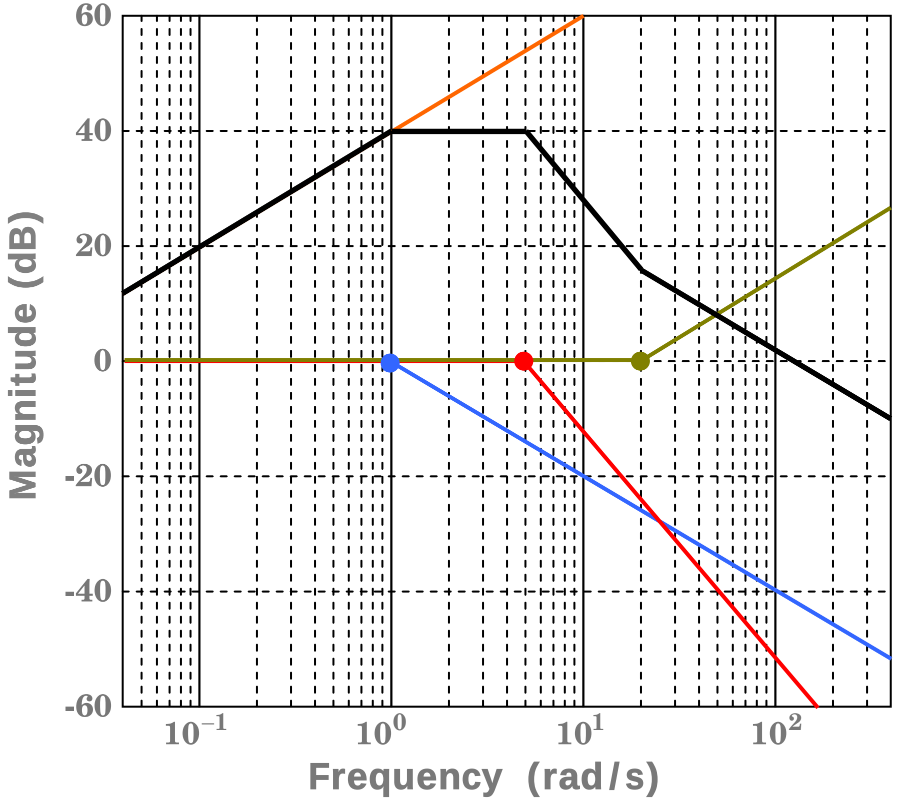
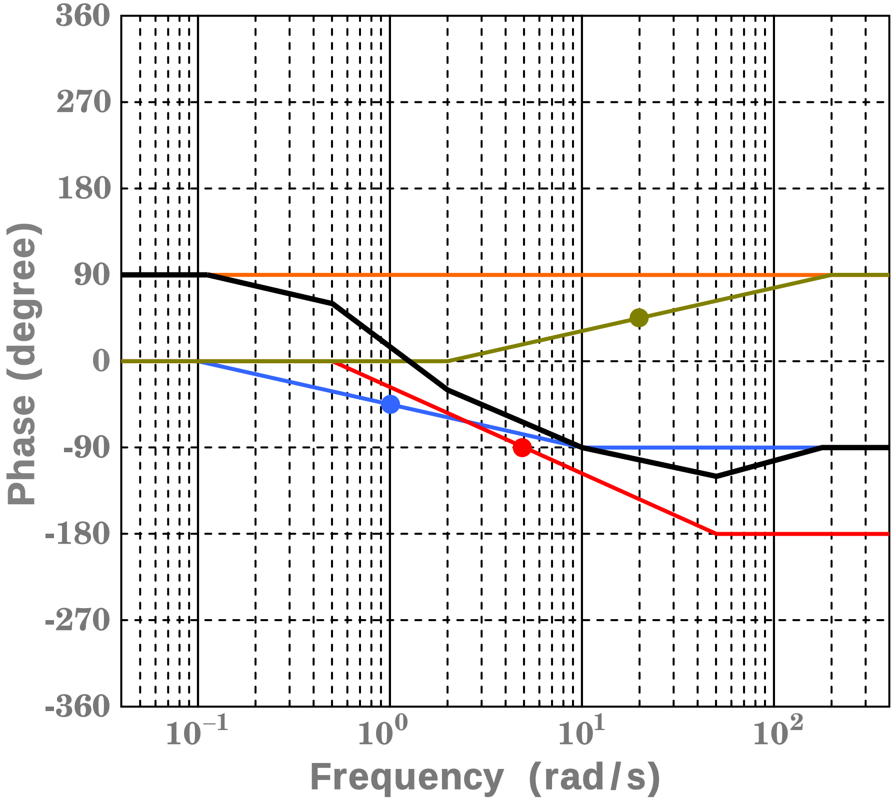
Bode Plots (Practice)

