

# **EE2023 Signals & Systems**

**Tables of** { **Fourier Transforms**  
**Laplace Transforms**  
**Trigonometric Identities**

**Fourier Series:** 
$$\begin{cases} X_k = \frac{1}{T} \int_{\tilde{t}}^{\tilde{t}+T} x(t) \exp(-j2\pi k t/T) dt \\ x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(j2\pi k t/T) \end{cases}$$

**Fourier Transform:** 
$$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi f t) dt \\ x(t) = \int_{-\infty}^{\infty} X(f) \exp(j2\pi f t) df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(f)$
Constant	$K$	$K\delta(f)$
Unit Impulse	$\delta(t)$	<b>1</b>
Unit Step	$u(t)$	$\frac{1}{2} \left[ \delta(f) + \frac{1}{j\pi f} \right]$
Sign (or Signum)	$\text{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(fT)$
Triangle	$\text{tri}\left(\frac{t}{T}\right)$	$T \text{sinc}^2(fT)$
Sine Cardinal	$\text{sinc}\left(\frac{t}{T}\right)$	$T \text{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f - f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2} [\delta(f - f_o) + \delta(f + f_o)]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2} [\delta(f - f_o) - \delta(f + f_o)]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5} \exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty} \delta(t - mT)$	$\frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES		
	Time-domain	Frequency-domain
Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{ \beta } X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t - t_o)$	$X(f) \exp(-j2\pi f t_o)$
Frequency shifting (Modulation)	$x(t) \exp(j2\pi f_o t)$	$X(f - f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t) x_2(t)$	$\int_{-\infty}^{\infty} X_1(\zeta) X_2(f - \zeta) d\zeta$ or $X_1(f) * X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta) x_2(t - \zeta) d\zeta$ or $x_1(t) * x_2(t)$	$X_1(f) X_2(f)$
Integration in the time-domain	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j2\pi f} X(f) + \frac{1}{2} X(0) \delta(f)$
		$\frac{1}{j2\pi f} X(f) \quad \text{if } X(0) = 0$

$$\text{Unilateral Laplace Transform: } X(s) = \int_{0^-}^{\infty} x(t) \exp(-st) dt$$

LAPLACE TRANSFORMS OF BASIC FUNCTIONS		
	$x(t)$	$X(s)$
Unit Impulse	$\delta(t)$	<b>1</b>
Unit Step	$u(t)$	$1/s$
Ramp	$t u(t)$	$1/s^2$
n <sup>th</sup> order Ramp	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
Damped Ramp	$t \exp(-\alpha t) u(t)$	$1/(s + \alpha)^2$
Exponential	$\exp(-\alpha t) u(t)$	$1/(s + \alpha)$
Cosine	$\cos(\omega_o t) u(t)$	$s/(s^2 + \omega_o^2)$
Sine	$\sin(\omega_o t) u(t)$	$\omega_o/(s^2 + \omega_o^2)$
Damped Cosine	$\exp(-\alpha t) \cos(\omega_o t) u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_o^2}$
Damped Sine	$\exp(-\alpha t) \sin(\omega_o t) u(t)$	$\frac{\omega_o}{(s + \alpha)^2 + \omega_o^2}$

LAPLACE TRANSFORM PROPERTIES		
	Time-domain	s-domain
Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(s) + \beta X_2(s)$
Time shifting	$x(t - t_o)$	$\exp(-st_o) X(s)$
Shifting in the s-domain	$\exp(s_o t) x(t)$	$X(s - s_o)$
Time scaling	$x(\alpha t)$	$\frac{1}{ \alpha } X\left(\frac{s}{\alpha}\right)$
Integration in the time-domain	$\int_{0^-}^t x(\zeta) d\zeta$	$\frac{1}{s} X(s)$
Differentiation in the time-domain	$\frac{dx(t)}{dt}$	$sX(s) - x(0^-)$
	$\frac{d^n x(t)}{dt^n}$	$s^n X(s) - \sum_{k=0}^{n-1} s^{n-1-k} \frac{d^k x(t)}{dt^k} \Big _{t=0^-}$
Differentiation in the s-domain	$-tx(t)$	$\frac{dX(s)}{ds}$
	$(-t)^n x(t)$	$\frac{d^n X(s)}{ds^n}$
Convolution in the time-domain	$\int_{-\infty}^{\infty} x_1(\zeta) x_2(t - \zeta) d\zeta$	$X_1(s) X_2(s)$
Initial value theorem	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$	
Final value theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$	

	$y_{step}(t)$	$Y_{step}(s)$	SYSTEM PARAMETERS
Step response of 1 <sup>st</sup> order system	$K \left[ 1 - \exp\left(-\frac{t}{T}\right) \right] u(t)$	$\frac{1}{s} \cdot \frac{K}{(sT + 1)}$	$\left( \begin{array}{l} T : \text{System Time-constant} \\ K : \text{System Steady-state (or DC) Gain} \end{array} \right)$
Step response of 2 <sup>nd</sup> order underdamped system: ( $0 < \zeta < 1$ )	$K \left[ 1 - \frac{\exp(-\omega_n \zeta t)}{(1 - \zeta^2)^{0.5}} \sin\left(\omega_n (1 - \zeta^2)^{0.5} t + \phi\right) \right] u(t)$	$\frac{1}{s} \cdot \frac{K \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$	$\left( \begin{array}{l} \omega_n : \text{System Undamped Natural Frequency} \\ \zeta : \text{System Damping Factor} \\ \omega_d : \text{System Damped Natural Frequency} \\ K : \text{System Steady-state (or DC) Gain} \end{array} \right) \left( \begin{array}{l} \sigma = \omega_n \zeta \\ \omega_d^2 = \omega_n^2 (1 - \zeta^2) \\ \omega_n^2 = \sigma^2 + \omega_d^2 \\ \tan(\phi) = \omega_d / \sigma \end{array} \right)$
	$K \left[ 1 - \left( \frac{\sigma^2 + \omega_d^2}{\omega_d^2} \right)^{0.5} \exp(-\sigma t) \sin(\omega_d t + \phi) \right] u(t)$	$\frac{1}{s} \cdot \frac{K (\sigma^2 + \omega_d^2)}{(s + \sigma)^2 + \omega_d^2}$	
2 <sup>nd</sup> order system - RESONANCE - ( $0 \leq \zeta < 1/\sqrt{2}$ )	RESONANCE FREQUENCY : $\omega_r = \omega_n (1 - 2\zeta^2)^{0.5}$		RESONANCE PEAK : $M_r = \left  H(j\omega_r) \right  = \frac{K}{2\zeta (1 - \zeta^2)^{0.5}}$

TRIGONOMETRIC IDENTITIES	
$\exp(\pm j\theta) = \cos(\theta) \pm j \sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$
$\cos(\theta) = \frac{1}{2} [\exp(j\theta) + \exp(-j\theta)]$	$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$
$\sin(\theta) = \frac{1}{j2} [\exp(j\theta) - \exp(-j\theta)]$	$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$	$\sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha) \cos(\beta) = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin^2(\theta) = \frac{1}{2} [1 - \cos(2\theta)]$	$\sin(\alpha) \cos(\beta) = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$
$\cos^2(\theta) = \frac{1}{2} [1 + \cos(2\theta)]$	$\mathbf{C} \cos(\theta) - \mathbf{S} \sin(\theta) = \sqrt{\mathbf{C}^2 + \mathbf{S}^2} \cos \left[ \theta + \tan^{-1} \left( \frac{\mathbf{S}}{\mathbf{C}} \right) \right]$