CHAPTER 13

Exercises

E13.1 The emitter current is given by the Shockley equation:

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

For operation with $i_E >> I_{ES}$, we have $\exp\left(\frac{v_{BE}}{V_T}\right) >> 1$, and we can write

$$i_E \cong I_{ES}\exp\left(\frac{v_{BE}}{V_T}\right)$$

Solving for v_{BE}, we have

$$v_{BE} \cong V_T \ln\left(\frac{i_E}{I_{ES}}\right) = 26\ln\left(\frac{10^{-2}}{10^{-14}}\right) = 718.4 \text{ mV}$$

$$v_{BC} = v_{BE} - v_{CE} = 0.7184 - 5 = -4.2816 \text{ V}$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{50}{51} = 0.9804$$

$$i_C = \alpha i_E = 9.804 \text{ mA}$$

$$i_B = \frac{i_C}{\beta} = 196.1\ \mu\text{A}$$

E13.2 $\beta = \dfrac{\alpha}{1-\alpha}$

α	β
0.9	9
0.99	99
0.999	999

E13.3 $i_B = i_E - i_C = 0.5 \text{ mA}$ $\quad \alpha = i_C / i_E = 0.95$ $\quad \beta = i_C / i_B = 19$

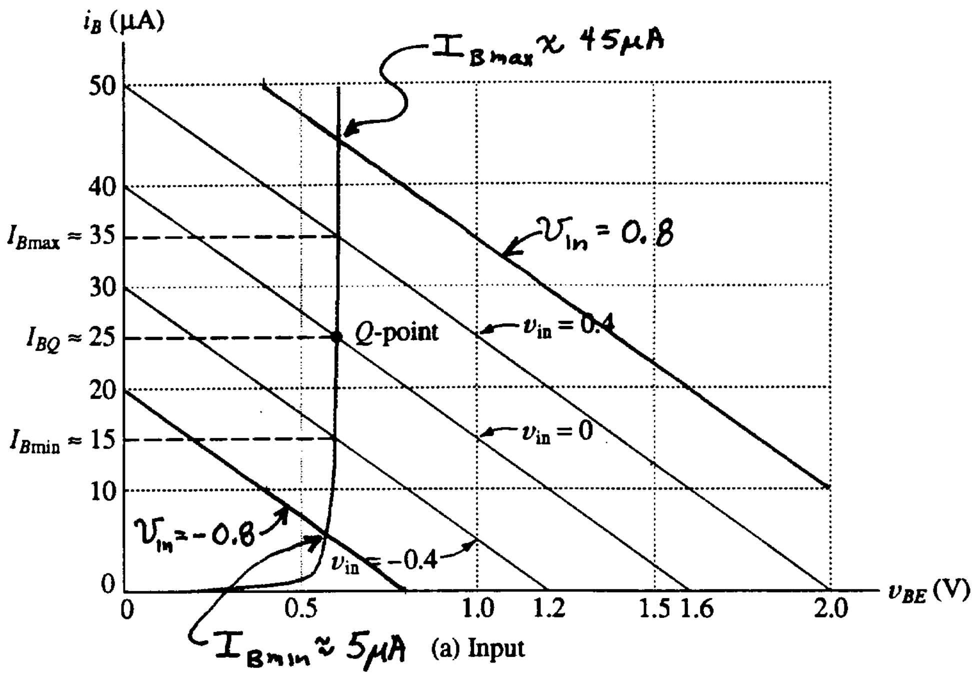
E13.4 The base current is given by Equation 13.8:

$$i_B = (1-\alpha)I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] = 1.961\times 10^{-16}\left[\exp\left(\frac{v_{BE}}{0.026}\right) - 1\right]$$

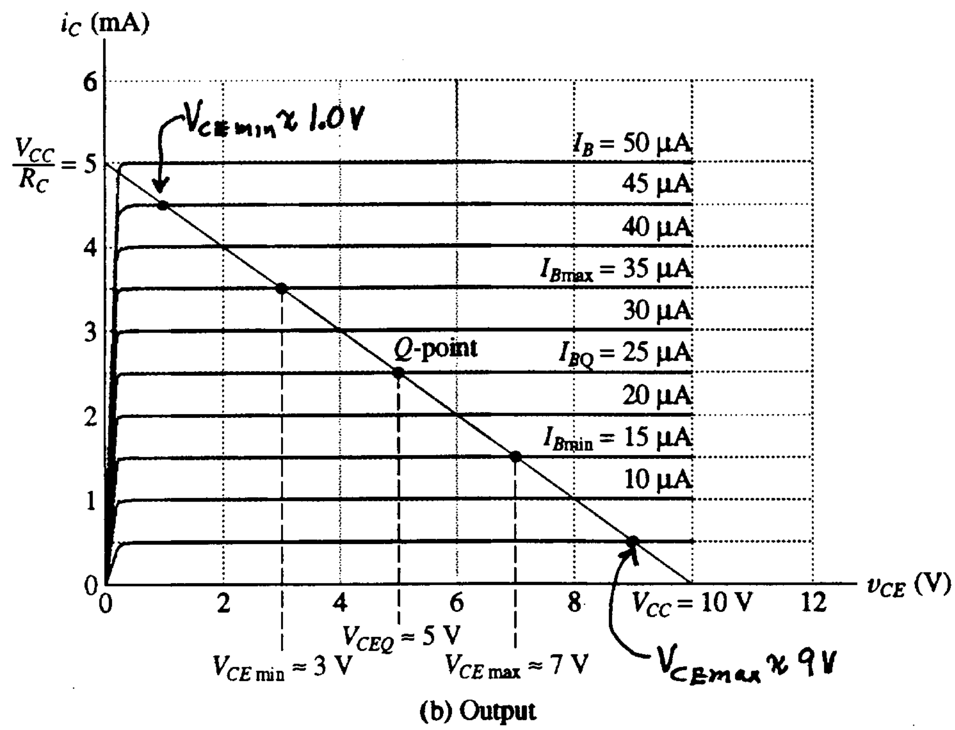
which can be plotted to obtain the input characteristic shown in Figure 13.6a. For the output characteristic, we have $i_C = \beta i_B$ provided that

$v_{CE} \geq$ approximately 0.2 V. For $v_{CE} \leq 0.2$ V, i_C falls rapidly to zero at $v_{CE} = 0$. The output characteristics are shown in Figure 13.6b.

E13.5 The load lines for $v_{in} = 0.8$ V and -0.8 V are shown:



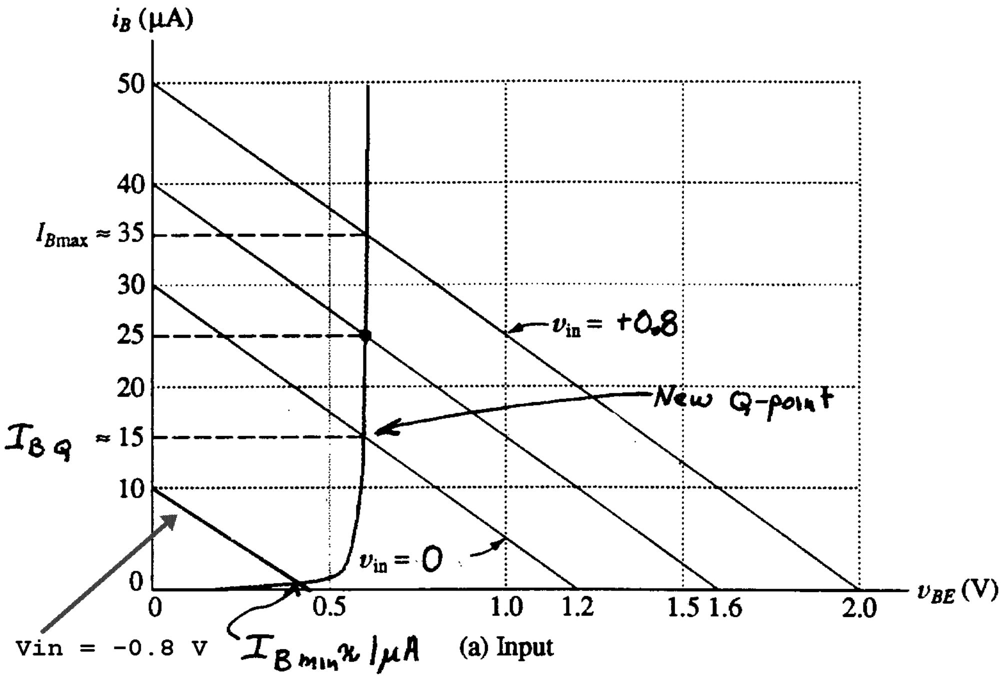
(a) Input



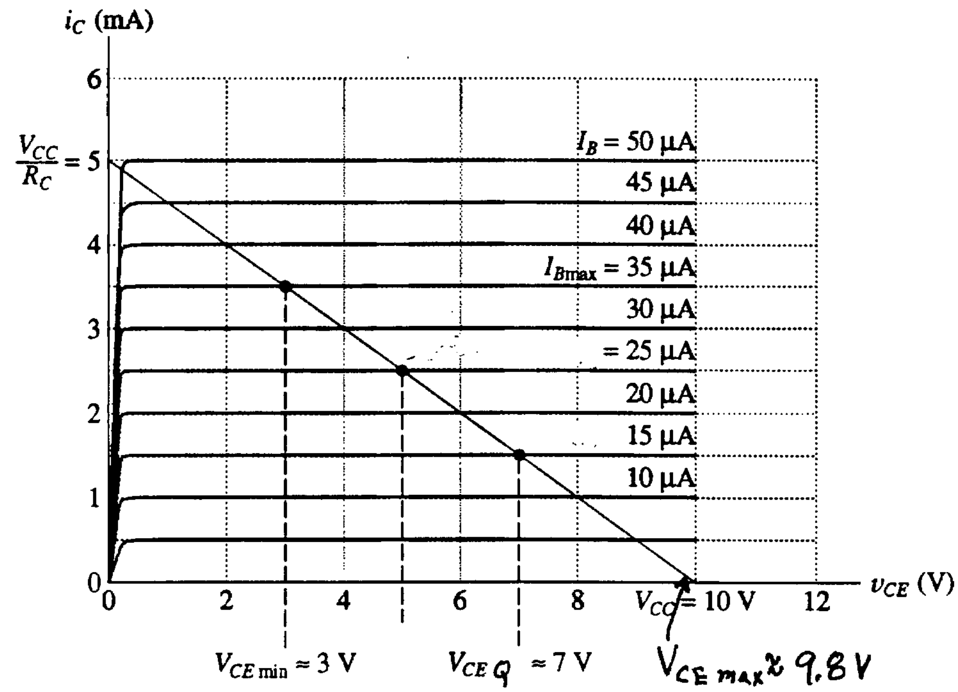
(b) Output

As shown on the output load line, we find $V_{CE\max} \cong 9$ V, $V_{CEQ} \cong 5$ V, and $V_{CE\min} \cong 1.0$ V.

E13.6 The load lines for the new values are shown:



(a) Input

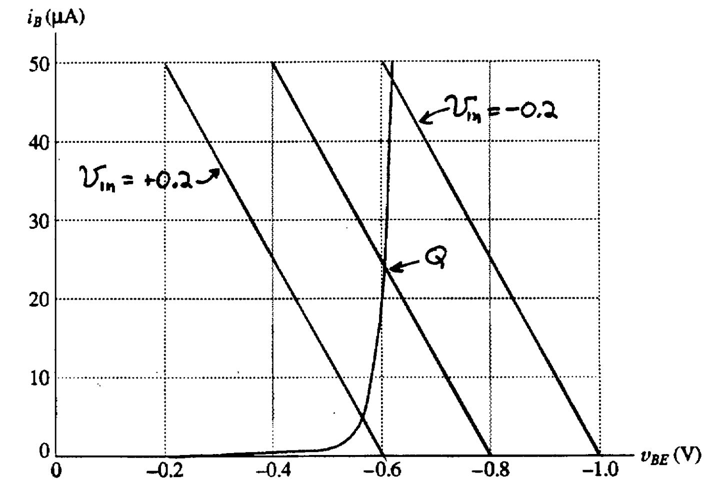


(b) Output

As shown on the output load line, we have $V_{CE\max} \cong 9.8$ V, $V_{CEQ} \cong 7$ V, and $V_{CE\min} \cong 3.0$ V.

E13.7 Refer to the characteristics shown in Figure 13.7 in the book. Select a point in the active region of the output characteristics. For example, we could choose the point defined by $v_{CE} = -6$ V and $i_C = 2.5$ mA at which we find $i_B = 50\ \mu$A. Then we have $\beta = i_C / i_B = 50$. (For many transistors the value found for β depends slightly on the point selected.)

E13.8 (a) Writing a KVL equation around the input loop we have the equation for the input load lines: $0.8 - v_{in}(t) - 8000 i_B + v_{BE} = 0$ The load lines are shown:

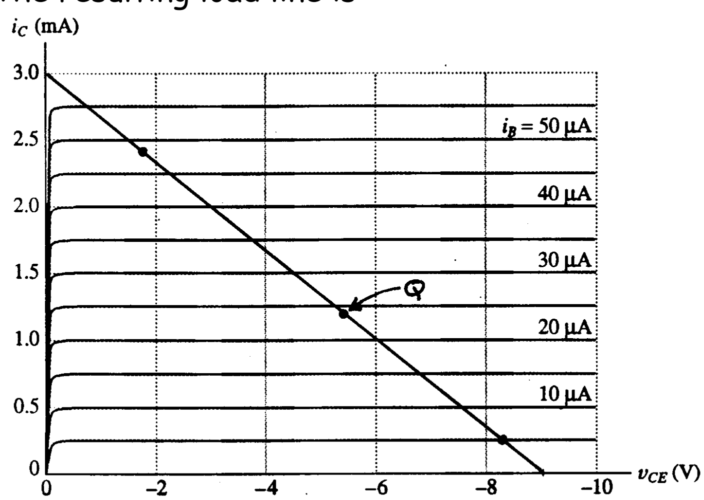


(a) Input

Then we write a KCL equation for the output circuit:

$$9 + 3000 i_C = v_{CE}$$

The resulting load line is:



(b) Output

From these load lines we find

$$I_{B\max} \cong 48\ \mu A,\ I_{BQ} \cong 24\ \mu A,\ I_{B\min} \cong 5\ \mu A,$$
$$V_{CE\max} \cong -1.8\ V,\ V_{CEQ} \cong -5.3\ \ V,\ V_{CE\min} \cong -8.3\ \ V$$

(b) Inspecting the load lines, we see that the maximum of v_{in} corresponds to $I_{B\min}$ which in turn corresponds to $V_{CE\min}$. Because the maximum of v_{in} corresponds to minimum V_{CE}, the amplifier is inverting. This may be a little confusing because V_{CE} takes on negative values, so the minimum value has the largest magnitude.

E13.9 (a) Cutoff because we have $V_{BE} < 0.5$ V and $V_{BC} = V_{BE} - V_{CE} = -4.5$ V which is less than 0.5 V.

(b) Saturation because we have $I_C < \beta I_B$.

(c) Active because we have $I_B > 0$ and $V_{CE} > 0.2$ V.

