Solutions to Tutorial 4

4.2 Here X = number of heads and there are 1, 4, 6, 4, 1 sequences corresponding to the values 0, 1, 2, 3, 4, respectively. Because the sequences are equally likely, the probabilities are

$$P(X=0)=1/16,\quad P(X=1)=1/4,\quad P(X=2)=6/16,\quad P(X=3)=1/4,\quad P(X=4)=1/16.$$

4.3 (a) Yes. $0 \le f(i) \le 1$, and $\sum_{i=1}^{4} f(i) \;=\; 1$.

(b) No. $\sum_{i=1}^{4} f(i) \;=\; 0.96 < 1$.

(c) No. $f(4) < 0$.

4.4 (a) No. $\sum_{i=0}^{4} f(i) \;=\; 10/14 < 1$.

(b) No. $f(2) \;=\; -1/4 < 0$.

(c) Yes. $0 \le f(i) \le 1$, and $\sum_{i=5}^{9} f(i) \;=\; 1$.

(d) No. $\sum_{i=1}^{5} f(i) \;=\; 35/50 < 1$.

4.5 Using the identity

$$(x-1)\sum_{i=0}^{n} x^i \;=\; x^{n+1}-1$$

or

$$\sum_{i=0}^{n} x^i \;=\; \frac{x^{n+1}-1}{x-1},$$

we have

$$\sum_{x=0}^{4} \frac{k}{2^x} \;=\; k\frac{(\frac{1}{2})^{4+1}-1}{\frac{1}{2}-1} \;=\; \frac{31k}{16}.$$

This must equal 1, so $k = 16/31$.

4.7

$$\begin{aligned} b(x;n,p) &= \binom{n}{x} p^x (1-p)^{n-x} \;=\; \frac{n!}{x!\,(n-x)!}\; p^x(1-p)^{n-x} \\ &= \frac{n!}{(n-x)!\,x!}\;(1-p)^{n-x}\,p^x \;=\; \binom{n}{n-x}(1-p)^{n-x}\,p^x \\ &= b(n-x;n,1-p) \end{aligned}$$

4.8 Using the result in Exercise 4.7,

$$\begin{aligned} B(x;n,p) &= \sum_{i=0}^{x} b(i;n,p) \;=\; \sum_{i=0}^{x} b(n-i;n,1-p) \\ &= \sum_{u=n-x}^{n} b(u;n,1-p) \;=\; 1-\sum_{u=0}^{n-x-1} b(u;n,1-p) \\ &= 1-B(n-x-1;n,p) \end{aligned}$$

4.9 (a) Assumptions appear to hold. Success is a home with a TV tuned to mayor's speech. The probability of success is the proportion of homes around city having a TV tuned to the mayor's speech.

(b) The binomial assumptions do not hold because the probability of a serious violation for the second choice depends on which plant is selected first.

Solutions to Tutorial 4

4.10 (a) Not the same probability for all facilities since the larger ones are more likely to experience accidents.

(b) Trials are not independent. If the second shift workers now that the first shift's production exceeded 560 units they are likely to work harder to achieve similar results.

4.11 (a) Success is person has a cold. Colds are typically passed around in families so trials would not be independent. Therefore, the binomial distribution does not apply.

(b) Success means projector does not work properly. The binomial assumptions do not hold because the probability of a success for the second choice depends on which projector is selected first.

4.15

$$b(2; 4, .75) = \begin{pmatrix} 4 \\ 2 \end{pmatrix} (.75)^2 (.25)^{4-2} = .2109.$$

4.16

$$b(4; 12, .4) = \begin{pmatrix} 12 \\ 4 \end{pmatrix} (.4)^4 (.6)^8 = 495 (.4)^4 (.6)^8 = .2128.$$

4.17 (a) $1 - B(11;\ 15,\ .7) = 1 - .7031 = .2969.$

(b) $B(6;\ 15,\ .7) = .0152.$

(c) $b(10;\ 15,\ .7) = B(10;\ 15,\ .7) - B(9;\ 15,\ .7) = .4845 - .2784 = .2061.$

