MA1506 TUTORIAL 8

Question 1

Despite her vast wealth, Tan Ah Lian continues to live with her mother. The latter does TAL's laundry [of course], including a large heavy jacket. TAL's mother hangs the jacket out to dry on a bamboo pole inserted into a socket outside the kitchen window. The pole has negligible mass but the wet jacket has a mass of M kg. It is suspended at a point exactly A metres along the pole, which is of length L metres. Assume that the usual beam parameters E and I are given for this pole, and assume for simplicity that the pole is perfectly horizontal before the jacket is attached. Use Laplace transforms to find the shape of the pole after it is loaded, assuming that it doesn't break. Draw a graph of y(x).

To do this, you need some information as follows. Let y(x) represent the shape of the pole, with x = 0 being the point where the pole is inserted in the socket. Then the third derivative at zero is $y'''(0) = Mg/EI$, because the shear force there has to balance the weight of the jacket. Similarly $y''(0) = -\frac{Mg}{EI} \times A$ because the torque at x = 0 has to balance the torque exerted by the jacket at x = A. [If you don't know the physics involved, just ignore this and take these initial conditions as given.] The function w(x) giving the force per unit length can be modelled by using a delta function [with units of 1/length], $w(x) = -Mg\delta(x - A)$. [Answer: for $x \leq A$ we have $y(x) = -\frac{Mg}{EI}\left[\frac{1}{2}x^2 A - \frac{1}{6}x^3\right]$, for $x \geq A$ we have $y(x) = -\frac{Mg}{EI}\left[\frac{1}{2}xA^2 - \frac{1}{6}A^3\right]$.]

Question 2

Tan Ah Lian attributes her enormous success to the fact that she never talked in class when she was an Engineering student at NUS. Unfortunately, a small but vociferous minority of other students do not share her virtues. One day in the lecture the prof announces that a certain gadget contains a circuit with a resistance, capacitance, and inductance in series, with values of R, C, L which were all stated, but TAL could not hear all of the numbers mentioned due to the incessant babbling of the talkative minority; all she could hear was that the resistance is 2 ohms. Undeterred, she steals back into the room after class and quickly switches the gadget on and off at t = 0, thus firing a short burst of voltage into it, and observes that the resulting current at $t > 0$ is $I(t) = e^{-t}cos(t) - e^{-t}sin(t)$ amperes. She then deduces what the prof must have said about the inductance and the capacitance. Follow her good example and also deduce these numbers. [See Chapter 2 and recall the formula for the Laplace transform of an integral.][Answer: L = 1 and C = 1/2 in the appropriate units.]

Question 3

The trace of a square matrix is defined as the sum of its diagonal entries. A square matrix is said to be traceless if its trace is zero. Show that any square matrix can be expressed as the sum of an antisymmetric matrix, a symmetric traceless matrix, and a multiple of the identity matrix. Express the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ as the sum of three matrices in this manner. [Answer: $\begin{pmatrix} -3/2 & 5/2 \\ 5/2 & 3/2 \end{pmatrix} + \begin{pmatrix} 5/2 & 0 \\ 0 & 5/2 \end{pmatrix} + \begin{pmatrix} 0 & -1/2 \\ 1/2 & 0 \end{pmatrix}$.]

Question 4

In the discussion of shear matrices in the notes, we assumed that the shearing forces were parallel to the x-axis. Find the matrix of a shear [shearing angle 30 degrees] when the shearing forces are parallel to an axis which makes an angle of 45 degrees with the x axis. [Hint: Rotate the axis down to the x-axis, do the shear there, and then rotate back up to the original direction.] Check that your matrix has determinant equal to 1, as it should [why?]. [Answer: $\begin{pmatrix} 1 - \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{2\sqrt{3}} & 1 + \frac{1}{2\sqrt{3}} \end{pmatrix}$.]

Question 5

The exponential of a square matrix B is defined to be the infinite series

$$e^B = I + B + \frac{B^2}{2!} + \frac{B^3}{3!} + \ldots..,$$

where I is the identity matrix. Show that the rotation matrix $\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$ can be expressed as $e^{\theta A}$, where A = $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [Hint: show that all powers of A are proportional either to the identity matrix or to A itself.]