5. Systems and Classification of Systems

In this chapter, we introduce the notion of systems and examine the way in which systems are classified into various categories in accordance with their properties.

5.1 Systems

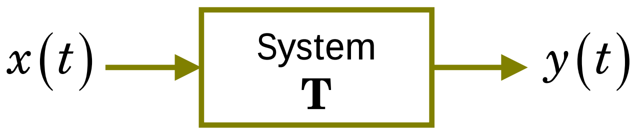
- Physical systems, in the broadest sense are interconnections of components, devices or subsystems. Examples are communication systems, mechanical systems, electronic systems, chemical processing systems, etc.

- In system studies, a system is a mathematical model of a physical process that relates the input (or **excitation**) signal to the output (or **response**) signal.

- With an input $x(t)$ and an output $y(t)$, the system may be viewed as a transformation (or mapping) of $x(t)$ into $y(t)$, mathematically denoted by

$$y(t) = \mathbf{T}\left[x(t)\right] \tag{5.1}$$

  and depicted by



  where $\mathbf{T}$ is the operator representing some well-defined transformation rule.

- Systems may be further classified into different categories according to their basic properties and the nature of their input and output signals.

5.1.1 Classification of Systems

A. *Systems with Memory and without Memory (Memoryless)*

- A system is said to be ***memoryless*** (or ***static***) if its output at a given time is dependent on only the input at that same time. Otherwise, the system is said to have ***memory*** (or to be ***dynamic***).

Example 5-1:

An example of a memoryless system is a resistor R with the current $i(t)$ flowing through it taken as the input and the voltage $v(t)$ across it as output. The input-output relationship (Ohm's law) of the resistor is

$$v(t) = R \cdot i(t).$$

Clearly, the output voltage at time t depends on only the value of the input current at time t.

An example of a system with memory is a capacitor C with the current $i(t)$ flowing through it taken as the input and the voltage $v(t)$ across it as output. The input-output relationship of the capacitor is

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau \qquad \text{or} \qquad i(t) = C\frac{dv(t)}{dt}$$

Clearly, the output voltage at time t depends on all values of the input current from $-\infty$ to t.

B. *Deterministic and Stochastic Systems*

- A system is called a ***deterministic system*** if its behavior can be accurately predicted.
- A system is called a ***stochastic system*** if its behavior cannot be accurately predicted.

C. *Causal and Noncausal Systems*

- A system is called ***causal*** (or ***non-anticipative***) if its output at the present time depends on only the present and/or past values of the input.
  - It is thus not possible for a causal system to produce an output before an input is applied to it.
  - All memoryless systems are causal, but not vice-versa.

Examples 5-2:

$$\left.\begin{array}{l} y(t) = x(t-1) \\ y(t) = \int_{t-3}^{t-1} x(\tau)\,d\tau \end{array}\right\} \textbf{Causal systems}$$

- A system called ***noncausal*** (or ***anticipative***) if its output at the present time depends on future values of the input.

Examples 5-3:

$$\left.\begin{array}{l} y(t) = x(t+1) \\ y(t) = \int_{t-2}^{t+1} x(\tau)\,d\tau \end{array}\right\} \textbf{Noncausal systems}$$

D. *Stable and Unstable Systems*

- A system is ***bounded-input/bounded-output (BIBO) stable*** if for any bounded input $x(t)$ as defined by

$$\left|x(t)\right| \le K; \quad \forall t, \tag{5.2}$$

the corresponding output $y(t)$ is also bounded as defined by

$$\left|y(t)\right| \le L; \quad \forall t, \tag{5.3}$$

where K and L are finite positive constants.

Example 5-4:

> *When a small horizontal force is applied to the pendulum, it starts to oscillate with decaying amplitude until it finally comes to rest. This happens because the pendulum gradually loses energy through its mechanical bearings and air resistance as it swings until the energy transferred to it by the applied force is completely expended. This is an example of a stable system.*

- An ***unstable*** system is one in which not all bounded inputs lead to a bounded output.

Example 5-5:

> *When the volume a PA system is set too high, we sometimes hear a sustained sound of rapidly increasing loudness, called howling, from the loudspeaker. This occurs even if the person is speaking softly into the microphone. This is due to the positive feedback received by the microphone from the loudspeaker, thus causing the system to electrically resonate. This can also happen when the microphone is placed too near to the loudspeaker, for instance, with a karaoke system. This is an example of an unstable system.*

E. *Linear and Nonlinear Systems*

- A ***linear system*** is one that satisfies the following two conditions:

$$\text{Additivity}: \quad \left\{ \begin{aligned} &\mathbf{T}\left[x_1(t)+x_2(t)\right] = \mathbf{T}\left[x_1(t)\right]+\mathbf{T}\left[x_2(t)\right] \\ &\qquad\qquad\qquad\qquad = y_1(t)+y_2(t) \\ &\cdots\cdots\cdots\cdots \quad \textit{for any signals } x_1(t) \textit{ and } x_2(t) \end{aligned} \right\} \tag{5.4}$$

$$\begin{array}{l}\text{Homogeneity} \\ \text{(or Scaling)}\end{array}: \left\{ \begin{aligned} &\mathbf{T}\left[\alpha x(t)\right] = \alpha \mathbf{T}\left[x(t)\right] = \alpha y(t) \\ &\cdots\cdots \quad \textit{for any signal } x(t) \textit{ and any scalar } \alpha \end{aligned} \right\} \tag{5.5}$$

  - (5.4) and (5.5) can be combined into a single condition as

$$\mathbf{T}\left[\alpha_1 x_1(t)+\alpha_2 x_2(t)\right] = \alpha_1 y_1(t)+\alpha_2 y_2(t). \tag{5.6}$$

    (5.6) is known as the ***superposition property***.

  - Another important property of *linear systems* is that ***a zero input yields a zero output***. This is readily shown by setting $\alpha = 0$ in (5.5).

- Any system that does not satisfy (5.4) and/or (5.5) is classified as a ***nonlinear system***.

F. *Time-Invariant and Time-Varying Systems*

- A system is ***time-invariant*** if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Hence, a *time-invariant* system has the property of

$$\mathbf{T}\left[x(t-\tau)\right] = y(t-\tau) \tag{5.7}$$

for any real value of τ.

- A general N^{th}-order linear constant-coefficient differential equation model for *time-invariant* system is given by

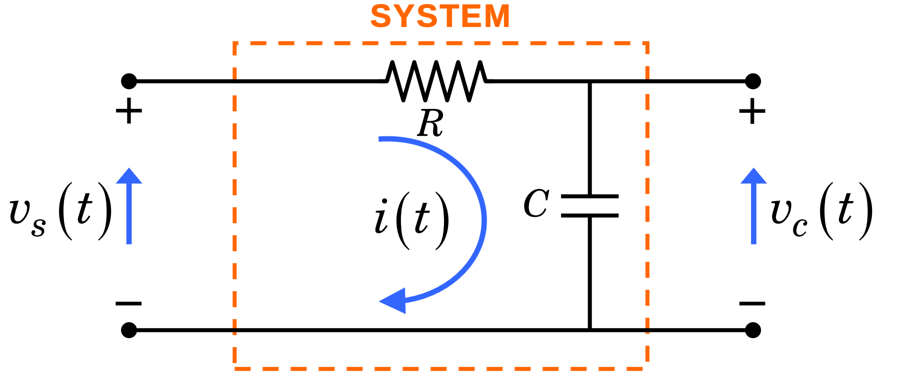
$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m} \tag{5.8}$$

where coefficients a_n and b_m are ***real constants*** and $x(t)$ and $y(t)$ are the input and output, respectively, of the system. The order N refers to the highest derivative of $y(t)$.

- A ***time-varying*** system is one which does not satisfy (5.7). In this case, the coefficients a_n and b_m in (5.8) are time-dependent and denoted by $a_n(t)$ and $b_m(t)$, respectively. When these coefficients take different values in time, the characteristics of the solution to the differential equation also changes.

Example 5-6:

Consider the RC circuit shown below where $x(t) = v_s(t)$ *is the input source voltage and* $y(t) = v_c(t)$ *the output voltage across the capacitor.*



Applying Kirchoff's law, we obtain

$$v_s(t) = R \cdot i(t) + v_c(t).$$

But $i(t) = C\dfrac{dv_c(t)}{dt}$. *Hence,*

$$RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)$$

or

$$RC\frac{dy(t)}{dt} + y(t) = x(t) \tag{5.9}$$

Notice that (5.9) is a special case of (5.8) with $N = 1$, $M = 0$, $a_0 = b_0 = 1$ *and* $a_1 = RC$.

5.2 Remarks

We often use mathematical tools to help us describe or model the behaviour of many different types of signals and systems. However, not all signals and not many systems can be described precisely by mathematics. But it is always possible to approximate their behaviour based on physical or natural laws of Physics. Even when it is possible to formulate a mathematical model to describe a signal or system, it is not always the case that they can be solved easily, especially when a closed-form solution is needed to predict the effects of system parameters on the output signal.

For this first course in Signals and Systems, we shall focus on

- **continuous-time**
- **deterministic**
- **causal**
- **linear**
- **time-invariant**

systems.

This class of systems can be described elegantly by mathematics. Their behaviours can be generalized easily and these lead to some nice properties that can be deduced for such systems, in many instances without having to solve their mathematical equations explicitly.