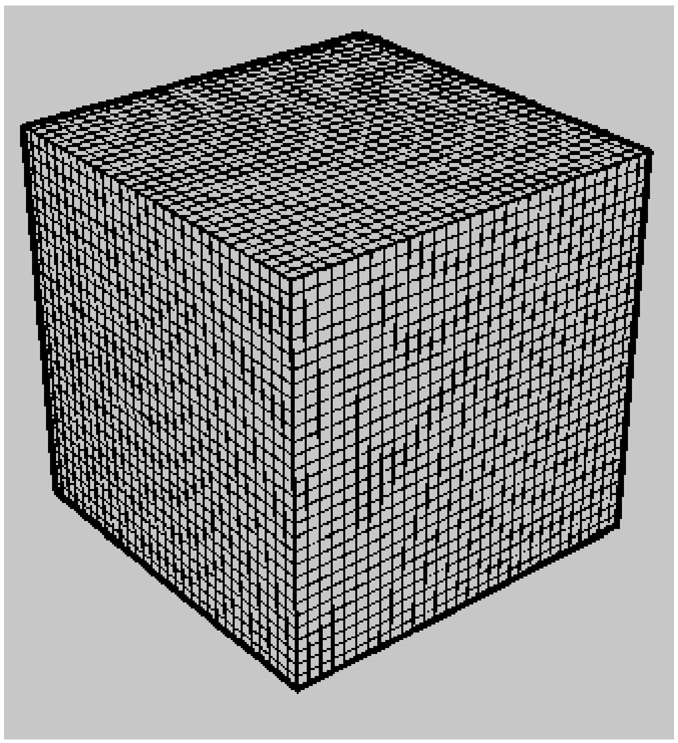
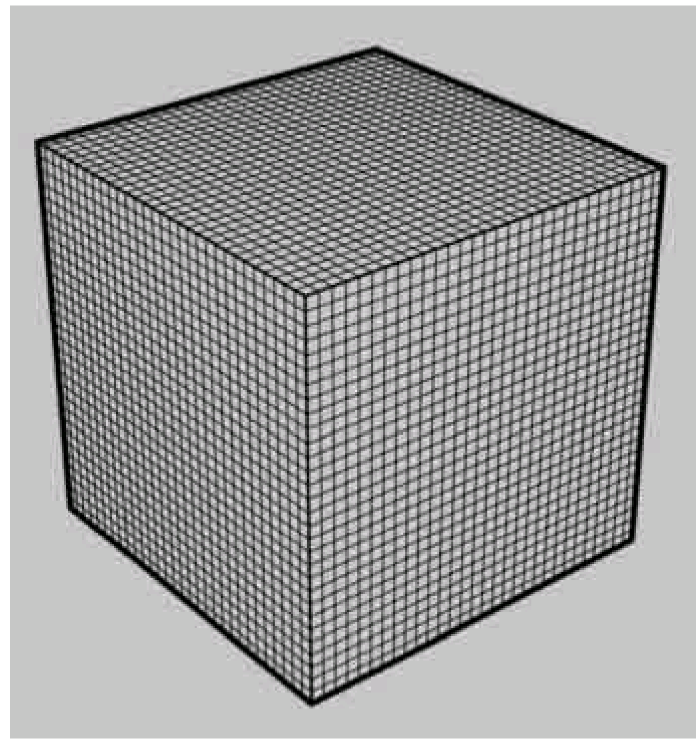
Today's Topics

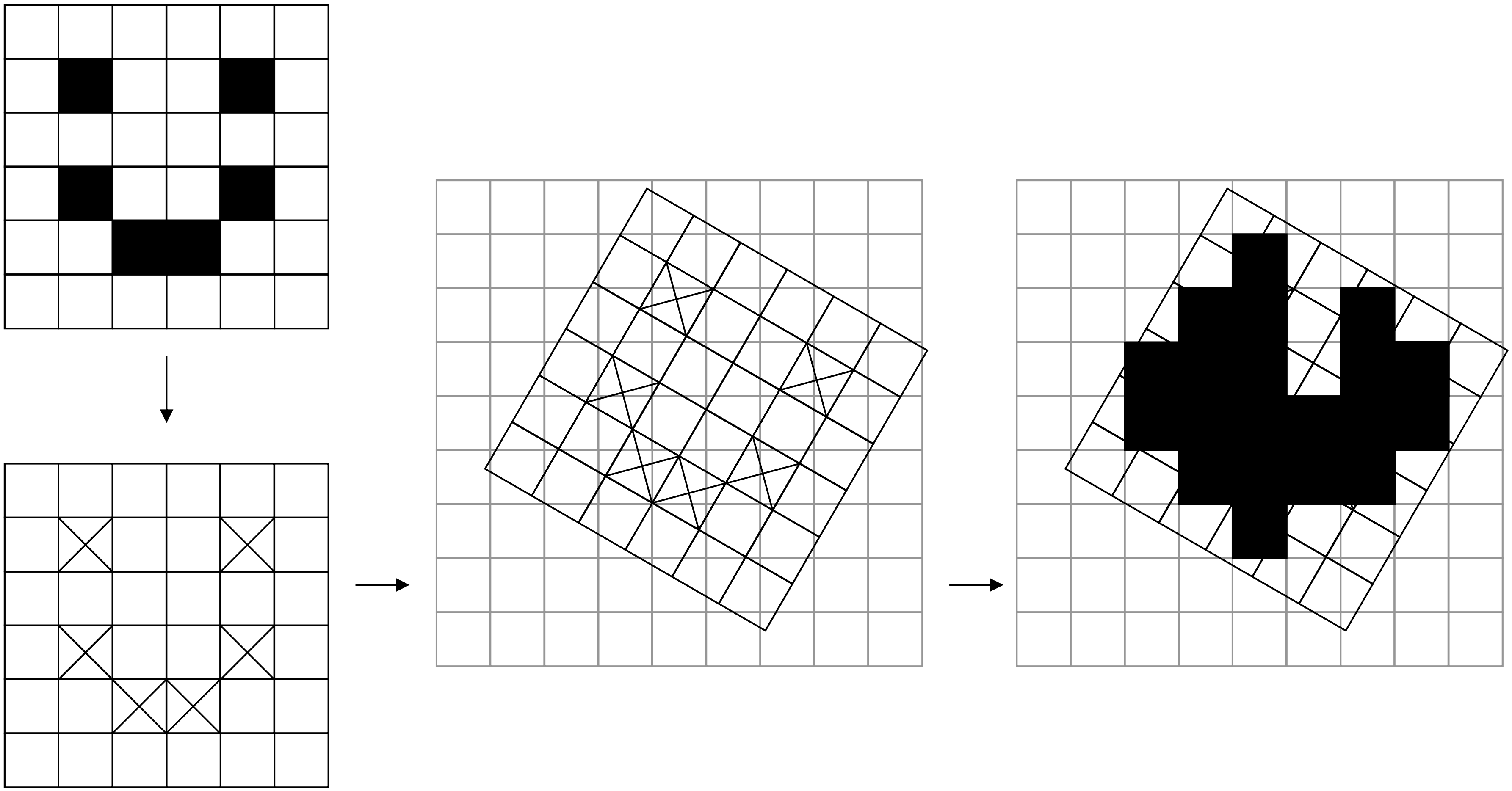
- Lecture this week and next week
  - Aliasing and Anti-aliasing
  - Digital Half toning
  - Bitmap Image Rotation
  - Fractals
  - Colors

Aliasing and Anti-aliasing





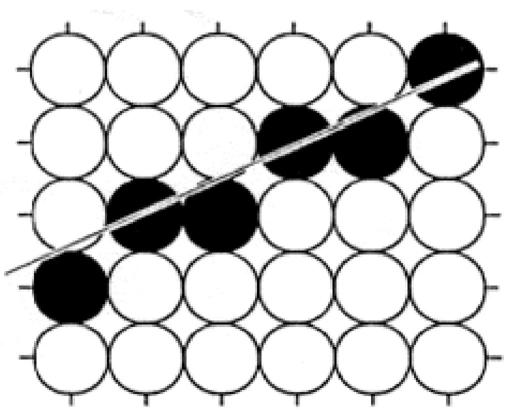
Aliasing Problem

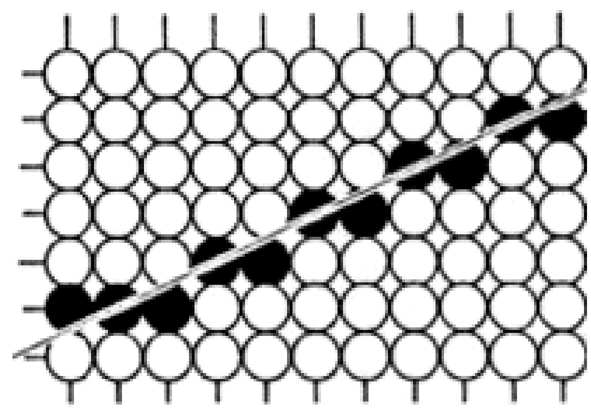


Anti-aliasing

- Aliasing:
  – Rasterized line segments and edges of polygons look jagged. Arises whenever attempting to present <u>Continuous repn.</u> of object with <u>discrete approximation</u>.

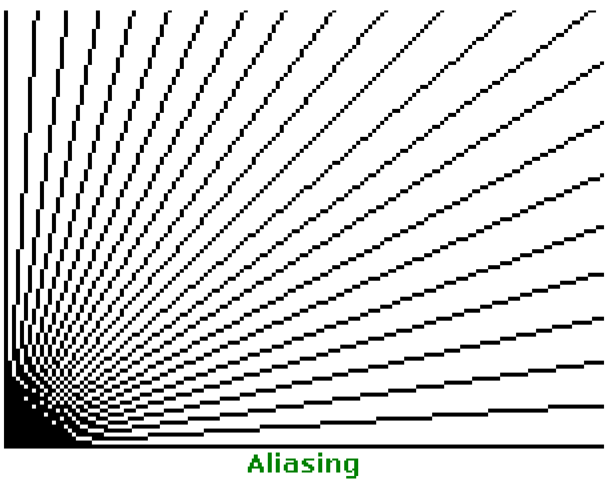
    (infinite resolution) (limited resolutionn)
- Not able to be eliminated by increasing resolution

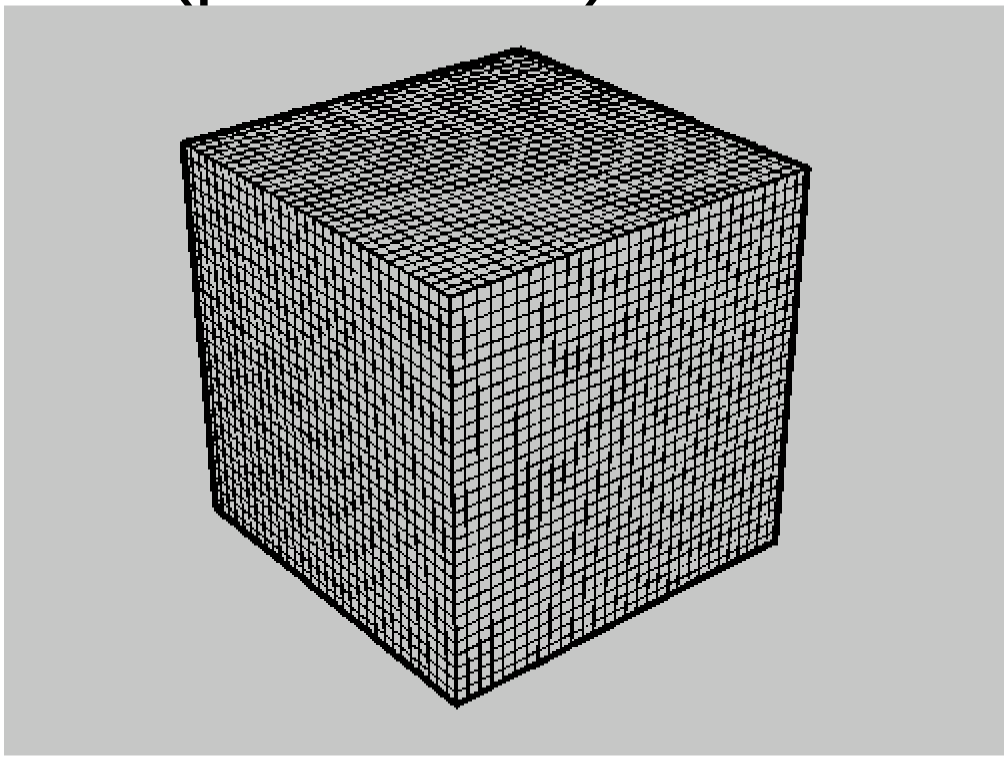




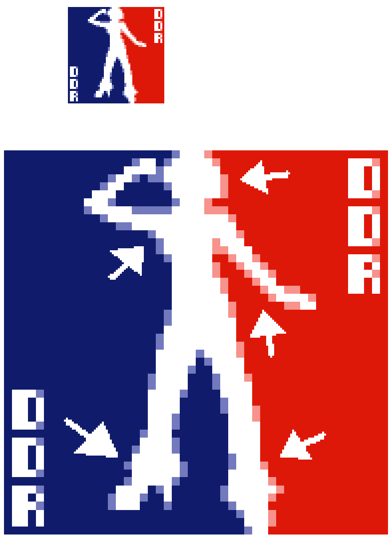
Aliasing Problem

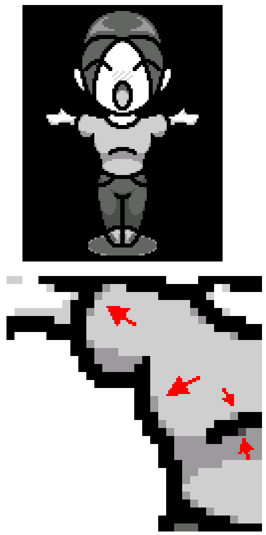
- Jagged edges (stair cases)
- Escalator effects
- Inconsistent brightness (pixel mud!)

