Problem 5.40 The rectangular loop shown in Fig. P5.40 is coplanar with the long, straight wire carrying the current $I = 20$ A. Determine the magnetic flux through the loop.

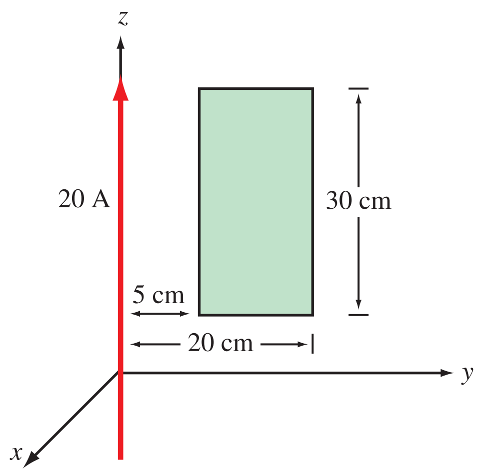

Figure P5.40: Loop and wire arrangement for Problem 5.40.

Solution: The field due to the long wire is, from Eq. (5.30),

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}}\,\frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}}\,\frac{\mu_0 I}{2\pi y}\,,$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}}$ becomes $-\hat{\mathbf{x}}$ and r becomes y.

The flux through the loop is along $-\hat{\mathbf{x}}$, and the magnitude of the flux is

$$\begin{aligned}
\Phi = \int_S \mathbf{B}\cdot d\mathbf{s} &= \frac{\mu_0 I}{2\pi}\int_{5\text{ cm}}^{20\text{ cm}} -\frac{\hat{\mathbf{x}}}{y}\cdot -\hat{\mathbf{x}}\,(30\text{ cm}\times dy) \\
&= \frac{\mu_0 I}{2\pi}\times 0.3\int_{0.05}^{0.2}\frac{dy}{y} \\
&= \frac{0.3\,\mu_0}{2\pi}\times 20\times\ln\left(\frac{0.2}{0.05}\right) = 1.66\times 10^{-6} \quad \text{(Wb)}.
\end{aligned}$$

Problem 6.6 The square loop shown in Fig. P6.6 is coplanar with a long, straight wire carrying a current

$$I(t) = 5\cos(2\pi \times 10^4 t) \quad \text{(A)}.$$

(a) Determine the emf induced across a small gap created in the loop.

(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

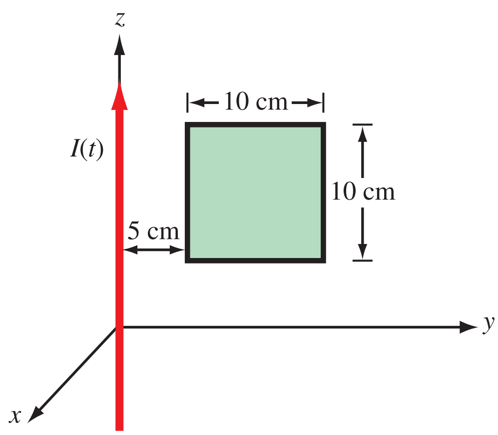

Figure P6.6: Loop coplanar with long wire (Problem 6.6).

Solution:

(a) The magnetic field due to the wire is

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}}\,\frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$ and $r = y$. The flux passing through the loop is

$$\begin{aligned}
\Phi = \int_S \mathbf{B}\cdot d\mathbf{s} &= \int_{5\text{ cm}}^{15\text{ cm}} \left(-\hat{\mathbf{x}}\,\frac{\mu_0 I}{2\pi y}\right)\cdot[-\hat{\mathbf{x}}\,10\text{ (cm)}]\;dy \\
&= \frac{\mu_0 I \times 10^{-1}}{2\pi}\ln\frac{15}{5} \\
&= \frac{4\pi \times 10^{-7} \times 5\cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1 \\
&= 1.1 \times 10^{-7}\cos(2\pi \times 10^4 t) \quad \text{(Wb)}.
\end{aligned}$$

$$
\begin{aligned}
V_{\text{emf}} = -\frac{d\Phi}{dt} &= 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7} \\
&= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t) \quad \text{(V)}.
\end{aligned}
$$

(b)

$$
I_{\text{ind}} = \frac{V_{\text{emf}}}{4+1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 1.38 \sin(2\pi \times 10^4 t) \quad \text{(mA)}.
$$

At $t = 0$, $\mathbf{B}$ is a maximum, it points in $-\hat{\mathbf{x}}$-direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be CCW when looking down on the loop, as shown in the figure.

Problem 6.11 The loop shown in P6.11 moves away from a wire carrying a current $I_1 = 10$ A at a constant velocity $\mathbf{u} = \hat{\mathbf{y}}7.5$ (m/s). If $R = 10\ \Omega$ and the direction of I_2 is as defined in the figure, find I_2 as a function of y_0, the distance between the wire and the loop. Ignore the internal resistance of the loop.

