

NATIONAL UNIVERSITY OF SINGAPORE
FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2004-2005

MA1505 MATHEMATICS I

November 2004 Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. **Write down your matriculation number neatly in the space provided below.** This booklet (and only this booklet) will be collected at the end of the examination. Do not insert any loose pages in the booklet.
2. This examination paper consists of **TEN (10)** questions and comprises **FORTY ONE (41)** printed pages.
3. Answer **ALL** questions. For each question, write your answer in the box and your working in the space provided inside the booklet following that question.
4. The marks for each question are indicated at the beginning of the question.
5. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Matriculation Number:

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For official use only. Do not write below this line.

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|---|---|---|---|---|---|---|---|---|----|
| Marks | | | | | | | | | | |

Question 1 (a) [5 marks]

Find the slope of the tangent line at the point $(2, -2)$ on the graph of $x^2y^2 - 2x = 4 - 4y$.

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| Answer 1(a) | |
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(Show your working below and on the next page.)

(More working space for Question 1(a))

Question 1 (b) [5 marks]

Find $\frac{1}{\pi} \left(f'(1) - \frac{1}{2\sqrt{3}} \right)$ if $f(x) = x \sin^{-1} \frac{x}{x+1}$.

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| Answer 1(b) | |
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(Show your working below and on the next page.)

(More working space for Question 1(b))

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Question 2 (a) [5 marks]

Given that the function $f(x) = \frac{x(3x-2)}{(x-1)(x-2)}$, where $x \in (1, 2)$, attains its absolute maximum value at the point $C \in (1, 2)$. Find the value of $(3 - \sqrt{2}) C$.

| | |
|------------------------------|--|
| Answer 2(a) | |
|------------------------------|--|

(Show your working below and on the next page.)

(More working space for Question 2(a))

Question 2 (b) [5 marks]

Find the value of

$$\lim_{x \rightarrow 0} \frac{\cos^2 8x - \cos^2 5x}{x^2}.$$

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| Answer 2(b) | |
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(Show your working below and on the next page.)

(More working space for Question 2(b))

Question 3 (a) [5 marks]

Find the volume of the solid obtained by revolving the region bounded by

$$y = \sqrt{x}, y = \frac{1}{x}, x = 1 \text{ and } x = 4$$

about the y -axis. Give your answer in terms of π .

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| Answer 3(a) | |
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(Show your working below and on the next page.)

(More working space for Question 3(a))

Question 3 (b) [5 marks]

Find the value of

$$\int_0^{\pi/3} (\sin^3 x) (\cos x) dx.$$

| | |
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| Answer 3(b) | |
|------------------------|--|

(Show your working below and on the next page.)

(More working space for Question 3(b))

Question 4 (a) [5 marks]

By using the Limit Comparison Test, or otherwise, determine whether the series

$$\sum_{n=1}^{\infty} \frac{2n+3}{5n^3-4n}$$

is convergent or divergent. Show clearly all your steps.

| | |
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| Answer 4(a) | |
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(Show your working below and on the next page.)

(More working space for Question 4(a))

Question 4 (b) [5 marks]

Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(3^n) (n!)^2}{(2n)!}$$

is absolutely convergent, conditionally convergent or divergent.
Show clearly all your steps.

| | |
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| Answer 4(b) | |
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(Show your working below and on the next page.)

(More working space for Question 4(b))

Question 5 (a) [5 marks]

Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (5x)^{2n}}{n}.$$

| | |
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| Answer 5(a) | |
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(Show your working below and on the next page.)

(More working space for Question 5(a))

Question 5 (b) [5 marks]

Let

$$f(x) = \int_0^{x^2} \tan^{-1} t \, dt.$$

Let $f(x) = \sum_{n=0}^{\infty} c_n (x-1)^n$ be the Taylor series representation for $f(x)$ about the point $a = 1$. Find the value of c_2 .

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| Answer 5(b) | |
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(Show your working below and on the next page.)

(More working space for Question 5(b))

Question 6 (a) [5 marks]

Let

$$f(x) = \begin{cases} 0 & \text{if } -2\pi < x < 0 \\ x^2 & \text{if } 0 < x < 2\pi. \end{cases}$$

Find the coefficient of $\cos x$ in the Fourier Series representation for $f(x)$.

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| Answer 6(a) | |
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(Show your working below and on the next page.)

(More working space for Question 6(a))

Question 6 (b) [5 marks]

Let $f(x) = \cos x$ for $0 < x < \pi$. Let $\sum_{n=1}^{\infty} b_n \sin nx$ be the Fourier Sine Series which represents $f(x)$. Find the value of $b_1 + b_2$.

| | |
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| Answer 6(b) | |
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(Show your working below and on the next page.)

(More working space for Question 6(b))

Question 7 (a) [5 marks]

Solve the differential equation

$$x \frac{dy}{dx} - y = 2x^2 \sin 2x$$

with the initial condition $y = \pi$ when $x = \pi$.

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|------------------------------|--|
| Answer 7(a) | |
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(Show your working below and on the next page.)

(More working space for Question 7(a))

Question 7 (b) [5 marks]

Solve the differential equation

$$y' = \frac{x^2 + xy + y^2}{x^2}$$

with the initial condition $y = 0$ when $x = 1$.

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| Answer 7(b) | |
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(Show your working below and on the next page.)

(More working space for Question 7(b))

Question 8 (a) [5 marks]

Solve the differential equation

$$9y'' - 6y' + y = 0$$

with the initial conditions that $y = 1$ and $y' = 3$ when $x = 0$.

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| Answer 8(a) | |
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(Show your working below and on the next page.)

(More working space for Question 8(a))

Question 8 (b) [5 marks]

Solve the differential equation

$$y'' - 5y' + 6y = 18x^2.$$

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| Answer 8(b) | |
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(Show your working below and on the next page.)

(More working space for Question 8(b))

Question 9 (a) [5 marks]

Find the Laplace transform

$$L(t \cos 2t).$$

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| Answer 9(a) | |
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(Show your working below and on the next page.)

(More working space for Question 9(a))

Question 9 (b) [5 marks]

Find the inverse Laplace transform

$$L^{-1} \left(\frac{2s^2 - 4}{(s - 2)(s + 1)(s - 3)} \right).$$

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| Answer 9(b) | |
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(Show your working below and on the next page.)

(More working space for Question 9(b))

Question 10 (a) [5 marks]

Solve the differential equation

$$\frac{d^2x}{dt^2} = 12(t-1)^2 U(t-1) , \quad x'(0) = x(0) = 0$$

where $U(t-1) = \begin{cases} 0 & \text{if } t < 1 \\ 1 & \text{if } t > 1. \end{cases}$

| | |
|-------------------------|--|
| Answer 10(a) | |
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(Show your working below and on the next page.)

(More working space for Question 10(a))

Question 10 (b) [5 marks]

Find the functions $x(t)$ and $y(t)$ which satisfy

$$\begin{cases} \frac{dx}{dt} - y = \frac{t^2}{2} \\ x - \frac{dy}{dt} = 0 \end{cases}$$

and the initial conditions $x(0) = 0, y(0) = 1$.

| | |
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| Answer 10(b) | |
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(Show your working below and on the next page.)

(More working space for Question 10(b))

END OF PAPER