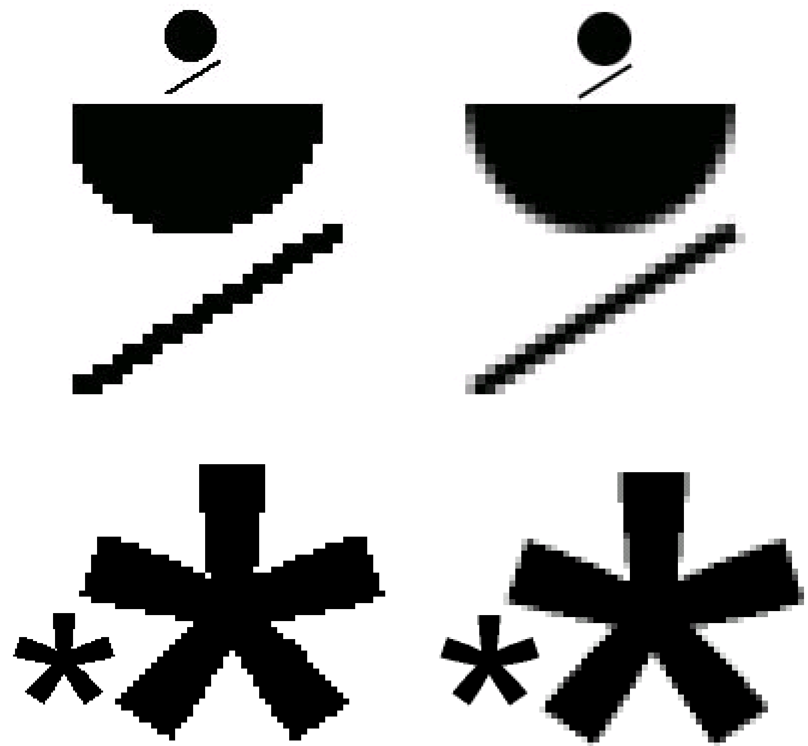




Anti-aliasing

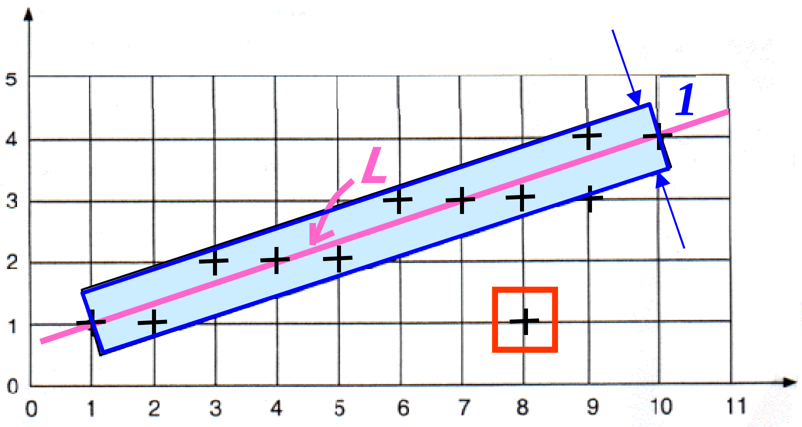


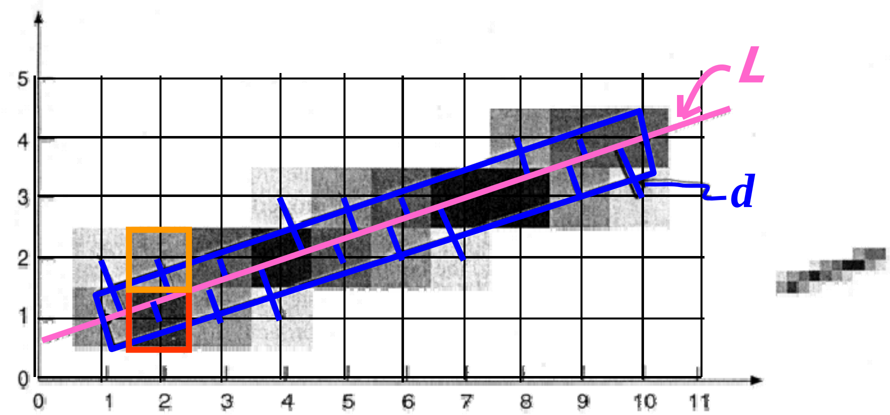




Anti-aliasing

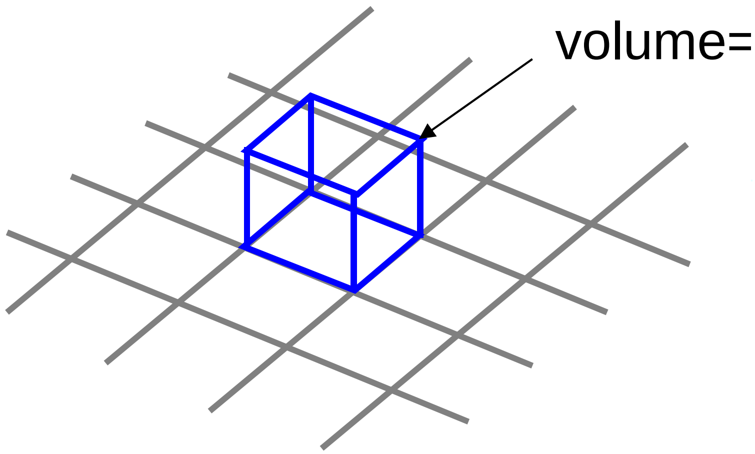
- Assume an idealized line segment in the frame buffer as being one pixel wide (a square to represent a pixel area)

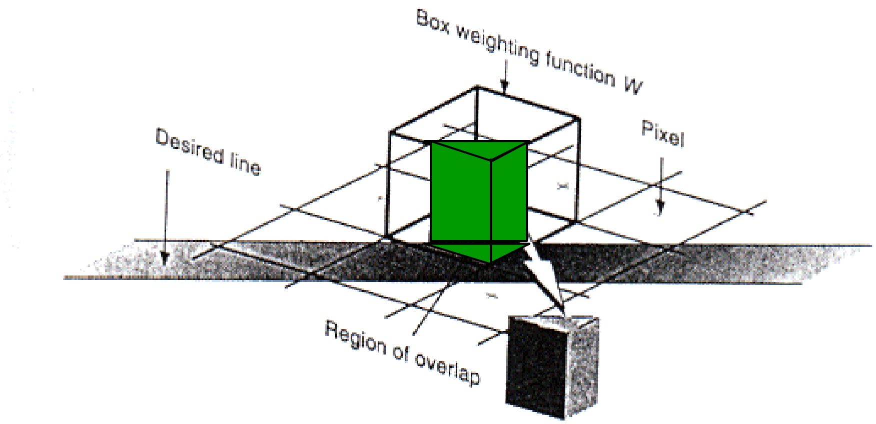




- An ideal line contributes to each pixel's intensity an amount proportional to the % of the pixel's tile it covers.
  - *e.g.* pixel (2,1) is ~70% black, pixel (2,2) ~25% black. In another words

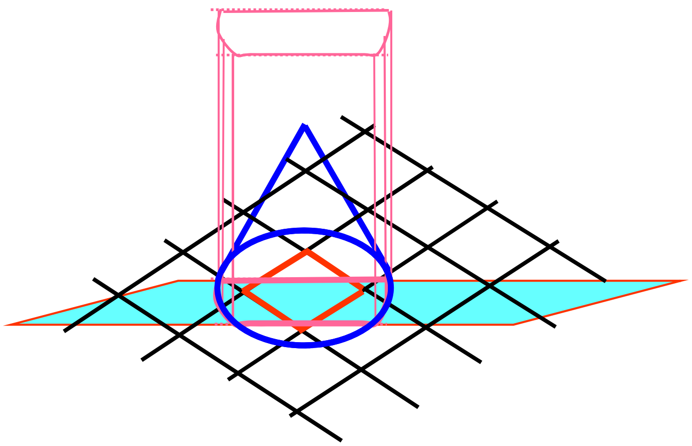
AA Viewed as Filtering

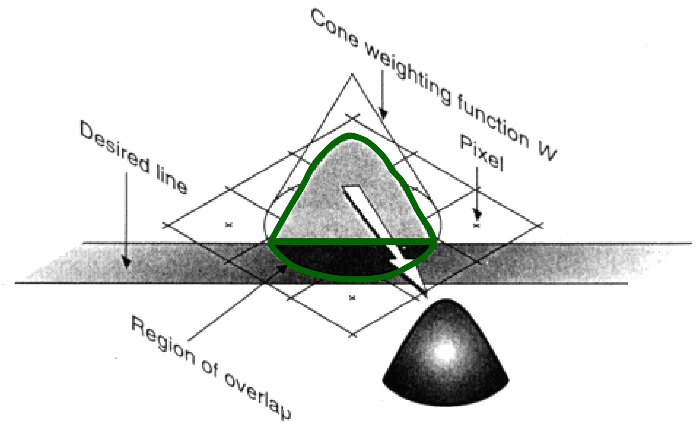




- The intensity of the pixel = the volume under the box covered by the line x the intensity/color of the line
- The box is an <u>un-weighted function</u>

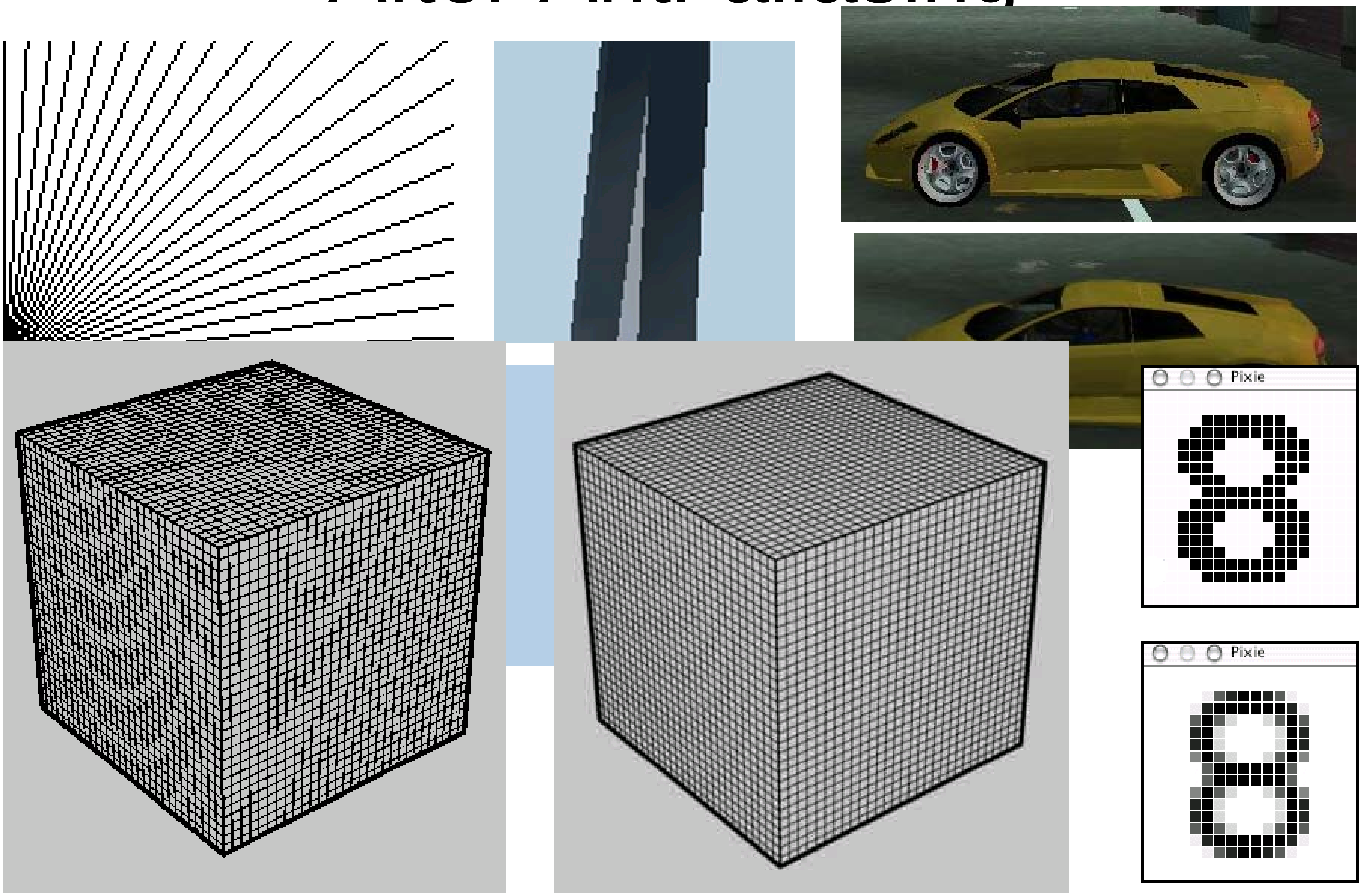
Weighted Pixel Mask





- Instead of a box, we use a cone that covers more than 1 pixel
- The intensity of the pixel = the volume under the cone covered by the line x the intensity/color of the line
- The cone is a <u>weighted function</u>

