EE2023 SIGNALS & SYSTEMS
PAST-YEAR EXAM ARCHIVE

Semester I : 2011/2012

w/ Numeric Answers appended

SECTION A : Answer ALL questions in this section

Q.1 Consider the circuit shown in Figure Q1-1 below.

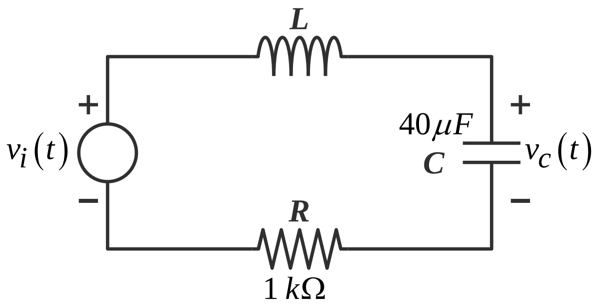

Figure Q1-1: RLC Circuit

(a) Find the transfer function, $G(s)=\dfrac{V_c(s)}{V_i(s)}$, in terms of L, R and C.
Assume that $V_c(s)=\mathcal{L}\{v_c(t)\}$ and $V_i(s)=\mathcal{L}\{v_i(t)\}$. (3 marks)

(b) Determine the value of L for which the circuit is critically damped. (3 marks)

(c) Sketch the impulse response of the circuit for the critically damped case. (4 marks)

Q.2 The signal $x(t)=10+10\cos\left(1000t+\dfrac{\pi}{8}\right)$ is sampled at five times the Nyquist frequency.

(a) What is the time interval between samples? (3 marks)

(b) How many samples are there in 1 second of this signal? (3 marks)

(c) Sketch the amplitude spectrum of the sampled signal. (4 marks)

Q.3 The spectrum of a signal $x(t)$ is shown in Figure Q3-1.

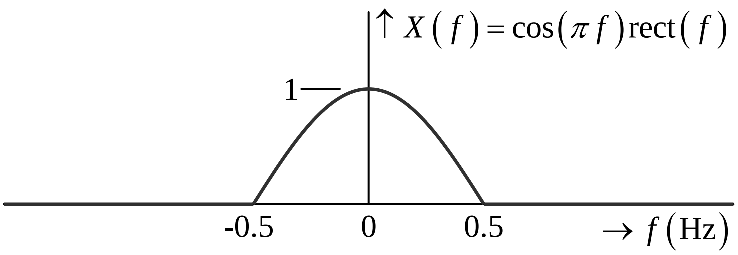

Figure Q3-1: Spectrum

(a) Calculate the 3dB bandwidth of $x(t)$. (4 marks)

(b) What is the DC value of $x(t)$? (3 marks)

(c) Sketch and label the spectrum of $y(t)=x(t)\cos(5\pi t)$. (3 marks)

Q.4 Consider a system modeled by the transfer function,

$$G(s)=\frac{K\left(-\frac{s}{\alpha}+1\right)}{\left(\frac{s}{\beta}+1\right)\left(\frac{s}{\gamma}+1\right)^2}.$$

Using the pole-zero map and Bode magnitude plot of $G(s)$ shown in Figure Q4-1, answer the following questions.

(a) Identify the corner frequencies $(\omega_1, \omega_2$ and $\omega_3)$ of the Bode magnitude plot for $G(s)$.

(3 marks)

(b) What is the value of the repeated pole? Justify your answer. (2 marks)

(c) Determine the DC gain, K. (2 marks)

(d) Is the system stable? Justify your answer. (3 marks)

