EEC 130A: Homework 5

Due: 3:30 pm, Feb. ~~12th~~14th, 2013

Happy Valentine's Day!

1. (4 points) (FAE P3.45) Vector field $\mathbf{E}$ is characterized by the following properties: (a) $\mathbf{E}$ points along $\hat{\mathbf{R}}$; (b) the magnitude of $\mathbf{E}$ is a function of only the distance from the origin; (c) $\mathbf{E}$ vanishes at the origin; and (d) $\nabla \cdot \mathbf{E} = 12$, everywhere. Find an expression for $\mathbf{E}$ that satisfies these properties.

2. (4 points) (FAE P3.47) For the vector field $\mathbf{E} = \hat{\mathbf{r}}10e^{-r} - \hat{\mathbf{z}}3z$, verify the divergence theorem for the cylindrical region enclosed by $r = 2$, $z = 0$, $z = 4$.

3. (4 points) (FAE P3.52) Verify Stokes's theorem for the vector field

$$\mathbf{B} = \left(\hat{\mathbf{r}}r\cos\phi + \hat{\boldsymbol{\phi}}\sin\phi\right)$$

by evaluating

(a) $\oint_c \mathbf{B} \cdot d\mathbf{l}$ over the semicircular contour shown in Fig. 1.

(b) $\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s}$ over the surface of the semicircle.

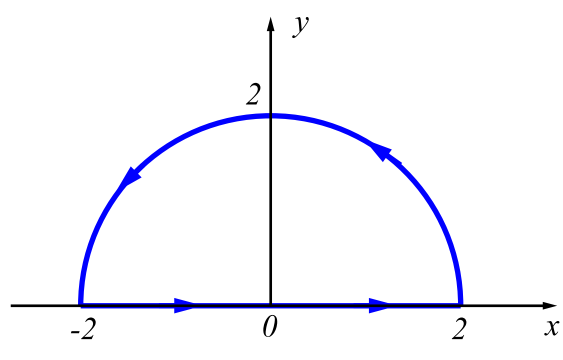

Figure 1: (FAE Fig. P3.52) Contour path for Problem. 3.

4. (4 points) (FAE P3.58) Find the Laplacian of the following scalar functions:

(a) $V_1 = 10r^3 \sin 2\phi$ (in cylindrical system)

(b) $V_2 = (2/R^2)\cos\theta\sin\phi$