Solutions to Tutorial 2

1. (a) $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
   (b) $A = \{HHH, HHT, HTH, HTT\}$
   (c) $B = \{HHH, HHT, THH, THT\}$
   (d) $C = \{HHH, THH, TTH, HTH\}$
   (e) $D = \{HHT, HTH, THH\}$
   (f) A∪B = {HHH, HHT, HTH, HTT, THH, THT}
   A∩B = {HHH, HHT}
   $\bar{A}$ = {THH, THT, TTH, TTT}
   A−B = {HTH, HTT}
   A∩B∩ C = {HHH}
   $A \cup \bar{B}$ = {HHH, HHT, HTT, HTH, TTH, TTT }
   A∩ D = {HHT, HTH}
2. A∪B =$(-\infty, 10]$,
   A∩B = (0, 5] ,
   $\bar{A}$ = (5, ∞),
   A−B = (-∞, 0],
   A∩B∩ C = ∅,
   $A \cup \bar{B}$ = (-∞, 5]∪ (10, ∞).
3. $P(A_1 \cup B_1)$ = 0.29+0.11+0.06
   P(A_1) = 0.11+0.29
   $P(A_1 - B_1)$ = 0.29

3.11 (a) Region 5 represents the event that the windings are improper, but the shaft size is not too large and the electrical connections are satisfactory.

(b) Regions 4 and 6 together represent the event that the electrical connections are unsatisfactory, but the windings are proper.

(c) Regions 7 and 8 together represent the event that the windings are proper and the electrical connections are satisfactory.

(d) Regions 1, 2, 3, and 5 together represent the event that the windings are improper.

3.12 (a) Region 8.

(b) Regions 1 and 2 together.

(c) Regions 2, 5, and 7 together.

(d) Regions 1, 2, 3, 4, and 6 together.

3.13 The following Venn diagram will be used in parts (a), (b), (c) and (d).

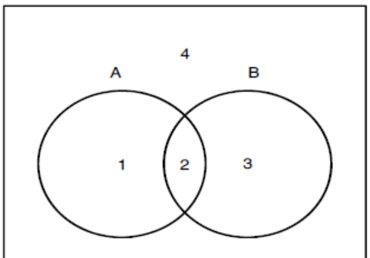

Fig. 1

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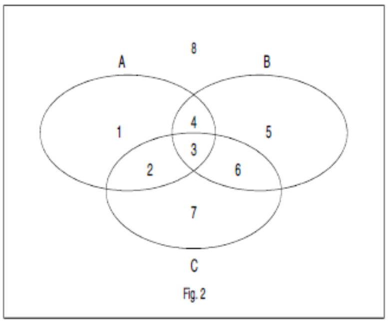
(a) $A \cap B$ is region 2 in Fig. 1. $\overline{(A \cap B)}$ is the region composed of areas 1, 3, and 4. $\overline{A}$ is the region composed of areas 3 and 4. $\overline{B}$ is the region composed of areas 1 and 4. $\overline{A} \cup \overline{B}$ is the region composed of areas 1, 3, and 4. This corresponds to $\overline{(A \cap B)}$.

(b) $A \cap B$ is the region 2 in the figure. A is the region composed of areas 1 and 2. Since $A \cap B$ is entirely contained in A, $A \cup (A \cap B) = A$.

(c) $A \cap B$ is region 2. $A \cap \overline{B}$ is region 1. Thus, $(A \cap B) \cup (A \cap \overline{B})$ is the region composed of areas 1 and 2 which is A.

(d) From part (c), we have $(A \cap B) \cup (A \cap \overline{B}) = A$. Thus, we must show that $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup (\overline{A} \cap B) = A \cup B$. A is the region composed of areas 1 and 2 and $\overline{A} \cap B$ is region 3. Thus, $A \cup (\overline{A} \cap B)$ is the region composed of areas 1, 2, and 3.



