NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 4 **SEMESTER II, AY 2010/2011**

1. A long distance running race had 15 participants, among them Amy, Bob and Charlie.
   (a) How many outcomes are possible if we know that Amy finished ahead of Bob and Bob finished ahead of Charlie?
   (b) How many outcomes are possible if we only know that Bob did not finish first or last among the three of Amy, Bob and Charlie?
2. We want to divide 12 children into four playgroups. However, there are two pairs of siblings among the children, and we do not want to put siblings in different groups. How many possibilities do we have?
3. How many ways are there to partition $[10]$ into blocks of sizes 3, 3 and 4?
4. What is the number of compositions of 24 into any number of parts so that each part is divisible by three?
5. Find the number of nonnegative integer solutions to the following equations
   (a) $x_1+x_2+5x_3+x_4=14$.
   (b) $x_1+x_2+x_3+x_4\leq 14$.
6. Prove the following identities
   (a) $\left\{ {n \atop n-2} \right\} = \binom{n}{3}+3\binom{n}{4}$ using the recurrence formula of Theorem 6.5.
   (b) $\left\{ {n \atop 3} \right\} = \frac{1}{2}(3^{n-1}+1)-2^{n-1}$ using in two ways: combinatorially and using the recurrence formula of Theorem 6.9.
7. (Angpao question: Angpaos or red packets are cash gifts given during festive occasions. The amount given is usually an even number.)

   During Chinese New Year, you received a total of \$48 in angpao money. (A lean year.) You remembered receiving angpaos of \$2, \$4, \$8 and \$10, but you could not recall exactly how many angpaos you received. How many different ways could you have received your \$48?

Answers

2. 34105
3. 2100
4. 128
5. 190 ; 3060