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NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

ST 2334 PROBABILITY and STATISTICS

(Semester 1 : AY 2007/2008)

November 2007 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6) Questions** and comprises **TWENTY SIX (26)** printed pages (inclusive of this cover page).
2. Candidates must answer **ALL** questions. The total mark for this paper is 120.
3. Please show work and answers in the space provided for each question. DO NOT use pencils to write answers.
4. This is a CLOSED BOOK examination. ONE A-4 size cheat sheet is allowed.
5. Hand in this booklet at the end of the examination.
6. Non-programmable calculators may be used.
7. Appendix A: Some key formulae
   Appendix B: Standardized Normal Distribution Table (*Z* Table)
   Appendix C: Student's *t*-Distribution Table (*t* Table)
   Appendix D: Table Random Digits

Matriculation No: ____________________

Seat Number: ____________________

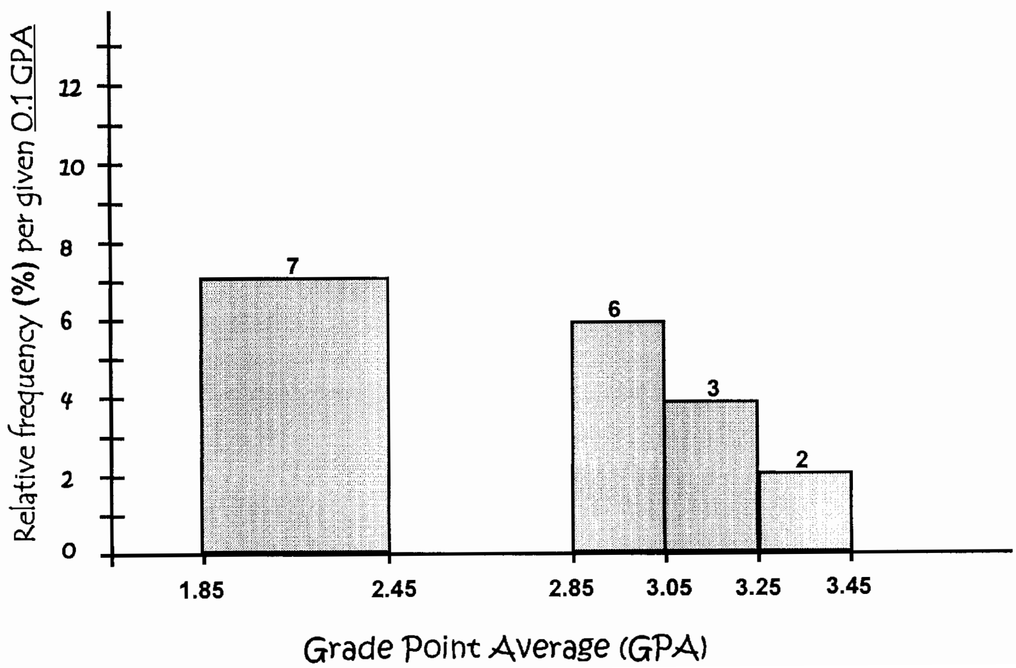
Question	1	2	3	4	5	6	Total
Marks							
Max	17	26	17	15	20	25	120

Question 1 (17 Marks)

(A) A certain faculty in a university has 4,000 students. An administrative support officer was asked to document the grade point averages (GPAs) of a sample of 50 students. Assume the students have the student IDs from 0001 to 4000.

(i) The administrative officer was asked to form a simple random sample by using the table of random digits (in Appendix D). Start with row 46, column 6 and read horizontally. List the student IDs of the first 3 students selected for the sample. (3 Marks)

(ii) Given below is the density histogram associated with GPAs of this sample of 50 students. Please complete the missing bar in the histogram and identify the bar height accordingly. (4 Marks)



Question 1 (Continued)

(iii) According to the histogram in Part (A)(ii), what proportion of the students had GPAs between 2.85 and 3.45? (3 Marks)

(B) Over the past years, a fertilizer production process has shown an average daily yield of 60 tons with a variance in daily yields of 100.

