National University of Singapore
Department of Mathematics

Semester II, 2010/2011 MA1506 Mathematics II Assignment

Instructions

(i) This assignment is due on 22nd March 2011 1:00pm.

(ii) This assignment counts towards 5% of your final grade.

(iii) Assignments should be typed and submitted on A4 sized writing paper. (Handwritten assignments with the required printouts are also accepted.)

(iv) The University takes a serious view of plagiarism. Any student found to have committed or aided and abetted the offence of plagiarism may be subject to disciplinary actions in accordance with Section 3 (l) of Statute 06 (Discipline) of the National University of Singapore at: `https://share.nus.edu.sg/registrar/info/statutes/Statute06.pdf`

(v) Include the following declaration together with your name, matriculation number, lecture group and signature on the first page of your assignment.
"I certify that the work submitted here represents solely my own efforts. I am aware of the University's regulations about, and penalties for, plagiarism."

(vi) Late assignments or those without the above declaration will not be graded.

(vii) Students should use either MATLAB or scilab (but not both) to answer their assignments.

(viii) Submission of assignment

- Assignments are to be submitted to the Mathematics General Office at S17 level 4 during office hours from 9am, 21th March to 1pm, 22nd March.
- Please place your assignments in the baskets/trays labeled according to your official lecture group A, B, C or D.
- Each lecture group has been assigned a grader. Students who wish to submit their assignments before 21th March, should contact their designated grader separately.

Group	Grader	Email
A	Samuel Seah	a0030907@nus.edu.sg
B	Hong Liu	a0078991@nus.edu.sg
C	Chen Weidong	chenweidong@nus.edu.sg
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(ix) Assignments submitted to the wrong group/person/grader run the risk of being misplaced.

(x) Students should contact their graders directly for issues concerning grading.

Questions: Total Marks: **40**.

Q1) (7 Marks)

The motion of a damped oscillator system is governed by

$$\ddot{x} + 4\dot{x} + kx = 0, \qquad x(0) = 5,\ \dot{x}(0) = 1.$$

(a) Find the equation of motion for each of $k = 3, 4$ and 8. Show all necessary workings.

(b) Print or write down the MATLAB or scilab code that will plot all three curves, that you obtained from (a), on the same graph over the interval $[0, 8]$. Use different colours or different types of lines for your graphs.

(c) Run your code to plot the graph. Include your own name as the title of this graph. In the Figure or graphic window, use File $\rightarrow$ Export, to save the graph as a jpeg or gif. Print out this graph and submit it together with your other answers.

Q2) (7 Marks)

The motion of a forced undamped oscillator system is governed by

$$10\ddot{x} + 99x = 50\cos\alpha t, \qquad x(0) = 0,\ \dot{x}(0) = 0,$$

where α, the forcing frequency of the motor, can be adjusted by the user.

(a) Compute ω the natural frequency of the oscillator and T the period (associated with ω.)

(b) Find the solutions $x_1(t)$ and $x_2(t)$ to the system, when the motor is adjusted to following forcing frequencies, $\alpha_1 = \pi$ and $\alpha_2 = 2.8$ respectively. Plot the solutions $x_1(t)$ and $x_2(t)$ from $t = 0$ to $t = 10T$, using different colours or different types of lines for your graphs. Include your own name as the title of this graph and submit your graph.

(c) Which solution corresponds to resonance like behaviour?

Remark: You may use the solution for $x(t)$ from your lecture notes. Do not use the numerical solvers in MATLAB or scilab.

Q3) (5 Marks)

The motion of a forced damped oscillator system is governed by

$$10\ddot{x} + 5\dot{x} + 120x = 50\cos\alpha t, \qquad x(0) = 0,\ \dot{x}(0) = 0,$$

where α, the forcing frequency of the motor, can be adjusted by the user.

(a) Plot the amplitude response curve of the system for the forcing frequency over the range $0 \leq \alpha \leq 30$. Include your own name as the title of this graph and submit your graph.

(b) Find the maximum amplitude and the value of α that gives rise to the maximum amplitude. Briefly explain how you arrived at your answer and leave your answer correct to 2 decimal places.

Remark: You may use the solution for $x(t)$ from your lecture notes. Do not use the numerical solvers in MATLAB or scilab.

Q4) (8 Marks)

Let $x(t)$ be the number of neutrons flying around inside a nuclear reactor at time t. It satisfies the differential equation

$$\frac{dx}{dt} = (x-1)^2 - x.$$

(a) Plot the direction field of $x(t)$ (Hint: You can use $0 \leq t, x \leq 4$. Adjust the interval values of t and x until the equilibriums can be seen clearly from your direction field.)

(b) Identify the equilibriums (if any) and their stability and plot the equilibrium lines into your direction field graph.

(c) Solve for the solution of $x(t)$ that satisfies the initial condition $x(0) = 2.4$. Plot this solution into your direction field graph. (Hint: Your solution should be a function given in terms of tanh.)

For this question, you should submit

i) your answer about the equilibriums;

ii) your solution for part (c);

iii) a graph of the direction field together with the equilibrium lines and the required solution in different colours or types of lines;

iv) the code you used.

Q5) (5 Marks)

The motion of a physical system is governed by the initial-value problem

$$\ddot{x} + \dot{x} + \frac{\sqrt{x}}{t} = 0, \qquad x(0.01) = 4,\ \dot{x}(0.01) = 10.$$

(a) Use the numerical ode solvers of MATLAB or scilab to approximate the solution of $x(t)$ for t ranging from 0.01 to 15. (Scilab users should specify t in intervals of 0.01.)

(b) What is the value of $x(15)$?

(c) Plot the graph of $x(t)$ and submit this graph and the code that you used for all the parts of this question. (Hint: We only want the curve of $x(t)$ and not $\dot{x}(t)$. See page 5 of lab 3 to find out how this can be done)

Q6) (8 Marks)

A small country has four residential districts: A, B, C and D. Initially, 70% of the population stays in A, while the remaining population is evenly distributed to the other three districts. A survey showed that every year, 20%, 10% and 10% of residents from A move to districts B, C and D respectively. For district B, 30%, 10% and 10% of residents move to districts A, C and D respectively. For district C, 10%, 30% and 10% of residents move to districts A, B and D respectively. Lastly, for district D, 30%, 10% and 10% of residents move to districts A, B and C respectively.

(a) Assuming the population remains constant, write down the probability matrix M that models the transitions of the population between the towns A, B, C and D.

(b) Write down the eigenvalues of the matrix M.

(c) What is the distribution of the population among the districts after five years? (Leave your answer as a percentage correct up to 1 decimal place.)