

## CHAPTER 12

### Exercises

- E12.1** (a)  $v_{GS} = 1 \text{ V}$  and  $v_{DS} = 5 \text{ V}$ : Because we have  $v_{GS} < V_{to}$ , the FET is in cutoff.
- (b)  $v_{GS} = 3 \text{ V}$  and  $v_{DS} = 0.5 \text{ V}$ : Because  $v_{GS} > V_{to}$  and  $v_{GD} = v_{GS} - v_{DS} = 2.5 > V_{to}$ , the FET is in the triode region.
- (c)  $v_{GS} = 3 \text{ V}$  and  $v_{DS} = 6 \text{ V}$ : Because  $v_{GS} > V_{to}$  and  $v_{GD} = v_{GS} - v_{DS} = -3 \text{ V} < V_{to}$ , the FET is in the saturation region.
- (d)  $v_{GS} = 5 \text{ V}$  and  $v_{DS} = 6 \text{ V}$ : Because  $v_{GS} > V_{to}$  and  $v_{GD} = v_{GS} - v_{DS} = 1 \text{ V}$  which is less than  $V_{to}$ , the FET is in the saturation region.

- E12.2** First we notice that for  $v_{GS} = 0$  or  $1 \text{ V}$ , the transistor is in cutoff, and the drain current is zero. Next we compute the drain current in the saturation region for each value of  $v_{GS}$ :

$$K = \frac{1}{2} KP(W/L) = \frac{1}{2} (50 \times 10^{-6})(80/2) = 1 \text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2$$

The boundary between the triode and saturation regions occurs at

$$v_{DS} = v_{GS} - V_{to}$$

$v_{GS} \text{ (V)}$	$i_D \text{ (mA)}$	$v_{DS} \text{ at boundary}$
2	1	1
3	4	2
4	9	3

In saturation,  $i_D$  is constant, and in the triode region the characteristics are parabolas passing through the origin. The apex of the parabolas are on the boundary between the triode and saturation regions. The plots are shown in Figure 12.7 in the book.

- E12.3** First we notice that for  $v_{GS} = 0$  or  $-1 \text{ V}$ , the transistor is in cutoff, and the drain current is zero. Next we compute the drain current in the saturation region for each value of  $v_{GS}$ :

$$K = \frac{1}{2} KP(W / L) = \frac{1}{2} (25 \times 10^{-6}) (200 / 2) = 1.25 \text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2$$

The boundary between the triode and saturation regions occurs at

$$v_{DS} = v_{GS} - V_{to}.$$

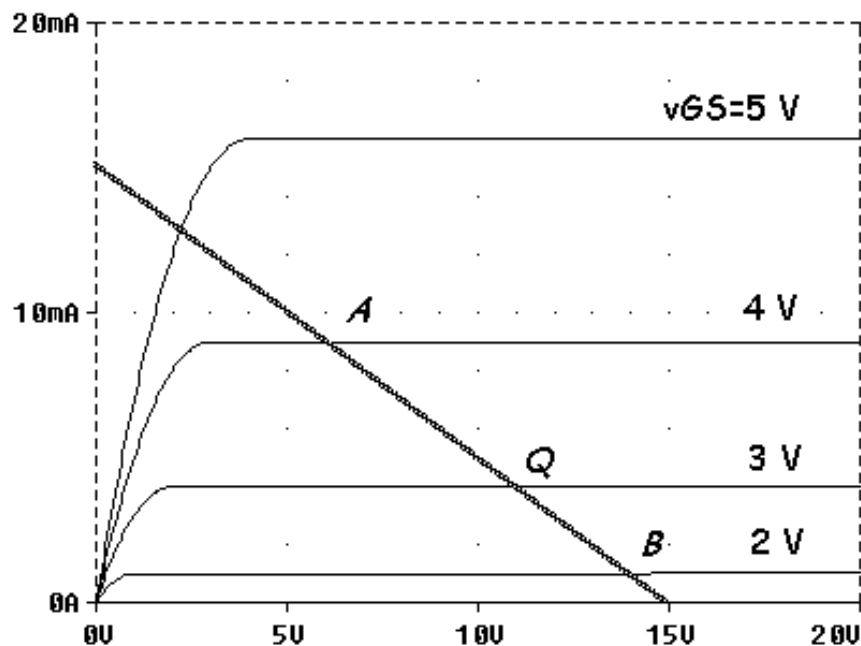
$v_{GS}$ (V)	$i_D$ (mA)	$v_{DS}$ at boundary
-2	1.25	-1
-3	5	-2
-4	11.25	-3

In saturation,  $i_D$  is constant, and in the triode region the characteristics are parabolas passing through the origin. The apex of the parabolas are on the boundary between the triode and saturation regions. The plots are shown in Figure 12.9 in the book.

