EE2011 Engineering Electromagnetics - Part CXD
Tutorial 6

Q1*

The electric field of a plane wave in free space is given by:

$$\mathbf{E}(x,t) = \hat{\mathbf{y}}50\cos\left(10^8 t + kx\right) \text{ V/m}$$

(i) Find the direction of the wave propagation.

(ii) Calculate the wave number *k* and the time t_1 required for the wave to travel a distance of $\lambda/2$.

(iii) Sketch the wave at $t = 0$, $T/4$, and $T/2$, where $T = 1/f$ is the period of the wave.

Q2*

The E-field of a uniform plane wave propagating in a dielectric nonmagnetic ($\mu = \mu_0$) medium is given by

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}2\cos\left(10^8 t - z/\sqrt{3}\right) \text{ (V/m)}$$

(i) Determine the frequency and wavelength of the wave.
(ii) What is the dielectric constant (or relative permittivity) of the medium?
(iii) Find an expression for the corresponding instantaneous H-field.

Q3

A 60-MHz plane wave traveling in the $-x$ direction in dry soil with relative permittivity $\varepsilon_r = 4$ and relative permeability $\mu_r = 1$ has an electric field polarized along the z-direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave's electric and magnetic fields.

Q4

An electromagnetic wave with the following instantaneous expression of electric field

$$\mathbf{E}(t) = \frac{1}{\sqrt{2}}(A\hat{\mathbf{y}} + \hat{\mathbf{z}})\cos\left[\frac{\beta}{\sqrt{2}}(y+z) - \omega t\right]$$

propagates in free space.

(i) Determine the x, y, z components of the wavenumber, k_x, k_y, k_z, respectively.

(ii) What is the wavenumber *k*? What is the unit vector $\hat{\mathbf{k}}$, i.e., the propagation direction?

(iii) Using the fact that $\mathbf{E}$ is perpendicular to $\hat{\mathbf{k}}$, determine the value of *A*.

(iv) What is the instantaneous expression of the magnetic field, $\mathbf{H}(t)$?

***** Questions to be discussed in the class.

For Q1 and Q2, which will be discussed in the tutorial class, the final solutions are given as follows. The full version of solutions will be distributed in due time.

Q1

(i) Direction of propagation: $-\hat{\mathbf{x}}$ direction

(ii) $\omega = 10^8$ rad/s, $k = \omega / c \approx 1/3$ m, $\lambda = \dfrac{2\pi}{k} = 6\pi$ m, $t_1 = \dfrac{\lambda/2}{c} = 31.42$ ns.

(iii) The wave has only the *y* component E_y.

$$E_y(x,t) = 50\cos\left(10^8 t + kx\right) = 50\cos\left(\omega t + kx\right) = 50\cos\left(\frac{2\pi}{T}t + kx\right)$$

$t = 0$: $E_y = 50\cos kx$

$t = T/4$: $E_y = 50\cos\left(\pi/2 + kx\right) = -50\sin kx$

$t = T/2$: $E_y = 50\cos\left(\pi + kx\right) = -50\cos kx$

Sketch the wave as the above equations (the sketch will be given in the full-version solution.)

Q2

The wave propagates in +z direction.

(i) $\omega = 10^8$ (rad/s) $\rightarrow$ $f = \dfrac{10^8}{2\pi} = 1.59\times10^7$ (Hz),

$k = \dfrac{1}{\sqrt{3}}$ (rad/m) $\rightarrow$ $\lambda = \dfrac{2\pi}{k} = 2\pi\sqrt{3}$ (m).

(ii) $\varepsilon_r = \left(\dfrac{ck}{\omega}\right)^2 = 3$.

(iii) $\mathbf{H}(z,t) = \dfrac{\sqrt{3}}{60\pi}\cos\left(10^8 t - z/\sqrt{3}\right)\hat{\mathbf{y}}$ (A/m)

***** Questions to be discussed in the class.