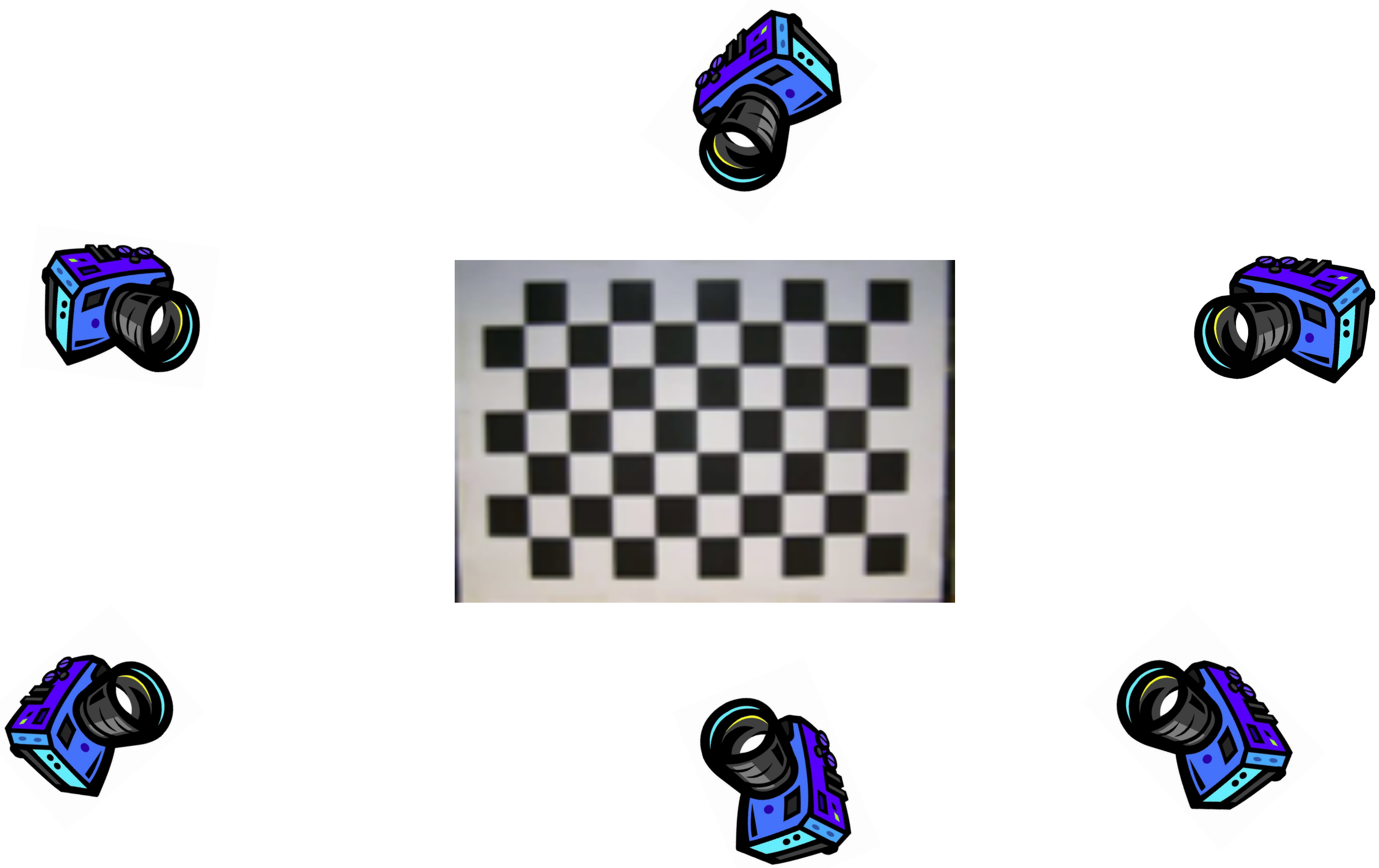
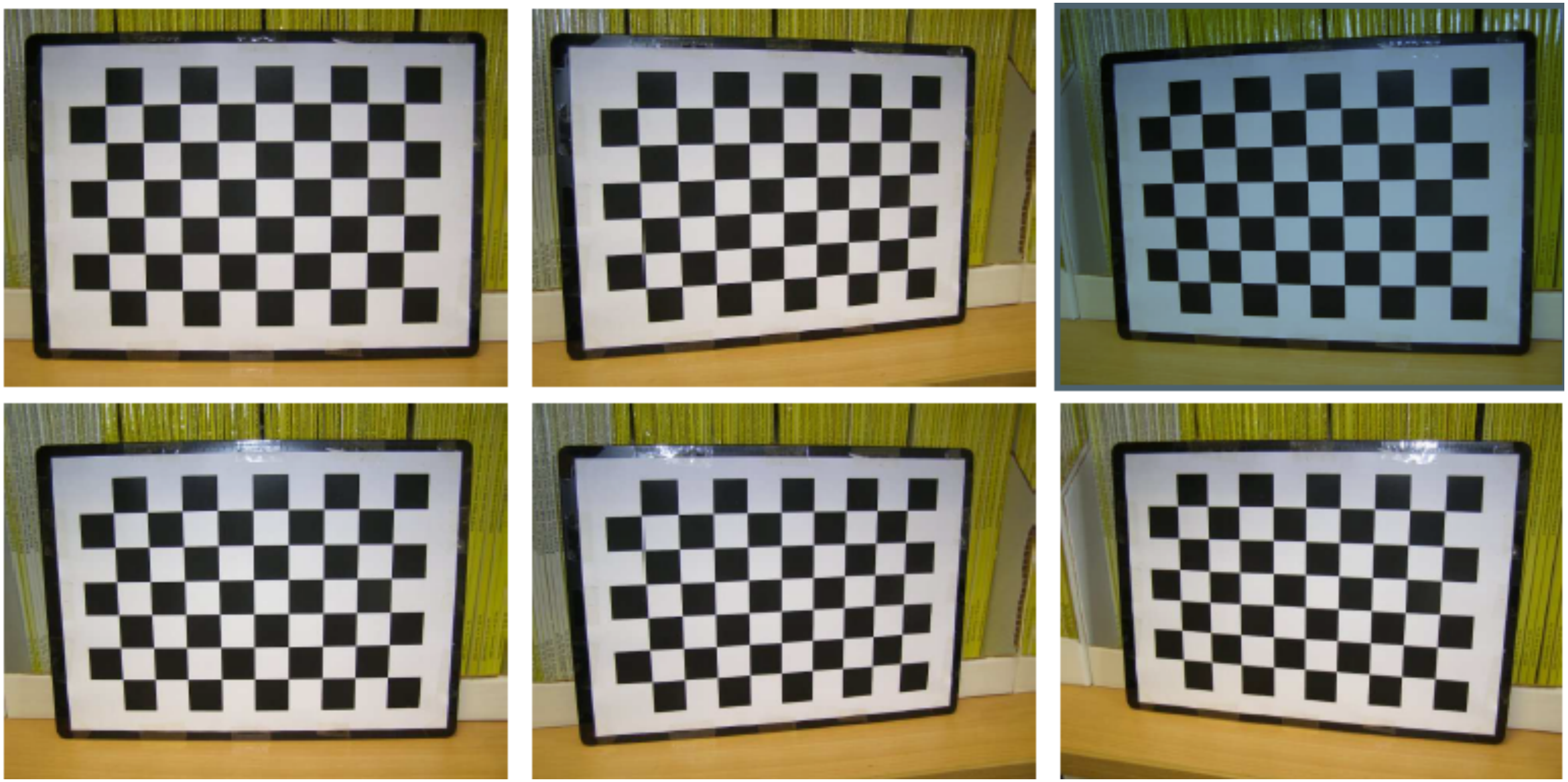
Summary of Intrinsic Parameter Calibration Procedure