After Anti-aliasing



Bitmap Image Rotation

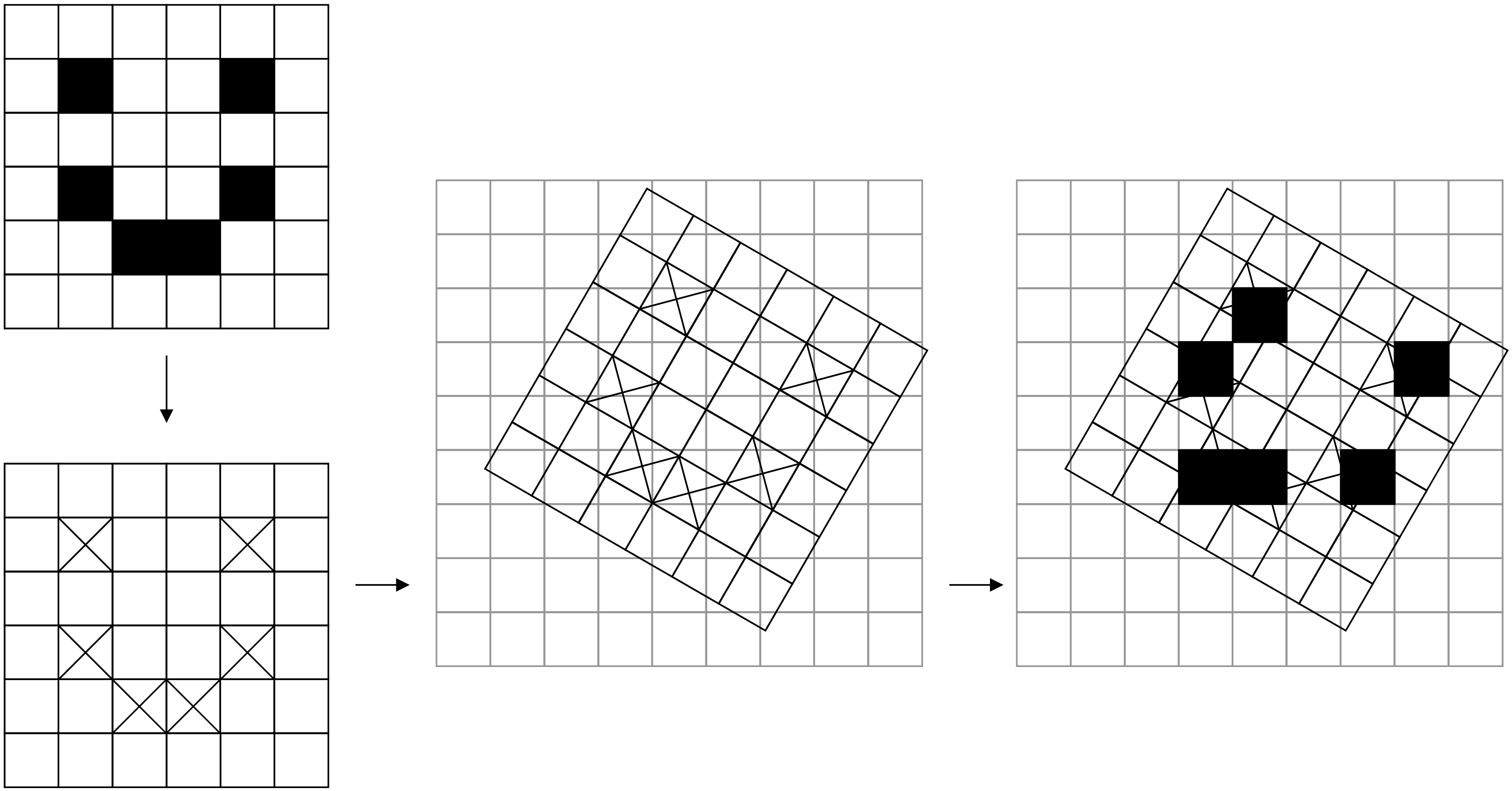
Rotated Image

Rotating an Image

- Problem: Given an image, rotate the image by an angle ϕ
- Naïvely rotating every pixel will cause serious aliasing problem

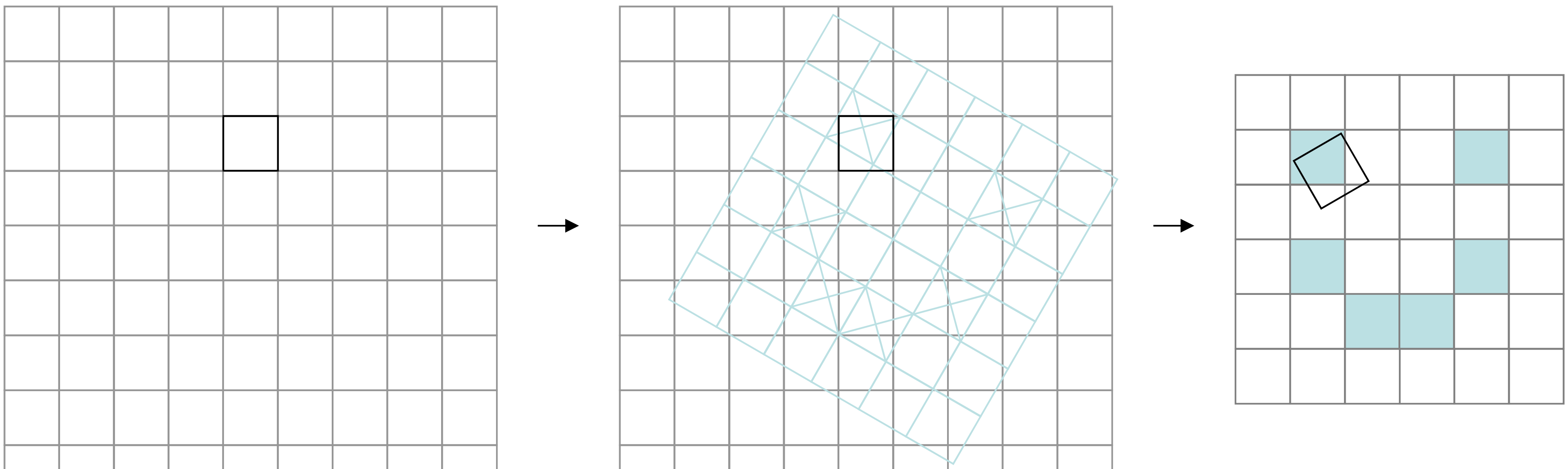
```
For every row in the picture
  For every pixel p in the row
     Let x be the coordinate of p,
     Calculate the new position R(φ) x
     Color the position R(φ) x by the color of p
  End for
End for
```

Aliasing Problem

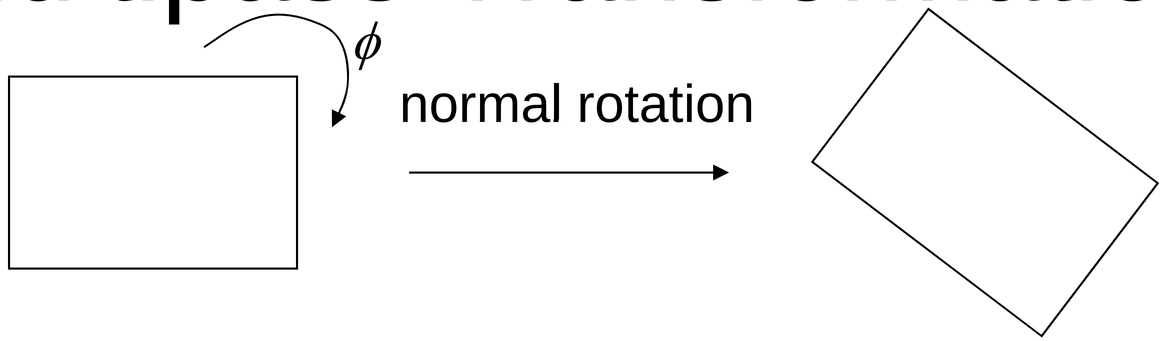


Another try: Pre-image of a Rotated Pixel

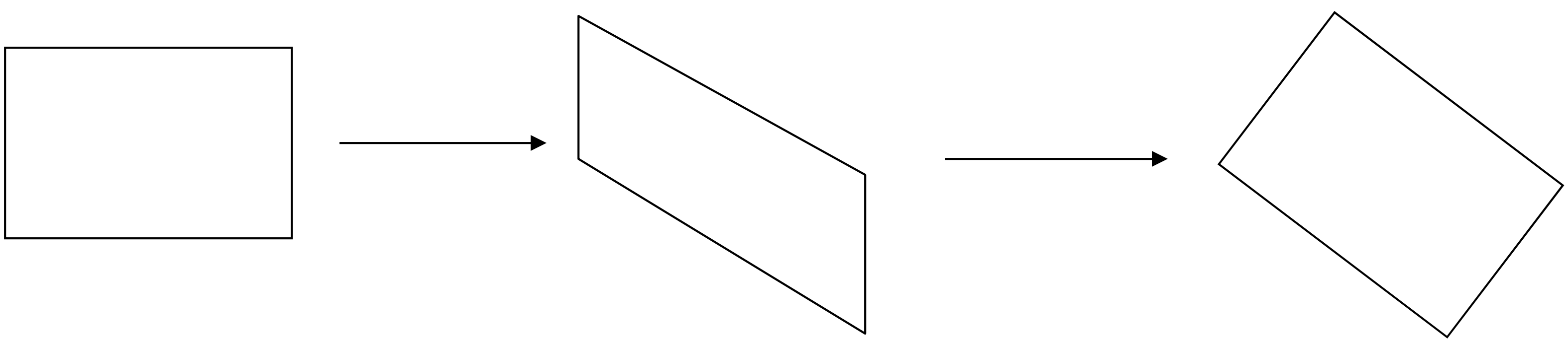
- We could find the pre-image of each pixel after rotation.
- However, it is too computational expensive



Solution:
Multipass Transformation



$$(r,s) = R(x,y) = (x \cos \phi - y \sin \phi, x \sin \phi + y \cos \phi)$$



$$(u,v) = A(x,y) = (x, f(x,y)) \qquad (r,s) = B(u,v) = (g(u,v), v)$$

such that: $(r,s) = (B(A(x,y)) = R(x,y)$

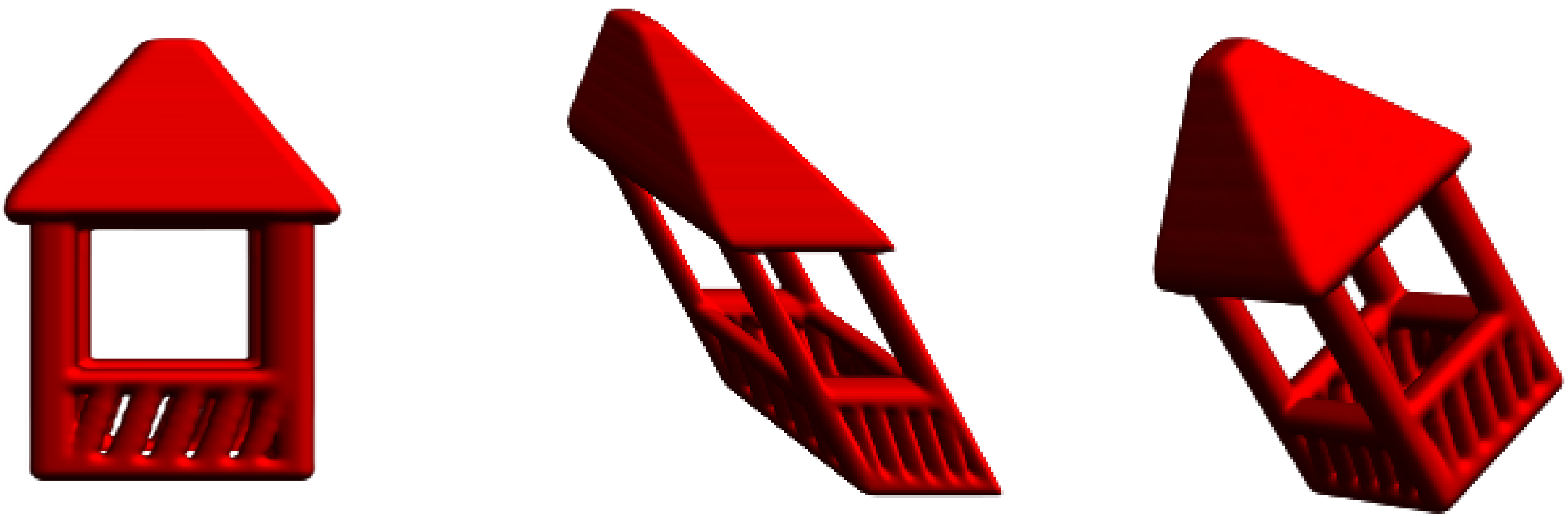
$$(u,v) = A(x,y) = (x, f(x,y))$$

$$(r,s) = B(u,v) = (g(u,v), v)$$

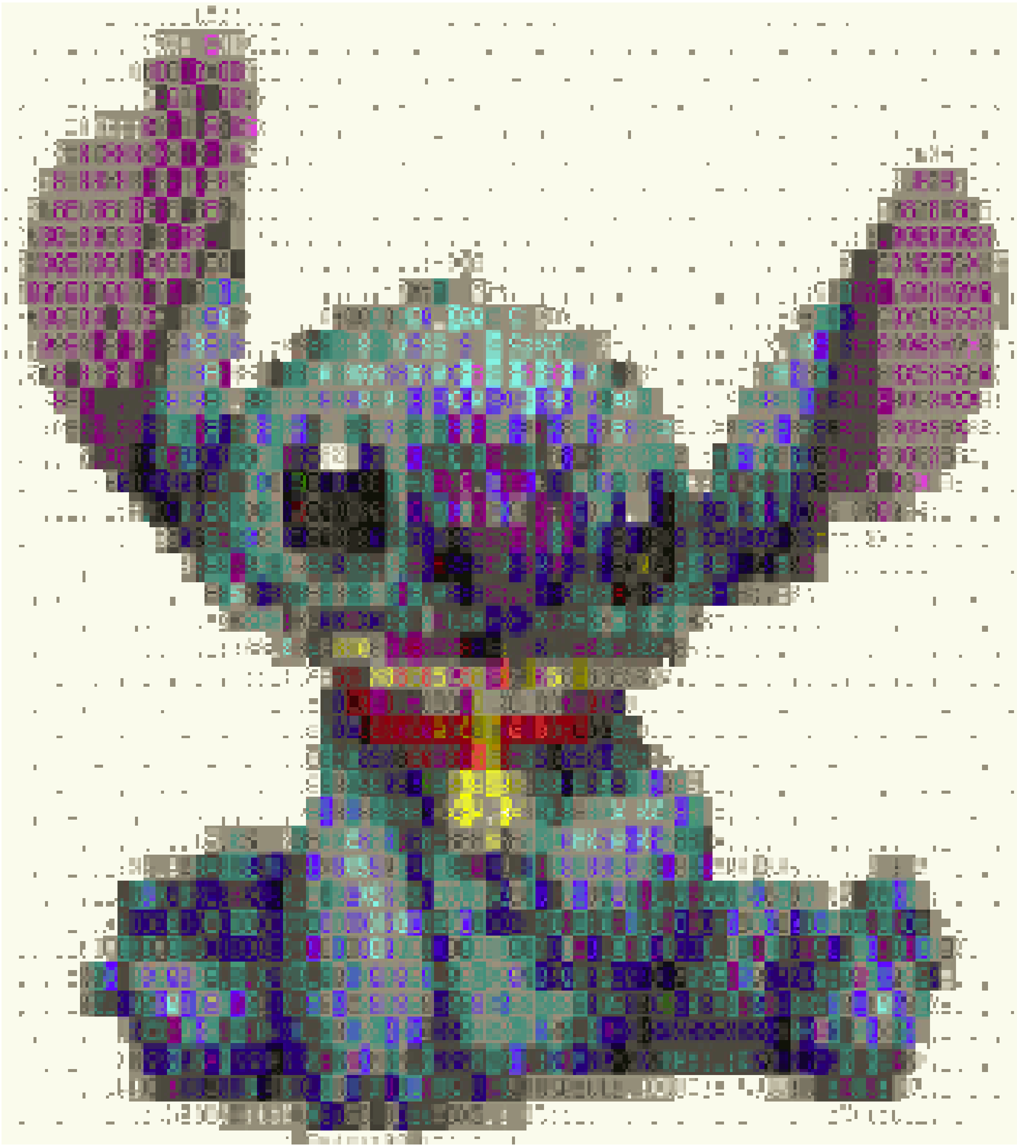
Computing $f(x,y)$ and $g(u,v)$

- $(u,v) = A(x,y) = (x, x \sin \phi + y \cos \phi)$
  - $f(x,y) = x \sin \phi + y \cos \phi$
- $(r,s) = B(u,v) = (g(u,v), v)$
  - $u = x,\ \ v = x \sin \phi + y \cos \phi$
  - $g(u,v) = x \cos \phi - y \sin \phi$?
  - $y = (v - x \sin \phi) / \cos \phi = (v - u \sin \phi) / \cos \phi$
  - $g(u,v) = u \cos \phi - \sin \phi\, (v - u \sin \phi) / \cos \phi$
  - $g(u,v) = u \sec \phi - v \tan \phi$

