NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 1 **SEMESTER II, AY 2010/2011**

1. Find the number of positive divisors of 540.
   (Remark: we always include 1 and n itself as positive divisors of n.)

2. How many 4-digit positive integers are there in which at least one digit occurs more than once?
   (Remark: Provide two different ways to arrive at your answer.)

3. Find the number of odd integers between 3000 and 8000 in which no digit is repeated.

4. There are 12 students queuing up to enter a party, five of them are female. In how many ways can the queue form if

   (i) there are no restrictions?

   (ii) the five ladies are together (in a block)?

   (iii) no two ladies are adjacent?

   (iv) there are exactly three ladies and no one else between two men A and B?

5. 7 men and 3 women attended a wedding dinner and were seated at the same (round) table. In how many ways could they have been seated if

   (i) there were no restrictions?

   (ii) the 3 women were together (in a block)?

   (iii) the 7 men were together (in a block)?

   (iv) each woman was married to one of the men (at the same table) and was seated next to her husband?

6. Give an algebraic and, if possible, a combinatorial proof of each of the following identities.

   (i) $(n)_{\underline{k}} = n(n-1)_{\underline{k-1}}$

   (ii) $(n)_{\underline{k}} = (n-r+1)(n)_{\underline{k-1}}$

   (iii) $(n+1)_{\underline{k}} = (n)_{\underline{k}} + k(n)_{\underline{k-1}}$

   (iv) $(-n)_{\underline{k}} = (-1)^k (n+k-1)_{\underline{k}}$

Answers

1. 24.
2. 4464
3. 1232
4. (i) 12! (ii) 8!5! (iii) $7!5! \times 56$ (iv) $8! \times 120$
5. (i) 9! (ii) 3!7! (iii) 3!7! (iv) $6! \times 8$