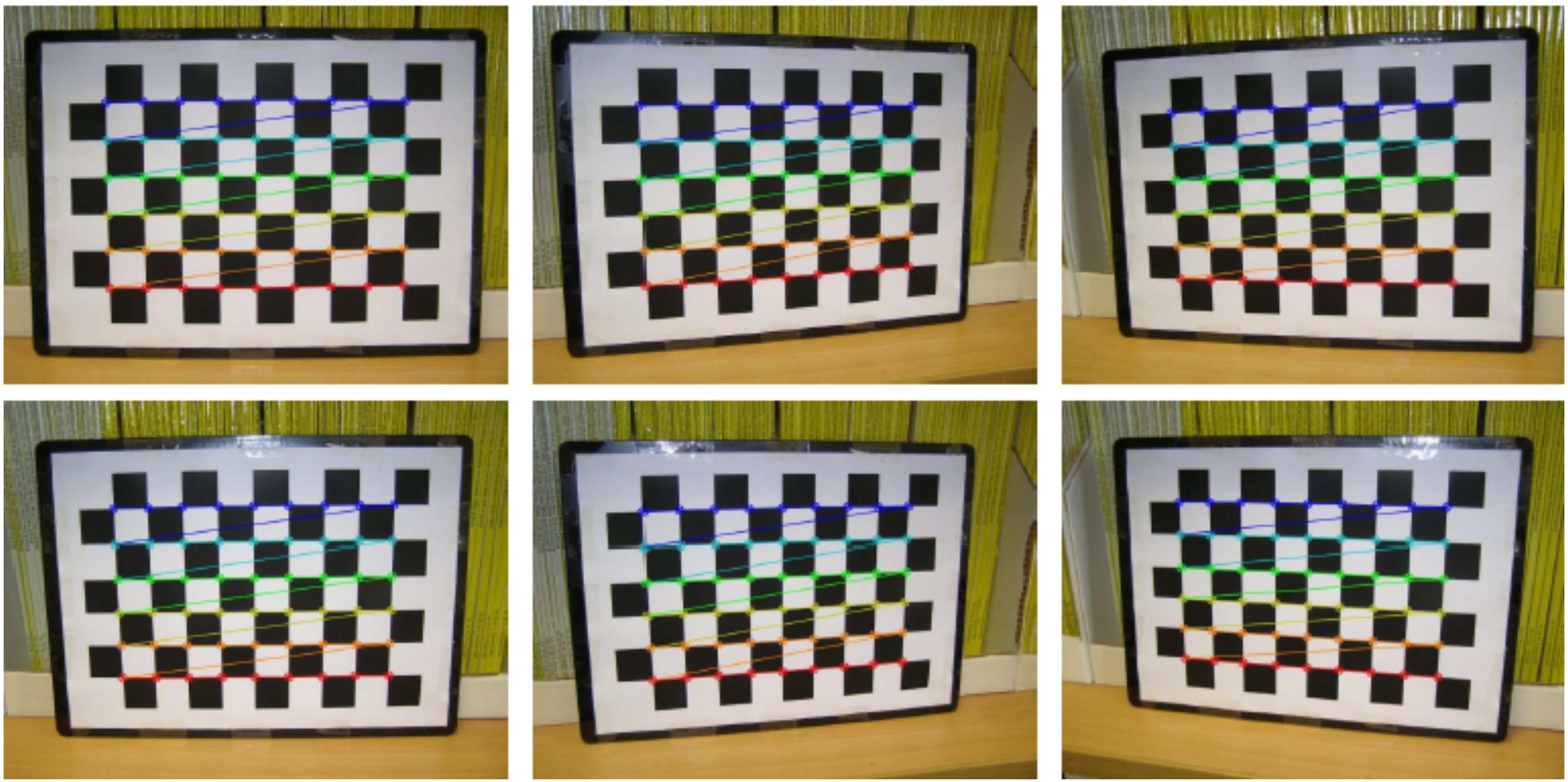
Step 1: Take images of checkerboard from various camera position and orientation





