EE3206/EE3206E INTRODUCTION TO COMPUTER VISION AND IMAGE PROCESSING

Tutorial Set F – Solution
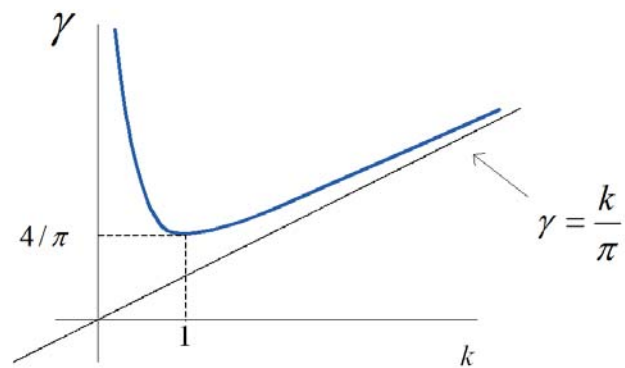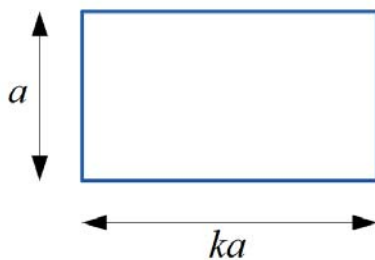

Question 1

Part (a)

Consider a rectangle with sides a, a, ka, ka. The compactness is

$$
\begin{aligned}
\gamma &= \frac{P^2}{4\pi A} \\
&= \frac{(2ka + 2a)^2}{4\pi \times ka^2} \\
&= \frac{4(k+1)^2}{4\pi k} \\
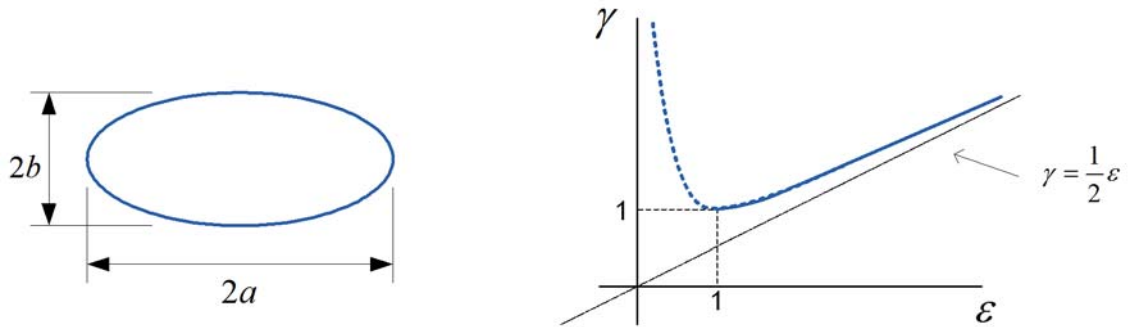&= \frac{(k+1)^2}{\pi k} \qquad (1)
\end{aligned}
$$

The minimum value of γ is $4/\pi$ when $k = 1$, i.e., a square.

Part (b)

$$\begin{aligned}
\gamma &= \frac{\pi^2 \times 2(a^2 + b^2)}{4\pi \times \pi ab} \\
&= \frac{a^2 + b^2}{2ab} \\
&= \frac{1 + \epsilon^2}{2\epsilon}, \qquad \text{where } \epsilon = \frac{a}{b} \geq 1
\end{aligned} \tag{2}$$

Minimum value of γ is 1, when $\epsilon = 1$, i.e., a circle.

Question 2

Procedure

1. Find a suitable threshold, T, by determining the valley between the two modes, or use the inter-means algorithm.

2. Threshold image at gray level T:

$$g(x,y) = \begin{cases} 1 & \text{if} \quad f(x,y) > T \\ 0 & \text{if} \quad f(x,y) \leq T. \end{cases} \tag{3}$$

3. Apply component labelling algorithm to $g(x,y)$ with $V = 1$.

4. This will result in 5 connected components labelled A to E. (As an additional step, small regions due to noise may be discarded by imposing a minimum-area requirement, or by using morphology.)

