NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA 1505 Mathematics I
Tutorial 3

1. Find the area of the following region.

   (a) The region bounded between $y = \frac{1}{2}\sec^2 x$, $y = -4\sin^2 x$, $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$.

   (b) The region in the first quadrant bounded by $y = x$, $y = \frac{1}{4}x^2$ and below $y = 1$.

   (c) The region bounded by $y = 4 - x^2$, $y = 2 - x$, $x = -2$ and $x = 3$.

   Ans. (a) $\frac{4}{3}\pi$ (b) $\frac{5}{6}$ (c) $\frac{49}{6}$

2. (a) Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

   (b) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

   Ans. (a) $\frac{64}{15}\sqrt{2}\pi$ (b) $\frac{8}{3}\pi$

3. Find the radius of convergence of the following series.

   (a) $\sum_{1}^{\infty} (-1)^n \frac{(x+2)^n}{n}$ (b) $\sum_{1}^{\infty} \frac{(3x-2)^n}{n}$ (c) $\sum_{1}^{\infty} (-1)^n (4x+1)^n$

   (d) $\sum_{0}^{\infty} \frac{3^n x^n}{n!}$ (e) $\sum_{1}^{\infty} n^n x^n$ (f) $\sum_{1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

   Ans. (a) 1 (b) 1/3 (c) 1/4 (d)∞ (e) 0 (f) 1/4

4. Find the sum of the geometric series inside the interval of convergence

$$1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 - + \cdots + (-\frac{x-3}{2})^n + \cdots .$$

   Ans. $\frac{2}{x-1}$

5. Find the Taylor series for the following functions:

   (a) $\dfrac{x}{1-x}$ at $x = 0$;

   (b) $\dfrac{1}{x^2}$ at $x = 1$;

   (c) $\dfrac{x}{1+x}$ at $x = -2$;

   Ans. (a) $\displaystyle\sum_{n=0}^{\infty} x^{n+1}$ (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n(n+1)(x-1)^n$ (c) $2 + \displaystyle\sum_{n=1}^{\infty}(x+2)^n$

6. Find a quadratic (2nd degree) polynomial to approximate each of the following functions near $x = 0$:

   (i) $e^{\sin x}$ and (ii) $\ln(\cos x)$.

   Ans. (i) $\dfrac{1}{2}x^2 + x + 1$ (ii) $-\dfrac{1}{2}x^2$

7. Let

$$S = \sum_{n=0}^{\infty} \frac{1}{n!\,(n+2)}.$$

   In this question, we will introduce two different ways to find the value of S, one by integration and the other by differentiation.

   (i) Integrate the Taylor series of xe^x to show that $S = 1$.

   (ii) Differentiate the Taylor series of $\frac{e^x-1}{x}$ to show that $S = 1$.