$$G(s) = K\frac{(s+a)(s+b)}{(s+c)(s+d)(s+e)} = K\frac{s(s+20)}{(s+1)(s+5)(s+5)}$$

$$G(s) = K_d s\frac{\left(\frac{s}{20}+1\right)}{(s+1)\left(\frac{s}{5}+1\right)\left(\frac{s}{5}+1\right)}; \quad \begin{cases} 20\log_{10} K_d\omega = K_{d,dB} \\ 20\log_{10} K_d(0.1) = 20 \\ K_d = \frac{Kb}{cde} = \frac{20}{25}K = 100 \\ \rightarrow K = 125 \end{cases}$$

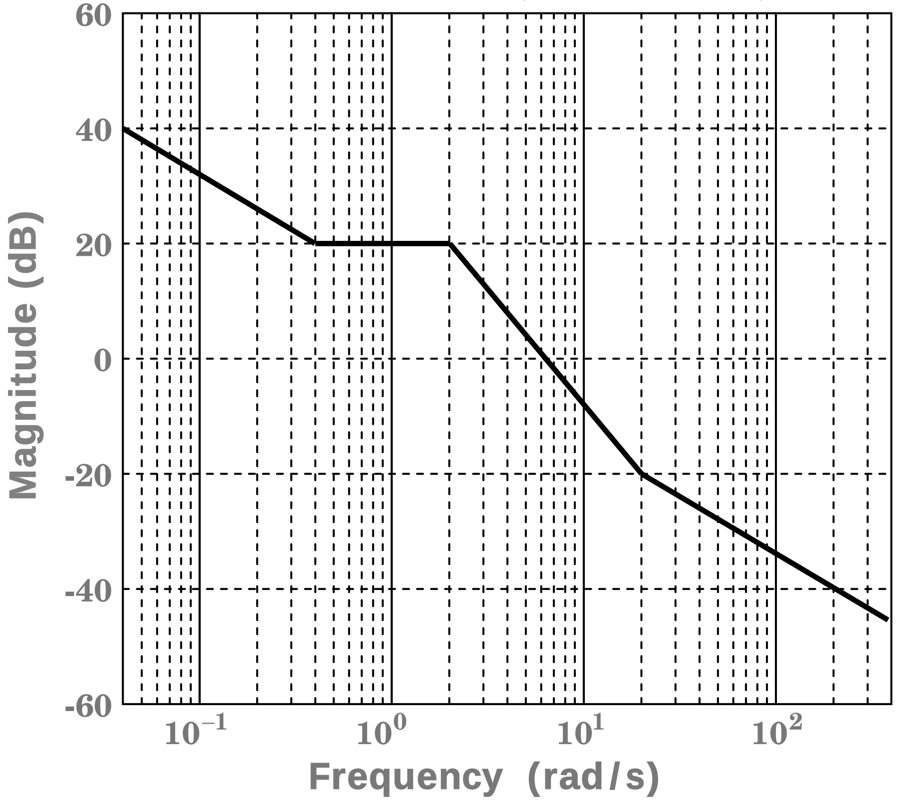




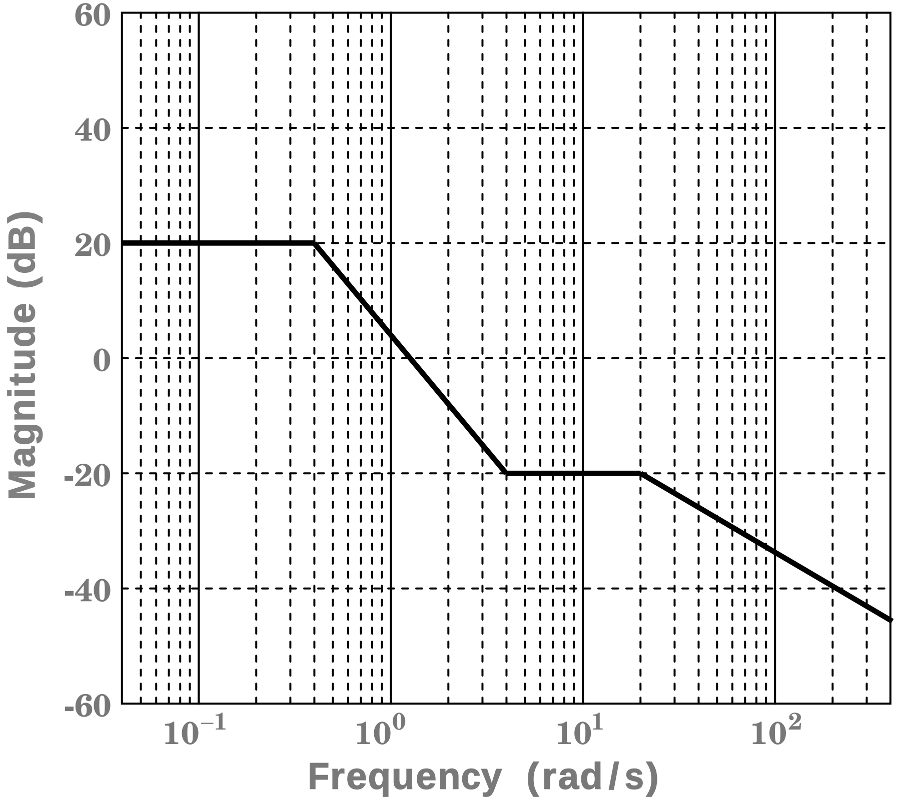
PRACTICE:

