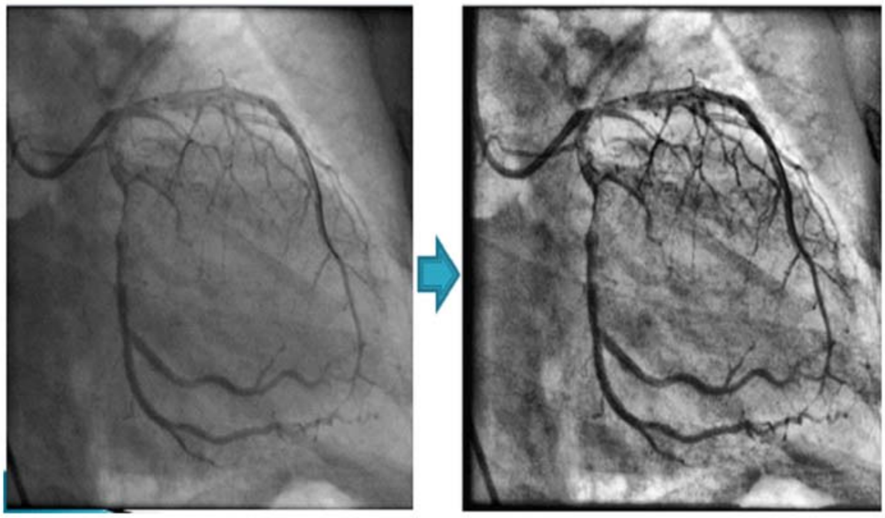
5 – IMAGE ENHANCEMENT (A)

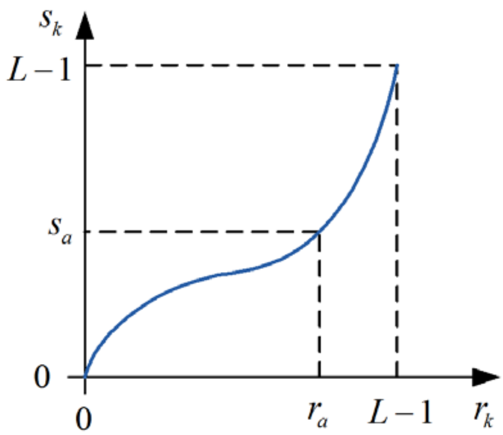
Image enhancement refers to accentuation or sharpening of image features such as edges, boundaries, or contrast to improve the image for display and analysis. The enhancement process does not increase the information content in the image data, but it does increase the dynamic range of the chosen features.

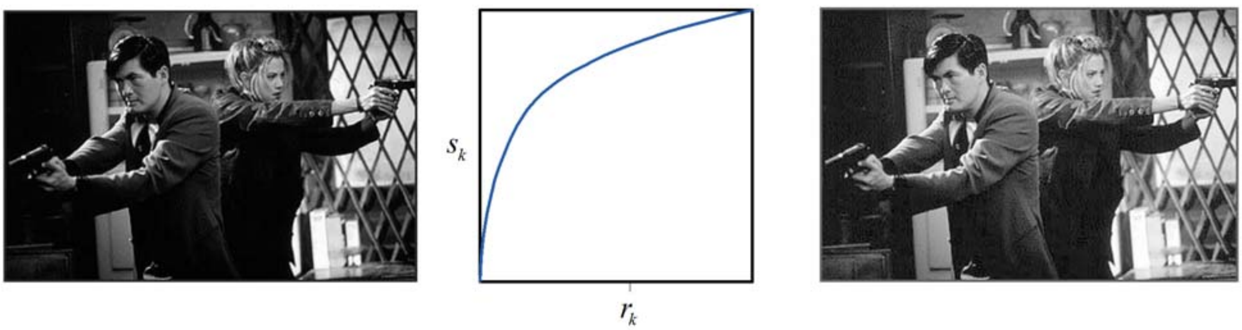


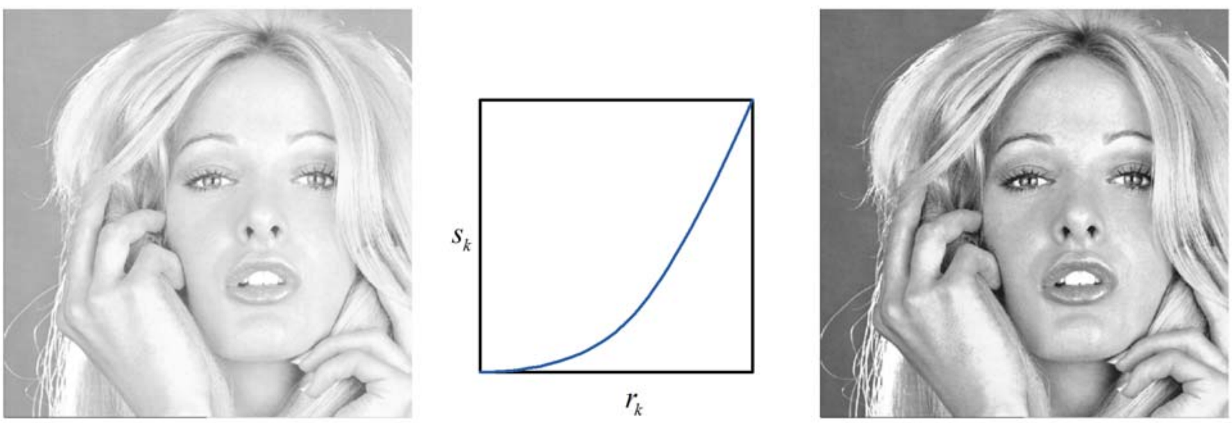
Gray-level Transformation Functions

Image intensities (or gray levels) ares mapped according to a transformation function:

$$
\begin{aligned}
s &= T(r) \quad \text{(continuous case)} \\
s_k &= T(r_k) \quad \text{(discrete)}
\end{aligned}
\tag{1}
$$





