Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(20 \times 10^{-2} \text{ m}) \times (20 \times 10^{3} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 1.33 \times 10^{-5} \text{ (negligible)}.$$

(b) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(50 \times 10^{3} \text{ m}) \times (60 \times 10^{0} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.01 \text{ (borderline)}.$$

(c) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(20 \times 10^{-2} \text{ m}) \times (600 \times 10^{6} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.40 \text{ (nonnegligible)}.$$

(d) $$\frac{l}{\lambda} = \frac{lf}{u_{\text{p}}} = \frac{(1 \times 10^{-3} \text{ m}) \times (100 \times 10^{9} \text{ Hz})}{3 \times 10^{8} \text{ m/s}} = 0.33 \text{ (nonnegligible)}.$$

Problem 2.3 Show that the transmission line model shown in Fig. P2.3 yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

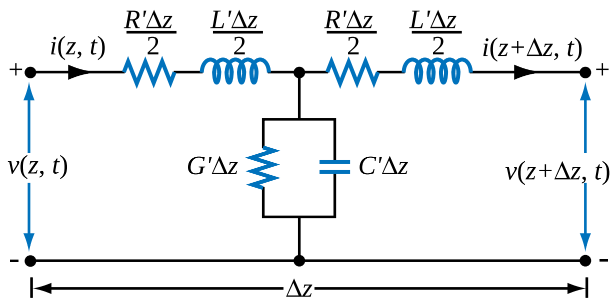

Figure P2.3: Transmission line model.

Solution: The voltage at the central upper node is the same whether it is calculated from the left port or the right port:

$$
\begin{aligned}
v(z+\tfrac{1}{2}\Delta z,t) &= v(z,t) - \tfrac{1}{2}R'\Delta z\, i(z,t) - \tfrac{1}{2}L'\Delta z \frac{\partial}{\partial t} i(z,t) \\
&= v(z+\Delta z,t) + \tfrac{1}{2}R'\Delta z\, i(z+\Delta z,t) + \tfrac{1}{2}L'\Delta z \frac{\partial}{\partial t} i(z+\Delta z,t).
\end{aligned}
$$

Recognizing that the current through the $G' \parallel C'$ branch is $i(z,t) - i(z+\Delta z,t)$ (from Kirchhoff's current law), we can conclude that

$$
i(z,t) - i(z+\Delta z,t) = G'\Delta z\, v(z+\tfrac{1}{2}\Delta z,t) + C'\Delta z \frac{\partial}{\partial t} v(z+\tfrac{1}{2}\Delta z,t).
$$

From both of these equations, the proof is completed by following the steps outlined in the text, ie. rearranging terms, dividing by Δz, and taking the limit as $\Delta z \to 0$.

Problem 2.4 A 1-GHz parallel-plate transmission line consists of 1.2-cm-wide copper strips separated by a 0.15-cm-thick layer of polystyrene. Appendix B gives $\mu_c = \mu_0 = 4\pi \times 10^{-7}$ (H/m) and $\sigma_c = 5.8 \times 10^7$ (S/m) for copper, and $\varepsilon_r = 2.6$ for polystyrene. Use Table 2-1 to determine the line parameters of the transmission line. Assume $\mu = \mu_0$ and $\sigma \simeq 0$ for polystyrene.

Solution:

$$R' = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{1.2 \times 10^{-2}} \left(\frac{\pi \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7} \right)^{1/2} = 1.38 \quad (\Omega/\text{m}),$$

$$L' = \frac{\mu d}{w} = \frac{4\pi \times 10^{-7} \times 1.5 \times 10^{-3}}{1.2 \times 10^{-2}} = 1.57 \times 10^{-7} \quad (\text{H/m}),$$

$$G' = 0 \qquad \text{because } \sigma = 0,$$

$$C' = \frac{\varepsilon w}{d} = \varepsilon_0 \varepsilon_r \frac{w}{d} = \frac{10^{-9}}{36\pi} \times 2.6 \times \frac{1.2 \times 10^{-2}}{1.5 \times 10^{-3}} = 1.84 \times 10^{-10} \quad (\text{F/m}).$$

Problem 2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad \text{(distortionless line)}.$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R'+j\omega L')(G'+j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'}+j\omega\right)\left(\frac{G'}{C'}+j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'}+j\omega\right)\left(\frac{R'}{L'}+j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'}+j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

From R'C'=L'G',
we get
R'/L'=G'/C'

Hence,

$$\alpha = \Re\mathfrak{e}(\gamma) = R'\sqrt{\frac{C'}{L'}}, \qquad \beta = \Im\mathfrak{m}(\gamma) = \omega\sqrt{L'C'}, \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L'+j\omega}{G'/C'+j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Problem 2.12 Generate a plot of Z_0 as a function of strip width w, over the range from 0.05 mm to 5 mm, for a microstrip line fabricated on a 0.7-mm–thick substrate with $\varepsilon_r = 9.8$.

Solution: