MA1506 TUTORIAL 4

Question 1

Find the equilibrium points for the following equations. Determine whether these equilibrium points are stable, and, if so, find the approximate angular frequency of oscillation around those equilibria.

(i) $\ddot{x} = \cosh(x)$ (ii) $\ddot{x} = \cos(x)$ (iii) $\ddot{x} = \tan(\sin(x))$.

Question 2

A certain electrical circuit has a resistor of resistance 5 ohms, an inductor of inductance 0.05 henries, and a capacitor of capacitance 4×10^{-4} farads. A varying voltage given by

$$V = 200\cos(100\,t)$$

is applied. Find the current in this circuit if the initial current and the initial charge on the capacitor are equal to zero.
[Answer: $-2.35\,e^{-50t}\cos(50\sqrt{19}t) + 22.13\,e^{-50t}\sin(50\sqrt{19}t) + 2.35\cos(100t) - 9.41\sin(100t)$]

Question 3

Consider a forced damped harmonic oscillator, as in Chapter 2 of the notes. Recall that the amplitude response function is a function of α, the input frequency. Find the maximum of this function. [Note: be careful, there are different answers when the frictional constant b is large and when it is small.] Show that, when b is so small that the dimensionless quantity $b^2/m^2\omega^2$ can be neglected, the maximal amplitude is proportional to $1/b$.

Question 4

A fully loaded large oil tanker can be modelled as a solid object with perfectly vertical sides and a perfectly horizontal bottom, so all horizontal cross-sections have the same area, equal to A. Archimedes' principle [http://en.wikipedia.org/wiki/Buoyancy] states that the upward force exerted on a ship by the sea is equal to the weight of the water pushed aside by the ship. Let ρ be the mass density of seawater, and let M be the mass of the ship, so that its weight is Mg, where g is 9.8 m/sec^2. When the ship is at rest, find the distance d from sea level to the bottom of the ship. This is called the **draught** of the ship.

Suppose now that the ship is *not* at rest; instead it is moving in the vertical direction. Let $d + x(t)$ be the distance from sea level to the bottom of the ship, where d is the draught as above. Show that, if gravity and buoyancy are the only forces acting on the ship, it will bob up and down with an angular frequency given by $\sqrt{\rho A g/M}$.

Next, suppose that in fact there is a small amount of friction between the sides of the ship and the seawater, as the ship moves up and down in the sea. The frictional force is equal to $-b\dot{x}$, where b is a constant and $\dot{x}$ is the downward speed of the ship. Furthermore, waves from a storm strike the ship and exert a vertical force $F_0\cos(\alpha t)$ on the ship, where F_0 is the amplitude of the wave force and α is the wave frequency. Find the most dangerous [= resonance] value of α. Let H be the height of the deck of the ship above sea level when the ship is at rest. For fixed values of F_0 and b, show how to design a ship that will be able to survive this storm, no matter what α may be. That is, explain how to choose safe values of the *things you control*, namely M and A.