5. For each R_i, compute m_{00}, m_{01} and m_{10} to obtain the centroid $(\bar{x}, \bar{y})$:

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$
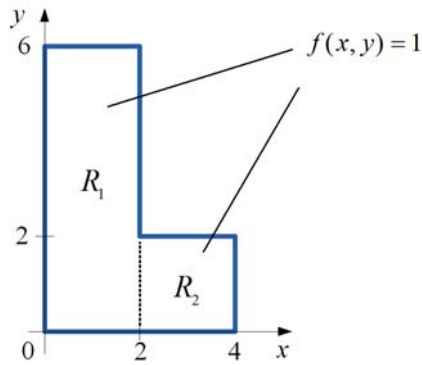
For example, for Region A:

$$m_{pq} = \sum \sum_{(x,y) \in A} x^p y^q f(x,y) \tag{4}$$
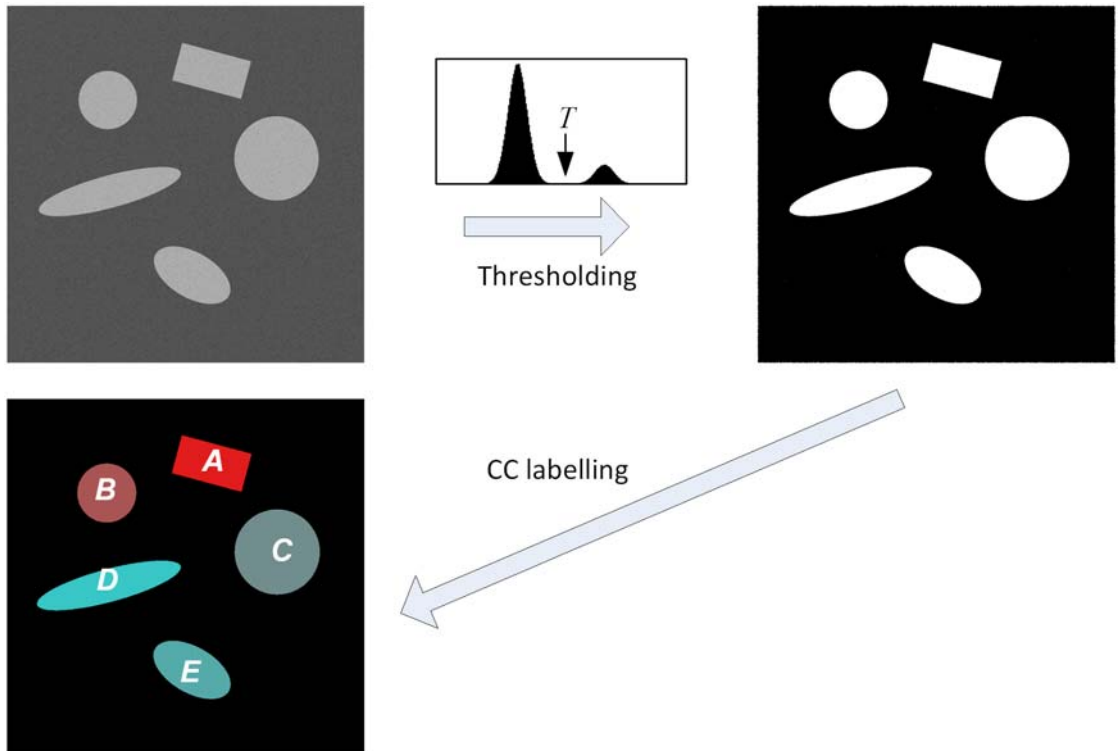
Question 3

