CS3230 : Tutorial - 9

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23/24-Oct-2012

Please drop your answer sheets in Bakh's office or Rahul's CQT office by 1 pm Tuesday, 23rd October, 2012.

1. We are given a sequence of n non-negative real numbers $p_1, p_2, \ldots, p_n$. We are supposed to determine $\max\{p(j) - p(i) \mid 1 \le i < j \le n\}$. Write a dynamic-programming algorithm (idea and pseudocode) running in linear time for this task.

2. Write an algorithm which takes as input array d of n denominations with $d[1] > d[2] > \ldots > d[n] = 1$ and an amount A. It outputs array C such that $C[j]$ (for $1 \le j \le A$) is the minimum number of coins used for amount j (using denominations in d). The algorithm must use storage only $O(n)$ (that is the storage used should not depend on A). Write the idea of the algorithm and then the pseudocode. What is the running time of your algorithm?

3. Consider the following algorithm:

   $power2(k)$ {

   $result = 2$

   for $i = 1$ to k

       $result = result * result$

   return $result$

   }

   Is this a polynomial time algorithm if k is given in unary ? Is this a polynomial time algorithm if k is given in binary ?

4. Show that if a decision problem A is in $\mathcal{P}$ (the class of decision problems solvable in polynomial time), then for every other decision problem B, we have $A \le_P B$ using a Karp reduction (a.k.a polynomial time many-one reduction), unless B or $\bar{B}$ is empty.

5. Take it granted that there exists an (infinite) listing of all polynomial time algorithms. Let P_k be the k-th algorithm in this listing. Take it granted that there exists a universal program U such that $U(x, k) = P_k(x)$ (that is the output of U on input (x, k) is the same as the output of P on input x) for every binary sting x and number natural number k. The running time of U on input (x, k) is polynomial in $|x| + k$. Consider the following algorithm:

   $diagp(k)$ {

   if $(U(k, k) == \text{yes})$

return no

else

return yes

}

Show that *diagp* is an exponential time algorithm and the language it decides is different from any language in P. From this conclude that $\mathcal{P} \neq \mathcal{EXP}$, where $\mathcal{EXP}$ is the class of decision problems that can be decided in exponential time. Above whenever number k is given as input to any program it is given using binary encoding.