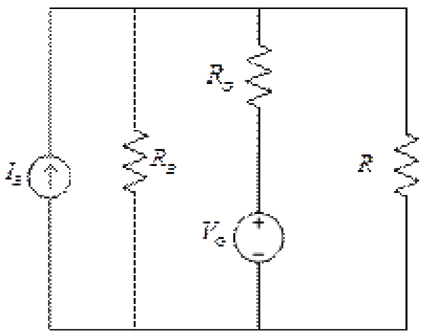
Tutorial 3

Q1:

Determine, using superposition, the voltage across R in the circuit of Figure Q1.

$I_S = 3A, R_S = 1\Omega, V_G = 15V, R_G = 1\Omega, R = 2\Omega$



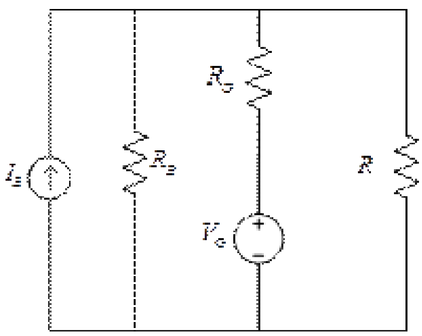
Given:
A circuit with one independent current source and one independent voltage source

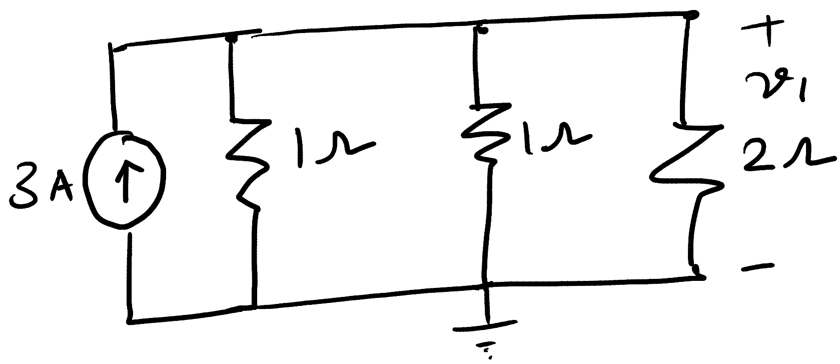
To find:
The voltage across the resistor R using superposition principle.

Analysis:
To evaluate the voltage with only one source at a time. Finally add up the two voltages

Solution:

1) Keep the current source and kill the voltage source by shorting it as shown.
   Apply node voltage analysis technique to solve for v.

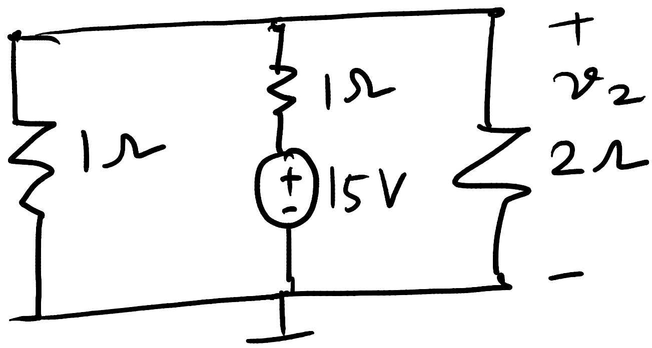




Applying KCL at the top node,

$$-3 + \frac{v_1}{1} + \frac{v_1}{1} + \frac{v_1}{2} = 0 \Rightarrow v_1 = \frac{3}{2 \cdot 5} V$$

2) Keep the voltage source and kill the current source by opening its terminals as shown in the figure



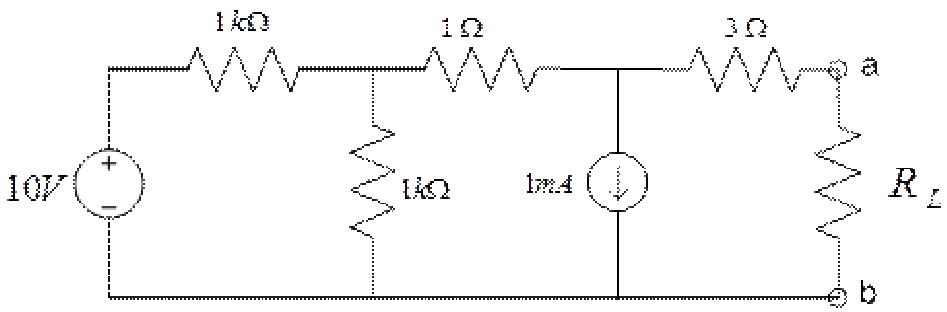
Again we apply non voltage analysis technique,

$$\frac{v_2}{1}+\frac{v_2-15}{1}+\frac{v_2}{2}=0 \Rightarrow v_2\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{2}\right)=\frac{15}{1} \Rightarrow v_2=\frac{15}{2\cdot 5}V$$

