Problem 2.20 A 300-Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor, as shown in Fig. P2.20. At 5 MHz, determine: (a) Γ, (b) S, (c) location of voltage maximum nearest to the load, and (d) location of current maximum nearest to the load.

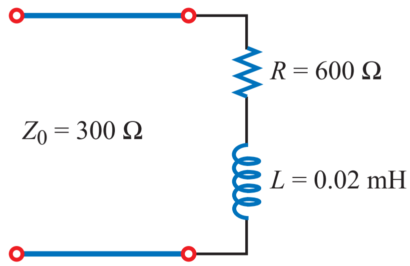


Figure P2.20: Circuit for Problem 2.20.

Solution:

(a)

$$\begin{aligned} Z_L &= R + j\omega L \\ &= 600 + j2\pi \times 5 \times 10^6 \times 2 \times 10^{-5} = (600 + j628)\ \Omega. \end{aligned}$$

$$\begin{aligned} \Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ &= \frac{600 + j628 - 300}{600 + j628 + 300} \\ &= \frac{300 + j628}{900 + j628} = 0.63e^{j29.6^\circ}. \end{aligned}$$

(b)

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.63}{1 - 0.63} = 4.4.$$

(c)

$$\begin{aligned} l_{max} &= \frac{\theta_r \lambda}{4\pi} \qquad \text{for } \theta_r > 0. \\ &= \left(\frac{29.6^\circ \pi}{180^\circ}\right) \frac{60}{4\pi}, \qquad \left(\lambda = \frac{3 \times 10^8}{5 \times 10^6} = 60 \text{ m}\right) \\ &= 2.46 \text{ m} \end{aligned}$$

(d) The locations of current maxima correspond to voltage minima and vice versa. Hence, the location of current maximum nearest the load is the same as location of voltage minimum nearest the load. Thus

$$\begin{aligned} l_{\min} &= l_{\max} + \frac{\lambda}{4}, \qquad \left(l_{\max} < \frac{\lambda}{4} = 15 \text{ m} \right) \\ &= 2.46 + 15 = 17.46 \text{ m}. \end{aligned}$$

Problem 2.30 Show that at the position where the magnitude of the voltage on the line is a maximum, the input impedance is purely real.

Solution: From Eq. (2.70), $d_{\max} = (\theta_r + 2n\pi)/2\beta$, so from Eq. (2.61), using polar representation for Γ,

$$\begin{aligned} Z_{\text{in}}(d_{\max}) &= Z_0 \left(\frac{1 + |\Gamma| e^{j\theta_r} e^{-j2\beta l_{\max}}}{1 - |\Gamma| e^{j\theta_r} e^{-j2\beta l_{\max}}} \right) \\ &= Z_0 \left(\frac{1 + |\Gamma| e^{j\theta_r} e^{-j(\theta_r + 2n\pi)}}{1 - |\Gamma| e^{j\theta_r} e^{-j(\theta_r + 2n\pi)}} \right) = Z_0 \left(\frac{1 + |\Gamma|}{1 - |\Gamma|} \right), \end{aligned}$$

which is real, provided Z_0 is real.

Problem 2.33 Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. P2.33.

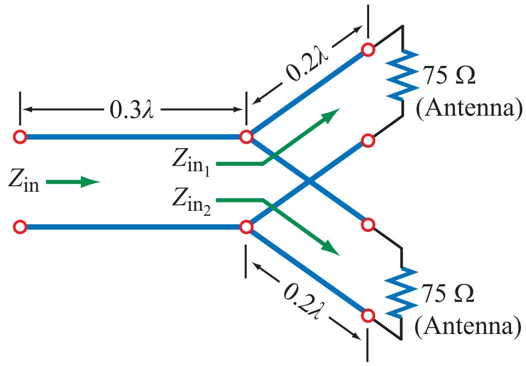


Figure P2.33: Circuit for Problem 2.33.

All lines are 50 Ω and lossless.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_{L}, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

Solution:

(a)

$$\begin{aligned} Z_{\text{in}_1} &= Z_0\left[\frac{Z_{\text{L}_1} + jZ_0\tan\beta l_1}{Z_0 + jZ_{\text{L}_1}\tan\beta l_1}\right] \\ &= 50\left\{\frac{75 + j50\tan[(2\pi/\lambda)(0.2\lambda)]}{50 + j75\tan[(2\pi/\lambda)(0.2\lambda)]}\right\} = (35.20 - j8.62)\ \Omega. \end{aligned}$$

(b)

$$Z'_{\text{L}} = \frac{Z_{\text{in}_1}Z_{\text{in}_2}}{Z_{\text{in}_1} + Z_{\text{in}_2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31)\ \Omega.$$