**E12.4** We have

$$v_{GS}(t) = v_{in}(t) + V_{GG} = \sin(2000\pi t) + 3$$

Thus we have  $V_{GS\max} = 4 \text{ V}$ ,  $V_{GSQ} = 3 \text{ V}$ , and  $V_{GS\min} = 2 \text{ V}$ . The characteristics and the load line are:



For  $v_{in} = +1$  we have  $v_{GS} = 4$  and the instantaneous operating point is  $A$ . Similarly for  $v_{in} = -1$  we have  $v_{GS} = 2$  V and the instantaneous operating point is at  $B$ . We find  $V_{DSQ} \cong 11$  V,  $V_{DS\min} \cong 6$  V,  $V_{DS\max} \cong 14$  V.

**E12.5** First, we compute

$$V_G = V_{DD} \frac{R_2}{R_1 + R_2} = 7 \text{ V}$$

$$\text{and } K = \frac{1}{2} KP(W/L) = \frac{1}{2} (50 \times 10^{-6})(200/10) = 0.5 \text{ mA/V}^2$$

As in Example 12.2, we need to solve:

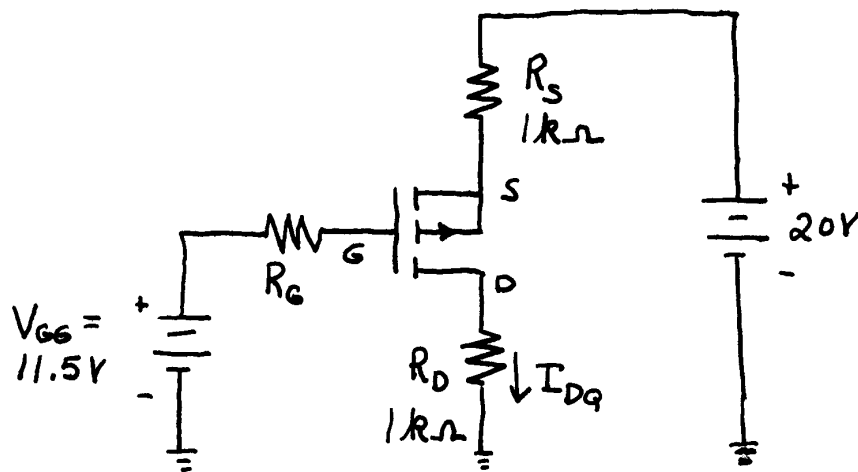
$$V_{GSQ}^2 + \left( \frac{1}{R_S K} - 2V_{to} \right) V_{GSQ} + (V_{to})^2 - \frac{V_G}{R_S K} = 0$$

Substituting values, we have

$$V_{GSQ}^2 - V_{GSQ} - 6 = 0$$

The roots are  $V_{GSQ} = -2$  V and 3 V. The correct root is  $V_{GSQ} = 3$  V which yields  $I_{DQ} = K(V_{GSQ} - V_{to})^2 = 2$  mA. Finally, we have  $V_{DSQ} = V_{DD} - R_S I_{DQ} = 16$  V.

**E12.6** First, we replace the gate bias circuit with its equivalent circuit:



Then we can write the following equations:

$$K = \frac{1}{2} KP(W/L) = \frac{1}{2} (25 \times 10^{-6})(400/10) = 0.5 \text{ mA/V}^2$$

$$V_{GG} = 11.5 = V_{GSQ} - R_S I_{DQ} + 20 \quad (1)$$

$$I_{DQ} = K(V_{GSQ} - V_{to})^2 \quad (2)$$

Using Equation (2) to substitute into Equation (1), substituting values, and rearranging, we have  $V_{GSQ}^2 - 16 = 0$ . The roots of this equation are  $V_{GSQ} = \pm 4$  V. However  $V_{GSQ} = -4$  V is the correct root for a PMOS transistor. Thus we have

