EE2011 Engineering Electromagnetics - Part CXD
Tutorial 7 - Solutions

Q1

$$3\delta_s = 1.2 \text{ km} = 1200 \text{ m}$$
$$\delta_s = 400 \text{ m}.$$

Hence,

$$\alpha = \frac{1}{\delta_s} = \frac{1}{400} = 2.5 \times 10^{-3} \quad \text{(Np/m)}.$$

Since $\varepsilon''/\varepsilon' \ll 1$, we can use low-loss approximation:

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{2\pi f \varepsilon_r'' \varepsilon_0}{2\sqrt{\varepsilon_r'}\sqrt{\varepsilon_0}}\sqrt{\mu_0} = \frac{\pi f \varepsilon_r''}{c\sqrt{\varepsilon_r}} = \frac{\pi f \times 10^{-2}}{3 \times 10^8 \sqrt{3}} = 6f \times 10^{-11} \text{Np/m}.$$

For $\alpha = 2.5 \times 10^{-3} = 6f \times 10^{-11}$,

$$f = 41.6 \text{ MHz}.$$

Since α increases with increasing frequency, the useable frequency range is

$$f \leq 41.6 \text{ MHz}.$$

Q2

(i) It is easy to see that $\omega = 10^{10}\pi$

$\frac{\sigma}{\omega\varepsilon} = \frac{4}{10^{10}\pi \times 80 \times 8.854 \times 10^{-12}} = 0.18$. In this case, the ratio $\frac{\sigma}{\omega\varepsilon}$ is not small enough to warrant the use of the low-loss dielectric formulas. Therefore the full formulas for α, β, and η_c have to be used.

$$\begin{aligned}
k &= \omega\sqrt{\mu_0\varepsilon_0\left(\varepsilon_r' - j\varepsilon_r''\right)} \\
&= \omega\sqrt{\mu_0\varepsilon_0\left(\varepsilon_r' - j\frac{\sigma}{\omega\varepsilon_0}\right)} \\
&= 941.03 - j83.9 \\
&\approx 300\pi - j84 = \beta - j\alpha
\end{aligned}$$

$$\begin{aligned}
\eta &= \sqrt{\frac{\mu_0}{\varepsilon_0\left(\varepsilon_r' - j\varepsilon_r''\right)}} = \sqrt{\frac{\mu_0}{\varepsilon_0\left(\varepsilon_r' - j\frac{\sigma}{\omega\varepsilon_0}\right)}} \\
&= 41.65 + j3.7136 = 41.8e^{j0.0283\pi} \quad (\Omega)
\end{aligned}$$

$$u_p = \omega/\beta = 33.3 \times 10^6 \quad \text{(m/s)},$$

$$\lambda = 2\pi/\beta = 0.67 \quad \text{(cm)},$$

$$\delta = 1/\alpha = 1.19 \quad \text{(cm)}.$$

(ii) $\mathbf{H}(y,t) = \hat{\mathbf{x}} H_0 e^{-\alpha y} \cos(\omega t - \beta y + \phi_0)$ A/m

where, $\alpha = 84$, $H_0 = 0.1$

Amplitude at y: $H_0 e^{-\alpha y} = 0.1 e^{-\alpha y} = 0.01$

$$\Rightarrow \quad e^{-\alpha y} = \frac{1}{10}, \quad y = \frac{1}{\alpha} \ln 10 = 2.74 \quad \text{(cm).}$$

(iii) Since the wave propagates in the +*y* direction, the H-field takes the form,

$$\mathbf{H}(y,t) = \hat{\mathbf{x}} H_0 e^{-\alpha y} \cos(\omega t - \beta y + \phi_0) \text{ A/m}$$

At $y = 0$, the H-field is given by

$\hat{\mathbf{x}}\, 0.1 \sin(10^{10} \pi t - \pi/3) = \hat{\mathbf{x}}\, 0.1 \cos(10^{10} \pi t - \pi/3 - \pi/2)$.

