CHAPTER 6

Exercises

E6.1 (a) The frequency of $v_{in}(t) = 2\cos(2\pi \cdot 2000t)$ is 2000 Hz. For this frequency $H(f) = 2\angle 60^\circ$. Thus, $\mathbf{V}_{out} = H(f)\mathbf{V}_{in} = 2\angle 60^\circ \times 2\angle 0^\circ = 4\angle 60^\circ$ and we have $v_{out}(t) = 4\cos(2\pi \cdot 2000t + 60^\circ)$.

(b) The frequency of $v_{in}(t) = \cos(2\pi \cdot 3000t - 20^\circ)$ is 3000 Hz. For this frequency $H(f) = 0$. Thus, $\mathbf{V}_{out} = H(f)\mathbf{V}_{in} = 0 \times 2\angle 0^\circ = 0$ and we have $v_{out}(t) = 0$.

E6.2 The input signal $v(t) = 2\cos(2\pi \cdot 500t + 20^\circ) + 3\cos(2\pi \cdot 1500t)$ has two components with frequencies of 500 Hz and 1500 Hz. For the 500-Hz component we have:

$$\mathbf{V}_{out,1} = H(500)\mathbf{V}_{in} = 3.5\angle 15^\circ \times 2\angle 20^\circ = 7\angle 35^\circ$$

$$v_{out,1}(t) = 7\cos(2\pi \cdot 500t + 35^\circ)$$

For the 1500-Hz component:

$$\mathbf{V}_{out,2} = H(1500)\mathbf{V}_{in} = 2.5\angle 45^\circ \times 3\angle 0^\circ = 7.5\angle 45^\circ$$

$$v_{out,2}(t) = 7.5\cos(2\pi \cdot 1500t + 45^\circ)$$

Thus the output for both components is

$$v_{out}(t) = 7\cos(2\pi \cdot 500t + 35^\circ) + 7.5\cos(2\pi \cdot 1500t + 45^\circ)$$

E6.3 The input signal $v(t) = 1 + 2\cos(2\pi \cdot 1000t) + 3\cos(2\pi \cdot 3000t)$ has three components with frequencies of 0, 1000 Hz and 3000 Hz.

For the dc component, we have

$$v_{out,1}(t) = H(0) \times v_{in,1}(t) = 4 \times 1 = 4$$

For the 1000-Hz component, we have:

$$\mathbf{V}_{out,2} = H(1000)\mathbf{V}_{in,2} = 3\angle 30^\circ \times 2\angle 0^\circ = 6\angle 30^\circ$$

$$v_{out,1}(t) = 6\cos(2\pi \cdot 1000t + 30^\circ)$$

For the 3000-Hz component:

$$\mathbf{V}_{out,3} = H(3000)\mathbf{V}_{in,3} = 0 \times 3\angle 0^\circ = 0$$

$$v_{out,3}(t) = 0$$

Thus, the output for all three components is

$$v_{out}(t) = 4 + 6\cos(2\pi \cdot 1000t + 30^\circ)$$

E6.4 Using the voltage-division principle, we have:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \times \frac{R}{R + j2\pi fL}$$

Then the transfer function is:

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{R}{R + j2\pi fL} = \frac{1}{1 + j2\pi fL/R} = \frac{1}{1 + jf/f_B}$$

E6.5 From Equation 6.9, we have $f_B = 1/(2\pi RC) = 200\text{ Hz}$, and from Equation 6.9, we have $H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1}{1 + jf/f_B}$.

For the first component of the input, the frequency is 20 Hz,
$H(f) = 0.995\angle -5.71^\circ$, $\mathbf{V}_{\text{in}} = 10\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 9.95\angle -5.71^\circ$
Thus the first component of the output is

$$v_{\text{out},1}(t) = 9.95\cos(40\pi t - 5.71^\circ)$$

For the second component of the input, the frequency is 500 Hz,
$H(f) = 0.371\angle -68.2^\circ$, $\mathbf{V}_{\text{in}} = 5\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 1.86\angle -68.2^\circ$
Thus the second component of the output is

$$v_{\text{out},2}(t) = 1.86\cos(40\pi t - 68.2^\circ)$$

For the third component of the input, the frequency is 10 kHz,
$H(f) = 0.020\angle -88.9^\circ$, $\mathbf{V}_{\text{in}} = 5\angle 0^\circ$, and $\mathbf{V}_{\text{out}} = H(f)\mathbf{V}_{\text{in}} = 0.100\angle -88.9^\circ$
Thus the third component of the output is

$$v_{\text{out},2}(t) = 0.100\cos(2\pi \times 10^4 t - 88.9^\circ)$$

Finally, the output with for all three components is:

$$v_{\text{out}}(t) = 9.95\cos(40\pi t - 5.71^\circ) + 1.86\cos(40\pi t - 68.2^\circ)$$
$$+ 0.100\cos(2\pi \times 10^4 t - 88.9^\circ)$$