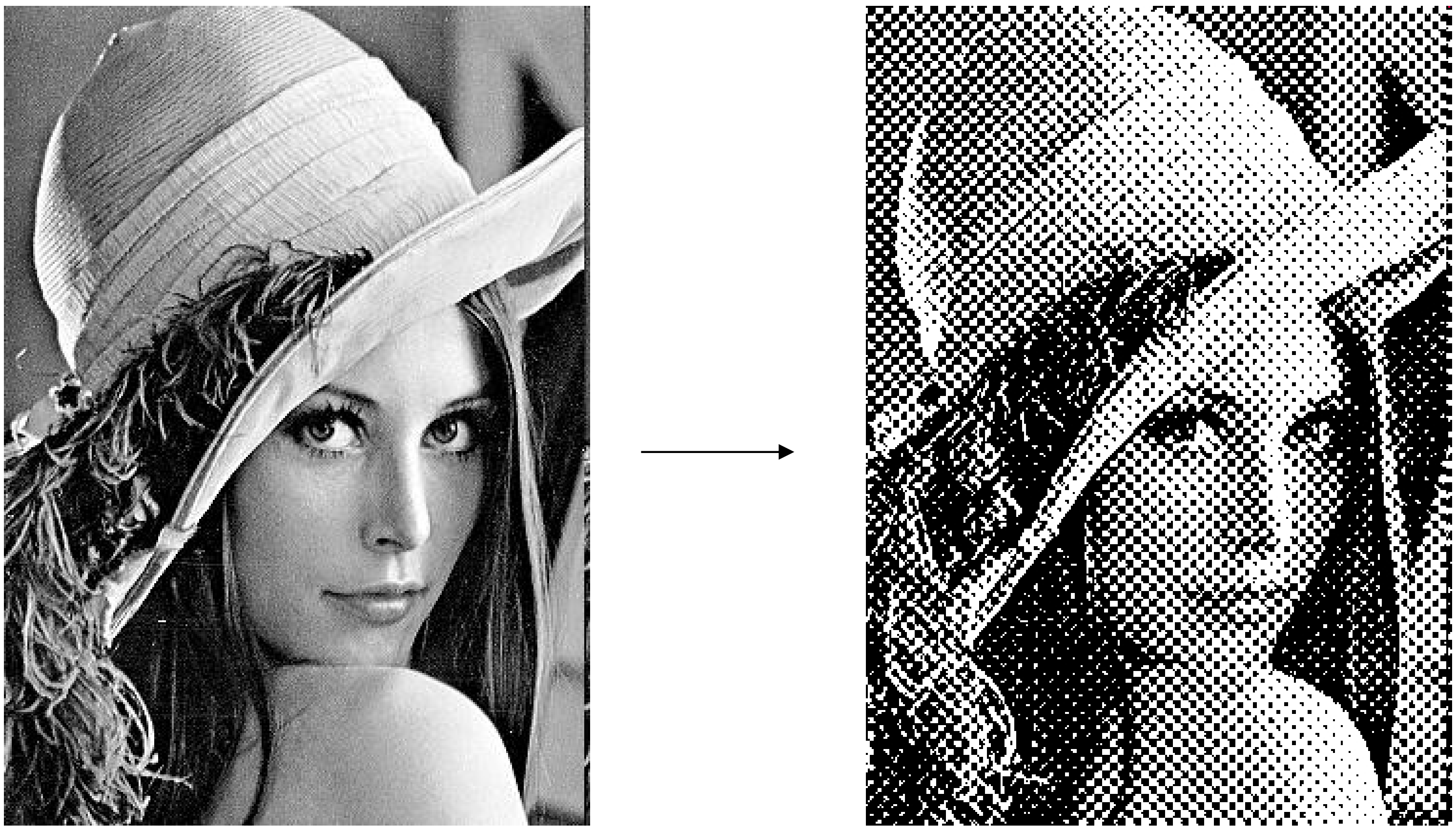
3D Volumetric Object



Digital Halftoning/Dithering



Digital Halftoning



Halftoning

- Limitation of Displaying Devices
  - Some display or hardcopy devices have limited levels of intensity
  - What if they are *bilevel* only, i.e. only black and white, e.g. newspaper printing

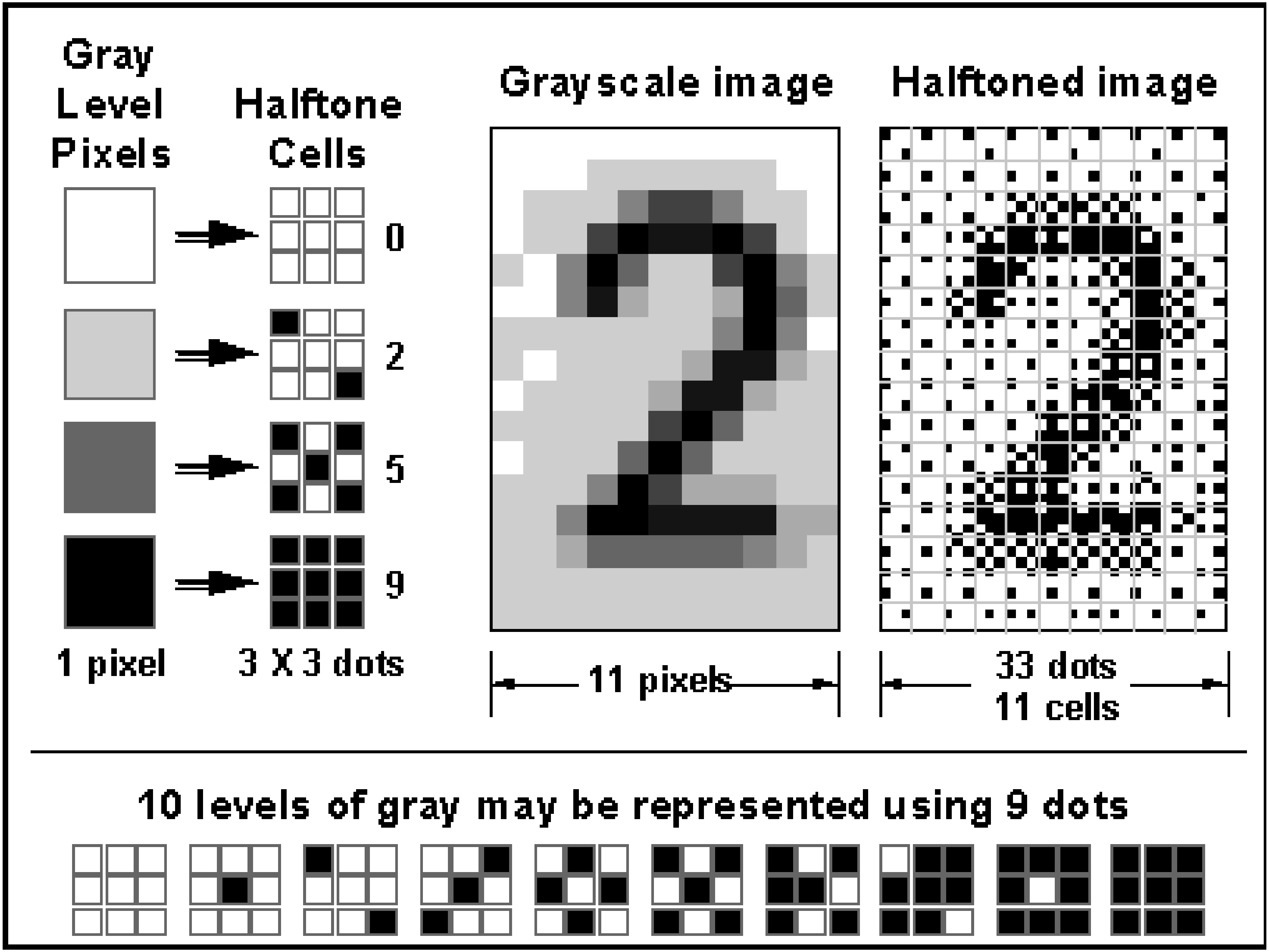
- Halftoning is the process of turning continuous tone grayscale or color images into a series of dots for printing that fool the eye.

Source Image

(Grey Scale)

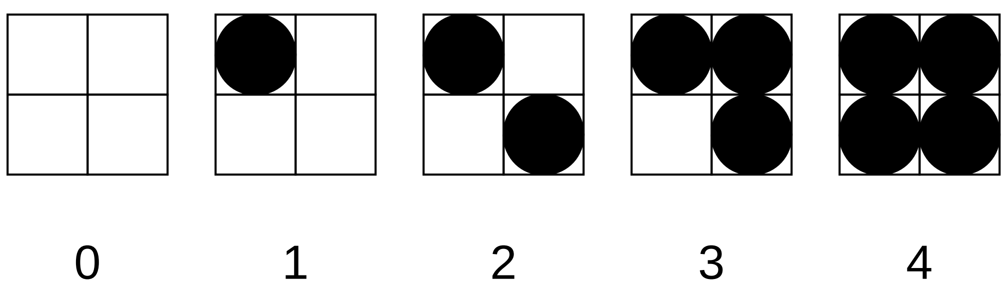
Bilevel

Display


Gray Level Pixels
Halftone Cells
0
2
5
9
1 pixel
3 X 3 dots
Grayscale image
Halftoned image
11 pixels
33 dots
11 cells
10 levels of gray may be represented using 9 dots

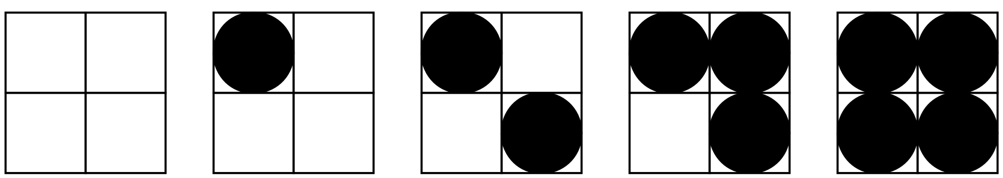
Halftone Approximation

- Use spatial integration that our eyes perform
- e.g. a 2 x 2 pixel area of a bilevel display can be used to produce five different intensity levels at the cost of halving the spatial resolution along each axis



Halftone Approximation

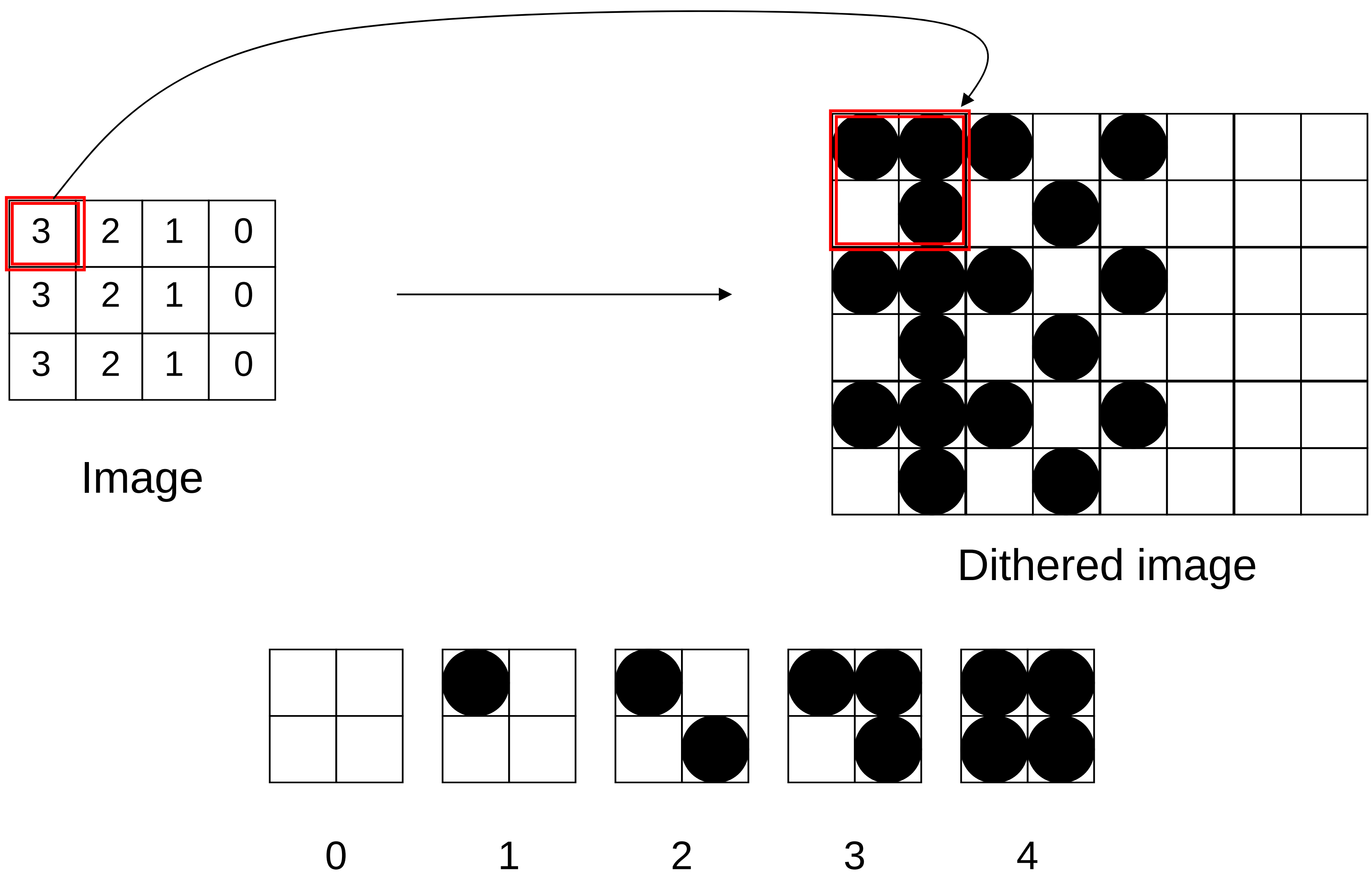
- This technique is called *halftoning* or *clustered-dot ordered dither*
- The pattern can be represented by the dither matrix:



0 1 2 3 4

$$\begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$$

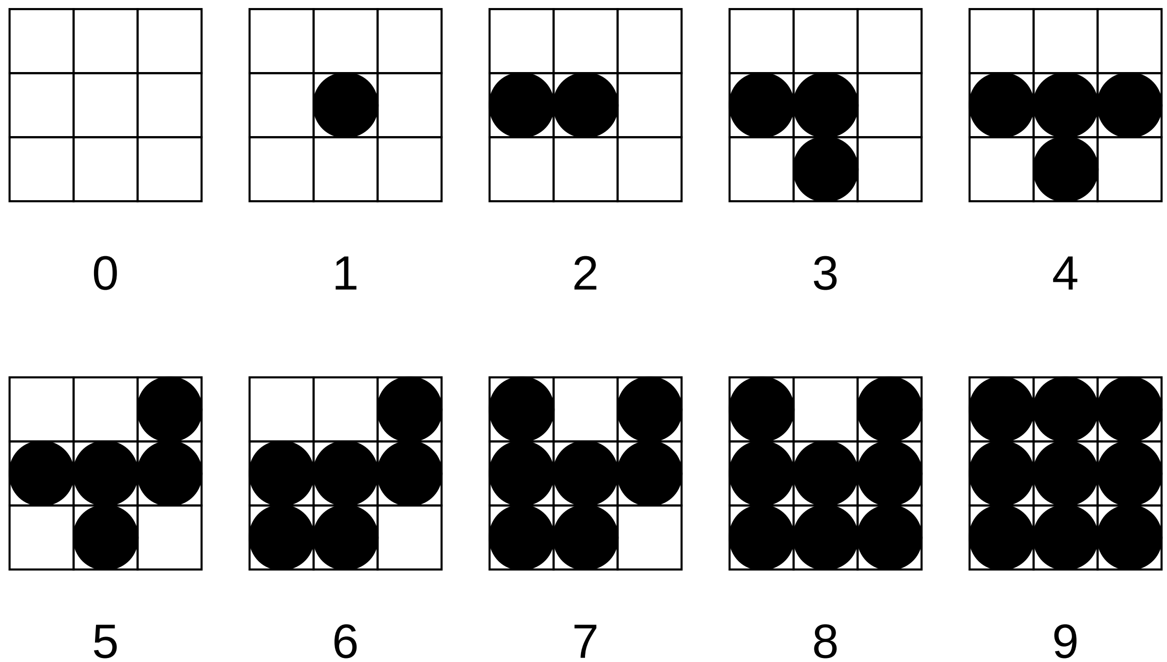
Example



Halftone Approximation

- An n x n group of bilevel pixels can provide n^2+1 intensity levels:

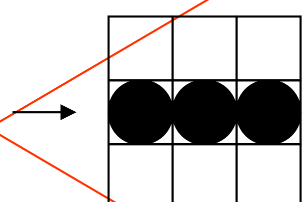
$$\begin{pmatrix} 6 & 8 & 4 \\ 1 & 0 & 3 \\ 5 & 2 & 7 \end{pmatrix}$$

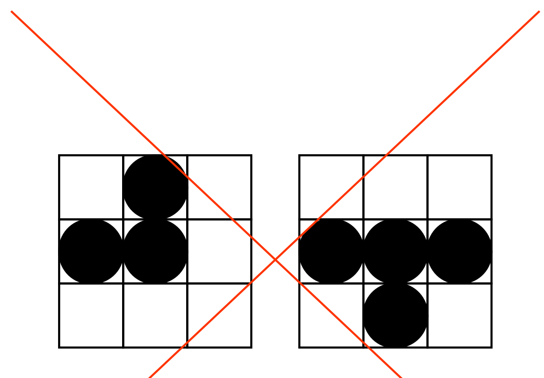


Halftone Approximation

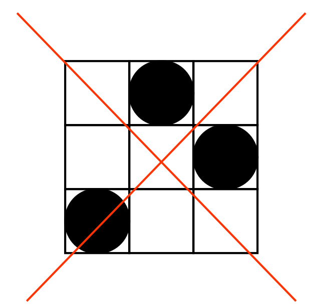
- Pattern properties:
  1. no artificial visual effects
  2. form a grow sequence
  3. grow outward from the center
  4. must be clustered

$$\begin{pmatrix} 3\ 4\ 5 \\ 0\ 1\ 2 \\ 7\ 8\ 9 \end{pmatrix}$$





3 4



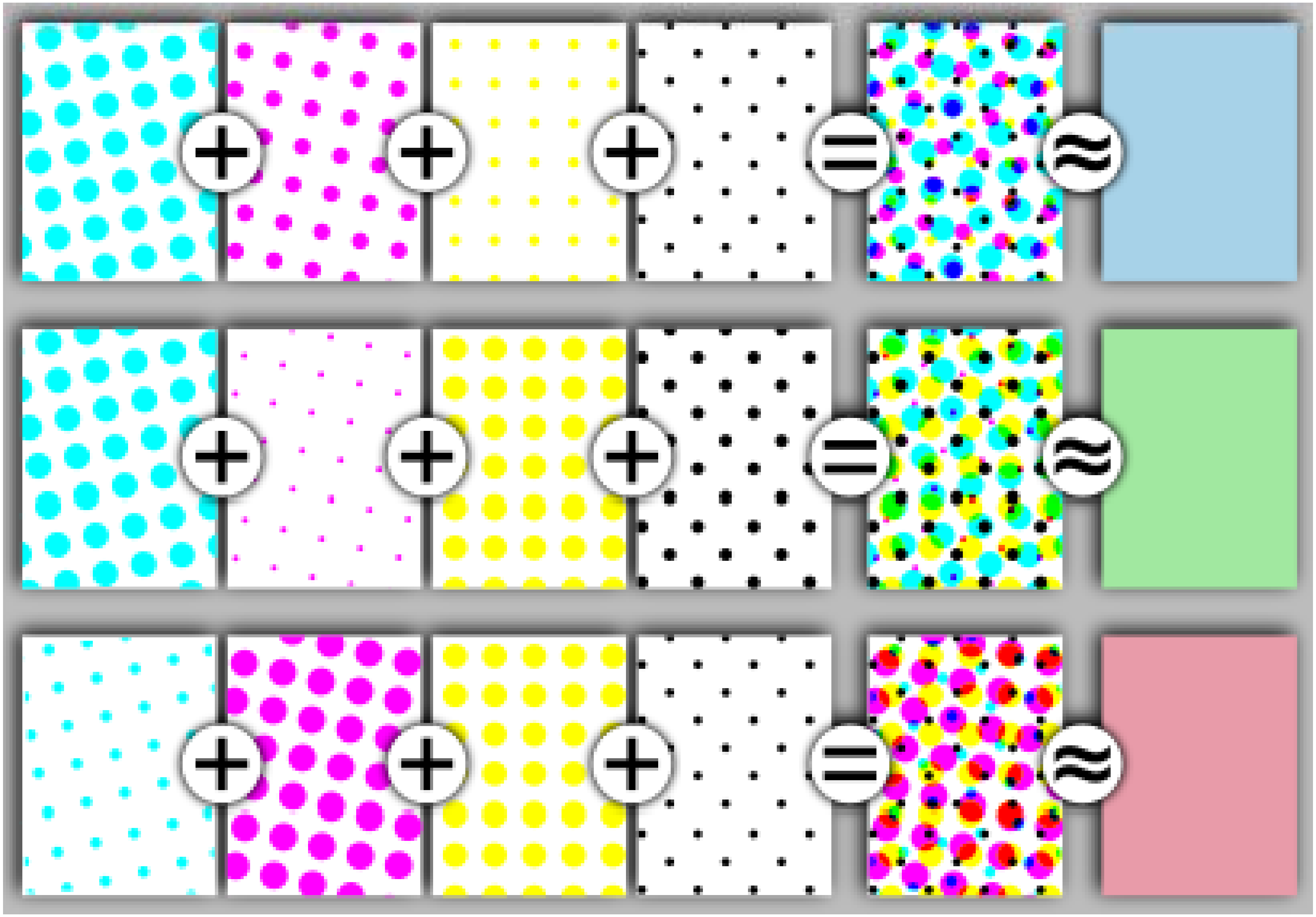
3

Halftone Approximation

- The technique is not limited to bilevel devices

$$\begin{array}{c}\begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline\end{array} \\ \\ 0\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 0 \\ \hline\end{array} \\ \\ 1\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 1 \\ \hline\end{array} \\ \\ 2\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 1 & 0 \\ \hline 1 & 1 \\ \hline\end{array} \\ \\ 3\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 1 & 1 \\ \hline\end{array} \\ \\ 4\end{array}$$

$$\begin{array}{c}\begin{array}{|c|c|}\hline 2 & 1 \\ \hline 1 & 1 \\ \hline\end{array} \\ \\ 5\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 2 & 1 \\ \hline 1 & 2 \\ \hline\end{array} \\ \\ 6\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 2 & 1 \\ \hline 2 & 2 \\ \hline\end{array} \\ \\ 7\end{array}\quad\begin{array}{c}\begin{array}{|c|c|}\hline 2 & 2 \\ \hline 2 & 2 \\ \hline\end{array} \\ \\ 8\end{array}$$



Halftoning

Georges-Pierre Seurat

Georges-Pierre Seurat

- ***Sunday Afternoon on the Island of La Grande Jatte*** shows people of all different classes in a park. The tiny juxtaposed dots of multi- colored paint *allow the eye of the viewer to blend colors optically*, rather than having the colors blended on the canvas or pre-blended as a material pigment. It took Seurat *two years* to complete this ten foot wide painting, and he spent much time in the park sketching to prepare for the work (there are about 60 studies).

ORDERED-DITHER APPROACH

Ordered-dither Approach

- What if the image and the display arrays are the same size?
  - Let the intensity of a pixel at (x,y) be $S(x,y)$
  - let $i = x \bmod n$ and $j = y \bmod n$
  - Color the pixel if $S(x,y) > D_{ij}$

0	2
3	1

→

0	2	0	2	0	2	0	2	0	2
3	1	3	1	3	1	3	1	3	1
0	2	0	2	0	2	0	2	0	2
3	1	3	1	3	1	3	1	3	1
0	2	0	2	0	2	0	2	0	2
3	1	3	1	3	1	3	1	3	1

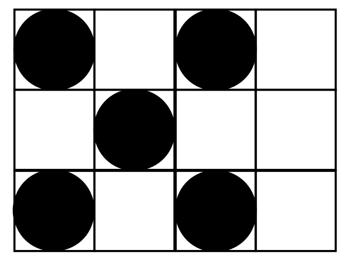
Example

3	2	1	0
3	2	1	0
3	2	1	0

Image

0	2	0	2
3	1	3	1
0	2	0	2

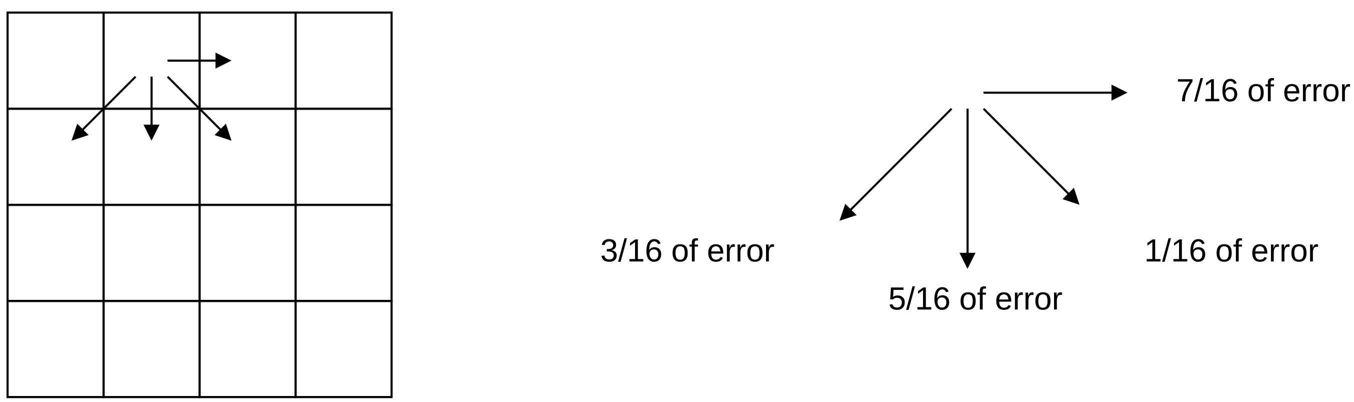
Dithering Matrices



Dithered image


Error Diffusion (Floyd-Steinberg)

- Problem: intensities are *lost*
- Solution: spread the error to its neighbors



Naïve Error Diffusion

(Non-Floyd-Steinberg)

Error Diffusion

(Floyd-Steinberg)

Error Diffusion

(Floyd-Steinberg)