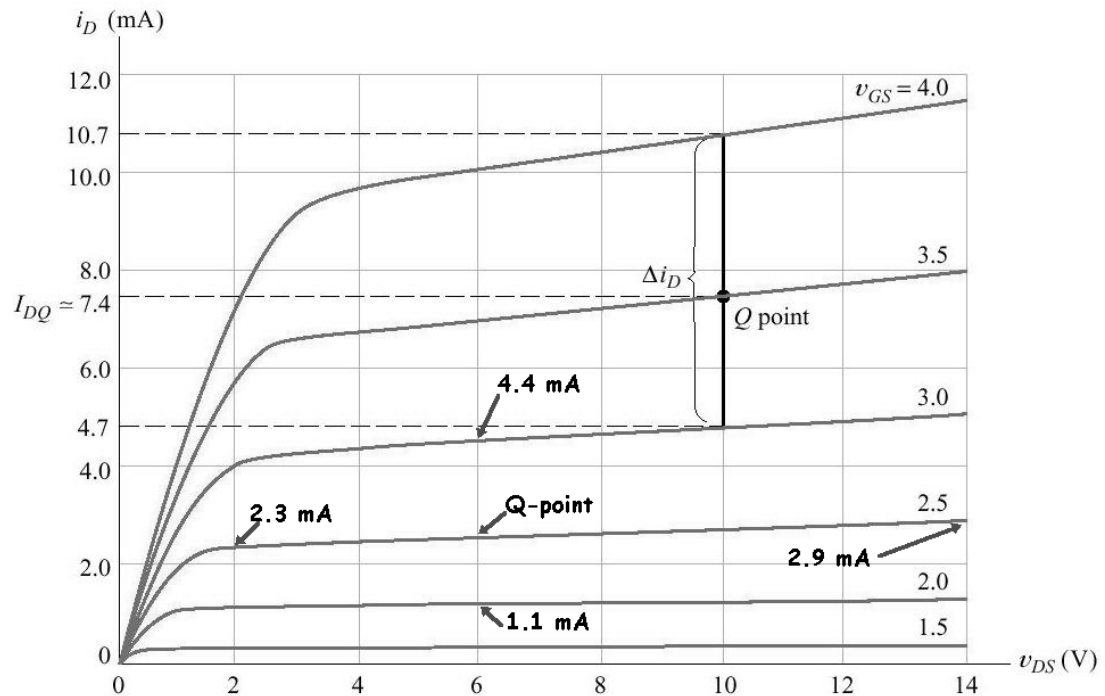
$$I_{DQ} = 4.5 \text{ mA}$$

and

$$V_{DSQ} = R_s I_{DQ} + R_D I_{DQ} - 20 = -11 \text{ V}.$$

**E12.7** From Figure 12.21 at an operating point defined by  $V_{GSQ} = 2.5$  V and  $V_{DSQ} = 6$  V, we estimate

$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{(4.4 - 1.1) \text{ mA}}{1 \text{ V}} = 3.3 \text{ mS}$$



$$1/r_d = \frac{\Delta i_D}{\Delta v_{GS}} \cong \frac{(2.9 - 2.3) \text{ mA}}{(14 - 2) \text{ V}} = 0.05 \times 10^{-3}$$

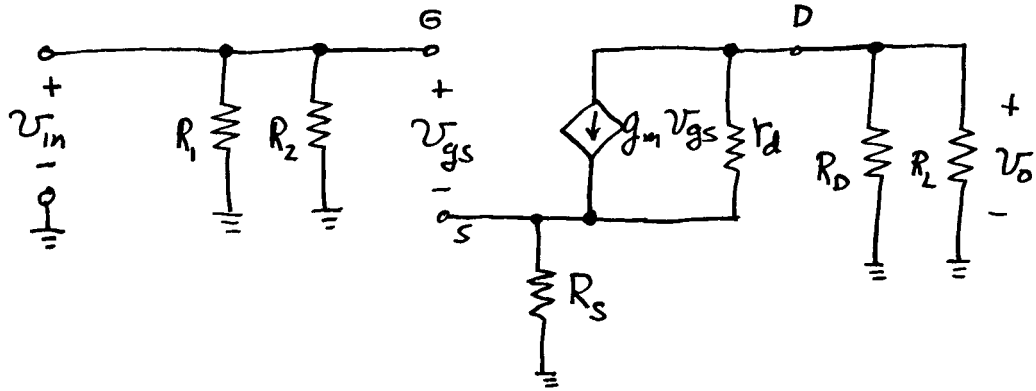
Taking the reciprocal, we find  $r_d = 20 \text{ k}\Omega$ .