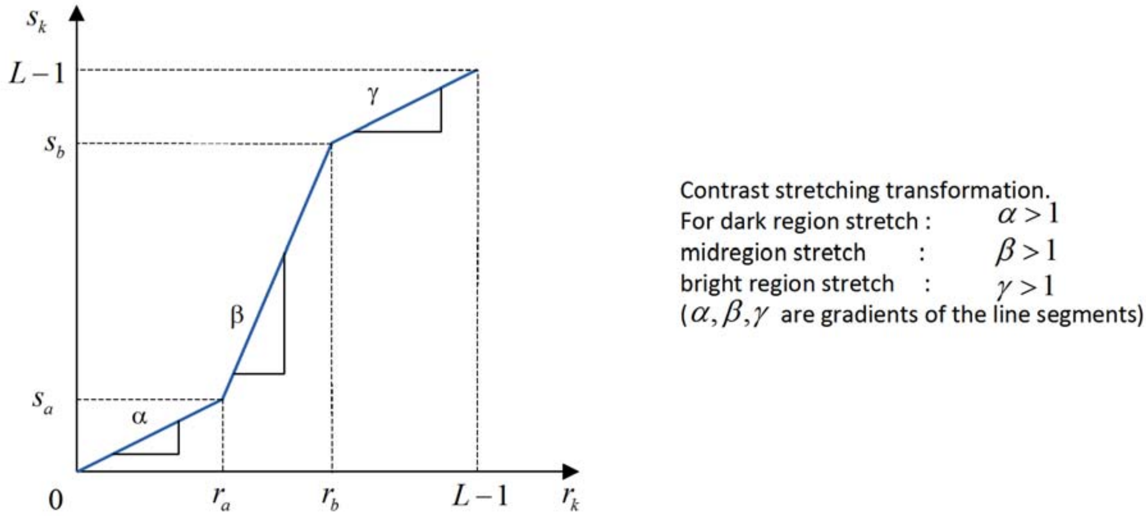


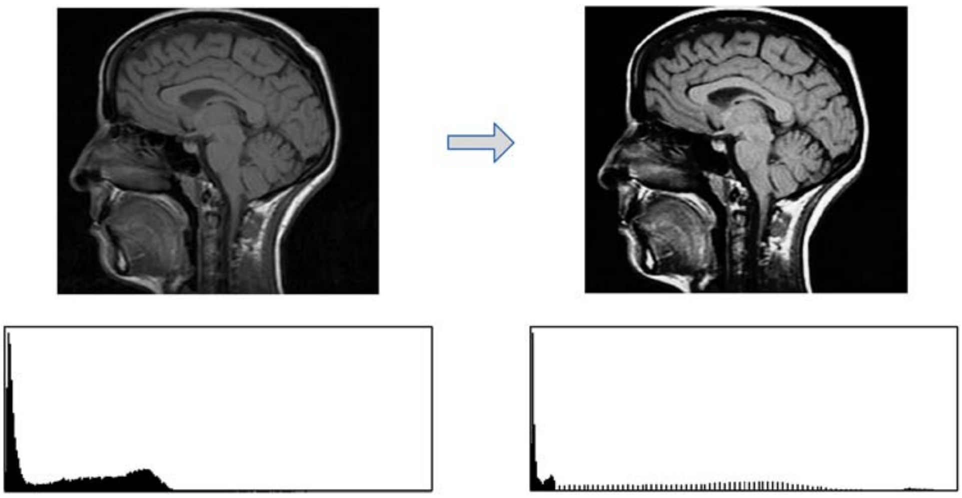
Contrast Stretching

Low-contrast images can result from poor or non-uniform lighting conditions, or due to nonlinearity or small dynamic range of the imaging sensor. A typical linear contrast stetching transformation is

$$s_k = \begin{cases} \alpha r_k & 0 \leq r_k < r_a \\ \beta(r_k - r_a) + s_a & r_a \leq r_k < r_b \\ \gamma(r_k - r_b) + s_b & r_b \leq r_k < L-1 \end{cases} \tag{2}$$

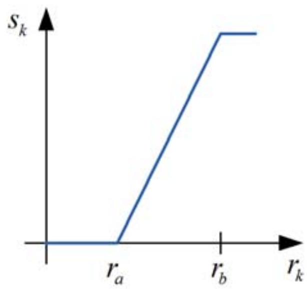
The slope of the transformation is chosen greater than unity in the region of stretch. The transformation parameters may be chosen such that the gray scale interval where the pxiels occur most frequently would be stretched most to improve overall visibility.

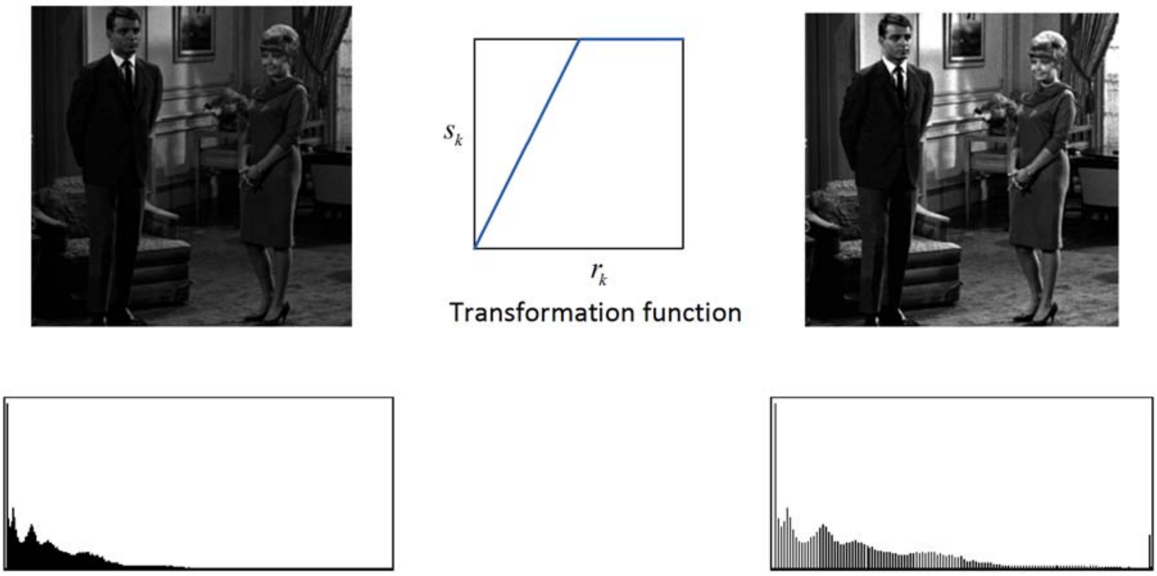




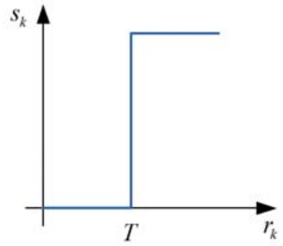
Clipping and Thresholding

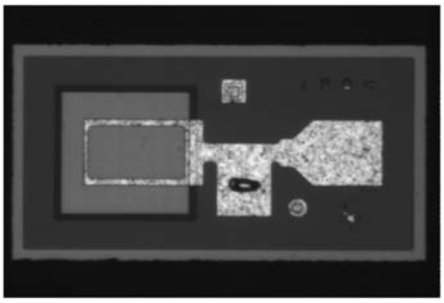
In clipping, α and γ are set to 0 in Eq. (2). This is useful for noise reduction when the input signal is known to lie in the range $[r_a, r_b]$.

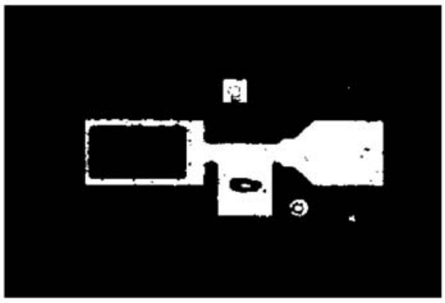


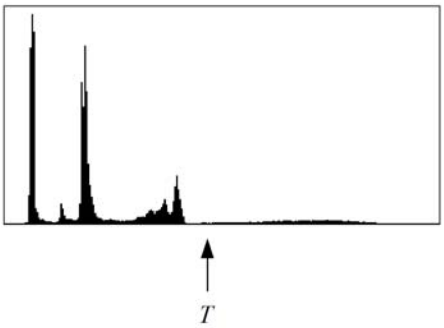


Thresholding is a special case of clipping where $r_a = r_b = T$ and the output becomes binary.







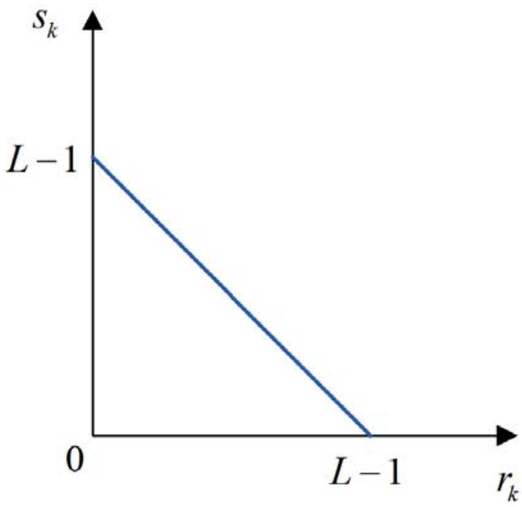


Digital Negative

A negative image can be obtained by reverse ordering of the gray levels according to the transformation

$$s_k = (L - 1) - r_k \tag{3}$$

Digital negatives are useful in the display of some images and in producing negative prints.



