EE2011 Engineering Electromagnetics - Part CXD
Tutorial 2 - Solutions

Q1

Successive voltage minima $=\lambda/2=25$ (cm), $\lambda=50$ (cm).

(a) $S=2 \Rightarrow |\Gamma_L|=\dfrac{S-1}{S+1}=\dfrac{1}{3}$

First voltage minimum occurs at $\ell_m=\dfrac{5}{50}=0.1\lambda$

$$\Rightarrow \quad \ell_m=0.1\lambda=\frac{\theta_L\lambda}{4\pi}+\frac{(2n+1)\lambda}{4} \quad \text{with } n=0$$
$$\Rightarrow \quad \theta_L=-0.15\times 4\pi=-1.885=-108^\circ$$

Therefore,

$$\Gamma_L=|\Gamma_L|e^{j\theta_L}=\frac{1}{3}e^{-j1.885}=-0.1030-j0.3170$$

(b) $\Gamma_L=\dfrac{Z_L-Z_0}{Z_L+Z_0}$

$$\Rightarrow \quad Z_L=Z_0\frac{1+\Gamma_L}{1-\Gamma_L}=50\times\frac{1+(-0.1030-j\,0.3170)}{1-(-0.1030-j\,0.3170)}=33.7429\text{-}j24.0682\ \Omega$$

(c) If $Z_L=0$, then $\Gamma_L=-\dfrac{Z_0}{Z_0}=-1$. That is $\theta_L=\pm\pi=\pm180^\circ$. As θ_L has to be in the range $[-\pi,\pi)$, we exclude the case of $\theta_L=+\pi$.

Therefore $\theta_L=-\pi=-180^\circ$. The voltage minimum positions are given by:

$$\ell_m=\frac{\theta_L\lambda}{4\pi}+\frac{(2n+1)\lambda}{4}, \quad n=0,1,2,\cdots$$

When $n=0$,
$$\ell_m=-\frac{\lambda}{4}+\frac{\lambda}{4}=0, \quad \text{(the load position, discarded)}$$
When $n=1$,
$$\ell_m=-\frac{\lambda}{4}+\frac{3\lambda}{4}=\frac{\lambda}{2}=25\text{ (cm)}, \quad \text{(the first minimum)}$$

Q2

The normalized load impedance $z_L = \frac{Z_L}{Z_0} = \frac{100 - j100}{50} = 2 - j2$, corresponding to point A in the Smith chart, reading 0.292λ.

(a)

The normalized input impedance is $z_{in} = \frac{Z_{in}}{Z_0} = \frac{12.5 - j12.7}{50} = 0.250 - j0.254$, which corresponds to point B in Smith chart, reading 0.459λ. The length of the transmission line is $0.459\lambda - 0.292\lambda = 0.167\lambda$.

(b)