For each of the systems given below, find a, b, c, d and K from the respective Bode Straight-line plots. What is the damping factor of the second order section in each of $G_1(s)$ and $G_2(s)$.

$$G_1(s) = K\frac{(s+a)(s+b)}{(s+c)\left(s^2+2s+d^2\right)}$$



$$G_2(s) = K\frac{(s+a)(s+b)}{(s+c)\left(s^2+d^2\right)}$$



Try these out before you unveil at the solutions on the next two pages.

SOLUTION TO $G_1(s)$

To unveil, delete the shield.

$$G_1(s) = K\frac{(s+a)(s+b)}{(s+c)\left(s^2+2s+d^2\right)} \qquad \left\{ \text{By inspection}: \quad c=0, \quad \underline{d=2}, \quad \underline{a=0.4}, \quad \underline{b=20} \right.$$

Therefore, $G_1(s) = K\dfrac{(s+0.4)(s+20)}{s\left(s^2+2s+4\right)} = K\dfrac{(s+0.4)(s+20)}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$

$$\left.\begin{aligned} \omega_n^2 &= 4 \\ 2\zeta\omega_n &= 2 \end{aligned}\right\} \rightarrow \underline{\zeta = 0.5}$$

$$G_1(s) = \frac{K_I}{s}\frac{\left(\frac{s}{0.4}+1\right)\left(\frac{s}{20}+1\right)}{\left(\frac{s^2}{4}+\frac{2s}{4}+1\right)} \quad \text{where} \quad K_I = \frac{K(0.4)20}{4} = 2K$$

At the point $(0.04\,rad/s, 40dB)$ on the Bode Magnitude Straight-line Plot, we have

$20\log_{10}\dfrac{K_I}{\omega}\bigg|_{\omega=0.04} = 40,$ which yields $K_I = 0.04\left(10^2\right) = 4.$ Therefore, $\underline{K = \dfrac{K_I}{2} = 2}.$

SUMMARY: $a=0.4, \quad b=20, \quad c=0, \quad d=2, \quad K=2, \quad \zeta=0.5$

SOLUTION TO $G_2(s)$

To unveil, delete the shield.

$$G_2(s) = K\frac{(s+a)(s+b)}{(s+c)\left(s^2+d^2\right)} \quad \left\{ \text{By inspection: } d=0.4,\ \underline{a=b=4},\ \underline{c=20} \right.$$

Therefore, $G_2(s) = K\dfrac{(s+4)^2}{(s+20)\left(s^2+0.4^2\right)} = K\dfrac{(s+4)^2}{(s+20)\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$

$2\zeta\omega_n = \mathbf{0} \rightarrow \underline{\zeta = 0}$

$$G_2(s) = K_{dc}\frac{\left(\frac{s}{4}+1\right)^2}{\left(\frac{s}{20}+1\right)\left(\frac{s^2}{0.4^2}+1\right)} \quad \text{where} \quad K_{dc} = \frac{K(4)^{\mathbf{2}}}{20(0.4)^2} = \mathbf{5}K$$

The DC gain shown on the Bode Magnitude Straight-line Plot is $20\ dB$,

or $20\log_{10} K_{dc} = 20$, which yields $K_{dc} = 10$. Therefore, $\underline{K = \dfrac{K_{dc}}{\mathbf{5}} = \mathbf{2}}$.

SUMMARY: $a=b=4,\ \ c=20,\ \ d=0.4,\ \ K=\mathbf{2}$