Part (a)

$$m_{pq} = \int\int x^p y^q f(x,y) dx dy = \int\int_{R_1,R_2} x^p y^q dx dy = \int\int_{R_1} x^p y^q dx dy + \int\int_{R_2} x^p y^q dx dy$$

$$f(x,y) = \begin{cases} 1 & (x,y) \in R_1, R_2 \\ 0 & \text{elsewhere} \end{cases}$$

Take the origin to be the lower left corner.

$$
\begin{aligned}
m_{00} &= \text{area of object} = 16 \\
m_{10} &= \int\int x \, dx \, dy = \int_0^6 \left\{ \int_0^2 x \, dx \right\} dy + \int_0^2 \left\{ \int_2^4 x \, dx \right\} dy \\
&= \int_0^6 \left[x^2/2 \right]_0^2 dy + \int_0^2 \left[x^2/2 \right]_2^4 dy \\
&= \int_0^6 2 \, dy + \int_0^2 6 \, dy \\
&= 12 + 12 \\
&= 24 \\
m_{01} &= \int\int y \, dx \, dy = \int_0^6 \left\{ \int_0^2 y \, dx \right\} dy + \int_0^2 \left\{ \int_2^4 y \, dx \right\} dy \\
&= \int_0^6 2y \, dy + \int_0^2 2y \, dy \\
&= \left[y^2 \right]_0^6 + \left[y^2 \right]_0^2 \\
&= 36 + 4 \\
&= 40
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= m_{10}/m_{00} = 1.5 \\
\bar{y} &= m_{01}/m_{00} = 2.5
\end{aligned}
$$

$$
\begin{aligned}
\mu_{11} &= \iint (x - 1.5)(y - 2.5)\, dx\, dy \\
&= \int_0^6 \left\{ \int_0^2 (x - 1.5)(y - 2.5)\, dx \right\} dy + \int_0^2 \left\{ \int_2^4 (x - 1.5)(y - 2.5)\, dx \right\} dy \\
&= -3 - 9 \\
&= -12
\end{aligned}
$$

$$
\begin{aligned}
\eta_{11} &= \mu_{11}/\mu_{00}^2 \\
&= -12/16^2 \\
&= -0.0469
\end{aligned}
$$

Alternatively, use

$$
\mu_{11} = m_{11} - \bar{y}\, m_{10}
$$

We have

$$
\begin{aligned}
m_{11} &= \iint xy\, dx\, dy = \int_0^6 \left\{ \int_0^2 xy\, dx \right\} dy + \int_0^2 \left\{ \int_2^4 xy\, dx \right\} dy \\
&= \int_0^6 \left[x^2/2 \right]_0^2 y\, dy + \int_0^2 \left[x^2/2 \right]_2^4 y\, dy \\
&= \int_0^6 2y\, dy + \int_0^2 6y\, dy \\
&= \left[y^2 \right]_0^6 + \left[3y^2 \right]_0^2 \\
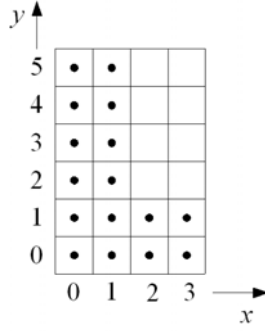&= 36 + 12 \\
&= 48
\end{aligned}
$$

$$
\begin{aligned}
\mu_{11} &= 48 - 2.5 \times 24 \\
&= -12
\end{aligned}
$$

$$
\begin{aligned}
\eta_{11} &= \mu_{11}/\mu_{00}^2 \\
&= -12/16^2 \\
&= -0.0469
\end{aligned}
$$

$$
\begin{aligned}
m_{20} &= \iint x^2 \, dx \, dy = \int_0^6 \left\{ \int_0^2 x^2 \, dx \right\} dy + \int_0^2 \left\{ \int_2^4 x^2 \, dx \right\} dy \\
&= 16 + \frac{112}{3} \\
&= 53.3333 \\
m_{02} &= \iint y^2 \, dx \, dy = \int_0^6 \left\{ \int_0^2 y^2 \, dx \right\} dy + \int_0^2 \left\{ \int_2^4 y^2 \, dx \right\} dy \\
&= \frac{432}{3} + \frac{16}{3} \\
&= 149.3333
\end{aligned}
$$

$$
\begin{aligned}
\mu_{20} &= m_{20} - \bar{x} m_{10} \\
&= 53.3333 - 1.5 \times 24 = 17.3333 \\
\mu_{02} &= m_{02} - \bar{y} m_{01} \\
&= 149.3333 - 2.5 \times 40 = 49.3333 \\
\eta_{20} &= \mu_{20} / \mu_{00}^2 = 0.0677 \\
\eta_{02} &= \mu_{02} / \mu_{00}^2 = 0.1927 \\
\phi_1 &= \eta_{20} + \eta_{02} \\
&= 0.26
\end{aligned}
$$

Part (b)

Choose origin at bottom left pixel.

$$m_{00} = 16$$

$$m_{10} = \sum\sum x^1 y^0 f(x,y) = \sum\sum x f(x,y) = \sum_{x=0}^{3} x \left\{ \sum_{y=0}^{5} f(x,y) \right\}$$

$$= 0 \times \sum_{y=0}^{5} f(x,y) + 1 \times \sum_{y=0}^{5} f(x,y) + 2 \times \sum_{y=0}^{5} f(x,y) + 3 \times \sum_{y=0}^{5} f(x,y)$$

$$= (0 \times 6) + (1 \times 6) + (2 \times 2) + (3 \times 2)$$

$$= 16$$

$$m_{01} = \sum\sum x^0 y^1 f(x,y) = \sum\sum y f(x,y) = \sum_{y=0}^{5} y \left\{ \sum_{x=0}^{3} f(x,y) \right\}$$

$$= (0 \times 4) + (1 \times 4) + (2 \times 2) + (3 \times 2) + (4 \times 2) + (5 \times 2)$$

$$= 32$$

$$\bar{x} = m_{10}/m_{00} = 1$$

$$\bar{y} = m_{01}/m_{00} = 2$$

$$\mu_{11} = \sum\sum (x-1)(y-2) f(x,y) = \sum_{y}(y-2)\left\{\sum_{x}(x-1)f(x,y)\right\}$$

$$= (-2)(-1+0+1+2) + (-1)(-1+0+1+2) + (0)(-1+0)$$

$$\quad +(1)(-1+0) + (2)(-1+0) + (3)(-1+0)$$

$$= -12$$

$$\eta_{11} = \mu_{11}/\mu_{00}^2 = -0.0469$$

$$
\begin{aligned}
m_{20} &= \sum\sum x^2 f(x,y) = \sum_x x^2 \left\{ \sum_y f(x,y) \right\} \\
&= (0^2 \times 6) + (1^2 \times 6) + (2^2 \times 2) + (3^2 \times 2) \\
&= 32 \\
m_{02} &= \sum\sum y^2 f(x,y) = \sum_y y^2 \left\{ \sum_x f(x,y) \right\} \\
&= (0^2 \times 4) + (1^2 \times 4) + (2^2 \times 2) + (3^2 \times 2) + (4^2 \times 2) + (5^2 \times 2) \\
&= 112
\end{aligned}
$$

$$
\begin{aligned}
\mu_{20} &= m_{20} - \bar{x} m_{10} \\
&= 32 - 1 \times 16 = 16 \\
\mu_{02} &= m_{02} - \bar{y} m_{01} \\
&= 112 - 2 \times 32 = 48
\end{aligned}
$$

$$
\begin{aligned}
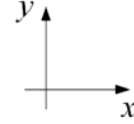\eta_{20} &= \mu_{20}/\mu_{00}^2 \\
&= 16/16^2 = 0.0625 \\
\eta_{02} &= \mu_{02}/\mu_{00}^2 \\
&= 48/16^2 = 0.1875 \\
\phi_1 &= \eta_{20} + \eta_{02} \\
&= 0.25
\end{aligned}
$$