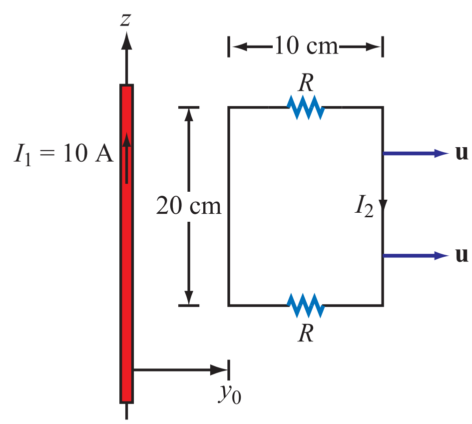

Figure P6.11: Moving loop of Problem 6.11.

Solution: Assume that the wire carrying current I_1 is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{\text{emf}}/2R$, where the induced voltage is due to motion of the loop and is given by Eq. (6.26):

$$V_{\text{emf}} = V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$

The magnetic field $\mathbf{B}$ is created by the wire carrying I_1. Choosing $\hat{\mathbf{z}}$ to coincide with the direction of I_1, Eq. (5.30) gives the external magnetic field of a long wire to be

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} \ .$$

For positive values of y_0 in the y-z plane, $\hat{\mathbf{y}} = \hat{\mathbf{r}}$, so

$$\mathbf{u} \times \mathbf{B} = \hat{\mathbf{y}}|\mathbf{u}| \times \mathbf{B} = \hat{\mathbf{r}}|\mathbf{u}| \times \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} = \hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r} \ .$$

Integrating around the four sides of the loop with $d\mathbf{l} = \hat{\mathbf{z}}\ dz$ and the limits of integration chosen in accordance with the assumed direction of I_2, and recognizing that only the two sides without the resistors contribute to $V^{\mathrm{m}}_{\mathrm{emf}}$, we have

$$\begin{aligned}
V^{\mathrm{m}}_{\mathrm{emf}} &= \int_0^{0.2} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r} \right) \Bigg|_{r=y_0} \cdot (\hat{\mathbf{z}}\, dz) + \int_{0.2}^{0} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r} \right) \Bigg|_{r=y_0+0.1} \cdot (\hat{\mathbf{z}}\, dz) \\
&= \frac{4\pi \times 10^{-7} \times 10 \times 7.5 \times 0.2}{2\pi} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \\
&= 3 \times 10^{-6} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \quad \text{(V)},
\end{aligned}$$

and therefore

$$I_2 = \frac{V^{\mathrm{m}}_{\mathrm{emf}}}{2R} = 150 \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \quad \text{(nA)}.$$

Problem 6.15 A coaxial capacitor of length $l = 6$ cm uses an insulating dielectric material with $\varepsilon_r = 9$. The radii of the cylindrical conductors are 0.5 cm and 1 cm. If the voltage applied across the capacitor is

$$V(t) = 50\sin(120\pi t) \quad \text{(V)}$$

what is the displacement current?

Solution:

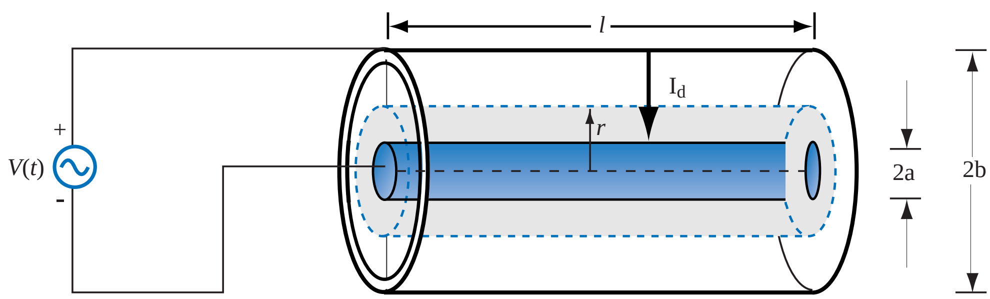

Figure P6.15:

To find the displacement current, we need to know **E** in the dielectric space between the cylindrical conductors. From Eqs. (4.114) and (4.115),

$$\mathbf{E} = -\hat{\mathbf{r}} \frac{Q}{2\pi\varepsilon r l},$$
$$V = \frac{Q}{2\pi\varepsilon l} \ln\left(\frac{b}{a}\right).$$

Hence,

$$\mathbf{E} = -\hat{\mathbf{r}} \frac{V}{r\ln\left(\frac{b}{a}\right)} = -\hat{\mathbf{r}} \frac{50\sin(120\pi t)}{r\ln 2} = -\hat{\mathbf{r}} \frac{72.1}{r} \sin(120\pi t) \quad \text{(V/m)},$$

$$\begin{aligned}\mathbf{D} &= \varepsilon\mathbf{E} \\ &= \varepsilon_r\varepsilon_0\mathbf{E} \\ &= -\hat{\mathbf{r}}\, 9 \times 8.85 \times 10^{-12} \times \frac{72.1}{r} \sin(120\pi t) \\ &= -\hat{\mathbf{r}} \frac{5.75 \times 10^{-9}}{r} \sin(120\pi t) \quad \text{(C/m}^2\text{)}.\end{aligned}$$