Digital negative transformation

Intensity Level Slicing

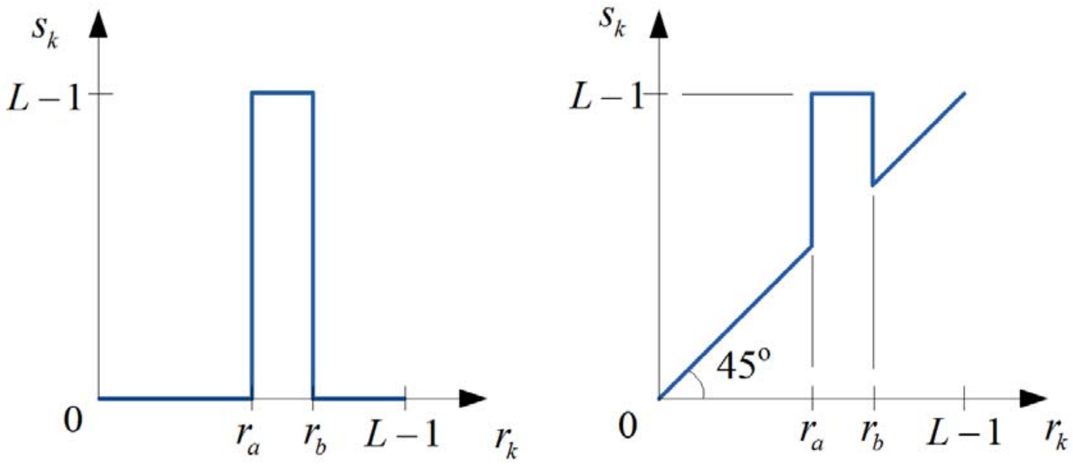
Without background:

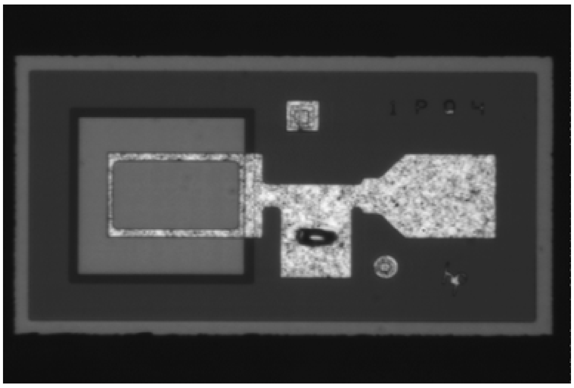
$$s_k = \begin{cases} L-1 & r_a \leq r_k \leq r_b \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

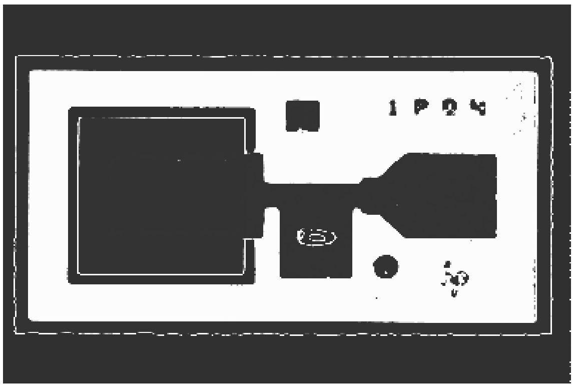
With background:

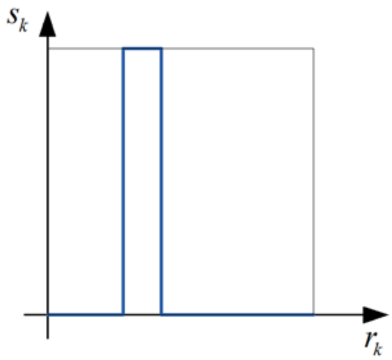
$$s_k = \begin{cases} L-1 & r_a \leq r_k \leq r_b \\ r_k & \text{otherwise} \end{cases} \tag{5}$$

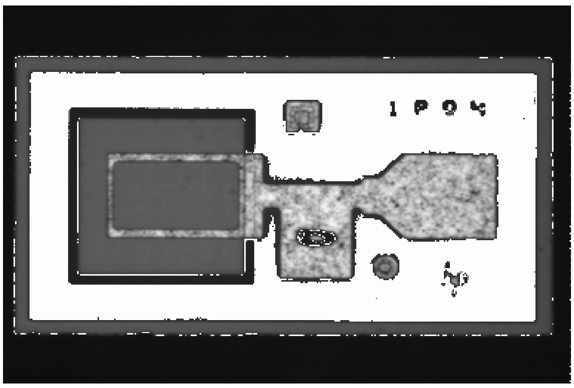
These transformations permit highlighting a specific range of gray levels in an image.

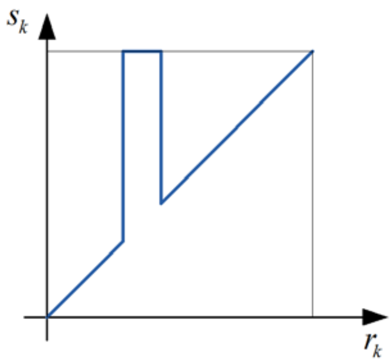







s_k
r_k




s_k
r_k

Log Transformation

The general form is

$$\begin{aligned} s &= c\log_{10}(r), \qquad r \geq 1 \qquad (6) \\ s &= c\log_{10}(1+r), \qquad r \geq 0 \qquad (7) \end{aligned}$$

where c is a scaling constant. For example, we may want $s = 255$ when $r = r_{max}$:

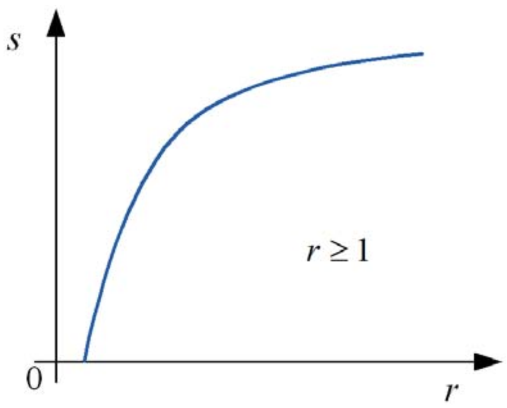
$$255 = c\log_{10}(1 + r_{max}) \qquad (8)$$

or

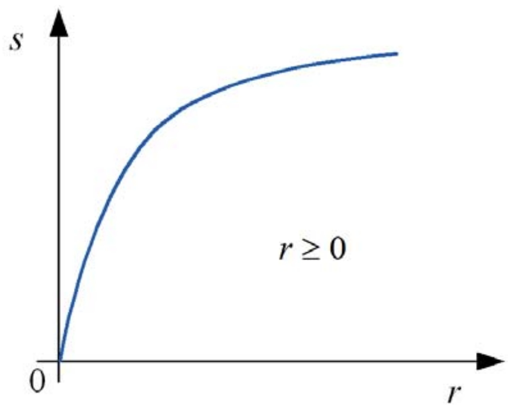
$$c = \frac{255}{\log_{10}(1 + r_{max})} \qquad (9)$$

Note that:

- at low gray levels, the contrast is enhanced.
- at high gray levels, the contrast is reduced.



