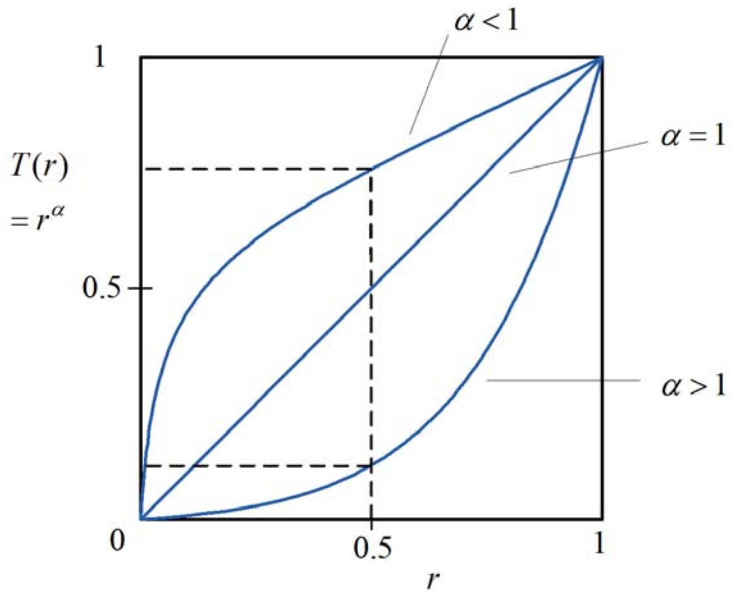
EE3206/EE3206E INTRODUCTION TO COMPUTER VISION AND IMAGE PROCESSING

Tutorial Set D – Solutions

Question 1



- $0 < \alpha < 1$: The image tends to get brighter as the gray levels get transformed to higher levels.
- $\alpha = 1$: No change.
- $\alpha > 1$: The image tends to get darker as the gray levels get transformed to lower levels.

