NFA to DFA Conversion

October 19, 2012

Converting an NFA to a DFA

Given:

A non-deterministic finite state machine (NFA)

Goal:

Convert to an equivalent <u>deterministic</u> finite state machine (DFA)

Why?

Faster recognizer!

Approach:

Consider simulating a NFA.

Work with sets of states.

IDEA: Each <u>state</u> in the DFA will correspond to a <u>set of</u> NFA states.

Worst-case:

There can be an exponential number $O(2^N)$ of sets of states.

The DFA can have exponentially many more states than the NFA ... but this is rare.

NFA to DFA

Input: A NFA
$$S = States = \{ s_0, s_1, ..., s_N \} = S_{NFA}$$

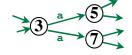
$$\delta = \text{Move function} = \text{Move}_{NFA}$$
Move'(S, a) \rightarrow Set of states

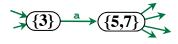
Output: A DFA
$$S = States = \{?, ?, ..., ?\} = S_{DFA}$$

$$\delta = Move function = Move_{DFA}$$

Move(s, a) \rightarrow Single state from S_{DEA}

Each state in S_{DFA} will be a set of states from the NFA Main Idea: $S_{DFA} = \{ \{...\}, \{...\}, ..., \{...\} \}$





NFA

DFA

(The names of the states is arbitrary and can be changed later, if desired.)



Algorithm: Convert NFA to DFA

We'll use...

Move_{NFA} (S, a) the transition function from NFA

ε-Closure(s) where s is a single state from NFA

ε-Closure (S) where S is a set of states from NFA

We'll construct...

 S_{nFA} the set of states in the DFA

Initially, we'll set **S**_{DFA} to {}

Add \mathbf{X} to \mathbf{S}_{DFA} where \mathbf{X} is some set of NFA states

Example: "Add (3,5,7) to S_{DFA} "

We'll "mark" some of the states in the DFA.

Marked = "We've done this one" ($\sqrt{}$)

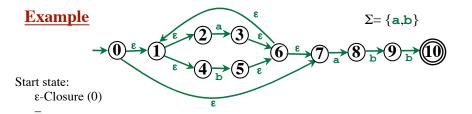
Unmarked = "Still need to do this one"

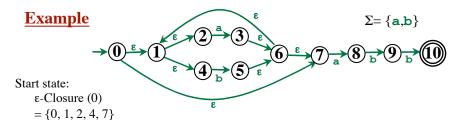
Move_{DFA} (T,b) The transition function from DFA

To add an edge to the growing DFA...

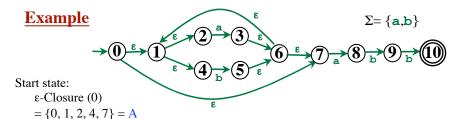
Set Move_{DFA}
$$(T,b)$$
 to S

...where **S** and **T** are sets of NFA states

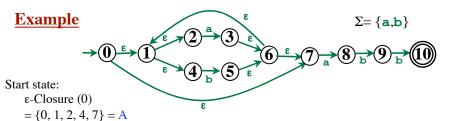




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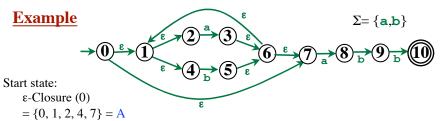
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$$Move_{DFA}(A, a)$$

=

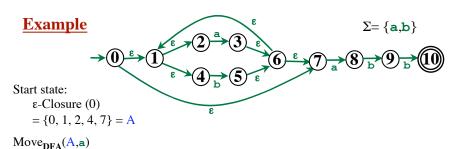
$$Move_{DFA}(A,b)$$



$$Move_{DFA}(A, a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

$$\mathsf{Move}_{DFA}({\color{red}A},\!{\color{blue}b})$$

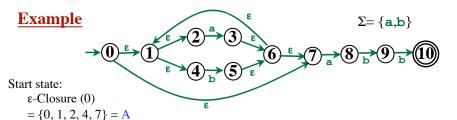


Move_{DFA}(A,b)

=

= ϵ -Closure (Move_{NFA}(A,a))

 $= \varepsilon$ -Closure ({3,8})



