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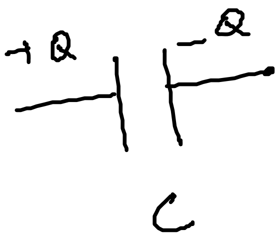
1. A voltage of 50V appears across a 10uF capacitor.
a) Determine the magnitude of net charge stored on each plate and total net charge on both the plates.
b) Calculate the energy stored in the capacitor.
c) If the capacitor is discharged by a steady current of 100uA. How long does it take to discharge the capacitor to 0V?

Solution :



$Q = CV$

$C = 10\,\mu F, \quad V = 50$

$Q = 10 \times 10^{-6} \times 50$

$= 5 \times 10^{-4}\ C.$

a) The net charge stored on each plate is 5×10^{-4} coulomb. One plate has positive charge and the other plate has negative charge.

The net charge on both the plates equals zero.

(b) Energy stored on the capacitor $= \frac{1}{2} CV^2$

$= \frac{1}{2} \times 10 \times 10^{-6} \times 50^2 = 12.5\ mJ.$

(c) If discharge rate $\frac{dQ}{dt} = -100 \times 10^{-6}\ A$

To discharge to 0V i.e. $Q_{final} = C \cdot V = 0$

Time taken $= \frac{Q_{final} - Q_{initial}}{dQ/dt} = \frac{0 - 5\times10^{-4}}{-100 \times 10^{\cancel{-4}\ -6}}$

$= 5$ sec.

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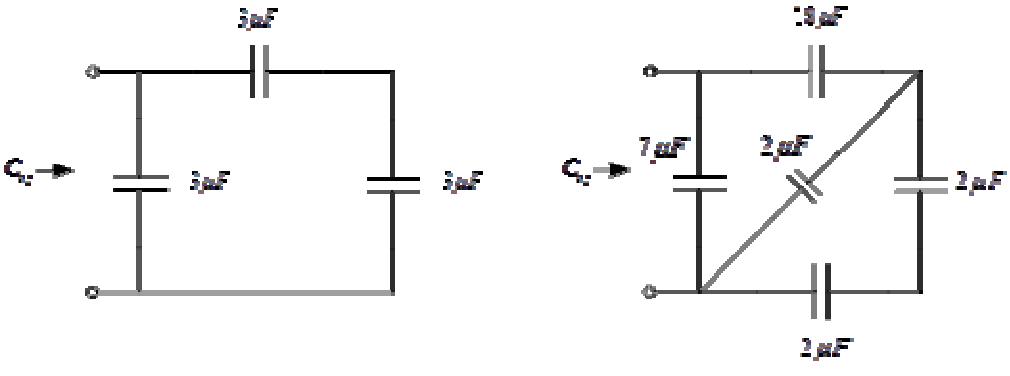
2. Find the equivalent capacitance for each of the circuits shown in the figure.

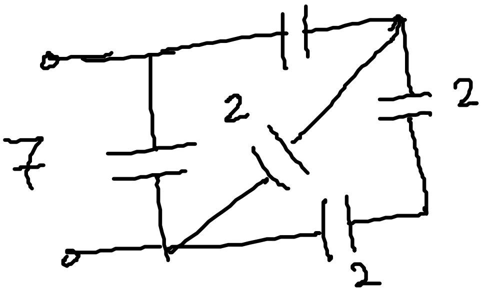


(a) (b)

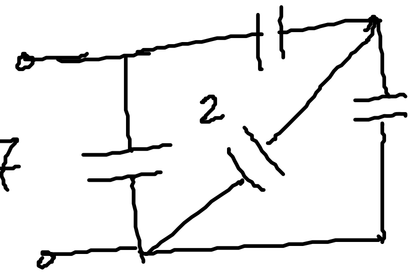
Soution:

(a) $C_{eq} = 3 + \frac{3\times 3}{3+3} = 3 + \frac{3}{2} = 4.5$ /

(b)

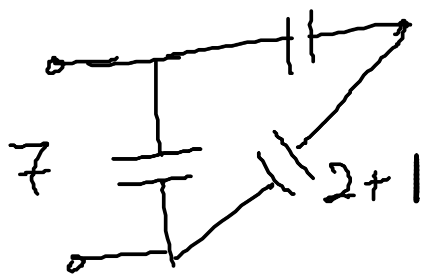


$\equiv$

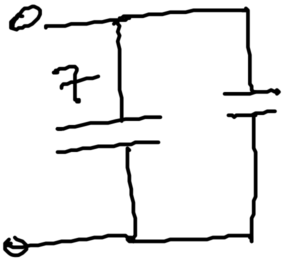


$\frac{2\times 2}{2+2} = 1$

$\equiv$

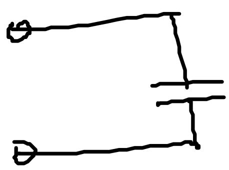


$2 + 1 = 3$



$\frac{18\times 3}{18+3}$ $\equiv$

$\equiv$



$7 + \frac{54}{21} = 9.57\ \mu F.$

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3. A constant voltage of 30V is applied to a 60 mH inductance. The current in the inductor was zero at t=0.
a) At what time does the current reach 2A?
b) What is energy stored in the inductor when the current is 2A?

Solution :

For inductor $V = L \frac{di}{dt}$.

If $V = 30V$, $L = 60 \times 10^{-3}$ H

Then $\frac{di}{dt} = \frac{V}{L} = \frac{30}{60 \times 10^{-3}} = 0.5 \times 10^{3}$ A/sec

a) Given $i_L = 0$ at $t = 0$.

