

## EE3206/EE3206E ICVIP

## Tutorial Set A – Solutions

## Question 1

	0	0	0	0	0	0	0	0	0	0	0	
	0	0	9	9	9	0	0	0	0	0	0	
	0	0	9	9	9	9	0	0	0	0	0	
	0	0	9	9	9	9	9	0	0	0	0	
	0	0	9	9	9	9	9	9	0	0	0	
	0	0	0	9	9	9	9	9	9	0	0	
	0	0	0	0	9	9	9	9	9	0	0	
	0	0	0	0	0	0	0	0	0	0	0	

$I$

	0	0	0	0	0	0	0	0	0	0	0	
	0	3	6	9	6	3	0	0	0	0	0	
	0	3	6	9	9	6	3	0	0	0	0	
	0	3	6	9	9	9	6	3	0	0	0	
	0	3	6	9	9	9	9	6	3	0	0	
	0	0	3	6	9	9	9	9	6	3	0	
	0	0	0	3	6	9	9	9	6	3	0	
	0	0	0	0	0	0	0	0	0	0	0	

$I_1$

	0	0	3	3	3	0	0	0	0	0	0	
	0	0	6	6	6	3	0	0	0	0	0	
	0	0	9	9	9	6	3	0	0	0	0	
	0	0	9	9	9	9	6	3	0	0	0	
	0	0	6	9	9	9	9	6	3	0	0	
	0	0	3	6	9	9	9	9	6	0	0	
	0	0	0	3	6	6	6	6	6	0	0	
	0	0	0	0	3	3	3	3	3	0	0	

$I_2$

- $M_1$  smooths (blurs) in a horizontal direction.
- $M_2$  smooths (blurs) in a vertical direction.

$$I_3$$
 $I_4$ 

- $M_3$  gives a strong (positive or negative) response at the pixels where there is a relatively large change in gray level.

$$\xrightarrow{M_1}$$

$$\xrightarrow{M_2}$$

## Question 2

Original

Pixels with value  
0 or 1

Four connected components under 4-connectivity

Two connected components under 8-connectivity

### Question 3

100	150	200	200	200
100	20	200	200	200
100	20	200	200	200
100	20	200	200	200
100	150	200	200	200

$I_1$

100	150	200	200	200
100	150	30	200	200
100	150	30	200	200
100	150	30	200	200
100	150	200	200	200

$I_2$

100	150	200	200	200
100	85	115	200	200
100	85	115	200	200
100	85	115	200	200
100	150	200	200	200

$A = 0.5(I_1 + I_2)$

0	0	0	0	0
0	130	170	0	0
0	130	170	0	0
0	130	170	0	0
0	0	0	0	0

$B = |I_1 - I_2|$

100	150	200	200	200
100	98	98	200	200
100	98	98	200	200
100	98	98	200	200
100	150	200	200	200

$C = 0.4I_1 + 0.6I_2$

## Question 4

### Part (a)

The image is given by

$$f(x, y) = i(x, y) \times r(x, y) \quad (1)$$

$$= 512 \exp(-x^2 - y^2) \quad (2)$$

The magnitude of  $f(x, y)$  spans the range 0 to 512; this range is quantised to 256 levels

$$0, 1, 2, \dots, 255$$

The scheme for uniform quantisation is shown in Figure 4a. The quantisation interval is

