National University of Singapore
Department of Mathematics
MA2214 Combinatorial Analysis

Tutorial 2 **Semester II, AY 2010/2011**

1. Find the number of ways to choose a pair of distinct numbers $\{a,b\}$ from the set $[50]$ such that (i) $|a-b|=5$ and (ii) $|a-b|\leq 5$.

2. In a class of 15 students, with 10 of them male, 9 students are chosen to form a committee. If there are be exactly 3 female committee members, how many different committees can be formed? If there are 9 different posts in the committee, how many different committees can be formed?

3. Use mathematical induction to prove the Binomial theorem.

4. A student works in a bookstore where he is required to work at least four and at most five days a week, at least one of which has to be a weekend day (Saturday or Sunday). How many different weekly work schedule can this student have?

5. The Singapore national lottery called TOTO (Tax On The Obtuse) works in the following way. To make a bet, you must pick 6 out of a possible 45 different numbers. A total of 6 numbers plus an additional number will be drawn. If your numbers match 4 or more winning numbers, you win a prize. The odds of winning according to the Singapore pools website is the following. Explain the odds.

Prize	Winning Numbers Matched	Odds
Group 1	6 numbers	1 in 8,145,060
Group 2	5 numbers plus the additional number	1 in 1,357,510
Group 3	5 numbers	1 in 35,724
Group 4	4 numbers plus the additional number	1 in 14,290
Group 5	4 numbers	1 in 772
Group 6	3 numbers plus the additional number	1 in 579
	Any prize	1 in 321

6. Give an algebraic and, if possible, a combinatorial proof of each of the following identities.

   (i) $r\binom{n}{r}=n\binom{n-1}{r-1}$.

   (ii) $\binom{n}{m}\binom{m}{r}=\binom{n}{r}\binom{n-r}{m-r}$.

   (iii) $\sum_{k=r}^{n}\binom{n}{k}\binom{k}{r}=\binom{n}{r}2^{n-r}$.

   (iv) $\sum_{r=1}^{n} r\binom{n}{r}=n\cdot 2^{n-1}$.

   (v) $\sum_{k=0}^{n}\binom{n}{k}\binom{n-k}{p-k}\binom{n-p}{q-k}=\binom{n}{p}\binom{n}{q}$.

Answers

1. 45 ; 235.

2. $\binom{10}{6}\binom{5}{3}$; $\binom{10}{6}\binom{5}{3}9!$

4. 50.