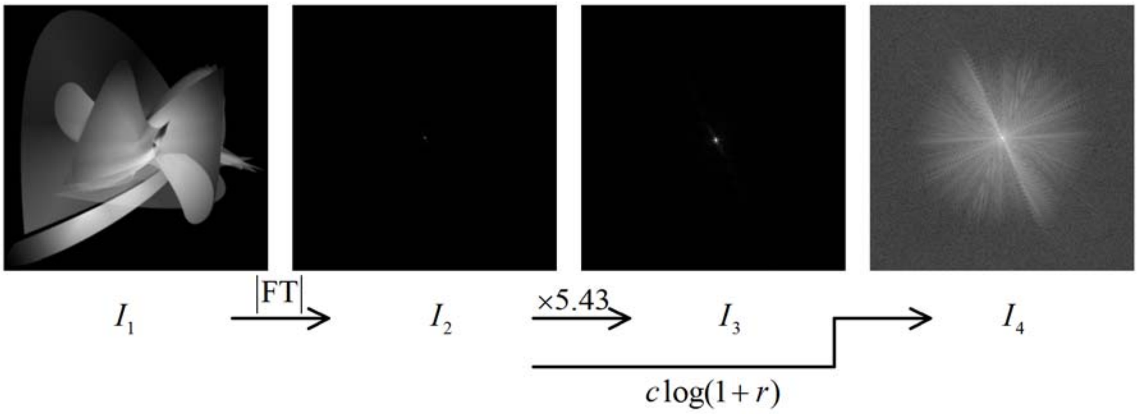
$s = \log_{10}(r)$



$s = \log_{10}(1 + r)$

Example

The log transformation is useful in improving the visual display of the Fourier transform of an image.



- The original image is I_1.
- I_2 shows $|F|$, the Fourier spectrum of I_1; its values range from about 0 to $\alpha = 47$.
- I_3 is obtained by a linear scaling of the values:

$$s = \frac{255}{\alpha} \times r = 5.43\, r$$

  so that the displayed values range from 0 to 255. We see that the central peak is visible but not the details.
- The details will be visible if we apply the log transformation of Eq. (7) to I_2. In this case, the scaling factor c is

$$c = \frac{255}{\log_{10}(1+\alpha)} = 152$$

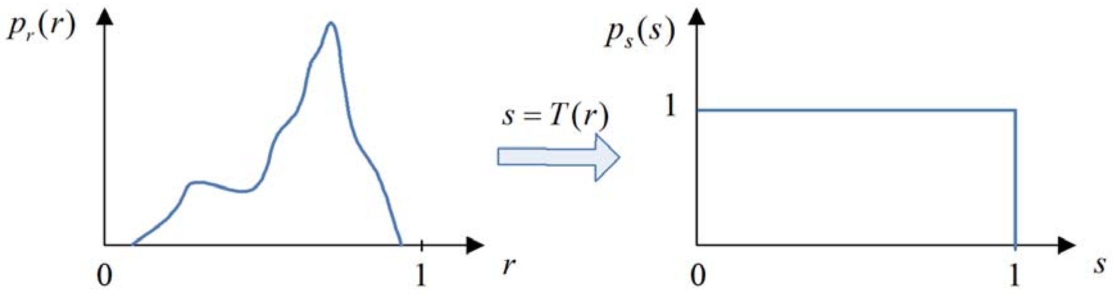
  The transformation function is thus

$$s = 152\ \log_{10}(1+r)$$

  which results in the displayed image I_4.

Histogram Equalization

A *continuous* image function may be characterized by its probability density function (PDF), $p_r(r)$. The variable r represents the intensity of pixels in an image, and is a normalized, continuous variable lying in the range $0 \leq r \leq 1$. The aim of histogram equalization is to enhance an image by equalizing its PDF (PDF is then uniform).



The transformation

$$s = T(r) \tag{10}$$

produces an intensity value s for every intensity value r in the input image. T satisfies the conditions

1. $T(r)$ is single-valued and monotonically increasing in the interval $[0, 1]$.
2. $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$.

Condition 1 preserves the order from black to white in the intensity scale; condition 2 guarantees a mapping that is consistent with the allowed 0 to 1 range of pixel values.

The inverse transformation function is:

$$r = T^{-1}(s) \tag{11}$$

where it is assumed that $T^{-1}(s)$ satisfies the two conditions above.

The intensity variables r and s are random quantities in the interval $[0, 1]$ which can be characterized by their probability density functions (PDFs) $p_r(r)$ and $p_s(s)$.

From probability theory, it follows that if $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies condition 1, then the PDFs $p_r(r)$ and $p_s(s)$ are related by

$$p_s(s)ds = p_r(r)dr \tag{12}$$

Since we require $p_s(s) = 1$,

$$\begin{aligned} ds &= p_r(r)dr & (13) \\ s &= \int p_r(r)dr & (14) \\ &= \int_0^r p_r(w)dw \qquad 0 \leq r \leq 1 & (15) \end{aligned}$$

where w is a dummy variable of integration. The rightmost side of the equation is the cumulative distribution function (CDF) of r. This transformation function satisfies the two conditions stated above.

Hence, the required transformation function is

$$s = T(r) = \int_0^r p_r(w)\, dw \qquad 0 \leq r \leq 1 \tag{16}$$

i.e.,

$$p_r(r) \xrightarrow{T(r)} p_s(s)$$

where $p_s(s) = 1$. Note that the transformation function given by Eq. (16) yields transformed intensities that always have a flat PDF, *independent* of the shape of $p_r(r)$.

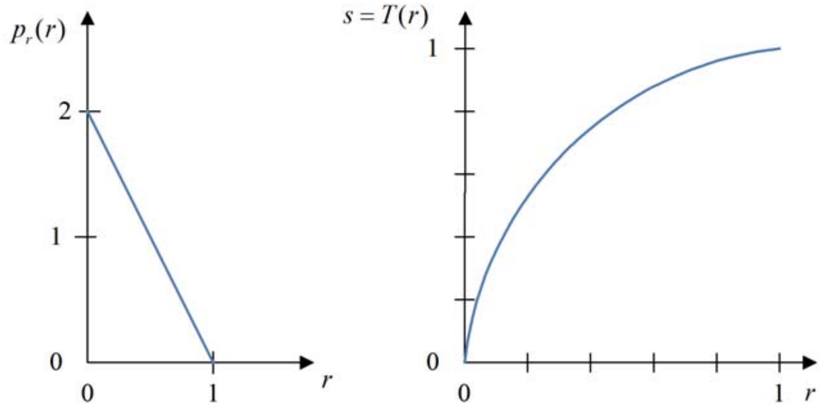
Example:

Suppose that $p_r(r)$ is given by

$$p_r(r) = \begin{cases} -2r+2 & 0 \le r \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Substitution of this expression in Eq. (16) yields

$$\begin{aligned} s &= T(r) = \int_0^r (-2w+2)\,dw \\ &= -r^2 + 2r. \end{aligned}$$



The PDF of s is obtained by using Eq. (12):

$$\begin{aligned} p_s(s) &= p_r(r)\,\frac{dr}{ds} \\ &= p_r(r) \times \frac{1}{ds/dr} \\ &= (-2r+2) \times \frac{1}{(-2r+2)} \\ &= 1 \qquad 0 \le s \le 1, \end{aligned}$$

We obtain the inverse transformation $r = T^{-1}(s)$ by solving for r in terms of s:

$$r = T^{-1}(s) = 1 \pm \sqrt{1-s}.$$

Since r lies in the interval $[0, 1]$, only the solution

$$r = T^{-1}(s) = 1 - \sqrt{1-s}$$

is valid.

