NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

ST2334 Probability and Statistics

(Semester 2 : AY 2004-2005)

27 April 2005 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and **EIGHT (8)** pages including this cover page.
2. The last three (3) pages are statistical tables that students can use. Students should not use or ask for any other tables.
3. Candidates must answer **ALL SEVEN (7)** questions. The total mark for this paper is 60.
4. This is a **CLOSE BOOK** examination. However, each student is allowed to bring a formula sheet, single sided of A4 size or smaller, on which student can write whatever he/she thinks is helpful.

Matriculation No: _____

Question	Marks Score	Max. Marks
1		12
2		8
3		8
4		8
5		8
6		8
7		8
Total		60

1. Let the random variable X have the following probability density function

$$f(x)=\begin{cases} c\exp(-2(x-1)) & x>1 \\ 0 & x\le 1 \end{cases}$$

   a. Find the constant c.

   b. Find the expected value E(X).

   c. Find the probability $P(X>2)$.

2. In a population, 30% people are left-handed. If 10 people are randomly surveyed, find the probability that

   a. There are at least 2 left-handed people.

   b. There are exactly 3 left-handed people.

3. In a particular city, traffic accidents arrive according to a Poisson process. On average, an accident occurs once in every 10 days in the particular city. Find the probability that

   a. There will be no accident in the next 10 days in the city.

   b. There will be at most two accidents in the next 10 days in the city.

4. In a particular region, the annual family incomes have an average of $12,300 and a standard deviation of $1900.

   a. Suppose the family incomes are normally distributed. If one family is randomly selected, what is the probability that this family has an annual income of $11,500 or less?

   b. If 49 families are randomly selected, what is the probability that the average income of these 49 families is $11,500 or less?

5. The mean potency reading for a drug is 9.2 when the drug is just out of the production line. Some believes that the drug potency is reduced after a 3-year storage period. To test the hypothesis, 12 samples were obtained and these samples were drawn from the capsules that were stored for three years:

```
8.9 9.0 9.1 8.3 9.2 9.0 8.4 9.2 9.0 8.7 9.3 9.1
Sample mean=8.933, s=0.314.
```

The distribution is assumed to be normal. Do the data provide sufficient evidence to support the hypothesis that the drug potency decreases after a three-year storage period? Answer this by **writing down** the null and the alternative hypotheses, **stating** whether to reject the null hypothesis or not, and **interpreting** your conclusion.

6. Among 200 randomly surveyed children who were given the MMR vaccine, 20 had mild fever after the vaccine. Find a 90% confidence interval for the proportion of children who would have mild fever if given the vaccine.

7. An experiment was conducted to compare the mean length of time required for the bodily absorption of two drugs used in the treatment of epilepsy, D_1 and D_2. A group of 20 epileptic persons was randomly selected for inclusion in the study. Ten persons were randomly assigned to receive an oral dosage of drug D_1 with the other ten receiving drug D_2. The length of time (in minutes) for the drug to reach a specified level in the blood was recorded and believed to be normally distributed. Partial Minitab results are provided below:

```
      N       Mean      Standard Dev (s)
D1   10       34.2       13.5
D2   10       44.0       18.5

Difference = mu D1 - mu D2
Estimate for difference:  -9.79
T-Test of difference = 0 (vs not =): T-Value = -1.35
Both use Pooled StDev = 16.2
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a. Find a 95% confidence interval for the difference between the mean lengths of time for the two drugs to reach the specified level in the blood.

b. Is there any significant difference between the two mean lengths of time for the two drugs? Why?

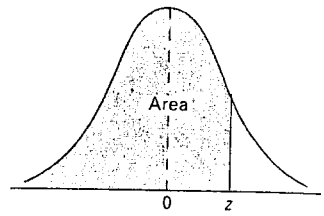

TABLE A.3 Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

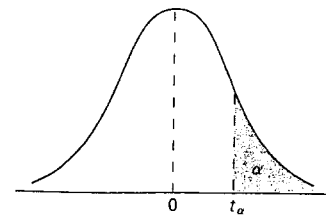

TABLE A.4 Critical Values of the *t*-Distribution

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

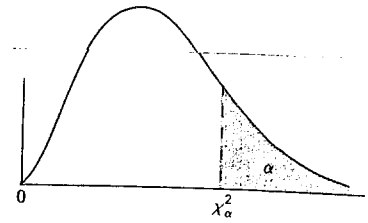

TABLE A.5 Critical Values of the Chi-Squared Distribution

	α									
v	**0.995**	**0.99**	**0.98**	**0.975**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.001**
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	3.841	5.024	5.412	6.635	7.879	10.827
2	0.0100	0.0201	0.0404	0.0506	5.991	7.378	7.824	9.210	10.597	13.815
3	0.0717	0.115	0.185	0.216	7.815	9.348	9.837	11.345	12.838	16.268
4	0.207	0.297	0.429	0.484	9.488	11.143	11.668	13.277	14.860	18.465
5	0.412	0.554	0.752	0.831	11.070	12.832	13.388	15.086	16.750	20.517
6	0.676	0.872	1.134	1.237	12.592	14.449	15.033	16.812	18.548	22.457
7	0.989	1.239	1.564	1.690	14.067	16.013	16.622	18.475	20.278	24.322
8	1.344	1.646	2.032	2.180	15.507	17.535	18.168	20.090	21.955	26.125
9	1.735	2.088	2.532	2.700	16.919	19.023	19.679	21.666	23.589	27.877
10	2.156	2.558	3.059	3.247	18.307	20.483	21.161	23.209	25.188	29.588
11	2.603	3.053	3.609	3.816	19.675	21.920	22.618	24.725	26.757	31.264
12	3.074	3.571	4.178	4.404	21.026	23.337	24.054	26.217	28.300	32.909
13	3.565	4.107	4.765	5.009	22.362	24.736	25.472	27.688	29.819	34.528
14	4.075	4.660	5.368	5.629	23.685	26.119	26.873	29.141	31.319	36.123
15	4.601	5.229	5.985	6.262	24.996	27.488	28.259	30.578	32.801	37.697
16	5.142	5.812	6.614	6.908	26.296	28.845	29.633	32.000	34.267	39.252
17	5.697	6.408	7.255	7.564	27.587	30.191	30.995	33.409	35.718	40.790
18	6.265	7.015	7.906	8.231	28.869	31.526	32.346	34.805	37.156	42.312
19	6.844	7.633	8.567	8.907	30.144	32.852	33.687	36.191	38.582	43.820
20	7.434	8.260	9.237	9.591	31.410	34.170	35.020	37.566	39.997	45.315
21	8.034	8.897	9.915	10.283	32.671	35.479	36.343	38.932	41.401	46.797
22	8.643	9.542	10.600	10.982	33.924	36.781	37.659	40.289	42.796	48.268
23	9.260	10.196	11.293	11.688	35.172	38.076	38.968	41.638	44.181	49.728
24	9.886	10.856	11.992	12.401	36.415	39.364	40.270	42.980	45.558	51.179
25	10.520	11.524	12.697	13.120	37.652	40.646	41.566	44.314	46.928	52.620
26	11.160	12.198	13.409	13.844	38.885	41.923	42.856	45.642	48.290	54.052
27	11.808	12.879	14.125	14.573	40.113	43.194	44.140	46.963	49.645	55.476
28	12.461	13.565	14.847	15.308	41.337	44.461	45.419	48.278	50.993	56.893
29	13.121	14.256	15.574	16.047	42.557	45.722	46.693	49.588	52.336	58.302
30	13.787	14.953	16.306	16.791	43.773	46.979	47.962	50.892	53.672	59.703

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