We obviously cannot just replace r by r_k to give

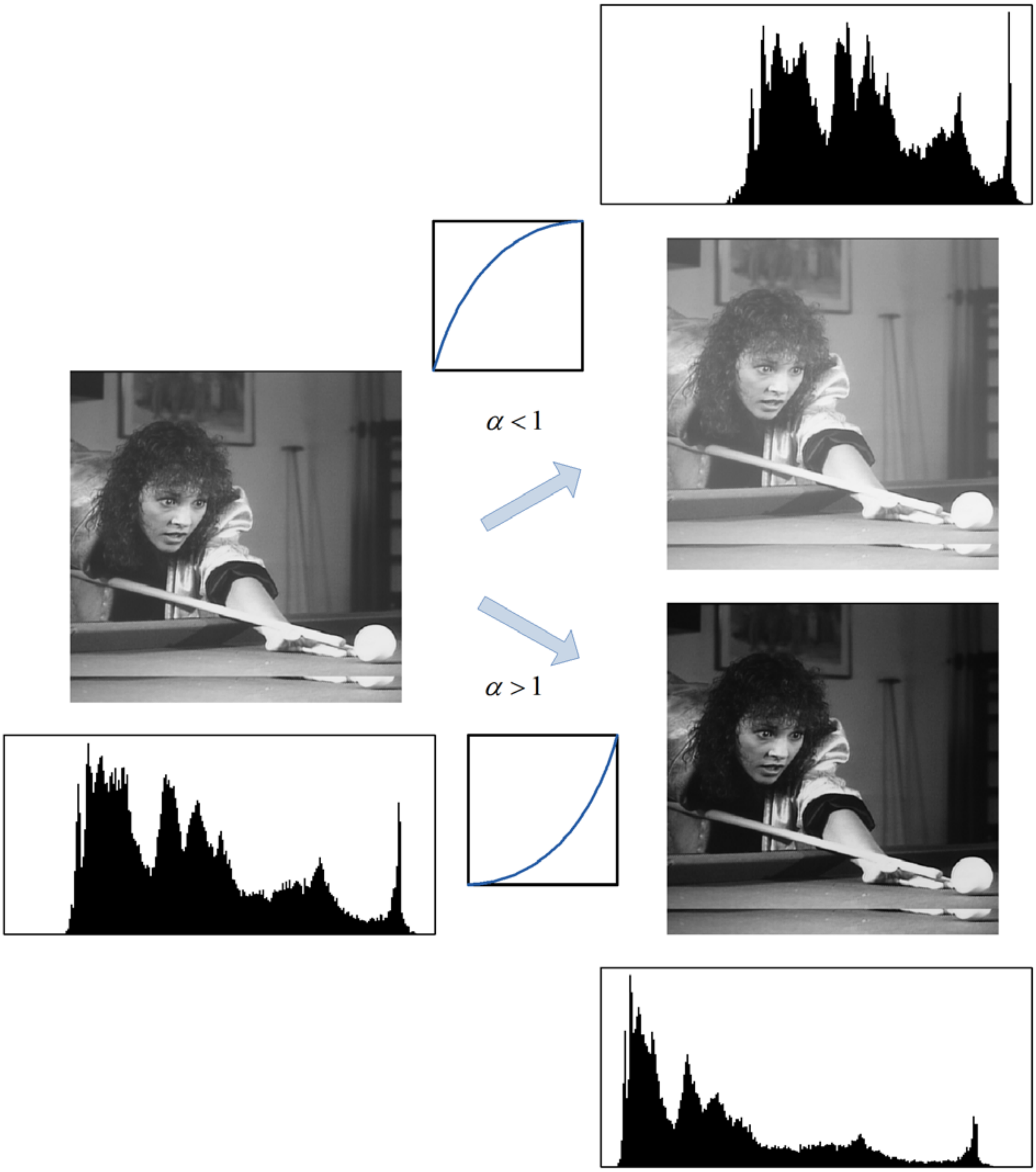
$$T_\alpha(r_k) = (r_k)^\alpha$$

since we know that

$$r_k = 255 \;\rightarrow\; s_k = 255$$

Hence, we normalise r_k by 255, then multiply by 255 after taking the exponential, i.e.,