Question 4

Part (a)

$$
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\ldots \; 0 & 0 & 1 & 1 & 2 & 2 \; \ldots \\
\ldots \; 0 & 0 & 1 & 1 & 2 & 2 \; \ldots \\
\ldots \; 2 & 2 & 0 & 0 & 1 & 1 \; \ldots \\
\ldots \; 2 & 2 & 0 & 0 & 1 & 1 \; \ldots \\
\ldots \; 1 & 1 & 2 & 2 & 0 & 0 \; \ldots \\
\ldots \; 1 & 1 & 2 & 2 & 0 & 0 \; \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}
$$

Texture P

$$
\mathbf{A}_{0,1} = \begin{bmatrix} 6 & 0 & 6 \\ 6 & 6 & 0 \\ 0 & 6 & 6 \end{bmatrix}
\qquad
\mathbf{A}_{0,2} = \begin{bmatrix} 0 & 0 & 12 \\ 12 & 0 & 0 \\ 0 & 12 & 0 \end{bmatrix}
$$

$$
\mathbf{C}_{0,1} = \begin{bmatrix} \frac{1}{6} & 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} \end{bmatrix}
\qquad
\mathbf{C}_{0,2} = \begin{bmatrix} 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}
$$

$$
\begin{aligned}
D & = \sum_{i=1}^{3} \sum_{j=1}^{3} (i-j)^2 c_{ij} \\
& = (1-1)^2 c_{11} + (1-2)^2 c_{12} + (1-3)^2 c_{13} \\
& + (2-1)^2 c_{21} + (2-2)^2 c_{22} + (2-3)^2 c_{23} \\
& + (3-1)^2 c_{31} + (3-2)^2 c_{32} + (3-3)^2 c_{33}
\end{aligned}
$$

For $\mathbf{C}_{0,1}$: $\quad D = 2^2 \times \frac{1}{6} + 1^2 \times \frac{1}{6} + 1^2 \times \frac{1}{6} = 1.0$

For $\mathbf{C}_{0,2}$: $\quad D = 2^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} = 2.0$

$$
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\ldots \ 0 & 1 & 2 & 0 & 1 & 2 \ \ldots \\
\ldots \ 1 & 2 & 0 & 1 & 2 & 0 \ \ldots \\
\ldots \ 2 & 0 & 1 & 2 & 0 & 1 \ \ldots \\
\ldots \ 0 & 1 & 2 & 0 & 1 & 2 \ \ldots \\
\ldots \ 1 & 2 & 0 & 1 & 2 & 0 \ \ldots \\
\ldots \ 2 & 0 & 1 & 2 & 0 & 1 \ \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}
$$

Texture Q

$$
\mathbf{A}_{-1,1} = \begin{bmatrix} 0 & 0 & 12 \\ 12 & 0 & 0 \\ 0 & 12 & 0 \end{bmatrix} \qquad\qquad \mathbf{A}_{1,1} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}
$$

$$
\mathbf{C}_{-1,1} = \begin{bmatrix} 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} \qquad\qquad \mathbf{C}_{1,1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}
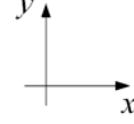$$

$$
\begin{aligned}
D \ &= \ \sum_{i=1}^{3}\sum_{j=1}^{3}(i-j)^2 c_{ij} \\
&= \ (1-1)^2 c_{11} + (1-2)^2 c_{12} + (1-3)^2 c_{13} \\
&+ \ (2-1)^2 c_{21} + (2-2)^2 c_{22} + (2-3)^2 c_{23} \\
&+ \ (3-1)^2 c_{31} + (3-2)^2 c_{32} + (3-3)^2 c_{33}
\end{aligned}
$$

