NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

EXAMINATION FOR
Semester 1 AY2011/2012

CS4243
COMPUTER VISION & PATTERN RECOGNITION

November 2011 Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

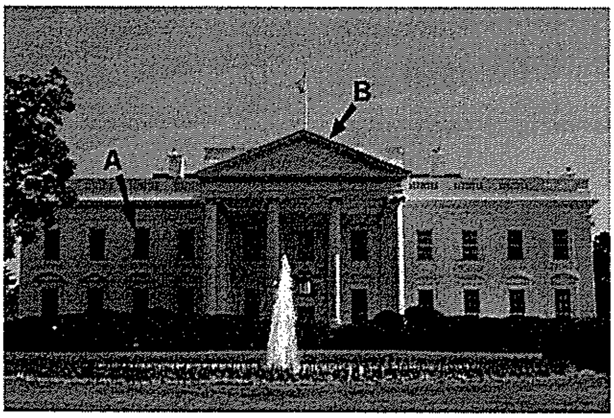
1. This examination paper contains **FIVE (5)** questions and comprises **FOURTEEN (14)** printed pages, including this page.
2. Answer **ALL** questions. The maximum mark is 100.
3. Write your answers in the space provided in this booklet. Use the reverse sides if necessary.
4. Write legibly. You may use pen or pencil.
5. This is an OPEN BOOK examination.
6. Please write your Matriculation Number below.

Matriculation No.: ______________________

This portion is for the examiner's use only.

Question	Marks	Remarks
Q1		
Q2		
Q3		
Q4		
Q5		
Total		

Q1: Image Mosaicking (15 marks)



As a computer vision expert in VisionTech Inc., you are given the task of mosaicking multiple images into a large image. Three of the images contain the house as shown above in their overlapping parts.

You recognises that your first task is to implement an algorithm that detects and matches good feature points in the various images. Your algorithm computes the auto-correlation matrix $\mathbf{A}$ at each point of the image I:

$$\mathbf{A} = \begin{bmatrix} \sum\limits_{W} I_x^2 & \sum\limits_{W} I_x I_y \\ \sum\limits_{W} I_x I_y & \sum\limits_{W} I_y^2 \end{bmatrix}, \tag{1}$$

followed by the eigenvalues λ_1 and λ_2 of $\mathbf{A}$.

1(a) (5 marks)

What is the relationship between λ_1 and λ_2 at the dark window corner labeled A? Is this point a good feature point for image mosaicking? Explain your answers.

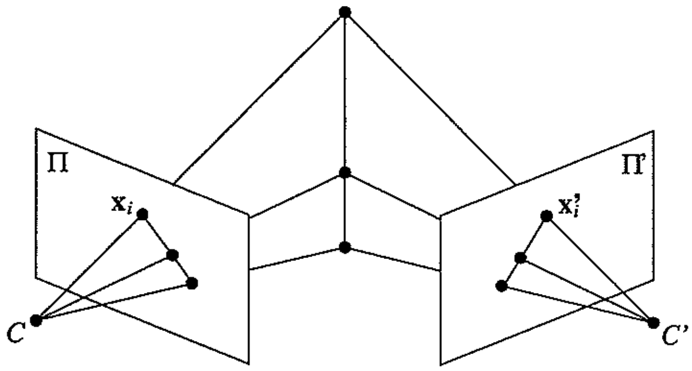
1(b) (5 marks)

What is the relationship between λ_1 and λ_2 at the top edge of the roof labeled B? Is this point a good feature point for image mosaicking? Explain your answers.

1(c) (5 marks)

Mark in the house image on the previous page a good feature point for image mosaicking. Explain why it is a good feature point.

Q2: Homography (20 marks)



In solving the image mosaicking problem, you discover that the corresponding image points $\mathbf{x}_i = [x_i, y_i]^\top$ and $\mathbf{x}_i' = [x_i', y_i']^\top$ in the overlapping part of two images Π and Π' are related by a homography $\mathbf{H}$ as follows:

$$w_i \tilde{\mathbf{x}}_i' = \mathbf{H}\tilde{\mathbf{x}}_i \tag{2}$$

which is

$$w_i \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}. \tag{3}$$

You can see in the figure above that a straight line in one image maps to a straight line in the other image. To be certain, you work on the mathematics to verify that your intuition is correct.