		$\frac{7}{16}$
$\frac{3}{16}$	$\frac{5}{16}$	$\frac{1}{16}$

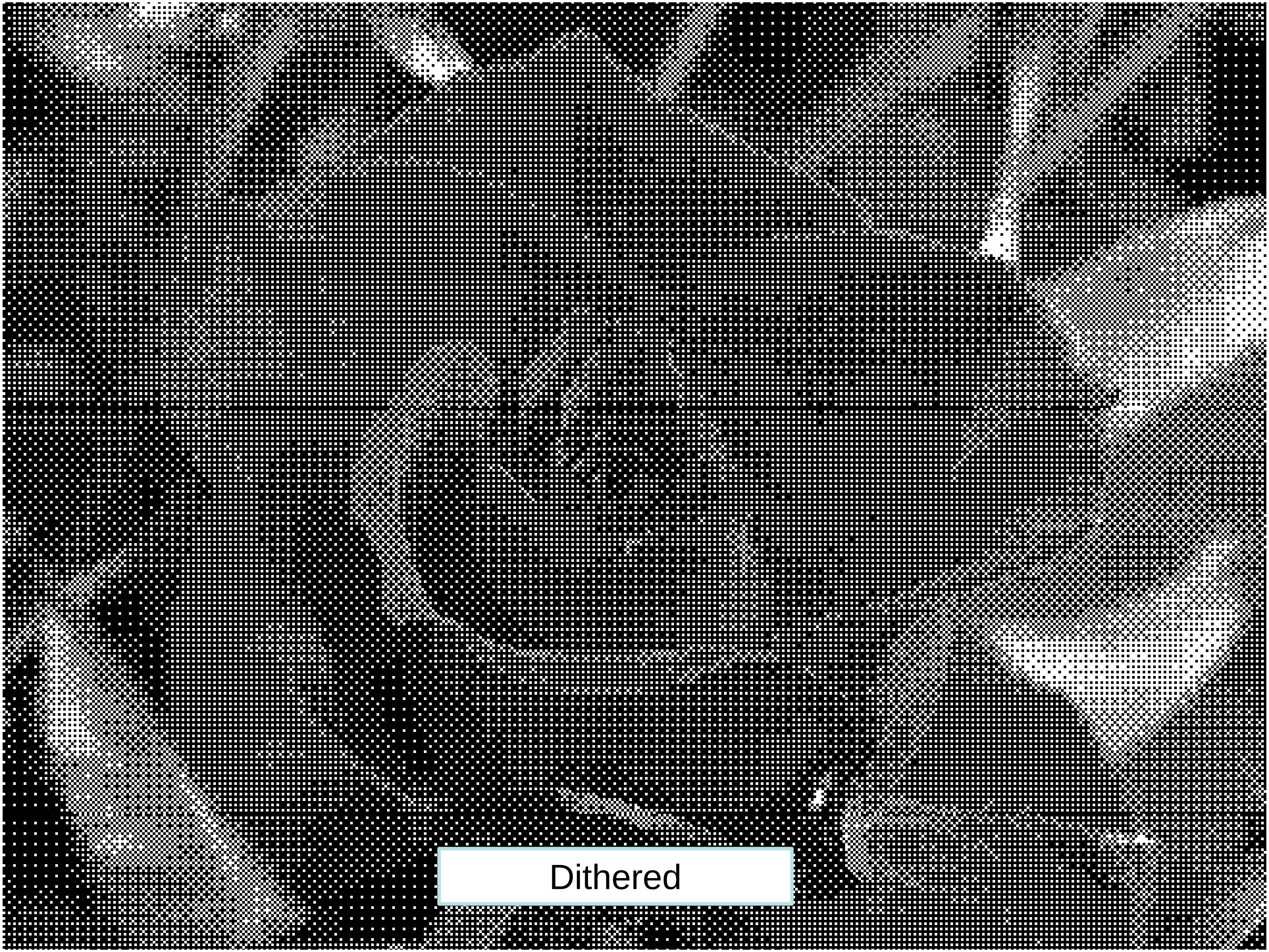
Error Diffusion

(JarvisJudiceNinke)

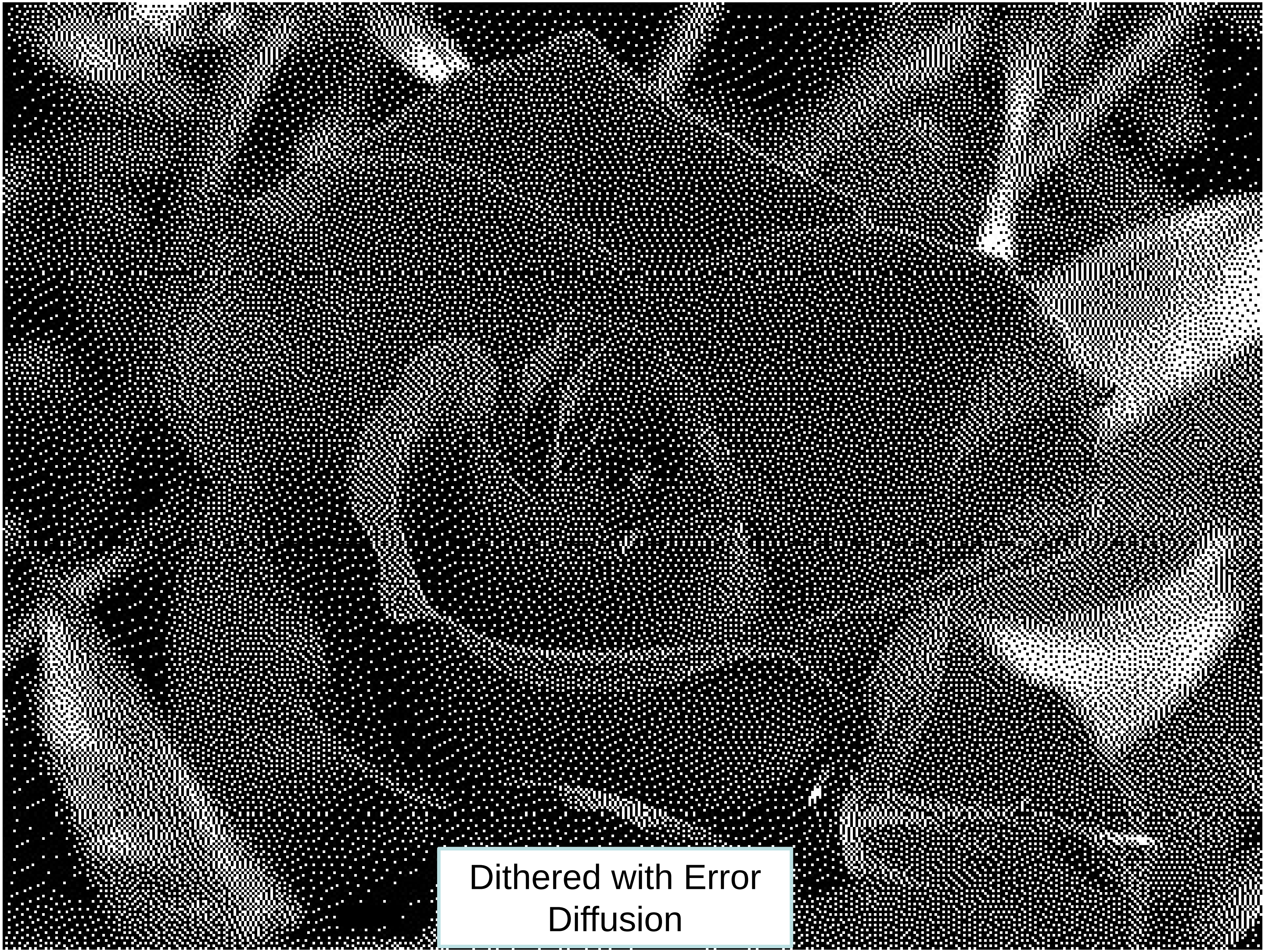
			$\frac{7}{48}$	$\frac{5}{48}$
$\frac{1}{16}$	$\frac{5}{48}$	$\frac{7}{48}$	$\frac{5}{48}$	$\frac{1}{16}$
$\frac{1}{48}$	$\frac{1}{16}$	$\frac{5}{48}$	$\frac{1}{16}$	$\frac{1}{48}$


