EE2011 Engineering Electromagnetics - Part CXD
Tutorial 6 - Solutions

Q1

(i) Direction of propagation: $-\hat{\mathbf{x}}$ direction

(ii) From the expression of the electric field, we have:

$\omega = 10^8$ rad/s, $k = \omega / c \approx 1/3$ m

Then,

$$\lambda = \frac{2\pi}{k} = 6\pi \text{ m},$$

$$t_1 = \frac{\lambda/2}{c} = 31.42 \text{ ns}.$$

(iii) The wave has only the *y* component E_y.

$$E_y(x,t) = 50\cos\left(10^8 t + kx\right) = 50\cos(\omega t + kx) = 50\cos\left(\frac{2\pi}{T}t + kx\right)$$

$t = 0$: $E_y = 50\cos kx$

$t = T/4$: $E_y = 50\cos(\pi/2 + kx) = -50\sin kx$

$t = T/2$: $E_y = 50\cos(\pi + kx) = -50\cos kx$

The wave at different times is sketched on next page:

Q2

The wave propagates in +z direction.

Phasor: $\mathbf{E}(z) = 2e^{-jz/\sqrt{3}}\hat{\mathbf{x}}$ (V/m)

(i) $\omega = 10^8$ (rad/s) $\rightarrow$ $f = \dfrac{10^8}{2\pi} = 1.59\times 10^7$ (Hz) ,

$$k = \frac{1}{\sqrt{3}} \text{ (rad/m)} \rightarrow \lambda = \frac{2\pi}{k} = 2\pi\sqrt{3} \text{ (m)} .$$

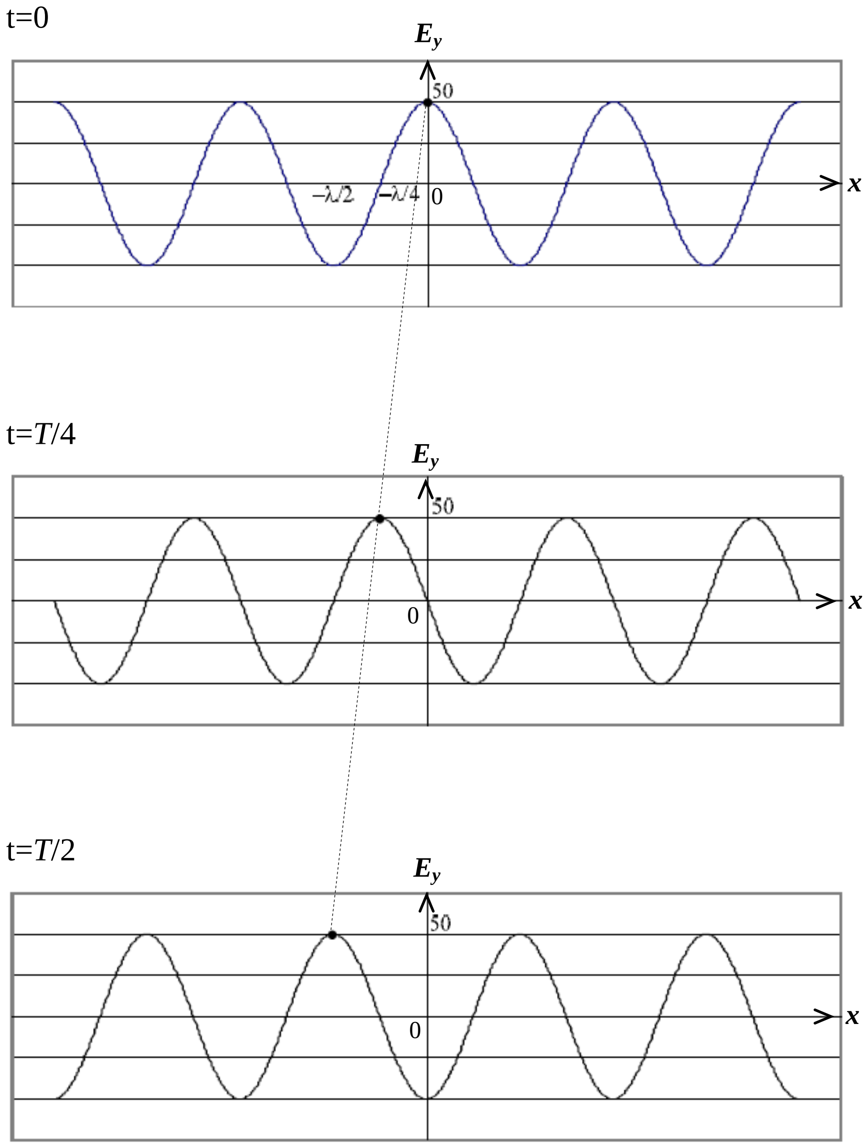
(ii) $u_p = \dfrac{\omega}{k} = \dfrac{c}{\sqrt{\varepsilon_r}}$ $\rightarrow$ $\varepsilon_r = \left(\dfrac{ck}{\omega}\right)^2 = 3$.