(i) If the yield were to fall below 50 tons tomorrow, should this outcome cause you to suspect abnormality in the production process? Show your work with necessary assumption. (3 Marks)

Question 1 (Continued)

(ii) If the average yield for the last 25 days were to fall below 50 tons, should this outcome cause you to suspect abnormality in the production process? Justify your answer. (4 Marks)

Question 2 (26 Marks)

(A) CPU time used by a company costs \$200 per hour. The CPU time used per week has a probability density function (measured in hours) given by

$$f(x) = \begin{cases} k \cdot x^2(4-x) & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

It is also known that the weekly amount of money spent on maintenance and repairs of IT facility is approximately normal with a mean of \$400 and a standard deviation of \$20.

(i) Find the value of *k*. (4 Marks)

(ii) Find the expected value of the weekly cost for CPU time used by this company. (4 Marks)

Question 2 (Continued)

(iii) Would you expect the weekly cost to exceed \$600 very often? Justify your answer. (4 Marks)

(iv) How much should be budgeted for the weekly maintenance and repairs of IT facility such that the probability of over-spending in a given week will not exceed 0.1. (5 Marks)

Question 2 (Continued)

(B) A radioactive mass emits particles according to a Poisson process at a mean rate of 12 particles per minute.

(i) Find the median waiting time until the next particle is emitted. (4 Marks)

(ii) The radioactive mass just emitted a particle. Find the probability that three particles will be emitted within the next 15 seconds. (5 Marks)

Question 3 (17 Marks)

(A) An electronic system has two components of different types (type A and type B) in joint operation. Let X and Y denote the lifetime, measured in hundreds of hours, of type A and type B components, respectively. The joint density function is defined as

$$f(x,y)=\begin{cases}\frac{1}{8}xe^{-(x+y)/2} & x>0 \text{ and } y>0\\ 0 & \text{otherwise}\end{cases}$$

(i) Find the probability that the type B component in this electronic system will last for more than 200 hours. (6 Marks)

Question 3 (Continued)

(ii) Are the lifetimes of type A and type B components independent? Justify your answer. (5 Marks)

Question 3 (Continued)

(B) A small grocery store has two checkout counters, L1 and L2. Let *X* and *Y* denote the proportion of the time that the checkout counters, L1 and L2, are busy during the opening hours on a given business day, respectively. Assume the joint probability density function of *X* and *Y* is given by

$$f(x,y)=\begin{cases}\dfrac{6}{7}(x+y)^2 & 0<x<1 \text{ and } 0<y<1\\[2ex] 0 & \text{otherwise}\end{cases}$$

What is the expected proportion of time that checkout counter L2 is busy given that checkout counter L1 is busy 40% of the time. (6 Marks)

Question 4 (15 Marks)

(A) A and B are two events. Check each of the following statements to see whether it is correct. (4 marks)

(i) $P(A \mid B) + P(\overline{A} \mid \overline{B}) = 1$

(ii) $P(A \mid B) + P(A \mid \overline{B}) = 1$

(iii) $P(A \mid B) + P(\overline{A} \mid B) = 1$

(B) Suppose you like four out of 15 songs on a compact disk (CD). When the random button selector on a CD player is used, each of the 15 songs is played once in a random order. Find the probability that

(i) You like both of the first two songs that are played. (2 marks)

(ii) You like exactly one of the first three songs played. (3 marks)

Question 4 (Continued)

(C) You are in a party of 18 people. A lucky draw box contains 18 balls, one is red and 17 are black. Each person is to take one ball from the bowl without replacement. The person who draws the red ball will be given a prize.

(i) If you have a choice of drawing first or last, which position would you choose? Justify your answer. (2 marks)

(ii) Suppose the bowl contains two red balls and 16 black balls and both people drawing the red balls are given prizes. Which position would you now choose? Justify your answer. (4 marks)

Question 5 (20 Marks)

The execution of technical instructions in a telecommunication company can be classified as completed successfully, partial completed or failed with probability 0.65, 0.25 and 0.10 respectively.