Time taken for i_L to reach 2A =

$$= \frac{2}{di/dt} = \frac{2}{0.5 \times 10^{3}}$$

$$= 4 \text{ mS.}$$

b) Energy stored in the inductor $= \frac{1}{2} L i^2$

$$= \frac{1}{2} \times 60 \times 10^{-3} \times 2^2$$

$$= 120 \times 10^{-3} \text{ J} = 120 \text{ mJ.}$$

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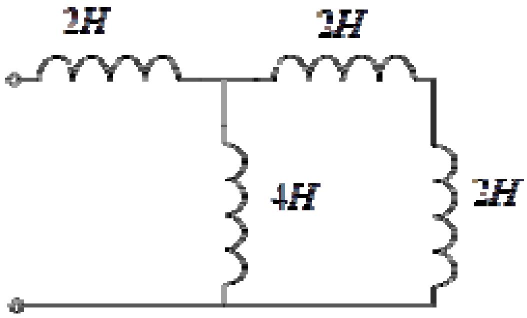
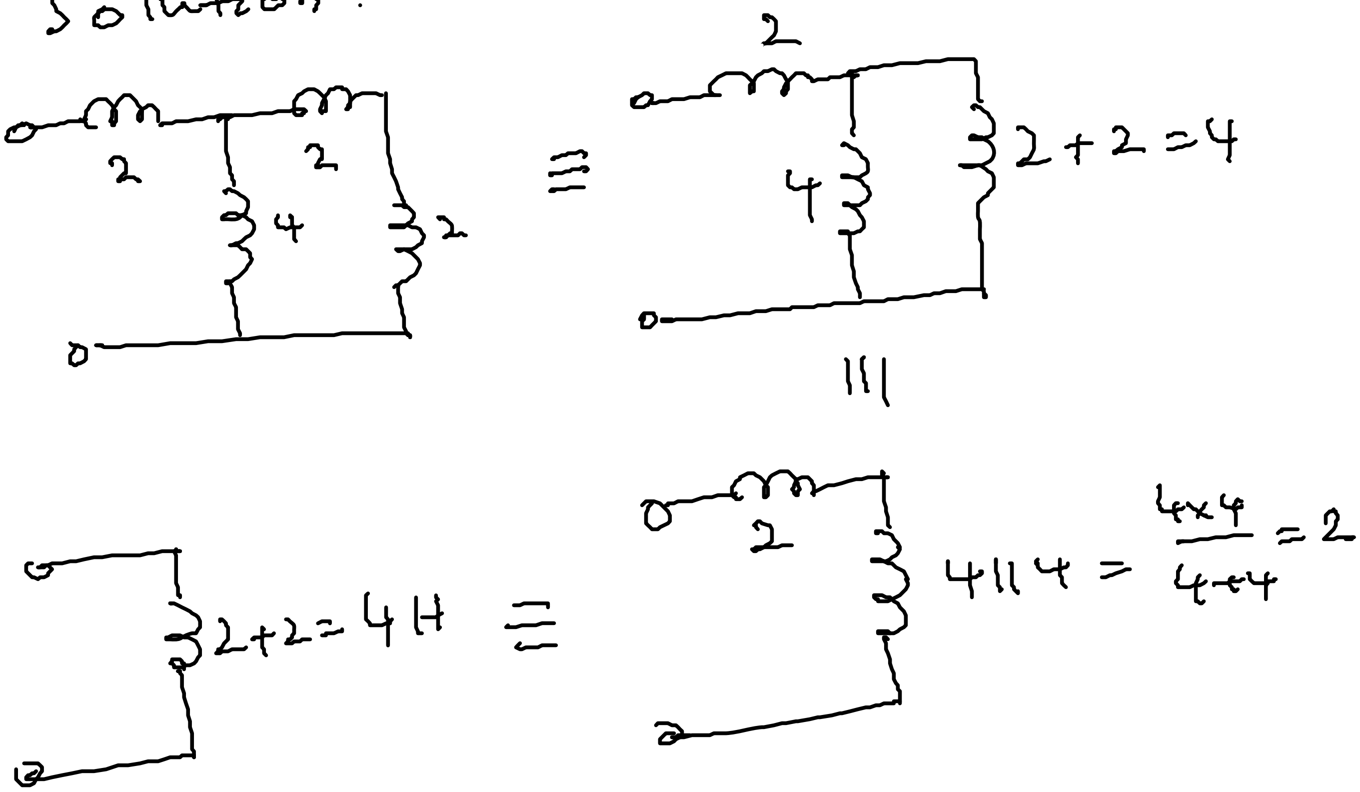
4. Find the equivalent inductance of the circuit below.



Solution :



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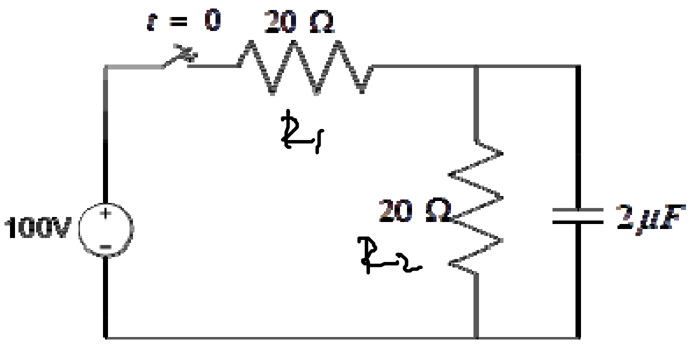
5. If the switch in the circuit is closed at t=0,
   i) Determine the current flowing through the resistors and the capacitor when t=0+ (immediately after the switch is closed).
   ii) What will be the current flow under steady state condition?
   iii) Determine the voltage across the capacitor under steady state condition.
   iv) Find an expression for the capacitor voltage as a function of time t>0.

   Assume that the capacitor is initially uncharged.



Solution

(i) As capacitor voltage can not change instantaneously $v_C(0^-) = v_C(0^+)$

$v_C(0^-)$ can be obtained by the DC analysis before switch was closed.

As can be seen $v_C(0^-) = 0$ before the switch was closed.

$v_C(0^+)$ i.e. capacitor voltage immediately after switch is closed will be zero.

∴ current in $R_2 = \frac{v_C(0^+)}{20} = 0\ A$.

current in $R_1 = \frac{100 - v_C(0^+)}{20} = 5A$.
$= i_C(0^+)$.