Original Image

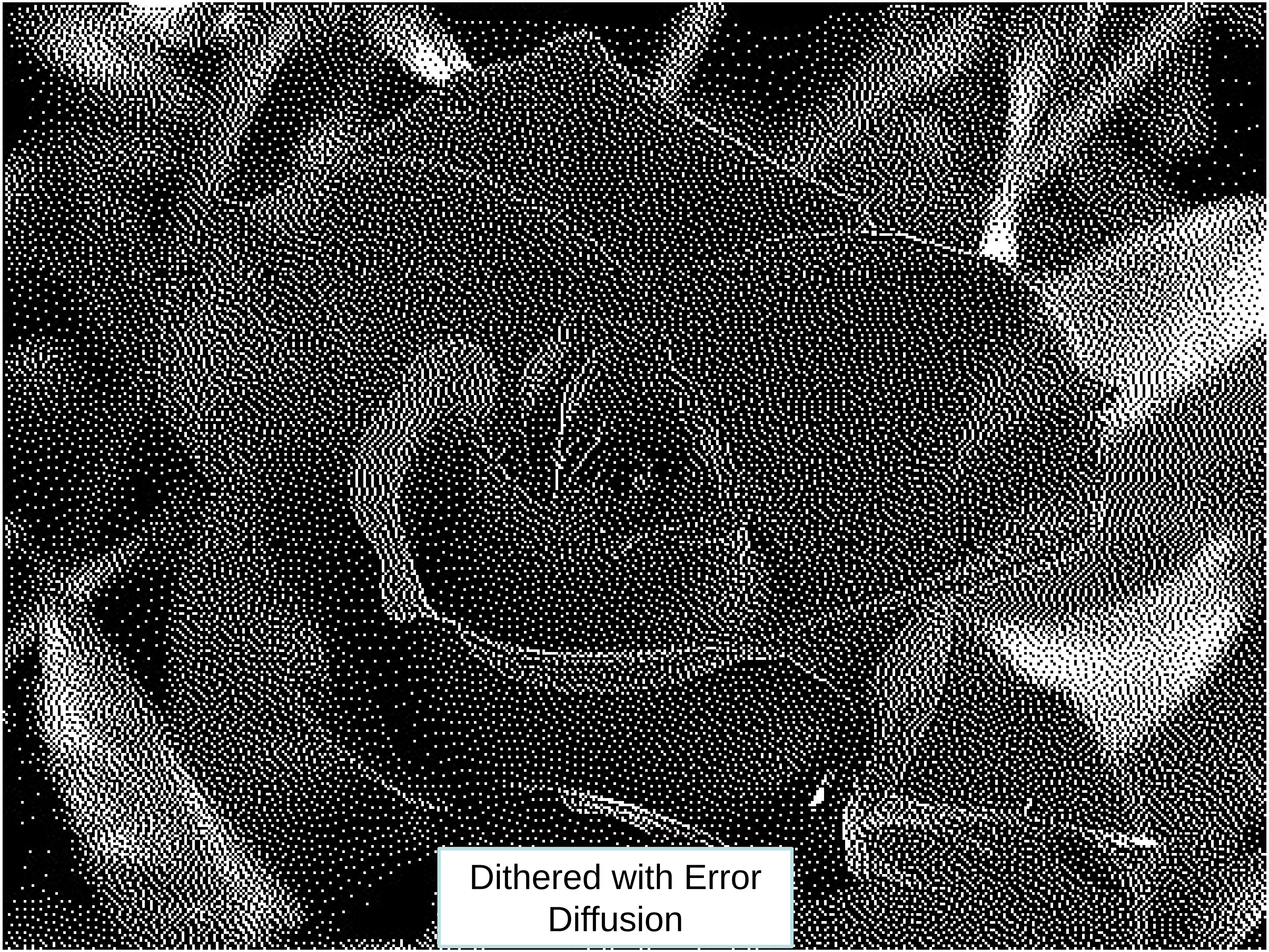

Bilevel



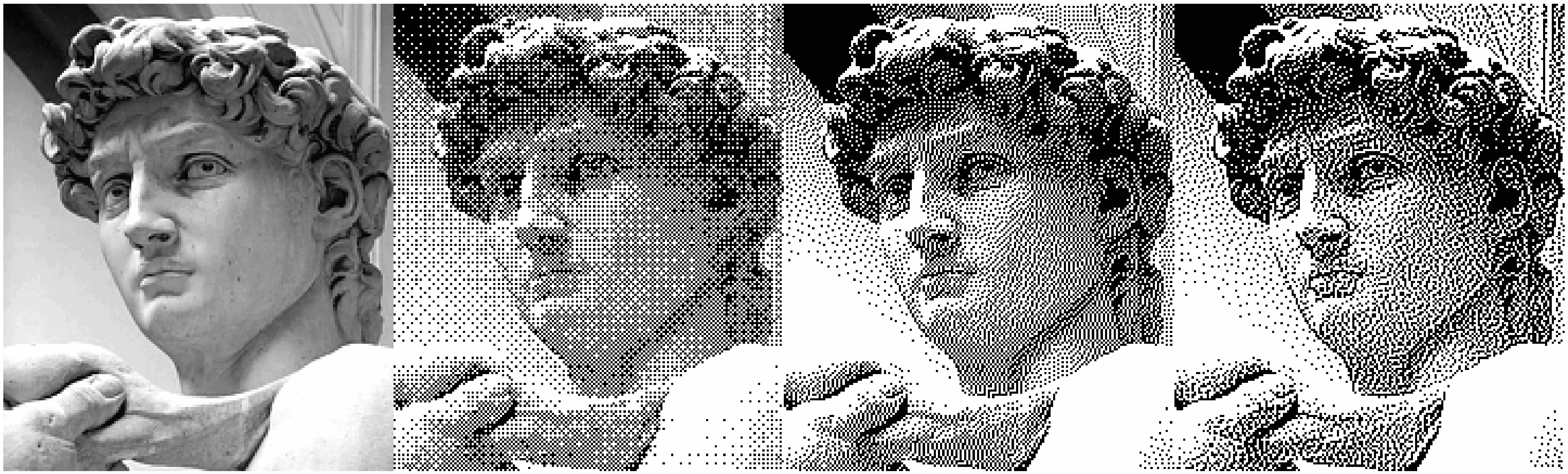
Dithered



Dithered with Error Diffusion

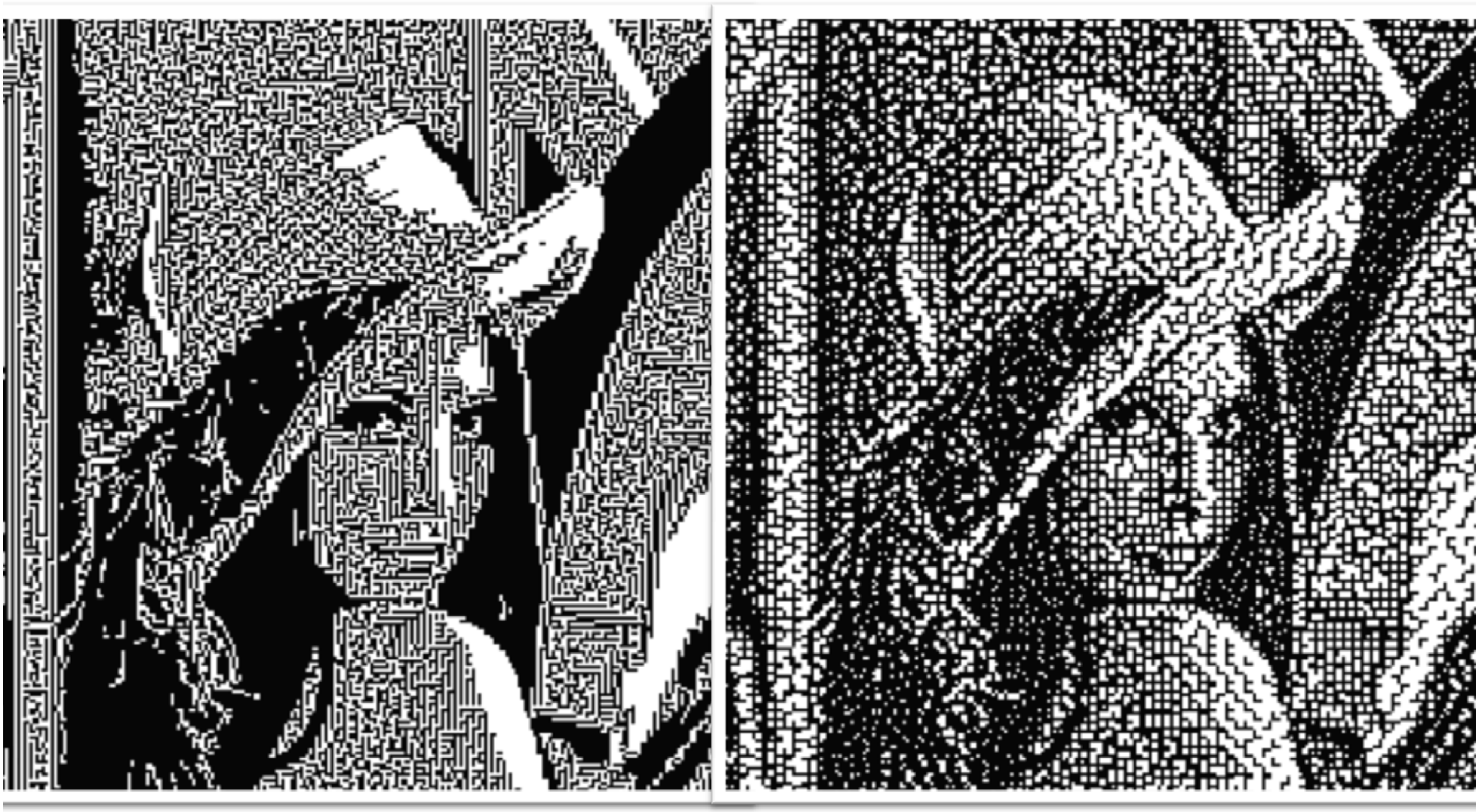


Dithered with Error Diffusion



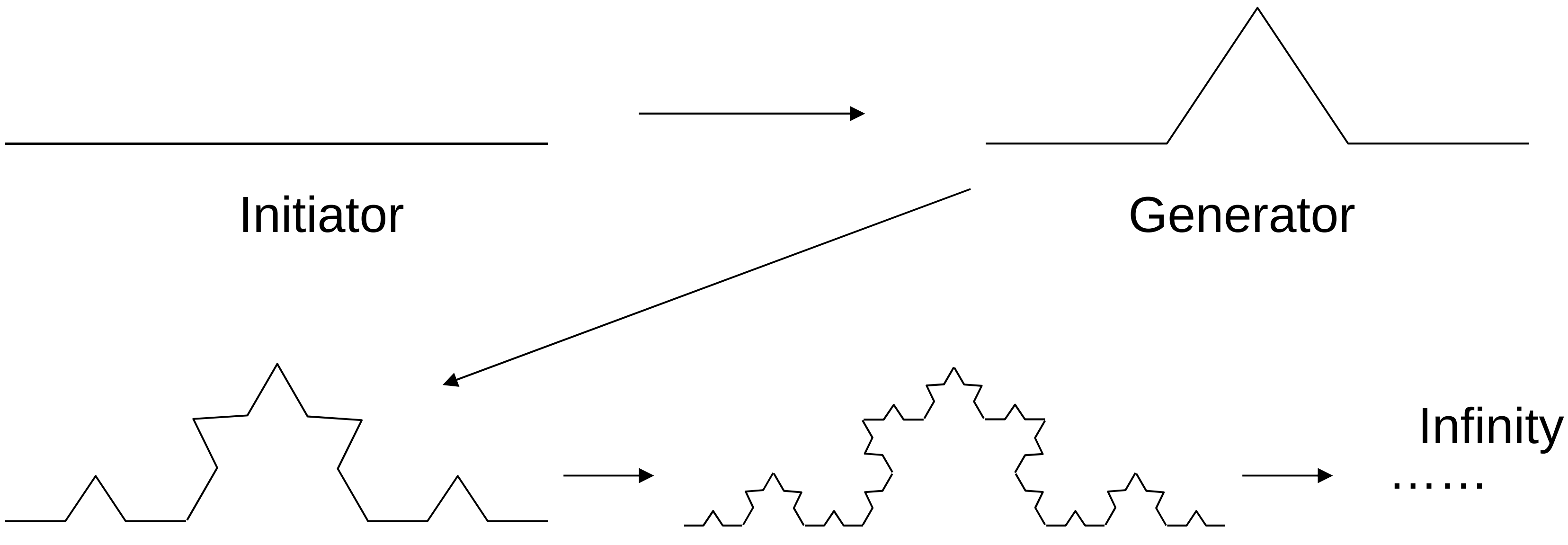
original (256 greys) ordered Floyd-Steinberg Jarvis

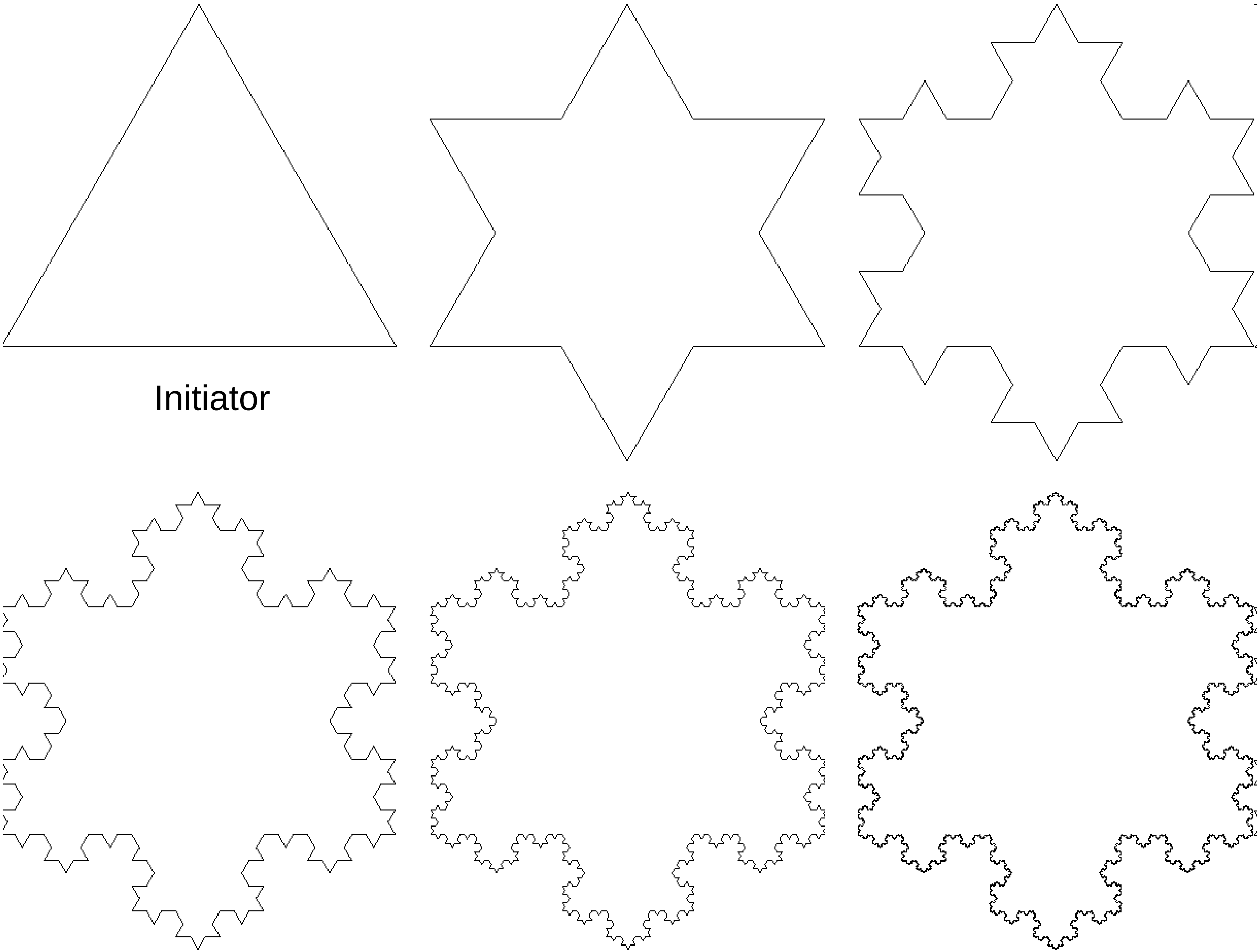
Special Effect (Diff. Error Diffusion)



