Question:

The selected questions in the Schaum's series as recommended by the lecturer are in angular frequency (ω) instead of cyclic frequency (f). I thought when we used the duality property, we need to account for the 2π factor when converting between f and ω, am I right?

Answer:

The Schaum's series has chosen to use ω instead of f. There is nothing wrong with that. In the SYSTEM part of this module we will also be using ω instead of f for spectral plots.

Yes, we have to account for the 2π factor in the duality property.

DUALITY property in the CYCLIC FREQUENCY (f) domain:

$$\textit{if}\quad \boldsymbol{x(t) \rightleftarrows X(f)}\quad \textit{then}\quad \boldsymbol{X(t) \rightleftarrows x(-f)}$$

Proof:

$$x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df$$

Interchanging the role of t *and* f, *we get*

$$x(f) = \int_{-\infty}^{\infty} X(t)\exp(j2\pi ft)\,dt\,.$$

Negating f, *we get*

$$x(-f) = \int_{-\infty}^{\infty} X(t)\exp(-j2\pi ft)\,dt = \Im_f\{X(t)\} \quad \text{-or-} \quad \boldsymbol{X(t) \rightleftarrows x(-f)}$$

where $\Im_f\{\cdot\}$ *denotes the Fourier transform from* t *to* f *domain.*

DUALITY property in the ANGULAR FREQUENCY (ω) domain:

$$\textit{if}\quad \boldsymbol{x(t) \rightleftarrows X(\omega)}\quad \textit{then}\quad \boldsymbol{X(t) \rightleftarrows 2\pi x(-\omega)}$$

Proof:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\exp(j\omega t)\,d\omega$$

Interchanging the role of t *and* ω, *we get*

$$x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t)\exp(j\omega t)\,dt$$

Negating ω, *we get*

$$x(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t)\exp(-j\omega t)\,dt = \frac{1}{2\pi}\Im_\omega\{X(t)\} \quad \text{-or-} \quad \boldsymbol{X(t) \rightleftarrows 2\pi x(-\omega)}$$

where $\Im_\omega\{\cdot\}$ *denotes the Fourier transform from* t *to* ω *domain.*

"Handout_ncs [Cyclic vs Angular Frequency]" has been revised to include some examples of Fourier transform and its properties in both f and ω domains for comparison. Do check this out at our EE2023 Group 2 (IVLE) course web.