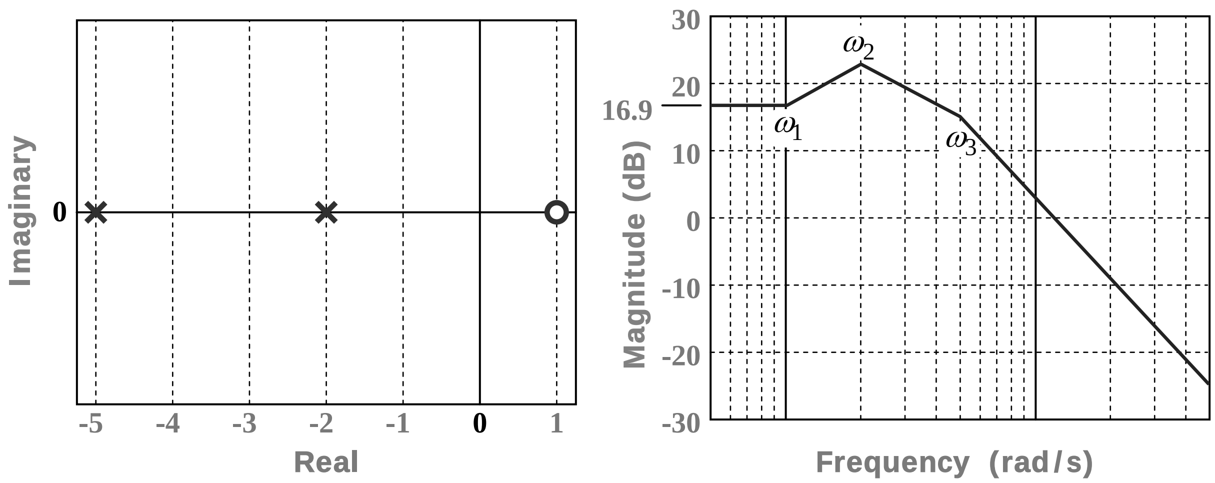

Figure Q4-1: Pole-Zero Map and Bode Magnitude Plot

SECTION B : Answer 3 out of the 4 questions in this section

Q.5 Consider the linear time invariant system shown in Figure Q5-1 below, where K and T are constants. The input, $x(t)$, is given by $x(t) = x_0 + \sin(t)$ and the corresponding steady state output, $y(t)$, is given by $y(t) = 4 + 2^{0.5}\sin(t - 0.5\pi)$.

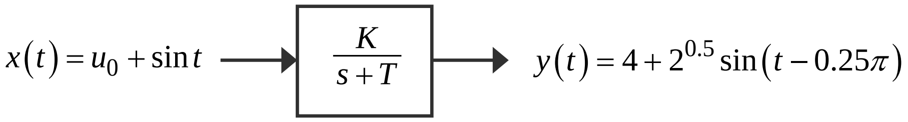

Figure Q5-1

(a) Find u_0, K and T. (6 marks)

(b) If K and u_0 remain unchanged but T is twice the value from part (a), explain qualitatively (without calculations) how the steady state output $y(t)$ will change. (6 marks)

(c) If $K = T = 2$ and $x(t) = 2t$, find the steady state error,

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}\left[x(t) - y(t)\right].$$

Verify your result using a second method. (8 marks)

Q.6 (a) Derive the Fourier transform of the pulse $x(t)$ shown in Figure Q6-1. (10 marks)

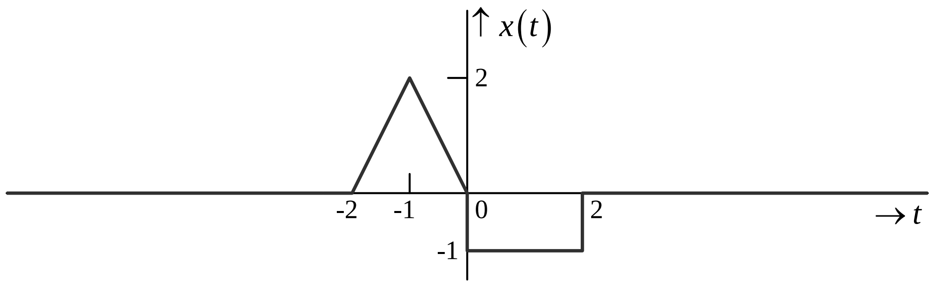

Figure Q6-1: Pulse

(b) The periodic signal $y(t)$, shown in Figure Q6-2, may be generated using $x(t)$. Using the Fourier transform of $x(t)$, derive the Fourier series coefficients, Y_k, of $y(t)$. (10 marks)

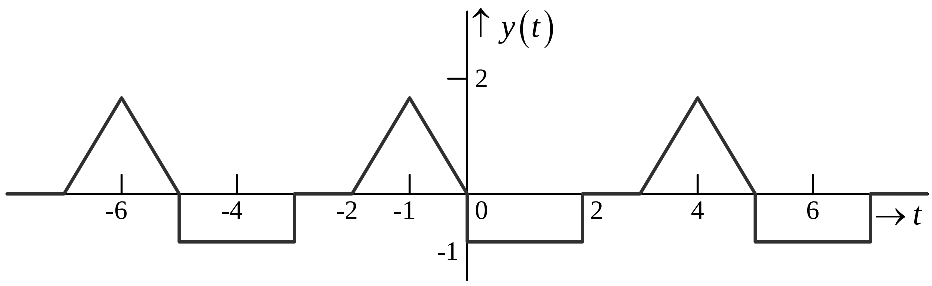

Figure Q6-2: Periodic Signal

Q.7 A pulse $p(t) = 5\text{sinc}^2(5t)$ is used as an acknowledgement signal in a communication system. Due to poor transmitter design, the 50 Hz hum from the a.c. power supply of the transmitter is superimposed on $p(t)$. As a result, $x(t) = \sin(100\pi t) + p(t)$ is transmitted instead of $p(t)$. At the receiver, $x(t)$ is first sampled into $x_s(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-0.05n)$ before being further processed.