We deal with **discrete variables** in digital image processing.

$$p_r(r_k) = \frac{n_k}{n} \tag{17}$$

where $0 \leq r_k \leq 1$ and $k = 0, 1, 2, \ldots, L-1$. In this equation

L	the number of discrete intensity levels
$p_r(r_k)$	the probability of intensity r_k
n_k	the number of times this intensity appears in the image
n	the total number of pixels in the image.

The plot of $p_r(r_k)$ versus r_k is the normalised histogram.

The discrete form of Eq. (16) is given by

$$\begin{aligned} s_k &= T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n} && (18) \\ &= \sum_{j=0}^{k} p_r(r_j) && (19) \end{aligned}$$

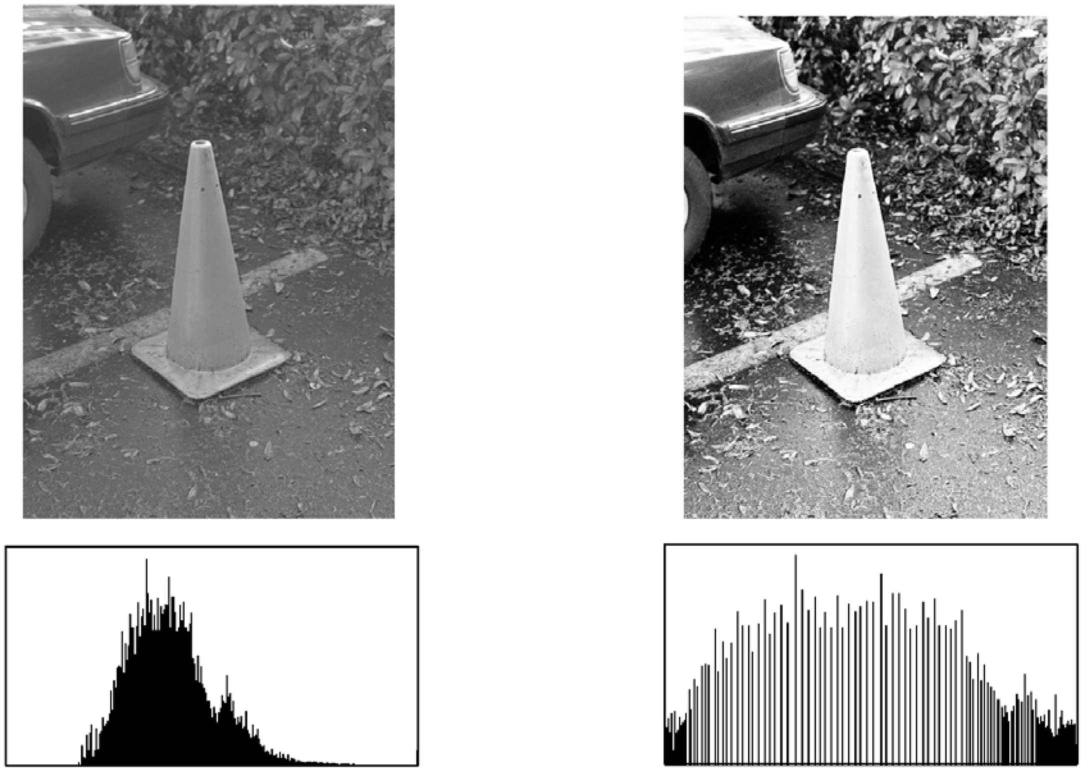
for $0 \leq r_k \leq 1$ and $k = 0, 1, 2, \ldots, L-1$. We have:

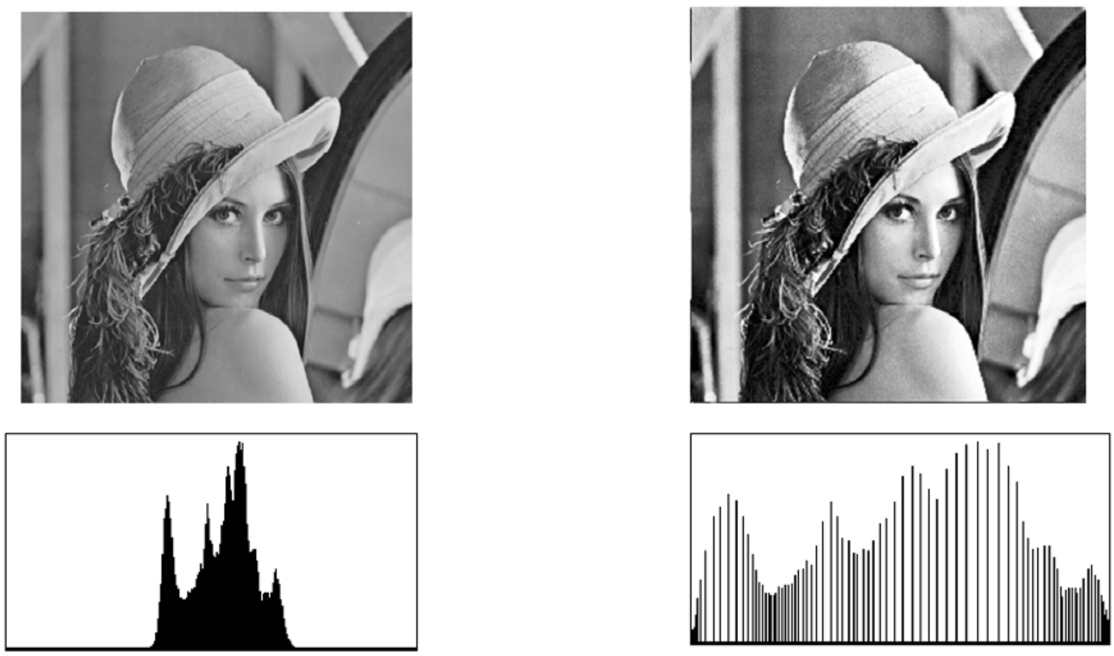
$$\begin{aligned} s_0 &= T(r_0) &= \textstyle\sum_{j=0}^{0} \frac{n_j}{n} = p_r(r_0) \\ s_1 &= T(r_1) &= \textstyle\sum_{j=0}^{1} \frac{n_j}{n} = p_r(r_0) + p_r(r_1) \\ \ldots \end{aligned}$$

The inverse discrete transformation is given by

$$r_k = T^{-1}(s_k) \qquad 0 \leq s_k \leq 1 \tag{20}$$

where both $T(r_k)$ and $T^{-1}(s_k)$ are assumed to satisfy conditions 1 and 2 above.

