

NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
MA2214 COMBINATORIAL ANALYSIS

TUTORIAL 7

SEMESTER II, AY 2010/2011

1. Prove that in any company of people there are two people who know the same number of people in that company. (Assume that knowing is a symmetric relation. If A knows B, then B knows A.)
2. In a company (of more than 3 people), the employees altogether speaks four languages, English, Chinese, Malay and Tamil. There is no language among the four that is spoken by every employee. However, for any subset of three employees, there is at least one language that is common to them. Prove that each employee must speak at least three languages.
3. Prove that $R(3, 4) = 9$.
4. How many positive integers not larger than 1000 are co-prime to both 7 and 8?
5. A total of 150 students read at least one of three modules algebra, analysis and combinatorics. The enrolment for the respective modules are 103 for algebra, 102 for analysis and 94 for combinatorics. At most 84 read both algebra and analysis, at most 65 read both algebra and combinatorics, and at most 75 read both analysis and combinatorics. Find the maximum number of students who read all three modules.
6. A year is a leap year if it is either (i) a multiple of 4 but not a multiple of 100; or (ii) a multiple of 400. For example, 2000 was a leap year, 2012 will be a leap year but not 2100. Find the number of leap years between 1000 and 3000 inclusive.
7. Three black, four blue and five red balls are arranged in a line. Find the total number of possible arrangements if balls of the same colour do not form a single block.
8. (Bonus question:)
Prove that in a set of 11 positive integers, none of which are divisible by 20, there are always two integers a and b so that at least one of $a + b$ and $a - b$ is divisible by 20.

Answers

4. 429
6. 485

7. 25762