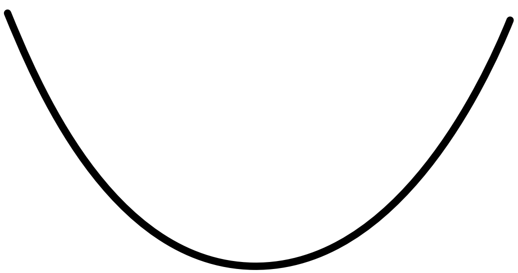
Points of Inflection

A point c is a point of inflection of the function f if f is continuous at c and there is an open interval containing c such that the graph of f changes from concave up (or down) before c to concave down (or up) after c.

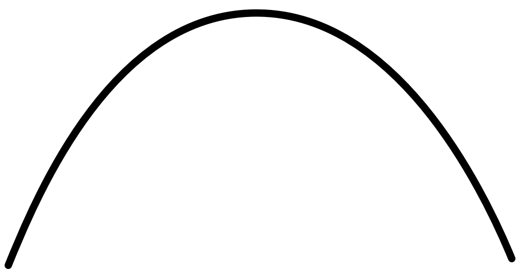
Question: How to test for Point of Inflection ??

A point c is a point of inflection of the function f if f is continuous at c and there is an open interval containing c such that the graph of f changes from concave up (or down) before c to concave down (or up) after c.



$$f''(x) > 0$$

Concave Up

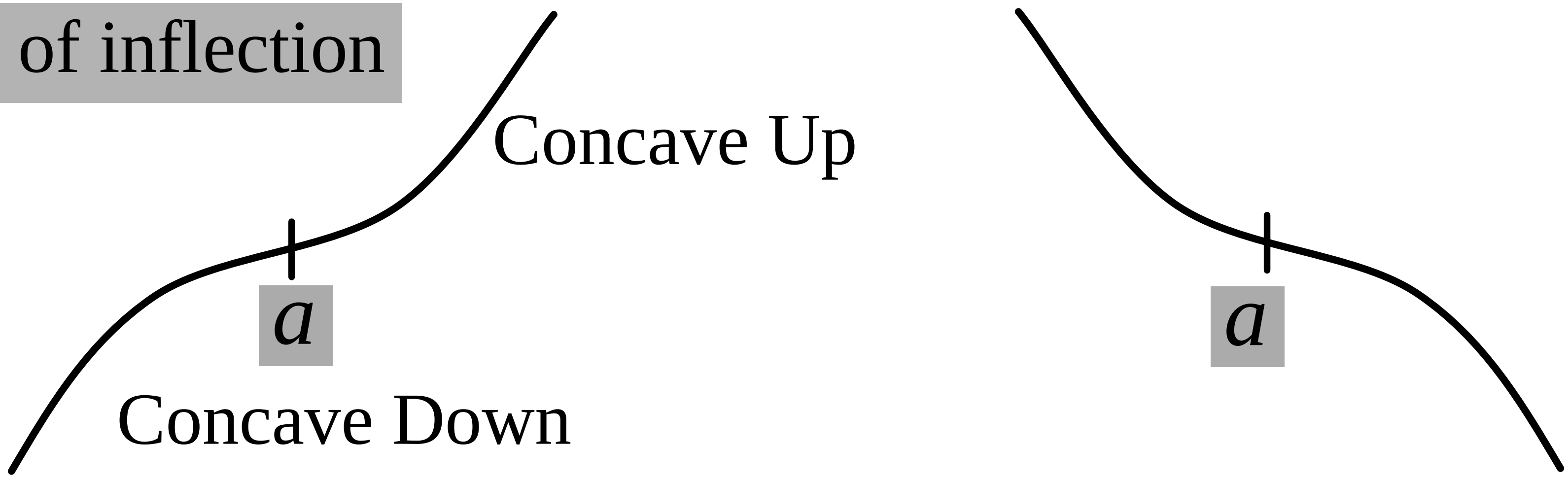


$$f''(x) < 0$$

Concave Down

What you have done in JC/High school

point of inflection



Pause and Think !!!