Total voltage, according to the superposition principle:

$$v=v_1+v_2=\frac{3}{2\cdot 5}+\frac{15}{2\cdot 5}=\frac{18}{2\cdot 5}=7.2V$$

Q2: Find the Thevenin equivalent circuit that the load () sees for the circuit of Figure Q2.

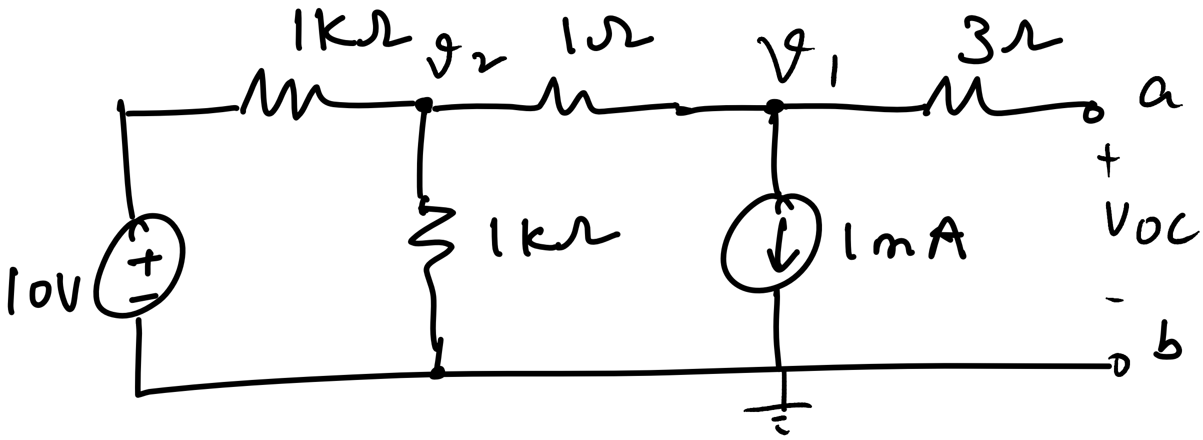


Analysis:

We need to find the open circuit voltage between points a and b.

Then we need to find the Thevenin resistance.

Solution



Let us apply node voltage analysis method. Note the reference node and the two unknown node voltages, v_1, v_2.

Writing KCL at v_1:

$$\frac{v_1 - v_2}{1} + 0.001 = 0 \Rightarrow v_1 - v_2 = -0.001 \qquad (1)$$

Writing KCL at node v_2:

$$\frac{v_2 - 10}{1000} + \frac{v_2}{1000} + \frac{v_2 - v_1}{1} = 0 \qquad (2)$$

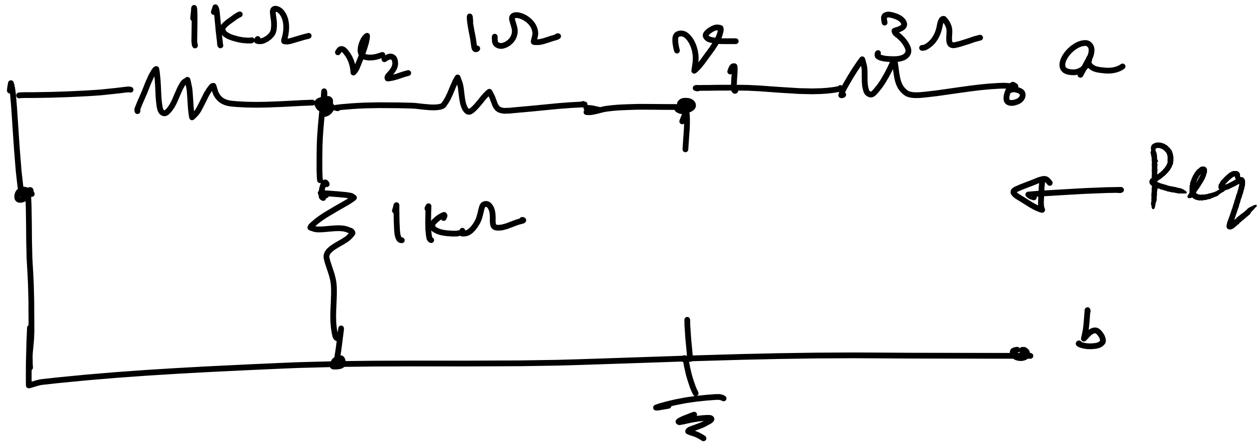
Putting the eqn(1) in eqn(2), and multiplying both sides by 1000, we get:

$$v_2 - 10 + v_2 + 1 = 0 \Rightarrow v_2 = \frac{9}{2} = 4 \cdot 5V.$$

Putting value of v_2 in eqn.(1), we get $v_1 = v_2 - 0.001 = 4.499V$.

Ii) To find the value of the Thevenin resistance

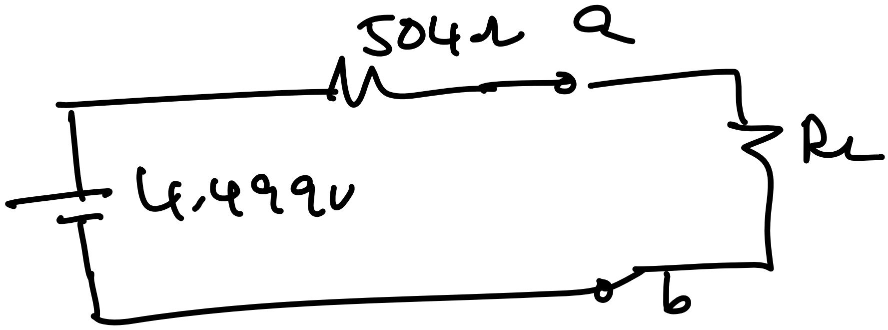
As the circuit contains independent sources only, we can kill the sources and calculate the equivalent resistance between the points a, b.



The equivalent resistance can be calculated as:

$$R_{eq} = 3 + 1 + \frac{1000 \times 1000}{1000 + 1000} = 504\Omega.$$

The Thevenin equivalent circuit is:



Q3:

The circuit shown in Figure Q3 is in the form of what is known as differential amplifier. Find the expression for v0 in terms of v1 and v2 using Thevenin's or Norton's theorem. Assume that the voltage sources v1 and v2 do not source any current.

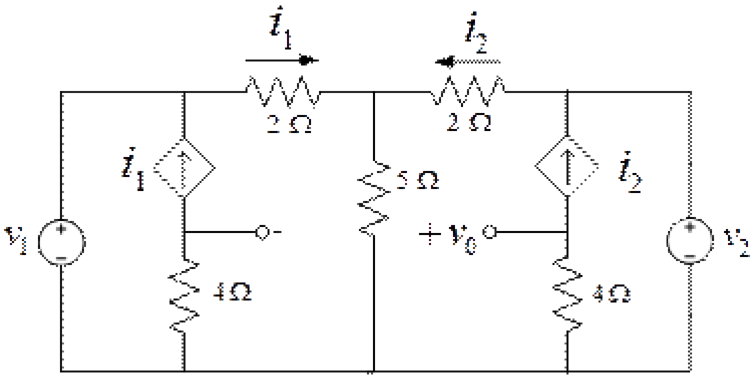


Fig. Q3

To find:

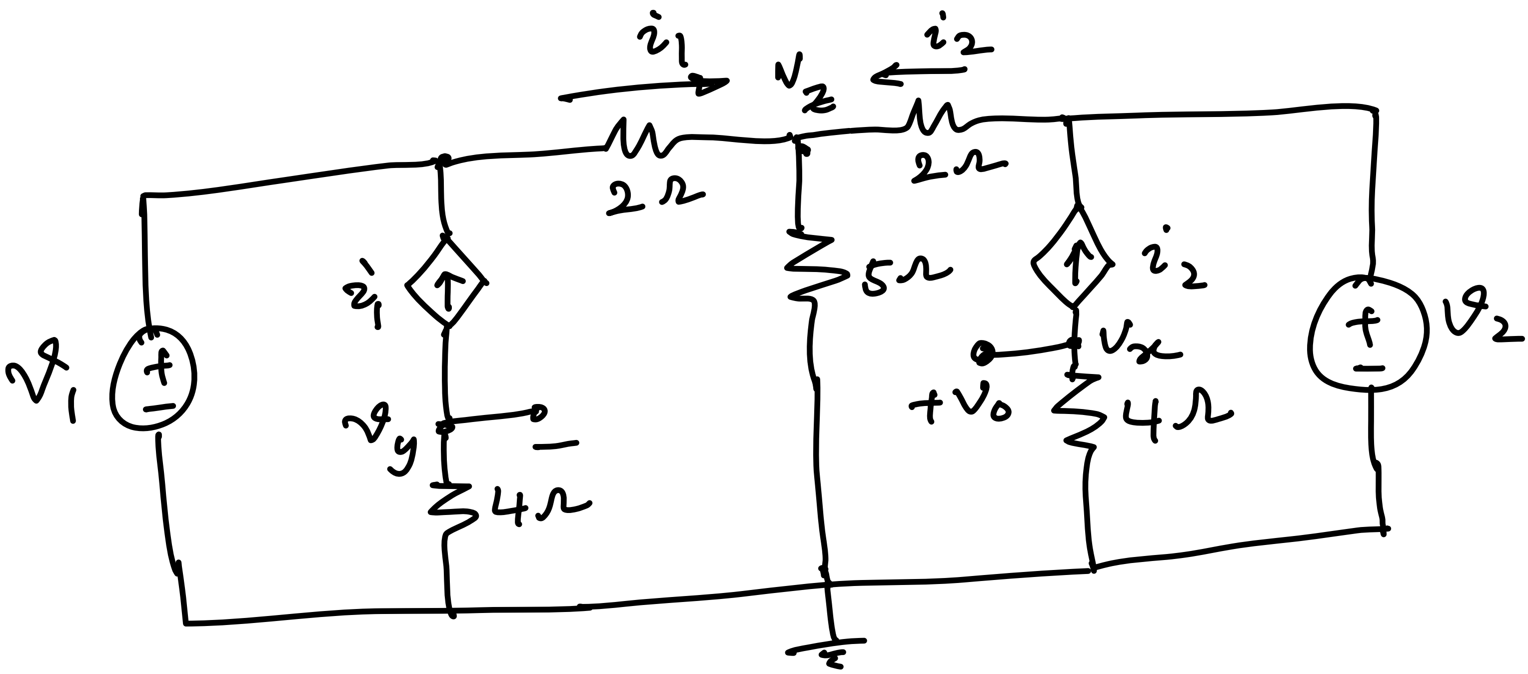
The output voltage v_0 in terms of the voltages v_1, v_2.

Analysis:

We find the open circuit voltage between the output terminals same as the Thevenin voltage .

Solution:

Apply Node voltage analysis technique.



Choose the reference node and assign voltages to the unknown node voltages v_x, v_y, v_z.

We can express the currents i_1, i_2:

$$i_1 = \frac{v_1 - v_z}{2}$$
$$i_2 = \frac{v_2 - v_z}{2}$$

Applying KCL at v_x:
$$\frac{v_x}{4} + i_2 = 0 \Rightarrow v_x = -4i_2 = -2(v_2 - v_z)$$

Applying KCL at v_y:
$$\frac{v_y}{4} + i_1 = 0 \Rightarrow v_y = -4i_1 = -2(v_1 - v_z)$$

$$v_0 = v_x - v_y = -2(v_2 - v_z) - \left(-2(v_1 - v_z)\right) = 2(v_1 - v_2)$$

Q4:

(i) Obtain the Thevenin's equivalent for the circuit (Figure Q4), which contains a dependent voltage source.
(ii) What should be the optimum value of a load resistor R_L to be connected between **a** and **b** so that the power delivered to it by the network is maximum?
(iii) What is the maximum power?
(iv) Also verify that the power delivered is less than the maximum power when $R_L = 0.8\ R_{Lop}$ and $1.2\ R_{Lop}$; where R_{Lop} is the optimum R_L for maximum power transfer.

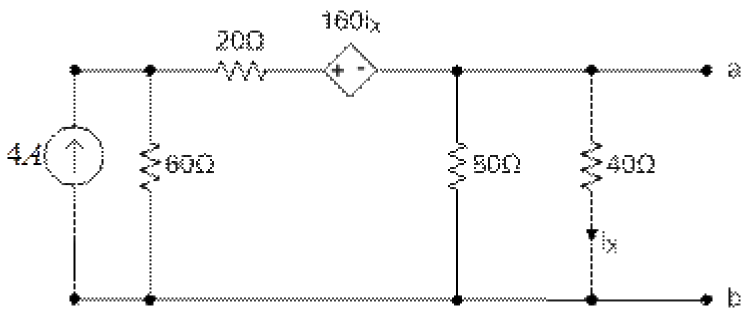


Fig. Q4

To find :

Thevenin's equivalent and then obtain the load at which power transfer is maximum.