4.18 (a) $b(1;\ 12,\ .05) = B(1;\ 12,\ .05) - B(0;\ 12,\ .05) = .8816 - (\ 5404 = .3412.$

(b) $B(2;\ 12,\ .05) = .9804.$

(c) $1 - B(1;\ 12,\ .05) = 1 - .8816 = .1184.$

4.19 (a) $P(18 \text{ are ripe}) = (.9)^{18} = .1501.$

(b) $1 - B(15;\ 18,\ .9) = 1 - .2662 = .7338.$

(c) $B(14;\ 18,\ .9) = .0982.$

4.20 (a) The probability that 1 or more components in a sample of 15 is defective when the true probability of being good is .95 is $1 - b(0; 15, .05) = 1 - (.95)^{15} = .5367.$

(b) The probability that 0 are defective when the true probability is .90 is $b(0; 15, .10) = .2059$.

(c) When the true probability is .80, we have $b(0; 15, .20) = .0352.$

4.21 (a) $B(2;\ 16,\ .05\ ) = .9571$

(b) $B(2;\ 16,\ .10\ ) = .7892$

(c) $B(2;\ 16,\ .15\ ) = .5614$

(d) $B(2;\ 16,\ .20\ ) = .3518$

Solutions to Tutorial 4

4.50

$$f(x+1;\lambda) = \frac{\lambda^{x+1}e^{-\lambda}}{(x+1)!} \quad \text{and} \quad f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Thus,

$$\frac{f(x+1;\lambda)}{f(x;\lambda)} = \frac{\lambda^{x+1}e^{-\lambda}}{(x+1)!}\,\frac{x!}{\lambda^x e^{-\lambda}} = \frac{\lambda}{x+1}.$$

4.57 $1 - F(12; 5.8) = 1 - .993 = .007.$

4.58 (a) $P(\text{ at least one request }) = 1 - F(0; .7) = 1 - .497 = .503.$

(b) λ for a 4-week period is 2.8 . Thus,

$$P(\text{at least 3 request in a 4-week period}) = 1 - F(2; 2.8) = 1 - .469 = .531$$

4.59 (a) $P(\text{at most 4 in a minute}) = F(4;\ 1.5\ ) = .981.$

(b) $P(\text{at least 3 in 2 minutes}) = 1 - F(2; 3) = 1 - .423 = .577.$

(c) $P(\text{at most 15 in 6 minutes}) = F(15; 9) = .978.$

4.61 P (fails after 1,200 times)

$$= \sum_{x=1201}^{\infty} (1-p)^{x-1}p = \frac{(1-p)^{1200}\,p}{1-(1-p)} = (1-p)^{1200}$$

where $p = .001$. Thus,

$$P(\text{fails after 1,200 times }) = (.999)^{1200} = .3010.$$

4.62 The required probability, given by the geometric distribution with $p = 0.02$, is

$$g(10; 0.02) = (0.98)^9(0.02)^1 = 0.0167$$

4.63 The required probability, given by the geometric distribution with $p = 0.10$, is

$$g(8; 0.1) = (0.9)^7(0.1)^1 = 0.0478$$

Solutions to Tutorial 4

4.65 We assume that the Poisson process with $\alpha = 0.01$ per hour applies.

(a) For a 4 hour time period, $\lambda = 0.01(4) = 0.04$.

$$f(1; 0.04) = \frac{(.04)^1}{1!} e^{-0.04} = 0.0384.$$

(b) We calculate $1 - f(0; 0.04) = 1 - e^{-0.04} = 0.0392$, or using Table 2,

$$1 - F(0; 0.04) = 1 - 0.961 = 0.039.$$

(c) For either of the two 4 hour time spans, the probability of exactly 1 customer is $f(1; 0.04) = 0.0384$. The two time intervals do not overlap so the counts are independent and we multiply the two probabilities

$$f(1; 0.04) \times f(1; 0.04) = (0.0384) \times (0.0384) = 0.0015$$