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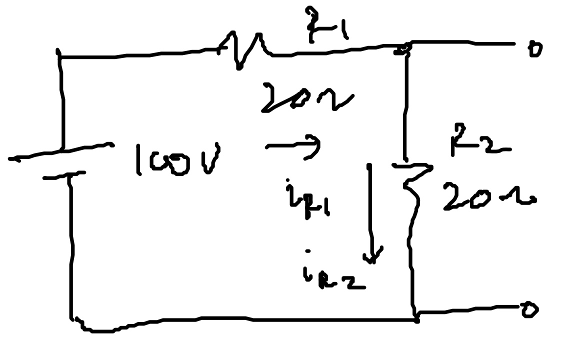
(ii) At steady-state, the capacitor will be open circuited.



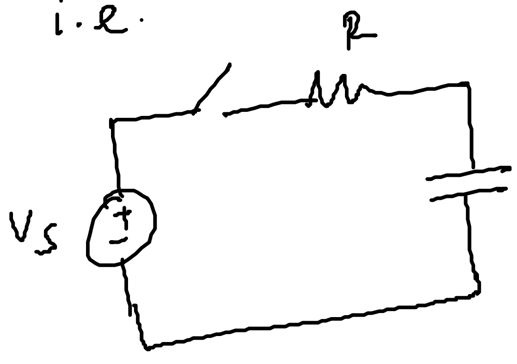
$v_C(\infty) = 100 \times \frac{20}{20+20} = 50V.$ (iii)

$i_C(\infty) = 0$

$i_{R1} = i_{R_2} = \frac{100}{20+20} = 2.5 A.$

(iii) $V_C(\infty) = 100 \times \frac{20}{20+20} = 50V$

(iv) To find the capacitor voltage as a function of time t i.e. $v_C(t)$ at $t = 0^+$, we shall put it in the standard form i.e.



where V_S is the Thevenin's voltage and R is the Thevenin's resistance with C as the Load.

$V_S = V_t = 50V.$

$R = R_t = 20 \| 20 = 10\,\Omega.$

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$$\tau = R \times C = 10 \times 2 \times 10^{-6} = 20 \times 10^{-6}\ \mu S.$$

$$v_C(0^+) = v_C(0^-) = 0\ V$$

$$v_C(\infty) = V_S = 50\ V$$

$$v_C(t) = v_C(0) \cdot e^{-t/\tau} + v_C(\infty) \cdot \left(1 - e^{-t/\tau}\right)$$

$$= 50\left(1 - e^{-t/20\times10^{-6}}\right) V.$$

Please note that we use the Thevenin equivalent to put it in the general form. then we can use the general solution.

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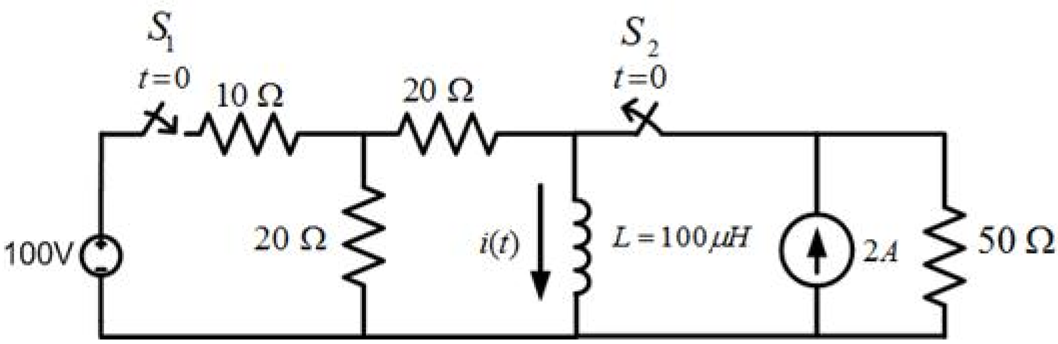
6. For the circuit given below, switch S2 was closed for a long time before t=0. At t=0, the switch S1 is closed and S2 is opened.
a. Find the inductor current i(t) at t=0+.
b. Find the time constant for t>=0.
c. Find an expression for i(t), and sketch the function.
d. Find i(t) for each of the following values of t zero, the time constant, twice the time constant, five times the time constant and ten times the time constant.

