NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 8 **SEMESTER II, AY 2010/2011**

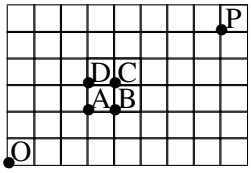
1. (a) Compute $\phi(360)$.
   (b) For $n > 2$ prove that $\phi(n)$ is even.
   (c) Show that $\phi(2n) = \begin{cases} \phi(n) & \text{if } n \text{ is odd;} \\ 2\phi(n) & \text{if } n \text{ is even.} \end{cases}$

2. n boys attends a gathering with both parents. In how many ways can the $3n$ people be divided into groups of 3 comprising one boy, one male parent and one female parent, such that no boy is together with both of his parents?

3. A man has 6 friends. At dinner in a certain restaurant, he has met each of them 12 times, every two of them 6 times, every three of them 4 times, every four of them 3 times, every five of them twice and all six only once. He has dined out 8 times without meeting any of them. How many times has he dined out altogether?

4. How many arrangement of the letters a,a,a,b,b,b,c,c,c are there such that
   (a) no three consecutive letters are the same?
   (b) no two consecutive letters are the same?

5. Consider the following street network.



   Find the number of shortest paths from O to P that
   (i) if the loop ABCD is closed;
   (ii) if the path must pass through exactly 1 of sections AB, BC, AD, DC ;
   (iii) if the path must pass through exactly 2 of sections AB, BC, AD, DC.

Answers

1. 96
3. 36
4. 1314 ; 174