Question:

Can a point of inflection also a max point at the same time?

Can a point of inflection also a max point at the same time?

Pause and Think !!!

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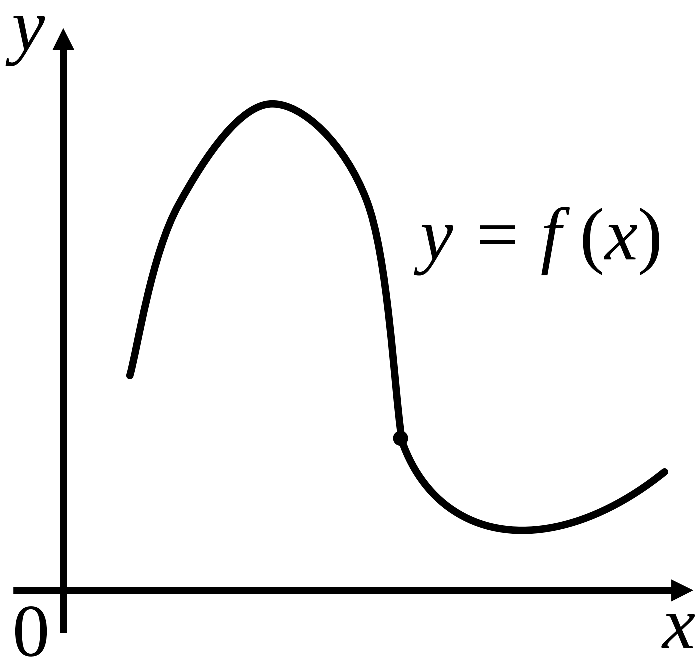
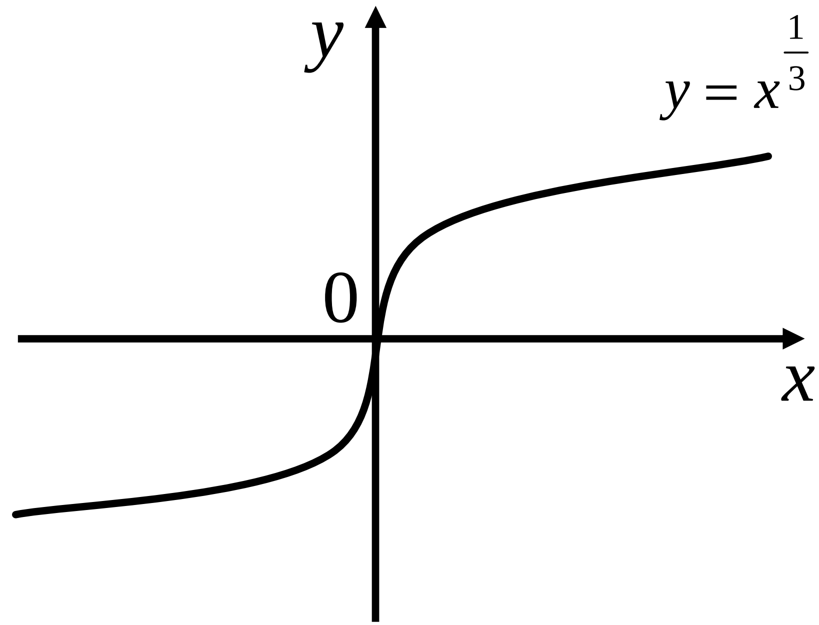
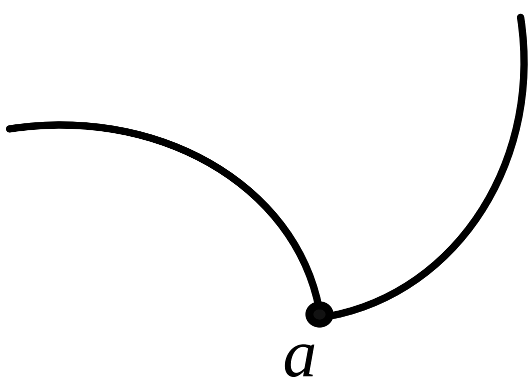
Note: The definition does not require that the function be differentiable at a point of inflection.