E13.10 (a) In this case ($\beta = 50$) the BJT operates in the active region. Thus the equivalent circuit is shown in Figure 13.18d. We have

$$I_B = \frac{V_{CC} - 0.7}{R_B} = 71.5\ \mu A \qquad I_C = \beta I_B = 3.575\ \text{mA}$$
$$V_{CE} = V_{CC} - R_C I_C = 11.43\ \text{V}$$

Because we have $V_{CE} > 0.2$, we are justified in assuming that the transistor operates in the active region.

(b) In this case ($\beta = 250$) ,the BJT operates in the saturation region. Thus the equivalent circuit is shown in Figure 13.18c. We have

$$V_{CE} = 0.2\ \text{V} \quad I_B = \frac{V_{CC} - 0.7}{R_B} = 71.5\ \mu A \quad I_C = \frac{V_{CC} - 0.2}{R_C} = 14.8\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are justified in assuming that the transistor operates in the saturation region.

E13.11 For the operating point to be in the middle of the load line, we want $V_{CE} = V_{CC}/2 = 10$ V and $I_C = \dfrac{V_{CC} - V_{CE}}{R_C} = 2$ mA . Then we have

(a) $I_B = I_C / \beta = 20\ \mu A \quad R_B = \dfrac{V_{CC} - 0.7}{I_B} = 965\ \text{k}\Omega$

(b) $I_B = I_C / \beta = 6.667\ \mu A \quad R_B = \dfrac{V_{CC} - 0.7}{I_B} = 2.985\ \text{M}\Omega$

E13.12 Notice that a *pnp* BJT appears in this circuit.

(a) For $\beta = 50$, it turns out that the BJT operates in the active region.

$$I_B = \frac{20 - 0.7}{R_B} = 19.3\ \mu A \qquad I_C = \beta I_B = 0.965\ \text{mA}$$

$$V_{CE} = R_C I_C - 20 = -10.35\ \text{V}$$

(b) For $\beta = 250$, it turns out that the BJT operates in the saturation region.

$$V_{CE} = -0.2\ \text{V} \quad I_B = \frac{20 - 0.7}{R_B} = 19.3\ \mu A \quad I_C = \frac{20 - 0.2}{R_C} = 1.98\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are assured that the transistor operates in the active region.

E13.13 $V_B = V_{CC}\dfrac{R_2}{R_1 + R_2} = 5\ \text{V} \qquad I_B = \dfrac{V_B - V_{BE}}{R_B + (\beta + 1)R_E}$

$I_C = \beta I_B \qquad V_{CE} = V_{CC} - R_C I_C - R_E(I_C + I_B)$

β	I_B (μA)	I_C (mA)	V_{CE} (V)
100	32.01	3.201	8.566
300	12.86	3.858	7.271

For the larger values of R_1 and R_2 used in this Exercise, the ratio of the collector currents for the two values of β is 1.205, whereas for the smaller values of R_1 and R_2 used in Example 13.7, the ratio of the collector currents for the two values of β is 1.0213. In general in the four-resistor bias network smaller values for R_1 and R_2 lead to more nearly constant collector currents with changes in β.

E13.14 $R_B = \dfrac{1}{1/R_1 + 1/R_2} = 3.333\ \text{k}\Omega \qquad V_B = V_{CC}\dfrac{R_2}{R_1 + R_2} = 5\ \text{V}$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = 14.13\ \mu A \quad I_{CQ} = \beta I_{BQ} = 4.239\ \text{mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300(26\ \text{mV})}{4.238\ \text{mA}} = 1840\ \Omega$$

$$R_L' = \frac{1}{1/R_L + 1/R_C} = 666.7\ \Omega \qquad A_v = -\frac{\beta R_L'}{r_\pi} = -108.7$$

$$A_{voc} = \frac{R_L \beta}{r_\pi} = -163.0 \qquad Z_{in} = \frac{1}{1/R_1 + 1/R_2 + 1/r_\pi} = 1186\ \Omega$$

$$A_i = A_v \frac{Z_{in}}{R_L} = -64.43 \qquad G = A_v A_i = 7004$$

$$Z_o = R_C = 1\ \text{k}\Omega$$

$$v_o = A_v v_{\text{in}} = A_v v_s \frac{Z_{\text{in}}}{Z_{\text{in}} + R_s} = -76.46 \sin(\omega t)$$

E13.15 First, we determine the bias point:

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 50.00\ \text{k}\Omega \qquad V_B = V_{CC} \frac{R_2}{R_1 + R_2} = 10\ \text{V}$$

$$I_{BQ} = \frac{V_B - V_{BE}}{R_B + (\beta + 1)R_E} = 14.26\ \mu A \qquad I_{CQ} = \beta I_{BQ} = 4.279\ \text{mA}$$

Now we can compute r_π and the ac performance.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300(26\ \text{mV})}{4.279\ \text{mA}} = 1823\ \Omega \qquad R_L' = \frac{1}{1/R_L + 1/R_E} = 666.7\ \Omega$$

$$A_v = \frac{R_L'(\beta + 1)}{r_\pi + (\beta + 1)R_L'} = 0.9910 \qquad A_{voc} = \frac{R_E(\beta + 1)}{r_\pi + (\beta + 1)R_E} = 0.9970$$

$$Z_{in} = \frac{1}{1/R_B + 1/[r_\pi + (\beta + 1)R_L']} = 40.10\ \text{k}\Omega \qquad A_i = A_v \frac{Z_{in}}{R_L} = 39.74$$

$$G = A_v A_i = 39.38 \qquad R_s' = \frac{1}{1/R_B + 1/R_s} = 8.333\ \text{k}\Omega$$

$$Z_o = \frac{1}{\dfrac{(\beta + 1)}{R_s' + r_\pi} + \dfrac{1}{R_E}} = 33.18\ \Omega$$

