NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

(Semester II: 2001–2002)

ST2334 PROBABILITY AND STATISTICS

April/May 2002 — Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions.
2. Answer **ALL** the questions. The number in [ ] indicates the number of marks allocated for that part. The total number of marks for this paper is 60 .
3. Write your answers in the spaces provided.
4. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
5. Candidates may bring in two A4-size (210 × 297 mm) help sheets.
6. Statistical tables are provided.

Matriculation No: ________________________

Question No.	1	2	3	4	5	**Total**
Mark						

1. A study was done using a scale, called the Boredom Proneness scale, to 25 male and 13 female college students:

Gender	sample size	sample mean	sample standard deviation
Male	$n_A = 25$	$\bar{X}_A = 10.40$	$S_A = 4.83$
Female	$n_B = 13$	$\bar{X}_B = 9.26$	$S_B = 4.68$

Assume that the two populations are approximately normal.

(a) Construct and explain the meaning of a 95% confidence interval for $\frac{\sigma_A^2}{\sigma_B^2}$. [6 marks]

(b) Can we assume their variances are equal, that is $\sigma_A^2 = \sigma_B^2$? Justify your answer based on the interval in (a). [2 marks]

(c) Do the data support the hypothesis that the mean Boredom Proneness scale is higher for men than for women? Use the 0.05 level of significance. [8 marks]

2. The proportion, p, of homes in Singapore which subscribe the cable television is believed to be 0.4. To test this claim a random sample of 10 homes is selected. If the number of homes that subscribe is 1 or less, we shall reject the hypothesis $p = 0.4$ in favor of $p < 0.4$.

   (a) Determine the level of significance of the test. [2 marks]

(b) Find the power of this test if the true p is 0.25. [2 marks]

(c) How large a sample is required if we want to be at least 90 % confident that our estimate of p is within 0.1? [2 marks]

...–6–

(d) Let X be the number of homes that subscribe in the 10 selected homes and $\hat{p} = X/10$. Is $\hat{p}^2$ an unbiased estimator of p^2? Why? [5 marks]

3. Two components of a minicomputer have the following joint density for their lifetimes X and Y (in years):

$$f(x,y) = \begin{cases} xe^{-x(1+y)}, & x \geq 0, y \geq 0 \\ 0, & otherwise. \end{cases}$$

What is the probability that the life time of at least one component exceeds 3 years? [5 marks]

4. Suppose a customer opening an account at the DBS bank will either open a savings or checking account but not both. Assume that cusotmers make decisions independently. From past experiences, customers who will open an account arrive according to a Poisson process. The number of customers (per day) arriving at the DBS bank for opening a savings account is a Poisson random variable with parameter $\lambda = 7$ and that for a checking account is also a Poisson random variable with parameter $\lambda = 3$. (The probability function of a Poisson distribution is: $p(x; \lambda) = e^{-\lambda}\lambda^x/x!$, for $x = 0, 1, 2, ...$). Let X_1 be the number of customers opening a savings account tomorrow, X_2 be the number of customers opening a checking account tomorrow.

   (a) What is the expected time between two successive customers opening an account ? [2 marks]

(b) Given that n customers will come and open an account tomorrow what is the conditional distribution of X_1? [5 marks]

(c) Find the probability that the account a customer opens at the DBS bank is a savings account. (Hint: use the result in (b)) [4 marks]

5. Anna and Maggie agree to meet at a certain location about 10:30 am. Suppose that Anna arrives at a time uniformly distributed between 10:15 and 10:45 am and Maggie independently arrives at a time uniformly distributed between 10:00 and 11:00 am. Let T_1 be the time Anna arrives and T_2 be the time Maggie arrives.

   (a) Find the joint density of (T_1, T_2) and sketch the domain where the joint density is not zero. [3 marks]

(b) What is the probability that Maggie arrives first? [6 marks]

(c) What is the expected time that the first person to arrive must wait? [6 marks]

(d) Are the two events, {Anna arrives first} and {Maggie arrives first}, independent? Why?
[2 marks]

—END OF PAPER—