NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION

(Semester II: 2002–2003)

ST2334 PROBABILITY AND STATISTICS

April 2003 — Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **NINE (9)** printed pages.
2. Answer **ALL** the questions. The number in [ ] indicates the number of marks allocated for that part. The total number of marks for this paper is 60.
3. Write your answers in the spaces provided.
4. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
5. Candidates may bring in **ONE (1)** handwritten A4-size (210 × 297 mm) help sheet.
6. Statistical tables are provided.

Matriculation No.: | | | | | | | | | | |

Question	1	2	3	4	5	**Total**
Marks	10	6	14	12	18	60
Scores						

QUESTION 1

Suppose that an insurance company classifies people into one of the 3 following classes: good risks, average risks and bad risks. Their records indicate that the numbers of accidents resulted from good, average and bad risk persons follow a Poisson distribution with means of 0.5, 1.0, and 2.0 accidents per year, respectively. Suppose 20% of the population are good risks, 50% are average risks and 30% are bad risks.

(a) What proportion of the people have at least one accident in a given year? [4 marks]

(b) Given that Tom, a policyholder, has had a total of one accident in the past two years, what is the probability that he is in fact a bad risk person? [6 marks]

QUESTION 2

A very expensive component is critical to the operation of an electrical system and must be replaced immediately upon failure. If lifetime of this type of component is normally distributed with a mean of 100 hours and a standard deviation of 30 hours, find the minimum number of these components that must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least 0.95?

[6 marks]

A farmer who has a piece of lumber of length 6 metres decides to build a pen in the shape of a triangle for his chickens. First he randomly makes a cut in the upper half of the log. Let X be the length of the longer piece. Then X is uniformly distributed in the interval $(3, 6)$. Then he randomly cuts another piece of length Y metres from this longer piece.

(a) Let $f(x, y)$ be the joint probability density function of X and Y. Prove that

$$f(x,y) = \begin{cases} \frac{1}{3x} & \text{if } (x,y) \in \mathbb{D}, \\ 0 & \text{otherwise.} \end{cases}$$

Determine and sketch the domain $\mathbb{D}$ of $f(x, y)$. [4 marks]

(b) Find the probability density function of Y.

[4 marks]

(c) Find the probability that the 3 pieces can be used to form a triangle. (Hint: Three segments form a triangle if and only if the length of *each* piece is less than the sum of the lengths of the remaining two pieces.)

[6 marks]

QUESTION 4

When evaluating a loan applicant, Mary, a DBS bank financial officer, is faced with the problem of granting loans to people who appear to be good risks and denying loans to people who appear to be bad risks. In fact, she is testing the null hypothesis

$$H_0 : \text{The applicant is a good risk}$$

against the alternative hypothesis

$$H_1 : \text{The applicant is a bad risk}$$

She commits a Type I error when she rejects an applicant who is actually a good risk; she commits a Type II error when she grants a loan to an applicant who is a bad risk. Let α be the significance level.

(a) For each of the following situations, should Mary set a high α value or a low α value? Explain your answers.

(i) Lending money is tight, interest rates are high and loan applicants are numerous. [2 marks]

(ii) Lending money is plentiful, interest rates are low and there is intense competition for loan applicants. [2 marks]

(b) In granting loans, Mary tends to be biased against male applicants because her past experience tells her that female applicants are better risks than male applicants. In 2000, DBS bank granted loans to 850 applicants; 250 of them are females. Of these 850 successful applicants, 69 male applicants and 23 female applicants turned out to be bad risks. At the 5% significance level, determine whether there is clear evidence that Mary should continue this practice.

[8 marks]

A special feed mix is claimed to yield the same weight gain for ducks and for hens. Suppose the weight gain (in kilograms), after being fed this special feed mix for 45 days, is normally distributed with mean μ and standard deviation 1.5σ for a duck, and mean μ and standard deviation σ for a hen. Let $X_1, X_2, \ldots, X_{10}$ be the weight gains of 10 randomly selected ducks, and $Y_1, Y_2, \ldots, Y_{15}$ be the weight gains of 15 randomly selected hens.

(a) To estimate μ using all available information, David is debating whether to use the average of the sample means,

$$\hat{\mu}_1 = \frac{\bar{X} + \bar{Y}}{2} = \frac{1}{2}\left(\frac{\sum_{i=1}^{10} X_i}{10} + \frac{\sum_{j=1}^{15} Y_j}{15} \right)$$

or to use the average of the weight gains of all 25 fowls,

$$\hat{\mu}_2 = \frac{\sum_{i=1}^{10} X_i + \sum_{j=1}^{15} Y_j}{25}$$

He seeks your professional help. Based on the biasedness and efficiency criteria, which one will you recommend?

[8 marks]

(b) David finds the idea of using two different types of fowl to estimate μ too difficult to comprehend. In fact, he does not even believe that the mean weight gain for ducks and the mean weight gain for hens should be the same. To prove his point, David collected data on a random sample of 10 ducks and a random sample of 15 hens. A summary of the data is given below:

$$\sum_{i=1}^{10} x_i = 23.2, \qquad \sum_{i=1}^{10} x_i^2 = 54.9, \qquad \sum_{j=1}^{15} y_j = 31.2, \qquad \sum_{j=1}^{15} y_j^2 = 65.8$$

Construct two 95% confidence intervals for μ, one using only data from ducks $(x_1, x_2, \ldots, x_{10})$ and one using only data from hens $(y_1, y_2, \ldots, y_{15})$. Based on these two intervals, do you agree with David? Explain. [10 marks]

END OF PAPER