E6.6 $|H(f)|_{\text{dB}} = 20\log|H(f)| = 20\log(50) = 33.98\text{ dB}$

E6.7 (a) $|H(f)|_{\text{dB}} = 20\log|H(f)| = 15\text{ dB}$

$$\log|H(f)| = 15/20 = 0.75$$
$$H(f) = 10^{0.75} = 5.623$$

(b) $\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right| = 30 \text{ dB}$

$\log\left|H(f)\right| = 30/20 = 1.5$

$H(f) = 10^{1.5} = 31.62$

E6.8 (a) $1000 \times 2^2 = 4000\,\text{Hz}$ is two octaves higher than 1000 Hz.

(b) $1000/2^3 = 125$ Hz is three octaves lower than 1000 Hz.

(c) $1000 \times 10^2 = 100$ kHz is two decades higher than 1000 Hz.

(d) $1000/10 = 100$ Hz is one decade lower than 1000 Hz.

E6.9 (a) To find the frequency halfway between two frequencies on a logarithmic scale, we take the logarithm of each frequency, average the logarithms, and then take the antilogarithm. Thus

$$f = 10^{[\log(100)+\log(1000)]/2} = 10^{2.5} = 316.2 \text{ Hz}$$

is half way between 100 Hz and 1000 Hz on a logarithmic scale.

(b) To find the frequency halfway between two frequencies on a linear scale, we simply average the two frequencies. Thus (100 + 1000)/2 = 550 Hz is halfway between 100 and 1000 Hz on a linear scale.

E6.10 To determine the number of decades between two frequencies we take the difference between the common (base-ten) logarithms of the two frequencies. Thus 20 Hz and 15 kHz are $\log(15 \times 10^3) - \log(20) = 2.875$ decades apart.

Similarly, to determine the number of octaves between two frequencies we take the difference between the base-two logarithms of the two frequencies. One formula for the base-two logarithm of z is

$$\log_2(z) = \frac{\log(z)}{\log(2)} \cong 3.322\log(z)$$

Thus the number of octaves between 20 Hz and 15 kHz is

$$\frac{\log(15 \times 10^3)}{\log(2)} - \frac{\log(20)}{\log(2)} = 9.551$$

E6.11 The transfer function for the circuit shown in Figure 6.17 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1/(j2\pi fC)}{R + 1/(j2\pi fC)} = \frac{1}{1 + j2\pi RCf} = \frac{1}{1 + jf/f_B}$$

in which $f_B = 1/(2\pi RC) = 1000$ Hz. Thus the magnitude plot is approximated by 0 dB below 1000 Hz and by a straight line sloping downward at 20 dB/decade above 1000 Hz. This is shown in Figure 6.18a in the book.

The phase plot is approximated by 0° below 100 Hz, by -90° above 10 kHz and by a line sloping downward between 0° at 100 Hz and -90° at 10 kHz. This is shown in Figure 6.18b in the book.

E6.12 Using the voltage division principle, the transfer function for the circuit shown in Figure 6.19 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{R}{R + 1/(j2\pi fC)} = \frac{j2\pi RC}{1 + j2\pi RCf} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = 1/(2\pi RC)$.

E6.13 Using the voltage division principle, the transfer function for the circuit shown in Figure 6.22 in the book is

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/(2\pi L)$.