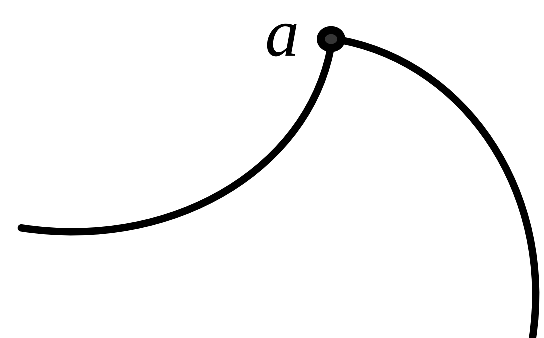






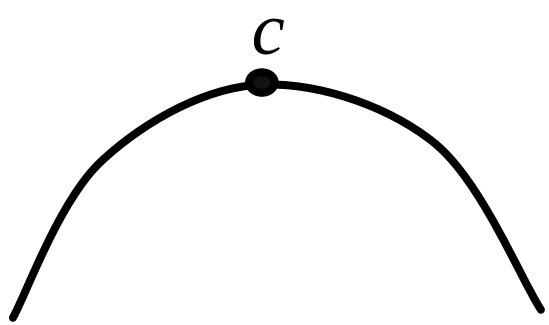
local min

local max



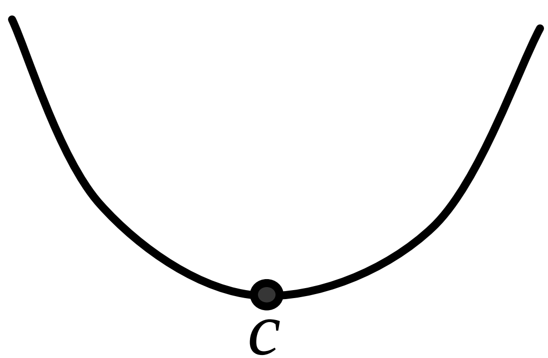
Second Derivative Test for Local Extreme Values

If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.



$f''(c) < 0$

If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

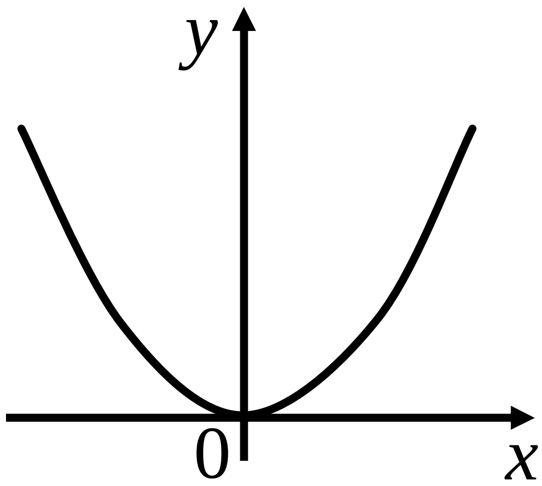


$f''(c) < 0$

Second Derivative Test - Note

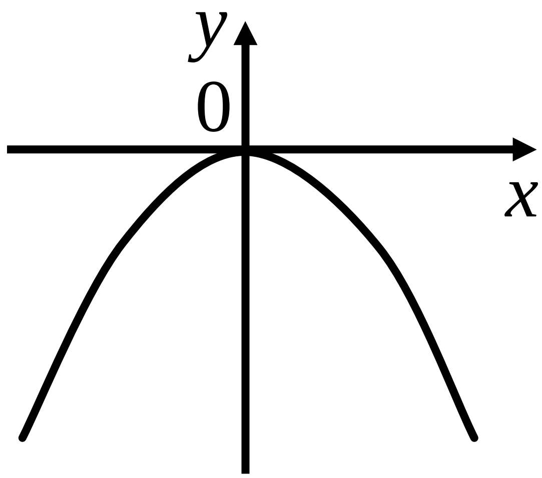
If $f'(c)=0$ and $f''(c)=0$, then the test fails.