Answers for Selected Problems

P13.6* $i_E = 9.3\ \text{mA}$

$\alpha = 0.9677$

$\beta = 30$

P13.7* $v_{BE} \cong 658.5\text{ mV}$

$v_{BC} = -9.341\text{ V}$

$\alpha = 0.9901$

$i_C = 9.901\text{ mA}$

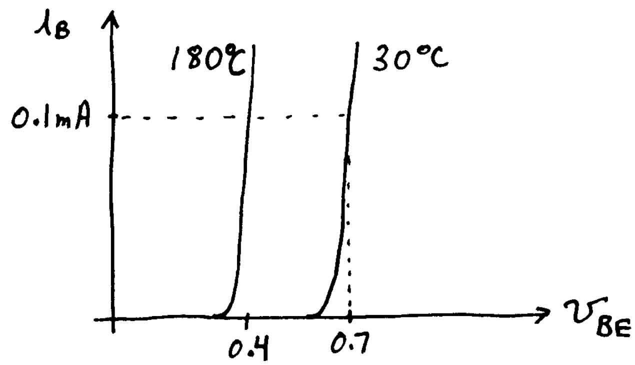
$i_B = 99.01\ \mu A$

P13.16* $I_{ESeq} = 2 \times 10^{-13}$ **A**

$\beta_{eq} = 100$

P13.18* At $180^\circ C$ and $i_B = 0.1\text{ mA}$, the base-to-emitter voltage is approximately:

$$v_{BE} = 0.7 - 0.002(180 - 30) = 0.4\text{ V}$$



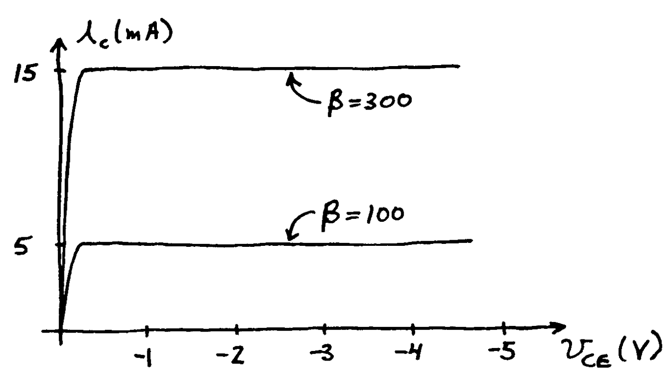
P13.19* $\beta = 400$

$\alpha = 0.9975$

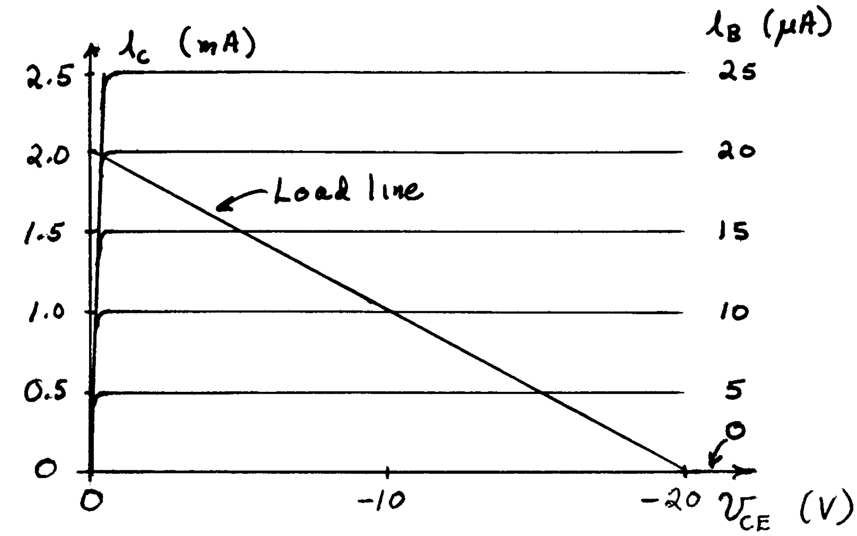
P13.24* $V_{CE\max} = 18.4\text{ V}, V_{CEQ} = 15.6\text{ V}$, and $V_{CE\min} = 12\text{ V}$

