EE3204/EE3204E Computer Communication Networks I (Part 1)

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Performance of CSMA/CD using a simplified model

Let T_f be the frame transmission time.

Let T_p be the end-to-end one way propagation delay.

Let τ be the duration of a (contention) slot which is given by $2\ T_p$.

Assume that time on medium is organized into cycles each of which consists of a collision/idle period (T_c) followed by a transmission period (T_f). A collision/idle period consists of a sequence of slots during which there are either collisions or no transmissions. Frame transmission occurs during the transmission period.

Therefore, the utilization (or effective throughput or efficiency) U is given by $U = \dfrac{T_f}{T_c + T_f}$

Let there be N nodes each of which attempts to transmit during a slot with probability p.

Let A be the probability that exactly one node transmits during a slot.

Let n_c be the mean number of slots in a collision/idle period.

Let $Pr[i]$ be the probability that the collision/idle period consists of i slots.

$$A = Np(1-p)^{N-1}$$

The mean number of slots in collision/idle period, n_c, is calculated as follows.

$$n_c = \sum_{i=0}^{\infty} i\ Pr[i]$$

$$= \sum_{i=1}^{\infty} i(1-A)^i A$$

$$n_c = \frac{1}{A} - 1$$

The utilization U is given by

$$\text{Utilization } U = \frac{T_f}{T_c + T_f}$$

$$= \frac{T_f}{n_c \tau + T_f}$$

$$= \frac{T_f}{2n_c T_p + T_f}$$

$$= \frac{1}{1 + 2an_c}$$

$$= \frac{1}{1 + 2a(\frac{1}{A} - 1)}$$

Calculation of Maximum Utilization

$A = Np(1 - p)^{N-1}$

Maximum utilization is achieved when A is maximized.

$A_{\max}$ can be determined by equating dA/dp to 0.

$dA/dp = N[-p(N-1)(1-p)^{N-2} + (1-p)^{N-1}] = 0$

$\Rightarrow p(N-1) = 1-p$

$\Rightarrow pN = 1$

$\Rightarrow \quad p = \frac{1}{N}$ (it can be shown that for this value of p, $d^2A/dp^2 < 0$)

Therefore A is maximized when $p = \frac{1}{N}$ and $A_{\max}$ is given by

$$A_{\max} = \left(1 - \frac{1}{N}\right)^{N-1}$$

The maximum utilization $U_{\max}$ is given by

$$U_{\max} = \frac{1}{1 + 2a\left(\frac{1}{A_{\max}} - 1\right)}$$

When N $\rightarrow \infty$ (i.e. large number of active nodes)

$\lim_{N\to\infty} A_{\max} = \lim_{N\to\infty}(1-\frac{1}{N})^{N-1} = \frac{1}{e}$, where $e = 2.72$ and the maximum utilization is given by

$$
\begin{aligned}
U_{\max} &= \frac{1}{1+2a\left(\frac{1}{A_{\max}}-1\right)} \\
&= \frac{1}{1+2a(e-1)} \\
&= \frac{1}{1+3.44a}
\end{aligned}
$$