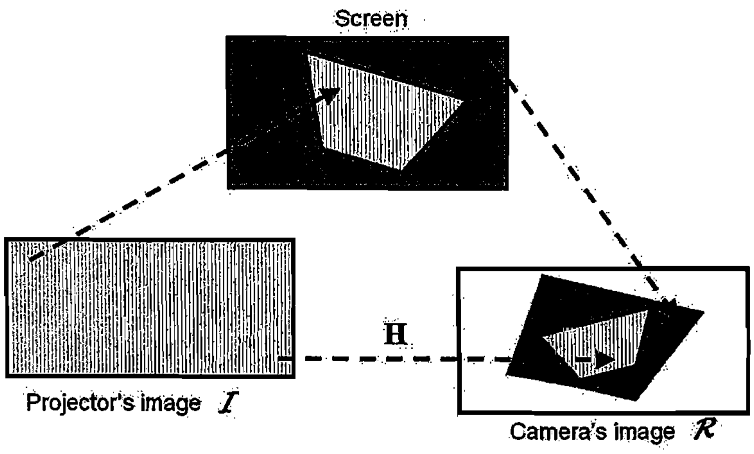

Figure 5: The three image planes.

Your task involves 3 main steps:

1. Calibrate the system to obtain the homography **H**.
2. Detect the laser dot on the screen.
3. Track the moving laser dot and draw it on the image $\mathcal{I}$.

P2(a). Explain how you would calibrate the system to obtain the homography **H**. [3 marks]

P2(b). Give an algorithm to detect and track the moving laser dot on the screen, and to draw the corresponding curve in the image. Assume that the laser dot is significantly brighter than the rest of the screen, and is large enough to be always visible in the camera.

Explain the key steps in your algorithm, stating any other assumptions that you make. Note that the camera sees both the laser dot and the curve currently being drawn, so you should explain how your system distinguishes between these two.

Also note that $\mathcal{I}$ and $\mathcal{R}$ have different spatial resolutions, so that two adjacent pixels in one image may have a gap in the other. Explain how to handle this so that the curve drawn is smooth and unbroken. Be careful to allow for the laser dot to abruptly jump to another location (because the user may make a sudden move). When this happens do not continue drawing the previous curve, but start a new one at the new location. [7 marks]

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