(A) If there are 16 instructions in a project, find the probability that

(i) Thirteen instructions are completed successfully. (4 marks)

(ii) Thirteen instructions are completed successfully and 2 instructions failed. (5 marks)

(iii) An engineer will be given a warning if two or more instructions in a project failed. Assuming that the projects are independent of each other, what is the probability that the engineer gets the first warning when he is working on his third project? (6 marks)

Question 5 (Continued)

(B) Of the 72 projects received today, 20 are categorized as critical. The chief engineer has been assigned 5 projects randomly, what is the probability that three critical projects are given to him? (5 marks)

Question 6 (25 Marks)

(A) A product is packaged using a machine with 24 filler heads numbered from 1 to 24, with the odd-numbered heads on one side of the machine and the even on the other side. To test whether the filler weights in grams for the even-numbered and odd-numbered head are from an identical distribution, one package is selected from each filler head and weighed. The average fill weights are 1076.75 gram for odd-numbered heads and 1072.33 gram for even-numbered heads with standard deviations of 29.30 gram and 26.24 gram respectively.

(i) What is your conclusion at 0.10 significance level? (6 marks)

(ii) What assumptions did you made in your above calculation? (3 marks)

Question 6 (Continued)

(B) A tire manufacturer would like to know the braking performance of a new type of tire. A test that used cars with the tires was conducted. The cars were running on a test track and the stopping distances were measured for stops made from 90 km per hour. 10 different cars were used, and the stopping distances for each car on both wet and dry pavement were recorded. Results are shown in the table.

Car number	Stopping Distance (meter)	
	Dry Pavement	Wet Pavement
1	68	100
2	74	110
3	68	96
4	67	83
5	65	91
6	67	87
7	67	101
8	64	90
9	68	96
10	79	103

(i) Construct a 95% confidence interval for the mean dry pavement stopping distance. (5 marks)

Question 6 (Continued)

(ii) Construct a 95% confidence interval for the mean difference of dry and wet pavement stopping distance. (5 marks)

(iii) Interpret the confidence interval in part (ii). (3 marks)

(C) A random sample $X_1, X_2, \ldots, X_n$ of size n is taken from a Poisson distribution with a mean of λ, $0 < \lambda < \infty$. Find the maximum likelihood estimator for λ. (3 marks)

***** **END OF PAPER** *****

Appendix A: Some Key formulae

Univariate Descriptive Measures –			
Mean:	$\mu = \frac{\sum x}{N}$ (population)	or	$\bar{x} = \frac{\sum x}{n}$ (sample)
Quartile positions:	$Q_1 = \frac{n+1}{4}$	$Q_2 = \frac{n+1}{2}$	$Q_3 = \frac{3(n+1)}{4}$
Standard deviation:	$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$ (population)	or	$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$ (sample)
Coefficient of variation ($c.v.$):	$cv = \frac{\sigma}{\mu} \times 100\%$ (population)	or	$cv = \frac{s}{\bar{x}} \times 100\%$ (sample)
For Discrete Random Variables:			
Expected Value	$E(X) = \mu_x = \sum_i x_i P(x_i)$		
Variance	$Var(X) = \sigma_x^{\,2} = \sum_i (x_i - \mu)^2 P(x_i) = E(X^2) - [E(X)]^2$		
Covariance	$Cov(X,Y) = \sigma_{XY} = \sum_i [x_i - E(X)][y_i - E(Y)]p(x_i y_i)$		
For linear transformation of a random variable:			
Expected Value	$E(a + bX) = a + bE(X)$		
Variance	$Var(a + bX) = b^2 Var(X)$		
For linear combination of 2 random variables:			
Expected Value	$E(aX + bY) = aE(X) + bE(Y)$		
Variance	$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X,Y)$		
Bayes' Theorem	$P(A) = \sum_{i=1}^{n} P(B_i) \bullet P(A \mid B_i)$ $P(B_r \mid A) = \frac{P(B_r) \bullet P(A \mid B_r)}{\sum_{i=1}^{n} P(B_i) \bullet P(A \mid B_i)}$		
Chebyshev's Theorem	$P(\mid X - \mu \mid \geq k\sigma) \leq \frac{1}{k^2}$		

Discrete Probability Distributions:

Binomial Distribution

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$$

n = sample size

x = # of successes in sample

$$E(X)=np$$

$$Var(X)=np(1-p)$$

Hypergeometric Distribution

$$P(X=x)=\frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$$

N = population size

a = # of successes in population

n = sample size

x = # of successes in sample

$$E(X)=n\frac{a}{N}$$

$$Var(X)=n\frac{a}{N}\left(1-\frac{a}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Poisson Distribution

$$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!} \qquad x=0,1,2,\ldots$$

$$E(X)=\lambda$$

$$Var(X)=\lambda$$

Geometric Distribution

$$P(X=x)=p(1-p)^{x-1} \qquad x=1,2,\ldots$$

$$E(X)=\mu_x=\frac{1}{p}$$

$$Var(X)=\sigma_x{}^2=\frac{1-p}{p^2}$$

Multinomial Distribution

$$P(X=x)=\frac{n!}{x_1!x_2!\cdots x_k!}p_1{}^{x_1}p_2{}^{x_2}\cdots p_k{}^{x_k}$$

Continuous Probability Distributions:

General Form

$$F(a)=P(X\le a)=\int_{-\infty}^{a}f(x)dx$$

where $-\infty<a<\infty$ and $f(x)$ is a density function

$$E(X)=\mu=\int_{-\infty}^{\infty}xf(x)dx$$

$$E(g(X))=\int_{-\infty}^{\infty}g(x)\cdot f(x)dx$$

$$Var(X)=\sigma^2=E[(X-\mu)^2]=E(X^2)-\mu^2$$

Normal Distribution	$X \sim N(\mu,\sigma^2)$	pdf: $f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$
	$Z=\frac{X-\mu}{\sigma}$	pdf: $f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$
Log-Normal Distribution	$\ln X \sim N(\alpha,\beta^2)$	if X is log-normally distributed
	$Z=\frac{\ln X-\alpha}{\beta}$	
	$\mu=e^{\alpha+\beta^2/2}$	$\sigma=\sqrt{e^{2\alpha+\beta^2}(e^{\beta^2}-1)}$
Exponential Distribution	$X \sim Exp(\lambda)$	pdf: $f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & x<0\end{cases}$
		where λ = success rate per unit
Gamma Distribution	Gamma Function:	$\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}e^{-x}dx$
		$\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$
		$\Gamma(\alpha)=(\alpha-1)!$
		$\Gamma(1/2)=\sqrt{\pi}$
	$X \sim Gamma(\alpha,\beta)$	pdf: $f(x)=\begin{cases}\frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x>0,\alpha>0,\beta>0\\ 0 & otherwise\end{cases}$
		$\mu=\alpha\beta$ $\quad\sigma=\sqrt{\alpha\beta^2}$
Weibull Distribution	$X \sim Weibull(\alpha,\beta)$	pdf: $f(x)=\begin{cases}\alpha\beta x^{\beta-1}e^{-\alpha x^\beta} & x>0,\alpha>0,\beta>0\\ 0 & otherwise\end{cases}$
		$F(X\le t)=1-e^{-\alpha t^\beta}$
		$\mu=\alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right)$
		$\sigma=\sqrt{\alpha^{-2/\beta}\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\}}$

Uniform Distribution $X \sim U(\alpha,\beta)$ $\mu = \dfrac{\alpha+\beta}{2}$ $\sigma = \sqrt{(\beta-\alpha)^2/12}$

Joint Probability Distributions:

$$Cov(X,Y) = \sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = E(XY) - \mu_X \cdot \mu_Y$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y}$$

Continuous – $f(x,y)$ is a joint density for random variables X, Y

$$P(a \le X \le b, c \le Y \le d) = \int_a^b \int_c^d f(x,y)\,dy\,dx$$ where a, b, c, d are constants

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$f_X(x \mid y) = \frac{f(x,y)}{f_Y(y)} \qquad f_Y(y \mid x) = \frac{f(x,y)}{f_X(x)}$$

Sampling Distributions:

Sampling Distribution for $\bar{X}$ – $\mu_{\bar{X}} = \mu$

$$\sigma_{\bar{X}} = \begin{cases} \dfrac{\sigma}{\sqrt{n}} & \text{if } \dfrac{n}{N} \le 0.05 \\[2ex] \dfrac{\sigma}{\sqrt{n}}\sqrt{\dfrac{N-n}{N-1}} & \text{if } \dfrac{n}{N} > 0.05 \end{cases}$$

