CHAPTER 3

Exercises

E3.1 $v(t) = q(t)/C = 10^{-6}\sin(10^5 t)/(2\times 10^{-6}) = 0.5\sin(10^5 t)$ V

$$i(t) = C\frac{dv}{dt} = (2\times 10^{-6})(0.5\times 10^5)\cos(10^5 t) = 0.1\cos(10^5 t)\text{ A}$$

E3.2 Because the capacitor voltage is zero at t = 0, the charge on the capacitor is zero at t = 0.

$$\begin{aligned} q(t) &= \int_0^t i(x)dx + 0 \\ &= \int_0^t 10^{-3}dx = 10^{-3}t \text{ for } 0 \le t \le 2\text{ ms} \\ &= \int_0^{2E-3} 10^{-3}dx + \int_{2E-3}^{t} -10^{-3}dx = 4\times 10^{-6} - 10^{-3}t \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

$$\begin{aligned} v(t) &= q(t)/C \\ &= 10^4 t \text{ for } 0 \le t \le 2\text{ ms} \\ &= 40 - 10^4 t \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= 10t \text{ for } 0 \le t \le 2\text{ ms} \\ &= -40\times 10^{-3} + 10t \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

$$\begin{aligned} w(t) &= Cv^2(t)/2 \\ &= 5t^2 \quad \text{for } 0 \le t \le 2\text{ ms} \\ &= 0.5\times 10^{-7}(40 - 10^4 t)^2 \text{ for } 2\text{ ms} \le t \le 4\text{ ms} \end{aligned}$$

in which the units of charge, electrical potential, power, and energy are coulombs, volts, watts and joules, respectively. Plots of these quantities are shown in Figure 3.8 in the book.

E3.3 Refer to Figure 3.10 in the book. Applying KVL, we have

$$v = v_1 + v_2 + v_3$$

Then using Equation 3.8 to substitute for the voltages we have

$$v(t)=\frac{1}{C_1}\int_0^t i(t)dt+v_1(0)+\frac{1}{C_2}\int_0^t i(t)dt+v_2(0)+\frac{1}{C_3}\int_0^t i(t)dt+v_3(0)$$

This can be written as

$$v(t)=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)\int_0^t i(t)dt+v_1(0)+v_2(0)+v_3(0) \qquad (1)$$

Now if we define

$$\frac{1}{C_{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right) \quad \text{and} \quad v(0)=v_1(0)+v_2(0)+v_3(0)$$

we can write Equation (1) as

$$v(t)=\frac{1}{C_{eq}}\int_0^t i(t)dt+v(0)$$

Thus the three capacitances in series have an equivalent capacitance given by Equation 3.25 in the book.

E3.4 (a) For series capacitances:

$$C_{eq}=\frac{1}{1/C_1+1/C_2}=\frac{1}{1/2+1/1}=2/3\ \mu\text{F}$$

(b) For parallel capacitances:

$$C_{eq}=C_1+C_2=1+2=3\ \mu\text{F}$$

E3.5 From Table 3.1 we find that the relative dielectric constant of polyester is 3.4. We solve Equation 3.26 for the area of each sheet:

$$A=\frac{Cd}{\varepsilon}=\frac{Cd}{\varepsilon_r\varepsilon_0}=\frac{10^{-6}\times15\times10^{-6}}{3.4\times8.85\times10^{-12}}=0.4985\ \text{m}^2$$

Then the length of the strip is

$$L=A/W=0.4985/(2\times10^{-2})=24.93\ \text{m}$$

E3.6 $$v(t)=L\frac{di(t)}{dt}=(10\times10^{-3})\frac{d}{dt}\left[0.1\cos\left(10^4t\right)\right]=-10\sin\left(10^4t\right)\ \text{V}$$

$$w(t)=\tfrac{1}{2}Li^2(t)=5\times10^{-3}\times\left[0.1\cos\left(10^4t\right)\right]^2=50\times10^{-6}\cos^2\left(10^4t\right)\ \text{J}$$

E3.7

$$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0) = \frac{1}{150\times10^{-6}}\int_0^t v(x)dx$$

$$= 6667\int_0^t 7.5\times10^6\, xdx = 25\times10^9 t^2 \text{ V for } 0 \le t \le 2\ \mu s$$

$$= 6667\int_0^{2E-6} 7.5\times10^6\, xdx = 0.1\text{ V for } 2\mu s \le t \le 4\ \mu s$$

$$= 6667\left(\int_0^{2E-6} 7.5\times10^6\, xdx + \int_{4E-6}^{t}(-15)dx\right) = 0.5 - 10^5 t\ \text{ V for } 4\mu s \le t \le 5\ \mu s$$

A plot of *i*(*t*) versus *t* is shown in Figure 3.19b in the book.