$|A_v| = 16$

P13.28*

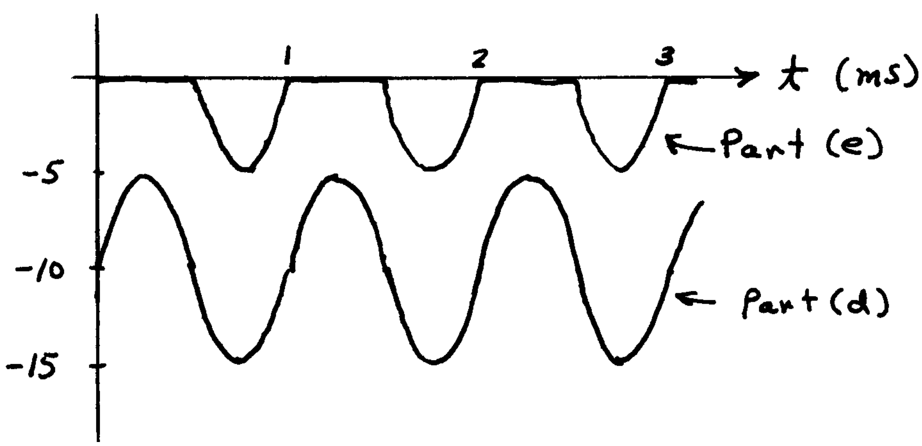


P13.29* (a) and (b)



(c) $I_{C\min} = 0.5\,\text{mA}, I_{CQ} = 1.0\,\text{mA}$, and $I_{C\max} = 1.5\,\text{mA}$
$V_{CE\min} = -15\,\text{V}, V_{CEQ} \cong -10\,\text{V},\ V_{CE\max} \cong -5\,\text{V}$

(d) and (e) The sketches of $v_{CE}(t)$ are:



P13.36* In the active region, the base-collector junction is reverse biased and the base-emitter junction is forward biased.

In the saturation region, both junctions are forward biased.

In the cutoff region, both junctions are reverse biased. (Actually, cutoff applies for slight forward bias of the base-emitter junction as well, provided that the base current is negligible.)

P13.41* 1. Assume operation in saturation, cutoff, or active region.

2. Use the corresponding equivalent circuit to solve for currents and voltages.

3. Check to see if the results are consistent with the assumption made in step 1. If so, the circuit is solved. If not, repeat with a different assumption.

P13.44* The results are given in the table:

Circuit	β	Region of operation	I_C (mA)	V_{CE} (volts)
(a)	100	active	1.93	10.9
(a)	300	saturation	4.21	0.2
(b)	100	active	1.47	5.00
(b)	300	saturation	2.18	0.2
(c)	100	cutoff	0	15
(c)	300	cutoff	0	15
(d)	100	active	6.5	8.5
(d)	300	saturation	14.8	0.2

P13.47* $R_B = 31.5\ \text{k}\Omega$ and $R_E = 753\ \Omega$

P13.49* $I_{C\max} = 0.952\ \text{mA}$

$I_{C\min} = 0.6667\ \text{mA}$

P13.56* $r_\pi = \dfrac{1581}{\sqrt{I_{CQ}}}$

For $I_{CQ} = 1$ mA, we obtain $r_\pi = 50\ \text{k}\Omega$.

P13.63*

	High impedance amplifier (Problem 13.57)	Low impedance amplifier (Problem 13.56)
I_{CQ}	0.0393 mA	3.93 mA
r_π	66.2 kΩ	662 Ω
A_v	-75.5	-75.5
A_{voc}	-151	-151
Z_{in}	54.8 kΩ	548 Ω
A_i	-41.4	-41.4
G	3124	3124
Z_o	100 kΩ	1 kΩ