$$s_k = T_\alpha(r_k) = 255\left(\frac{r_k}{255}\right)^\alpha$$

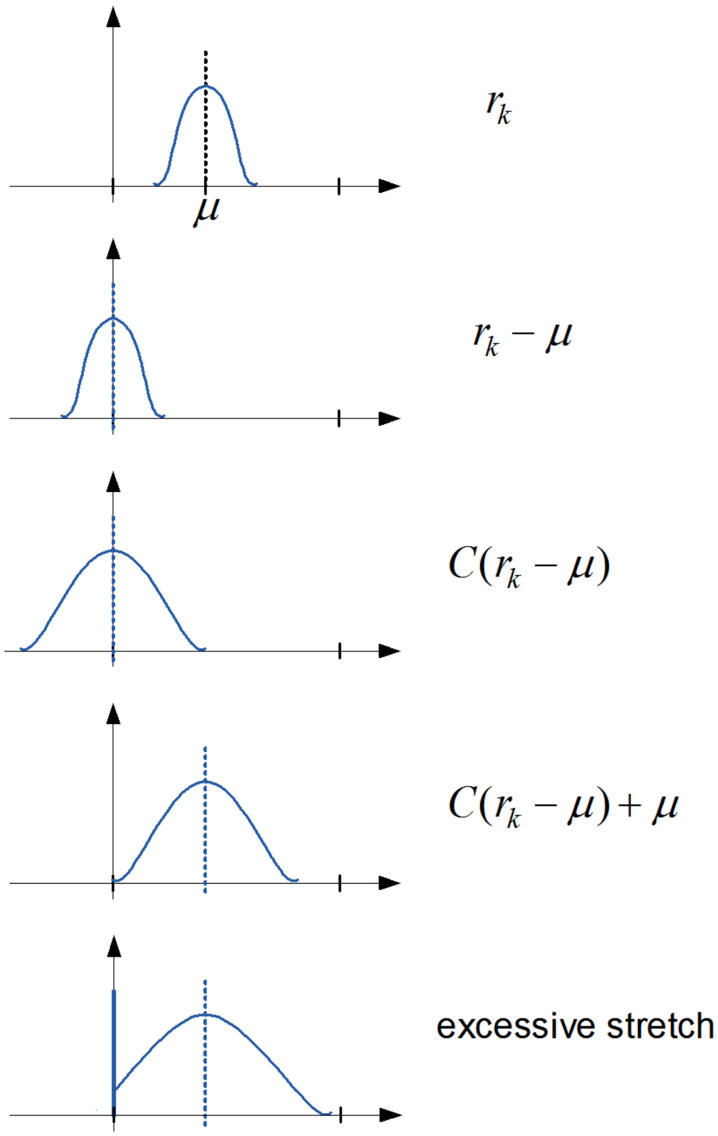

$\alpha < 1$
$\alpha > 1$

Question 2

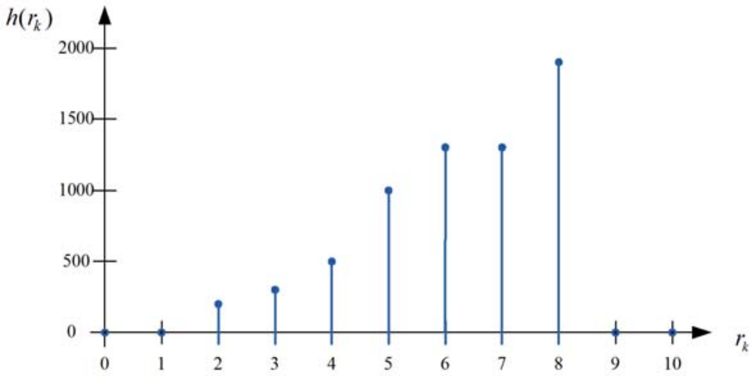
The transformation function is

$$s_k = C(r_k - \mu) + \mu$$

The histogram is shifted such that its mean is at the origin. It is then stretched and shifted back by the same amount. C is chosen such that the stretched histogram occupies the full range of gray levels (without significant clipping occurring).



Gray level:	0	1	2	3	4	5	6	7	8	9	10
Number of pixels:	0	0	200	300	500	1000	1300	1300	1800	0	0



We first compute the image mean

$$\begin{aligned}\mu &= \frac{1}{N}\sum_k n_k r_k \\ &= \frac{1}{6400}(0(0) + 1(0) + 2(200) + 3(300) + \ldots) \\ &= 6.19\end{aligned}$$

C has to satisfy two constraints:

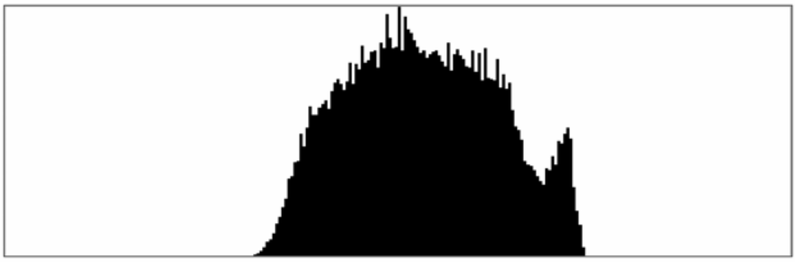
1. For $r_k = 2$, $s_k \geq 0$

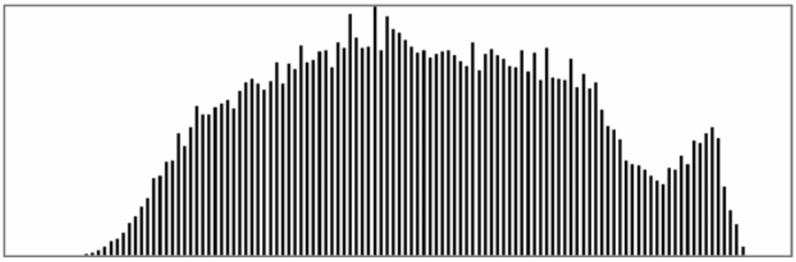
$$\begin{aligned}\text{i.e.} \quad C(2 - 6.19) + 6.19 &\geq 0 \\ \text{or} \quad C &\leq 1.5\end{aligned}$$