$$\Delta G = \frac{512}{256} = 2 \quad (3)$$

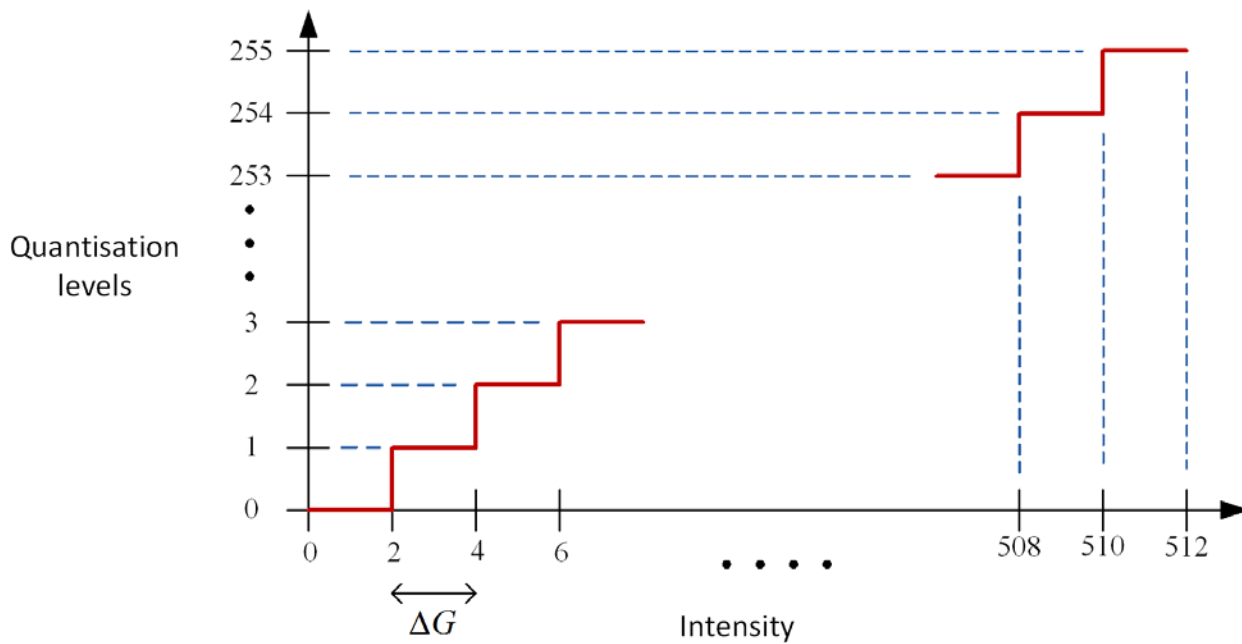


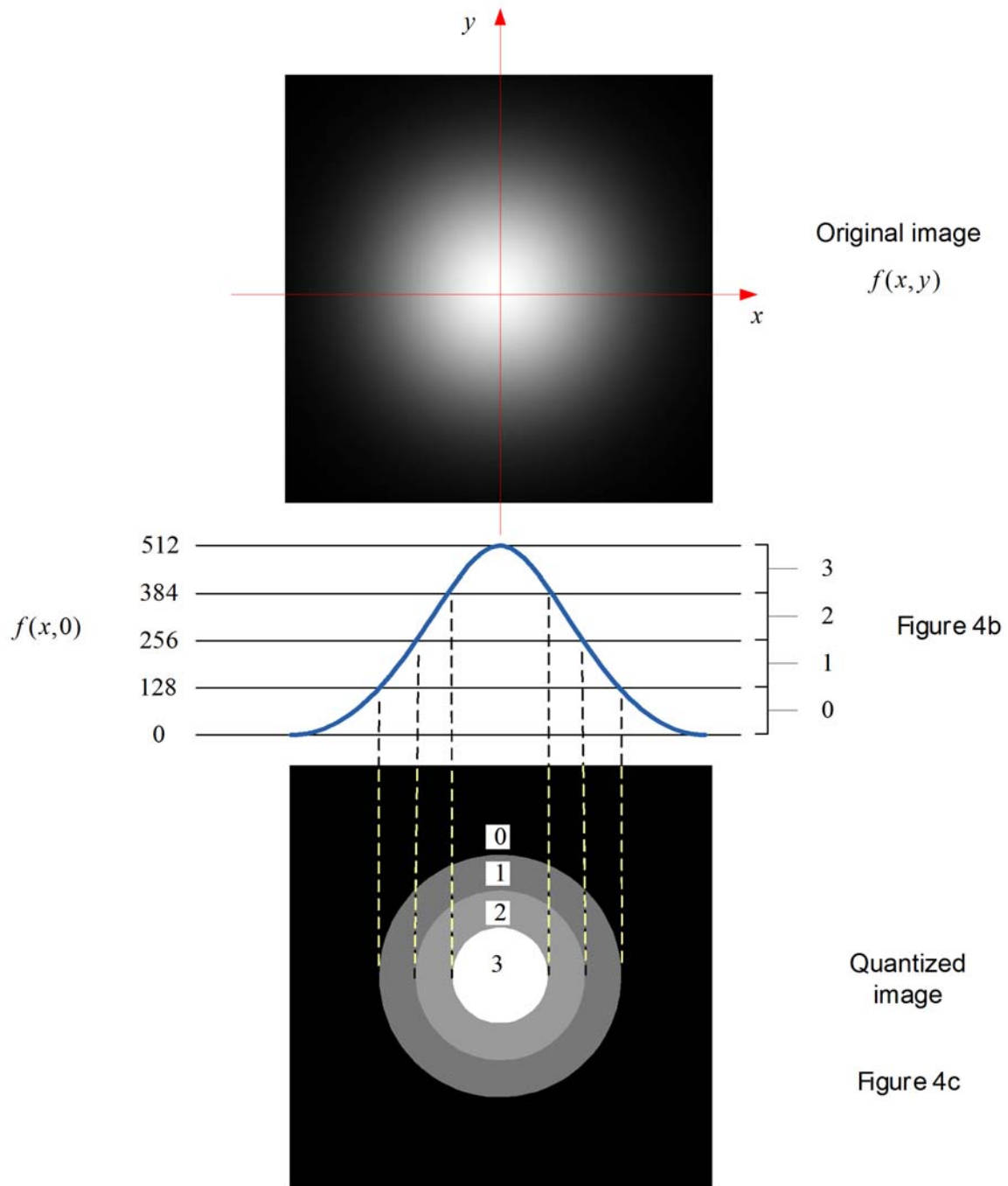
Figure 4a

## Part (b)

The use of two bits produces 4 