(c)

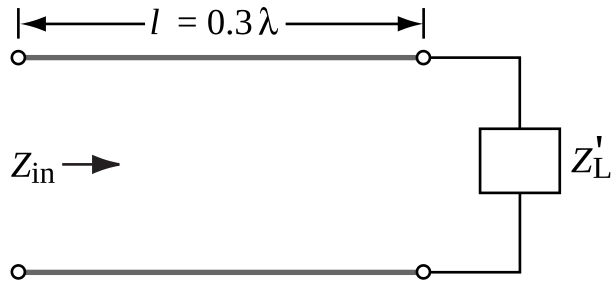


Figure P2.33: (b) Equivalent circuit.

$$Z_{\text{in}} = 50\left\{\frac{(17.60 - j4.31) + j50\tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31)\tan[(2\pi/\lambda)(0.3\lambda)]}\right\} = (107.57 - j56.7)\ \Omega.$$

Problem 2.50 Use the Smith chart to determine the input impedance Z_{in} of the two-line configuration shown in Fig. P2.50.

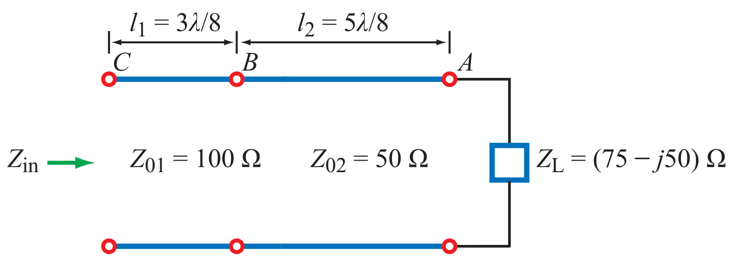
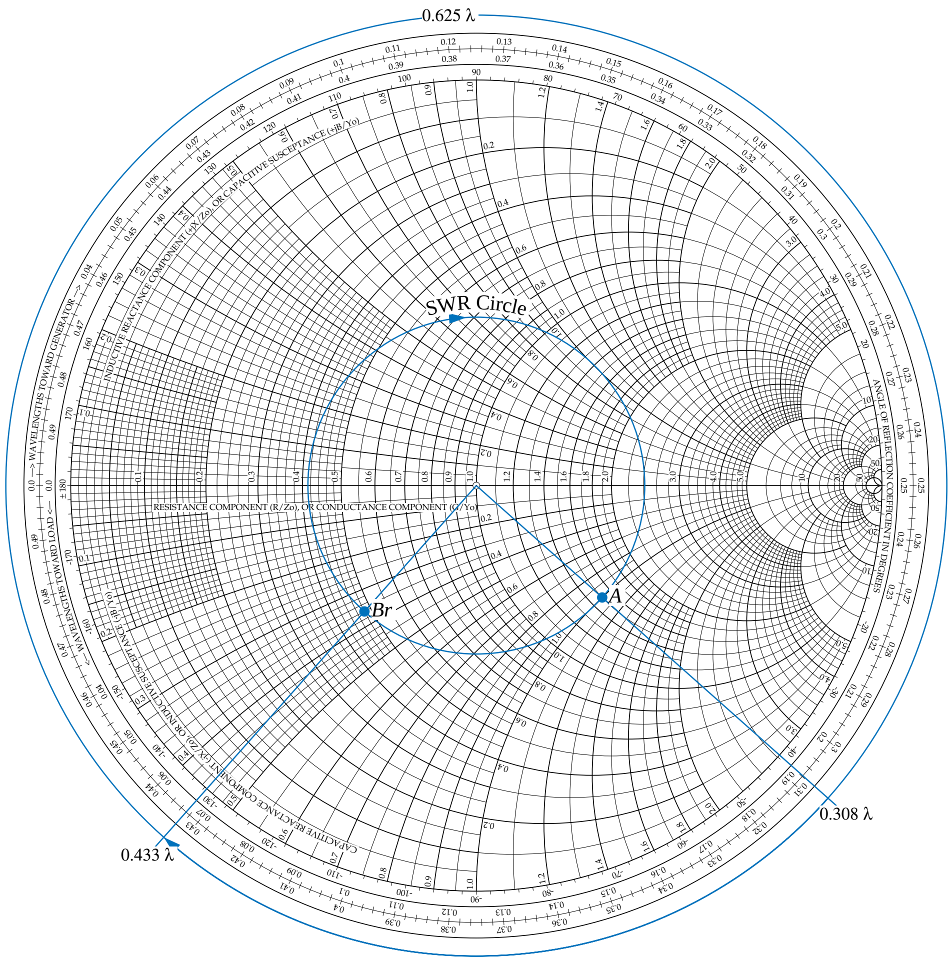


Figure P2.50: Circuit for Problem 2.50.

