**NATIONAL UNIVERSITY OF SINGAPORE**

**FACULTY OF SCIENCE**

**SEMESTER I EXAMINATION 2000-2001**

# ST2334 PROBABILITY AND STATISTICS

October / November 2000 – Time allowed: 2 hours

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## INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **TWO (2)** sections: Section A and Section B. It contains a total of **TEN (10)** questions and comprises **FIVE (5)** printed pages.

2. Answer **ALL** the **FIVE (5)** questions in Section A. Section A carries a total of 60 marks. The marks for each question are indicated at the end of each question.

3. Answer not more than **FOUR (4)** questions in Section B. Section B carries a total of 40 marks. Each question in Section B carries 10 marks.

4. Candidates may use non-programmable calculators. However, they should lay out systematically the various steps in the calculations.

5. Statistical tables are provided.

**Section A** (60%): **Answer all the following FIVE questions.**

1. A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let $X$ denote the number of hoses being used on the self-service island at a particular time and let $Y$ denote the number of hoses on the full-service island in use at that time. The joint probability distribution of $X$ and $Y$ is given in the following table.

| $p(x, y)$ | | $y$ = 0 | 1 | 2 |
|---|---|---|---|---|
| | 0 | 0.10 | 0.04 | 0.02 |
| $x$ | 1 | 0.08 | 0.20 | 0.06 |
| | 2 | 0.06 | 0.14 | 0.30 |

(i) Let A = $\{X \neq 0$ and $Y \neq 0\}$. Compute the probability of the event A.

(ii) Find the marginal probability distribution of $X$.

(iii) Find the cumulative distribution function of $X$.

**(8 marks)**

2. The joint probability density function of $X$ and $Y$ is given by

$$f_{X,Y}(x,y) = e^{-y^2/2} \quad \text{for } 0 < x < y < \infty.$$

(i) Verify that $f_{X,Y}(x,y)$ is a joint probability density function.

(ii) Are $X$ and $Y$ independent? Justify your answer.

(iii) Find $E(X)$.

**(18 marks)**

3. Samples arriving at a certain laboratory may be given a number of tests to determine the presence of trace amounts of Chemical X. If the result of test A is positive, then the sample is subjected to test B. If the result of test B is positive, then it is subjected to test C. If the result of test C is positive, then it is assumed that the sample contains Chemical X. Suppose that the probabilities of false positives for tests A, B, and C, are .05, .10, and .15, respectively, and the probabilities of false negatives are .01, .02, and .03, respectively. Furthermore, suppose that 10% of samples arriving at the laboratory contain trace amounts of Chemical X.

(i) Find the probability that the laboratory will falsely assume the presence of Chemical X in a sample.

(ii) What percent of samples tested by the laboratory are reported to contain no trace of Chemical X?

**(12 marks)**

4. We have to test the claim of a publisher that the average number of misprints per page of a book, $\mu$, is 1. If a randomly chosen page of that book contains more than two misprints then the null hypothesis $\mu = 1$ is rejected.

(i) What is the probability of committing a type-I error?

(ii) Find the probability of committing a type-II error when the alternative hypothesis is $\mu = 2$.

**(8 marks)**

5. Let $X_1, \ldots, X_n$ be a random sample from the uniform distribution over the interval $(0, \theta)$, where $\theta > 0$ and is unknown. Let $W = \min(X_1, \ldots, X_n)$.

(i) Find the cumulative distribution function of $W$.

(ii) Find the probability density function of $W$.

**(14 marks)**

## Section B (40%): Answer not more than FOUR questions.

6. A machine consists of 3 components. It will not function if any of the components fails. Suppose the components will fail on a given day are independent. The probability of each of the components will fail on a given day is respectively 0.05, 0.1 and 0.15. Find the probability that the machine will not function on a given day.

**(10 marks)**

7. An urn holds 60 red marbles and 40 white marbles. Two sets of 30 marbles are drawn from the urn and their colours are noted. Each marble in a set is drawn at a time with replacement. What is the approximate probability that the number of red marbles of the two sets differ by 8 or more marbles?

**(10 marks)**

8. A study has been made to compare the nicotine contents of two brands of cigarettes. The data are summarized below:

| Brand | Sample Size | Sample Mean | Sum of squares |
|---|---|---|---|
| Brand X | $n_1 = 10$ | $\bar{x} = 3.1$ | $\sum_{i=1}^{n_1} x_i^2 = 98.35$ |
| Brand Y | $n_2 = 8$ | $\bar{y} = 2.7$ | $\sum_{i=1}^{n_2} y_i^2 = 61.75$ |

Assume that the two sets of data are independent random samples from normal populations.

(i) Construct a 98% confidence interval for the ratio of the standard deviations of measurements made with brands X and Y, $\sigma_1/\sigma_2$.

(ii) On testing $H_0$: $\sigma_1 = \sigma_2$, what conclusion can you draw from part (i)?

**(10 marks)**

9. Suppose that $X$ has an exponential distribution with mean $\mu$. If

$$P(X \leq 1) = P(X > 1),$$

what is the variance of $X$?

**(10 marks)**

10. Suppose a company psychologist administers a set of tests to each of ten supervisors before a management-training programme begins and then administers a similar set of tests at the end of the programme. The tests are designed to measure supervisory skills, with higher scores indicating increased skill. The results of the tests are shown in the table.

(i) Test whether the data indicate that the training programme is effective. Use $\alpha = 0.10$.

(ii) State the assumption concerning the distribution of the data in your analysis if there is any.

| Supervisor | Before training programme | After training programme |
|---|---|---|
| 1 | 63 | 78 |
| 2 | 93 | 92 |
| 3 | 84 | 91 |
| 4 | 72 | 80 |
| 5 | 65 | 69 |
| 6 | 72 | 85 |
| 7 | 91 | 99 |
| 8 | 84 | 82 |
| 9 | 71 | 81 |
| 10 | 80 | 87 |

**(10 marks)**

**-- END OF PAPER --**