For $\mathbf{C}_{-1,1}$: $\quad D = 2^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} = 2.0$

For $\mathbf{C}_{1,1}$: $\qquad\qquad\qquad D = 0$

Part (b)

Texture P

$$
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots\; 0 & 0 & 1 & 1 & 2 & 2 \;\cdots \\
\cdots\; 0 & 0 & 1 & 1 & 2 & 2 \;\cdots \\
\cdots\; 2 & 2 & 0 & 0 & 1 & 1 \;\cdots \\
\cdots\; 2 & 2 & 0 & 0 & 1 & 1 \;\cdots \\
\cdots\; 1 & 1 & 2 & 2 & 0 & 0 \;\cdots \\
\cdots\; 1 & 1 & 2 & 2 & 0 & 0 \;\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
& & f & & &
\end{array}
\qquad\Rightarrow\qquad
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots\; 0 & 1 & 0 & 1 & 0 & \text{-}2 \;\cdots \\
\cdots\; 0 & 1 & 0 & 1 & 0 & \text{-}2 \;\cdots \\
\cdots\; 0 & \text{-}2 & 0 & 1 & 0 & 1 \;\cdots \\
\cdots\; 0 & \text{-}2 & 0 & 1 & 0 & 1 \;\cdots \\
\cdots\; 0 & 1 & 0 & \text{-}2 & 0 & 1 \;\cdots \\
\cdots\; 0 & 1 & 0 & \text{-}2 & 0 & 1 \;\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
& & g & & &
\end{array}
$$

Texture Q

$$
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots\; 0 & 1 & 2 & 0 & 1 & 2 \;\cdots \\
\cdots\; 1 & 2 & 0 & 1 & 2 & 0 \;\cdots \\
\cdots\; 2 & 0 & 1 & 2 & 0 & 1 \;\cdots \\
\cdots\; 0 & 1 & 2 & 0 & 1 & 2 \;\cdots \\
\cdots\; 1 & 2 & 0 & 1 & 2 & 0 \;\cdots \\
\cdots\; 2 & 0 & 1 & 2 & 0 & 1 \;\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
& & f & & &
\end{array}
\qquad\Rightarrow\qquad
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots\; 1 & 1 & \text{-}2 & 1 & 1 & \text{-}2 \;\cdots \\
\cdots\; 1 & \text{-}2 & 1 & 1 & \text{-}2 & 1 \;\cdots \\
\cdots\; \text{-}2 & 1 & 1 & \text{-}2 & 1 & 1 \;\cdots \\
\cdots\; 1 & 1 & \text{-}2 & 1 & 1 & \text{-}2 \;\cdots \\
\cdots\; 1 & \text{-}2 & 1 & 1 & \text{-}2 & 1 \;\cdots \\
\cdots\; \text{-}2 & 1 & 1 & \text{-}2 & 1 & 1 \;\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
& & g & & &
\end{array}
$$

$h_{g,P}$: points at $(-2, 1/6)$, $(0, 3/6)$, $(1, 2/6)$ over axis a_i.

$h_{g,Q}$: points at $(-2, 1/3)$, $(1, 2/3)$ over axis a_i.
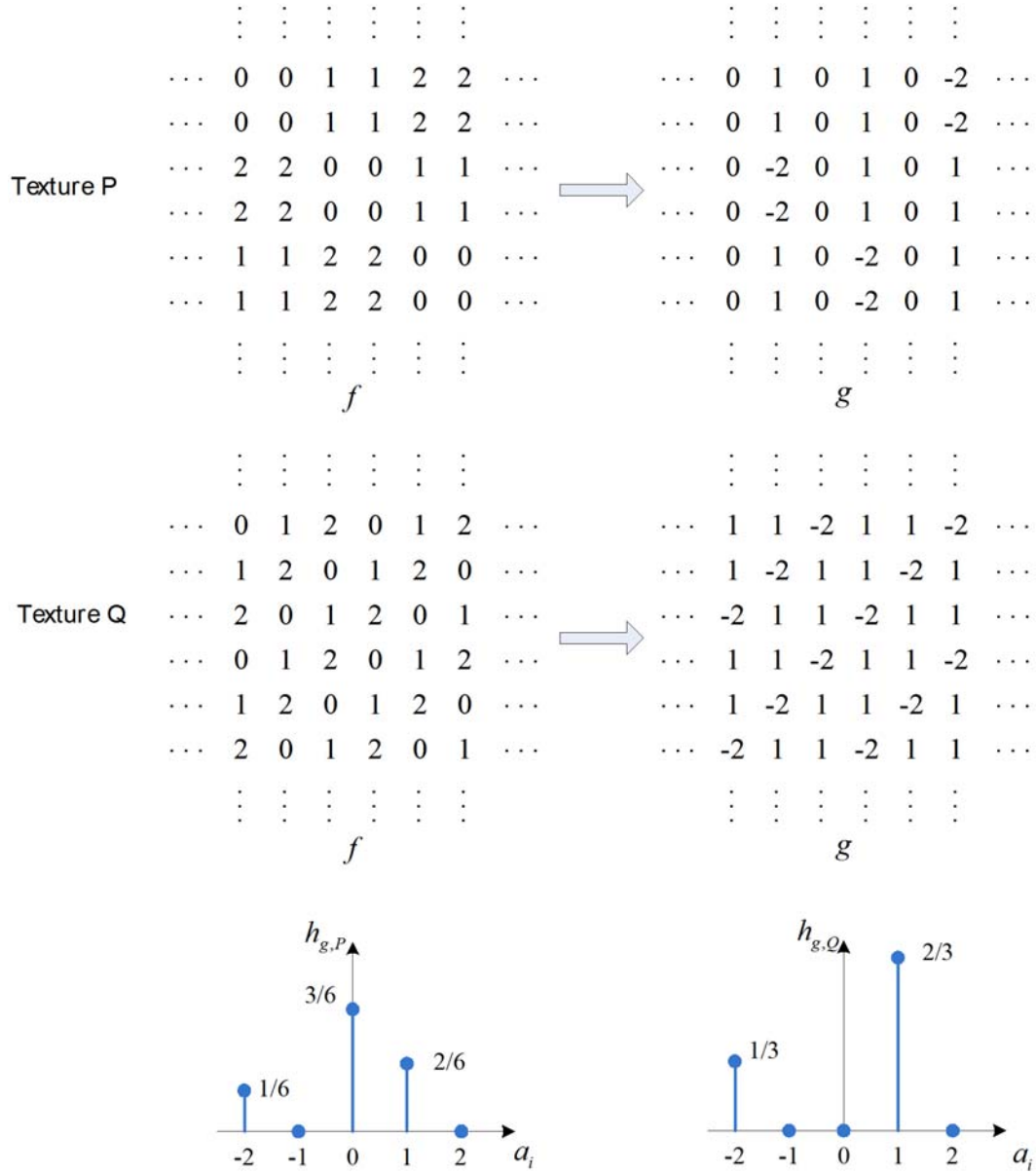
The nth moment of a about its mean, m is

$$
\mu_n(a) = \sum_{i=1}^{K}(a_i - m)^n p(a_i) \quad \text{where} \quad m = \sum_{i=1}^{K} a_i p(a_i)
$$

For the resulting histograms $h_{g,P}$ and $h_{g,Q}$, we compute m and μ_2 (the variance).

	m	μ_2
Texture P: $h_{g,P}$	0	1
Texture Q: $h_{g,Q}$	0	2

Hence, the textures can be differentiated by using the μ_2 values.