E6.14 A first-order filter has a transfer characteristic that decreases by 20 dB/decade below the break frequency. To attain an attenuation of 50 dB the signal frequency must be 50/20 = 2.5 decades below the break frequency. 2.5 decades corresponds to a frequency ratio of $10^{2.5} = 316.2$. Thus to attenuate a 1000 Hz signal by 50 dB the high-pass filter must have a break frequency of 316.2 kHz. Solving Equation 6.22 for capacitance and substituting values, we have

$$C = \frac{1}{2\pi f_B R} = \frac{1}{2\pi \times 1000 \times 316.2 \times 10^3} = 503.3\,\text{pF}$$

E6.15 $C = \dfrac{1}{\omega_0^2 L} = \dfrac{1}{(2\pi f_0)^2 L} = \dfrac{1}{(2\pi 10^6)^2 10 \times 10^{-6}} = 2533\,\text{pF}$

$R = \omega_0 L / Q_s = 1.257\ \Omega$

$B = f_0 / Q_s = 20\,\text{kHz}$

$f_L \cong f_0 - B/2 = 990\,\text{kHz}$

$f_H \cong f_0 + B/2 = 1010\,\text{kHz}$

E6.16 At resonance we have

$$\mathbf{V}_R = \mathbf{V}_s = 1\angle 0^\circ$$
$$\mathbf{V}_L = j\omega_0 L\mathbf{I} = j\omega_0 L\mathbf{V}_s / R = jQ_s\mathbf{V}_s = 50\angle 90^\circ \text{ V}$$
$$\mathbf{V}_C = (1/j\omega_0 C)\mathbf{I} = (1/j\omega_0 C)\mathbf{V}_s / R = -jQ_s\mathbf{V}_s = 50\angle -90^\circ \text{ V}$$

E6.17 $$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \times 5\times 10^6)^2 470\times 10^{-12}} = 2.156\ \mu\text{H}$$
$$Q_s = f_0 / B = (5\times 10^6)/(200\times 10^3) = 25$$
$$R = \frac{1}{\omega_0 C Q_s} = \frac{1}{2\pi\times 5\times 10^6\times 470\times 10^{-12}\times 25} = 2.709\ \Omega$$

E6.18 $$f_0 = \frac{1}{2\pi\sqrt{LC}} = 711.8\text{ kHz} \qquad Q_p = \frac{R}{\omega_0 L} = 22.36 \qquad B = f_0 / Q_p = 31.83\text{ kHz}$$

E6.19 $$Q_p = f_0 / B = 50 \qquad L = \frac{R}{\omega_0 Q_p} = 0.3183\ \mu\text{H} \qquad C = \frac{Q_p}{\omega_0 R} = 795.8\text{ pF}$$

E6.20 A second order lowpass filter with f_0 = 5 kHz is needed. The circuit configuration is shown in Figure 6.34a in the book. The normalized transfer function is shown in Figure 6.34c. Usually we would want a filter without peaking and would design for Q = 1. Given that L = 5 mH, the other component values are

$$R = \frac{2\pi f_0 L}{Q} = 157.1\ \Omega \qquad C = \frac{1}{(2\pi f_0)^2 L} = 0.2026\ \mu\text{F}$$

The circuit is shown in Figure 6.39 in the book.

E6.21 We need a bandpass filter with $f_L = 45$ kHz and $f_H = 55$ kHz. Thus we have

$$f_0 \cong \frac{f_L + f_H}{2} = 50\text{ kHz} \qquad B = f_H - f_L = 10\text{ kHz} \qquad Q = f_0 / B = 5$$

$$R = \frac{2\pi f_0 L}{Q} = 62.83\ \Omega \qquad C = \frac{1}{(2\pi f_0)^2 L} = 10.13\text{ nF}$$

The circuit is shown in Figure 6.40 in the book.

E6.22 The files Example_6_8 and Example_6_9 can be found in the MATLAB folder on the OrCAD disk. The results should be similar to Figures 6.42 and 6.44.

E6.23 (a) Rearranging Equation 6.56, we have

$$\frac{\tau}{T} = \frac{a}{1-a} = \frac{0.9}{1-0.9} = 0.9$$

Thus we have $\tau = 9T$.

(b) From Figure 6.49 in the book we see that the step response of the digital filter reaches 0.632 at approximately $n = 9$. Thus the speed of response of the *RC* filter and the corresponding digital filter are comparable.