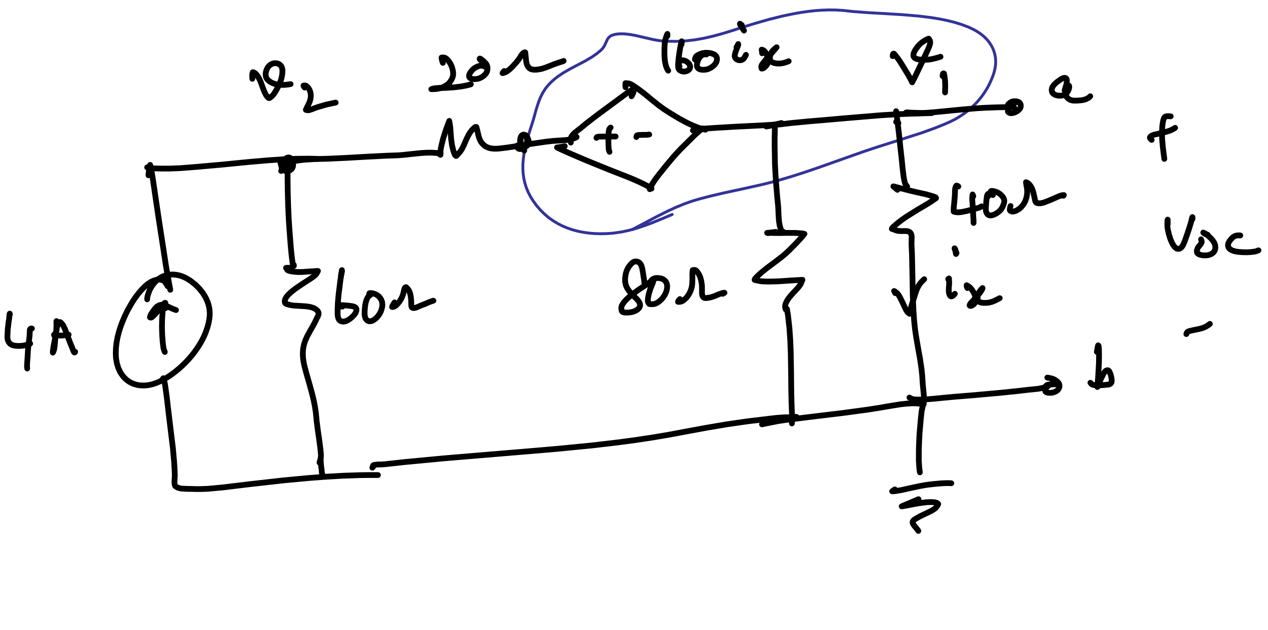
Analysis:

We can apply node voltage analysis to find the Thevenin's voltage.

As there is a dependent source, we can use test source method to obtain the Thevenin's resistance.

Solution:

(i) Find the open circuit voltage. Apply node voltage analysis.



Note the choice of the reference node and the unknown node voltages v_1, v_2.

We can write the current i_x as $i_x = \frac{v_1}{40}$.

Then the dependent voltage source becomes $160i_x = 160x\frac{v_1}{40} = 4v_1$.

Applying KCL at the super node surrounding v_1:

$$\frac{v_1}{40} + \frac{v_1}{80} + \frac{(v_1 + 160i_x) - v_2}{20} = 0$$

Replacing i_x we get $v_1 + 160ix = 5v_1$.

