**NATIONAL UNIVERSITY OF SINGAPORE**

**FACULTY OF SCIENCE**

**SEMESTER II EXAMINATION 1999-2000**

# ST2334 PROBABILITY AND STATISTICS

April / May 2000 – Time allowed: 2 hours

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**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **TWO (2)** sections : Section A and Section B. It contains a total of **TEN (10)** questions and comprises **FIVE (5)** printed pages.

2. Answer **ALL** the **FIVE (5)** questions in Section A. The marks for each question are indicated at the end of each question.

3. Answer not more than **FOUR (4)** questions in Section B. Each question in Section B carries 10 marks.

4. Candidates may use non-programmable calculators. However, they should lay out systematically the various steps in the calculations.

5. Statistical tables are provided.

**Section A (60%): Answer all the following FIVE questions.**

1. A ball is randomly chosen from an urn containing 1 white, 1 black, 1 red and 1 blue balls. Suppose that one will win

| | | |
|---|---|---|
| prize 1 | if a white | ball is selected, |
| prize 2 | if a black | ball is selected, |
| prize 3 | if a red | ball is selected, and |
| prize 1, 2 and 3 | if a blue | ball is selected. |

Let the events $A_1 = \{\text{one wins prize } 1\}$,

$A_2 = \{\text{ one wins prize } 2\}$ and

$A_3 = \{\text{ one wins prize } 3\}$.

Are $A_1$, $A_2$ and $A_3$ mutually independent?

**(5 marks)**

2. One urn contains 7 white balls and 3 black balls, and a second urn contains 4 white balls and 2 black balls. A ball is selected from each urn, and is placed in a bag containing 5 white and 6 black balls. What is the probability of drawing a white ball from the bag?

**(5 marks)**

3. A box contains 20 chips. There are three chips marked with a number 3, nine chips marked with a number 9, and eight chips marked with a number 27. If 100 chips are selected randomly with replacement, find the approximate probability that the product of the numbers observed will be between $3^{210}$ and $3^{230}$.

**(15 marks)**

4. Let the joint probability density function of $(X,Y)$ be

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}, & \text{if } 0 < y < 2x,\ 0 < y < 4-2x,\ 0 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find

(i) the marginal probability distribution of $X$,

(ii) the condition probability distribution of $Y$ given $X = 1.5$, and

(iii) determine whether the two variables are independent.

**(15 marks)**

5. The strength of right-hand grip was known to be normally distributed. The strength in kilograms was measured for a group of 35 adult men using a dynamometer. Calculations based on the data yield

$$\sum_{i=1}^{35} X_i = 1365 \text{ kg} \qquad \text{and} \qquad \sum_{i=1}^{35} X_i^2 = 53779 \text{ kg}^2.$$

(i) Test $H_0 : \mu = 41$ kg against $H_1 : \mu < 41$ kg at 1% level of significance.

(ii) If the population variance is known to be 9, find the sample size n so that the probability of committing a Type II error is approximately 0.05 when the population mean $\mu$ is 38 kg.

**(20 marks)**

## Section B (40%): Answer not more than FOUR questions.

6. If the probability density function of a random variable $X$ is given by

$$f(x) = \begin{cases} 2(1-x), & 0< x<1, \\ 0, & \text{elsewhwere,} \end{cases}$$

(i) show that $E(X^r) = \dfrac{2}{(r+1)(r+2)}$ ;

(ii) and use this result or otherwise to evaluate $E[(2X+1)^2]$.

**(10 marks)**

7. A supermarket has two customers waiting to pay for their purchase at counter I and one customer waiting to pay at counter II. Let $X$ and $Y$ denote the numbers of customers who purchase more than \$50 of groceries at respective counter. Suppose $X$ and $Y$ are independent binomial random variables, the probability of a customer spending more than \$50 equal to 0.2 for counter I and 0.3 for counter II. Find the probability that not more than one of the three customers spends in excess of \$50.

**(10 marks)**

8. Suppose that the random variable $X$ has possible values 1,2, 3, ... and

$$\mathrm{P}(X = x) = \mathrm{p}\,(1-\mathrm{p})^{x-1},$$

where $0 < \mathrm{p} < 1$ and $x = 1, 2, \ldots$ .

If s and t are any two positive integers, show that

$$\mathrm{P}(X \geq \mathrm{s}+\mathrm{t} \mid X \geq \mathrm{s}) = \mathrm{P}(X > \mathrm{t})$$

**(10 marks)**

9. A machine packs flour into bags. Suppose the masses in gram of a random sample of 10 bags are found to be

1293, 1380, 1614, 1497, 1340, 1643, 1466, 1094, 1270 and 1028.

Find a 95% confidence interval for $\sigma^2$. It is assumed that the masses are normally distributed.

**(10 marks)**

10. Let $U$ be a uniform random variable on the interval (0, 1). Find the expected value of the random variable

$$X = -\alpha(1 - \ln U).$$

**(10 marks)**

**-- END OF PAPER --**