NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 6: SUGGESTED SOLUTIONS **SEMESTER II, AY 2010/2011**

1. First check that for valid partitions in both sets, the conditions $4 \leq n-4 \leq 16$ and $3 \leq n-5 \leq 15$ give the same range $8 \leq n \leq 20$.

   For any partition in A we add a first part of 4 to create a partition of n into exactly 5 parts with 4 as its largest part. Taking the conjugate map, this becomes a partition of n into exactly 4 parts, with 5 as its largest part. Be removing the first part, we obtain a partition in B. This map is well-defined and invertible.

2.
$$\begin{array}{rcl;{\quad}rcl}
10 & \leftrightarrow & 5+5 & 9+1 & \leftrightarrow & 9+1 \\
8+2 & \leftrightarrow & 1+1+1+1+1+1+1+1+1+1 & 7+3 & \leftrightarrow & 7+3 \\
6+4 & \leftrightarrow & 3+3+1+1+1+1 & 7+2+1 & \leftrightarrow & 7+1+1+1 \\
4+3+2+1 & \leftrightarrow & 3+1+1+1+1+1+1+1 & 6+3+1 & \leftrightarrow & 3+3+3+1 \\
5+4+1 & \leftrightarrow & 5+1+1+1+1+1 & 5+3+2 & \leftrightarrow & 5+3+1+1
\end{array}$$

3. The least $n = 7$. To show that 6 is not enough, simply consider $1,2,5,6,9,10$.

4.

5.

6.

7.