Example of images taken

Step 2: Detect the inner corners of the checkerboard pattern. The coordinates of these corners are the (u_p, v_p)



Step 3: Convert the (u_p, v_p) into 3D coordinates, with the z-coordinates set to zero (i.e. the XY plane of the 3D frame is aligned with the checkerboard).

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} \approx \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_p & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

Step 4: For each image, align the 3D reference frame of the checkerboard with the 3D reference frame of the camera

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} \approx \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$

Step 5: Form the equation linking camera image coordinates and the checkerboard coordinates

$$\begin{bmatrix} u_c \\ v_c \\ 1 \end{bmatrix} \approx \begin{bmatrix} f_c k_u & 0 & u_0 & 0 \\ 0 & f_c k_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_p & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_c \\ v_c \\ 1 \end{bmatrix} \approx \underbrace{\begin{bmatrix} f_c k_u & 0 & u_0 \\ 0 & f_c k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_p & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}_c H_p} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

Step 6: Rearranging the equation

$$\begin{bmatrix} \frac{1}{f_c k_u} & 0 & \frac{-u_0}{f_c k_u} \\ 0 & \frac{1}{f_c k_v} & \frac{-v_0}{f_c k_v} \\ 0 & 0 & 1 \end{bmatrix} {}_c H_p \begin{bmatrix} \frac{1}{k_p} & 0 & 0 \\ 0 & \frac{1}{k_p} & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}$$

Let $${}_c H_p = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

Step 7: Make use of orthonormal properties of rotation matrix to form 2 constraints

$$\left(\frac{1}{f_c k_u}\right)^2 (h_{11} - u_0 h_{31})(h_{12} - u_0 h_{32}) + \left(\frac{1}{f_c k_v}\right)^2 (h_{21} - v_0 h_{31})(h_{22} - v_0 h_{32}) + h_{31} h_{32} = 0$$

$$\left(\frac{1}{f_c k_u}\right)^2 ((h_{11} - u_0 h_{31})^2 - (h_{12} - u_0 h_{32})^2) + \left(\frac{1}{f_c k_v}\right)^2 ((h_{21} - v_0 h_{31})^2 - (h_{22} - v_0 h_{32})^2) + h_{31}^2 - h_{32}^2 = 0$$

Step 8: Let $\theta_1 = f_c k_u \quad \theta_2 = f_c k_v \quad \theta_3 = u_0 \quad \theta_4 = v_0$

$$g_1(\theta, H) = \left(\frac{1}{\theta_1}\right)^2 (h_{11} - \theta_3 h_{31})(h_{12} - \theta_3 h_{32}) + \left(\frac{1}{\theta_2}\right)^2 (h_{21} - \theta_4 h_{31})(h_{22} - \theta_4 h_{32}) + h_{31} h_{32}$$

$$g_2(\theta, H) = \left(\frac{1}{\theta_1}\right)^2 ((h_{11} - \theta_3 h_{31})^2 - (h_{12} - \theta_3 h_{32})^2) + \left(\frac{1}{\theta_2}\right)^2 ((h_{21} - \theta_4 h_{31})^2 - (h_{22} - \theta_4 h_{32})^2) + h_{31}^2 - h_{32}^2$$

Form an objective function to be minimized:

$$e(\theta) = \sum_{i=1}^{N} (g_1(H_i, \theta)^2 + g_2(H_i, \theta)^2)$$

Step 9: Wait a minute… how did we get H ?

we know that

$$\begin{bmatrix} u_c \\ v_c \\ 1 \end{bmatrix} \approx \underbrace{\begin{bmatrix} f_c k_u & 0 & u_0 \\ 0 & f_c k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_p & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}_c H_p} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

so we can write

$$\begin{bmatrix} \alpha u_c \\ \alpha v_c \\ \alpha \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

Step 10: Rewrite the equation into the following form

$$\begin{bmatrix} u_p & v_p & 1 & 0 & 0 & 0 & -u_c u_p & -u_c v_p & -u_c \\ 0 & 0 & 0 & u_p & v_p & 1 & -v_c u_p & -v_c v_p & -v_c \\ & & \vdots & & & & \vdots & & \vdots \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve for h_{ij} (for each image) using SVD