E3.8 Refer to Figure 3.20a in the book. Using KVL we can write:

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

Using Equation 3.28 to substitute, this becomes

$$v(t) = L_1\frac{di(t)}{dt} + L_2\frac{di(t)}{dt} + L_3\frac{di(t)}{dt} \qquad (1)$$

Then if we define $L_{eq} = L_1 + L_2 + L_3$, Equation (1) becomes:

$$v(t) = L_{eq}\frac{di(t)}{dt}$$

which shows that the series combination of the three inductances has the same terminal equation as the equivalent inductance.

E3.9 Refer to Figure 3.20b in the book. Using KCL we can write:

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

Using Equation 3.32 to substitute, this becomes

$$i(t) = \frac{1}{L_1}\int_0^t v(t)dt + i_1(0) + \frac{1}{L_2}\int_0^t v(t)dt + i_2(0) + \frac{1}{L_3}\int_0^t v(t)dt + i_3(0)$$

This can be written as

$$v(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_0^t v(t)dt + i_1(0) + i_2(0) + i_3(0) \qquad (1)$$

Now if we define

$$\frac{1}{L_{eq}} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) \quad \text{and} \quad i(0) = i_1(0) + i_2(0) + i_3(0)$$

we can write Equation (1) as

$$i(t) = \frac{1}{L_{eq}} \int_0^t v(t)dt + i(0)$$

Thus, the three inductances in parallel have the equivalent inductance shown in Figure 3.20b in the book.

E3.10 Refer to Figure 3.21 in the book.

(a) The 2-H and 3-H inductances are in series and are equivalent to a 5-H inductance, which in turn is in parallel with the other 5-H inductance. This combination has an equivalent inductance of 1/(1/5 + 1/5) = 2.5 H. Finally the 1-H inductance is in series with the combination of the other inductances so the equivalent inductance is 1 + 2.5 = 3.5 H.

(b) The 2-H and 3-H inductances are in series and have an equivalent inductance of 5 H. This equivalent inductance is in parallel with both the 5-H and 4-H inductances. The equivalent inductance of the parallel combination is 1/(1/5 + 1/4 + 1/5) = 1.538 H. This combination is in series with the 1-H and 6-H inductances so the overall equivalent inductance is 1.538 + 1 + 6 = 8.538 H.

E3.11 The MATLAB commands including some explanatory comments are:

```
% We avoid using i alone as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t iC qC vC % Define t, iC, qC and vC as symbolic objects.
iC = 0.5*sin((1e4)*t);
ezplot(iC, [0 3*pi*1e-4])
qC=int(iC,t,0,t); % qC equals the integral of iC.
figure % Plot the charge in a new window.
ezplot(qC, [0 3*pi*1e-4])
vC = 1e7*qC;
figure % Plot the voltage in a new window.
ezplot(vC, [0 3*pi*1e-4])
```

The plots are very similar to those of Figure 3.5 in the book. An m-file (named Exercise_3_11) containing these commands can be found in the MATLAB folder on the OrCAD disk.

E.12 The MATLAB commands including some explanatory comments are:

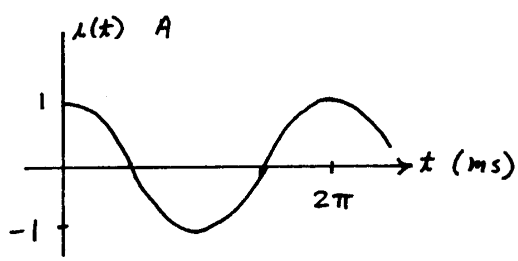
```
% We avoid using i by itself as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t vC iC pC wC % Define t, vC, iC, pC and wC as symbolic objects.
vC = 1000*t*(heaviside(t)- heaviside(t-1)) + ...
    1000*(heaviside(t-1) - heaviside(t-3)) + ...
    500*(5-t)*(heaviside(t-3) - heaviside(t-5));
ezplot(vC, [0 6])
iC = (10e-6)*diff(vC, 't'); % iC equals C times the derivative of vC.
figure % Plot the current in a new window.
ezplot(iC, [0 6])
pC = vC*iC;
figure % Plot the power in a new window.
ezplot(pC, [0 6])
wC = (1/2)*(10e-6)*vC^2;
figure % Plot the energy in a new window.
ezplot(wC, [0 6])
```

The plots are very similar to those of Figure 3.6 in the book. An m-file (named Exercise_3_12) containing these commands can be found in the MATLAB folder on the OrCAD disk.