Sampling Distribution for $\hat{p}$ – $\mu_{\hat{p}} = p$

$$\sigma_{\hat{p}} = \begin{cases} \sqrt{\dfrac{p(1-p)}{n}} & \text{if } \dfrac{n}{N} \le 0.05 \\[2ex] \sqrt{\dfrac{p(1-p)}{n}}\sqrt{\dfrac{N-n}{N-1}} & \text{if } \dfrac{n}{N} > 0.05 \end{cases}$$

One-sample Inference:

Test statistic:

$$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \quad \text{or} \quad t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$$

(1-α)100% Confidence Interval for μ

$$\bar{X} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$$

Two-sample Inference:

(Unequal Variance)

Test statistic:

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \mu_\Delta}{\sqrt{\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}}}$$

(1-α)100% Confidence Interval for μ_1 - μ_2

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm t_{\frac{\alpha}{2},v} \sqrt{\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}}$$

$$v = \frac{\left(\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}\right)^2}{\frac{\left(s_1^{\ 2}/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^{\ 2}/n_2\right)^2}{n_2 - 1}}$$

(Equal Variance)

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^{\ 2} + (n_2 - 1)s_2^{\ 2}}{n_1 + n_2 - 2}}$$

Test statistic:

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \mu_\Delta}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

(1-α)100% Confidence Interval for μ_1 - μ_2

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm t_{\frac{\alpha}{2},v}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$v = n_1 + n_2 - 2$$

Matched Pairs Analysis

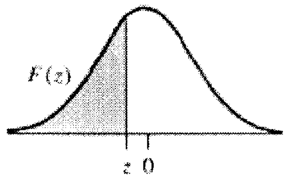
Test statistic:

$$t = \frac{\overline{X}_d}{s_d / \sqrt{n}}$$

(1-α)100% Confidence Interval for μ_d

$$\overline{X}_d \pm t_{\alpha/2, n-1} \frac{s_d}{\sqrt{n}}$$

Appendix B: Standard Normal Table (*Z* Table)



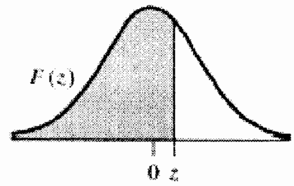
Standard Normal Distribution Function

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−5.0	0.0000003									
−4.0	0.00003									
−3.5	0.0002									
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0006	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

* Entries in the table represent area under the standard normal density curve from – ∞ to z

Appendix B: (Continued from the previous page)



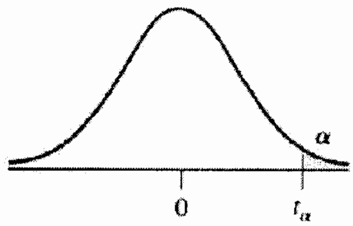
Standard Normal Distribution Function

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\,dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5973	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998									
4.0	0.99997									
5.0	0.9999997									

* Entries in the table represent area under the standard normal density curve from $-\infty$ to z

