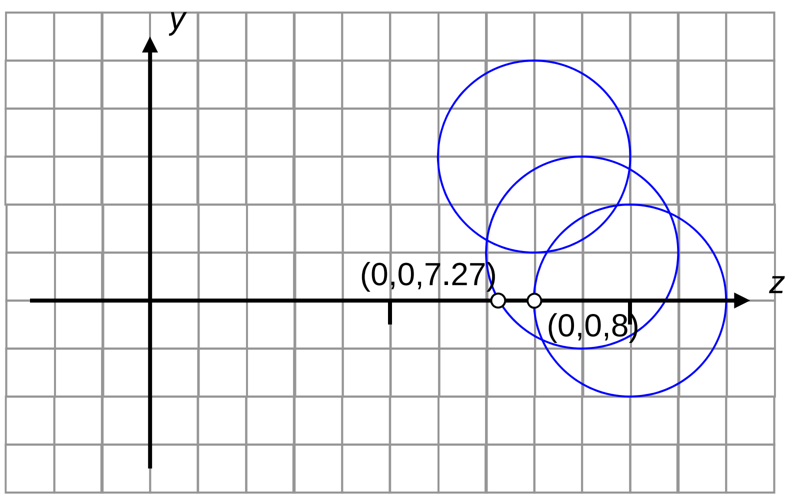
CS3241COMPUTER GRAPHICS

Tutorial 7

Question #1

- There are three spheres in the space with centers (0,0,10), (0,3,8) and (0,1,9). They all have the same radius of 2 units. A ray starts tracing from the eye which is at the origin, (0,0,0) with direction (0,0,1). Which is the first sphere that is hit by the ray?



Question #1

- Eye: (0, 0, 0)
- Direction: (0, 0, 1)
- The ray equation is

$$l(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x(t) = 0 \\ y(t) = 0 \\ z(t) = t \end{pmatrix}$$

Question #1

- Sphere equation

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

- a, b, c is the sphere's center
- r is radius.

- First sphere located (0,0,10) and r=2

$$(x-0)^2+(y-0)^2+(z-10)^2=2^2$$

Question #1

- Ray equation:

$$l(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x(t) = 0 \\ y(t) = 0 \\ z(t) = t \end{pmatrix}$$

- Sphere1 equation

$$(x-0)^2 + (y-0)^2 + (z-10)^2 = 2^2$$

- Substitute l(t) into Sphere 1 equation to get

$$(0-0)^2 + (0-0)^2 + (t-10)^2 = 2^2$$

- Solve to get t=8 or 12

Question #1

- Ray equation:

$$l(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x(t) = 0 \\ y(t) = 0 \\ z(t) = t \end{pmatrix}$$

- Sphere2 equation

$$(x-0)^2 + (y-3)^2 + (z-8)^2 = 2^2$$

- Substitute l(t) into Sphere 2 equation to get

$$(0-0)^2 + (0-3)^2 + (t-8)^2 = 2^2$$

- Undefined! Therefore ray doesn't intersect sphere 2.

Question #2

- Ray equation:

$$l(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x(t) = 0 \\ y(t) = 0 \\ z(t) = t \end{pmatrix}$$