Then,

$$v_1\left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20}\right) - \frac{v_2}{20} = 0 \Rightarrow v_1\frac{23}{4} - v_2 = 0 \qquad (1)$$

Applying KCL at v_2:

$$-4 + \frac{v_2}{60} + \frac{v_2 - (v_1 + 160i_x)}{20} = 0$$

Replacing $v_1 + 160ix = 5v_1$:

$$-\frac{5}{20}v_1 + v_2\left(\frac{1}{60} + \frac{1}{20}\right) = 4 \Rightarrow -\frac{v_1}{4} + \frac{v_2}{15} = 4 \qquad (2)$$

(1)+(2)x23=>

$$-v_2 + v_2 \cdot \frac{23}{15} = 4 \times 23 \Rightarrow v_2 = \frac{23 \times 15}{2}$$

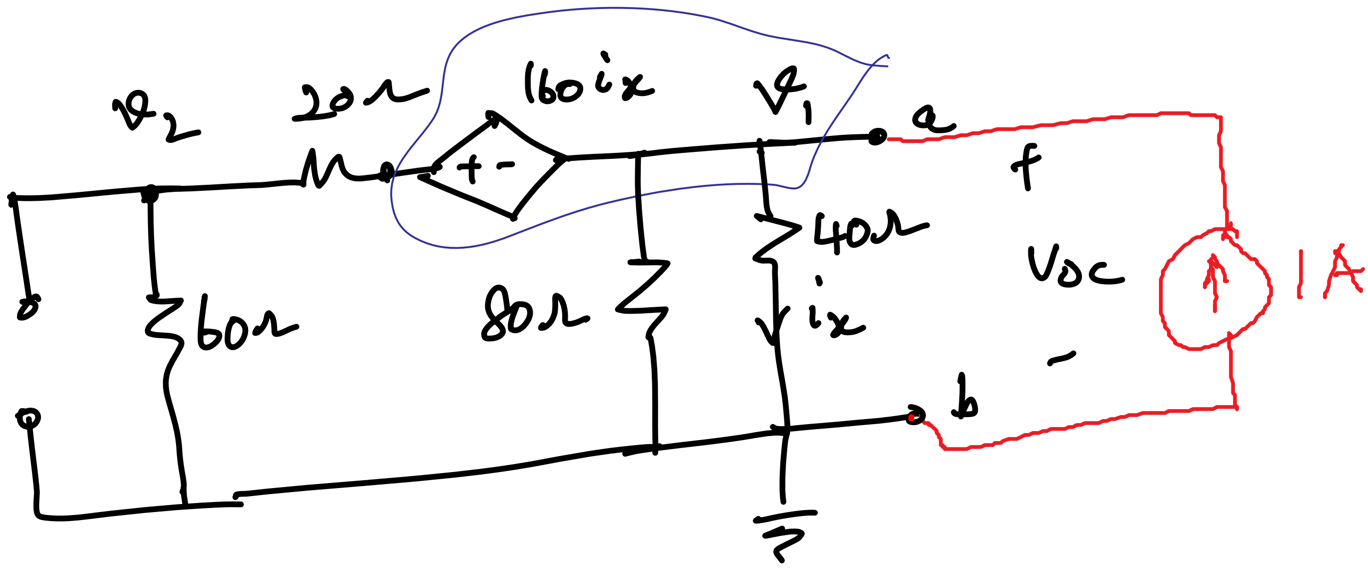
Putting the value of v_2 in eqn(1), we get:

$$v_1 = v_2 \times \frac{4}{23} = 30V.$$

Thus the open circuit voltage $v_{oc} = v_1 = 30V$.

(ii) Find the Thevenin resistance using 'Test source method'.

We shall kill the independent current source of 4A and connect a current source of 1A to the terminals and then calculate the voltage across them.



Applying node voltage analysis method. Note the choice of the reference node and the unknown node voltages.

We can write the current i_x as $i_x = \frac{v_1}{40}$.

Then the dependent voltage source becomes $160i_x = 160x\frac{v_1}{40} = 4v_1$.

Applying KCL at the super node surrounding v_1:

$$-1 + \frac{v_1}{40} + \frac{v_1}{80} + \frac{(v_1+160i_x) - v_2}{20} = 0 \Rightarrow v_1\left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20}\right) - \frac{v_2}{20} = 1 \Rightarrow v_1\frac{23}{4} - v_2 = 20 \qquad (1)$$

Applying KCL at v_2:

$$\frac{v_2}{60} + \frac{v_2 - (v_1 + 160i_x)}{20} = 0$$

Replacing $v_1 + 160ix = 5v_1$:

$$-\frac{5}{20}v_1 + v_2\left(\frac{1}{60} + \frac{1}{20}\right) = 0 \Rightarrow -\frac{v_1}{4} + \frac{v_2}{15} = 0 \qquad (2)$$