E6.24 Writing a current equation at the node joining the resistance and capacitance, we have

$$\frac{y(t)}{R} + C\frac{d[y(t)-x(t)]}{dt} = 0$$

Multiplying both sides by R and using the fact that the time constant is $\tau = RC$, we have

$$y(t) + \tau\frac{dy(t)}{dt} - \tau\frac{dx(t)}{dt} = 0$$

Next we approximate the derivatives as

$$\frac{dx(t)}{dt} \cong \frac{\Delta x}{\Delta t} = \frac{x(n)-x(n-1)}{T} \quad \text{and} \quad \frac{dy(t)}{dt} \cong \frac{\Delta y}{\Delta t} = \frac{y(n)-y(n-1)}{T}$$

which yields

$$y(n) + \tau\frac{y(n)-y(n-1)}{T} - \tau\frac{x(n)-x(n-1)}{T} = 0$$

Solving for $y(n)$, we obtain

$$y(n) = a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

in which

$$a_1 = b_0 = -b_1 = \frac{\tau/T}{1+\tau/T}$$

E6.25 (a) Solving Equation 6.58 for d and substituting values, we obtain

$$d = \frac{f_s}{2f_{notch}} = \frac{10^4}{2\times 500} = 10$$

(b) Repeating for $f_{notch} = 300$ Hz, we have

$$d = \frac{f_s}{2f_{notch}} = \frac{10^4}{2 \times 300} = 16.67$$

However, d is required to be an integer value so we cannot obtain a notch filter for 300 Hz exactly for this sampling frequency. (Possibly other more complex filters could provide the desired performance.)

Answers for Selected Problems

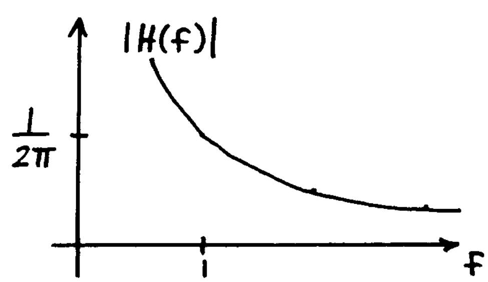
P6.8* $v_{out}(t) = 10 + 3.5\cos(2\pi 2500t - 15^\circ) + 2.5\cos(2\pi 7500t - 135^\circ)$

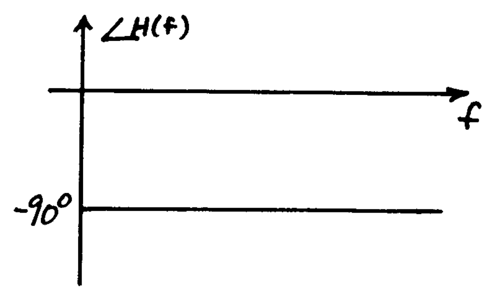
P6.11* $H(5000) = 0.5\angle 45^\circ$

P6.12* $f = 250 \text{ Hz}$ $\qquad H(250) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = 3\angle -45^\circ$

P6.13* $v_o(t) = 2$

P6.14* $H(f) = \frac{-j}{2\pi f}$



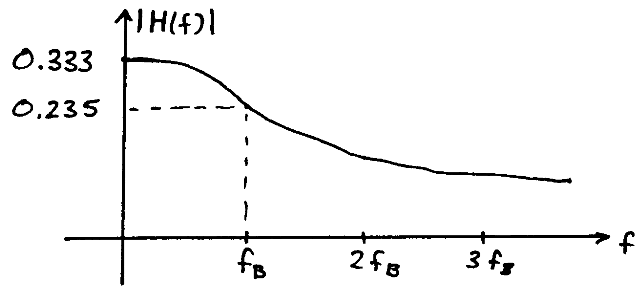


P6.23* For $\angle H(f) = -1^\circ$, we have $f = 0.01746 f_B$.
For $\angle H(f) = -10^\circ$, we have $f = 0.1763 f_B$.
For $\angle H(f) = -89^\circ$, we have $f = 57.29 f_B$.

P6.25* $v_{out}(t) = 4.472\cos(500\pi t - 26.57^\circ) + 3.535\cos(1000\pi t - 45^\circ)$
$\qquad + 2.236\cos(2000\pi t - 63.43^\circ)$