gray levels, 0, 1, 2 and 3, spanning the intensity range 0 to 512. A cross section of  $f(x, y)$  through  $(0, 0)$  is shown in Figure 4b. The equation describing the profile can be obtained from Eq.(2):

$$f(x, 0) = 512 \exp(-x^2) \quad (4)$$

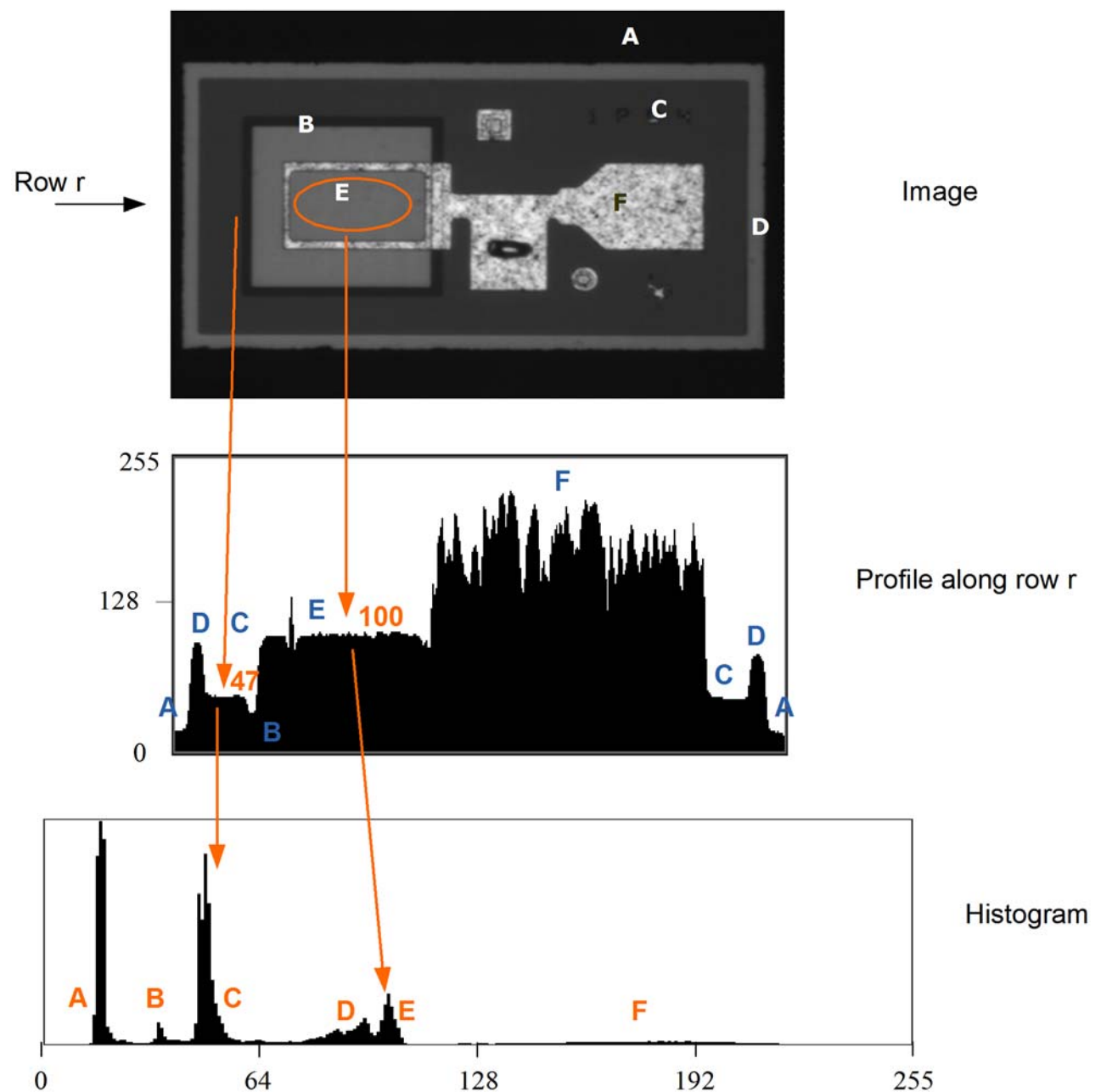
A sketch of the quantised image is shown in Figure 4c. Note the false contours.