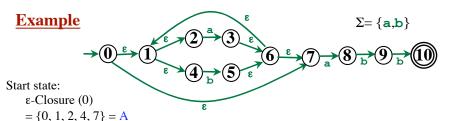
$Move_{DFA}(A,a)$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\}$$

$$Move_{DFA}(A,b)$$



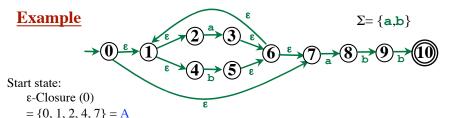
$$Move_{DFA}(A, a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

=
$$\varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$

$$Move_{DFA}(A,b)$$



$Move_{DFA}(A, a)$

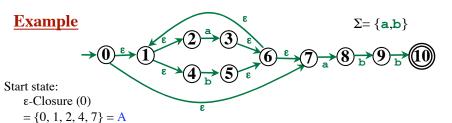
=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$

$Move_{DFA}(A,b)$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,b))



$$Move_{DFA}(A, a)$$

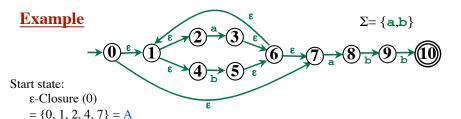
=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,b))

=
$$\varepsilon$$
-Closure ({5})



$$Move_{DFA}(A,a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

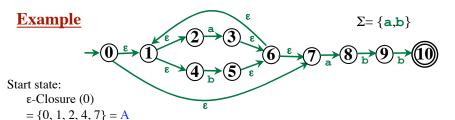
$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = B$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,b))

$$= \varepsilon$$
-Closure ({5})

$$= \{1,2,4,5,6,7\} = C$$



$Move_{DFA}(A,a)$

=
$$\varepsilon$$
-Closure (Move_{NFA}(A,a))

=
$$\varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$

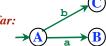
$Move_{DFA}(A,b)$

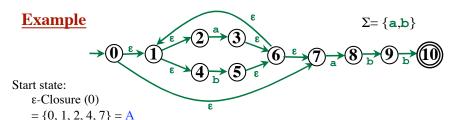
=
$$\varepsilon$$
-Closure (Move_{NFA}(A,b))

=
$$\varepsilon$$
-Closure ({5})

$$= \{1,2,4,5,6,7\} = \mathbb{C}$$

So far:





$Move_{DFA}(A,a)$

= ε -Closure (Move_{NFA}(A,a))

= ε -Closure ({3,8})

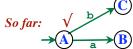
 $= \{1,2,3,4,6,7,8\} = B$

$Move_{DFA}(A,b)$

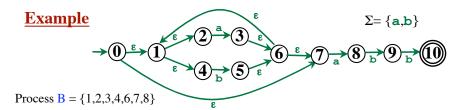
= ε -Closure (Move_{NFA}(A,b))

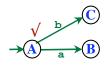
= ε -Closure ({5})

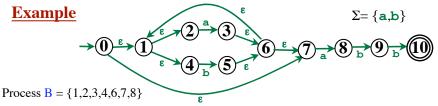
 $= \{1,2,4,5,6,7\} = C$

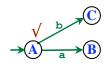


A is now done; mark it! B and C are unmarked. Let's do B next...

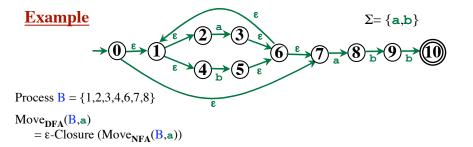


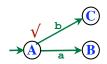


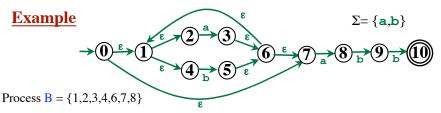




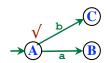
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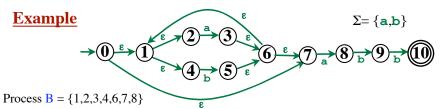






$$\begin{aligned} & \text{Move}_{\mathbf{DFA}}(\mathbf{B}, \mathbf{a}) \\ &= \epsilon\text{-Closure} \left(\text{Move}_{\mathbf{NFA}}(\mathbf{B}, \mathbf{a}) \right) \\ &= \epsilon\text{-Closure} \left(\{3, 8\} \right) \\ &- \end{aligned}$$