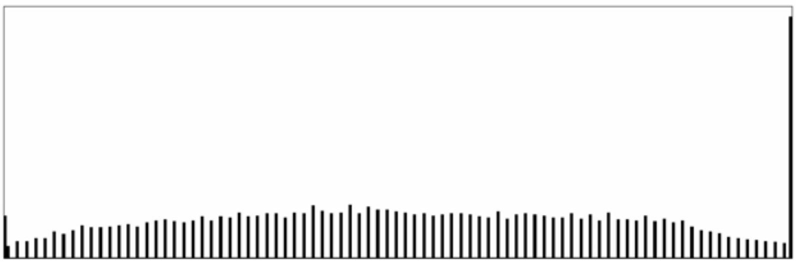
2. For $r_k = 8$, $s_k \leq 10$

$$\begin{aligned}\text{i.e.} \quad C(8 - 6.19) + 6.19 &\leq 10 \\ \text{or} \quad C &\leq 2.1\end{aligned}$$

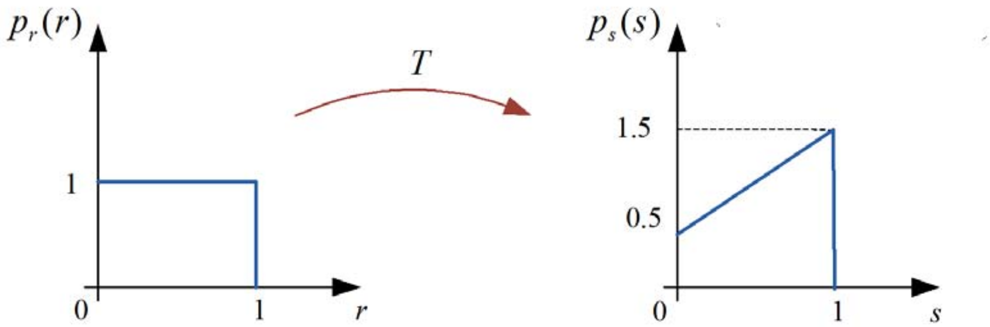
Therefore, we choose $C = 1.5$.





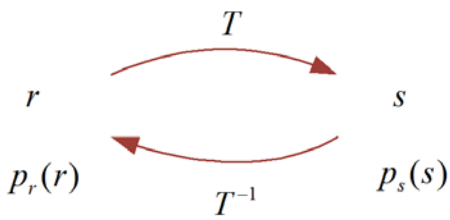


Question 3



$$p_s(s) = s + 0.5$$

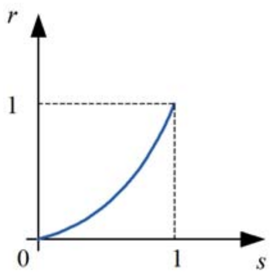
We first find the transformation $T^{-1}(s)$ that equalises $p_s(s)$:



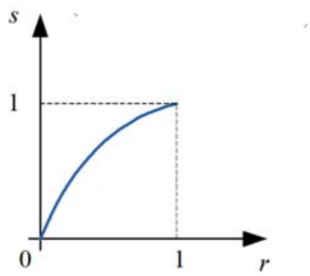
$$\begin{aligned}
r &= T^{-1}(s) \\
&= \int_0^s p_s(w)dw \\
&= \int_0^s (w + 0.5)dw \\
&= \left[\frac{1}{2}w^2 + \frac{1}{2}w\right]_0^s \\
&= \frac{1}{2}s^2 + \frac{1}{2}s
\end{aligned}$$

The transformation that is applied to $p_r(r)$ to give $p_s(s)$ is

$$\begin{aligned}
s &= T(r) \\
&= -\frac{1}{2} \pm \frac{\sqrt{1+8r}}{2} \\
&= -\frac{1}{2} + \frac{\sqrt{1+8r}}{2} \qquad \text{since } r = 0 \Rightarrow s = 0
\end{aligned}$$