(iii) $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \dfrac{120\pi}{\sqrt{\varepsilon_r}} = \dfrac{120\pi}{\sqrt{3}}$ (Ω) ,

$$\mathbf{H}(z) = \frac{1}{\eta}\hat{\mathbf{k}}\times\mathbf{E} = \frac{\sqrt{3}}{120\pi}2e^{-jz/\sqrt{3}}\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \frac{2\sqrt{3}}{120\pi}e^{-jz/\sqrt{3}}\hat{\mathbf{y}}$$

$$\mathbf{H}(z,t) = \mathrm{Re}\left\{\mathbf{H}(z)e^{j\omega t}\right\} = \mathrm{Re}\left\{\frac{2\sqrt{3}}{120\pi}e^{-jz/\sqrt{3}}e^{j10^8 t}\right\}\hat{\mathbf{y}}$$

$$= \frac{\sqrt{3}}{60\pi}\cos\left(10^8 t - z/\sqrt{3}\right)\hat{\mathbf{y}} \text{ (A/m)}$$



Question 1: Part (iii)

Q3

For $f = 60\text{MHz}=6\times10^7\text{Hz}$, $\varepsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r\mu_0} = \frac{2\pi\times6\times10^7}{3\times10^8}\sqrt{4} = 0.8\pi \quad \text{(rad/m)}$$

Given that $\mathbf{E}$ points along $\hat{\mathbf{z}}$ and wave travel is along $-\hat{\mathbf{x}}$, we can write

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}E_0\cos(2\pi\times6\times10^7 t + 0.8\pi x + \phi_0) \quad \text{(V/m)}$$

where E_0 and ϕ_0 are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi \quad (\Omega)$$

With $\mathbf{E}$ along $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ along $-\hat{\mathbf{x}}$, we have

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E}$$

or

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\frac{E_0}{\eta}\cos(1.2\pi \times 10^8 t + 0.8\pi x + \phi_0) \quad \text{(A/m)}$$

Hence,

$$\frac{E_0}{\eta} = 10 \quad \text{(mA/m)}$$

$$E_0 = 10 \times 60\pi \times 10^{-3} = 0.6\pi \quad \text{(V/m)}$$

Also,

$$H(-0.75\text{m}, 0) = 7 \times 10^{-3} = 10\cos(-0.8\pi \times 0.75 + \phi_0) \times 10^{-3}$$

which leads to $\phi_0 = 0.6\pi \pm \cos^{-1} 0.7$

Thus $\phi_0 = 153.6^\circ$ or $\phi_0 = 62.4^\circ$

Hence,

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}0.6\pi\cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad \text{(V/m)}$$

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}10\cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad \text{(mA/m)}$$

or

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}0.6\pi\cos(1.2\pi \times 10^8 t + 0.8\pi x + 62.4^\circ) \quad \text{(V/m)}$$

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}10\cos(1.2\pi \times 10^8 t + 0.8\pi x + 62.4^\circ) \quad \text{(mA/m)}$$

Q4

$$\mathbf{E}(t) = \frac{1}{\sqrt{2}}(A\hat{\mathbf{y}} + \hat{\mathbf{z}})\cos\left[\omega t - \frac{\beta}{\sqrt{2}}(y+z)\right]$$

(i) $k_x = 0, k_y = \frac{\beta}{\sqrt{2}}, k_z = \frac{\beta}{\sqrt{2}}$

(ii) $k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \beta$, $\hat{\mathbf{k}} = \mathbf{k}/k = (k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}})/k = (\frac{\beta}{\sqrt{2}}\hat{\mathbf{y}} + \frac{\beta}{\sqrt{2}}\hat{\mathbf{z}})/\beta = \frac{1}{\sqrt{2}}(\hat{\mathbf{y}} + \hat{\mathbf{z}})$

(iii) $\mathbf{E} \cdot \hat{\mathbf{k}} = 0 \Rightarrow \frac{1}{\sqrt{2}}(A\hat{y} + \hat{z}) \cdot \frac{1}{\sqrt{2}}(\hat{\mathbf{y}} + \hat{\mathbf{z}}) = 0 \Rightarrow A + 1 = 0 \Rightarrow A = -1$

(iv)

$$\begin{aligned}
\mathbf{H}(t) &= \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E}(t) \\
&= \frac{1}{\eta_0}\left(\frac{1}{\sqrt{2}}\left(\hat{\mathbf{y}}+\hat{\mathbf{z}}\right)\right) \times \frac{1}{\sqrt{2}}(-\hat{\mathbf{y}}+\hat{\mathbf{z}})\cos\left[\frac{\beta}{\sqrt{2}}(y+z)-\omega t\right] \\
&= \frac{1}{\eta_0}\hat{\mathbf{x}}\cos\left[\frac{\beta}{\sqrt{2}}(y+z)-\omega t\right]
\end{aligned}$$