Solutions to Tutorial 8

7.3 Since the sample is fairly large, the error is, with approximately 95 percent confidence, less than or equal to

$$z_{.025}\frac{s}{\sqrt{n}} = 1.96 \cdot \frac{1.250}{\sqrt{52}} = .340$$

7.4 The 95 percent confidence interval is given by

$$\bar{x} \pm z_{.025} \cdot \frac{s}{\sqrt{n}} = 1.865 \pm (1.96)(1.250)/\sqrt{52} = 1.865 \pm .340$$

or 1.525 to 2.205 hours. We are 95 % confident that the mean amount of labor required to produce an order is between 1.525 and 2.205 hours.

7.10 We need to choose n such that

$$z_{.025}\frac{\sigma}{\sqrt{n}} = 3.$$

Since $\sigma = 20$, and $z_{.025} = 1.96$,

$$\sqrt{n} = \frac{1.96 \cdot 20}{3} = 13.067.$$

Thus, $n = 170.7 \simeq 171$. Thus, the required sample size is 171.

7.11 Since $P\left(|X - \mu| \leq z_{.005}\sigma/\sqrt{n}\right) = .99$, we need to choose n such that

$$z_{.005}\frac{\sigma}{\sqrt{n}} = .25.$$

In this case, $\sigma = 1.40$, $\quad z_{.005} = 2.575$. Thus,

$$n = \left(\frac{(2.575) \cdot 1.40}{.25}\right)^2 = 207.9 \simeq 208.$$

7.12 The sample size $n = 9$ is small so we use $t_{.025} = 2.306$ for 8 degrees of freedom. We first calculate $\overline{x} = 1.334$ and $s = 0.674$ The error is, with approximately 95 percent confidence, less than or equal to

$$t_{.025}\frac{s}{\sqrt{n}} = 2.306 \cdot \frac{0.674}{\sqrt{9}} = 0.518$$

7.13 The sample size $n = 9$ is small so we use $t_{.025} = 2.306$ for 8 degrees of freedom. We first calculate $\overline{x} = 1.334$ and $s = 0.674$ Since the population is normal, the 95 percent confidence interval for the mean product volume is given by

$$\bar{x} \pm t_{.025} \cdot \frac{s}{\sqrt{n}} = 1.334 \pm 2.306 \cdot \frac{0.674}{\sqrt{9}} = 1.334 \pm 0.518$$

or from 0.816 to 1.852. We are 95 % confident that the mean product volume is between .816 and 1.852 gal.

7.14 The sample size $n = 9$ is small so we use $t_{.025} = 2.306$ for 8 degrees of freedom. We first calculate $\overline{x} = 114.00$ and $s = 8.34$ The error is, with approximately 95 percent confidence, less than or equal to

$$t_{.025}\frac{s}{\sqrt{n}} = 2.306 \cdot \frac{8.34}{\sqrt{9}} = 6.41$$

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7.15 The sample size $n = 9$ is small so we use $t_{.025} = 2.306$ for 8 degrees of freedom. We first calculate $\overline{x} = 114.00$ and $s = 8.34$ Since the population is normal, the 95 percent confidence interval for the key performance indicator is given by

$$\bar{x} \pm t_{.025} \cdot \frac{s}{\sqrt{n}} = 114.00 \pm 2.306 \cdot \frac{8.34}{\sqrt{9}} = 114.00 \pm 6.41$$

or 107.59 to 120.41. We are 95 % confident that the mean of the key performance indicator is between 107.59 and 120.41.

7.20 Since the data are a small sample from a normal population, we use the small sample confidence interval for μ with $\bar{x} = .5060$, $s = .0040$, $n = 10$, and $t_{.025}$ with 9 degrees of freedom, which is equal to 2.262. Thus, the 95 percent confidence interval is

$$.5060 - (2.262)\frac{.0040}{\sqrt{10}} \quad < \quad \mu \quad < .5060 + (2.262)\frac{.0040}{\sqrt{10}}$$

or, $.5031 \quad < \quad \mu \quad < \quad .5089$. We are 95 % confident that the mean diameter of the ball bearings is between .5031 and .5089 cm.

7.21 We are given $n = 36$, $\overline{x} = 3.5$ and $s = 0.8$.

(a) The sample size is large so the 90 % confidence interval is

$$\bar{x} \pm z_{.05} \cdot \frac{s}{\sqrt{n}} = 3.5 \pm 1.645 \cdot \frac{0.8}{\sqrt{36}} = 3.5 \pm 0.219$$

or 3.28 to 3.72. We are 090 % confident that the mean freshness is between 3.28 and 3.72.

(b) The population mean μ for all customers is unknown so we never know if it is covered by a particular confidence interval.

(c) Before we sample, the probability is .90 that the interval will cover μ. By the long run relative frequency interpretation of probability, if we take many different samples and calculate a 90 % confidence interval for each, about 90 % of the time they will cover μ.

7.23 We are given $n = 12$, $\overline{x} = 7.2$ and $s = 1.2$.

(a) The sample size is small so we assume that the population is normal. We use $t_{.025} = 2.201$ for 11 degrees of freedom. The 95 % confidence interval is

$$\bar{x} \pm t_{.025} \cdot \frac{s}{\sqrt{n}} = 7.2 \pm 2.201 \cdot \frac{1.2}{\sqrt{12}} = 7.2 \pm .76$$

or 6.4 to 8.0. We are 95 % confident that the mean stamping pressure is between 6.44 and 7.96 thousand psi.

(b) The population mean μ of maximum pressures from all possible occasions, is unknown so we never know if it is covered by a particular confidence interval.

(c) The sample size was small so we assumed the population was normal.

(d) Before we sample, the probability is .95 that the interval will cover μ. By the long run relative frequency interpretation of probability, if we take many different samples and calculate a 95% confidence interval for each, about 95 % of the time they will cover μ.

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7.24 The mean of the sample is 2.1. The standard deviation is .5372. Since the sample is from a normal population, we can use the small sample confidence interval with $\bar{x} = 2.1$, $s = .5372$, $n = 8$, and $t_{.025}$

with 7 degrees of freedom equals 2.365. Thus, the 95 percent confidence interval is

$$2.1 - (2.365)(.5372)/\sqrt{8} \quad < \quad \mu \quad < \quad 2.1 + (2.365)(.5372)/\sqrt{8}$$

or, $1.6508 \quad < \quad \mu \quad < \quad 2.5492$. We are 95 % confident that the mean of suspended organic matter is between 1.65 and 2.55 micrograms/m^3

7.27 Let X have a binomial distribution with a probability of success p.

(a) X/n is an unbiased estimator of p since the expected value of X/n is

E(X) = np, and E(X/n) = p

According to the definition, X/n is an unbiased estimator of p

(b) $(X+1)/(n+2)$ is not an unbiased estimator of p since

$$E\left(\frac{X+1}{n+2}\right) = \frac{1}{n+2}E(X+1) = \frac{1}{n+2}[E(X)+1] = \frac{np+1}{n+2} \neq p$$