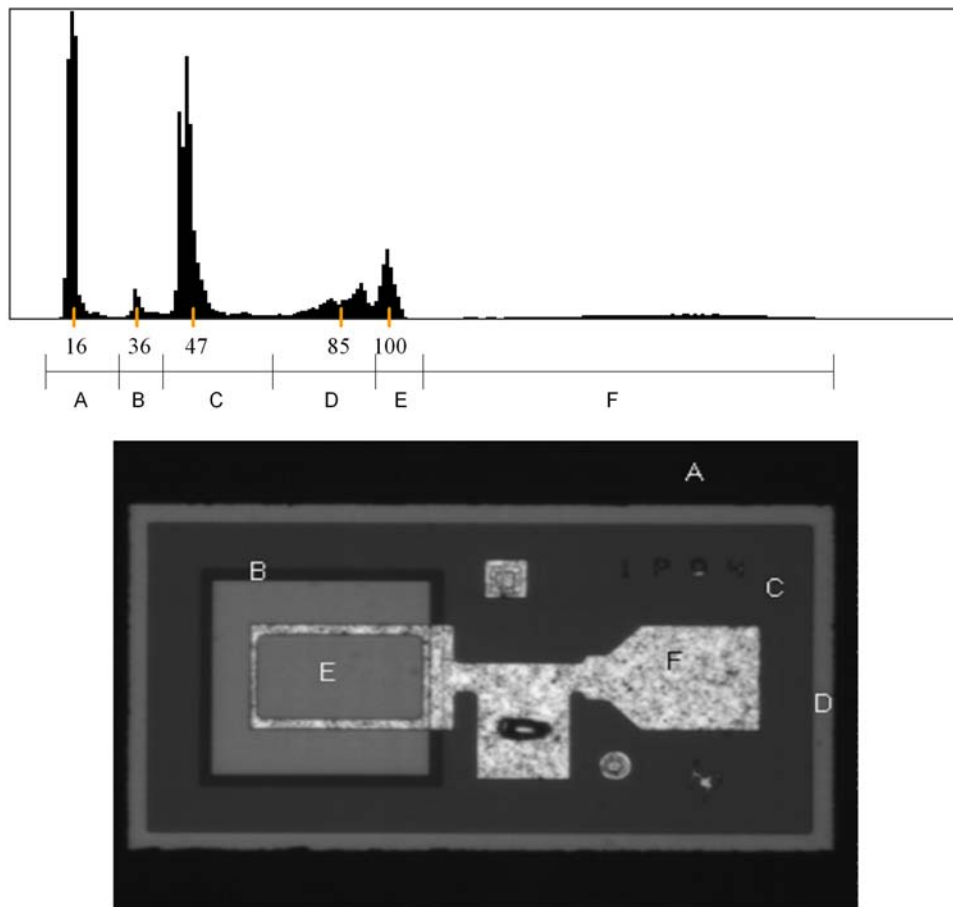
## Question 5

### Part (a)

Six distinct modes (A to F) can be readily seen in the histogram. By examining the gray levels, it is possible to identify the region in the image corresponding to each mode. For example, from the profile, we see that the small square region has a gray level of about 100, which corresponds to mode E.

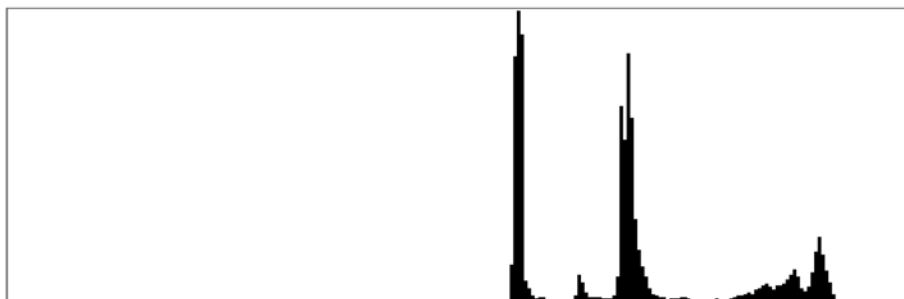


We can relate all the histogram modes to the corresponding regions of the image in a similar manner.



### Part (b)

The histogram is shifted to the right by 128. Note that clipping occurs at gray level 255.



## Question 6

This histogram is

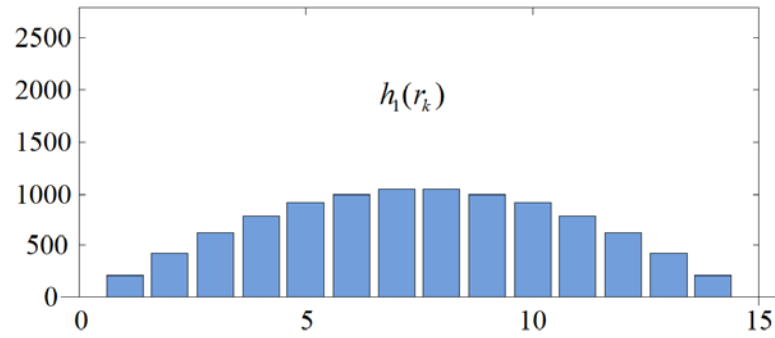
$$h_1(r_k) = A \sin\left(\frac{\pi}{15}k\right) \quad (5)$$

The summation  $\sum_{k=0}^{15} h_1(r_k)$  is equal to the number of pixels in the image, 10,000. Hence

$$\sum_{k=0}^{15} h_1(r_k) = A \sum_{k=0}^{15} \sin\left(\frac{\pi}{15}k\right) = 10,000 \quad (6)$$

from which we obtain  $A = 1051$ .

$r_k$	$h_1(r_k)$
0	0
1	219
2	428
3	618
4	781
5	910
6	1000
7	1045
8	1045
9	1000
10	910
11	781
12	618
13	428
14	219
15	0





To apply the transformation, we first have to determine the mapping of the individual gray levels. This is done by using the transformation function, which is given by the equations of the two straight-line segments,  $AB$  and  $BC$ :

$$\begin{aligned} AB : \quad s_k &= \frac{2}{5}r_k & 0 \leq r_k \leq 10 \\ BC : \quad s_k &= \frac{11}{5}r_k - 18 & 10 \leq r_k \leq 15 \end{aligned} \quad (7)$$