The displacement current flows between the conductors through an imaginary cylindrical surface of length l and radius r. The current flowing from the outer conductor to the inner conductor along $-\hat{\mathbf{r}}$ crosses surface **S** where

$$\mathbf{S} = -\hat{\mathbf{r}}\, 2\pi r l.$$

Hence,

$$\begin{aligned} I_{\text{d}} = \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{S} &= -\hat{\mathbf{r}} \frac{\partial}{\partial t} \left(\frac{5.75 \times 10^{-9}}{r} \sin(120\pi t) \right) \cdot (-\hat{\mathbf{r}}\, 2\pi r l) \\ &= 5.75 \times 10^{-9} \times 120\pi \times 2\pi l \cos(120\pi t) \\ &= 0.82 \cos(120\pi t) \quad (\mu\text{A}). \end{aligned}$$

Alternatively, since the coaxial capacitor is lossless, its displacement current has to be equal to the conduction current flowing through the wires connected to the voltage sources. The capacitance of a coaxial capacitor is given by (4.116) as

$$C = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)} .$$

The current is

$$I = C \frac{dV}{dt} = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)} [120\pi \times 50 \cos(120\pi t)] = 0.82 \cos(120\pi t) \quad (\mu\text{A}),$$

which is the same answer we obtained before.

Problem 6.16 The parallel-plate capacitor shown in Fig. P6.16 is filled with a lossy dielectric material of relative permittivity ε_r and conductivity σ. The separation between the plates is d and each plate is of area A. The capacitor is connected to a time-varying voltage source $V(t)$.

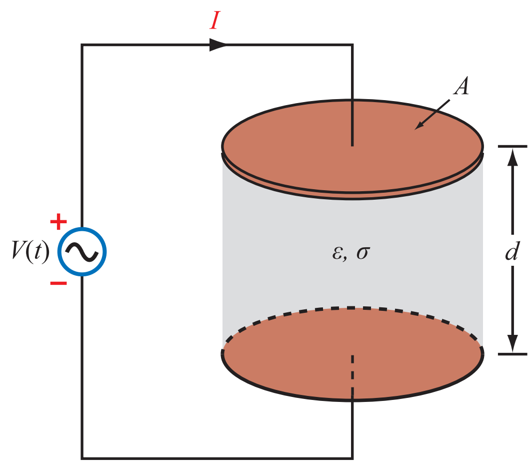

Figure P6.16: Parallel-plate capacitor containing a lossy dielectric material (Problem 6.16).

(a) Obtain an expression for I_c, the conduction current flowing between the plates inside the capacitor, in terms of the given quantities.

(b) Obtain an expression for I_d, the displacement current flowing inside the capacitor.

(c) Based on your expressions for parts (a) and (b), give an equivalent-circuit representation for the capacitor.

(d) Evaluate the values of the circuit elements for $A = 4$ cm^2, $d = 0.5$ cm, $\varepsilon_r = 4$, $\sigma = 2.5$ (S/m), and $V(t) = 10\cos(3\pi \times 10^3 t)$ (V).

Solution:

(a)

$$R = \frac{d}{\sigma A}, \qquad I_c = \frac{V}{R} = \frac{V\sigma A}{d}.$$

(b)

$$E = \frac{V}{d}, \qquad I_d = \frac{\partial D}{\partial t} \cdot A = \varepsilon A \frac{\partial E}{\partial t} = \frac{\varepsilon A}{d} \frac{\partial V}{\partial t}.$$

(c) The conduction current is directly proportional to V, as characteristic of a resistor, whereas the displacement current varies as $\partial V/\partial t$, which is characteristic of a capacitor. Hence,

$$R = \frac{d}{\sigma A} \qquad \text{and} \qquad C = \frac{\varepsilon A}{d}.$$

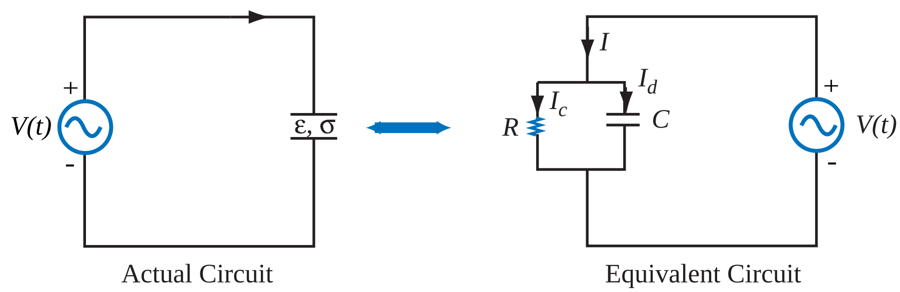

Figure P6.16: (a) Equivalent circuit.

(d)

$$R = \frac{0.5 \times 10^{-2}}{2.5 \times 4 \times 10^{-4}} = 5\ \Omega,$$

$$C = \frac{4 \times 8.85 \times 10^{-12} \times 4 \times 10^{-4}}{0.5 \times 10^{-2}} = 2.84 \times 10^{-12}\ \text{F}.$$