(e) In Fig. 2, $A \cup B$ is the region composed of areas 1, 2, 3, 4, 5, and 6. $A \cup C$ is the region composed of areas 1, 2, 3, 4, 6, and 7, so $(A \cup B) \cap (A \cup C)$ is the region composed of areas 1, 2, 3, 4, and 6. $B \cap C$ is the region composed of areas 3, and 6, and A is the region composed of areas 1, 2, 3, and 4. Thus, $A \cup (B \cap C)$ is the region composed of areas 1, 2, 3, 4, and 6. Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

3.15 The tree diagram is given in Figure 3.3, where S = Spain, U = Uruguay, P = Portugal and J = Japan.

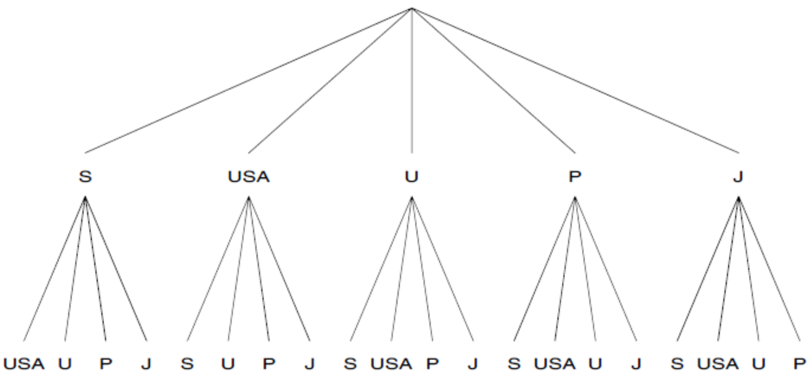


Figure 3.3: The tree diagram for Exercise 3.15.

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3.17 There are $(6)(4)(3) = 72$ ways.

3.19 (a) There men and women can be chosen in

$$_6C_2 = \binom{6}{2} = \frac{6!}{4!\,3!} = 15 \quad \text{and} \quad _4C_2 = \binom{4}{2} = \frac{4!}{2!\,2!} = 6$$

ways so there are $15 \times 6 = 90$ different project teams consisting of 2 men and 2 women.

(b) We are restricted from the choice of having the two women in question both selected giving a total of $6 - 1 = 5$ choices for two women. The number of project teams is reduced to $15 \times 5 = 75$.

3.20 $_9P_3 = 9 \cdot 8 \cdot 7 = 504$.

3.21 $6! = 720$.

3.23 Since order does not matter, there are

$$_{15}C_2 = \binom{15}{2} = \frac{15!}{13!\,2!} = 105$$

ways.

3.24 There are

$$_{18}C_4 = \binom{18}{4} = \frac{18!}{4!\,14!} = 3,060 \qquad \text{ways.}$$

3.25 There are $_{12}C_3 = 220$ ways to draw the three rechargeable batteries.

There are $_{11}C_3 = 165$ ways to draw none are defective.

(a) The number of ways to get the one that is defective is $220 - 165 = 55$.

(b) There are 165 ways not to get the one that is defective.

3.26 (a) There are $_{10}C_3 = 120$ ways to get no defective batteries.

(b) There are $2 \cdot {_{10}C_2} = 90$ ways to get 1 defective battery.

(c) There are $_{10}C_1 = 10$ ways to get both defective batteries.

3.27 There are $_8C_2$ ways to choose the electric motors and $_5C_2$ ways to choose the switches. Thus, there are

$$_8C_2 \cdot {_5C_2} = 28 \cdot 10 = 280$$

ways to choose the motors and switches for the experiment.

3.28 (a) Using the long run relative frequency approximation to probability, we estimate the probability

$$P\,[\,\text{Warranty repair required}\,] = \frac{287}{2756} = 0.104$$

(b) Using the data from last year, the long run relative frequency approximation to probability gives the estimate

$$P\,[\,\text{Receive season ticket}\,] = \frac{6000}{8400} = 0.714$$

One factor is the expected quality of the team next year. If the team is expected to be much better next year more students will apply for tickets.

3.30 There are 250 numbers divisible by 200. Thus, the probability is $250/50{,}000 = 1/200$.

3.31 There are $18 + 12 = 30$ cars. Thus, there are ${}_{30}C_4$ ways to choose the cars for inspection. There are ${}_{18}C_2$ ways to get the compacts and ${}_{12}C_2$ ways to get the intermediates. Thus, the probability is:

$$\frac{\binom{18}{2}\binom{12}{2}}{\binom{30}{4}} = \frac{10,098}{27,405} = .368.$$

3.33 The number of students enrolled in the statistics course or the operations research course is $92+63-40 = 115$. Thus, $160 - 115 = 45$ are not enrolled in either course.