P13.67* $I_{CQ} = \beta I_{BQ} = 6.41\text{ mA}$

$r_\pi = 405\,\Omega$

$A_v = 0.98$

$A_{voc} = 0.996$

$Z_{in} = 4.36\text{ k}\Omega$

$A_i = 8.61$

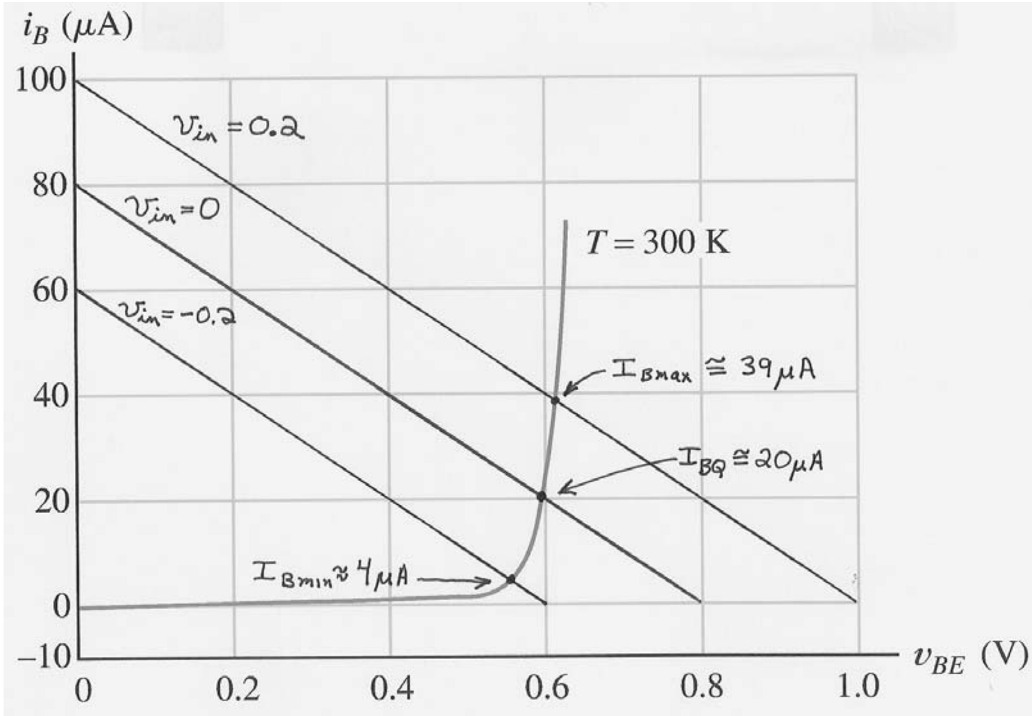
$G = 8.51$

$Z_o = 12.1\,\Omega$

Practice Test

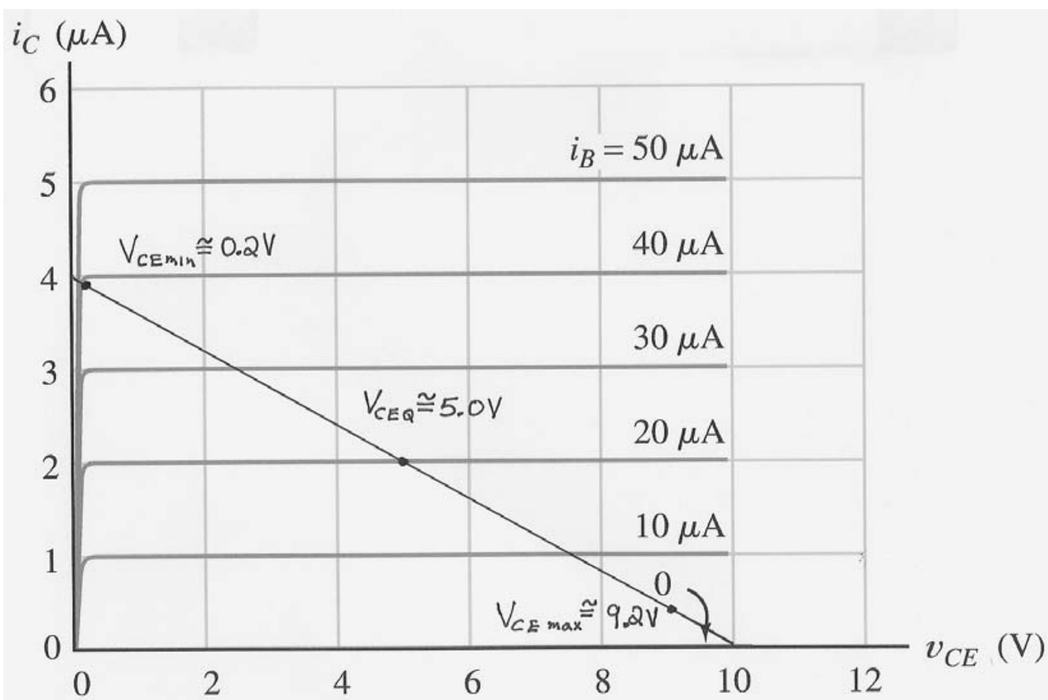
T13.1 **a.** 3, **b.** 2, **c.** 5, **d.** 7 and 1 (either order), **e.** 10, **f.** 7, **g.** 1, **h.** 7, **i.** 15, **j.** 12, **k.** 19.

T13.2 First, we construct the load lines on the input characteristics for $v_{in} = 0$, –0.2 V, and +0.2 V:



(a) Input characteristic

At the intersections of the characteristic with the load lines, we find the minimum, *Q*-point, and maximum values of the base current as shown. Then, we construct the load line on the collector characteristics:



(b) Output characteristics

Interpolating between collector characteristics when necessary, we find $V_{CE\min} \cong 0.2\text{ V}, V_{CEQ} \cong 5.0\text{ V}$, and $V_{CE\max} \cong 9.2\text{ V}$.

T13.3 $\alpha = \dfrac{I_{CQ}}{I_{EQ}} = \dfrac{1.0}{1.04} = 0.9615 \quad I_{BQ} = I_{EQ} - I_{CQ} = 0.04 \text{ mA}$

$$\beta = \frac{I_{CQ}}{I_{BQ}} = \frac{\alpha}{1-\alpha} = 25 \qquad r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{25 \times 0.026}{0.001} = 650\ \Omega$$

The small-signal equivalent circuit is shown in Figure 13.26.

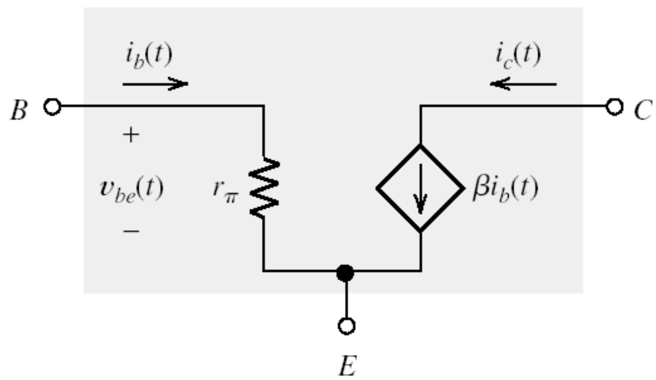
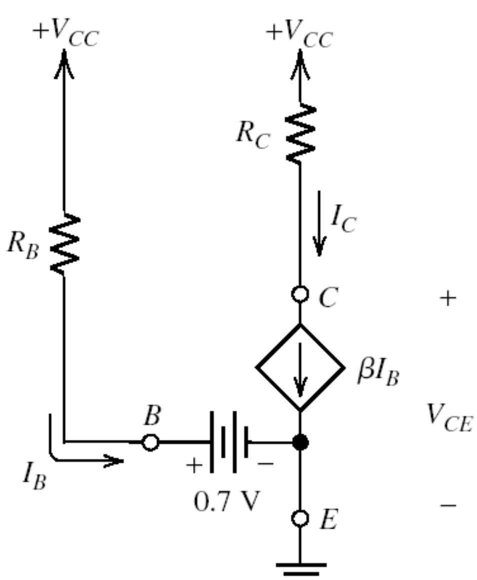