$$\text{E12.8} \quad g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{Q\text{-point}} = \left. \frac{\partial}{\partial v_{GS}} K(v_{GS} - V_{to})^2 \right|_{Q\text{-point}} = 2K(V_{GSQ} - V_{to})$$

$$\text{E12.9} \quad R'_L = \frac{1}{1/r_d + 1/R_D + 1/R_L} = R_D = 4.7 \text{ k}\Omega$$

$$A_{voc} = -g_m R'_L = -(1.77 \text{ mS}) \times (4.7 \text{ k}\Omega) = -8.32$$

**E12.10** For simplicity we treat  $r_d$  as an open circuit and let  $R'_L = R_D \parallel R_L$ .



$$v_{in} = v_{gs} + R_s g_m v_{gs}$$

$$v_o = -R'_L g_m v_{gs}$$

$$A_v = \frac{v_o}{v_{in}} = \frac{-R'_L g_m}{1 + R'_L g_m}$$

$$\text{E12.11} \quad R'_L = R_D \parallel R_L = 3.197 \text{ k}\Omega$$

$$A_v = \frac{v_o}{v_{in}} = \frac{-R'_L g_m}{1 + R'_L g_m} = \frac{-(3.197 \text{ k}\Omega)(1.77 \text{ mS})}{1 + (2.7 \text{ k}\Omega)(1.77 \text{ mS})} = -0.979$$

**E12.12** The equivalent circuit is shown in Figure 12.28 in the book from which we can write

$$v_{in} = 0 \quad v_{gs} = -v_x \quad i_x = \frac{v_x}{R_s} + \frac{v_x}{r_d} - g_m v_{gs} = \frac{v_x}{R_s} + \frac{v_x}{r_d} + g_m v_x$$

Solving, we have

$$R_o = \frac{v_x}{i_x} = \frac{1}{g_m + \frac{1}{R_s} + \frac{1}{r_d}}$$

**E12.13** Refer to the small-signal equivalent circuit shown in Figure 12.30 in the book. Let  $R'_L = R_D \parallel R_L$ .

$$v_{in} = -v_{gs}$$

$$v_o = -R'_L g_m v_{gs}$$

$$A_v = v_o / v_{in} = R'_L g_m$$

$$i_{in} = v_{in} / R_s - g_m v_{gs} = v_{in} / R_s + g_m v_{in}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m + 1/R_s}$$

If we set  $v(t) = 0$ , then we have  $v_{gs} = 0$ . Removing the load and looking back into the amplifier, we see the resistance  $R_D$ . Thus we have  $R_o = R_D$ .