P6.30* $f_B = 11.94$ Hz

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1/3}{1 + j(f/f_B)}$$

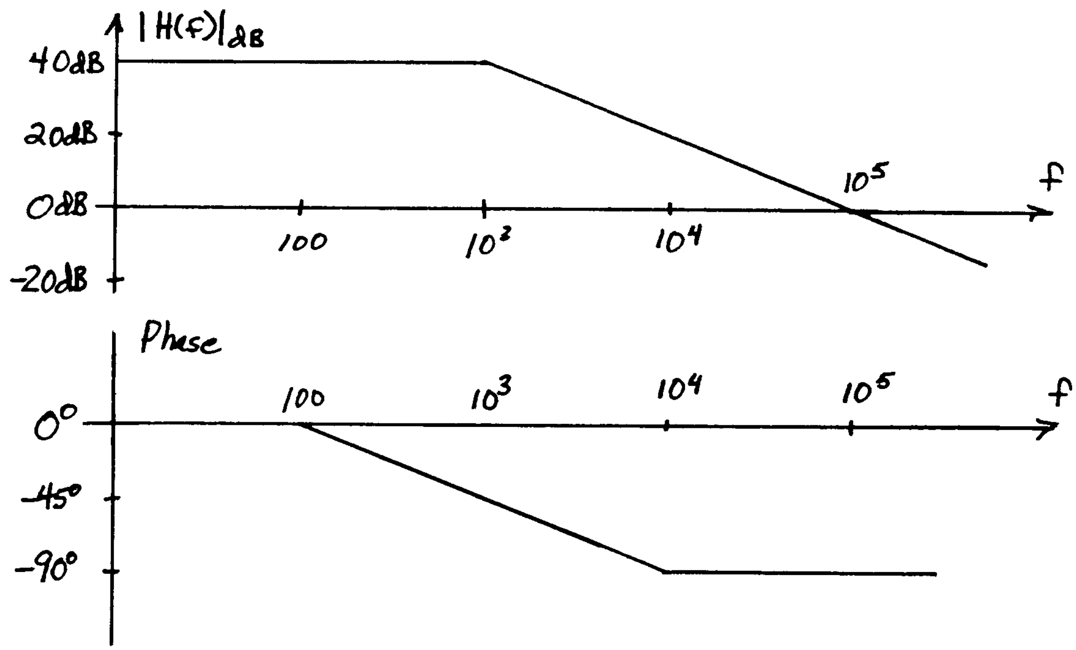


P6.40* (a) $|H(f)| = 0.3162$ (b) $|H(f)| = 3.162$

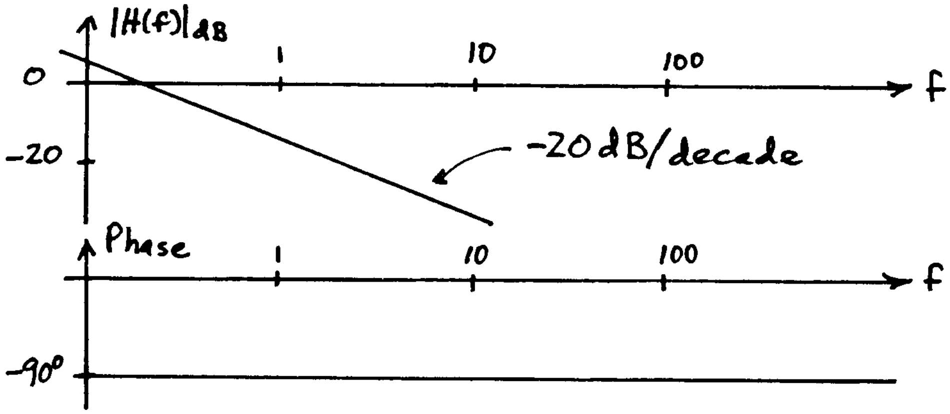
P6.41* (a) 547.7 Hz (b) 1550 Hz

P6.46* (a) $H(f) = \dfrac{1}{[1 + j(f/f_B)]^2}$ (b) $f_{3dB} = 0.6436 f_B$

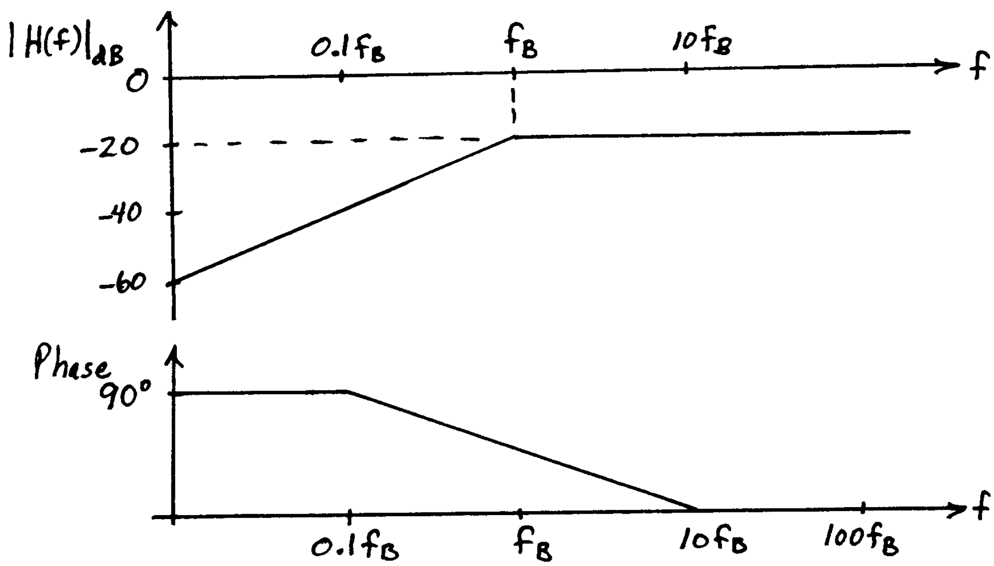
P6.52*



P6.60*



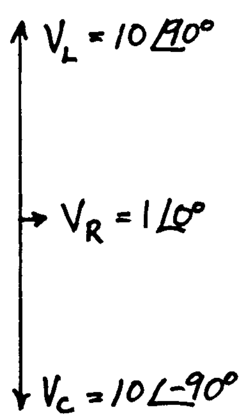
P6.64*



P6.65* $v_{out}(t) = 3.536\cos(2000\pi t + 45^\circ)$