Figure 13.26 Small-signal equivalent circuit for the BJT.

T13.4 (a) It turns out that, in this case ($\beta = 50$), the BJT operates in the active region. The equivalent circuit is:



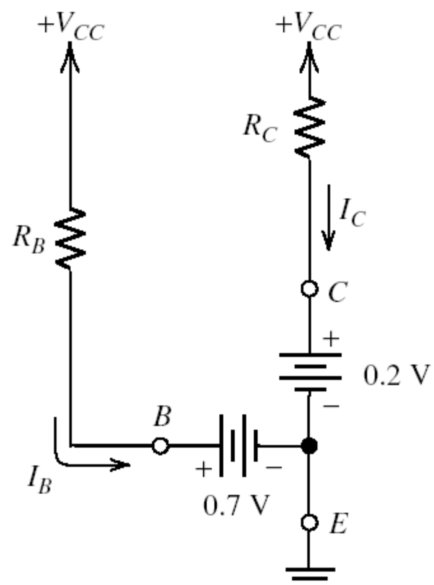
in which we have $V_{CC} = 9$ V, $R_C = 4.7$ kΩ, and $R_B = 470$ kΩ.
We have

$$I_B = \frac{V_{CC} - 0.7}{R_B} = 17.66\ \mu A \qquad I_C = \beta I_B = 0.8830\ \text{mA}$$

$$V_{CE} = V_{CC} - R_C I_C = 4.850\ \text{V}$$

Because we have $V_{CE} > 0.2$, we are justified in assuming that the transistor operates in the active region.

(b) In this case ($\beta = 250$), the BJT operates in the saturation region. The equivalent circuit is:

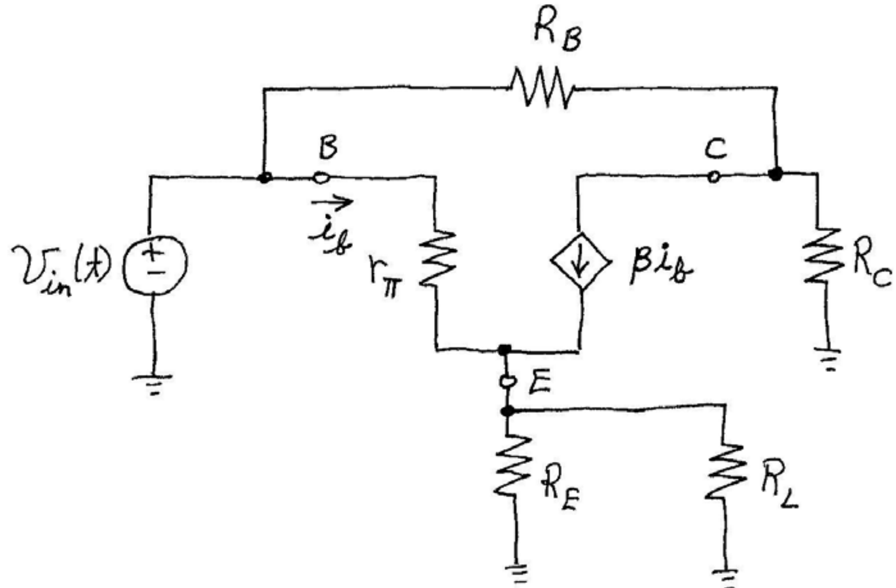


We have

$$V_{CE} = 0.2\ \text{V} \quad I_B = \frac{V_{CC} - 0.7}{R_B} = 17.66\ \mu A \quad I_C = \frac{V_{CC} - 0.2}{R_C} = 1.872\ \text{mA}$$

Because we have $\beta I_B > I_C$, we are justified in assuming that the transistor operates in the saturation region.

T13.5 We need to replace V_{CC} by a short circuit to ground, the coupling capacitances with short circuits, and the BJT with its equivalent circuit. The result is:



T13.6 This problem is similar to parts of Example 13.8.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{120(26\,\text{mV})}{4\,\text{mA}} = 780\,\Omega$$

$$R'_L = \frac{1}{1/R_L + 1/R_C} = 1.579\,\text{k}\Omega$$

$$A_v = -\frac{\beta R'_L}{r_\pi} = -243.0$$

$$R_B = \frac{1}{1/R_1 + 1/R_2} = 31.97\,\text{k}\Omega$$

$$Z_{in} = \frac{1}{1/R_B + 1/r_\pi} = 761.4\,\Omega$$