Appendix C: Student's *t* –Table



ν	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.00833$	$\alpha = 0.00625$	$\alpha = 0.005$	ν
1	3.078	6.314	12.706	31.821	38.204	50.923	63.657	1
2	1.886	2.920	4.303	6.965	7.650	8.860	9.925	2
3	1.638	2.353	3.182	4.541	4.857	5.392	5.841	3
4	1.533	2.132	2.776	3.747	3.961	4.315	4.604	4
5	1.476	2.015	2.571	3.365	3.534	3.810	4.032	5
6	1.440	1.943	2.447	3.143	3.288	3.521	3.707	6
7	1.415	1.895	2.365	2.998	3.128	3.335	3.499	7
8	1.397	1.860	2.306	2.896	3.016	3.206	3.355	8
9	1.383	1.833	2.262	2.821	2.934	3.111	3.250	9
10	1.372	1.812	2.228	2.764	2.870	3.038	3.169	10
11	1.363	1.796	2.201	2.718	2.820	2.891	3.106	11
12	1.356	1.782	2.179	2.681	2.780	2.934	3.055	12
13	1.350	1.771	2.160	2.650	2.746	2.896	3.012	13
14	1.345	1.761	2.145	2.624	2.718	2.864	2.977	14
15	1.341	1.753	2.131	2.602	2.694	2.837	2.947	15
16	1.337	1.746	2.120	2.583	2.673	2.813	2.921	16
17	1.333	1.740	2.110	2.567	2.655	2.793	2.898	17
18	1.330	1.734	2.101	2.552	2.639	2.775	2.878	18
19	1.328	1.729	2.093	2.539	2.625	2.759	2.861	19
20	1.325	1.725	2.086	2.528	2.613	2.744	2.845	20
21	1.323	1.721	2.080	2.518	2.602	2.732	2.831	21
22	1.321	1.717	2.074	2.508	2.591	2.720	2.819	22
23	1.319	1.714	2.069	2.500	2.582	2.710	2.807	23
24	1.318	1.711	2.064	2.492	2.574	2.700	2.797	24
25	1.316	1.708	2.060	2.485	2.566	2.692	2.787	25
26	1.315	1.706	2.056	2.479	2.559	2.684	2.779	26
27	1.314	1.703	2.052	2.473	2.553	2.676	2.771	27
28	1.313	1.701	2.048	2.467	2.547	2.669	2.763	28
29	1.311	1.699	2.045	2.462	2.541	2.663	2.756	29
inf.	1.282	1.645	1.960	2.326	2.394	2.498	2.576	inf.

Appendix D: Table of Random Digits (Partial Table)

Row 41	4611	9861	7916	9305	2074	9462	0254	4827	9198	3974
42	1093	3784	4190	6332	1175	8599	9735	8584	6581	7194
43	3374	3545	6865	8819	3342	1676	2264	6014	5012	2458
44	3650	9676	1436	4374	4716	5548	8276	6235	6742	2154
45	7292	5749	7977	7602	9205	3599	3880	9537	4423	2330
46	2353	8319	2850	4026	3027	1708	3518	7034	7132	6903
47	1094	2009	8919	5676	7283	4982	9642	9235	8167	3366
48	0568	4002	0587	7165	1094	2006	7471	0940	4366	9554
49	5606	4070	5233	4339	6543	6695	5799	5821	3953	9458
50	8285	7537	1181	2300	5294	6892	1627	3372	1952	3028
51	2444	9039	4803	8658	1590	2420	2547	2470	8179	4617
52	5748	7767	2800	6289	2814	8281	1549	9519	3341	1192
53	7761	8583	0852	5619	6864	8506	9643	7763	9611	1289
54	6838	9280	2654	0812	3988	2146	5095	0150	8043	9079
55	6440	2631	3033	9167	4998	7036	0133	7428	9702	1376
56	8829	0094	2887	3802	5497	0318	5168	6377	9216	2802
57	9845	4796	2951	4449	1999	2691	5328	7674	7004	6212
58	5072	9000	3887	5739	7920	6074	4715	3681	2721	2701
59	9035	0553	1272	2600	3828	8197	8852	9092	8027	6144
60	5562	1080	2222	0336	1411	0303	7424	3713	9278	1818
61	2757	2650	8727	3953	9579	2442	8041	9869	2887	3933
62	6397	1848	1476	0787	4990	4666	1208	2769	3922	1158
63	9208	7641	3775	4279	1282	1840	5999	1806	7809	5885
64	2418	9289	6120	8141	3908	5577	3590	2317	8975	4593
65	7300	9006	5659	8258	3662	0332	5369	3640	0563	7939
66	6780	2535	8916	3245	2256	4350	6064	2438	2002	1272
67	2914	7309	4045	7513	3195	4166	0878	5184	6680	2655
68	0868	8657	8118	6340	9452	7460	3291	5778	1167	0312
69	7994	6579	6461	2292	9554	8309	5036	0974	9517	8293
70	8587	0764	6687	9150	1642	2050	4934	0027	1376	5040
71	8016	8345	2257	5084	8004	7949	3205	3972	7640	3478
72	5581	5775	7517	9076	4699	8313	8401	7147	9416	7184
73	2015	3364	6688	2631	2152	2220	1637	8333	4838	5699
74	7327	8987	5741	0102	1173	7350	7080	7420	1847	0741
75	3589	1991	1764	8355	9684	9423	7101	1063	4151	4875