(1)+(2)x23=>

$$-v_2 + v_2 \cdot \frac{23}{15} = 20 \Rightarrow v_2 = \frac{20 \times 15}{8}$$

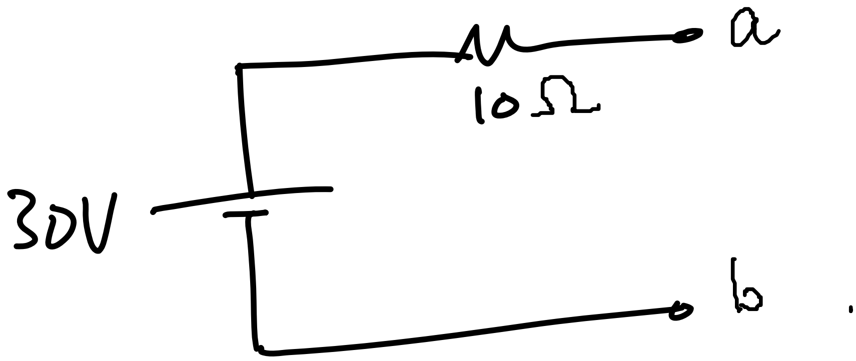
Putting the value of v_2 in eqn(2), we get:

$$-\frac{v_1}{4} + \frac{20 \times 15}{15 \times 8} = 0 \Rightarrow v_1 = 10V.$$

Thevenin resistance is obtained as:

$$R_t = \frac{10V}{1A} = 10\Omega.$$

Thevenin equivalent circuit will be:



(ii)

According to maximum power transfer theorem, the load resistance for maximum power transfer would be $R_L = R_t = 10\Omega$

(iii)

Maximum power delivered to the load at $R_L = 10\Omega$

$$(P_L)_{max} = \left(\frac{30}{10+10}\right)^2 \times 10 = \frac{9}{4} \times 10 = 22.5W$$

(iii)

At $R_L = 0.8 \times 10 = 8\Omega$:

$$P_L = \left(\frac{30}{10+8}\right)^2 \times 8 = 22.22W$$

At $R_L = 1.2 \times 10 = 12\Omega$:

$$P_L = \left(\frac{30}{10+12}\right)^2 \times 12 = 22.31W$$

Hence, power is maximum at $R_L = 10\Omega$.

Q5.

In the circuit given in Figure Q5, Vs models the voltage produced by the generator in a power plant, and Rs model the losses in the generator, distribution wire, and transformers. The three resistances model the various loads connected to the system by a customer. How much does the voltage across the total load change when the customer connects the third load R3 in parallel with the other two loads?