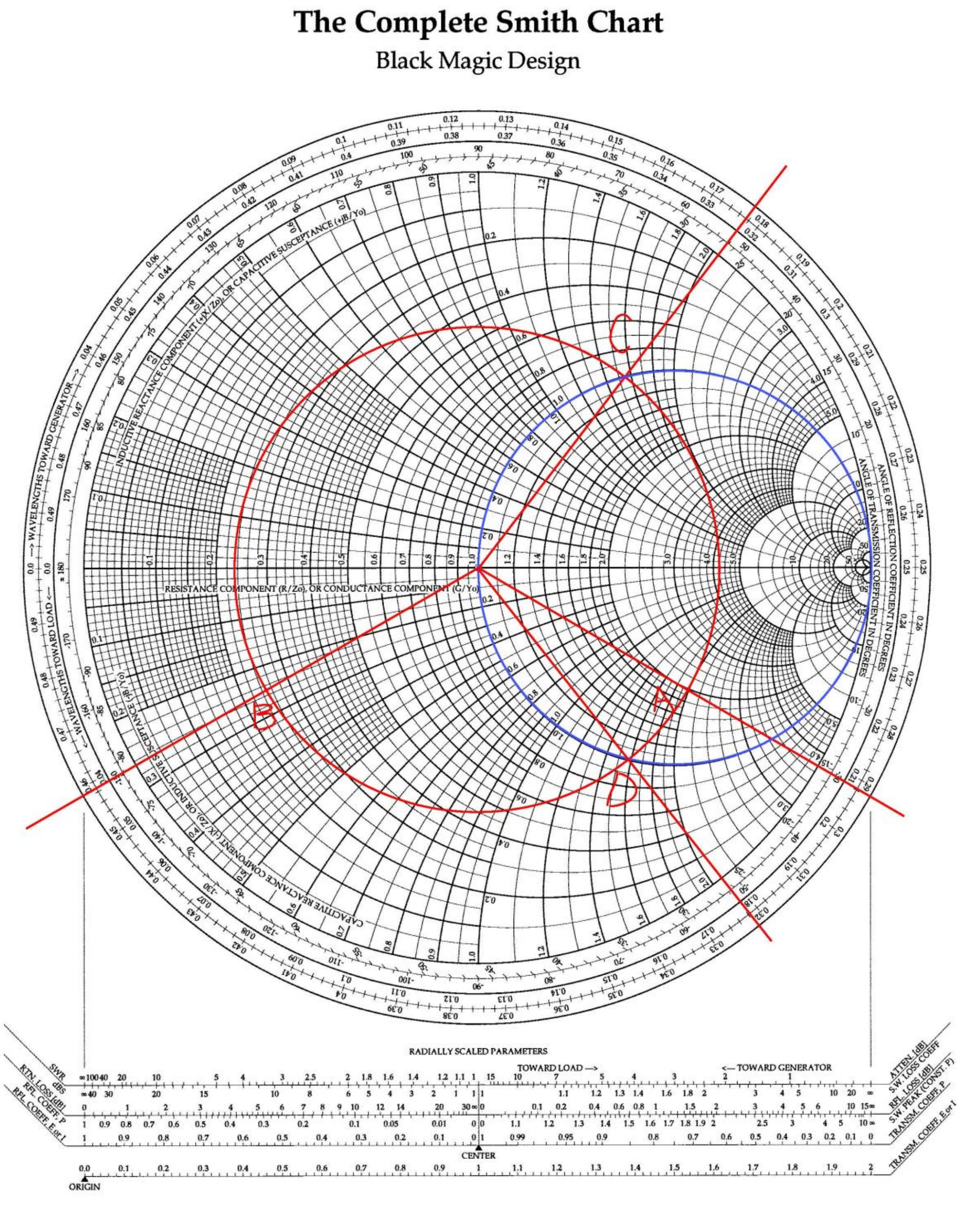
For the resultant input impedance, the normalized real part is 1, which is shown as the blue circle in the Smith Chart. The red circle intercepts the blue one at points C and D, as shown in the Figure below. We read

Point C: 0.178 λ

Point D: 0.322 λ

Thus, the length of the transmission line is

$0.178\lambda + 0.5\lambda - 0.292\lambda = 0.386\lambda$ or $0.322\lambda - 0.292\lambda = 0.03\lambda$



Smith chart solutions to Q2

Q3

First find:

$$z_L = (9 + j12)/50 = 0.18 + j0.24$$

z_L corresponds to point A, with 0.038λ

(a)

$0.038\lambda + 0.65\lambda = 0.688\lambda,$ which is equivalent to 0.188λ ($0.688\lambda - 0.5\lambda$) on Smith Chart, reaching point B.

At point B, read: $z_{in} = 1 + j2$

$Z_{in} = z_{in}Z_0 = 50 + j100\ \ \Omega$

(b)