**E12.14** See Figure 12.34 in the book.

**E12.15** See Figure 12.35 in the book.

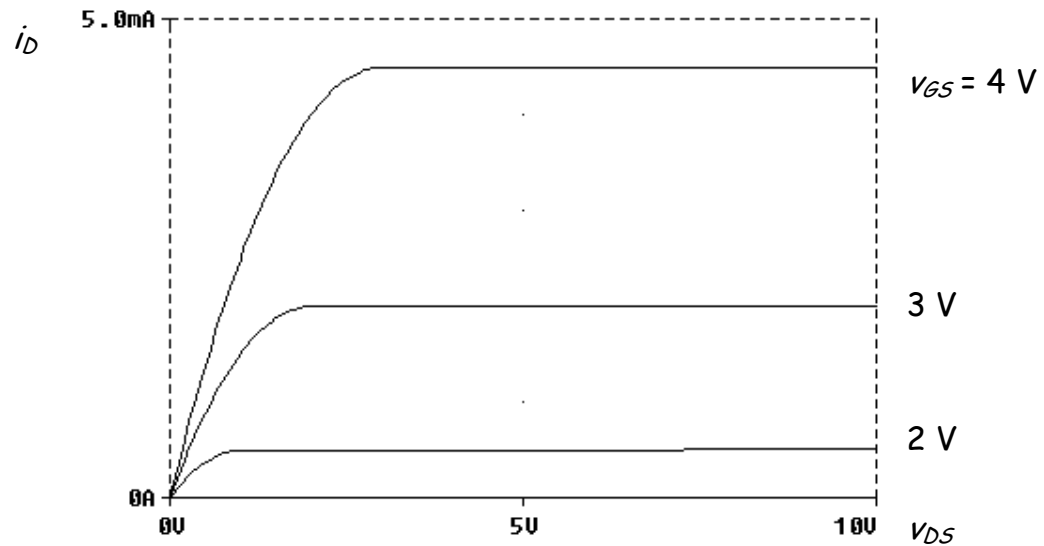
### Answers for Selected Problems

**P12.3\*** (a) Saturation  $i_D = 2.25 \text{ mA}$

(b) Triode  $i_D = 2 \text{ mA}$

(c) Cutoff  $i_D = 0$

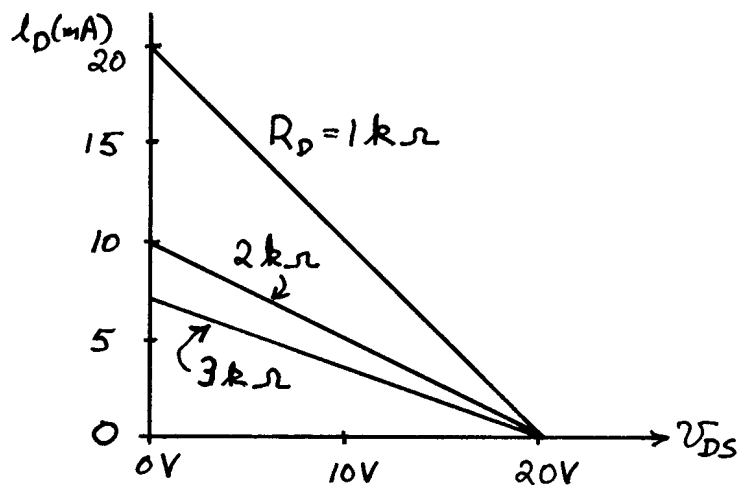
P12.4\*



P12.11\*  $V_{to} = 1.5$  V  
 $K = 0.8$  mA/V<sup>2</sup>

P12.15\*  $v_{GS} = -2.5$  V

P12.17\*



The load line rotates around the point  $(V_{DD}, 0)$  as the resistance changes.

P12.19\* The gain is zero.

P12.21\*  $R_{D\max} = 3.778$  k $\Omega$

**P12.27\***  $I_{DQ} = 3.432 \text{ mA}$   
 $V_{DSQ} = 16.27 \text{ V}$

**P12.28\***  $R_S = 3 \text{ k}\Omega$   
 $R_2 = 2 \text{ M}\Omega$

**P12.29\***  $R_S = 400 \Omega$   $R_1 = 2.583 \text{ M}\Omega$ .

**P12.34\***  $V_{DSQ} = V_{GSQ} = 5.325 \text{ V}$   
 $I_{DQ} = 4.675 \text{ mA}$

**P12.40\***  $g_m = 2KV_{DSQ}$

**P12.41\***  $r_d = \frac{1}{2K(V_{GSQ} - V_{to} - V_{DSQ})}$

**P12.50\*** (a)  $V_{GSQ} = 3 \text{ V}$   
 $I_{DQ} = 10 \text{ mA}$   
 $g_m = 0.01 \text{ S}$

(b)  $A_v = -5$   
 $R_{in} = 255 \text{ k}\Omega$   
 $R_o = 1 \text{ k}\Omega$

**P12.53\***  $R_o = \frac{1}{1/R_D + g_m} = 253 \Omega$

**P12.56\***  $R_S = 3.382 \text{ k}\Omega$

$A_v = 0.6922$

$R_{in} = 666.7 \text{ k}\Omega$

$R_o = 386.9 \Omega$



## Practice Test

**T12.1** Drain characteristics are plots of  $i_D$  versus  $v_{DS}$  for various values of  $v_{GS}$ .

First, we notice that for  $v_{GS} = 0.5 \text{ V}$ , the transistor is in cutoff, and the drain current is zero, because  $v_{GS}$  is less than the threshold voltage  $V_{to}$ . Thus, the drain characteristic for  $v_{GS} = 0.5 \text{ V}$  lies on the horizontal axis.