P6.72*

$$f_0 = 1.125\text{ MHz}$$
$$Q_s = 10$$
$$B = 112.5\text{ kHz}$$
$$f_H \cong 1.181\text{ MHz}$$
$$f_L \cong 1.069\text{ MHz}$$



P6.75* $L = 79.57\ \mu\text{H}$ $\quad \mathbf{V}_C = 20\angle -90^\circ$

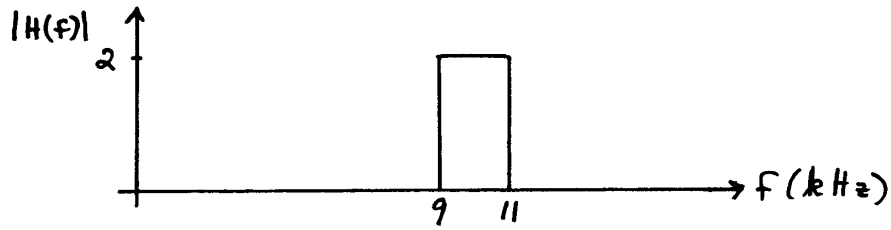
$C = 318.3\ \text{pF}$

P6.79* $f_0 = 1.592\ \text{MHz}$

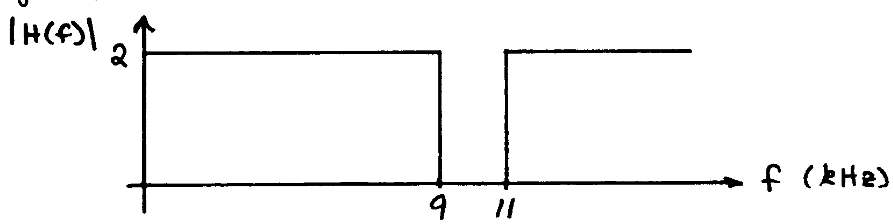
$Q_p = 10.00$

$B = 159.2\ \text{kHz}$

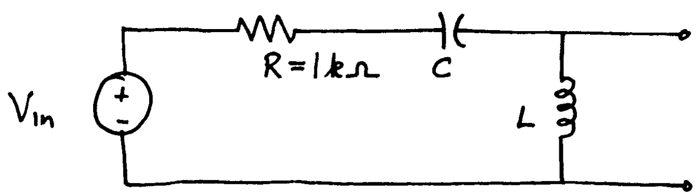
P6.84* Bandpass filter:



Band-reject filter:



