# NATIONAL UNIVERSITY OF SINGAPORE

## SCHOOL OF COMPUTING

**EXAMINATION FOR**
**Semester 1 AY2008/2009**

**CS3230 – Design and Analysis of Algorithms**

Nov/Dec 2008 Time Allowed: 2 Hours

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## INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **NINE (9)** questions and comprises **FOURTEEN (14)** printed pages, including this page. **BOTH** sides of the page are used. The total marks is 100.
2. Answer **ALL** questions within the space provided in this booklet.
3. This is an **Open Book** examination. You may cite any result in the textbook, lecture notes, tutorials, or labs.
4. Fill in your **Matriculation Number** here :

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| Examiners' Use Only | | |
|---|---|---|
| **Question** | **Marks** | **Remarks (if any)** |
| Q1 (18) | | |
| Q2 (12) | | |
| Q3 (10) | | |
| Q4 (10) | | |
| Q5 (10) | | |
| Q6 (15) | | |
| Q7 (10) | | |
| Q8 (10) | | |
| Q9 (5) | | |
| **Total** (100) | | |

**Q1 (Growth Rates of Functions)** (total 18 marks)

Indicate whether the first function of each of the following pairs has a smaller, same or larger order of growth (to within a constant multiple) than the second function.

a) $5^{n-1}$ and $5^n$
b) $n(n+50)$ and $1000n^2$
c) $(n-1)!$ and $n!$
d) $100n^3$ and $0.01n^4$
e) $\log_{10} n$ and $\log_2 n$
f) $\log_2^2 n$ and $\log n^2$

**(Answer 3 marks each)**

**Q2 (Decrease-and-Conquer)** (total 12 marks)

A graph is said to be *bipartite* if all its vertices can be partitioned into two disjoint subsets $X$ and $Y$ so that every edge connects a vertex in $X$ with a vertex in $Y$ (i.e., the graph is *2-colorable*).

a) Describe a DFS-based algorithm for checking whether a graph is bipartite.
**(Answer 6 marks)**

b) Describe a BFS-based algorithm for checking whether a graph is bipartite.
**(Answer 6 marks)**

**Q2 (continued)**

**Q3 (Divide-and-Conquer)** (total 10 marks)

a) What is the largest number of key comparisons made by a binary search in searching for a key in the following array? **(Answer 5 marks)**

$$3, 14, 27, 31, 39, 42, 55, 70, 74, 81, 85, 93, 98$$

b) List all the keys of this array that will require the largest number of key comparisons when searched for by a binary search. **(Answer 5 marks)**

**Q3 (continued)**

**Q4 (Transform-and-Conquer)** (total 10 marks)

Consider an array of $n$ integer numbers which can be positive and/or negative. Design an algorithm that determines whether the array contains two elements whose sum is 0. For example, for the array $-6, 3, 1, 4, -3, 7$ the answer is yes $(3 + (-3) = 0)$.

a) Describe an algorithm (in pseudo-code) such that it runs in better than quadratic efficiency. **(Answer 5 marks)**

b) Describe how to modify the algorithm from a) such that it works not just for a sum of 0, but an arbitrary value $s$ of the sum. **(Answer 5 marks)**

**Q4 (continued)**

**Q5 (Graphs)** (total 10 marks)

Explain what adjustments, if any, need to be made in Dijkstra's algorithm and/or in an underlying graph to solve the following problems.

a) Find a shortest path between two given vertices of a weighted graph or digraph. (This variation is called the *single-pair shortest-path problem.*) **(Answer 5 marks)**

b) Find the shortest paths to a given vertex from each other vertex of a weighted graph or digraph. (This variation is called the *single-destination shortest-paths problem.*) **(Answer 5 marks)**

**Q6 (Dynamic Programming)** (total 15 marks)

*Shortest Path Counting.* Consider a small chess board of $4 \times 4$ squares. A chess piece can move horizontally or vertically to any square in the same row or in the same column of a chess board. Let $P(i, j)$ be the number of the chess piece's shortest paths from square $(1, 1)$ to square $(i, j)$. For example, the number of paths to square $(1, 3)$ is 1, i.e., there is only one shortest path to go from $(1, 1)$ to $(1, 3)$. Find the number of shortest paths by which the chess piece can move from one corner (say $(1, 1)$) of the chess board to the diagonally opposite corner ($(4, 4)$).

a) Draw the $4 \times 4$ chess board and write the number of shortest paths into each square. **(Answer 5 marks)**

b) Write down the recurrence relation for $P(i, j)$ which can be used in a dynamic programming solution. **(Answer 5 marks)**

c) Use elementary combinatorics to solve the problem. **(Answer 5 marks)**

**Q6 (continued)**

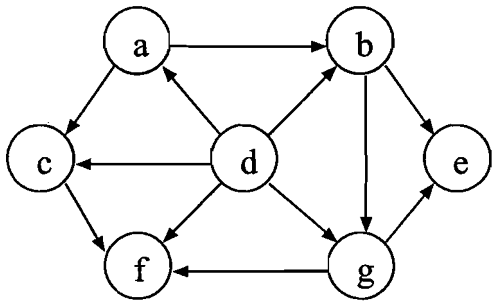
**Q7 (Branch-and-Bound)** (total 10 marks)

Solve the following instance of the knapsack problem. Execute a best-first *branch-and-bound* algorithm and write down the state-space tree based on the data given in the following table. $W = 16$. Use the bounding function given in the lecture. **(Answer 10 marks)**

| item | weight | value |
|---|---|---|
| 1 | 10 | \$100 |
| 2 | 7 | \$63 |
| 3 | 8 | \$56 |
| 4 | 4 | \$12 |

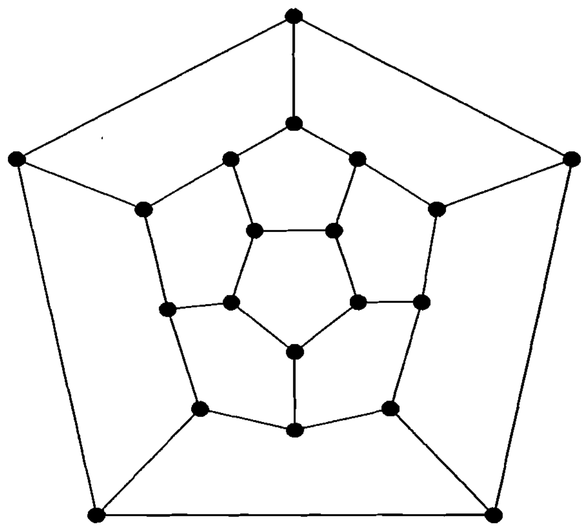
**Q8 (Topological Sorting)** (total 10 marks)

Apply the DFS-based algorithm to solve the topological sorting problem for the following digraph. Please write down the intermediate steps of processing. Start with vertex $a$. **Answer (10 marks)**



**Q9 (Icosian Game)** (total 5 marks)

A famous puzzle – invented by the renown Irish mathematician Sir William Hamilton (1805–1865) – was presented to the world under the name of Icosian Game. Find the *Hamiltonian circuit* for the graph given below. **(Answer 5 marks)**



**End of Questions**