CS3230 : Tutorial - 7

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Deadline 1 pm, 9-Oct-2012, slip below the door of S15-04-01

1. Suppose you are visiting a carnival and there are n activities in which you can participate. Activity number i has a starting time $S[i]$ and an ending time $E[i]$. Suppose you want to maximize the number of activities that you participate in (here if you start an activity, then you must stay with it until it finishes; you cannot participate in two activities simultaneously.) Write a dynamic-programming algorithm to solve this problem. How does your algorithm differ from a Greedy algorithm you have seen before for the same problem (what was the problem called before ?) What is the worst case time of your algorithm?

2. Write idea of a dynamic-programming algorithm to find a shortest cycle in a directed graph. What is the worst case time of your algorithm ?

3. Define
$$F = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$
Show that
$$F^n = \begin{pmatrix} f_{n-1} & f_n \\ f_n & f_{n+1} \end{pmatrix}, \quad n \geq 1$$
where f_n is the n-th Fibonacci number and f_0 is defined to be 0.

   Write an algorithm that calculates the n-th Fibonacci number (for n that is a power of 2) using at most $O(\log n)$ arithmetic operations.

4. Given sets of values $l_1, l_2, \ldots, l_n$ and $h_1, h_2, \ldots, h_n$ write a dynamic programming algorithm to find values $p_1, p_2, \ldots, p_n$ maximizing $\sum_{i=1}^{n} p_i$ under the following rules:

   (a) For each $i \in [n] : p_i = h_i$ or l_i or 0.

   (b) For each $i \in [n-1]$: If $p_{i+1} = h_{i+1}$ then $p_i = 0$.

5. The n-th Catalan number is defined to be:
$$C_n = \sum_{i=1}^{n} C_{i-1} C_{n-i}$$
for all $n \geq 1$ and $C_0 = 1$. Prove that the number of ways to group the product $M_1 \times \cdots \times M_n, \quad n \geq 2$ (each M_i is a matrix) is equal to C_{n-1}. Show that $C_n = \Omega(2^n)$. For example for product of three matrices $M_1 \times M_2 \times M_3$, different ways to group are $(M_1 \times (M_2 \times M_3))$ and $((M_1 \times M_2) \times M_3)$.