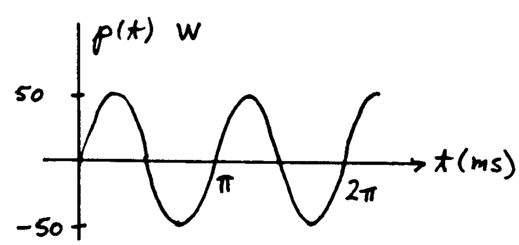
Answers for Selected Problems

P3.5* $\Delta t = 2000\text{ s}$

P3.6* $$i(t) = C\frac{dv}{dt} = 10^{-5}\frac{d}{dt}(100\sin 1000t) = \cos(1000t)$$

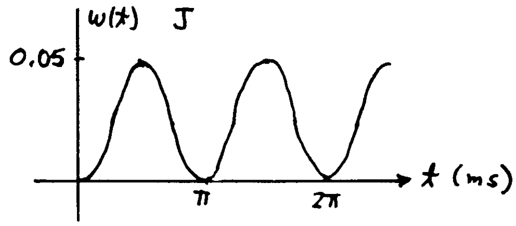


$$p(t) = v(t)i(t)$$



$$= 100\cos(1000t)\sin(1000t)$$
$$= 50\sin(2000t)$$

$$w(t) = \frac{1}{2}C[v(t)]^2$$
$$= 0.05\sin^2(1000t)$$



P3.7* At $t = 0$, we have $p(0) = -60$ mW. Because the power has a negative value, the capacitor is delivering energy.

At $t = 1$ s, we have $p(1) = 120$ mW. Because the power is positive, we know that the capacitor is absorbing energy.

P3.8* $V = 51.8$ kV

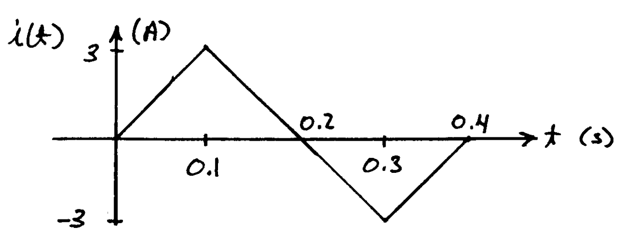
P3.24* (a) $C_{eq} = 2\ \mu$F
(b) $C_{eq} = 8\ \mu$F

P3.25* $C = 198\ \mu$F
$I_{battery} = 19.8\ \mu A$
Ampere-hour rating of the battery is 0.867 Ampere hours

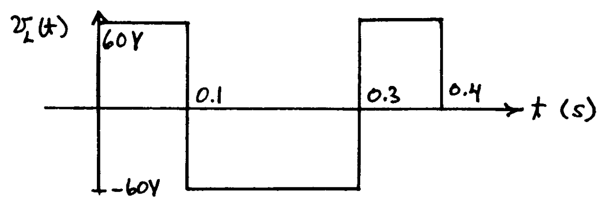
P3.31* $C = 0.398\ \mu$F

P3.32* $W_1 = 500\ \mu$J $\quad C_2 = 500$ pF
$V_2 = 2000$ V $\quad W_2 = 1000\ \mu$J
The additional energy is supplied by the force needed to pull the plates apart.

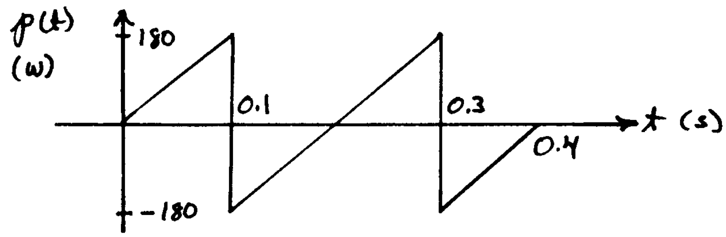
P3.43* $L = 2$ H



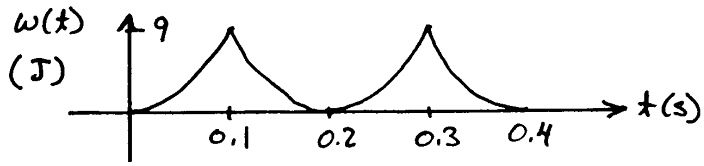
$$v_L(t) = L\frac{di_L(t)}{dt}$$



$$p(t) = v_L(t)i_L(t)$$



$$w(t) = \frac{1}{2}L[i_L(t)]^2$$



P3.44* $t_x = 1\ \mu s$

P3.45* $v_L = 10\text{ V}$

P3.60* (a) $L_{eq} = 3\text{ H}$

(b) $L_{eq} = 6\text{ H}$

P3.61* $i_1(t) = \dfrac{L_2}{L_1 + L_2}i(t) = \dfrac{2}{3}i(t)$

$i_2(t) = \dfrac{L_1}{L_1 + L_2}i(t) = \dfrac{1}{3}i(t)$

P3.72* (a) $L_{eq} = L_1 + 2M + L_2$

(b) $L_{eq} = L_1 - 2M + L_2$

Practice Test

T3.1

$$v_{ab}(t) = \frac{1}{C}\int_0^t i_{ab}(t)dt + v_C(0) = 10^5\int_0^t 0.3\exp(-2000t)dt$$