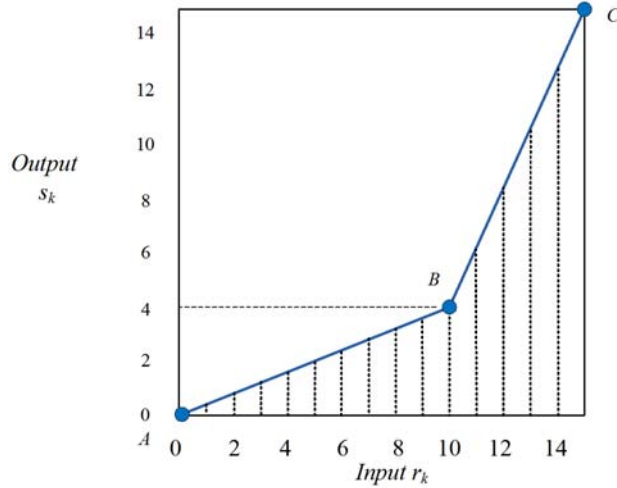
The histogram ( $h_2$ ) of the transformed image is then readily obtained, bearing in mind that the gray levels  $r$  and  $s$  take integer values. For example,

$$\begin{aligned} r_k = 0 &\rightarrow s_k = 0 \\ r_k = 1 &\rightarrow s_k = 0.4 \rightarrow 0 \quad (\text{nearest integer}) \end{aligned}$$

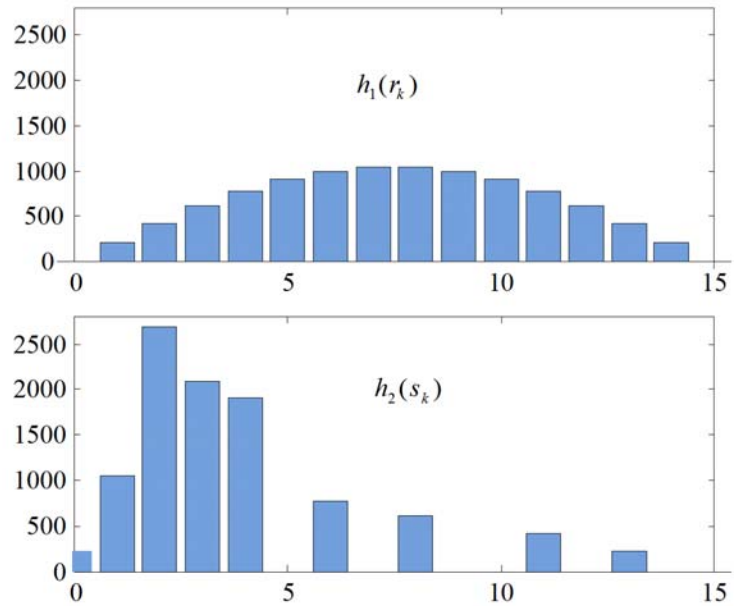
Hence, the number of pixels in output gray level  $s_k = 0$  is

$$h_2(0) = h_1(0) + h_1(1) = 0 + 219 = 219$$

We apply the transformation to all the gray levels and tabulate the results.



$r_k$	$h_1(r_k)$	$s_k$	$h_2(s_k)$
0	0	0	219
1	219	0	
2	428	1	1046
3	618	1	
4	781	2	2691
5	910	2	
6	1000	2	
7	1045	3	2090
8	1045	3	
9	1000	4	1910
10	910	4	
11	781	6	781
12	618	8	618
13	428	11	428
14	219	13	219
15	0	15	0



The  $n$ th moment of  $r_k$  about the mean is defined as

$$\mu_n = \sum_{k=0}^{15} (r_k - m)^n p(r_k) \quad (8)$$

where the mean is

$$m = \sum_{k=0}^{15} r_k p(r_k)$$

For example,

$$\mu_2 = (r_0 - m)^2 p(r_0) + (r_1 - m)^2 p(r_1) + \dots + (r_{15} - m)^2 p(r_{15}) \quad (9)$$

Applying the equations to the input and output *normalized histograms*,

	<u>Input image</u>	<u>Output image</u>
$m$	7.50	3.75
$\mu_2$	10.49	7.69
$\mu_3$	0	34.3

- The pixels in the output image are generally concentrated in the lower gray levels; hence the mean is lower and the image appears darker overall.
- The output image has a lower contrast (smaller  $\mu_2$ ) due to the concentration of pixels about gray levels 2 and 3.
- $\mu_3$  is called skewness, which is a measure of the lack of symmetry, with  $\mu_3 = 0$  indicating perfect symmetry. Histogram  $h_2$  is asymmetrical, hence its  $\mu_3 \neq 0$ , unlike histogram  $h_1$ .