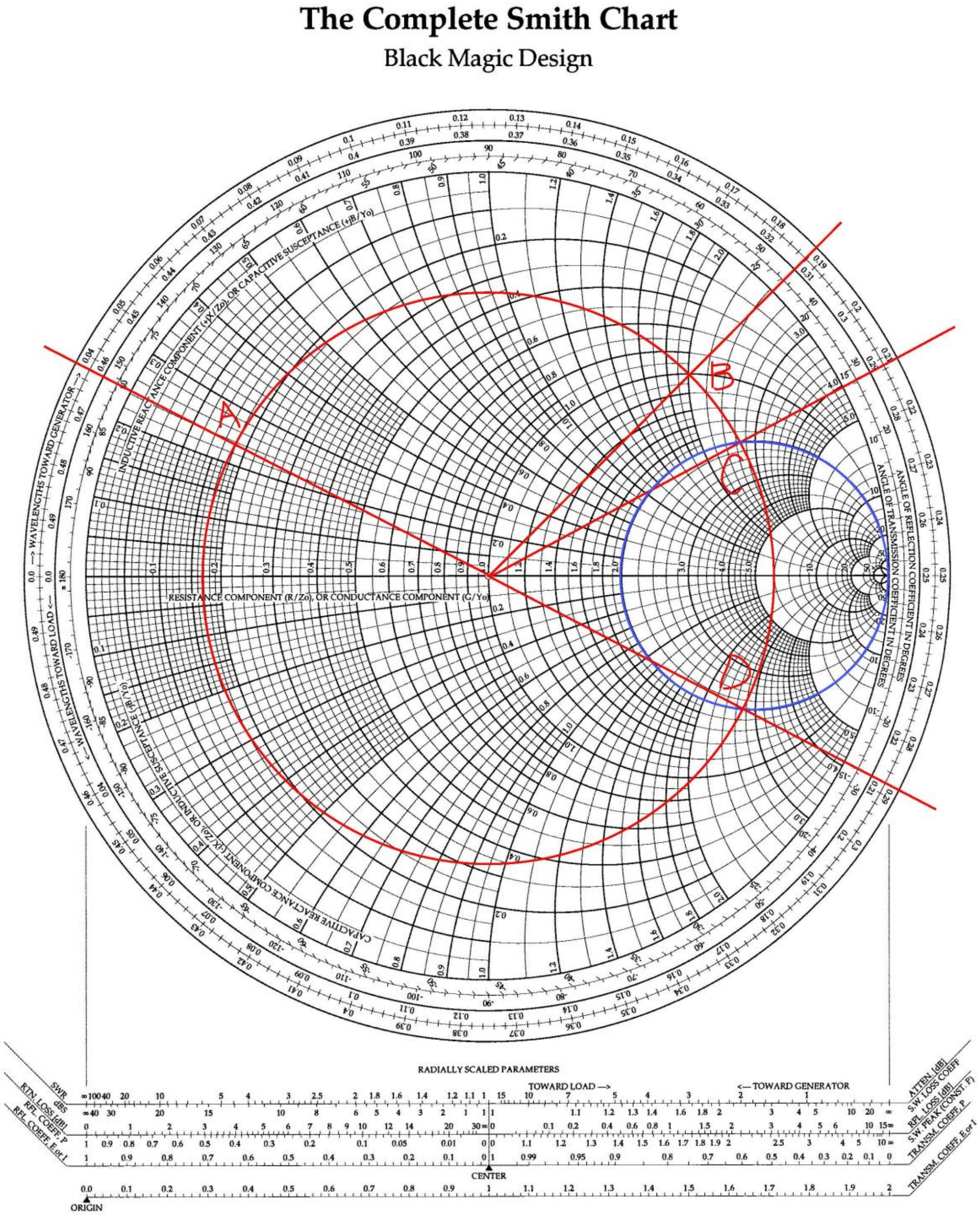
For the resultant input impedance, the normalized real part is 100/50 = 2, which is shown as the blue circle in the Smith Chart. The red circle intercepts the blue one at points C and D, as shown in the Figure below. We read

Point C: 0.212 λ

Point D: 0.288 λ

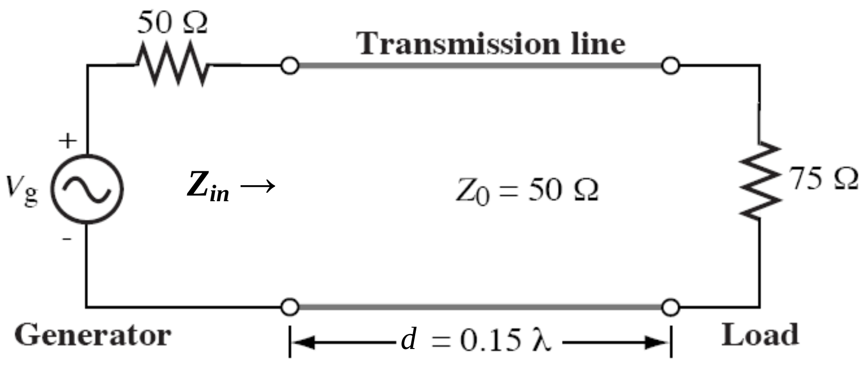
Thus, the length of the transmission line is

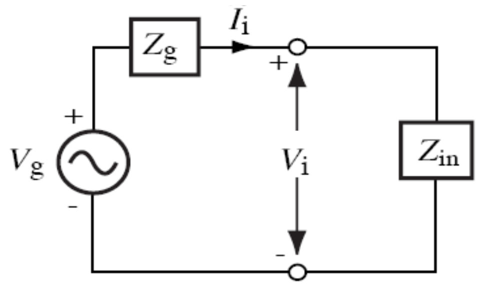
$0.212\lambda - 0.038\lambda = 0.174\lambda$ or $0.288\lambda - 0.038\lambda = 0.25\lambda$