3.35 (a) Yes. $P(A) + P(B) + P(C) + P(D) = 1$.

(b) No. $P(A) + P(B) + P(C) + P(D) = 1.02 > 1$.

(c) No. $P(C) = -.06 < 0$.

(d) No. $P(A) + P(B) + P(C) + P(D) = 15/16 < 1$.

(e) Yes. $P(A) + P(B) + P(C) + P(D) = 1$.

3.41 (a) $P(\overline{A}) = 1 - P(A) = 1 - .26 = .74$.

(b) $P(A \cup B) = P(A) + P(B) = .26 + .45 = .71$, since A and B are mutually exclusive.

(c) $P(A \cap \overline{B}) = P(A) = .26$, since A and B are mutually exclusive.

(d) $P(\overline{A} \cap \overline{B}) = P(\overline{(A \cup B)}) = 1 - P(A \cup B) = 1 - .26 - .45 = .29$.

3.44 (a) P(at most 4 complaints)$=.01 + .03 + .07 + .15 + .19 = .45$.

(b) P(at least 6 complaints)$=.14 + .12 + .09 + .02 = .37$.

(c) P(from 5 to 8 complaints)$=.18 + .14 + .12 + .09 = .53$.

3.45 (a) $15/32$ (b) $13/32$ (c) $5/32$ (d) $23/32$ (e) $8/32$ (f) $9/32$.

3.47 (a) "At least one award" is the same as "design or efficiency award". Thus, the probability is $.16 + .24 - .11 = .29$.

(b) This the probability of "at least one award" minus the probability of both awards or $.29-.11 = .18$.

3.48 (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .35 + .65 - .12 = .88$.

(b) $P(\overline{A} \cap B) = P(B) - P(A \cap B) = .65 - .12 = .53$.

(c) $P(A \cap \overline{B}) = P(A) - P(A \cap B) = .35 - .12 = .23$.

(d) $P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - .12 = .88$.

3.49

$$\begin{aligned}
&P(A \cup B \cup C) = 1 - .11 = .89, && P(A) = .24 + .06 + .04 + .16 = .5,\\
&P(B) = .19 + .06 + .04 + .11 = .4, && P(C) = .09 + .16 + .04 + .11 = .4,\\
&P(A \cap B) = .06 + .04 = .1, && P(A \cap C) = .16 + .04 = .2,\\
&P(B \cap C) = .04 + .11 = .15, && P(A \cap B \cap C) = .04.
\end{aligned}$$

Thus, the following equation must equal to .89:

$$.5 + .4 + .4 - .1 - .2 - .15 + .04 = .89.$$

This proves the formula.

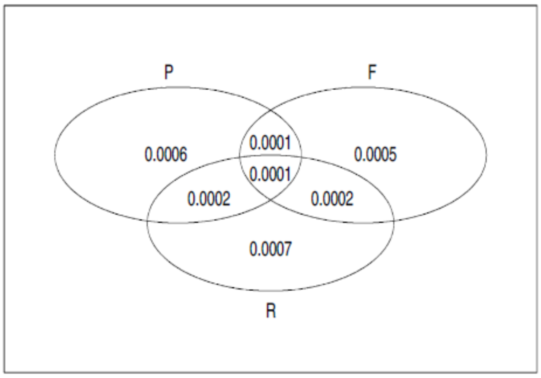

Figure 3.6: P = Processing, F = Filing and R = Retrieving.

3.50 These probabilities are shown in Figure 3.6.

The probability of at least one of these errors is:

$$.0006 + .0001 + .0001 + .0002 + .0002 + .0005 + .0007 = .0024.$$

We can also use the formula given in Exercise 3.49 to calculate the probability:

$$P(P \cup F \cup R) = .001 + .0009 + .0012 - .0002 - .0003 - .0003 + .0001 = .0024.$$