Electrical & Computer Engineering Department
National University of Singapore

EE2011 Engineering Electromagnetics
Tutorial 4: Electric Fields
(for Week 7 beginning 27 February 2012)

The tutorial discussion will focus on Questions 2 and 3 (which are marked by asterisks *).

1. BASICS

(a) Two spheres (each carrying charge q_k where k = 1 or 2) are connected by a metallic wire as shown in Figure 1(a). Given that the radius r_k of each sphere is much smaller than the separation of the two spheres, show that $\frac{E_1}{E_2} = \frac{r_2}{r_1}$ where E_k is the electric field normal to the surface of sphere k.

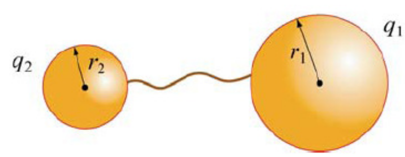

Figure 1(a)

(b) A flexible rod (which has been uniformly charged) is bent into the shape of a quarter-circle. If the rod has a linear charge density of λ, determine the electric field intensity vector at O which is at a distance of R from the quarter-circular arc.

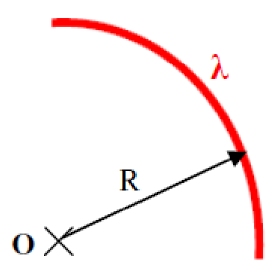

Figure 1(b)

(c) Depicted in Figure 1(c) is a length L of truncated cone where r_A and r_B are the radii of the circular cross-sections at A and B respectively. Show that the resistance for current flowing from A to B is given by $\frac{\rho L}{\pi r_A r_B}$ where ρ is the resistivity of the material.

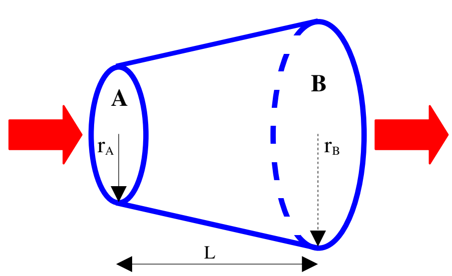

Figure 1(c)

2. * A sphere (of radius a) has a volume charge density of $\sigma(0<r<a) = \frac{\sigma_0 r}{a}$ where σ_0 is a constant and r is the distance from the center of the sphere.

(a) Derive expressions for the electric field inside the sphere (where $r < a$) and outside the sphere (where $r > a$).

(b) The charged sphere is placed concentrically inside a metallic spherical shell (of inner radius b and outer radius c). Derive expressions for the electric field in the exterior region (where $r > c$) for the following cases:

(i) when the spherical shell is left unearthed

(ii) after the spherical shell has subsequently been earthed.

3. * Figure 3 depicts a sphere (of radius a) which has been placed concentrically inside a spherical shell (of inner radius b and outer radius c). There are no free charges in the interior space (where $a < r < b$) between the two conductors.

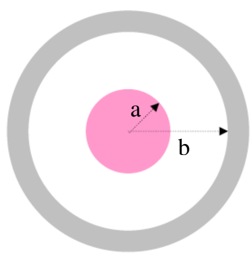

Figure 3

For such a structure (with spherical symmetry), the electric potential V in the interior space is governed by the following second-order differential equation:

$$\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0$$

(a) Solve this differential equation for V in the interior space given that the sphere is held at a potential V_0 while the shell has been earthed.

(b) Hence, derive an expression for the capacitance of this structure.

4. Figure 4 depicts the two-dimensional cross-section of a long prism-like structure. The L-shaped side AOB has been earthed whereas the hypotenuse side CD is held at a potential of 10 V.

Apply an appropriate numerical technique to estimate the potentials at the grid nodes identified by (1, 2) and (2, 2) where O is the origin of the two-dimensional Cartesian coordinate system represented in Figure 4 by faint lines.

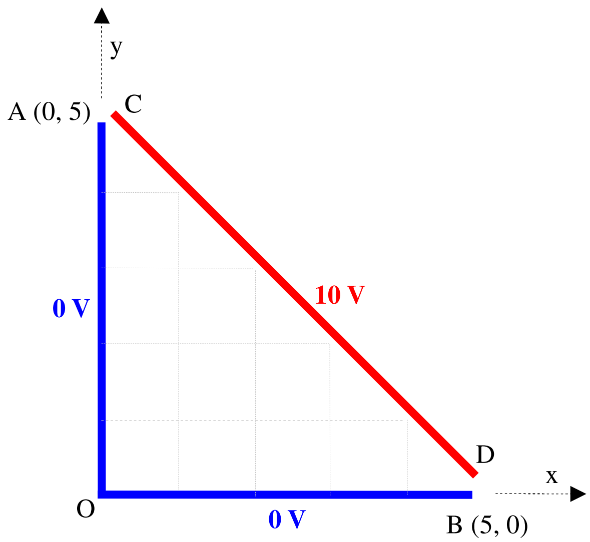

Figure 4

Answers:

1(b) $-\frac{\lambda}{4\pi\varepsilon_o R}\left(\hat{u}_x+\hat{u}_y\right)$

2. $\vec{E}(r<a)=\frac{\sigma_0 r^2}{4\varepsilon_0 a}\hat{u}_r$ and $\vec{E}(r>a)=\frac{\sigma_0 a^3}{4\varepsilon_0 r^2}\hat{u}_r$

3. $\frac{4\pi\varepsilon_0 ab}{b-a}$

4. 3.64 V and 6.82 V