2003 TEST AND ANSWERS

NUS National University of Singapore

Dean's Office, Faculty of Science (ANSWER SHEET)

FORM CC1

USE 2B PENCILS ONLY

INSTRUCTIONS

Suggested answers to each question are given in the question paper. Choose an answer and shade the corresponding circle.

EXAMPLES OF SHADING

CORRECT ● INCORRECT ⊗ ⊘ ⊖ ⊖

SECTION A STUDENT'S NAME

MODULE : MA1505

YEAR/SEMESTER: 2003

DATE:

SECTION B : MATRICULATION NUMBER

U

1. Write your matriculation number here.
2. Now SHADE the corresponding circle in the grid for each digit or letter.

S/NO 27892

SECTION C : ANSWERS

	1	2	3	4	5
1	A	B	C	●	E
2	●	B	C	D	E
3	A	●	C	D	E
4	A	●	C	D	E
5	A	B	●	D	E
6	●	B	C	D	E
7	A	B	C	D	●
8	A	B	C	●	E
9	A	B	C	D	●
10	A	B	●	D	E
11	●	B	C	D	E
12	A	●	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
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90	A	B	C	D	E
91	A	B	C	D	E
92	A	B	C	D	E
93	A	B	C	D	E
94	A	B	C	D	E
95	A	B	C	D	E
96	A	B	C	D	E
97	A	B	C	D	E
98	A	B	C	D	E
99	A	B	C	D	E
100	A	B	C	D	E

1. $\lim_{x\to -2}\frac{f(x)-f(-2)}{x-(-2)}=\lim_{x\to -2}\frac{(-x)^3-(-2)^3}{x-(-2)}=\frac{d}{dx}(-x^3)\Big|_{x=-2}=-12$

2. $f'(x)=9(x^3-2x^2+3x+5)^8(3x^2-4x+3)$

$f''(x)=72(x^3-2x^2+3x+5)^7(3x^2-4x+3)^2+9(x^3-2x^2+3x+5)^8(6x-4)$

$f''(-1)=-7200-90=-7290$

3. $32xy+16x^2y'=(\sec^2(x-2y))(1-2y')$

at $(\frac{\pi}{4},0)$, $16(\frac{\pi^2}{16})y'=\frac{1}{\cos^2\frac{\pi}{4}}(1-2y')\Rightarrow \pi^2y'=2-4y'$

$(\pi^2+4)y'=2\Rightarrow y'=\frac{2}{\pi^2+4}$

4. $f'(x)=-\frac{(x+1)^2-2}{(x^2+1)^2}=0\Rightarrow x=-1\pm\sqrt{2}$

$f(1)=1,\ f(\frac{1}{5})=\frac{15}{13},\ f(\sqrt{2}-1)=\frac{1}{2(\sqrt{2}-1)},\ f(1)\le f(\frac{1}{5})\le f(\sqrt{2}-1)$

5. $\lim_{x\to\frac{\pi}{2}}\frac{1+\sin 3x}{1+\cos 2x}=\lim_{x\to\frac{\pi}{2}}\frac{3\cos 3x}{-2\sin 2x}=\lim_{x\to\frac{\pi}{2}}\frac{-9\sin 3x}{-4\cos 2x}=\frac{-9(-1)}{-4(-1)}$

6. $\ln y=x\ln x\Rightarrow y'=y(1+\ln x)\Rightarrow y''=y'(1+\ln x)+y(\frac{1}{x})$

$y''=y[(1+\ln x)^2+\frac{1}{x}]=x^x[(1+\ln x)^2+\frac{1}{x}]>0$ if $x>0$

7. $\int\frac{1}{x^2}\tan\frac{1}{x}\sec^2\frac{1}{x}\,dx=-\int\tan\frac{1}{x}\sec^2\frac{1}{x}\,d(\frac{1}{x})$

$=-\int\tan\frac{1}{x}\,d(\tan\frac{1}{x})=-\frac{1}{2}\tan^2\frac{1}{x}+C$

8. $\frac{d}{d(x^2)}\int_2^{x^2}(\ln t)^2\,dt\,\frac{d(x^2)}{dx}=(\ln x^2)^2(2x)=4(\ln x)^2(2x)$

9. $\int_{-2}^{1}|x+1|\,dx=\int_{-2}^{-1}-(x+1)\,dx+\int_{-1}^{1}x+1\,dx=-(\frac{1}{2}x^2+x)\Big|_{-2}^{-1}+\frac{1}{2}x^2+x\Big|_{-1}^{1}$

$=-[\frac{1}{2}-1-(2-2)]+\frac{1}{2}+1-(\frac{1}{2}-1)=\frac{1}{2}+2=\frac{5}{2}$

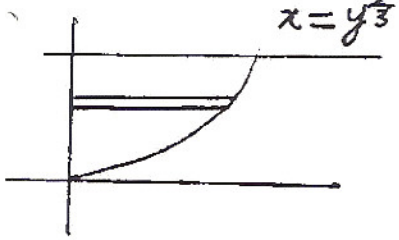
10. $\int\frac{\ln x}{x^2}\,dx=(\ln x)(-\frac{1}{x})-\int(-\frac{1}{x})(\frac{1}{x})\,dx=-\frac{1}{x}\ln x-\frac{1}{x}+C$

