Remarks on tutorial 3

Note that Q2(d) = Q3(a). Use the method of undetermined coefficients to solve Q2(b)

Q4 (A) Chain Rule $\frac{df(x)}{dy} = \frac{df(x)}{dx}\frac{dx}{dy}$

Apply Chain Rule to the right-hand side of the following equation

$$\frac{d^2y}{dx^2} = \frac{d}{dy}\left[\frac{1}{2}(y')^2(x)\right]$$

(B) Suppose that the earth were to stop moving. Then the earth would fall towards the sun according to

$$\ddot{r} = -\frac{GM}{r^2}$$

Use the first part result of Q4, we get

$$\frac{d}{dr}\left[(r')^2\right] = -\frac{GM}{r^2}$$

$$d\left[(r')^2\right] = -\frac{GM}{r^2}dr$$

Now Integrate both sides

Remember $\int df(x) = \int f'(x)dx = f(x)$

(C) From (B) , we have

$$(\dot{r})^2 = 2GM\left(\frac{1}{r} - \frac{1}{R}\right) \qquad \dot{r} = -\sqrt{2GM\left(\frac{1}{r} - \frac{1}{R}\right)}$$

(D) Initial condition

$$t = 0, r = R, \dot{r} = 0$$

(E) Now find t when $r = \frac{2}{3}R$

i.e., reach the orbit of Venus

(F) We need to find the integral

$$\int \frac{1}{\sqrt{\frac{1}{r} - \frac{1}{R}}} dr$$

Let x=r/R Then we need to find the integral

$$\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$\int \frac{1}{\sqrt{\frac{1}{x}-1}}\,dx = \int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

$$= \sin^{-1}(\sqrt{x}) - \sqrt{x(1-x)}$$

From online integrator

or

$$\frac{1}{\sqrt{1-x}} = (1-x)^{-\frac{1}{2}}$$

$$= 1 + \frac{1}{2}x + \frac{1}{2}\frac{3}{4}x^2 + \frac{1}{2}\frac{3}{4}\frac{5}{6}x^3 + \cdots$$

for $-1 < x < 1$

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = \int \left(\sqrt{x} + \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}\frac{3}{4}x^{\frac{5}{2}} + \cdots\right)dx$$

$$\approx \int \left(\sqrt{x} + \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}\frac{3}{4}x^{\frac{5}{2}}\right)dx$$

Wolfram *Mathematica*
ONLINE INTEGRATOR

HOW TO ENTER INPUT RANDOM EXAMPLE

$\int$ sqrt[x]/sqrt[1-x]

Traditional Form Input Form Output Fo

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

$$\sin^{-1}(\sqrt{x}) - \sqrt{-(x-1)\,x}$$

Time to compute: 0.02 sec

: ArcSin[x]: inverse sine [properties]

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