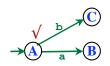


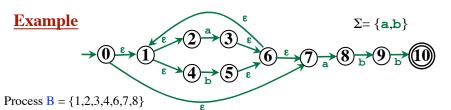
$$Move_{\mathbf{DFA}}(\mathbf{B}, \mathbf{a})$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$



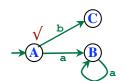


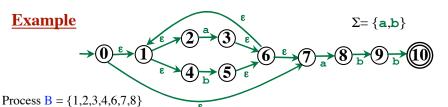
$$\mathsf{Move}_{\mathbf{DFA}}({\color{blue}\mathbf{B}},\!\mathbf{a})$$

=
$$\epsilon$$
-Closure (Move_{NFA}(B,a))

$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = B$$





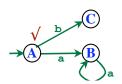
$$Move_{DFA}(B,a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

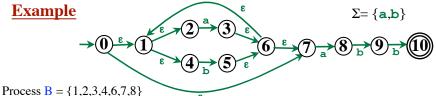
$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$

$$Move_{DFA}(B,b)$$







ε

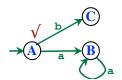
$$Move_{DFA}(B,a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

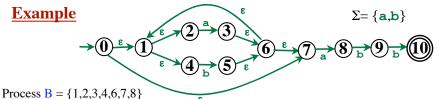
=
$$\varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = B$$

$$= \varepsilon$$
-Closure (Move_{NFA}(B,b))







$$Move_{DFA}(B,a)$$

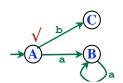
=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

$$= \varepsilon$$
-Closure ({3,8})

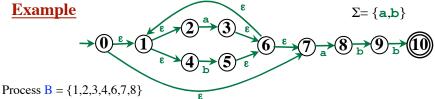
$$= \{1,2,3,4,6,7,8\} = B$$

$$= \varepsilon$$
-Closure (Move_{NFA}(B,b))

=
$$\varepsilon$$
-Closure ({5,9})







$$Move_{DFA}(B,a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

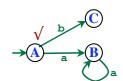
$$= \varepsilon$$
-Closure ({3,8})

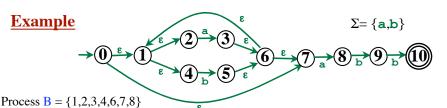
$$= \{1,2,3,4,6,7,8\} = \mathbf{B}$$

$$= \varepsilon$$
-Closure (Move_{NFA}(B,b))

=
$$\varepsilon$$
-Closure ({5,9})

$$= \{1,2,4,5,6,7,9\} = D$$





$$Move_{DFA}(B,a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

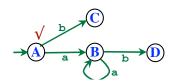
$$= \varepsilon$$
-Closure ({3,8})

$$= \{1,2,3,4,6,7,8\} = B$$

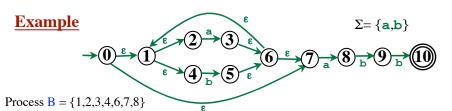
$$= \varepsilon$$
-Closure (Move_{NFA}(B,b))

=
$$\varepsilon$$
-Closure ({5,9})

$$= \{1,2,4,5,6,7,9\} = D$$



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$$Move_{DFA}(B,a)$$

=
$$\varepsilon$$
-Closure (Move_{NFA}(B,a))

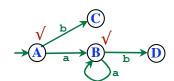
$$= \varepsilon$$
-Closure ({3,8})

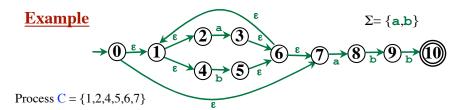
$$= \{1,2,3,4,6,7,8\} = B$$

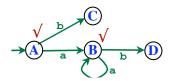
$$= \varepsilon$$
-Closure (Move_{NFA}(B,b))

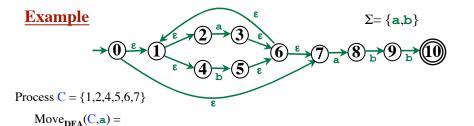
=
$$\varepsilon$$
-Closure ($\{5,9\}$)