11. $y=(1-\sqrt{x})^2\ge 0$, $y=0$ if and only if $x=1$

Area $=\int_0^1(1-\sqrt{x})^2\,dx=\int_0^1 1+x-2\sqrt{x}\,dx=x+\frac{1}{2}x^2-2x^{\frac{3}{2}}\cdot\frac{2}{3}\Big|_0^1$

12. volume $=\int_0^1\pi(x-0)^2\,dy=\int_0^1\pi y^{\frac{2}{3}}\,dy$

$=\pi(\frac{3}{5}y^{\frac{5}{3}})\Big|_0^1=\frac{3}{5}\pi$



1. Let $f(x) = |x|^3$. Then $f'(-2) =$

(A) ∞
(B) 12
(C) 0
(D) -12
(E) $-\infty$

2. Let $f(x) = \left(x^3 - 2x^2 + 3x + 5\right)^9$. Then $f''(-1) =$

(A) -7290
(B) -7200
(C) -6390
(D) -6300
(E) -4590

3. Find the slope of the tangent line to the curve $1+16x^2y = \tan(x - 2y)$ at the point $\left(\frac{\pi}{4}, 0\right)$.

(A) $\frac{1}{\pi^2}$
(B) $\frac{2}{\pi^2+4}$
(C) $\frac{-1}{\pi^2+4}$
(D) $\frac{-2}{\pi^2}$
(E) $\frac{1}{\pi^2+2}$

4. Let $f(x)$ be a function defined by

$$f(x) = \frac{x+1}{x^2+1}$$

where $x \in \left[\frac{1}{5}, 1\right]$. Let M and m denote the absolute maximum value and absolute minimum value respectively of f in this interval. Then

(A) $M = \frac{1}{2(\sqrt{2}-1)}$, $m = -\frac{1}{2(\sqrt{2}+1)}$

(B) $M = \frac{1}{2(\sqrt{2}-1)}$, $m = 1$

(C) $M = 1$, $m = -\frac{1}{2(\sqrt{2}+1)}$

(D) $M = \frac{1}{2(\sqrt{2}-1)}$, $m = \frac{15}{13}$

(E) $M = \frac{15}{13}$, $m = -\frac{1}{2(\sqrt{2}+1)}$

5. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{1+\sin 3x}{1+\cos 2x}$.

(A) The limit does not exist

(B) $\frac{3}{4}$

(C) $\frac{9}{4}$

(D) $-\frac{3}{4}$

(E) $-\frac{9}{4}$

6. For the function $y = x^x$, $x \in (0, \infty)$, its second derivative $\frac{d^2y}{dx^2}$

(A) is always positive

(B) is always negative

(C) has exactly one zero at $x = 1$

(D) has exactly one zero at $x = e$

(E) has more than one zeros

7. $\int \frac{1}{x^2} \tan\frac{1}{x} \sec^2\frac{1}{x} dx =$

(A) $-\frac{1}{2}\tan\frac{1}{x}\sec\frac{1}{x} + C$

(B) $-\frac{1}{2}\sec\frac{1}{x} + C$

(C) $\frac{1}{2}\sec^2\frac{1}{x} + C$

(D) $-\frac{1}{2}\tan\frac{1}{x} + C$

(E) $-\frac{1}{2}\tan^2\frac{1}{x} + C$

8. If $x > 0$, then $\frac{d}{dx}\int_2^{x^2} (\ln t)^2 dt =$

(A) $4x\,(\ln x)$

(B) $4x\,(\ln x)^2$

(C) $4(x\ln x)^2$

(D) $8x\,(\ln x)^2$

(E) $8\,(x\ln x)^2$

9. $\int_{-2}^{1} \sqrt{(x+1)^2} dx =$

(A) $-\frac{3}{2}$

(B) $\frac{15}{4}$

(C) $\frac{3}{2}$

(D) $\frac{3}{4}$

(E) $\frac{5}{2}$

10. $\int \frac{\ln x}{x^2}\, dx =$

(A) $\frac{1}{x}(1+\ln x)+C$

(B) $-\frac{1}{x}(-1+\ln x)+C$

(C) $-\frac{1}{x}(1+\ln x)+C$

(D) $\frac{1}{x^2}(1-\ln x)+C$

(E) $-\frac{1}{x^2}(1+\ln x)+C$

11. Find the area of the region in the first quadrant bounded by the x-axis, the y-axis and the curve $\sqrt{x}+\sqrt{y}=1$.

(A) $\frac{1}{6}$

(B) $\frac{2}{5}$

(C) $\frac{1}{3}$

(D) $\frac{3}{4}$

(E) $\frac{5}{8}$

12. Let R denote the region in the first quadrant bounded by $y=x^3$, $y=1$ and $x=0$. Find the volume generated by revolving the region R about the y-axis.

(A) $\frac{4\pi}{3}$

(B) $\frac{3\pi}{5}$

(C) $\frac{2\pi}{3}$

(D) $\frac{5\pi}{6}$

(E) $\frac{3\pi}{2}$

END OF PAPER