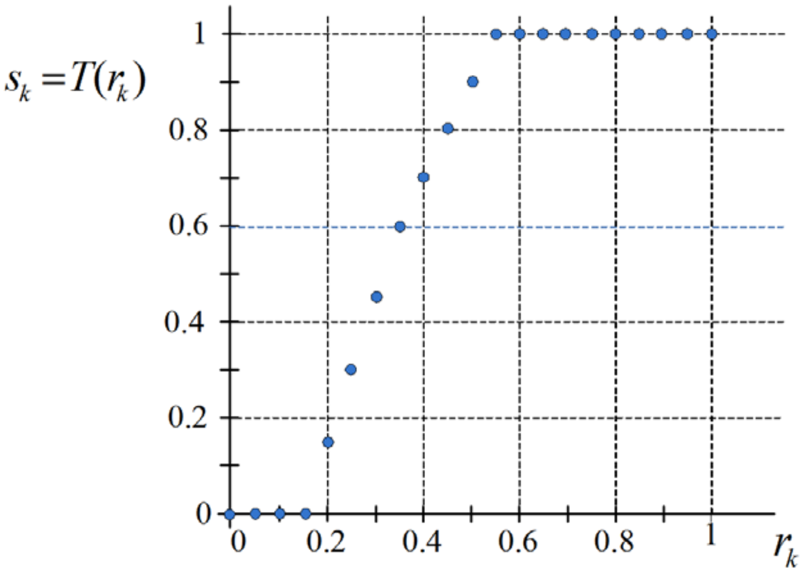
$r = T^{-1}(s)$

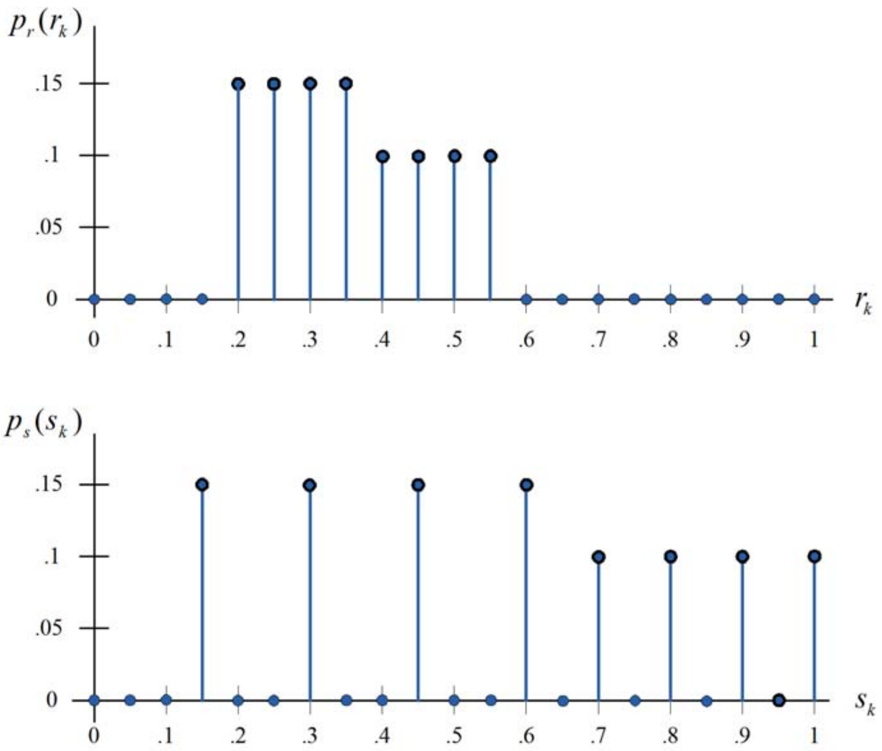


$s = T(r)$

Question 4

k	r_k	$p_r(r_k)$	s_k	$p_s(s_k)$
0	0	0	0	
1	0.05	0	0	0
2	0.1	0	0	
3	0.15	0	0	
4	0.2	0.15	0.15	0.15
5	0.25	0.15	0.3	0.15
6	0.3	0.15	0.45	0.15
7	0.35	0.15	0.6	0.15
8	0.4	0.1	0.7	0.1
9	0.45	0.1	0.8	0.1
10	0.5	0.1	0.9	0.1
11	0.55	0.1	1	
12	0.6	0	1	
13	0.65	0	1	
14	0.7	0	1	
15	0.75	0	1	0.1
16	0.8	0	1	
17	0.85	0	1	
18	0.9	0	1	
19	0.95	0	1	
20	1	0	1	