We easily find that $H_0 = 0.1$, $\phi_0 = -5\pi/6$, and $\omega = 10^{10}\pi$. Thus, we have the phasor form of **H**(*y*):

$$\mathbf{H}(y) = \hat{\mathbf{x}} 0.1 e^{-\alpha y} e^{-j\beta y} e^{-j5\pi/6} \text{ A/m}$$

The phasor form of **E**(*y*) is then:

$$\begin{aligned}\mathbf{E}(y) &= \eta_c \mathbf{H}(y) \times \hat{\mathbf{y}} \\ &= \hat{\mathbf{x}} \times \hat{\mathbf{y}} 41.8 e^{j0.0283\pi} 0.1 e^{-\alpha y} e^{-j\beta y} e^{-j5\pi/6} \\ &= \hat{\mathbf{z}} 4.18 e^{-\alpha y} e^{-j\beta y} e^{-j2.5291} \text{ V/m}\end{aligned}$$

(iv)

$$\begin{aligned}\mathbf{H}(0.5,t) &= \hat{\mathbf{x}}\, 0.1\, e^{-84\times 0.5} \sin(10^{10}\pi t - 300\pi \times 0.5 - \pi/3) \\ &= \hat{\mathbf{x}}\, 5.75 \times 10^{-20} \sin(10^{10}\pi t - 150\pi - \pi/3) \\ &= \hat{\mathbf{x}}\, 5.75 \times 10^{-20} \sin(10^{10}\pi t - \pi/3) \quad \text{A/m}\end{aligned}$$

The instantaneous form of **E**(*y*,*t*) is:

$$\begin{aligned}\mathbf{E}(y,t) &= \mathrm{Re}\{\mathbf{E}(y) e^{j\omega t}\} \\ &= \hat{\mathbf{z}} 4.18 e^{-\alpha y} \cos(\omega t - \beta y - 2.5291) \\ &= \hat{\mathbf{z}} 4.18 e^{-\alpha y} \sin(\omega t - \beta y - 0.9583) \text{ V/m}\end{aligned}$$

Hence,

$$\begin{aligned}\mathbf{E}(0.5,t) &= \hat{\mathbf{z}} 4.18 e^{-\alpha \times 0.5} \sin(10^{10}\pi t - 300\pi \times 0.5 - 0.9583) \\ &= \hat{\mathbf{z}} 4.18 e^{-84\times 0.5} \sin(10^{10}\pi t - 150\pi - 0.9583) \\ &= \hat{\mathbf{z}} 2.53 \times 10^{-18} \sin(10^{10}\pi t - 0.9583) \text{ V/m}\end{aligned}$$

Q3

(i) The conduction and displacement current densities are given by $\mathbf{J_c} = \sigma \mathbf{E}$ and

$$\mathbf{J_D} = j\omega\varepsilon'\mathbf{E}.$$

Thus

$$\frac{|\mathbf{J_c}|}{|\mathbf{J_D}|} = \frac{\sigma}{\omega\varepsilon'} = \frac{\sigma}{\omega\varepsilon_r'\varepsilon_0} = 108 \gg 1$$

Since $\tan\delta = \frac{\sigma}{\omega\varepsilon} \gg 1$, this is a good conductor.

(ii) $\alpha \approx \sqrt{\frac{\omega\mu\sigma}{2}} = 0.435$ N

The skin depth is $\delta = \frac{1}{\alpha} = 2.3$ m.

Q4

(i) Since $f = 5\times10^6$ Hz, we obtain $\omega = 10^7\pi$ rad/s.

Here $\frac{\sigma}{\omega\varepsilon'} \approx 200 >> 1$. We may therefore approximate seawater as a good conductor at this frequency.

$$\alpha = \beta = \sqrt{\pi f \mu_r \mu_0 \sigma} = 8.89 \text{ Np/m or rad/m}$$

$$\eta = (1+j)\sqrt{\frac{\pi f \mu_r \mu_0}{\sigma}} = \frac{\pi}{\sqrt{2}}(1+j) = \pi e^{j\pi/4} \quad \Omega$$

$$u_p = \frac{\omega}{\beta} = 3.53\times10^6 \text{ m/s}$$

$$\lambda = \frac{2\pi}{\beta} = 0.707 \text{ m}$$

$$\delta = 1/\alpha = 0.112 \text{ m}$$

(ii) $\exp(-2\alpha z_1) = \frac{10^{-4}}{1}$ $\qquad z_1 = \frac{1}{2\alpha}\ln 10^4 = 0.518$ m