$y=x^4$



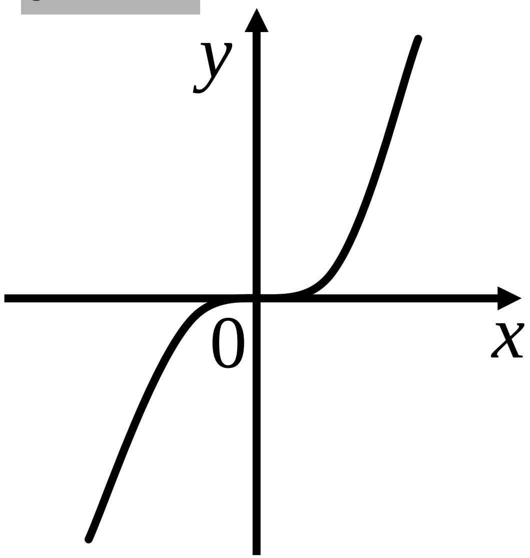
$\frac{dy}{dx}=4x^3$

$y=-x^4$



$\frac{dy}{dx}=-4x^3$

$y=x^3$



$\frac{dy}{dx}=3x^2$

Note : In all 3 cases, $y'(0)=y''(0)=0$

Optimization Problems

Optimization Problems

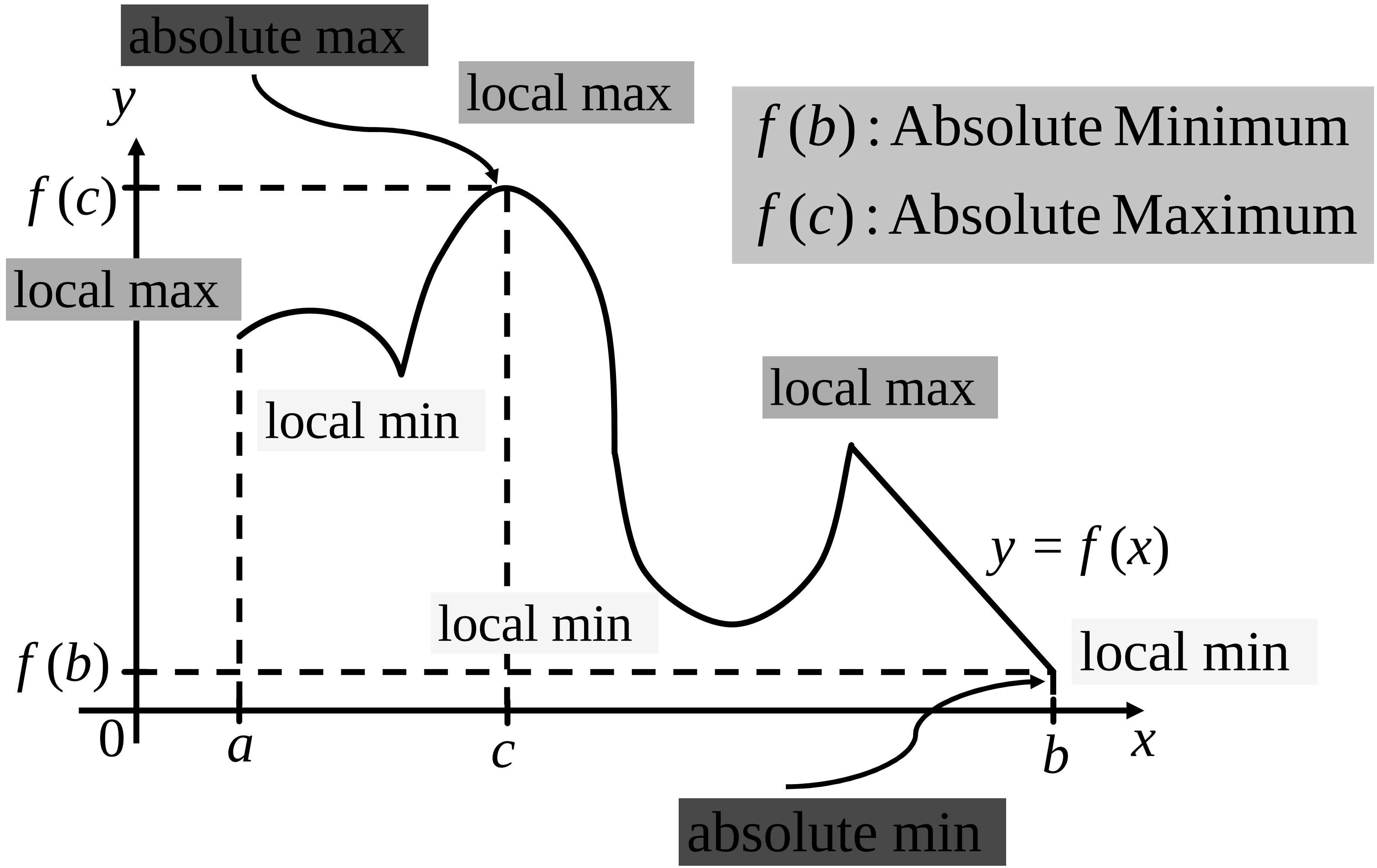
- Finding Absolute Extreme Values

Step 1: Find all the critical points of the function in the interior.

Step 2: Evaluate the functions at its critical points and at the end points of its domain.

Step 3: The largest and smallest of these values will be the absolute maximum and minimum values respectively.

Finding Absolute Extreme Values

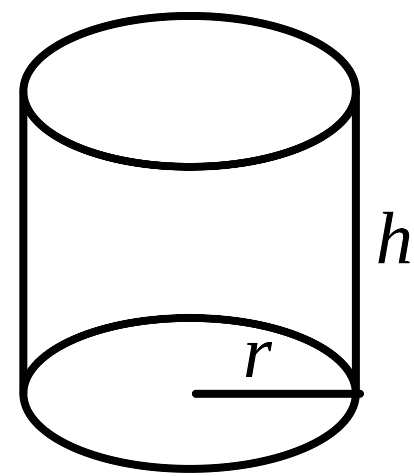


Absolute Extreme Values - Example

We are asked to design a 1000cm^3 shaped like a right circular cylinder. What dimensions will use the least material?

Let r be the radius of the circular base

and h the height of the can.



$$\text{Volume} = p r^2 h = 1000.$$

$$\text{Note that}: \quad h = \frac{1000}{p r^2}.$$

$$\text{The surface area } A = 2pr^2 + 2prh = 2pr^2 + \frac{2000}{r}, \;\; r > 0.$$

Absolute Extreme Values - Example

Note that : $h = \dfrac{1000}{p r^2}.$

Surface area $A = 2p r^2 + 2p rh = 2p r^2 + \dfrac{2000}{r}, \quad r > 0.$

$A' = 4p r - \dfrac{2000}{r^2}$

Solve $A' = 0$, we have $r = \left(\dfrac{500}{p}\right)^{\frac{1}{3}}$

$A'' = 4p + \dfrac{4000}{r^3} > 0$ since $r > 0$

Thus, $r = \left(\dfrac{500}{p}\right)^{\frac{1}{3}}$ leads to mimimum of A.

This value of r gives $h = 2r$.