$p_r(r_k)$
.15
.1
.05
0
r_k
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1
$p_s(s_k)$
.15
.1
.05
0
s_k
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1

Question 5

In the frequency domain, the output image after one application of the filter is

$$G_1(u,v) = H(u,v) \times F(u,v) = e^{-\omega^2/2\sigma^2}F(u,v)$$

Applying the filter k times will result in

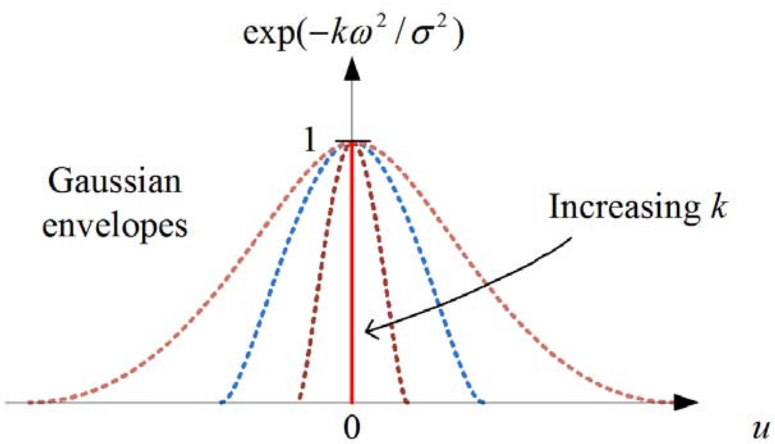
$$G_k(u,v) = [e^{-\omega^2/2\sigma^2}]^k F(u,v) = e^{-k\omega^2/2\sigma^2}F(u,v)$$

As k increases, the function $\exp(-k\omega^2/2\sigma^2)$ tends towards an impulse, i.e., it is equal to 1 at $(0,0)$ and 0 elsewhere. Thus, $G_k(u,v)$ will be zero everywhere except at the origin where it is equal to $F(0,0)$:

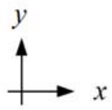
$$G_k(u,v) = \begin{cases} F(0,0) & (u,v) = (0,0) \\ 0 & \text{elsewhere} \end{cases}$$

Hence, with repeated filtering, the output image will take on a constant value equal to the average value of the input image:

$$g_k(x,y) = \bar{f}(x,y)$$



Question 6



0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	0	0	0
0	0	1	1	1	1	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	0	0
+	0	0	0	0	0	0	0	0	0	0

$f(x,y)$

0	0	0	0	0	0	0	0	0	0	0
0	1	1	-1	-1	0	0	0	0	0	0
0	3	3	-2	-3	-1	0	0	0	0	0
0	4	4	-1	-3	-3	-1	0	0	0	0
0	4	4	0	-1	-3	-3	-1	0	0	0
0	4	4	0	0	-1	-3	-3	-1	0	0
0	4	4	0	0	0	-1	-3	-3	-1	0
0	3	3	0	0	0	0	-1	-3	-2	0
+	1	1	0	0	0	0	0	-1	-1	0

$G_x(x,y)$

0	0	0	0	0	0	0	0	0	0	0
0	-1	-3	-3	-1	0	0	0	0	0	0
0	-1	-3	-4	-3	-1	0	0	0	0	0
0	0	0	-1	-3	-3	-1	0	0	0	0
0	0	0	0	-1	-3	-3	-1	0	0	0
0	0	0	0	0	-1	-3	-3	-1	0	0
0	0	0	0	0	0	-1	-3	-3	-1	0
0	1	3	4	4	4	4	3	1	0	0
+	1	3	4	4	4	4	4	3	1	0
0	0	0	0	0	0	0	0	0	0	0

