Solutions to Tutorial 5

4.79 (a) $P($ 2 or more defects $) = f(2) + f(3) = .03 + .01 = .04.$

(b) 0 is more likely since its probability $f(0) = .89$ is much larger than that of its complement $1 - .89 = .11.$

4.80 (a) $P($ requests exceed number of rooms $) = f(3) + f(4) = .25 + .08 = .33.$

(b) $P($ requests less than number of rooms $) = f(0) + f(1) = .07 + .15 = .22.$

(c) If 1 room is added, to make a total of 3 rooms, $P($ requests exceed number of rooms $) = f(4) = .08$ which satisfies the requirement.

4.81 (a) $\mu = 0 \times .07 \ + \ 1 \times .15 \ + \ 2 \times .45 \ + \ 3 \times .25 \ + \ 4 \times .08 \ = \ 2.12.$

(b) We first calculate

$$0^2 \times .07 \ + \ 1^2 \times .15 \ + \ 2^2 \times .45 \ + \ 3^2 \times .25 \ + \ 4^2 \times .08 \ = \ 5.48$$

so variance $= 5.48 - (2.12)^2 = .9856$

(c) standard deviation $= \sqrt{.9856} = .9928$ rooms

4.84 This probability is given by geometric distribution. The probability of a miss is $p = 1 - .90 = .10$.

$$g(7; .05) = (.9)^6(.1) = .048.$$

4.86 (a) $b(16; 18, .85) = B(16; 18, .85) - B(15; 18, .85) = .7759 - .5203 = .2556$

(b) $1 - B(13; 18, .85) = 1 - .1206 = .8794$

(c) $1 - B(15; 18, .85) = 1 - .5203 = .4797$

4.88 (a) The mean is given by:

$$\mu = 0(.216) + 1(.432) + 2(.288) + 3(.064) = 1.2.$$

(b) Using the special formula for the binomial mean

$$\mu = np = 3(.4) = 1.2$$

4.89 (a) The variance is given by:

$$\begin{aligned} \sigma^2 &= (0 - 1.2)^2(.216) + (1 - 1.2)^2(.432) + (2 - 1.2)^2(.288) + (3 - 1.2)^2(.064) \\ &= .72 \end{aligned}$$

(b) Using the special formula for the binomial variance

$$\sigma^2 = np(1 - p) = 3(.4)(.6) = .72$$

Solutions to Tutorial 5

4.90 We use the special formulas for the binomial mean and variance.

(a)

$$\mu = np = 440(.5) = 220$$

$$\sigma^2 = np(1-p) = 440(.5)(.5) = 110$$

so $\sigma = \sqrt{110} = 10.488$

(b)

$$\mu = np = 300(\frac{1}{6}) = 50$$

$$\sigma^2 = np(1-p) = 300(\frac{1}{6})(\frac{5}{6}) = 41.667$$

so $\sigma = 6.46$

(c)

$$\mu = np = 700(.03) = 21$$

$$\sigma^2 = np(1-p) = 700(.03)(.97) = 20.37$$

so $\sigma = 4.51$

4.91 Here $n = 100$, $p = 0.02$ so $np = 2$. Using $\lambda = 2$, the approximate probability is $f(1;2)) = 2e^{-2}/1! =$ 0.2707. Alternatively

$$f(1;2) = F(1;2) - F(1;2) = .406 - .135 = .271.$$

4.94 $n = 10{,}000$, $p = .00004$, $np = .4$.

$$1 - F(1;.4) = 1 - .938 = .062.$$

4.95 $\lambda = 0.6$ for three weeks. The probability is

$$f(0;6) = (.6)^0 e^{-.6}/0! = .5488.$$