You know that a set of points $\mathbf{x}_i$ on a line in 2D plane can be represented by the equation

$$a_1 x_i + a_2 y_i + a_3 = 0 \tag{4}$$

for some parameters a_1, a_2 and a_3. Then, the corresponding points $\mathbf{x}_i'$ also form a line that can be represented by a similar equation

$$b_1 x_i' + b_2 y_i' + b_3 = 0 \tag{5}$$

with a different set of parameters b_1, b_2 and b_3.

Verify that the above reasoning is correct in the following manner (turn to the next page).

2(a) (10 marks)

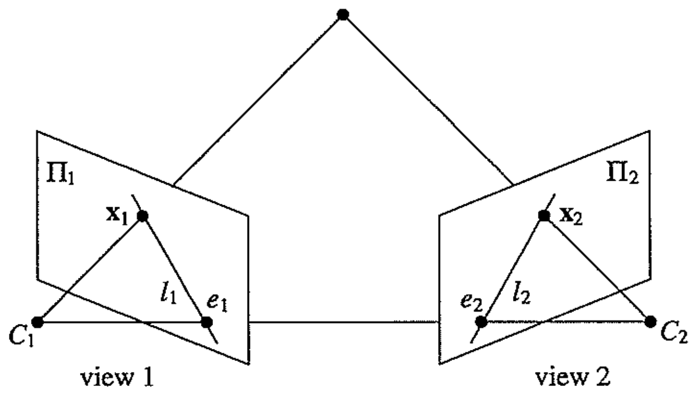
Expand Equation 3 and substitute it into Equation 5.

2(b) (10 marks)

Rearrange the equations you obtained in Q2(a) into the form of Equation 4. What are the expressions for the parameters a_1, a_2 and a_3 in terms of b_j and h_{jk}, $j, k = 1, 2, 3$?

Denoting $\mathbf{a} = [a_1, a_2, a_3]^\top$ and $\mathbf{b} = [b_1, b_2, b_3]^\top$, write down the relationship between $\mathbf{a}$ and $\mathbf{b}$ in matrix form.

Q3: Multiple-View Methods (20 marks)



With three views of a 3D scene, the normalized image point $\bar{\mathbf{x}}_2$ in view 2 is related to the corresponding normalized image point $\bar{\mathbf{x}}_1$ in view 1 by the epipolar constraint:

$$\bar{\mathbf{x}}_2^\top \mathbf{E}_{21} \bar{\mathbf{x}}_1 = 0. \tag{6}$$

In deriving Equation 6, the world coordinate frame is located at the camera centre of view 1 and aligned with camera frame 1. The 3D coordinates $\mathbf{X}_2$ of a scene point in camera frame 2 is related to the 3D coordinates $\mathbf{X}_1$ of the same point in camera frame 1 by the rigid transformation:

$$\mathbf{X}_2 = \mathbf{R}_{21}\mathbf{X}_1 + \mathbf{T}_{21}. \tag{7}$$

Then, the essential matrix $\mathbf{E}_{21} = [\mathbf{T}_{21}]_\times \mathbf{R}_{21}$.

Similarly, the epipolar constraint between view 3 and view 2 is:

$$\bar{\mathbf{x}}_3^\top \mathbf{E}_{32} \bar{\mathbf{x}}_2 = 0, \tag{8}$$

with

$$\mathbf{X}_3 = \mathbf{R}_{32}\mathbf{X}_2 + \mathbf{T}_{32}, \quad \mathbf{E}_{32} = [\mathbf{T}_{32}]_\times \mathbf{R}_{32}. \tag{9}$$

(Turn to the next page.)