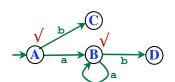
$$= \{1,2,4,5,6,7,9\} = D$$



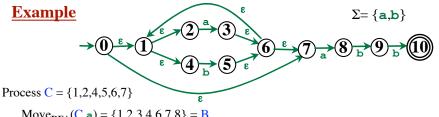






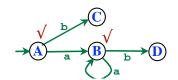


 $Move_{DFA}(C,b) =$

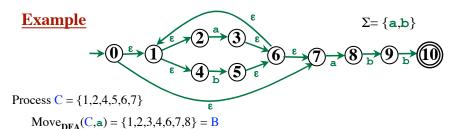


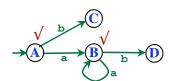
$$Move_{DFA}(C,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(C,b) =$

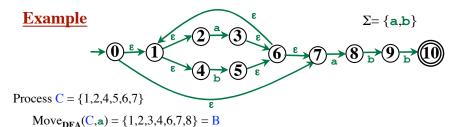


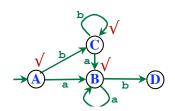
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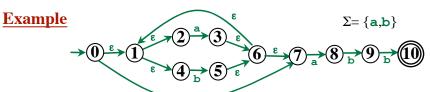


 $Move_{DFA}(C,b) = \{1,2,4,5,6,7\} = C$





 $Move_{DFA}(C,b) = \{1,2,4,5,6,7\} = C$



Process
$$C = \{1,2,4,5,6,7\}$$

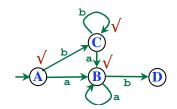
$$Move_{DFA}(C,a) = \{1,2,3,4,6,7,8\} = B$$

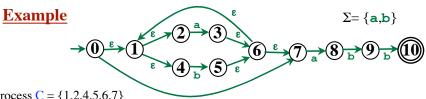
$$Move_{DFA}(C,b) = \{1,2,4,5,6,7\} = C$$

Process
$$D = \{1,2,4,5,6,7,9\}$$

$$Move_{\mathbf{DFA}}(\mathbf{D}, \mathbf{a}) =$$

$$Move_{DFA}(D,b) =$$





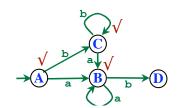
Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

Process
$$D = \{1,2,4,5,6,7,9\}$$

$$Move_{DFA}(D,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(D,b) =$



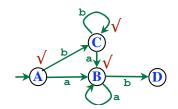


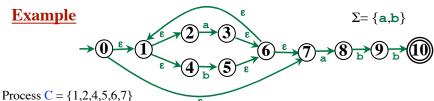
Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

Process
$$D = \{1,2,4,5,6,7,9\}$$

Move_{DFA}(D,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(D,b) = $\{1,2,4,5,6,7,10\}$ = E





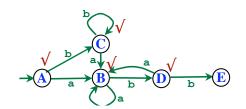
Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

Process
$$D = \{1,2,4,5,6,7,9\}$$

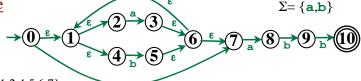
$$Move_{DFA}(D,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(D,b) = \{1,2,4,5,6,7,10\} = E$





$$\Sigma = \{a,b\}$$



Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

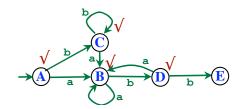
Process
$$D = \{1,2,4,5,6,7,9\}$$

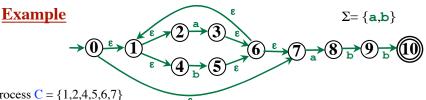
Process
$$\mathbf{E} = \{1, 2, 4, 5, 6, 7, 10\}$$

$$Move_{DFA}(D,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(D,b) = \{1,2,4,5,6,7,10\} = E$

$$Move_{DFA}(E,a) = Move_{DFA}(E,b) =$$





Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

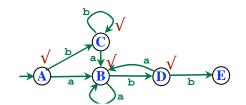
Process
$$D = \{1,2,4,5,6,7,9\}$$

Move_{DFA}(D,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(D,b) = $\{1,2,4,5,6,7,10\}$ = E

