Chapter 11

1. 2002/2003 Sem 2 Q10(a)

   Use the method of separation of variable to obtain solutions $u(x, y)$ of the equation

   $$u_x + u_y = 2(x + y)u.$$

   Ans: $u(x, y) = Ce^{kx-ky+x^2+y^2}$

Chapter 11

1. **2002/2003 Sem 2 Q10(a)**

   Let $u(x, y) = X(x)Y(y)$ so that $u_x = X'Y$ and $u_y = XY'$.

   Hence $u_x + u_y = 2(x + y)u$ becomes

$$X'Y + XY' = 2(x + y)XY$$

   or

$$\frac{X'}{X} - 2x = -\frac{Y'}{Y} + 2y = k$$

$$\frac{X'}{X} = k + 2x \Rightarrow \int \frac{dX}{X} = \int (k + 2x)dx \Rightarrow \ln|X| = kx + x^2 + c \Rightarrow X = Ae^{(k+x)x}$$

and

$$\frac{Y'}{Y} = -k + 2y \Rightarrow \int \frac{dY}{Y} = \int (-k + 2y)dy \Rightarrow \ln|Y| = -ky + y^2 + d \Rightarrow Y = Be^{(-k+y)y}$$

So

$$u(x, y) = Ce^{(kx - ky + x^2 + y^2)}$$

where $C = AB$.

3. **2005/2006 Sem 2 Q4**

Use the method of separation of variables to find $u(x, y)$ that satisfies the partial differential equation

$$u_{xy} + \frac{\sin y}{x+2} u = 0,$$

given that $u\left(2, \frac{\pi}{2}\right) = 10$ and $u\left(7, \frac{\pi}{2}\right) = 15$.

Ans: $u(x, y) = 5(x+2)^{1/2} e^{\frac{1}{2}\cos y}$

3. **2005/2006 Sem 2 Q4**

Let $u(x, y) = X(x)Y(y)$. Then the p.d.e can be rewritten as:

$$\begin{aligned} u_{xy} \; + \; \frac{\sin y}{x+2} u \; &= \; 0 \\ \Rightarrow \quad X'(x)Y'(y) + \frac{\sin y}{x+2} X(x)Y(y) &= 0 \\ \Rightarrow \quad X'(x)Y'(y) &= -\frac{\sin y}{x+2} X(x)Y(y) \\ \Rightarrow \quad (x+2)\frac{X'}{X} &= -\sin y \frac{Y}{Y'} \end{aligned}$$

The two sides of the last equation above gives two o.d.e's

$$\frac{X'}{X} = \frac{k}{(x+2)} \quad \text{and} \quad \frac{Y'}{Y} = -\frac{1}{k}\sin y.$$

Solve each of the two o.d.e's separately:

$$\int \frac{dX}{X} = \int \frac{k}{(x+2)}dx \Rightarrow \ln|X| = k\ln(x+2) + A \Rightarrow X = c_1(x+2)^k$$

and

$$\int \frac{dY}{Y} = \int -\frac{1}{k}\sin y dy \Rightarrow \ln|Y| = \frac{1}{k}\cos y + B \Rightarrow Y = c_2 e^{\frac{1}{k}\cos y}.$$

So $u(x,y) = C(x+2)^k e^{\frac{1}{k}\cos y}$.

To solve for C and k,

$$u(2, \frac{\pi}{2}) = C \cdot 4^k e^{\frac{1}{k} \cos \frac{\pi}{2}} = 10 \Rightarrow C \cdot 4^k = 10 - - - (1)$$

$$u(7, \frac{\pi}{2}) = C \cdot 9^k e^{\frac{1}{k} \cos \frac{\pi}{2}} = 15 \Rightarrow C \cdot 9^k = 15 - - - (2)$$

On solving (1) and (2) we get $C = 5$ and $k = 1/2$.

i.e. $u(x, y) = 5(x + 2)^{1/2} e^{2 \cos y}$