**Question:**

According to the Pg 2-36 of the lecture notes, we use $\operatorname{sgn}(t)$ to derive the FT of $u(t)$. Why is it that the same approach when applied directly to $u(t)$ does not work, as shown below:

$$\underbrace{\left(\frac{d}{dt}u(t)=\delta(t)\right)\rightarrow\left(j2\pi f\cdot\Im\{u(t)\}=1\right)}_{\text{Taking Fourier transform of both sides}}\rightarrow\left(\Im\{u(t)\}=\frac{1}{j2\pi f}\text{, which is wrong}\right)$$

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**Answer:**

The differentiation property works for all cases if applied 'directly'. For example,

$$\left.\begin{aligned}&\frac{d}{dt}u(t)=\delta(t)\\&\Im\left\{\frac{d}{dt}u(t)\right\}=j2\pi f\cdot\Im\{u(t)\}=j2\pi f\cdot\left(\frac{1}{j2\pi f}+\frac{1}{2}\delta(f)\right)=1+\underbrace{j\pi f\delta(f)}_{0}=1\\&\Im\{\delta(t)\}=1\end{aligned}\right\}\text{ which is consistent}$$

If $y(t)=\frac{d}{dt}x(t)$, then $Y(f)=j2\pi f\cdot X(f)$ with $X(f)$ given. However, in general, this does not imply that $X(f)=\frac{1}{j2\pi f}Y(f)$ with $Y(f)$ given. This is because $\frac{d}{dt}x(t)$ will remove the dc component of $x(t)$ and the Fourier transform of the dc value will be missing in $X(f)$. (*This is similar to the integration property of the Fourier transform which has a condition attached.*)

Now, $u(t)$ has a dc value of 0.5. If we use the method suggested in the question, then this dc value of 0.5 will be removed by the differentiation operation. This accounts for the missing $\frac{1}{2}\delta(f)$ in the solution.

Last but not least, $\operatorname{sgn}(t)$ has no dc value. Therefore the method suggested in the question applies.

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