CS2020 – Data Structures and Algorithms Accelerated

Recitation Week12 – DP

Finding the Correct States and Transitions

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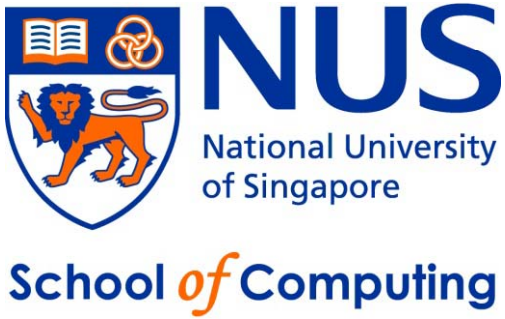

DP Exercises

- The hardest part of solving a DP problem is in finding the correct DP states (or in another word, the vertices in implicit DAG) and the correct transitions/recurrences (the edges in implicit DAG)
- In the next 40-45 minutes, we will look at two DP problems
  - Determining distinct states and the space complexity
  - Determining overlapping transitions/recurrences and the time complexity
  - Implementation: bottom-up (toposort) or top-down

How do you add? (1)

- http://uva.onlinejudge.org/external/109/10943.html
- Distinct States:
  - N? (1 <= N <= 100)
  - K? (1 <= K <= 100)
  - Both N and K? (Space Complexity: |N| * |K| = 100^2 = 10000)
- Overlapping Transitions/Recurrences:
  - ways(N, 1) = 1 // no choice
  - ways(N, K) = $\sum$ ways(N - split, K - 1), for all split $\in$ [0 .. N]
  - Cyclic? Overlap?
  - Time Complexity: |N| * |K| * |N| = 100^3 = 1M

How do you add? (2)

- Implementation (See UVa10943.java)
  - Top-down
    - Straightforward
    - Add check when entering recursion
    - Assign value to memo table before exiting the recursion
  - Bottom-up
    - Find topological order
    - Either using toposort algorithm,
      or by identifying the correct loop order
    - Process the DAG edges (transitions) according to this order

Headmaster's Headache (1)

- http://uva.onlinejudge.org/external/108/10817.html
- Distinct States:
  - Teacher?
    - M serving teachers? (20?)
    - N new applicants? (100?)
  - Set of subjects (two copies per subject)? (2^{8x2} = 65536)
  - N applicants x set of two copies/subject?
  - Space Complexity = (|N| * $2^{\wedge|S|x2}$ = 100x65536 = 6M)

Headmaster's Headache (2)

- Transitions:
  - cost(id, (1 << (2 * S)) - 1) = 0 // no need to hire anymore
  - cost(N, bitmask) = INF // no more applicant
  - cost(id, bitmask) = min(salary[id] + cost(id + 1, newmask),
    cost(id + 1, bitmask))
  - Explanation of newmask
    - Suppose teacher id can teach subject 1 (red), 2 (green), and 3 (blue)
    - Suppose current bitmask = 001011
    - newmask = 101111
  - Cyclic? Overlap?
  - Time Complexity: (|N| * $2^{|S| \times 2}$ * |S| = 6M * 8 = 48 M)

Headmaster's Headache (3)

- Implementation (See UVa10817.java)
  - Easier in top-down format