

NATIONAL UNIVERSITY OF SINGAPORE
FACULTY OF SCIENCE
SEMESTER 2 EXAMINATION 2005-2006

MA1505 Mathematics I

April 2006

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **TEN (10)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. Each question carries 10 marks.
3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1 [10 marks]

Let

$$f(x) = e^{-\frac{x^2}{2}}.$$

Find all solutions to the equation

$$f''(x) = 0.$$

Question 2 [10 marks]

Let

$$f(x) = \frac{x}{(x+10)^2}$$

where $x > 0$. Find the absolute maximum value of $f(x)$.

Question 3 [10 marks]

Evaluate

$$\int_0^a \frac{4x^2}{(1+x^2)^2} dx,$$

where a is a positive constant.

Question 4 [10 marks]

Evaluate

$$\int_0^{2\pi} (\sin^3 x)(\cos^{15} x) dx.$$

Question 5 [10 marks]

Let $f(x)$ be a function defined on the interval $[0, \pi]$ by the formula

$$f(x) = \begin{cases} \sin 2x, & 0 \leq x \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

and let $\sum_{n=1}^{\infty} b_n \sin nx$ be the Fourier Sine Series which represents $f(x)$. Find

the value of $b_2 + b_3 + b_{2006}$. Give your answer in terms of π .

Question 6 [10 marks]

At time $t = 0$ a tank contains 1 lb of salt dissolved in 100 gal of water.

Assume that water containing 1 lb of salt per gallon is entering the tank at a rate of 3 gal/min and the well stirred solution is leaving the tank at the same rate. Find the time at which the mixture in the tank contains 2 lbs of salt.

Question 7 [10 marks]

A roast beef, initially at 10°C , is placed in a 200°C oven at 5:00pm. After 75 minutes it is found that the temperature of the roast beef is 50°C . What time should you remove the roast beef if you want it to be medium rare (i.e. its temperature is 65°C)? You may use Newton's law of cooling which states that the rate of change of the surface temperature of an object is proportional to the difference between the temperature of the object and the temperature of its surroundings at that time.

Question 8 [10 marks]

Solve the initial value problem $x \frac{dy}{dx} = 2y + x^3 \cos 4x$, $y\left(\frac{\pi}{2}\right) = 2\pi^2$.

Question 9 [10 marks]

Find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 4y = 8x^2.$$

Question 10 [10 marks]

Use Laplace Transform to solve the following initial value problem

$$\begin{cases} y'' + y' - 2y = xe^{-2x}, \\ y(0) = y'(0) = 0. \end{cases}$$

Show clearly all the steps in your solution.

END OF PAPER