- Sphere3 equation

$$(x-0)^2 + (y-1)^2 + (z-9)^2 = 2^2$$

- Substitute l(t) into Sphere 3 equation to get

$$(0-0)^2 + (0-1)^2 + (t-9)^2 = 2^2$$

- Solve to get t = 10.73 or 7.27

Question #2

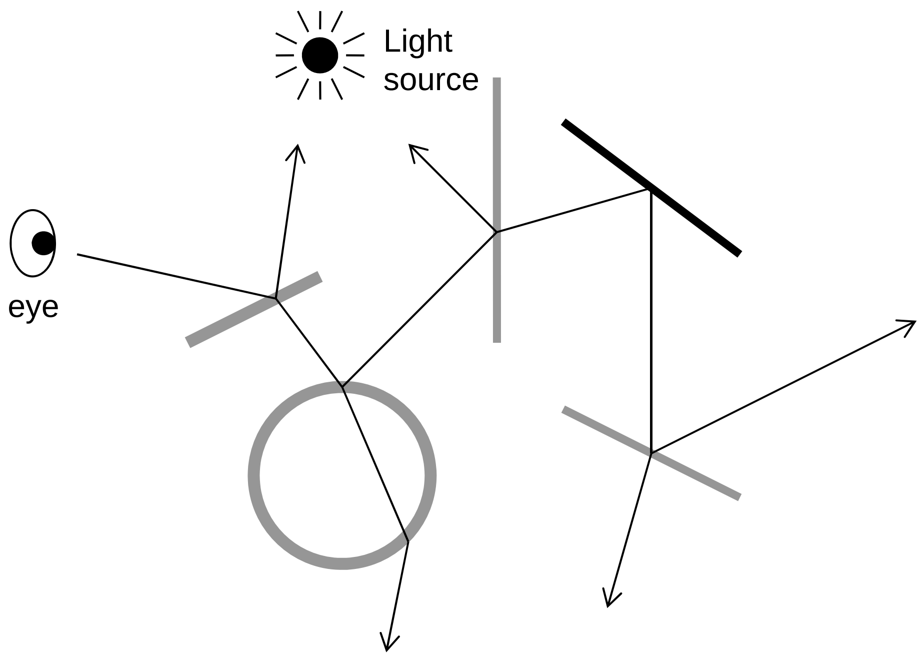
- Sphere 1: t=8, t=12
- Sphere 3: t=10.73, t=7.27

- Smallest t value is 7.27.
- Therefore, ray first intersects sphere 3, with the point of intersection being

$$l(7.27) = \begin{pmatrix} 0 \\ 0 \\ 7.27 \end{pmatrix}$$

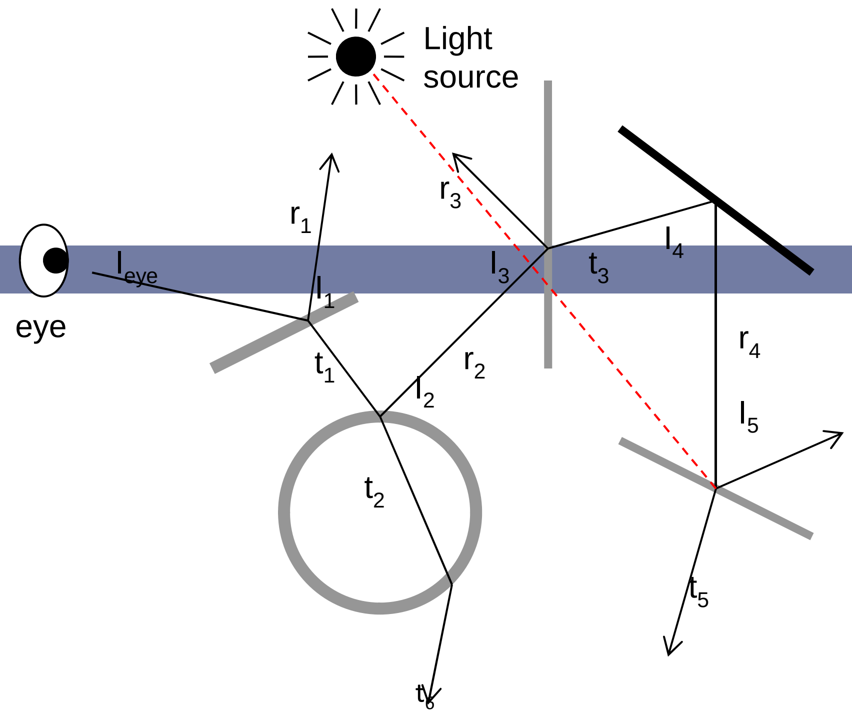
Question #2

- Draw the ray tree according to the following ray in the scene. There are altogether five positions on the objects which require the computation of light intensity. State which of these points need to include all three terms of the Phong illumination model since some of them only need to include the ambient term only. Assume there is no internal reflection in the sphere.