P6.88*



$L = 1.592\ \text{mH}$ $\quad C = 1592\ \text{pF}$

P6.104* $L = \dfrac{Q_s}{\omega_0}$ and $C = \dfrac{1}{\omega_0 Q_s}$

$$y(n) = \frac{\omega_0 T + 2Q_s}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T} y(n-1) - \frac{Q_s}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T} y(n-2)$$
$$+ \frac{\omega_0 T}{Q_s + \omega_0^2 T^2 Q_s + \omega_0 T}[x(n) - x(n-1)]$$

Practice Test

T6.1 All real-world signals (which are usually time-varying currents or voltages) are sums of sinewaves of various frequencies, amplitudes, and phases. The transfer function of a filter is a function of frequency that shows how the amplitudes and phases of the input components are altered to produce the output components.

T6.2 Applying the voltage-division principle, we have:

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R}$$
$$= \frac{j(f/f_B)}{1 + j(f/f_B)}$$

in which $f_B = R/2\pi L = 1000$ Hz. The input signal has components with frequencies of 0 (dc), 500 Hz, and 1000 Hz. The transfer function values for these frequencies are: $H(0) = 0$, $H(500) = 0.4472\angle 63.43°$, and $H(1000) = 0.7071\angle 45°$. Applying the transfer function values to each of the input components, we have

$H(0) \times 3 = 0$, $H(500) \times 4\angle 0° = 1.789\angle 63.43°$, and

$H(1000) \times 5\angle -30° = 3.535\angle 15°$. Thus, the output is

$$v_{out}(t) = 1.789\cos(1000\pi t - 63.43°) + 3.535\cos(2000\pi t + 15°)$$

T6.3 (a) The slope of the low-frequency asymptote is +20 dB/decade.

(b) The slope of the high-frequency asymptote is zero.

(c) The coordinates at which the asymptotes meet are 20log(50) = 34 dB and 200 Hz.

(d) This is a first-order highpass filter.

(e) The break frequency is 200 Hz.

T6.4 (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 1125\,\text{Hz}$

(b) $Q_s = \dfrac{2\pi f_0 L}{R} = 28.28$

(c) $B = \dfrac{f_0}{Q_s} = 39.79\,\text{Hz}$

(d) At resonance, the impedance equals the resistance, which is $5\,\Omega$.

(e) At dc, the capacitance becomes an open circuit so the impedance is infinite.

(f) At infinite frequency the inductance becomes an open circuit, so the impedance is infinite.

T6.5 (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 159.2\,\text{kHz}$

(b) $Q_p = \dfrac{R}{2\pi f_0 L} = 10.00$

(c) $B = \dfrac{f_0}{Q_p} = 15.92\,\text{kHz}$

(d) At resonance, the impedance equals the resistance which is $10\,\text{k}\Omega$.

(e) At dc, the inductance becomes a short circuit, so the impedance is zero.

(f) At infinite frequency the capacitance becomes a short circuit, so the impedance is zero.

T6.6 (a) This is a first-order circuit because there is a single energy-storage element (L or C). At very low frequencies, the capacitance approaches an open circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the capacitance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus, we have a first-order lowpass filter.

(b) This is a second-order circuit because there are two energy-storage elements (L or C). At very low frequencies, the capacitance approaches an open circuit, the inductance approaches a short circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the inductance approaches an open circuit, the capacitance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus we have a second-order lowpass filter.

(c) This is a second-order circuit because there are two energy-storage elements (L or C). At very low frequencies, the inductance approaches a short circuit, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At very high frequencies, the capacitance approaches a short circuit, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. At the resonant frequency, the LC combination becomes an open circuit, the current is zero, $\mathbf{V}_{out} = 0$, and $|H| = 0$. Thus, we have a second-order band-reject (or notch) filter.

(d) This is a first-order circuit because there is a single energy-storage element (L or C). At very low frequencies, the inductance approaches a short circuit, $\mathbf{V}_{out} = 0$, and $|H| = 0$. At very high frequencies the inductance approaches an open circuit, the current is zero, $\mathbf{V}_{out} = \mathbf{V}_{in}$ and $|H| = 1$. Thus we have a first-order highpass filter.

T6.7 One set of commands is:

```
f = logspace(1,4,400);
H = 50*i*(f/200)./(1+i*f/200);
semilogx(f,20*log10(abs(H)))
```

Other sets of commands are also correct. You can use MATLAB to see if your commands give a plot equivalent to: