NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

120

**ST 2334 PROBABILITY AND STATISTICS**

(Semester 2: AY 2007/2008)

April 2008 - Time Allowed: 2 Hours

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**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **SIX (6) Questions** and comprises **TWENTY ONE (21)** printed pages (inclusive of this cover page).
2. Candidates must answer **ALL** questions. The total mark for this paper is 120.
3. Please show work and answers in the space provided for each question. DO NOT use pencils to write answers.
4. This is a CLOSED BOOK examination. ONE A-4 size cheat sheet is allowed.
5. Hand in this booklet at the end of the examination.
6. Non-programmable calculators may be used.
7. Appendix A: Some key formulae
   Appendix B: Standardized Normal Distribution Table (*Z* Table)
   Appendix C: Student's *t*-Distribution Table (*t* Table)

**Matriculation No** : ______________________

**Seat Number** : ______________________

| Question | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Marks | | | | | | | |
| Max | 12 | 22 | 20 | 25 | 16 | 25 | 120 |

**Question 1 (12 Marks)**

The speed of the cars, in kilometre per hour on a certain stretch of the Orchid Road for 40 randomly selected cars is given below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 46 | 44 | 49 | 28 | 28 | 39 | 76 | 58 | 38 | 34 |
| 43 | 99 | 28 | 52 | 79 | 29 | 57 | 32 | 60 | 38 |
| 46 | 27 | 87 | 41 | 39 | 55 | 32 | 38 | 67 | 62 |
| 27 | 47 | 43 | 84 | 55 | 78 | 39 | 67 | 33 | 66 |

(i) Draw a stem-and-leaf plot for the above data. [4 Marks]

(ii) Identify the shape of the distribution. [2 Marks]

**Question 1 (Continued)**

(iii) Are there any outliers? [3 Marks]

(iv) Which measure of the central tendency is most suitable for this dataset? Why? [3 Marks]

**Question 2 (22 Marks)**

(A) Two dice are rolled in a game. You win if the sum of the outcomes of two dice is seven or eleven and you lose if the sum is two, three or twelve. You keep rolling until one of these sums occurs. Using conditional probability, find the probability of winning this game. [5 marks]

(B) There are three production lines in a factory, two of them are new. All three production lines produce components at the same rate. The old production line has 8% defective rate while the new production lines have only 3% defective rate. The components are shipped to customers in 100-unit lots. A buyer received a lot and tested five components. One failed. What is the probability that the lot was produced by

(i) The old line? [6 Marks]

(ii) One of the new lines? [2 Marks]

**Question 2 (Continued)**

(C) Of the six robots available, two have electronic defects, another one has defect in the memory and only three are in good working order. A sample of two robots is selected at random. Let $X$ be the number of robots with electronic defects, and $Y$ be the number of robots with defect in memory in the sample.

(i) Find the probability of at most one defect in the sample. [3 Marks]

(ii) Find the marginal distribution of $X$. [3 marks]

(iii) Find the conditional probability distribution of $Y$ given $X = 1$. [3 marks]

**Question 3 (20 Marks)**

85% of the students in a university is right handed (use right hand to complete most tasks such as writing) and 14% is left handed. The remaining are ambidextrous (use both hands equally well). There are 25 students at the bus stop outside the library. It is known that on average, 3 buses arriving at the bus stop in every 20 minutes.

(i) Find the probability that there are 20 right handed students at the bus stop. [4 marks]

(ii) Find the probability that there are 22 right handed students and 1 ambidextrous student at the bus stop. [4 marks]

**Question 3 (Continued)**

(iii) Find the probability that at least 2 buses arriving in a 10-minute interval.
[4 marks]

(iv) Find the probability that the waiting time for the next bus is at least 30 minutes.
[4 marks]

(v) What assumptions do you need to compute the above probabilities? [4 marks]

**Question 4 (25 Marks)**

STAT is a factory that produces electronic components. Currently there are three types of components in production, i.e. *Type A*, *Type B* and *Type C*. The lifetimes of the components are modelled closely with different distributions. The lifetimes of *Type A* are log-normally distributed with parameter $\alpha$ = 1 and $\beta$ = 0.5 year, the lifetimes of *Type B* are normally distributed with $\mu$ = 20 and $\sigma$ = 3 months while the lifetimes of *Type C* has a Weibull distribution with $\alpha$ = 1.5 and $\beta$ = 0.0001 hours.

(i) Find the probability that a *Type A* component lasts longer than four years. [4 Marks]

(ii) Find the third quartile of the lifetime of *Type A* components. [4 Marks]

(iii) The manufacturer is thinking to give a warranty for the lifetimes of *Type B*. if the component fails within the warranty period, the customer can get a free replacement of a new component. How long the manufacturer should specify as the warranty period so that at most 5% of the products will be replaced under warranty? [4 Marks]

**Question 4 (Continued)**

(iv) Find the probability that a *Type C* component lasts for more than 10,000 hours? [4 Marks]

(v) Show that the expected lifetime of *component C* is $\alpha^{-\frac{1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right)$. [4 marks]

(vi) *Type C* will be produced only when there is an order. Sometimes, the total quantity of orders is too large and some of the orders cannot be fulfilled within the time. Over many observations, the manufacturer found that the proportion of the orders being fulfilled could be modelled by a beta distribution with $\alpha = 4$ and $\beta = 2$. Find the probability that the manufacturer can fulfil at least 90% of the orders. [5 Marks]

**Question 5 (16 Marks)**

(A) Let $X_1, X_2, \ldots, X_n$ be a sample from the geometric distribution. Find the maximum likelihood estimator for the parameter. [5 marks]

(B) Refer to the data in question 1.

(i) Construct a 90% confidence interval for the percentage of all drivers who drive below 40 kilometres per hour on a certain stretch of Orchid road. Leave your answers up to two decimal places. [5 marks]

**Question 5 (Continued)**

(ii) Interpret the confidence interval above. [3 marks]

(iii) How large a sample will we need to be at least 98% confident that the error of the estimate is at most 5%? [3 marks]

**Question 6 (25 Marks)**

The human resources unit of a company is interested to know whether the time spent in employment is different for men and women. A sample is taken and some descriptive statistics for the weekly number of hours spent in employment are given below.

| Group | Size | Mean | Standard deviation |
|---|---|---|---|
| Men | 14 | 31.8 | 22.6 |
| Women | 15 | 18.4 | 20.0 |

(i) Does it seem plausible that time spent in employment has a normal distribution for each gender? Explain. [3 marks]

(ii) State the null and alternative hypotheses. [2 marks]

(iii) Compute the standard error for the estimate. [2 marks]

(iv) What is the degrees of freedom for the test statistic? [2 marks]

**Question 6 (Continued)**

(v) Compute the test statistic. [2 marks]

(vi) What is the P-value? Interpret. [4 marks]

(vii) Is the test significant at the 0.05 significance level? What is the conclusion? [2 marks]

**Question 6 (Continued)**

(viii) What is the possible error you could have made? Explain. [2 marks]

(ix) Construct a 80% confidence interval for the difference in average time spent in employment between men and women. [3 marks]

(x) What assumptions do you need for the above inference? [3 marks]

***** END OF PAPER *****

## Appendix A: Some Key formulae

| | | | |
|---|---|---|---|
| Mean: | $\mu = \dfrac{\sum x}{N}$ (population) | *or* | $\bar{x} = \dfrac{\sum x}{n}$ (sample) |
| Quartile positions: | $Q_1 = \dfrac{n+1}{4}$ | $Q_2 = \dfrac{n+1}{2}$ | $Q_3 = \dfrac{3(n+1)}{4}$ |
| Standard deviation: | $\sigma = \sqrt{\dfrac{\sum (x_i - \mu)^2}{N}}$ (population) | *or* | $s = \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n-1}}$ (sample) |
| Coefficient of variation ($c.v.$): | $cv = \dfrac{\sigma}{\mu} \times 100\%$ (population) | *or* | $cv = \dfrac{s}{\bar{x}} \times 100\%$ (sample) |

For Discrete Random Variables:

Expected Value $E(X) = \mu_x = \sum_i x_i P(x_i)$

Variance $Var(X) = \sigma_x{}^2 = \sum_i (x_i - \mu)^2 P(x_i) = E(X^2) - [E(X)]^2$

Covariance $Cov(X,Y) = \sigma_{XY} = \sum_i [x_i - E(X)][y_i - E(Y)]p(x_i y_i)$

For linear transformation of a random variable:

Expected Value $E(a + bX) = a + bE(X)$

Variance $Var(a + bX) = b^2 Var(X)$

For linear combination of 2 random variables:

Expected Value $E(aX + bY) = aE(X) + bE(Y)$

Variance $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X,Y)$

Bayes' Theorem

$$P(A) = \sum_{i=1}^{n} P(B_i) \bullet P(A \mid B_i)$$

$$P(B_r \mid A) = \frac{P(B_r) \bullet P(A \mid B_r)}{\sum_{i=1}^{n} P(B_i) \bullet P(A \mid B_i)}$$

Chebyshev's Theorem $P(|X - \mu| \geq k\sigma) \leq \dfrac{1}{k^2}$

Discrete Probability Distributions:

**Binomial Distribution**

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$$

$n$ = sample size

$x$ = # of successes in sample

$$E(X)=np$$

$$Var(X)=np(1-p)$$

**Hypergeometric Distribution**

$$P(X=x)=\frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$$

$N$ = population size
$a$ = # of successes in population
$n$ = sample size
$x$ = # of successes in sample

$$E(X)=n\frac{a}{N}$$

$$Var(X)=n\frac{a}{N}\left(1-\frac{a}{N}\right)\left(\frac{N-n}{N-1}\right)$$

**Poisson Distribution**

$$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!} \qquad x=0,1,2,\ldots$$

$$E(X)=\lambda$$

$$Var(X)=\lambda$$

**Geometric Distribution**

$$P(X=x)=p(1-p)^{x-1} \qquad x=1,2,\ldots$$

$$E(X)=\mu_x=\frac{1}{p}$$

$$Var(X)=\sigma_x^{\,2}=\frac{1-p}{p^2}$$

**Multinomial Distribution**

$$P(X=x)=\frac{n!}{x_1!x_2!\cdots x_k!}p_1^{\,x_1}p_2^{\,x_2}\cdots p_k^{\,x_k}$$

Continuous Probability Distributions:

**General Form**

$$F(a)=P(X\le a)=\int_{-\infty}^{a}f(x)dx$$

where $-\infty<a<\infty$ and $f(x)$ is a density function

$$E(X)=\mu=\int_{-\infty}^{\infty}xf(x)dx$$

$$E(g(X))=\int_{-\infty}^{\infty}g(x)\cdot f(x)dx$$

$$Var(X)=\sigma^2=E[(X-\mu)^2]=E(X^2)-\mu^2$$

**Normal Distribution**

$$X\sim N(\mu,\sigma^2) \qquad \text{pdf}:\ f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$$

$$Z=\frac{X-\mu}{\sigma} \qquad \text{pdf}:\ f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$$

| | | |
|---|---|---|
| Log-Normal Distribution | $\ln X \sim N(\alpha,\beta^2)$<br>$Z=\frac{\ln X-\alpha}{\beta}$ | if $X$ is log-normally distributed<br>$\mu=e^{\alpha+\beta^2/2}$ $\sigma=\sqrt{e^{2\alpha+\beta^2}(e^{\beta^2}-1)}$ |
| Exponential Distribution | $X \sim Exp(\lambda)$ | pdf : $f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & x<0\end{cases}$<br>where $\lambda$ = success rate per unit |
| Gamma Distribution | Gamma Function:<br>$X \sim Gamma(\alpha,\beta)$ | $\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}e^{-x}dx$<br>$\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$ and $\Gamma(\alpha)=(\alpha-1)!$<br>pdf : $f(x)=\begin{cases}\frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x>0,\alpha>0,\beta>0\\ 0 & otherwise\end{cases}$<br>$\mu=\alpha\beta$ $\sigma=\sqrt{\alpha\beta^2}$ |
| Beta Distribution | $X \sim Beta(\alpha,\beta)$ | pdf : $f(x)=\begin{cases}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\bullet\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1} & 0<x<1,\alpha>0,\beta>0\\ 0 & otherwise\end{cases}$<br>$\mu=\frac{\alpha}{\alpha+\beta}$ $\sigma=\sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}$ |
| Weibull Distribution | $X \sim Weibull(\alpha,\beta)$ | pdf : $f(x)=\begin{cases}\alpha\beta x^{\beta-1}e^{-\alpha x^\beta} & x>0,\alpha>0,\beta>0\\ 0 & otherwise\end{cases}$<br>$F(X\le t)=1-e^{-\alpha t^\beta}$<br>$\mu=\alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right)$<br>$\sigma=\sqrt{\alpha^{-2/\beta}\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\}}$ |
| Uniform Distribution | $X \sim U(\alpha,\beta)$ | $\mu=\frac{\alpha+\beta}{2}$ $\sigma=\sqrt{(\beta-\alpha)^2/12}$ |

Continuous – $f(x,y)$ is a joint density for random variables $X$, $Y$

$P(a\le X\le b, c\le Y\le d)=\int_a^b\int_c^d f(x,y)dydx$ where a, b, c, d are constants

$f_X(x)=\int_{-\infty}^{\infty}f(x,y)dy$ $\qquad f_Y(y)=\int_{-\infty}^{\infty}f(x,y)dx$

$f_X(x\mid y)=\frac{f(x,y)}{f_Y(y)}$ $\qquad f_Y(y\mid x)=\frac{f(x,y)}{f_X(x)}$

Sampling Distributions for $\overline{X}$: $\mu_{\overline{X}} = \mu$ $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$

Sampling Distributions for $\hat{p}$: $\mu_{\hat{p}} = p$ $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$

Finite population correction factor is $\left(\dfrac{N-n}{N-1}\right)$

One-sample Inference:

Test statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$$

100(1-α)% Confidence Interval for $p$

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \quad \text{or} \quad t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}}$$

100(1-α)% Confidence Interval for μ

$$\overline{X} \pm t_{\alpha/2, n-1}\frac{s}{\sqrt{n}}$$

Two-sample Inference:

Test statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - \delta_0}{\sqrt{p_0(1-p_0)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

100(1-α)% Confidence Interval for $p_1 - p_2$

$$\hat{p}_1 - \hat{p}_2 \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

(Unequal Variance)

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \mu_\Delta}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

100(1-α)% Confidence Interval for $\mu_1 - \mu_2$

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm t_{\frac{\alpha}{2}, v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad v = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \dfrac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$$

(Equal Variance)

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \mu_\Delta}{\sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

100(1-α)% Confidence Interval for $\mu_1 - \mu_2$

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm t_{\frac{\alpha}{2}, v}\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$v = n_1 + n_2 - 2$$

Matched Pairs Analysis

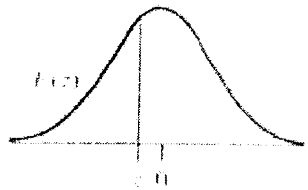
Test statistic:

$$t = \frac{\overline{X}_d}{s_d/\sqrt{n}}$$

100(1-α)% Confidence Interval for $\mu_d$

$$\overline{X}_d \pm t_{\alpha/2, n-1}\frac{s_d}{\sqrt{n}}$$

## Appendix B: Standard Normal Table (*Z* Table)



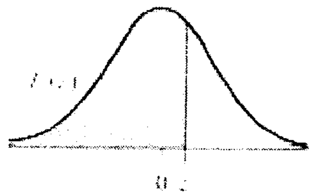
Standard Normal Distribution Function

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −5.0 | 0.0000003 | | | | | | | | | |
| −4.0 | 0.00003 | | | | | | | | | |
| −3.5 | 0.0002 | | | | | | | | | |
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0006 | 0.0003 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| −0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| −0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| −0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| −0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| −0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |

* Entries in the table represent area under the standard normal density curve from $-\infty$ to $z$

**Appendix B: (Continued from the previous page)**



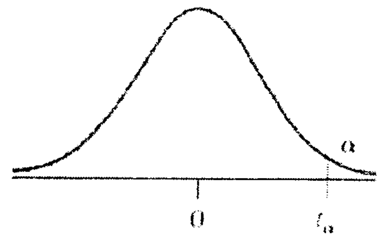
Standard Normal Distribution Function

$$F(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-t^2/2}\,dt$$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | | | | | | | | | |
| 4.0 | 0.99997 | | | | | | | | | |
| 5.0 | 0.9999997 | | | | | | | | | |

* Entries in the table represent area under the standard normal density curve from $-\infty$ to $z$

## Appendix C: Student's *t* –Table




| $\nu$ | $\alpha = 0.10$ | $\alpha = 0.05$ | $\alpha = 0.025$ | $\alpha = 0.01$ | $\alpha = 0.00833$ | $\alpha = 0.00625$ | $\alpha = 0.005$ | $\nu$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 38.204 | 50.923 | 63.657 | 1 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 7.650 | 8.860 | 9.925 | 2 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 4.857 | 5.392 | 5.841 | 3 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 3.961 | 4.315 | 4.604 | 4 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 3.534 | 3.810 | 4.032 | 5 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.288 | 3.521 | 3.707 | 6 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.128 | 3.335 | 3.499 | 7 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.016 | 3.206 | 3.355 | 8 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 2.934 | 3.111 | 3.250 | 9 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 2.870 | 3.038 | 3.169 | 10 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 2.820 | 2.891 | 3.106 | 11 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 2.780 | 2.934 | 3.055 | 12 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 2.746 | 2.896 | 3.012 | 13 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.718 | 2.864 | 2.977 | 14 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.694 | 2.837 | 2.947 | 15 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.673 | 2.813 | 2.921 | 16 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.655 | 2.793 | 2.898 | 17 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.639 | 2.775 | 2.878 | 18 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.625 | 2.759 | 2.861 | 19 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.613 | 2.744 | 2.845 | 20 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.602 | 2.732 | 2.831 | 21 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.591 | 2.720 | 2.819 | 22 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.582 | 2.710 | 2.807 | 23 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.574 | 2.700 | 2.797 | 24 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.566 | 2.692 | 2.787 | 25 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.559 | 2.684 | 2.779 | 26 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.553 | 2.676 | 2.771 | 27 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.547 | 2.669 | 2.763 | 28 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.541 | 2.663 | 2.756 | 29 |
| inf. | 1.282 | 1.645 | 1.960 | 2.326 | 2.394 | 2.498 | 2.576 | inf. |