$$v_{ab}(t) = -15\exp(-2000t)\Big|_0^t$$

$$v_{ab}(t) = 15 - 15\exp(-2000t) \text{ V}$$

$$w_C(\infty) = \frac{1}{2}Cv_C^2(\infty) = \frac{1}{2}10^{-5}(15)^2 = 1.125 \text{ mJ}$$

T3.2 The 6-μF and 3-μF capacitances are in series and have an equivalent capacitance of

$$C_{eq1} = \frac{1}{1/6 + 1/3} = 2\ \mu\text{F}$$

C_{eq1} is in parallel with the 4-μF capacitance, and the combination has an equivalent capacitance of

$$C_{eq2} = C_{eq1} + 4 = 6\ \mu\text{F}$$

C_{eq2} is in series with the 12-μF and the combination, has an equivalent capacitance of

$$C_{eq3} = \frac{1}{1/12 + 1/6} = 4\ \mu\text{F}$$

Finally, C_{eq3} is in parallel with the 1-μF capacitance, and the equivalent capacitance is

$$C_{eq} = C_{eq3} + 1 = 5\ \mu\text{F}$$

T3.3

$$C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{80\times 8.85\times 10^{-12}\times 2\times 10^{-2}\times 3\times 10^{-2}}{0.1\times 10^{-3}} = 4248 \text{ pF}$$

T3.4

$$v_{ab}(t) = L\frac{di_{ab}}{dt} = 2\times 10^{-3}\times 0.3\times 2000\cos(2000t) = 1.2\cos(2000t) \text{ V}$$

The maximum value of sin(2000t) is unity. Thus the peak current is 0.3 A, and the peak energy stored is

$$w_{peak} = \frac{1}{2}Li_{peak}^2 = \frac{1}{2}\times 2\times 10^{-3}(0.3)^2 = 90\ \mu\text{J}$$

T3.5 The 2-H and 4-H inductances are in parallel and the combination has an equivalent inductance of

$$L_{eq1} = \frac{1}{1/2 + 1/4} = 1.333 \text{ H}$$

Also, the 3-H and 5-H inductances are in parallel, and the combination has an equivalent inductance of

$$L_{eq2} = \frac{1}{1/3 + 1/5} = 1.875 \text{ H}$$

Finally, L_{eq1} and L_{eq2} are in series. The equivalent inductance between terminals a and b is

$$L_{eq} = L_{eq1} + L_{eq2} = 3.208 \text{ H}$$

T3.6 For these mutually coupled inductances, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$
$$v_2(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

in which the currents are referenced into the positive polarities. Thus the currents are

$$i_1(t) = 2\cos(500t) \quad \text{and} \quad i_2(t) = -2\exp(-400t)$$

Substituting the inductance values and the current expressions we have

$$v_1(t) = -40 \times 10^{-3} \times 1000 \sin(500t) - 20 \times 10^{-3} \times 800 \exp(-400t)$$
$$v_1(t) = -40 \sin(500t) - 16 \exp(-400t) \text{ V}$$

$$v_2(t) = 20 \times 10^{-3} \times 1000 \sin(500t) - 30 \times 10^{-3} \times 800 \exp(-400t)$$
$$v_2(t) = 20 \sin(500t) - 24 \exp(-400t) \text{ V}$$

T3.7 One set of commands is

```
syms vab iab t
iab = 3*(10^5)*(t^2)*exp(-2000*t);
vab = (1/20e-6)*int(iab,t,0,t)
subplot(2,1,1)
ezplot(iab, [0 5e-3]), title('\iti_a_b\rm (A) versus \itt\rm (s)')
subplot(2,1,2)
ezplot(vab, [0 5e-3]), title('\itv_a_b\rm (V) versus \itt\rm (s)')
```

The results are

$$v_{ab} = \frac{15}{4} - \frac{15}{4}\exp(-2000t) - 7500\exp(-2000t) - 7.5\times10^{6}t^{2}\exp(-2000t)$$