3(a) (10 marks)

View 3 is also related to view 1 by an epipolar constraint:

$$\bar{\mathbf{x}}_3^\top \mathbf{E}_{31} \bar{\mathbf{x}}_1 = 0 \tag{10}$$

with $\mathbf{E}_{31} = [\mathbf{T}_{31}]_\times \mathbf{R}_{31}$. Derive the expressions of $\mathbf{R}_{31}$ and $\mathbf{T}_{31}$ in terms of $\mathbf{R}_{32}$, $\mathbf{T}_{32}$, $\mathbf{R}_{21}$ and $\mathbf{T}_{21}$.

3(b) (10 marks)

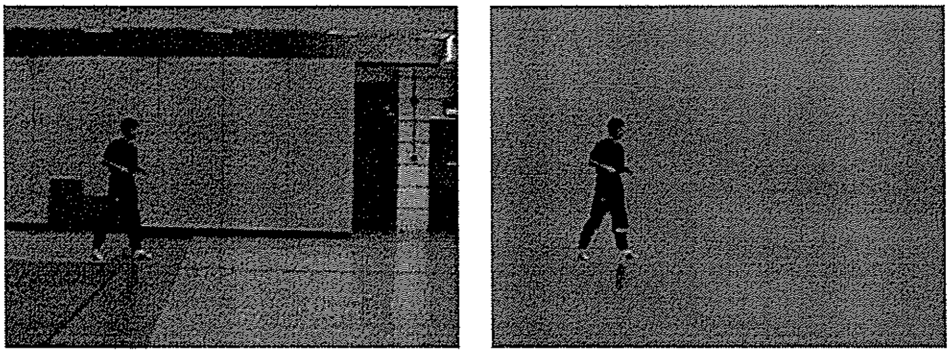
A 3D scene point $\mathbf{X}$ can be recovered from the triangulation of the corresponding image points in the three views (refer to the lecture notes):

$$\mathbf{X} = \left[\sum_{k=1}^{3} \mathbf{M}_k \right]^{-1} \sum_{k=1}^{3} \mathbf{M}_k \mathbf{C}_k, \tag{11}$$

where $\mathbf{M}_k = \mathbf{I} - \mathbf{v}_k \mathbf{v}_k^\top$, and $\mathbf{v}_k$ is the unit vector along the projection line from the 3D scene point to the image point in view k.

Write the expressions of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ in terms of the rotation and translation matrices $\mathbf{R}_{ij}$ and $\mathbf{T}_{ij}$. The camera projection matrix $\mathbf{K}$ is the same for the three views.

Q4: Background Removal (25 marks)



(a) With background. (b) Without background.

You worked with Double Exposure digital effects company and you are given the task of removing the background in a video sequence. After studying the video, you realise that the background is not stationary. Fortunately, the background moves at a constant velocity, so you can devise a background removal algorithm without having to stabilise and destabilise the video. Moreover, the background area is quite large, and it occupies more than 70% of the image area.

4(a) (15 marks)

Outline an algorithm that detects the background that is moving at constant velocity in the video.

(You may use this page for your answer.)

4(b) (10 marks)

Extend the algorithm for Q4(a) to detect the moving background for the general case, that is the area it occupies is not known in advance.

Q5: Camera Calibration (20 marks)

The perspective projection equation that projects a 3D point $\mathbf{X}_i = [X_i, Y_i, Z_i]^\top$ in the camera coordinate frame to an image point $\mathbf{x}_i = [x_i, y_i]^\top$ in the image frame is given by

$$\rho_i \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}. \tag{12}$$

Given n pairs of known corresponding points $\mathbf{x}_i$ and $\mathbf{X}_i$, $i = 1, \ldots, n$, you can recover the camera's intrinsic parameters f_x, f_y, s, c_x and c_y by forming a system of linear equations

$$\mathbf{D}\,\mathbf{k} = \mathbf{b} \tag{13}$$

where $\mathbf{D}$ is a matrix, $\mathbf{b}$ is a column vector, and $\mathbf{k}$ is the following column vector

$$\mathbf{k} = \begin{bmatrix} f_x \\ f_y \\ s \\ c_x \\ c_y \end{bmatrix}. \tag{14}$$

5(a) (10 marks)

Write down the matrix entries in $\mathbf{b}$.

5(b) (10 marks)

Write down the matrix entries in **D**.

— END OF PAPER —