Solution:



Smith Chart 1

Starting at point A, namely at the load, we normalize Z_L with respect to Z_{02}:

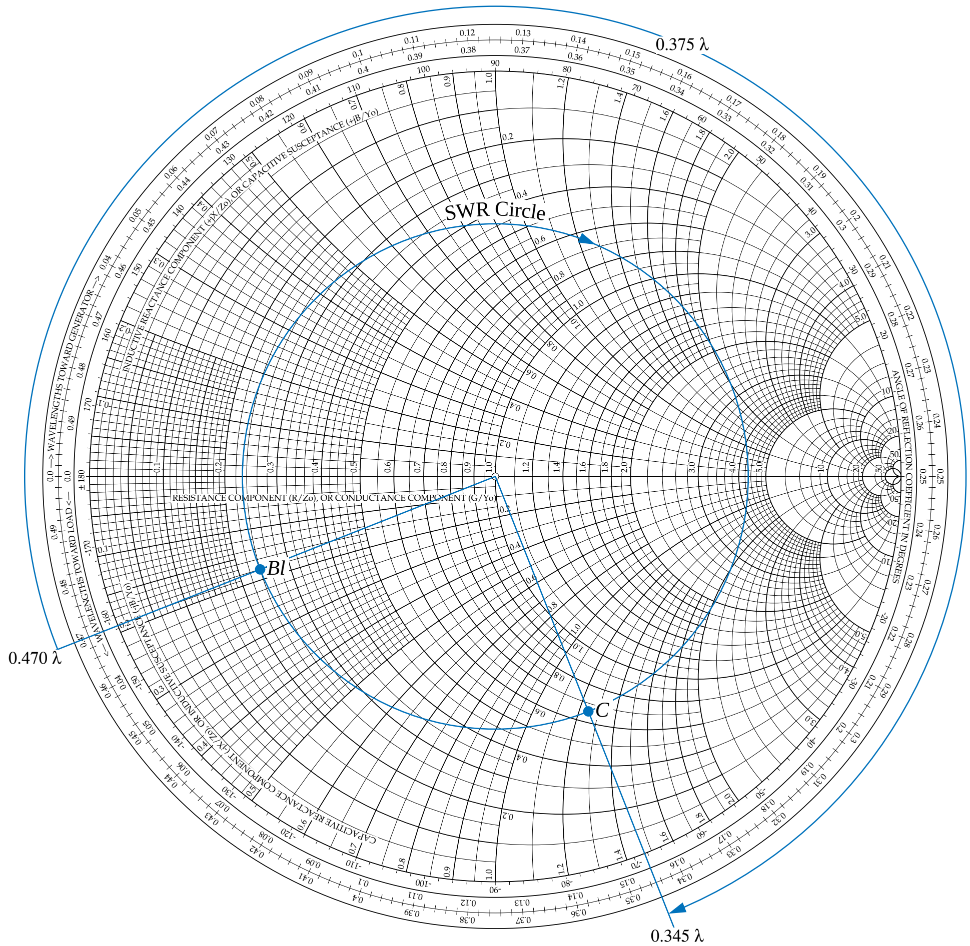
$$z_L = \frac{Z_L}{Z_{02}} = \frac{75 - j50}{50} = 1.5 - j1. \qquad \text{(point } A \text{ on Smith chart 1)}$$

From point A on the Smith chart, we move on the SWR circle a distance of $5\lambda/8$ to point B_r, which is just to the right of point B (see figure). At B_r, the normalized input impedance of line 2 is:

$$z_{in2} = 0.48 - j0.36 \qquad \text{(point } B_r \text{ on Smith chart)}$$

Next, we unnormalize z_{in2}:

$$Z_{in2} = Z_{02} z_{in2} = 50 \times (0.48 - j0.36) = (24 - j18)\ \Omega.$$



Smith Chart 2

To move along line 1, we need to normalize with respect to Z_{01}. We shall call this z_{L1}:

$$z_{L1} = \frac{Z_{in2}}{Z_{01}} = \frac{24 - j18}{100} = 0.24 - j0.18 \qquad \text{(point } B_\ell \text{ on Smith chart 2)}$$

After drawing the SWR circle through point B_ℓ, we move $3\lambda/8$ towards the generator, ending up at point C on Smith chart 2. The normalized input impedance of line 1 is:

$$z_{in} = 0.66 - j1.25$$

which upon unnormalizing becomes:

$$Z_{in} = (66 - j125)\ \Omega.$$

Problem 2.53 A lossless 50-Ω transmission line is terminated in a load with $Z_L = (50 + j25)\ \Omega$. Use the Smith chart to find the following:

(a) The reflection coefficient Γ.

