NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATTION

ST2334 PROBABILITY AND STATISTICS

(Semester 2 2006/2007)

April 2007 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **Seven (7) Questions** and comprises **Ten (10)** printed pages.
2. Answer **ALL** questions. All answers must be written in the space provided for each question. Hand in this answer book at the end of the examination.
3. Please write your **matriculation number** on this cover.
4. Candidates may use non-programmable calculators.
5. One handwritten A4-size help sheet is allowed.
6. Statistical tables are provided.

MATRICULATION NO. : _____

Question	Marks	Max
1		15
2		15
3		10
4		10
5		18
6		22
7		10
TOTAL:		100

Question 1 (15 marks)

The owner of a restaurant that serves continental food wants to study characteristics of customers of his restaurant. In particular, he decides to focus on the amount of money spent by customers. A random sample of 20 customers gives an average amount of money spent \$28 and a sample standard deviation of \$5.

(a) Let μ be the population mean amount spent per customer in the restaurant. Find a point estimate of the mean μ.

(b) Construct a 95% confidence interval for the mean μ.

(c) Do you require any assumption in forming the confidence interval in part (b)? Explain.

(d) The owner of the restaurant believes that the mean amount spent per customer in the restaurant is \$30. Based on the confidence interval obtained in part (b), does it appear that the owner's hypothesis is supported? Explain.

Question 2 (15 marks)

A study of iron deficiency among infants compared samples of infants following different feeding regimens. One group contained breast-fed infants, while the children in another group were fed a standard baby formula without any iron supplements. The following is the summary results on blood hemoglobin levels at 12 months of age.

Group	Sample size	Sample mean	Sample standard deviation
Breast-fed, (1)	$n_1 = 13$	$\bar{x}_1 = 13.3$	$s_1 = 1.7$
Formula, (2)	$n_2 = 19$	$\bar{x}_2 = 12.4$	$s_2 = 1.8$

(a) At the 5% significance level, is there any evidence of a difference between the variances in hemoglobin level in the two groups of babies?

(b) Based on the result of part (a), conduct an appropriate test of hypothesis to determine whether the mean hemoglobin level is higher among breast-fed babies? Use $\alpha = 0.05$.

(c) What assumptions must be made in order the procedures in parts (a) and (b) to be valid?

Question 3 (10 marks)

In a certain community, 8% of all adults over 50 have diabetes. A health service in this community correctly diagnoses 95% of all persons with diabetes as having the disease and incorrectly diagnoses 2% of all persons without diabetes as having the disease.

(a) Find the probability that the community health service will diagnose an adult over 50 as having diabetes.

(b) What is the probability that a person over 50 diagnosed by the health service as having diabetes actually has the disease?

Question 4 (10 marks)

A notice is sent to all owners of a certain type of automobile, asking them to bring their cars to a dealer for the presence of a particular manufacturing defect. Suppose that only 1% of such cars have the defect. Company A has 10 of this type of cars.

(a) What is the expected value and standard deviation of number of cars in company A that have the defect?

(b) What is the probability that at most 1 of company A's cars have the defect?

(c) Suppose that a taxi company B has 200 of this type of cars. What is the probability that at most 3 of its taxis have defect?

Question 5 (18 marks)

A grocery store sells X hundred kilograms of rice every day, where the cumulative distribution function of the random variable X is given as follows.

$$F_X(x) = \begin{cases} 0, & x < 0; \\ \frac{1}{18}x^2, & 0 \le x < 3; \\ \frac{1}{18}(-x^2 + 12x - 18), & 3 \le x < 6; \\ 1 & x \ge 6. \end{cases}$$

Suppose that this grocery store's total sales of rice do not reach 600 kilograms on any given day.

(a) What is the probability that the store sells between 200 and 400 kilograms of rice next Thursday?

(b) What is the probability that the store sells over 300 kilograms of rice next Thursday?

(c) We are given that the store sold at lest 300 kilograms of rice last Friday. What is the probability that it did not sell more than 400 kilograms on that day?

(d) Find the probability density function of X.

(e) Find $E(X)$ and $V(X)$.

Question 5 (Continued) [Blank page for answers]

Question 6 (22 marks)

An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X be the number of points earned in the first part, and Y be the number of points earned in the second part. Suppose that the joint probability function is given as follows.

$f_{x,y}(x, y)$		y		
		0	5	10
x	0	0.05	0.10	0.05
	5	0.07	0.18	0.25
	10	0.03	0.12	0.15

(a) Find the marginal probability functions of X and Y.
(b) What is the conditional probability function of Y given that $X = 10$? Hence find the expected value of Y given that $X = 10$.
(c) Let $A = \{X \neq 0 \text{ and } Y \neq 0\}$. Compute the probability of event A.
(d) If the score recorded in the grade book is the total number of points earned in the two parts, what is the expected recorded score $E(X + Y)$?
(e) If the maximum score of the two scores is recorded, what is the expected recorded score?

Question 6 (Continued) [Blank page for answers]

Question 7 (10 marks)

Suppose that the expectation and the variance of a random variable, X, are n and $2n$ respectively, where $n \geq 2$. Show that $\Pr(0 < X < 2n)$ is at least equal to $(n - 2)/n$.

- END OF PAPER -