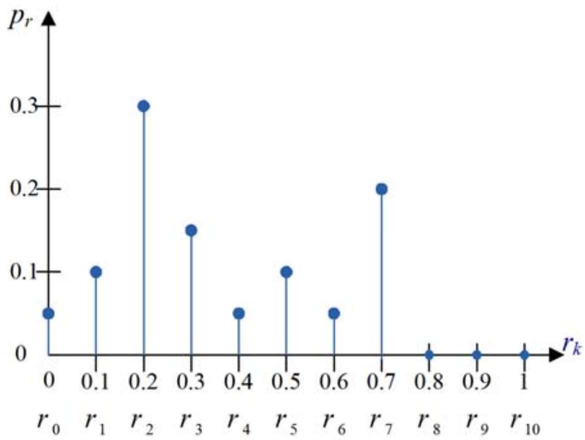




Example

Consider an 11-level input image of size 100×100.

k	r_k	n_k	$p_r(r_k) = n_k/n$
0	0	500	0.05
1	0.1	1000	0.10
2	0.2	3000	0.30
3	0.3	1500	0.15
4	0.4	500	0.05
5	0.5	1000	0.10
6	0.6	500	0.05
7	0.7	2000	0.20
8	0.8	0	0
9	0.9	0	0
10	1	0	0



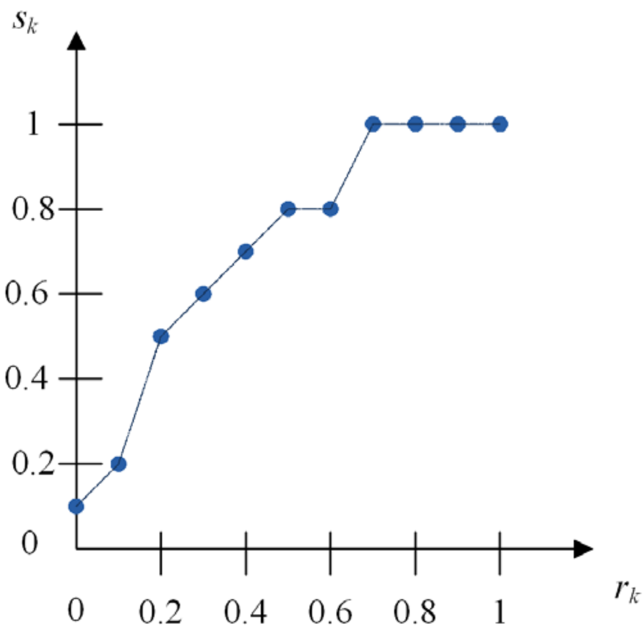
The transformation function is obtained from

$$
\begin{aligned}
s_k &= T(r_k) = \sum_{j=0}^{k} p_r(r_j) \\
r_0 \to s_0 &= T(r_0) = \sum_{j=0}^{0} p_r(r_j) = p_r(r_0) = 0.05 \to 0.1 \\
r_1 \to s_1 &= T(r_1) = \sum_{j=0}^{1} p_r(r_j) = p_r(r_0) + p_r(r_1) = 0.05 + 0.10 = 0.15 \to 0.2
\end{aligned}
$$

$$
\begin{aligned}
s_2 &= 0.45 \to 0.5 \\
s_3 &= 0.60 \to 0.6 \\
s_4 &= 0.65 \to 0.7 \\
s_5 &= 0.75 \to 0.8 \\
s_6 &= 0.80 \to 0.8 \\
s_7 &= 1.00 \to 1 \\
s_8 &= 1.00 \to 1 \\
s_9 &= 1.00 \to 1 \\
s_10 &= 1.00 \to 1
\end{aligned}
$$

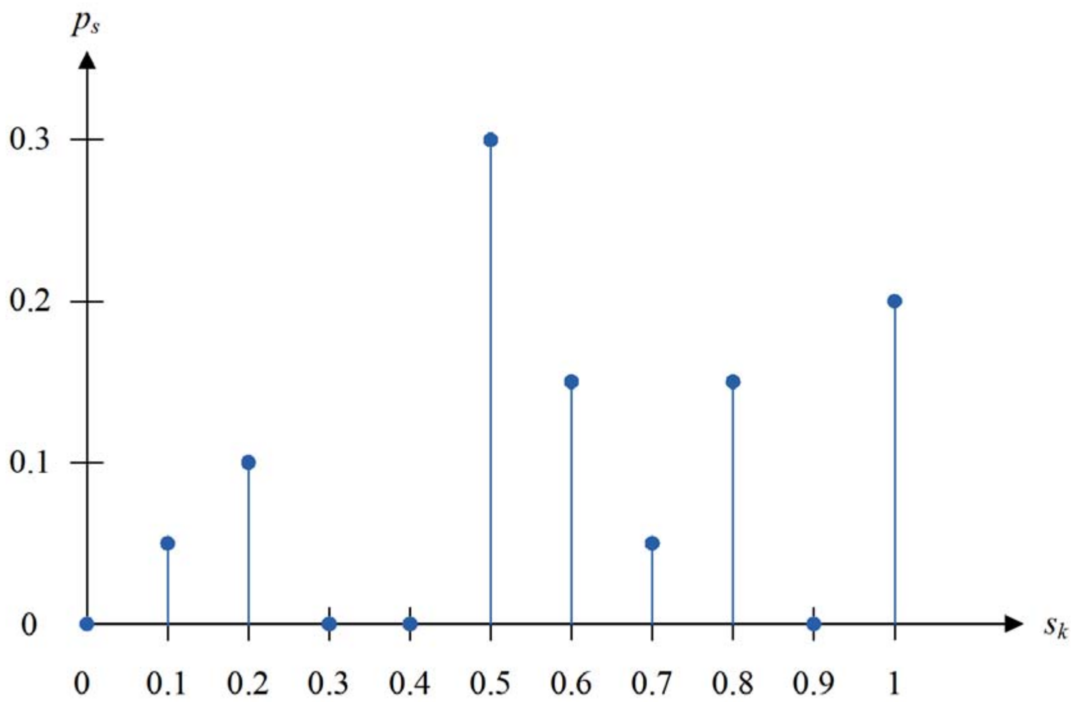
Note: The transformed values must be assigned to the closest valid level.

k	$p_r(r_k)$	r_k		s_k	$p_s(s_k)$
0	0.05	0	$\rightarrow$	0.1	.05
1	0.10	0.1	$\rightarrow$	0.2	.10
2	0.30	0.2	$\rightarrow$	0.5	.30
3	0.15	0.3	$\rightarrow$	0.6	.15
4	0.05	0.4	$\rightarrow$	0.7	.05
5	0.10	0.5	$\rightarrow$	0.8	.15
6	0.05	0.6	$\rightarrow$	0.8	
7	0.20	0.7	$\rightarrow$	1	.20
8	0	0.8	$\rightarrow$	1	
9	0	0.9	$\rightarrow$	1	
10	0	1	$\rightarrow$	1	



Transformation function

s_k	$p_s(s_k)$
0	0
0.1	$p_r(r_0) = 0.05$
0.2	$p_r(r_1) = 0.10$
0.3	0
0.4	0
0.5	$p_r(r_2) = 0.3$
0.6	$p_r(r_3) = 0.15$
0.7	$p_r(r_4) = 0.05$
0.8	$p_r(r_5) + p_r(r_6) = 0.15$
0.9	0
1	$p_r(r_7) + p_r(r_8) + p_r(r_9) + p_r(r_{10}) = 0.2$



Histogram Specification

It is sometimes desirable to specify particular histograms capable of highlighting certain gray-level ranges in an image.

Let $p_r(r)$ and $p_z(z)$ be the original and desired PDFs, respectively. Suppose that a given image is first histogram equalized using Eq. (16):

$$s = T(r) = \int_0^r p_r(w)\, dw. \tag{21}$$

If the desired image were available, its levels could also be equalized by using the transformation function

$$v = G(z) = \int_0^z p_z(w)\, dw. \tag{22}$$

Note that $p_s(s)$ and $p_v(v)$ are *identical* uniform densities. Thus, instead of using v in the inverse process, we use the uniform levels s obtained from the original image, and the resulting levels, $z = G^{-1}(s)$, would have the desired PDF.

