NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

(Semester I: 2001-02)

ST2334 PROBABILITY AND STATISTICS

November 2001 – Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **TWO (2)** sections : Section A and Section B. It contains a total of **NINE (9)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** the **FOUR** questions in Section A. The marks for each question are indicated at the end of each question.

3. Answer not more than **FOUR (4)** questions in Section B. Each question in Section B carries 10 marks.

4. This is a CLOSED BOOK examination.

5. Candidates may use non-programmable calculators. However, they should lay out systematically the various steps in the calculations.

6. Statistical tables are provided.

Section A (60%): **Answer all the following FOUR questions.**

1. Suppose that random variables X and Y follow the joint probability density function given by
$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{3}(x+2y), & \text{for } 0 < x < 1,\ 0 < y < 1, \\ 0, & \text{elesewhere.} \end{cases}$$

(i) Find the marginal probability density function of X. **(4 marks)**

(ii) Find $P(Y < \frac{1}{2} \mid X = \frac{3}{4})$. **(7 marks)**

(iii) Are the random variables X and Y independent? Why? **(4 marks)**

2. Let X have the uniform distribution on $(0,1)$.

(i) Find the cumulative distribution function of X^2. **(6 marks)**

(ii) Find the probability density function of $Y = X^2$. **(4 marks)**

(iii) Find $E(X^2)$. **(4 marks)**

3. To compare two kinds of bumper guards, six of each kind were mounted on a certain make compact car. Then each car was run into a concrete wall at 5 miles per hour, and the following are the costs of the repairs (in dollars):

Bumper guard 1:	127	168	143	165	122	139
Bumper guard 2:	154	135	132	171	153	149

(i) Test whether it is reasonable to assume that the two populations sampled have equal variances. Use 0.02 level of significance. **(10 marks)**

(ii) Based on the result in part (i), test whether the difference between the means of these two samples is significant. Use 0.01 significance level. **(9 marks)**

(iii) What assumptions did you make in the above two tests. **(2 marks)**

4. The sampling distribution of a sample range, R, is given by

$$f_R(r) = \begin{cases} \dfrac{2}{\theta^2}(\theta - r), & \text{for } 0 < r < \theta, \\ 0, & \text{elesewhere.} \end{cases}$$

Find the value of c, where c is greater than 1, so that

$$R < \theta < cR$$

is a $(1 - \alpha)$ 100% confidence interval for θ. **(10 marks)**

Section B (40%): Answer not more than following FOUR questions.

5. A computer center has three printers A, B and C which print at different speeds. Files are sent to the first available printer. The probabilities that a file is sent to printers A, B and C are 0.6, 0.3 and 0.1 respectively. Occasionally a printer will jam and destroy a printout. The probabilities that printers A, B and C will jam given that a file has been routed to one of them are 0.01, 0.05 and 0.04 respectively.

   (i) Find the probability that a printer will jam. **(7 marks)**

   (ii) When a printer jams, what is the probability that printer A is involved? **(3 marks)**

6. Let $X_1, \ldots, X_n$ be independent random variables having the chi-square distribution with 1 degree of freedom, and $Y = X_1 + X_2 + \ldots + X_n$.

   (i) Find the mean and variance of $W = \dfrac{Y}{n}$. **(6 marks)**

   (ii) What is the distribution of $\dfrac{W-1}{\sqrt{2/n}}$ for large n? Give reason to support your result. **(4 marks)**

7. (i) A coffee-maker is regulated so that it takes an average of 5.8 minutes to brew a cup of coffee with a standard deviation of 0.6 minutes. According to Chebyshev's theorem, what percentage of the times that this coffee-maker is used will the brewing time take anywhere from 4.6 minutes to 7 minutes? **(5 marks)**

   (ii) Give your reason why we apply Chebyshev's theorem in part (i) **(2 marks)**

   (iii) Do you think that the percentage found in part (i) is accurate? Explain. **(3 marks)**

8. A single observation of a random variable having an exponential distribution is used to test the null hypothesis that the mean of the distribution is $\mu = 2$ against the alternative that it is $\mu = 5$. If the null hypothesis is accepted if and only if the observed value of the random variable is less than 3, find the probabilities of type I and type II errors.

   (10 marks)

9. The starting salaries of newly appointed lecturers who have just graduated with doctorates are to be compared for the fields of business and engineering. The following salary statistics were obtained from independent random samples of newly appointed lecturers in each of these fields:

Sample	Field	Sample Size	Sample Mean	Sample standard deviation
1	Business	170	$52,250	$6,541
2	Engineering	190	$48,212	$6,056

   (i) Construct a 90% confidence interval for the difference in the mean salaries for the two fields. **(7 marks)**

   (ii) Interpret the above confidence interval. **(3 marks)**

-- END OF PAPER --