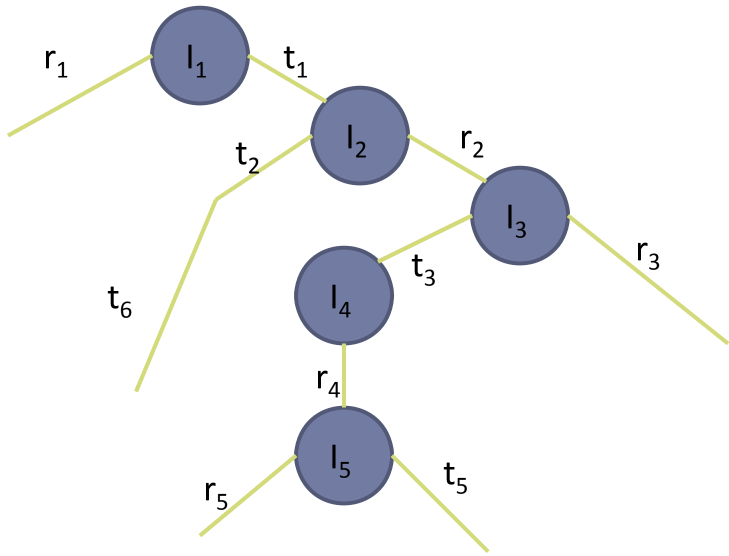


Question #2

- We need to include all the three terms in the Phong model for I_1, I_2, and I_3 only.

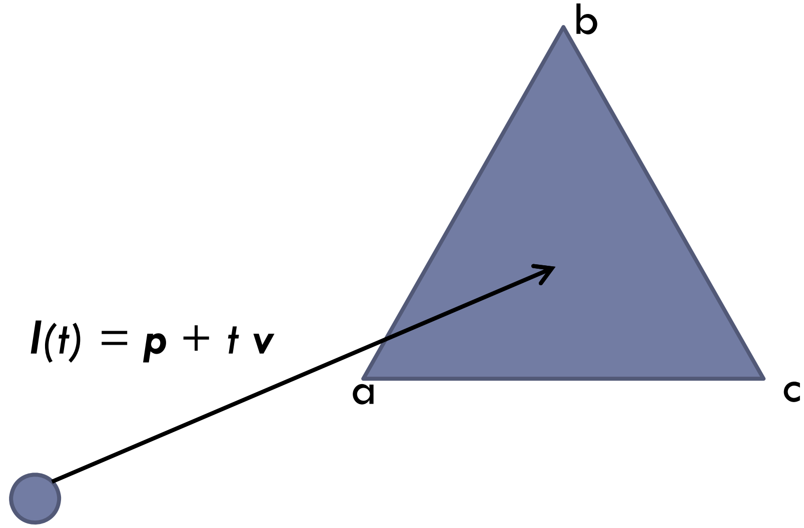


- Because for I_4 and I_5 The shadow ray intersects some objects in the scene. Therefore I_p is not in the Phong equation



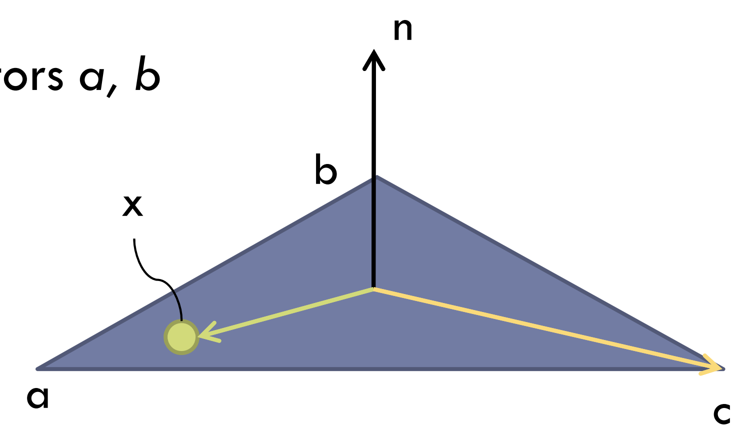
Question #3

- Given three vertices $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ in the space, they form a triangle *abc*. A light ray, $\boldsymbol{l}(t)$, shooting from a point $\boldsymbol{p}$ with direction $\boldsymbol{v}$ can be represented by the line $\boldsymbol{l}(t) = \mathbf{p} + t\,\mathbf{v}$ in which t is a real number. How to determine if the light ray $\boldsymbol{l}(t)$ hits the triangle *abc*?