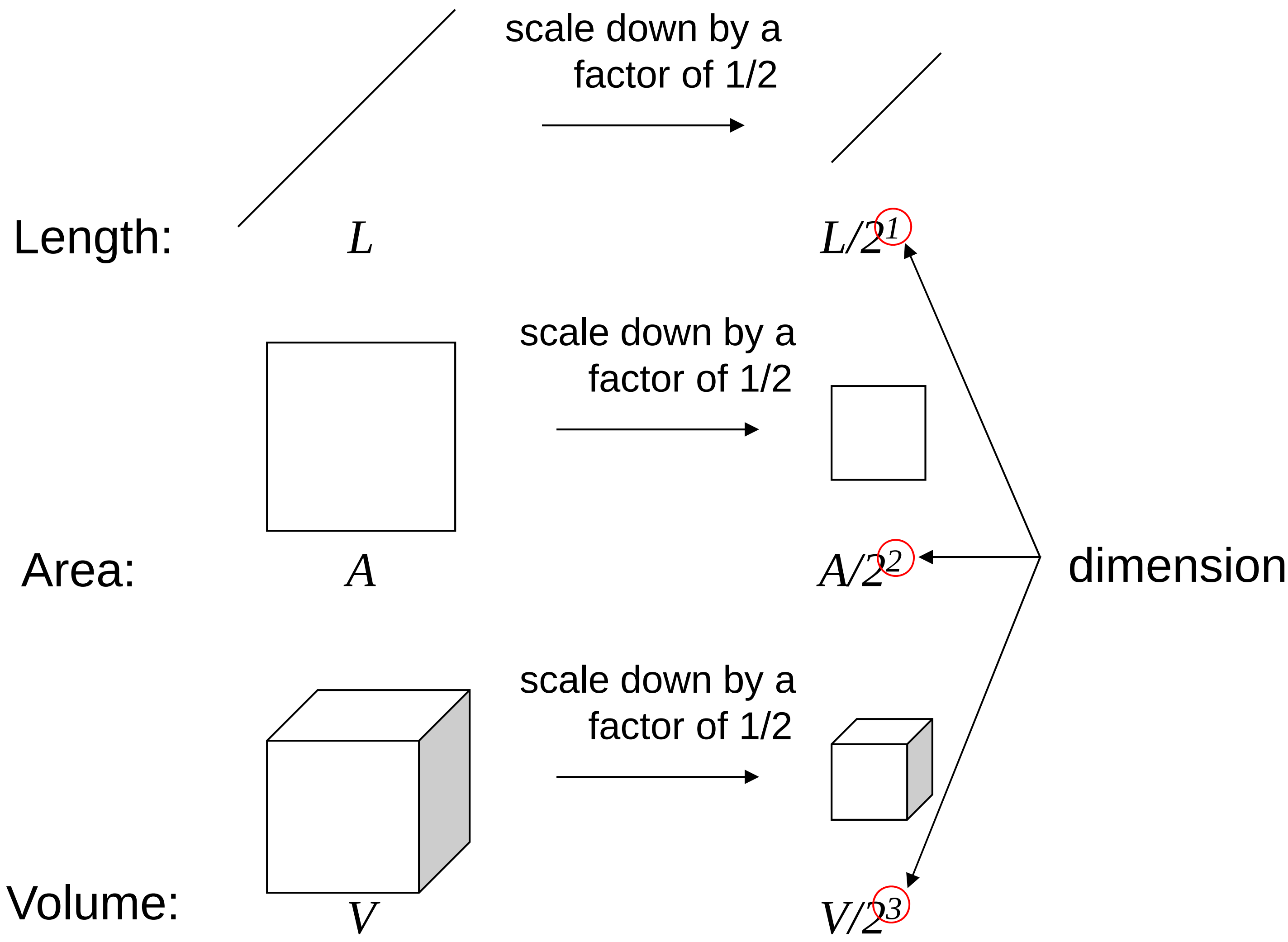
Fractals

Fractal Curves




Initiator

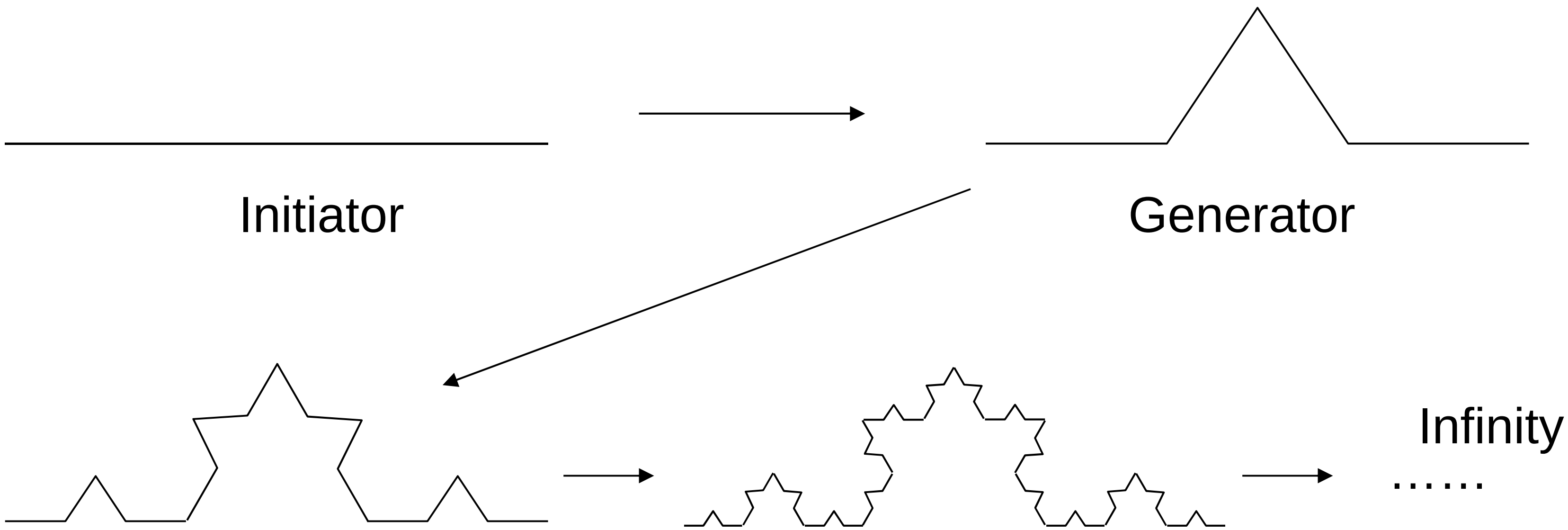
‘Normal’ Dimensions



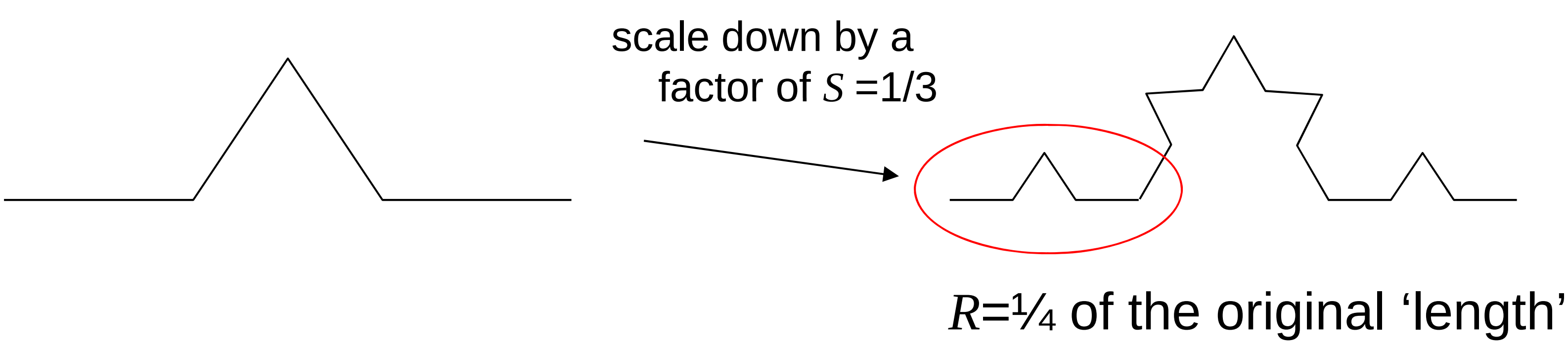
‘Normal’ Dimension

- ‘Volume’ ration: R
  - e.g. ½, 1/9, 1/125
- Scaling factor: S
  - e.g. ½
- If d is the dimension of the object, we have the following relationship:
  - $R^{1/d} = S$

Fractal Curves



Fractal Dimension

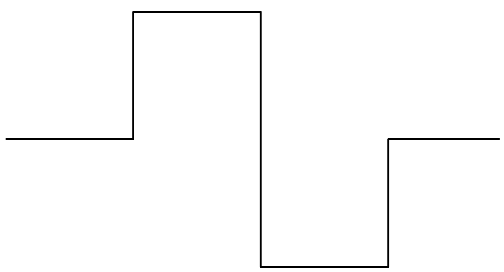


$$R^{1/d} = S$$

$$(1/4)^{1/d} = (1/3)$$

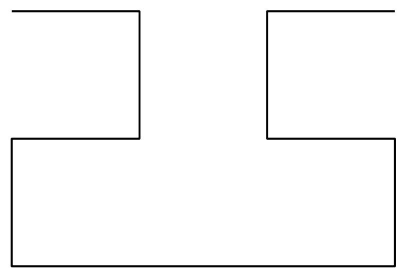
$$d = 1.26\ldots$$

Fractal Dimension



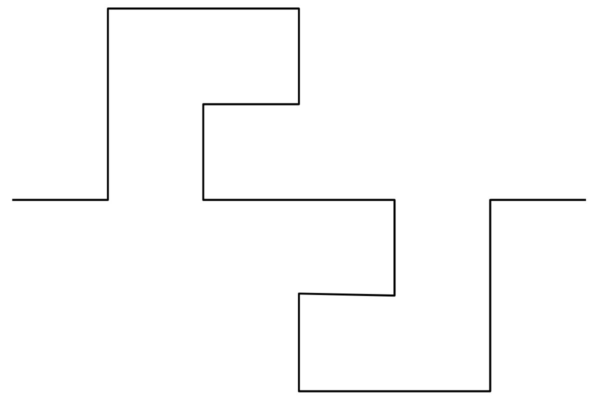
$(1/8)^{1/d} = (1/4)$

$d = 1.5$



$(1/11)^{1/d} = 1/3$

$d = 2.183\ldots$

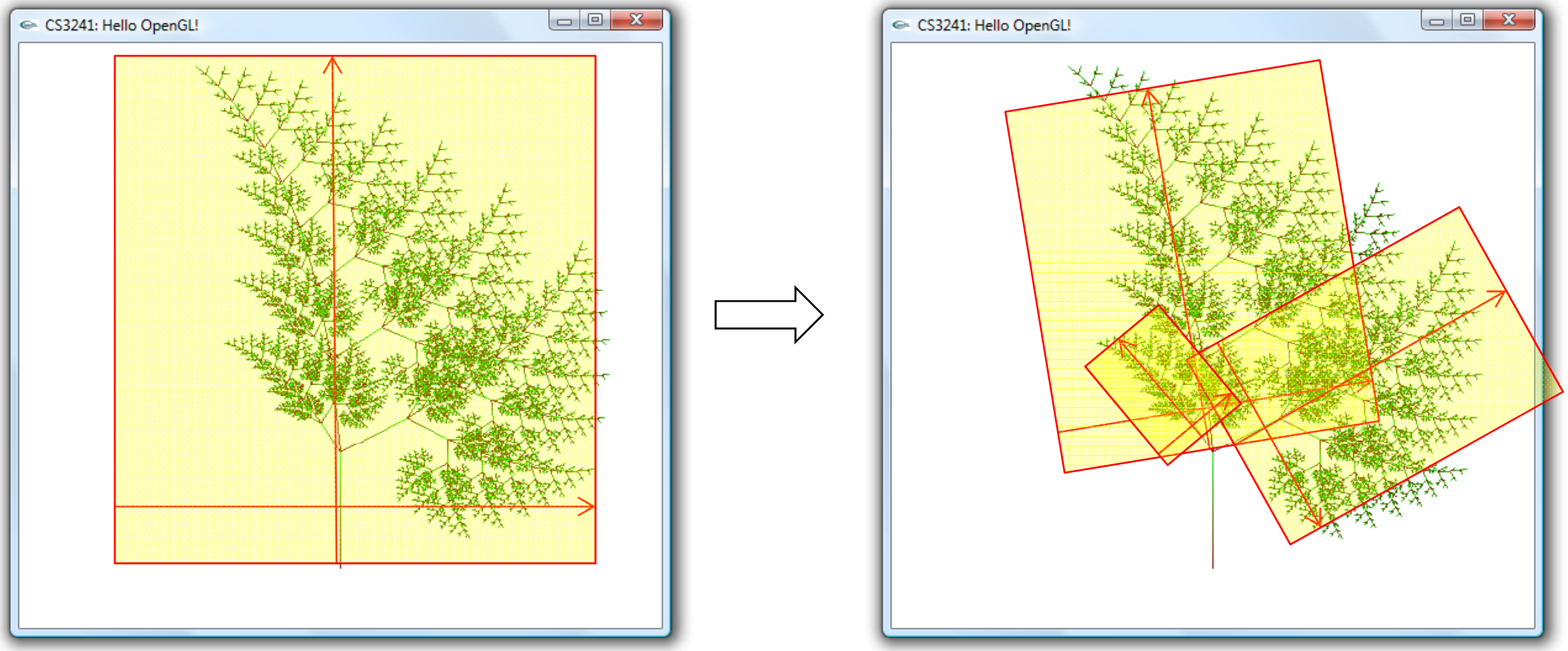


$(1/18)^{1/d} = (1/6)$

$d = 1.613\ldots$

Space filling curve

Fractal Plants

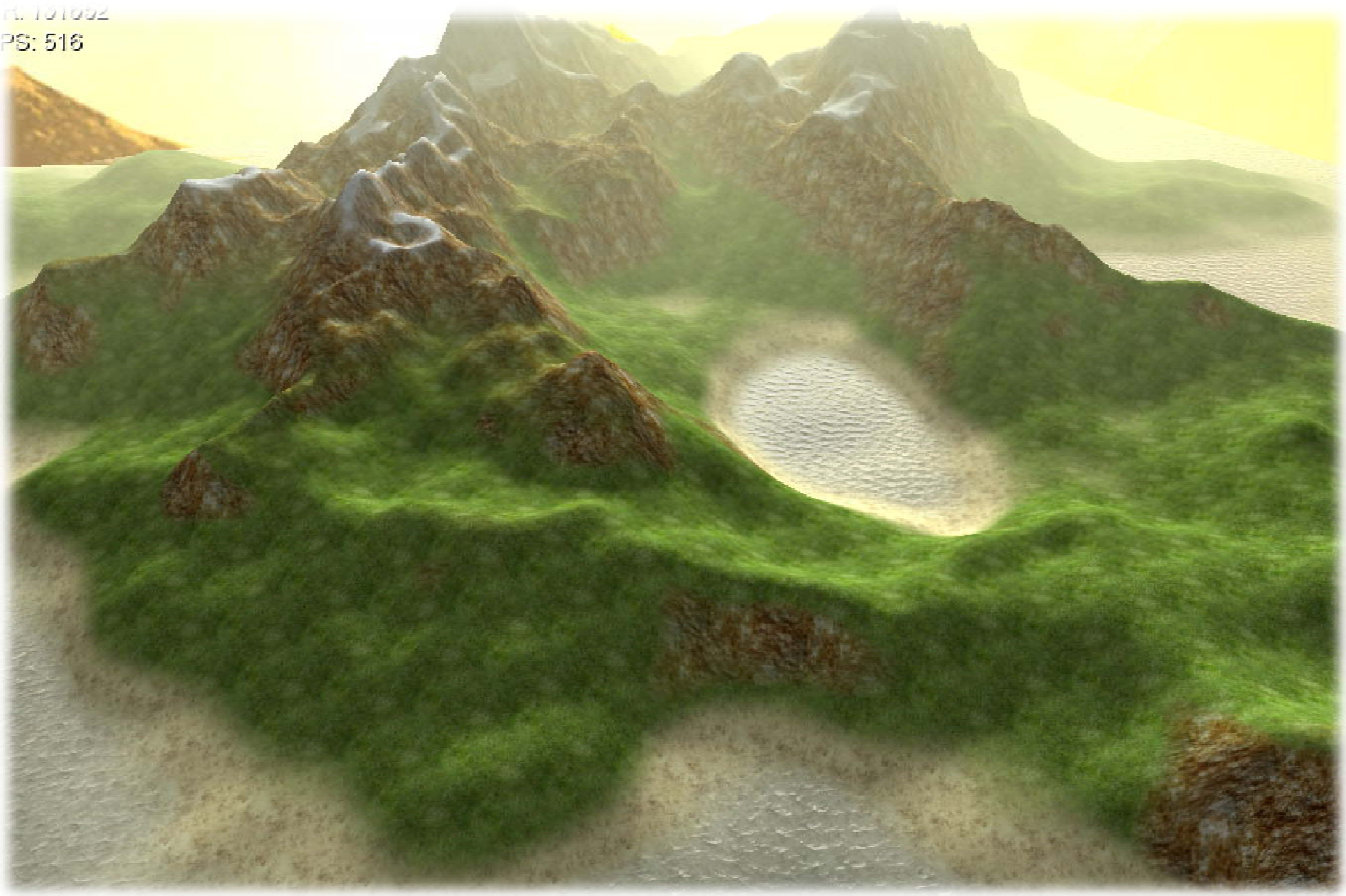


Fractal Trees

Fractal Trees

Figure 1: Photorealistically rendered images of the synthetic sample trees: Tree I: complex tree; Tree II: young lime tree; Tree III: conifer.

Fractal Terrains



Fractal Terrains