<u>Smith chart solutions to Q3</u>

Q4





(a)

$$
\begin{aligned}
Z_{in} = Z(\ell = d) &= Z_0 \frac{Z_L + jZ_0 \tan(kd)}{Z_0 + jZ_L \tan(kd)} \\
&= 50 \frac{75 + j50 \tan(2\pi \times 0.15)}{50 + j75 \tan(2\pi \times 0.15)} \\
&= 41.25 - j16.35 \ \Omega
\end{aligned}
$$

(b)

$$I_i = \frac{V_g}{Z_g + Z_{in}} = \frac{100}{50 + 41.25 - j16.35} = 1.08e^{j0.1773} \quad \text{(A)}$$

$$V_i = I_i Z_{in} = 1.08e^{j0.1773}(41.25 - j16.3) = 47.86e^{-j0.2000} \quad \text{(V)}$$

(c)

$$P_{in} = \frac{1}{2}\mathrm{Re}\{V_i I_i^*\} = \frac{1}{2}\mathrm{Re}\{47.86e^{-j0.2000} \times 1.08e^{-j0.1773}\} = 24 \quad \text{(W)}$$

(d)

The general expressions of the voltage and current on the transmission line are:

$$V(\ell) = I_L\left[Z_L \cos(k\ell) + jZ_0 \sin(k\ell)\right]$$

$$I(\ell) = \frac{I_L}{Z_0}\left[Z_0 \cos(k\ell) + jZ_L \sin(k\ell)\right]$$

$$I_i = I(\ell = d) = I_L\left[\cos(kd) + j\frac{Z_L}{Z_0}\sin(kd)\right]$$

Hence,

$$I_L = \frac{I_i}{\cos(kd) + j\frac{Z_L}{Z_0}\sin(kd)} = \frac{1.08e^{j0.1773}}{\cos(2\pi\times 0.15) + j\frac{75}{50}\sin(2\pi\times 0.15)} = 0.80e^{-j0.9424} \quad \text{(A)}$$

$$V_L = I_L Z_L = 60e^{-j0.9424} \quad \text{(V)}$$

The various powers are calculated as:

$$P_g = \frac{1}{2}\mathrm{Re}\left\{V_g I_i^*\right\} = \frac{1}{2}\mathrm{Re}\left\{100\times 1.08e^{-j0.1773}\right\} = 53.15 \quad \text{(W)}$$

$$P_L = \frac{1}{2}\mathrm{Re}\left\{V_L I_L^*\right\} = \frac{1}{2}\mathrm{Re}\left\{60e^{-j0.9425}\times 0.8e^{j0.9425}\right\} = 24 \quad \text{(W)} \quad \text{Note: indeed the same as } P_{in} \text{ in part (c)}$$

$$P_{\mathrm{int}} = \frac{1}{2}\mathrm{Re}\left\{Z_g I_i I_i^*\right\} = \frac{1}{2}\left|I_i\right|^2 \mathrm{Re}\left\{Z_g\right\} = \frac{1}{2}\times 1.08^2\times 50 = 29.16 \quad \text{(W)}$$

Hence,

$$P_g = P_L + P_{\mathrm{int}} \quad \text{(within numerical error)}$$

Power conservation is satisfied.

Q5 (Optional)

Input impedance of the antenna $Z_{in} = 35 + j10\,\Omega$

Normalized input impedance is $z_{in} = \dfrac{Z_{in}}{Z_0} = \dfrac{35 + j10}{50} = 0.7 + j0.2$

Normalized input admittance is $y_{in} = \dfrac{1}{z_{in}} = 1.32\text{-}j0.38$

<u>Steps:</u>

1. Enter y_{in} into the Smith chart as shown. Note that y_{in} is diagonally opposite z_{in}.
2. Turn clockwise (towards generator) to point *A* which gives the first solution. Point *A's* normalized admittance is 1.0-*j*0.44. The distance traveled from y_{in} to point *A* measuring on the Smith chart is d_1 which is

$$d_1 = (0.3585 - 0.3045)\lambda = 0.054\lambda$$

   This is the position of the stub from the load.
3. To cancel the reactive part of -*j*0.44, an open-circuit parallel stub, whose normalized admittance at the open end is at point *S* on the Smith chart, must have

a length ℓ_1 measuring (towards generator) from point S to point S_A on the Smith chart in order to give a normalized admittance of $j0.44$. That is,

$$\ell_1 = 0.0660\lambda$$

This is the length of the stub.