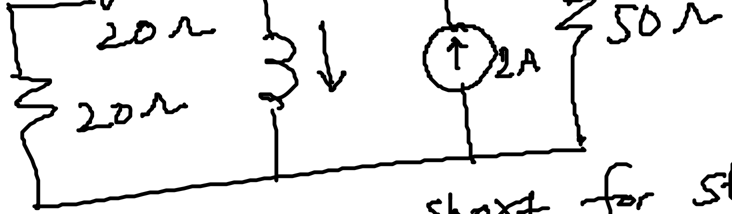


Solution

① To find inductor current $t=0^+$.

$i_L(0^+) = i_L(0^-)$ as inductor current can not change instantaneously.

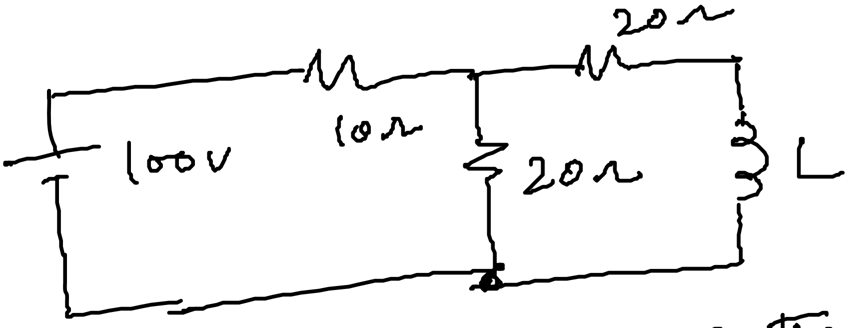
Before $t=0$, with S_1 open and S_2 closed, the circuit is:



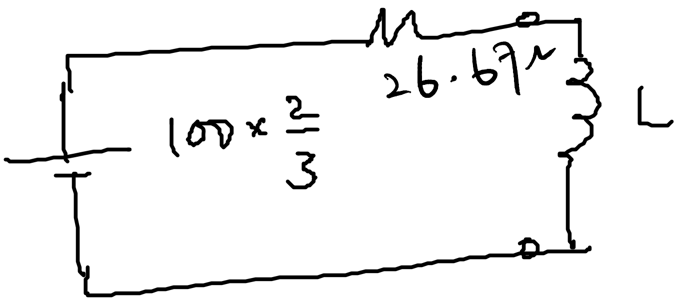
As inductor acts as a short for steadystate, the inductor current = 2A.

$i_L(0^+) = i_L(0^-) = 2A$.

After $t=0$, when S_1 is closed and S_2 is opened, the circuit becomes:



Thevenin equivalent of the circuit will be



which is in the standard form.

$$\tau = \frac{L}{R} = \frac{100 \times 10^{-6}}{26.67}$$
$$= 3.75 \mu S.$$

$$i_L(0^+) = 2A$$
$$i_L(\infty) = \frac{100 \times 0.5}{20}$$ as L acts as a short circuit in steady-state
$$= 2.5A$$

$$i_L(t) = i_L(0) \cdot e^{-t/\tau} + i_L(\infty) \cdot (1 - e^{-t/\tau})$$
$$= 2e^{-t/3.75\times10^{-6}} + 2.5\left(1 - e^{-t/3.75\times10^{-6}}\right) A.$$

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(d). At $= \tau$, $i_L = 2 \times e^{-1} + 2.5\,(1 - e^{-1})$
$= 2.316$ A

At $t = 2\tau$, $i_L = 2 \times e^{-2} + 2.5\,(1 - e^{-2})$
$= 2.432$ A

At $t = 5\tau$, $i_L = 2 \times e^{-5} + 2.5\,(1 - e^{-5})$
$= 2.497$ A

Change 3.33 to 2.5

At $t = 10\tau$, $i_L = 2 \times e^{-10} + \cancel{3.33}\,(1 - e^{-10})$
$= 2.4998$ A