Solutions to Tutorial 9

7.39 (a) The manufacturer wants to establish that mean time, μ, to set up a computer is less than 2 hours so that becomes the alternative hypothesis.

$$H_0 : \mu = 2 \qquad H_1 : \mu < 2$$

(b) If $\mu = 1.9$ the alternative hypothesis holds and the only possible error is failing to reject H_0. That is, the mean time to set up a computer is less than 2 hours but we fail to reject the null hypothesis that it is 2 hours.

(c) If $\mu = 2.0$ the null hypothesis holds and the only possible error is to reject H_0. That is, the mean time to set up a computer is 2 hours but we conclude that it is less than 2 hours.

7.40 (a) The manufacturer wants to establish that mean mileage before failure, μ, is greater than 50,000 miles so that becomes the alternative hypothesis.

$$H_0 : \mu = 50,000 \qquad H_1 : \mu > 50,000$$

(b) If $\mu = 55,000$ the alternative hypothesis holds and the only possible error is failing to reject H_0. That is, the mean mileage to failure is greater than 50,000 but we fail to reject the null hypothesis that it is 50,000.

(c) If $\mu = 50,000$ the null hypothesis holds and the only possible error is to reject H_0. That is, the mean mileage to failure is 50,000 miles but we conclude that it is greater than 50,000 miles.

7.41 (a) We want to show that the mean flying time, μ, is different from 56 minutes so that becomes the alternative hypothesis.

$$H_0 : \mu = 56 \qquad H_1 : \mu \neq 56$$

(b) If $\mu = 50$ the alternative hypothesis holds and the only possible error is failing to reject H_0. That is, the mean flying time is different from 56 but we fail to reject the null hypothesis that it is 56.

(c) If $\mu = 56$ the null hypothesis holds and the only possible error is to reject H_0. That is, the mean flying time is 56 minutes but we conclude that it is different from 56 minutes.

7.42 (a) The company wants to show that the mean life, μ, is greater than 183 days so that becomes the alternative hypothesis.

$$H_0 : \mu = 183 \qquad H_1 : \mu > 183$$

(b) If $\mu = 190$ days, the alternative hypothesis holds and the only possible error is failing to reject H_0. That is, the mean is greater than 183 days but we fail to reject the null hypothesis that it is 183 days.

7.43 (a) We should make the engineers prove that the bridge is safe so that should be the alternative hypothesis. The null hypothesis then asserts it is unsafe. These hypothesis would need to be translated into a statement about a parameter, perhaps the probability of failure in next three years.

(b) The value $\alpha = .05$, or 1 in 20 chances of saying a unsafe bridge is safe, is too high for me. Even $\alpha = .01$, or 1 in 100 bridges, seems high when talking about personal safety.

Solutions to Tutorial 9

7.47 We can assume from past experience that the standard deviation of the drying times is 2.4 minutes. The null hypothesis is that the mean $\mu = 20$. We reject the null hypothesis if $\bar{X} > 20.50$ minutes.

(a) the probability of a Type I error is the probability that $\bar{X} > 20.50$ when $\mu = 20$. Using a normal approximation to the distribution of the sample mean, this probability is given by

$$1 - F\left(\frac{20.50 - 20}{2.4/\sqrt{36}}\right) = 1 - F(1.25) = 1 - .8944 = .1056$$

(b) The probability of a Type II error when $\mu = 21$ is the probability that $\bar{X} < 20.50$ when $\mu = 21$. Using a normal approximation to the distribution of the sample mean, this probability is given by

$$F\left(\frac{20.5 - 21}{2.4/\sqrt{36}}\right) = F(-1.25) = 0.$$

7.48 (a) Proceeding as in Exercise 7.47, the probability of a Type I error is given by

$$1 - F\left(\frac{20.75 - 20}{2.4/\sqrt{50}}\right) = 1 - F(2.21) = .014$$

(b) The probability of a Type II error is

$$F\left(\frac{20.75 - 21}{2.4/\sqrt{50}}\right) = F(-.737) = .2327.$$

7.49 Using the normal approximation for the distribution of the sample mean, we need to find c such that

$$F\left(\frac{c - 100}{12/\sqrt{40}}\right) = .01.$$

Using the normal table, we see that

$$\frac{c - 100}{12/\sqrt{40}} = -2.33$$

So, we reject the null hypothesis if x-bar < c, where

c = 95.58