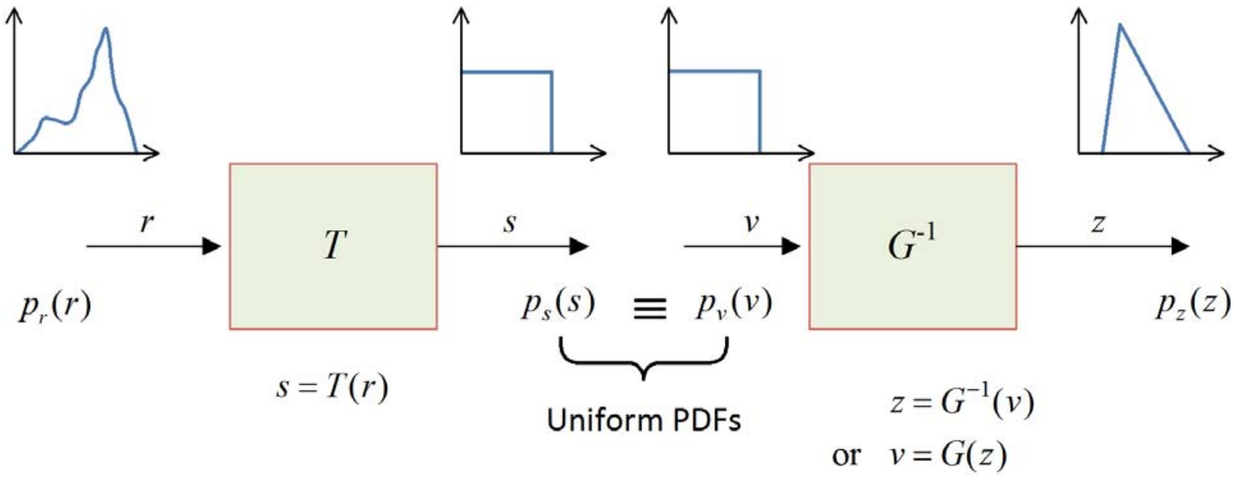
The procedure is summarized as follows:

1. Equalize the levels of the original image using Eq. (16).
2. Specify the desired density function and obtain $G(z)$ using Eq. (22).
3. Apply the inverse transformation function, $z = G^{-1}(s)$, to the levels obtained in step 1.

The two transformations required for histogram specification can be combined into a single transformation:

$$z = G^{-1}(s) = G^{-1}[T(r)] \tag{17}$$

which relates r to z.



$$z = G^{-1}(v) = G^{-1}(s) = G^{-1}[T(r)]$$

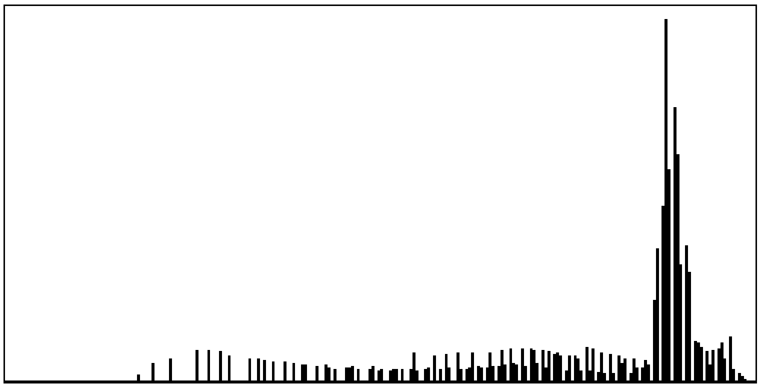
The **discrete formulation** is as follows:

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) \quad (23)$$

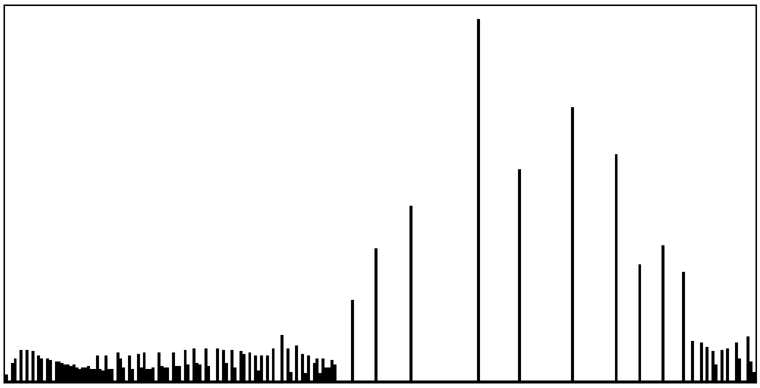
$$G(z_i) = \sum_{j=0}^{i} p_z(z_j) \quad (24)$$

$$z_i = G^{-1}(s_i) \quad (25)$$

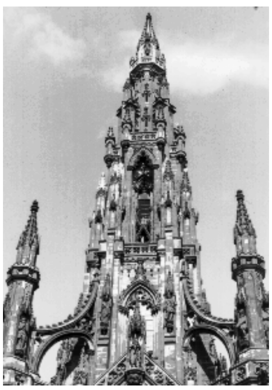
where $p_r(r_j)$ is computed from the input image and $p_z(z_j)$ is specified.

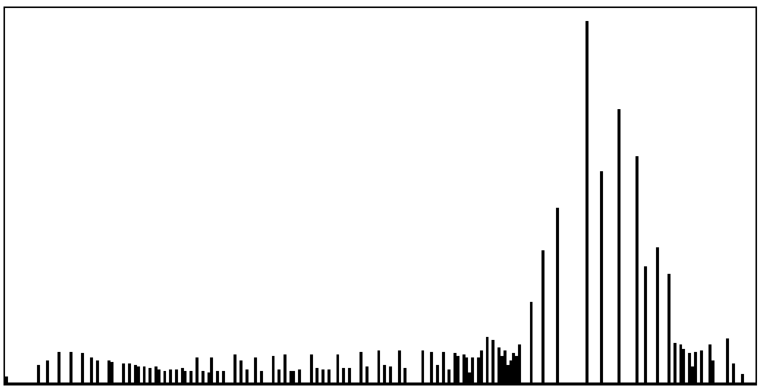


Original



Histogram-equalised

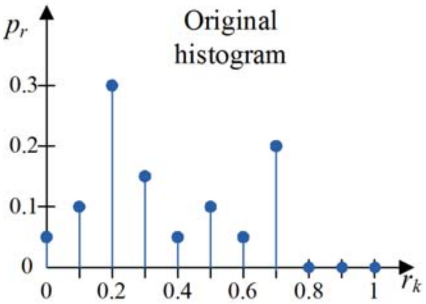




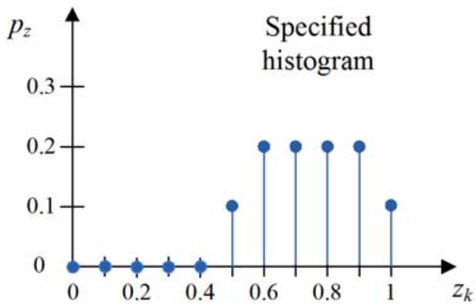
After transformation

Example

Consider a 100×100 11-level image with histogram $p_r(r_k)$. It is desired to transform the image to one with histogram $p_z(z_k)$.



r_k	$p_r(r_k)$		z_k	$p_z(z_k)$
0	0.05		0	0
0.1	0.10		0.1	0
0.2	0.30		0.2	0
0.3	0.15		0.3	0
0.4	0.05	⇨	0.4	0
0.5	0.10		0.5	0.1
0.6	0.05		0.6	0.2
0.7	0.20		0.7	0.2
0.8	0		0.8	0.2
0.9	0		0.9	0.2
1	0		1	0.1



