NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA1506 Laboratory 1 (scilab)
Comments and Suggested Solutions
Semester 2 2010/2011

Exercise 1

1. Find the value of $e^{0.5}$, correct to 13 decimal places.

```
--> format(20) ; exp(0.5)

ans =

   1.64872127070012819
```

2. Evaluate $\sin^2(23.195) + \sqrt{\tanh(0.12)}$, correct to 13 decimal places.

```
--> sin(23.195)^2 + sqrt(tanh(0.12))

ans =

   1.21686993920416731
```

3. Plot the function

$$y(t) = e^{-t/2}\cos(2t), \qquad 0 \leq t \leq 10.$$

```
--> t=0:0.01:10;
--> y=exp(-t/2).*cos(2*t);
--> plot(t,y)
```

Comment: You will learn in chapter 2 that this is an example of an underdamped solution of a damped, unforced oscillator.

4. Plot these three functions on the same graph:

$$\sinh(t), \cosh(t), \tanh(t) \qquad -2 \le t \le 2.$$

```
--> t=-2:0.01:2;
--> plot(t,sinh(t))
--> plot(t,cosh(t),'g')
--> plot(t,tanh(t),'r')
```

5. Which of the following IVP is stable? (i.e. the solutions converge for small changes in the initial value.)

   (i) $\dfrac{dy}{dx} - y = -4e^{-x}, \qquad y(0) = 2.$

   (ii) $\dfrac{dy}{dx} - 2y = -6e^{-x}, \qquad y(0) = 2.$

   (iii) $\dfrac{dy}{dx} + 2y = 2e^{-x}, \qquad y(0) = 2.$

   <u>Comments</u>
   (iii) is a stable d.e. You are supposed to solve each of the three IVP and plot curves with slightly different values of $y(0)$. If you use $y(0) = 2$ exactly, all three IVP give the same solution $y = 2e^{-x}$! Identifying stable d.e. is important in practical applications because you usually have to come up with your own d.e. and initial values. The actual values are determined by experiments and are unlikely to be exact. The study of how slight variations in initial conditions drastically affect solutions forms part of what is known as chaos theory.

6. Find the solution for the following nonhomogeneous d.e.

$$x'' + 2x' + 2x = 2\cos(t), \qquad x(0) = x'(0) = 0.$$

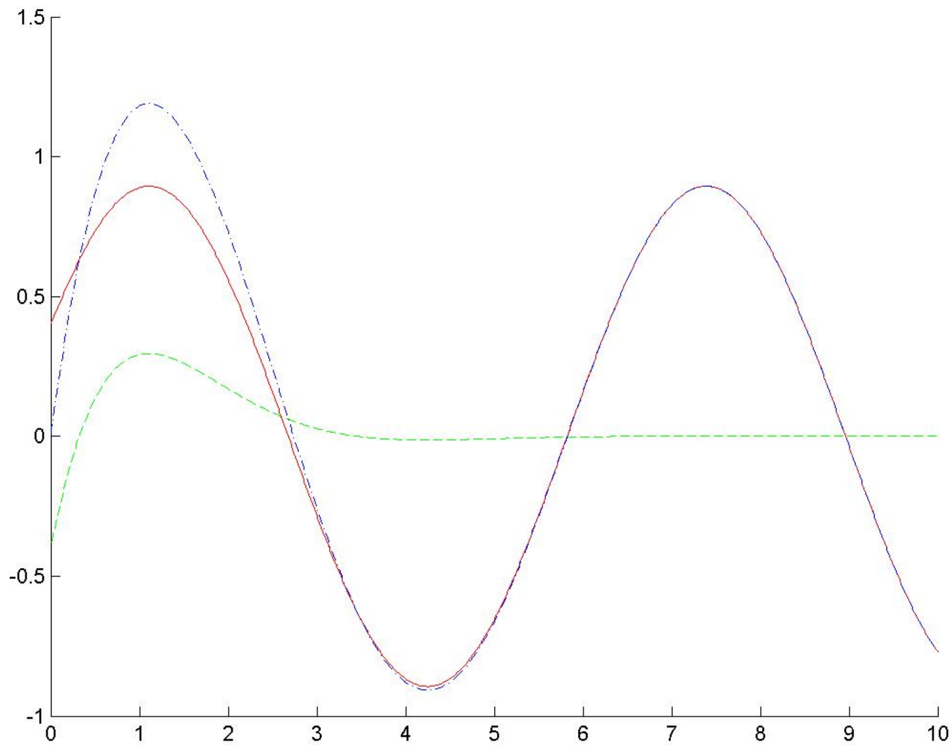
   Using different colours, plot a graph containing the three curves $x(t)$, the homogenous solution $x_h(t)$ and the particular solution $x_p(t)$. Use t between 0 to 10 for your horizontal range.

   <u>Comments</u>
   The main aim of this question is to let students practise plotting multiple graphs. One possible set of matlab commands would be

```
--> t=0:0.01:10;
--> xh=-2/5*exp(-t).*cos(t)+6/5*exp(-t).*sin(t);
--> xp=2/sqrt(5)*cos(t-atan(2));
--> x=xh+xp;
```

```
--> plot(t,xh,'g--')
--> plot(t,xp,'r-')
--> plot(t,x,'b-.')
```

Note that you could have used the particular solution $x_p = \frac{2}{5}\cos(t) + \frac{4}{5}\sin(t)$, instead of $x_p = \frac{2}{\sqrt{5}}\cos(t - \tan^{-1}(2))$.



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