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In this assignment, you will model the dynamics of a vehicle, analyze its behaviour and design a control system to automatically control the position of this vehicle with respect to some reference position.

1. Dynamics of a Vehicle

Consider the vehicle on a horizontal surface, as illustrated in Figure 1.

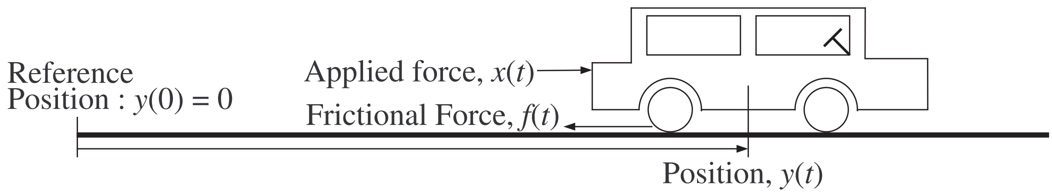

Figure 1: Vehicle on a horizontal surface

Suppose the position of the vehicle at time t, relative to some reference point, is $y(t)$. Let M be the mass of the vehicle, $x(t)$ is the applied force and $f(t)$ is the frictional forces opposing the motion of the vehicle. For simplicity, it may be assumed that the frictional force is proportional to the vehicle speed i.e.

$$f(t) = K_f \frac{\mathrm{d}y(t)}{\mathrm{d}t}$$

where K_f is the coefficient representing frictional losses (Unit of k_f is $\frac{\mathrm{N}}{m/s}$).

The behaviour of the vehicle is determined by the parameters, M and K_f. Hence, in order to analyse the behaviour of the vehicle, you need these values. These parameters may be obtained as follows :

- Mass of the vehicle, M is $(1 + 0.1D)$ kg
- Friction coefficient, K_f, is $\left(0.6 + \frac{C+D}{50}\right) \frac{\mathrm{N}}{m/s}$

where C and D may be derived by reading the last 4 digits of your student matriculation number. Equate the last two digits to C and the remaining two to D. For example if your last 4 digits of your matriculation number is 1234, then choose $C = 34$ and $D = 12$. With the values of M and K_f determined in this manner, you may proceed to answer the following questions.

i. Using Newton's second law, derive the differential equation that relates the vehicle position, $y(t)$, and the applied force, $x(t)$.

ii. Write down the transfer function of the vehicle relating its position, $Y(s) = \mathcal{L}\{y(t)\}$, and applied force, $X(s) = \mathcal{L}\{x(t)\}$.

iii. Determine the poles of the transfer function found in (ii) above. Sketch the pole-zero diagram.

iv. Based on your understanding of the results in (ii) and (iii), sketch the expected trajectory of the vehicle's position when a constant unit force is applied to it at $t = 0$.

v. Explain the role of each parameter in the dynamics of the vehicle.

2. Analysis of the vehicle position control system

A feedback control system is developed by using an infra-red sensor to measure the vehicle's position, and then using the following expression to determine the force, $x(t)$, that should be applied to the vehicle

$$x(t) = K_1[y_r(t) - y(t)] \tag{1}$$

where $y_r(t)$ is the desired position at time t and K_1 is a constant to be designed. Figure 2 shows the block diagram of the resulting *automatic position control system* that relates the desired position, $y_r(t)$, and the actual position, $y(t)$ of the vehicle.

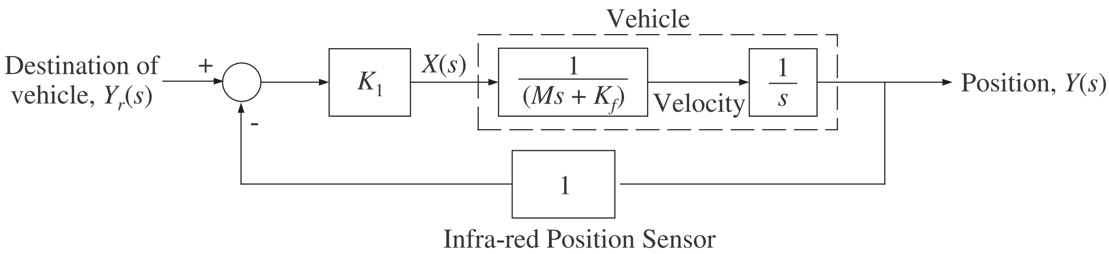

Figure 2: Vehicle position control system

i. Derive the transfer function relating $Y_r(s) = \mathcal{L}\{y_r(t)\}$ to $Y(s) = \mathcal{L}\{y(t)\}$. You may use $K_1 = 0.9$.

ii. Determine the poles of the automatic controlled vehicle.

iii. If the vehicle is now at the start position when $t = 0$ and it needs to move to a location that is 100 m away from the starting point. How can you model the input signal, $y_r(t)$?

iv. Derive and sketch the trajectory of the vehicle under closed loop control.

v. Suppose the vehicle is now loaded in such a manner that its closed loop controlled behaviour is altered significantly. You may take "significant" to mean a change in response from oscillatory to non-oscillatory or non-oscillatory to oscillatory - depending on the parameters of the vehicle you have. Describe a scenario (by giving values for relevant parameters) in which a significant change in behaviour can happen. Analyze this behaviour and sketch the trajectory of the vehicle.

vi. Is it possible to re-design the value of K_1 to get a better response from your vehicle? Support your answer with an analysis.