Next, we compute the drain current in the saturation region for  $v_{GS} = 4 \text{ V}$ .

$$K = \frac{1}{2} KP(W/L) = \frac{1}{2} (80 \times 10^{-6})(100/4) = 1 \text{ mA/V}^2$$

$$i_D = K(v_{GS} - V_{to})^2 = K(4 - 1)^2 = 9 \text{ mA for } v_{DS} > v_{GS} - V_{to} = 3 \text{ V}$$

Thus, the characteristic is constant at 9 mA in the saturation region.

The transistor is in the triode region for  $v_{DS} < v_{GS} - V_{to} = 3 \text{ V}$ , and the drain current (in mA) is given by

$$i_D = K[2(v_{GS} - V_{to})v_{DS} - v_{DS}^2] = 6v_{DS} - v_{DS}^2$$

with  $v_{DS}$  in volts. This plots as a parabola that passes through the origin and reaches its apex at  $i_D = 9 \text{ mA}$  and  $v_{DS} = 3 \text{ V}$ .

The drain characteristic for  $v_{GS} = 4 \text{ V}$  is identical to that of Figure 12.11 in the book.

**T12.2** We have  $v_{GS}(t) = v_{in}(t) + V_{GG} = \sin(2000\pi t) + 3 \text{ V}$ . Thus, we have  $V_{GS\max} = 4 \text{ V}$ ,  $V_{GSQ} = 3 \text{ V}$ , and  $V_{GS\min} = 2 \text{ V}$ . Writing KVL around the drain circuit, we have

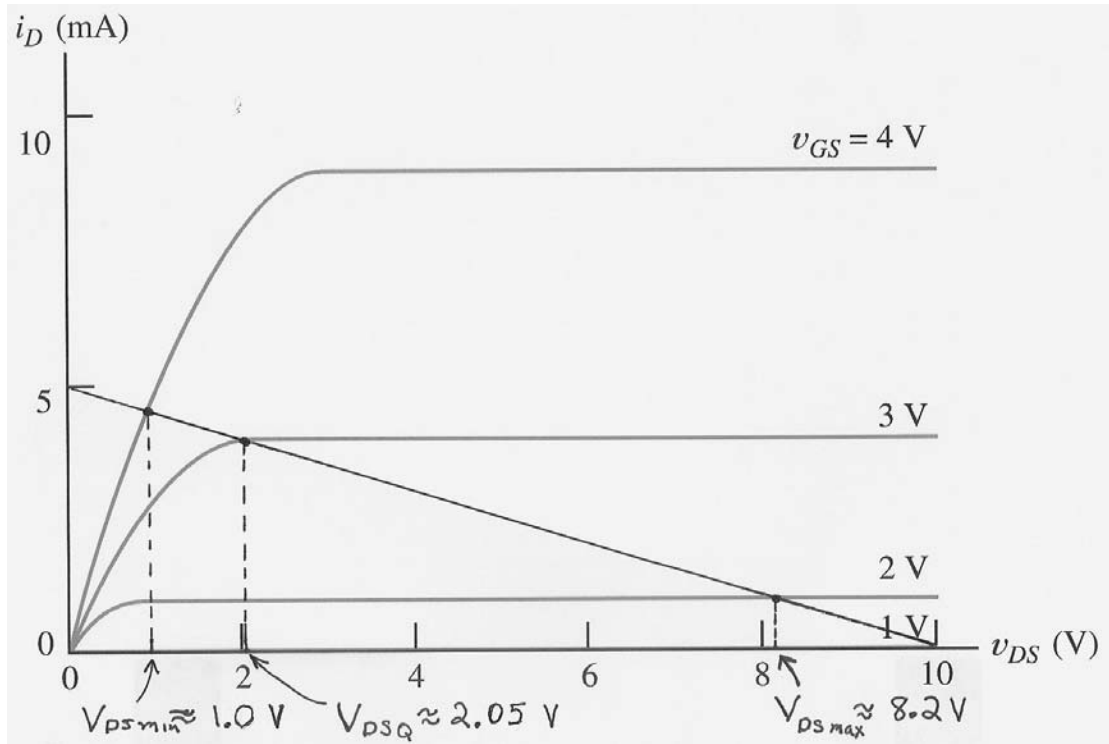
$$V_{DD} = R_D i_D + v_{DS}$$

With voltages in volts, currents in mA, and resistances in  $\text{k}\Omega$ , this becomes

$$10 = 2i_D + v_{DS}$$

which is the equation for the load line.