Question #3

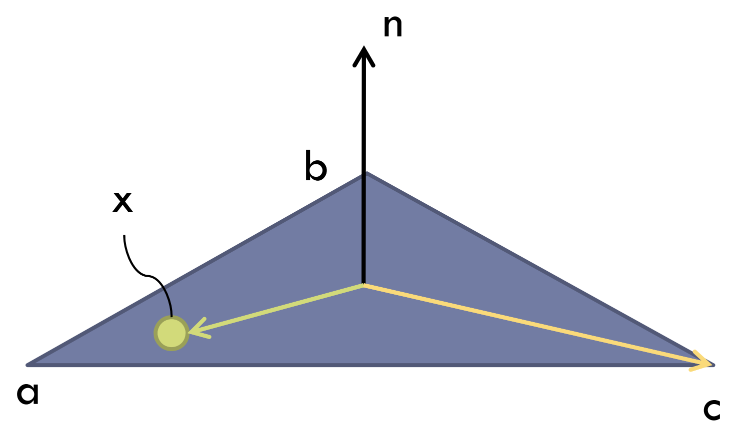
- Triangle is on a plane with point x on the plane satisfying the equation $\langle n,x\rangle=\langle n,q\rangle$ in which q is any point on the plane.
  - $\langle a,b\rangle$ is the dot product of the two vectors a, b
  - *q can be a, b, or c.*



1. Compute this plane
   - Needs normal n
   - $n = ac \times ab$
2. Compute the intersection of the line $l(t)$ and the plane
3. Check if intersection is in *abc*

Question #3

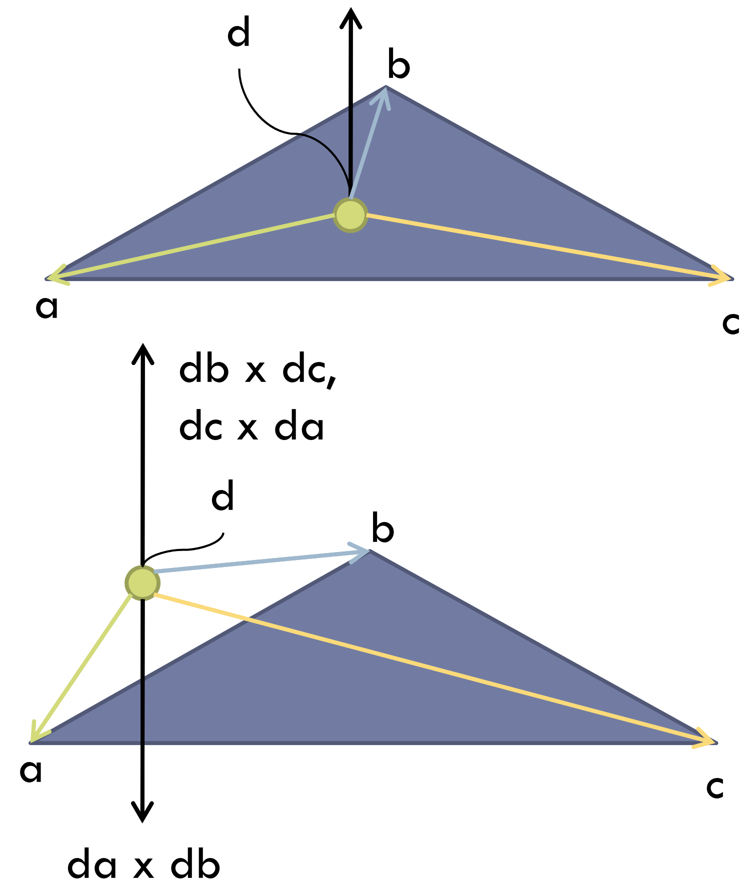
- Step 2: Compute the intersection of the line $l(t)$ and the plane



- Substitute the formula of $l(t)$ in $<n,x>=<n,q>$.
- Let $q = a$, substitute l(t) into x.
- Solving $<n,l(t)> = <n,a>$, find t
- Intersection point obtained by substituting the value of t into $l(t)$.

Question #3

- Step 3: Check if intersection is in *abc*



- let *d* be the intersection point of the light ray and the plane spanned by the triangle *abc*.
- Point d is within the triangle *abc*, if the three cross products *da* x *db*, *db* x *dc*, *dc* x *da* have <u>same directions</u>.