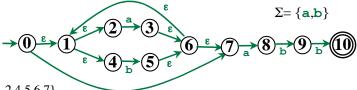
Process
$$\mathbf{E} = \{1, 2, 4, 5, 6, 7, 10\}$$

$$Move_{DFA}(E,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(E,b) =$







Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

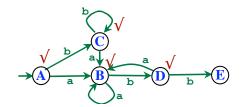
Process
$$D = \{1,2,4,5,6,7,9\}$$

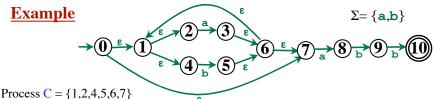
$$Move_{DFA}(D,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(D,b) = \{1,2,4,5,6,7,10\} = E$

Process
$$\mathbf{E} = \{1,2,4,5,6,7,10\}$$

Move_{DFA}(E,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(E,b) = $\{1,2,4,5,6,7\}$ = C





Process
$$C = \{1,2,4,5,6,7\}$$

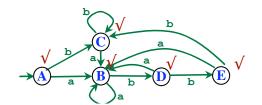
Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

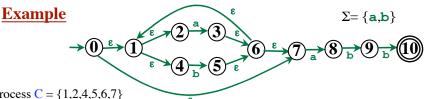
Process
$$D = \{1,2,4,5,6,7,9\}$$

Move_{DFA}(D,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(D,b) = $\{1,2,4,5,6,7,10\}$ = E

Process
$$\mathbf{E} = \{1, 2, 4, 5, 6, 7, 10\}$$

Move_{DFA}(E,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(E,b) = $\{1,2,4,5,6,7\}$ = C





Process
$$C = \{1,2,4,5,6,7\}$$

Move_{DFA}(C,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

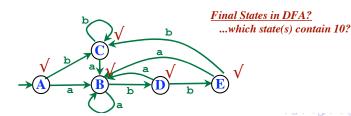
Process
$$D = \{1,2,4,5,6,7,9\}$$

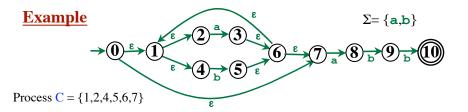
$$Move_{DFA}(D,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(D,b) = \{1,2,4,5,6,7,10\} = E$

Process
$$\mathbf{E} = \{1,2,4,5,6,7,10\}$$

Move_{DFA}(E,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(E,b) = $\{1,2,4,5,6,7\}$ = C





Process
$$D = \{1, 2, 4, 5, 6, 7, 9\}$$

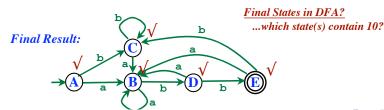
$$Move_{DFA}(D,a) = \{1,2,3,4,6,7,8\} = B$$

 $Move_{DFA}(D,b) = \{1,2,4,5,6,7,10\} = E$

Move_{DFA}(C,a) = $\{1,2,3,4,6,7,8\}$ = B Move_{DFA}(C,b) = $\{1,2,4,5,6,7\}$ = C

Process
$$\mathbf{E} = \{1, 2, 4, 5, 6, 7, 10\}$$

Move_{DFA}(E,a) =
$$\{1,2,3,4,6,7,8\}$$
 = B
Move_{DFA}(E,b) = $\{1,2,4,5,6,7\}$ = C



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October 19, 2012

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NFA to DFA Conversion

Algorithm: Convert NFA to DFA

```
S_{DEA} = \{\}
Add \varepsilon-Closure(s_0) to S_{DFA} as the start state
Set the only state in SDEA to "unmarked"
while Spra contains an unmarked state do
  Let T be that unmarked state
                                             A set of NFA states
  Mark T
  for each a in \Sigma do
                                                 Everywhere you could
    S = \varepsilon-Closure (Move<sub>NFA</sub> (T, a))
                                                possibly get to on an a
    if S is not in SDEA already then
       Add S to SDEA (as an "unmarked" state)
    endIf
    Set Movener (T, a) to S
                                         i.e, add an edge to the DFA...
  endFor
endWhile
for each S in SDFA do
  if any s∈s is a final state in the NFA then
    Mark S an a final state in the DFA
  endIf
endFor
```