4. There is another possible solution. This is achieved by turning (towards generator) from y_{in} to point B on the Smith chart. Point B's normalized admittance is $1.0+j0.44$. The distance traveled from y_{in} to point B measuring on the Smith chart is d_2 which is

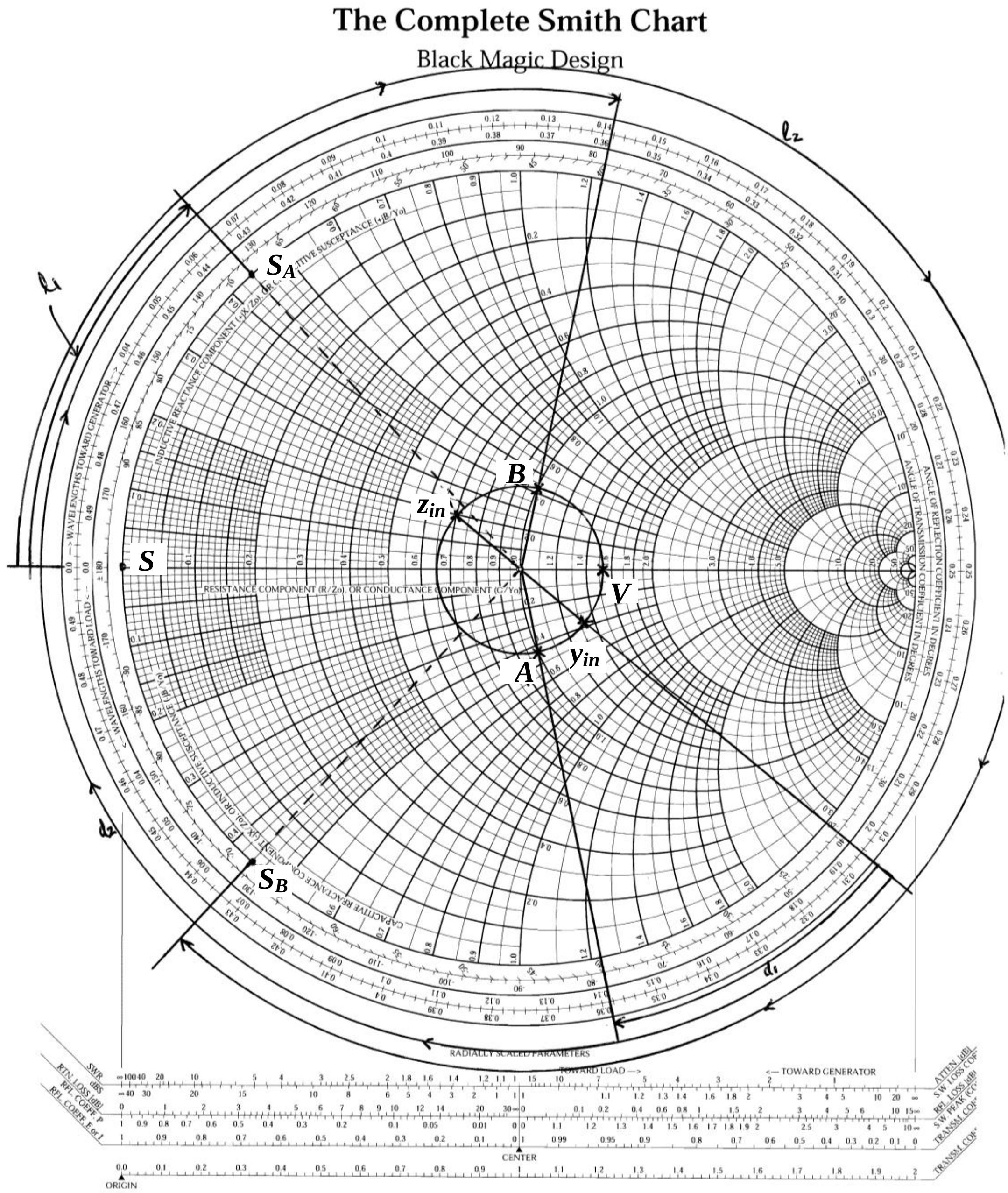
$$d_2 = (0.500 - 0.3045)\lambda + 0.1415\lambda = 0.337\lambda$$

This is the position of the stub from the load for the second solution.

5. To cancel the reactive part of $j0.44$, an open-circuit parallel stub, whose normalized admittance at the open end is at point S on the Smith chart, must have a length ℓ_2 measuring (towards generator) from point S to point S_B on the Smith chart in order to give a normalized admittance of $-j0.44$. That is,

$$\ell_2 = 0.434\lambda$$

This is the length of the stub for the second solution.



Smith chart solutions to Q5