(a) Find the spectrum of $p(t)$. (6 marks)

(b) Sketch and label the spectrum of $x_s(t)$. (7 marks)

(c) In theory, can $p(t)$ be perfectly recovered from $x_s(t)$? If 'NO', explain why. If 'YES', explain how it can be done in the least expensive way from the standpoint of practical implementation.

(7 marks)

Q.8 A second-order system, $G(s) = \dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}e^{-0.1s}$, has the following responses:

- Figure Q8-1 shows the unit step response of $G(s)$.
- When the input signal is $x(t) = 10\cos(9t - 13.16°)$, the steady-state output signal is $\lim_{t\to\infty} y(t) = 192.2\cos(9t - 180°)$.

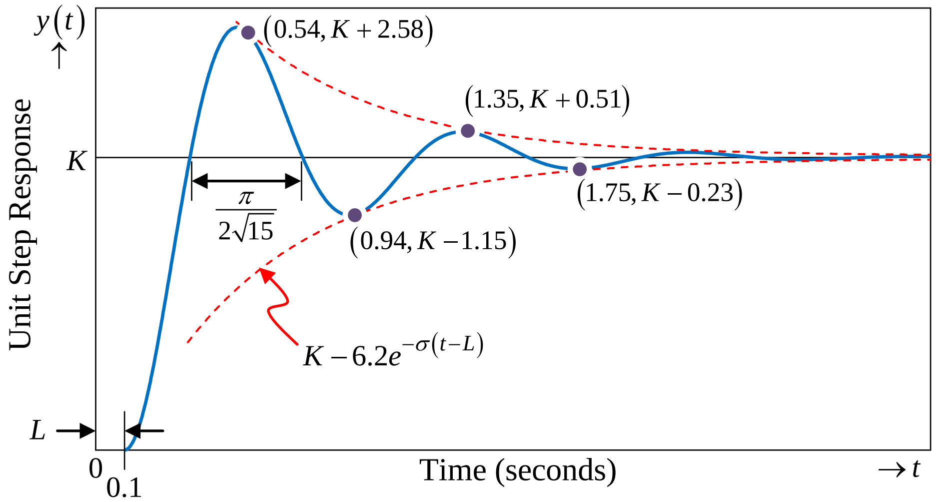

Figure Q8-1: Unit Step response of $G(s)$

(a) Using Figure Q8-1, show that the damping ratio (ζ) and undamped natural frequency (ω_n) of $G(s)$ is 0.25 and 8 rad/s, respectively. (12 marks)

(b) Derive the steady-state value of the unit step response shown in Figure Q8-1?

(8 marks)

END OF QUESTIONS

NUMERIC ANSWERS

Section A

Q.1 (a) $G(s) = \dfrac{1}{s^2LC + sRC + 1}$

(b) $L = \dfrac{R^2C}{4} = 10\,\mathrm{H}$

(c) Sketch: $Kt\exp(-Ct)$ where K and C are positive constants

Q.2 (a) $\dfrac{\pi}{5000}$ $(\text{or } 0.0006283)$ s

(b) $\dfrac{5000}{\pi}$ $(\text{or } 1591.5)$ *samples*

(c) Sketch: $5\delta\left(f + \dfrac{500}{\pi}\right) + 10\delta(f) + 5\delta\left(f - \dfrac{500}{\pi}\right)$

Q.3 (a) $B_{3dB} = 0.25$ Hz

(b) DC value = 0

(c) Sketch: $0.5X(f - 2.5) + 0.5X(f + 2.5)$

Q.4 (a) Corner frequencies: $\omega_1 = 1$ rad/s, $\omega_2 = 2$ rad/s, $\omega_3 = 5$ rad/s

(b) Repeated pole: $s = -2$

(c) $K = 16.9\ dB = 10^{16.9/20} = 7$

(d) System is stable (?)

Section B

Q.5 (a) $u_0 = 2$, $K = 2$, $T = 1$

(c) Steady state error = 1 (Verify using FVT)

Q.6 (a) $X(f) = 2.\exp(j2\pi f).\mathrm{sinc}^2(f) - 2.\exp(-j2\pi f).\mathrm{sinc}(2f)$

(b) $Y_k = 0.4\exp(j0.4\pi k)\mathrm{sinc}^2(0.2k) - 0.4\exp(-j0.4k\pi).\mathrm{sinc}(0.4k)$

Q.7 (a) $P(f) = \mathrm{tri}(f/5)$

(b) $X_S(f) = 20\sum_{k=-\infty}^{\infty} \mathrm{tri}\left(\dfrac{f - 20k}{5}\right)$

(c) Use LPF with ideal passband from 0 to 5 *Hz*, and ideal stopband from 15 *Hz* onwards.

Q.8 (b) $K = 6$