$$V_s = 110V, R_s = 19m\Omega, R_1 = R_2 = 930m\Omega, R_3 = 100m\Omega$$

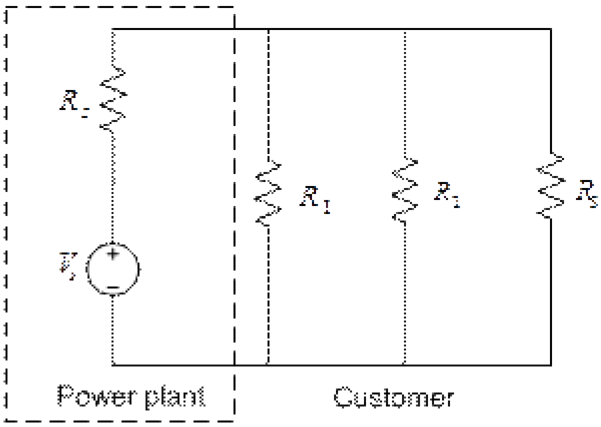


Fig. Q5

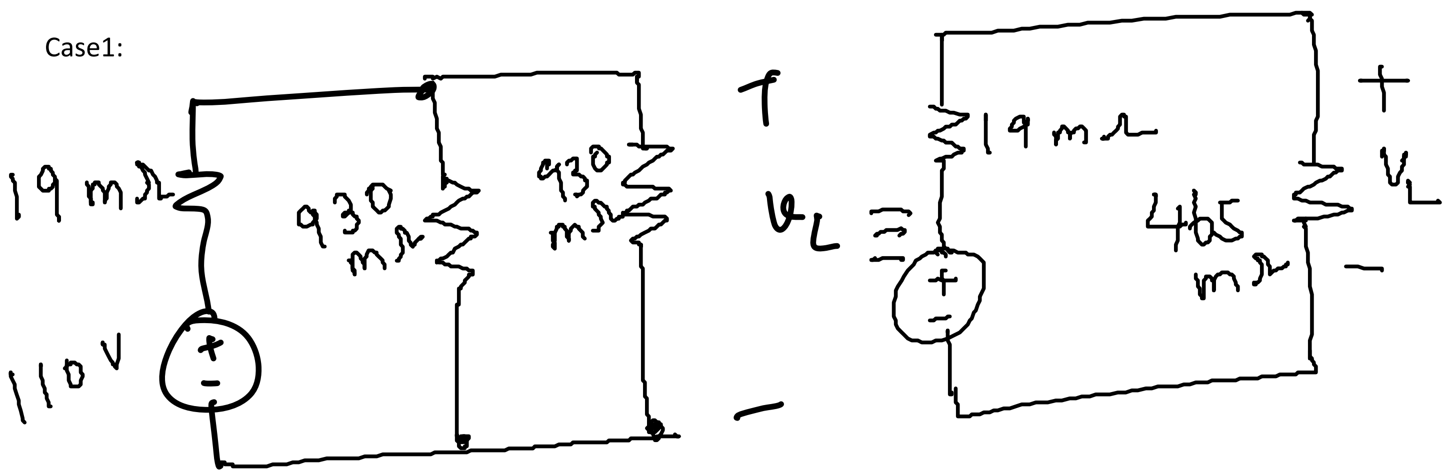
Given a model of a source as an ideal voltage source with a series resistance and the loads modelled as resistances.

To find the change in the voltage when an extra load is connected across the supply.

Analysis:

We shall find the load voltage in the two cases:

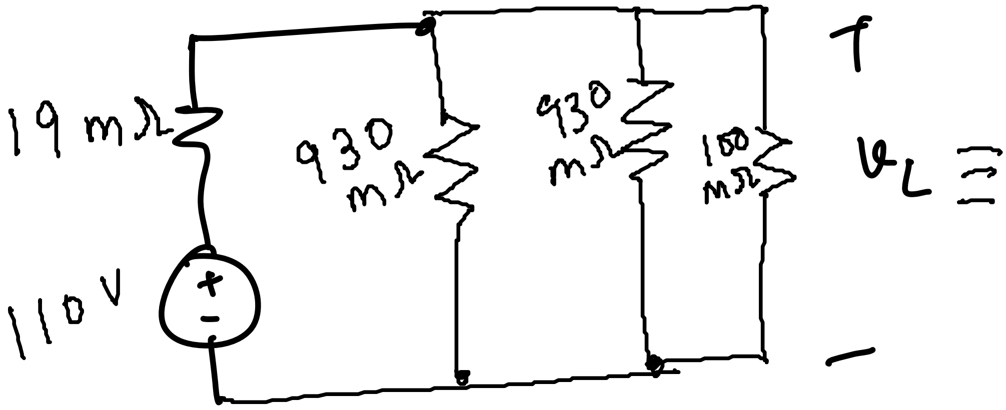
Case1:

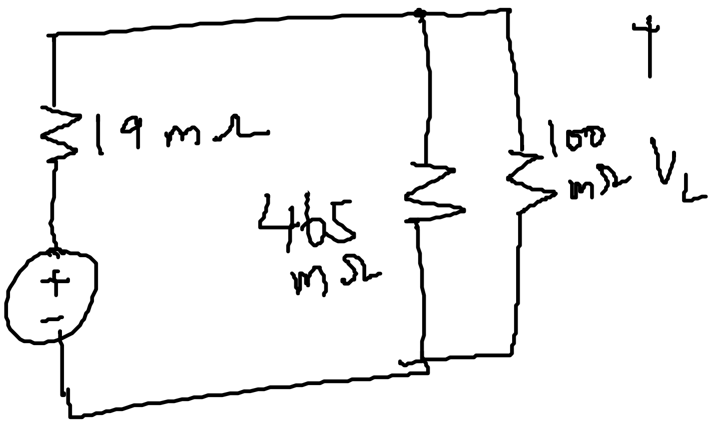


We can use parallel rule for the resistance followed by voltage divider rule:

$$v_L = \frac{465}{465 + 19} \times 110 = 105.68V$$

Case1:





$$465 \parallel 100 = \frac{465 \times 100}{465 + 100} = 82.30 m\Omega$$

We can use voltage divider rule:

$$v_L = \frac{82.30}{82.30 + 19} \times 110 = 89.37V$$

It can be noted that when the large load (equivalent low resistance)is connected across the system, the output voltage drops substantially. This phenomenon is called 'source loading'.