$G_y(x,y)$

0	0	0	0	0	0	0	0	0	0	0
0	1.4	3.2	3.2	1.4	0	0	0	0	0	0
0	3.2	4.2	4.5	4.2	1.4	0	0	0	0	0
0	4.0	4.2	1.4	4.2	4.2	1.4	0	0	0	0
0	4.0	4.0	0	1.4	4.2	4.2	1.4	0	0	0
0	4.0	4.0	0	0	1.4	4.2	4.2	1.4	0	0
0	4.0	4.0	0	0	0	1.4	4.2	4.2	1.4	0
0	3.2	4.2	4.0	4.0	4.0	4.0	3.2	3.2	2.0	0
+	1.4	3.2	4.0	4.0	4.0	4.0	4.0	3.2	1.4	0
0	0	0	0	0	0	0	0	0	0	0

Gradient magnitude

$= \sqrt{(G_x)^2 + (G_y)^2}$

x	x	x	x	x	x	x	x	x	x	x
x	-45	-72	-108	-135	x	x	x	x	x	x
x	-18	-45	-117	-135	-135	x	x	x	x	x
x	0	0	-135	-135	-135	-135	x	x	x	x
x	0	0	x	-135	-135	-135	-135	x	x	x
x	0	0	x	x	-135	-135	-135	-135	x	x
x	0	0	x	x	x	-135	-135	-135	-135	x
x	18	45	90	90	90	90	108	162	180	x
+	45	72	90	90	90	90	90	108	135	x
x	x	x	x	x	x	x	x	x	x	x

Gradient angle

$= \operatorname{atan}(G_y / G_x)$

(in degrees)

x: undefined

Note:

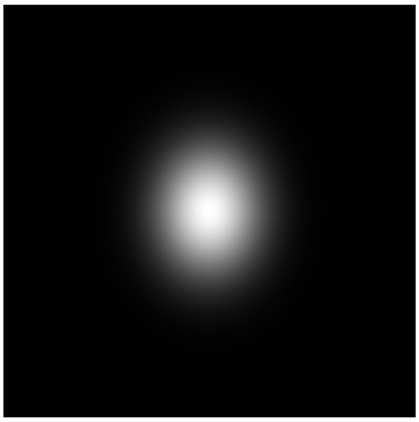
- edge strength depends on edge orientation
- diagonal edges give a stronger response compared to vertical and horizontal edges

Question 7

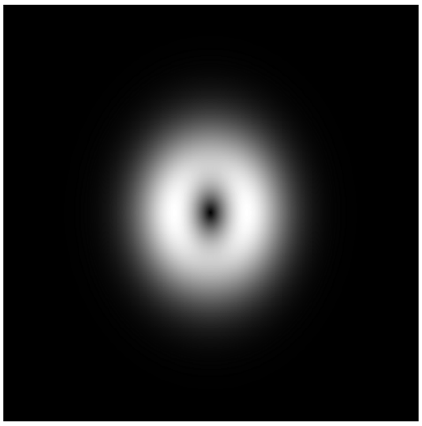
$$\begin{aligned} f(x,y) &= \exp(-ax^2 - by^2) \\ G_x(x,y) &= \frac{\partial f}{\partial x} \\ &= -2ax\exp(-ax^2 - by^2) \\ G_y(x,y) &= \frac{\partial f}{\partial y} \\ &= -2by\exp(-ax^2 - by^2) \end{aligned}$$

Hence

$$\begin{aligned} |\mathbf{G}(x,y)| &= [G_x^2 + G_y^2]^{1/2} \\ &= 2\sqrt{a^2x^2 + b^2y^2}\exp(-ax^2 - by^2) \\ \theta(x,y) &= \tan^{-1}(G_y/G_x) \\ &= \tan^{-1}(by/ax) \\ \mathbf{G}(x,y) &= 2\sqrt{a^2x^2 + b^2y^2}\exp(-ax^2 - by^2)\ \angle\tan^{-1}(by/ax) \end{aligned}$$



$f(x,y)$



$|\mathbf{G}(x,y)|$