Solutions to Tutorial 3

3.58 (a)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.06 + .04}{.06 + .04 + .19 + .11} = .25.$$

(b)

$$P(B|\overline{C}) = \frac{P(B \cap \overline{C})}{P(\overline{C})} = \frac{.19 + .06}{1 - .4} = .417.$$

(c)

$$P(A \cap B|C) = \frac{P(A \cap B \cap C)}{P(C)} = \frac{.04}{.4} = .1.$$

(d)

$$P(B \cup C|\overline{A}) = \frac{P((B \cup C) \cap \overline{A})}{P(\overline{A})} = \frac{.09 + .11. + .19}{1 - .5} = .78.$$

(e)

$$P(A|B \cup C) = \frac{P(A \cap (B \cup C))}{P(B \cup C)} = \frac{.06 + .04 + .16}{.06 + .04 + .16 + .19 + .11 + .09} = \frac{.26}{.65} = .4$$

(f)

$$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{.04}{.04 + .11} = .267.$$

(g)

$$P(A \cap B \cap C|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = .267.$$

(h)

$$P(A \cap B \cap C|B \cup C) = \frac{P(A \cap B \cap C)}{P(B \cup C)} = \frac{.04}{.65} = .062.$$

3.62 Let A be the event that a bottle is free of major defects and B be the event that it is free of minor blemishes. Then, $P(A) = .98$, $P(B) = .96$, $P(A \cap B) = .95$.

(a) $P(B|A) = P(B \cap A)/P(A) = .95/.98 = .969$.

(b) $P(A|B) = P(A \cap B)/P(B) = .95/.96 = .990$.

3.63 (a) $P(A|B) = P(A \cap B)/P(B) = .24/.40 = .6 = P(A)$.

(b) $P(A|\overline{B}) = P(A \cap \overline{B})/P(\overline{B}) = (P(A) - P(A \cap B))/(1 - P(B))$
$= (.60 - .24)/(1 - .40) = .6 = P(A)$.

(c) $P(B|A) = P(B \cap A)/P(A) = .24/.60 = .4 = P(B)$.

(d) $P(B|\overline{A}) = P(B \cap \overline{A})/P(\overline{A}) = (P(B) - P(B \cap A))/(1 - P(A))$
$= (.40 - .24)/(1 - .60) = .4 = P(B)$.

Solutions to Tutorial 3

3.64 (a) The probability of getting an error on the first draw is $4/24$. The probability of getting an error on the second draw given that there was an error on the first draw is $3/23$. Thus, the probability that both will contain errors is:

$$\frac{4}{24} \cdot \frac{3}{23} = \frac{1}{46} = .0217.$$

(b) The probability that neither will contain errors is :

$$\frac{20}{24} \cdot \frac{19}{23} = \frac{95}{138} = .688.$$

3.65 (a) The probability of drawing a Seattle-bound part on the first draw is $45/60$. The probability of drawing a Seattle-bound part on the second draw given that a Seattle-bound part was drawn on the first draw is $44/59$. Thus, the probability that both parts should have gone to Seattle is:

$$\frac{45}{60} \cdot \frac{44}{59} = .559.$$

(b) Using an approach similar to (a), the probability that both parts should have gone to Vancouver is:

$$\frac{15}{60} \cdot \frac{14}{59} = .059.$$

(c) The probability that one should have gone to Seattle and one to Vancouver is 1 minus the sum of the probability in parts (a) and (b) or .381.

3.66 The two digits chosen are independent.

(a) $P($ two $5's) = 0.1 \times 0.1 = 0.01$. Alternatively, only 1 of the 100 outcomes is (5, 5).

(b) $P($ first a 5 then a number less than 5 $) = 0.1 \times 0.5 = 0.05$ since five digits 0, 1, 2, 3, and 4 are less than 5. Alternatively, only 5 of the 100 possible pairs satisfy the specified condition.

3.67 A and B are independent if and only if $P(A)P(B) = P(A \cap B)$. Since (0.60)(0.45)= 0.27, they are independent.

3.68 By the definition of odds, $P(M) = 3/8$, $P(N) = 2/3$, and $P(M \cap N) = 1/5$. Since

$$P(M)P(N) = (3/8)(2/3) = 1/4 \neq 1/5 \quad \text{the events M and N are not independent.}$$

3.69 (a) Each head has probability $1/2$, and each toss is independent. Thus, the probability of 8 heads is $(1/2)^8 = 1/256$.

(b) $P(\text{three 3's and then a 4 or 5}) = (1/6)^3(1/3) = 1/648$.

(c) $P(\text{five questions answered correctly}) = (1/3)^5 = 1/243$.

3.70 (a) $P(\text{first three are blanks}) = (3/6)(2/5)(1/4) = 1/20$.

(b) $P(R_2R_3S_4R_5|R_1) = (.8)(.8)(1-.8)(.6) = .0768$, where R_i means it rained on day i, and S_i means it was sunny on day i.

(c) $P(\text{not promptly next 3 months} \mid \text{promptly this month}) = (1-.90)(.50)(.50) = .025$.

(d) $P(\text{4 picked do not meet standards}) = (5/12)(4/11)(3/10)(2/9) = .0101$.

Solutions to Tutorial 3

3.71 Using the law of total probability, $P(\text{new worker meets quota}) = (.80)(.83) + (.20)(.35) = .734$.

3.72

$$\begin{aligned}&P(\text{attended training program} \mid \text{meets quota})\\ &\quad = \frac{P(\text{attended training program and meets quota})}{P(\text{meets quota})} = \frac{(.80)(.83)}{.734} = .905.\end{aligned}$$

3.73 $P(\text{car had bad tires}) = (.20)(.10)+(.20)(.12)+(.60)(.04)=.068$.

3.74 $P(\text{from agency } F \mid \text{bad tires}) = P(\text{from agency } F \text{ and bad tires})/P(\text{bad tires})$
$= (.60)(.04)/.068 = .353$.

3.75 (a) $P(A) = (.4)(.3) + (.6)(.8) = .60$.

(b) $P(B|A) = P(B \cap A)/P(A) = (.4)(.3)/(.60) =.20$.

(c) $P(B|\overline{A}) = P(B \cap \overline{A})/P(\overline{A}) = (.4)(.7)/(.4) =.70$.

3.78 (a)

$$P(\text{ V gets job}) = (3/4)(1/3) + (1/4)(3/4) = .4375.$$

(b)

$$\begin{aligned}&P(\text{ W did not bid } \mid \text{ V gets job})\\ &\quad = P(\text{ W did not bid and V gets job})/P(\text{V gets job})\\ &\quad = (1/4)(3/4)/(.437) = .4286.\end{aligned}$$

3.79 Let A be the event that the test indicates corrosion inside of the pipe and C be the event that corrosion is present. We are given $P(A|C) = .7$, $P(A|\overline{C}) = .2$, and $P(C) = .1$.

(a) By Bayes' theorem

$$\begin{aligned}P(C|A) &= \frac{P(A|C)P(C)}{P(A|C)P(C) + PA|\overline{C})P(\overline{C})}\\ &= \frac{.7 \times .1}{.7 \times .1 + .2 \times .9} = \frac{.07}{.07 + .18} = .28\end{aligned}$$

(b)

$$P(C|\overline{A}) = \frac{P(\overline{A}|C)P(C)}{P(\overline{A}|C)P(C) + P(\overline{A}|\overline{C})P(\overline{C})}$$

$$= \frac{(1 - .7) \times .1}{(1 - .7) \times .1 + (1 - .2) \times .9} = \frac{.03}{.03 + .72} = .04$$