(b) The standing-wave ratio.

(c) The input impedance at 0.35λ from the load.

(d) The input admittance at 0.35λ from the load.

(e) The shortest line length for which the input impedance is purely resistive.

(f) The position of the first voltage maximum from the load.

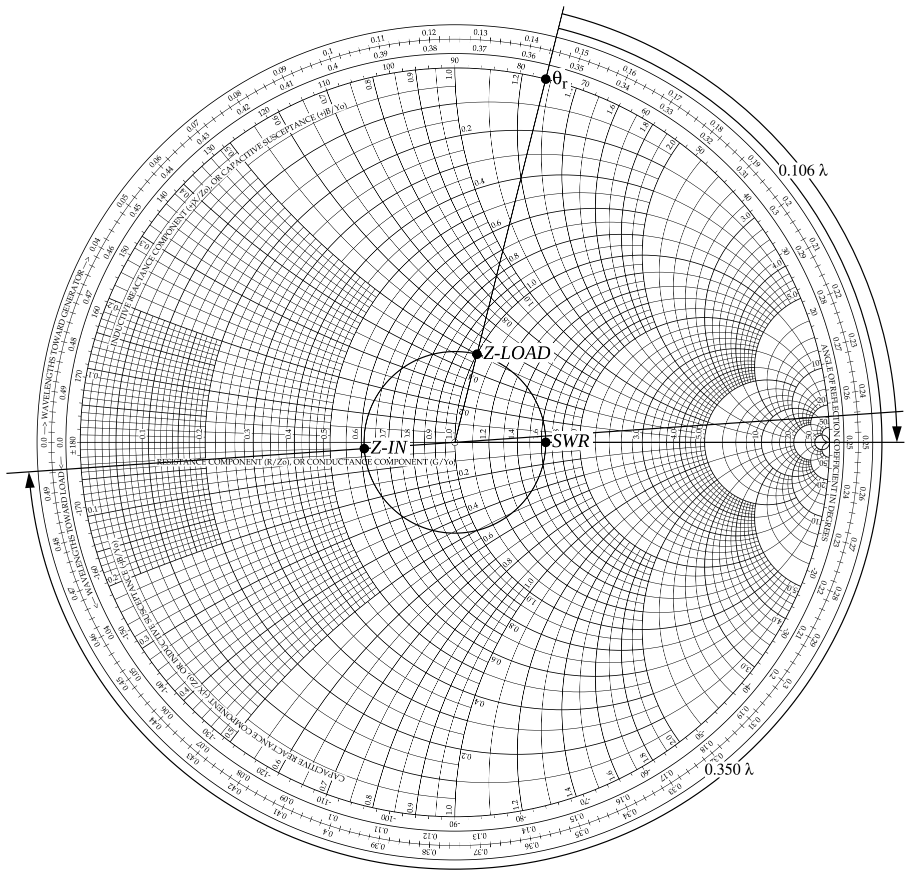


Figure P2.53: Solution of Problem 2.53.

Solution: Refer to Fig. P2.53. The normalized impedance

$$z_L = \frac{(50 + j25)\ \Omega}{50\ \Omega} = 1 + j0.5$$

is at point *Z-LOAD*.

(a) $\Gamma = 0.24e^{j76.0^\circ}$ The angle of the reflection coefficient is read of that scale at the point θ_r.

(b) At the point *SWR*: $S = 1.64$.

(c) Z_{in} is 0.350λ from the load, which is at 0.144λ on the wavelengths to generator scale. So point *Z-IN* is at $0.144\lambda + 0.350\lambda = 0.494\lambda$ on the WTG scale. At point *Z-IN*:

$$Z_{in} = z_{in} Z_0 = (0.61 - j0.022) \times 50\ \Omega = (30.5 - j1.09)\ \Omega.$$

(d) At the point on the SWR circle opposite *Z-IN*,

$$Y_{\text{in}} = \frac{y_{\text{in}}}{Z_0} = \frac{(1.64 + j0.06)}{50\ \Omega} = (32.7 + j1.17)\ \text{mS}.$$

(e) Traveling from the point *Z-LOAD* in the direction of the generator (clockwise), the SWR circle crosses the $x_L = 0$ line first at the point *SWR*. To travel from *Z-LOAD* to *SWR* one must travel $0.250\lambda - 0.144\lambda = 0.106\lambda$. (Readings are on the wavelengths to generator scale.) So the shortest line length would be 0.106λ.

(f) The voltage max occurs at point *SWR*. From the previous part, this occurs at $z = -0.106\lambda$.