The characteristics and the load line are:



The results of the load-line analysis are  $V_{DSmin} \cong 1.0$  V,  $V_{DSQ} \cong 2.05$  V, and  $V_{DSmax} \cong 8.2$  V.

**T12.3** Because the gate current is zero, we can apply the voltage division principle to determine the voltage at the gate with respect to ground.

$$V_G = \frac{10 \text{ k}\Omega}{(10 + 30) \text{ k}\Omega} \times 12 = 3 \text{ V}$$

For the transistor, we have

$$K = \frac{1}{2} KP(W/L) = \frac{1}{2} (80 \times 10^{-6})(100/4) = 1 \text{ mA/V}^2$$

Because the drain voltage is 12 V, which is higher than the gate voltage, we conclude that the transistor is operating in the saturation region.

Thus, we have

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

$$I_{DQ} = (V_{GSQ} - 1)^2 = 0.5 \text{ mA}$$

Solving, we have  $V_{GSQ} = 1.707$  V or  $V_{GSQ} = 0.293$  V. However,  $V_{GSQ}$  must be larger than  $V_{to}$  for current to flow, so the second root is extraneous.

Then, the voltage across  $R_S$  is  $V_S = V_G - V_{GSQ} = 1.293$  V. The current through  $R_S$  is  $I_{DQ}$ . Thus, the required value is  $R_S = 1.293 / 0.5 = 2.586$  k $\Omega$ .

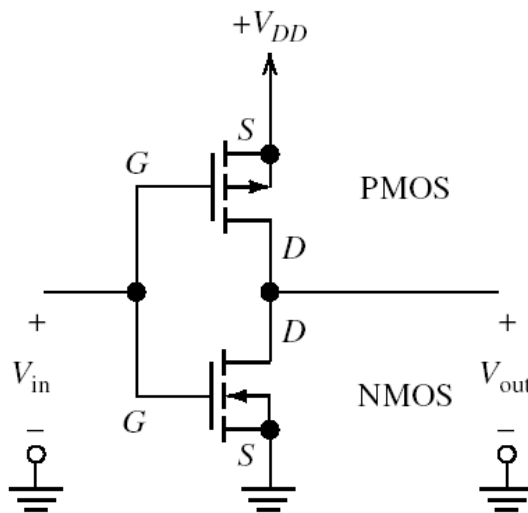
**T12.4** This transistor is operating with constant  $v_{DS}$ . Thus, we can determine  $g_m$  by dividing the peak ac drain current by the peak ac gate-to-source voltage.

$$g_m = \left. \frac{\Delta i_D}{\Delta v_{GS}} \right|_{v_{DS}=V_{DSQ}} = \frac{0.05 \text{ mA}}{0.02 \text{ V}} = 2.5 \text{ mS}$$

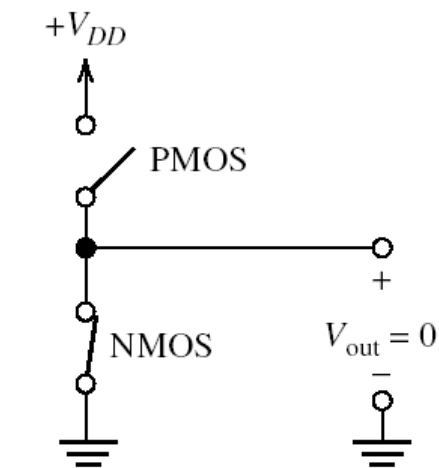
The  $Q$ -point is  $V_{DSQ} = 5 \text{ V}$ ,  $V_{GSQ} = 2 \text{ V}$ , and  $I_{DQ} = 0.5 \text{ mA}$ .

**T12.5** (a) A dc voltage source is replaced with a short circuit in the small-signal equivalent. (b) A coupling capacitor becomes a short circuit. (c) A dc current source is replaced with an open circuit, because even if an ac voltage appears across it, the current through it is constant (i.e., zero ac current flows through a dc current source).

**T12.6** See Figure 12.31(b) and (c) in the text.



(b) Circuit diagram



(c) Equivalent circuit with  $V_{in}$  high