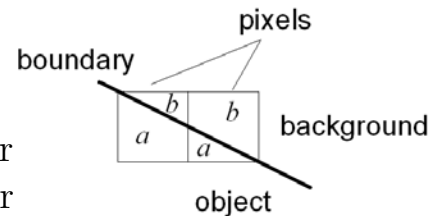
## Question 7

### Part (a)

The intensity,  $I$ , of a pixel depends on the proportion of the object area ( $a$ ) to the background area ( $b$ ) in a photoelement. We have

$$I = \frac{a \times 80 + b \times 20}{a + b}$$

It is easy to see that the ratio  $a : b$  is either  $3 : 1$  or  $1 : 3$ . Hence we obtain the image of Figure 7a. After thresholding at 50, we obtain the binary image of Figure 7b.



### Part (b)

The result is shown in Figure 7c. It should be noted that the two triangles do not have the same shape. Sampling at higher resolutions will lead to a more accurate representation of the object.

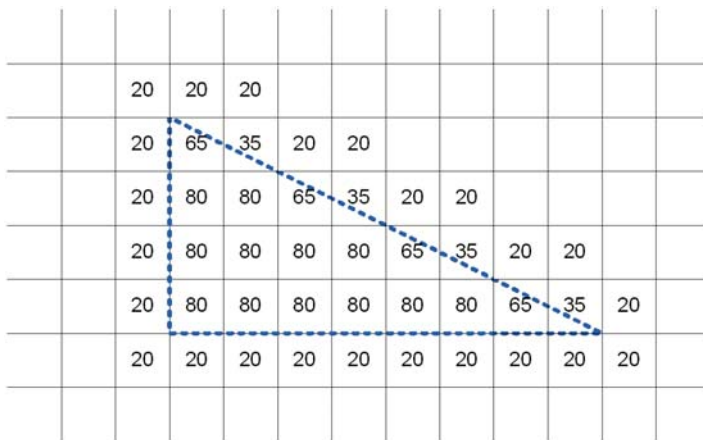


Figure 7a

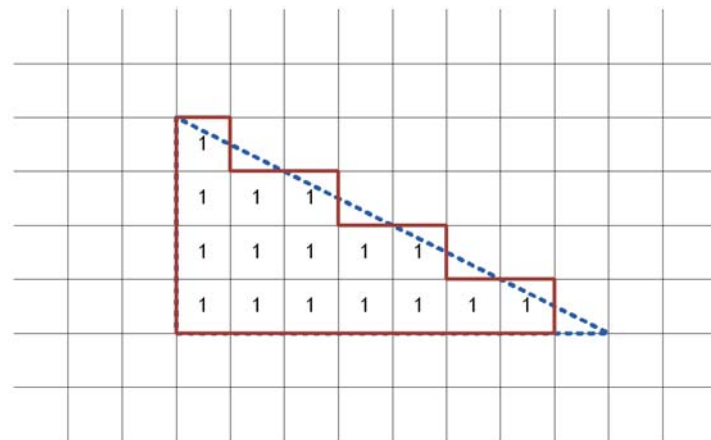


Figure 7b

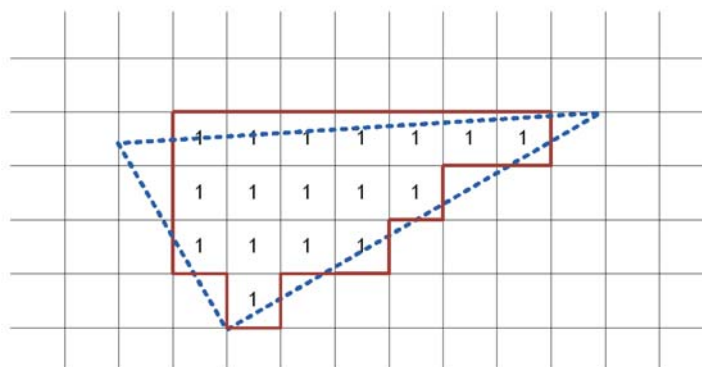


Figure 7c