DC and Steady-state Gain

DC Gain and Steady-state Gain of a stable LTI System with transfer function $G(s)$

- **DC Gain** $= \boldsymbol{G(0)}$

  This is the system gain at zero frequency $(\omega = 0)$ and can be computed directly from $G(s)$ by setting $s = j\omega = 0$.

- **Steady-state Gain**

  The steady-state gain is dependent on the input. Therefore, steady-state gain is, in general, not a system parameter.

  In this module, unless otherwise specified, the term steady-state gain is used with the implicit assumption that the system input is a unit step function.

  If the input is a unit step function, then the steady-state gain is equal to the DC gain.

Proof:

System unit step input at steady-state $\Big\}$ $\quad x_{ss}(t) = \lim_{t\to\infty} u(t) = 1$

System unit step response at steady-state $\Big\}$ $\quad y_{ss}(t) = \underbrace{\lim_{s\to 0} s \overbrace{\left[\frac{1}{s}G(s)\right]}^{\mathcal{L}\left\{\substack{\text{Unit Step}\\ \text{Response}}\right\}}}_{\text{FVT}} = G(0)$

$$\therefore \quad \textbf{Steady-state Gain} = \frac{y_{ss}(t)}{x_{ss}(t)} = \frac{G(0)}{1} = G(0) = \textbf{DC Gain}$$