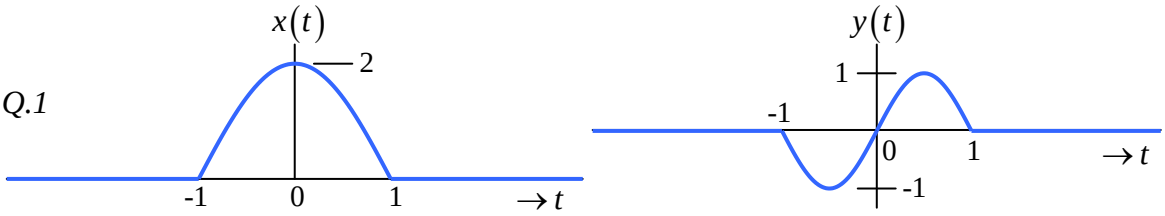
EE2023 TUTORIAL 3 (PROBLEMS)

Q.1 A half-cosine pulse $x(t)$ and a sine pulse $y(t)$ are shown in Fig.Q.1.

Fig.Q.1

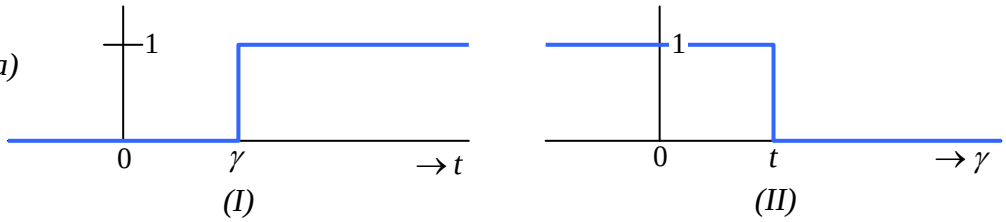


(a) Derive the spectrum of $x(t)$ using the forward Fourier transform equation and show how the derivation can be simplified by applying relevant Fourier transform properties.

(b) Using the results of Part-(a), determine the spectrum of $y(t)$.

Q.2 *(a)* Show that Fig.Q.2(a)(I) and Fig.Q.2(a)(II) are plots of the same function $u(t-\gamma)$, where $u(\cdot)$ denotes the unit step function. Hence, express $\int_{-\infty}^{t} x(\gamma)d\gamma$ as a convolution integral.

Fig.Q.2(a)

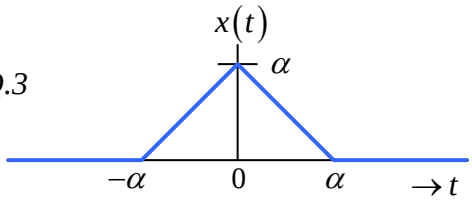


(b) Evaluate $\cos(t)u(t) \circledast u(t)$ where $\circledast$ denotes convolution.

(c) Find the Fourier transform of $\alpha \cdot \text{sinc}(\alpha t)$ and show that $\lim_{\alpha \to \infty} \alpha \cdot \text{sinc}(\alpha t) = \delta(t)$.

Q.3 Fig.Q.3 shows the plot of a triangular pulse $x(t)$.

Fig.Q.3



Determine the spectra of $\frac{dx(t)}{dt}$ and $x(t)$. Express $x(t)$ as a function of $\text{rect}(\cdot)$.

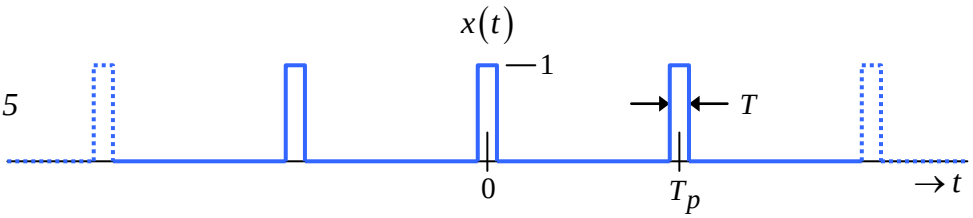
Q.4 The spectrum of a lowpass energy signal $x(t)$ is given by $X(f) = \exp(-\alpha|f|)$ where α is a positive constant.

(a) The 99% energy containment bandwidth of a signal is defined as the smallest bandwidth that contains at least 99% of the total signal energy. Find the 99% energy containment bandwidth of $x(t)$?

(b) Find the 3dB bandwidth of $x(t)$. How many percent of the total energy of $x(t)$ does its 3dB bandwidth contain?

Q.5 A military lookout tower uses a laser pointer as a make-shift signaling device to communicate with a base camp. The laser pointer's built-in ON-OFF pushbutton switch is replaced by an electronic switch which is activated by a signal $x(t)$. The output of the laser pointer has the form $y(t) = x(t) \cdot \mu\cos(2\pi f_c t)$ where μ and f_c are the amplitude and frequency of the laser beam when $x(t)$ has a value of 1. Unless there is an incident, the laser pointer continuously sends short pulses of light, spaced at regular interval, back to the base camp to indicate a 'No Incident' situation. The $x(t)$ used for signaling 'No Incident' is shown in Fig.Q.5.

Fig.Q.5



(a) Derive the continuous-frequency spectrum of $x(t)$. Sketch and label the spectrum.

(b) The 99% power containment bandwidth of a signal is defined as the smallest bandwidth that contains at least 99% of the average signal power. Provide a formula for computing the 99% power containment bandwidth of $x(t)$?

(c) Assume $T >> 1/f_c$. What is the average power of the laser output $y(t)$ and how it can be controlled?

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

The 1st Edition can be found in the following link:
http://www.kousik.net/wp-content/uploads/2010/10/Schaums-Outline-Series-Signals_Systems.pdf

Selected solved-problems: 5.19-to-5.27, 5.32, 5.34, 5.40, 5.42, 5.42, 5.57

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.