Step 1: *Compute histogram-equalisation mappings*

(a) $r \rightarrow s$

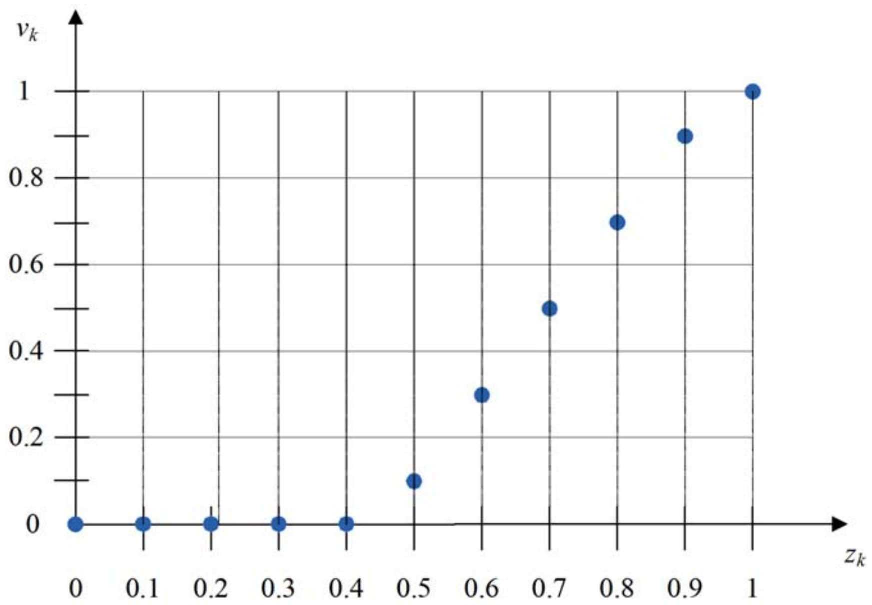
k	$p_r(r_k)$	r_k		s_k	$p_s(s_k)$
0	0.05	0	$\rightarrow$	0.1	.05
1	0.10	0.1	$\rightarrow$	0.2	.10
2	0.30	0.2	$\rightarrow$	0.5	.30
3	0.15	0.3	$\rightarrow$	0.6	.15
4	0.05	0.4	$\rightarrow$	0.7	.05
5	0.10	0.5	$\rightarrow$	0.8	.15
6	0.05	0.6	$\rightarrow$	0.8	
7	0.20	0.7	$\rightarrow$	1	.20
8	0	0.8	$\rightarrow$	1	
9	0	0.9	$\rightarrow$	1	
10	0	1	$\rightarrow$	1	

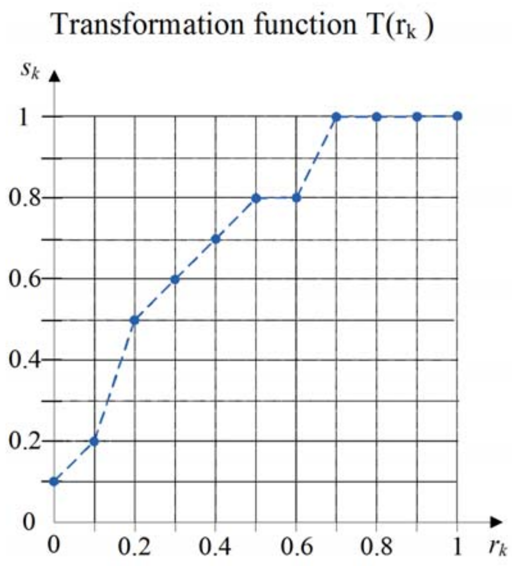
(b) $z \rightarrow v$

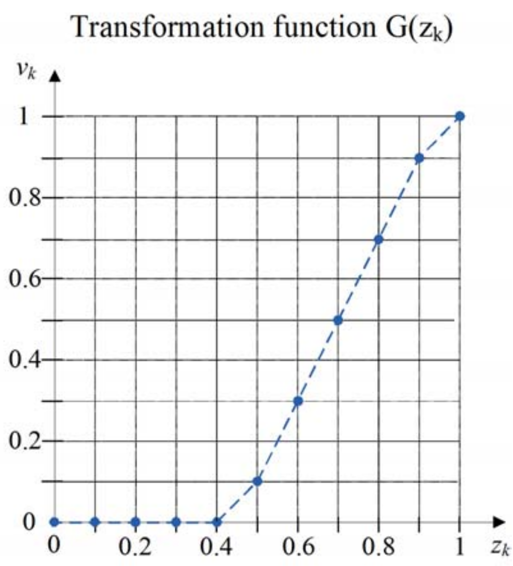
Compute the transformation function

$$v_k = G(z_k) = \sum_{j=0}^{k} p_z(z_j)$$

$$\begin{array}{lllllll}
z_0 & \longrightarrow & v_0 & = & G(z_0) & = & 0 \\
z_1 & \longrightarrow & v_1 & = & G(z_1) & = & 0 \\
z_2 & \longrightarrow & v_2 & = & G(z_2) & = & 0 \\
z_3 & \longrightarrow & v_3 & = & G(z_3) & = & 0 \\
z_4 & \longrightarrow & v_4 & = & G(z_4) & = & 0 \\
z_5 & \longrightarrow & v_5 & = & G(z_5) & = & 0.1 \\
z_6 & \longrightarrow & v_6 & = & G(z_6) & = & 0.3 \\
z_7 & \longrightarrow & v_7 & = & G(z_7) & = & 0.5 \\
z_8 & \longrightarrow & v_8 & = & G(z_8) & = & 0.7 \\
z_9 & \longrightarrow & v_9 & = & G(z_9) & = & 0.9 \\
z_{10} & \longrightarrow & v_{10} & = & G(z_{10}) & = & 1
\end{array}$$

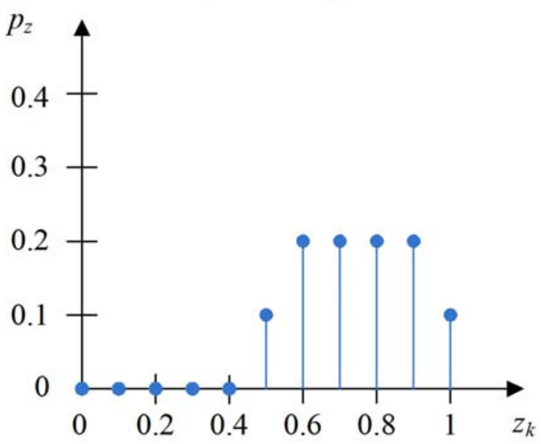




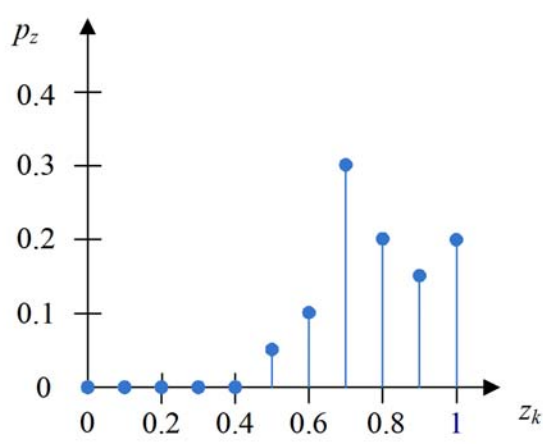


<u>Step 2</u>: *Compute mappings* $r \to s$, $v \to z$

$p_r(r_k)$	r_k		s_k		z_k	$p_z(z_k)$
0.05	0	→	0.1	→	0.5	0.05
0.10	0.1	→	0.2	→	0.6	0.10
0.30	0.2	→	0.5	→	0.7	0.30
0.15	0.3	→	0.6	→	0.8	0.20
0.05	0.4	→	0.7	→	0.8	
0.10	0.5	→	0.8	→	0.9	0.15
0.05	0.6					
0.20	0.7	→	1	→	1	0.2
0	0.8					
0	0.9					
0	1					



Specified histogram



Resulting histogram