CG1108 AY2010/11 Sem2

Lecture 7

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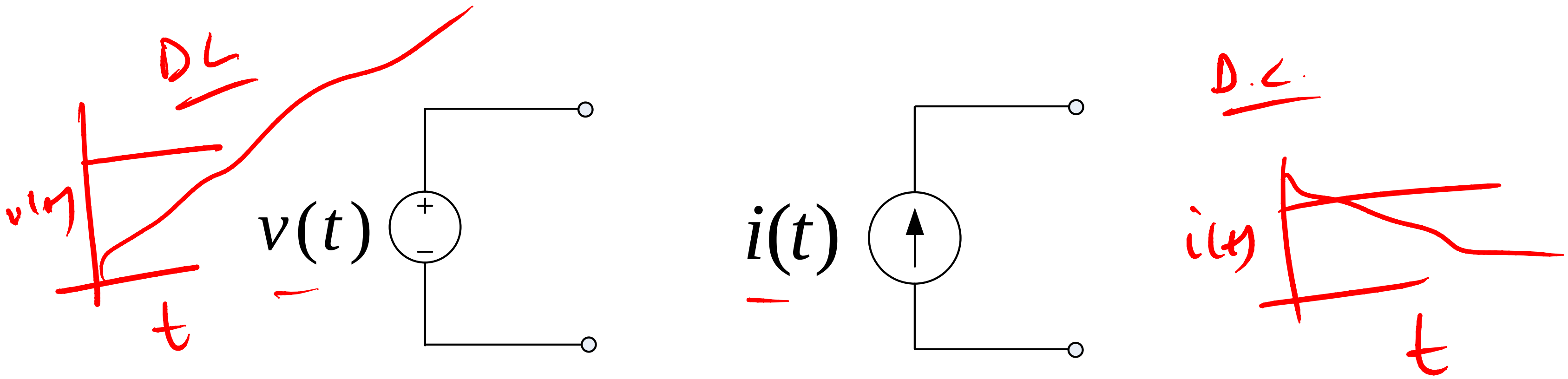
Lecture 6 review

Learning objectives

- Time-varying sources
- RMS (Root Mean Square) value
- Sinusoids
- Complex algebra
- Phasor
- Impedances
- AC ckt analysis with phasors and impedances

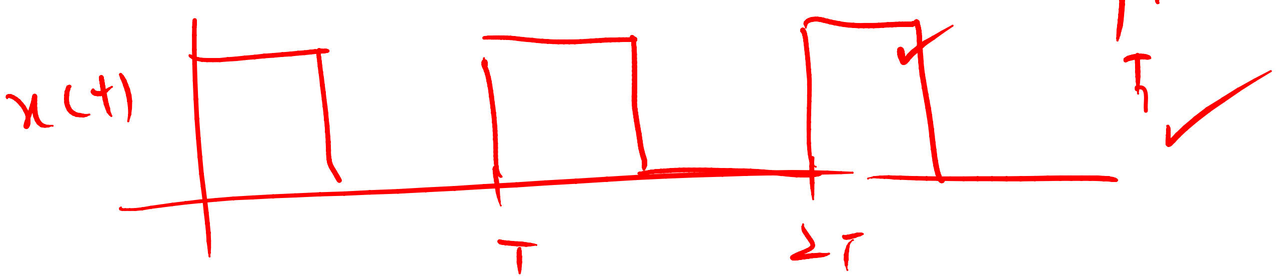
Time dependent sources



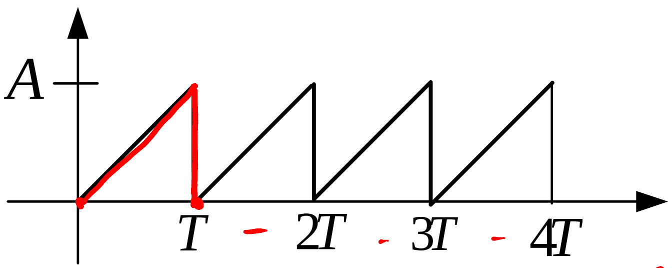
- Periodic functions

$$x(t) = x(t + nT), \quad n = 1,2,3,\ldots$$

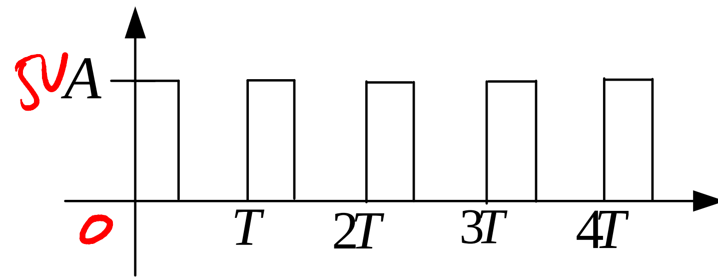
T = Time period



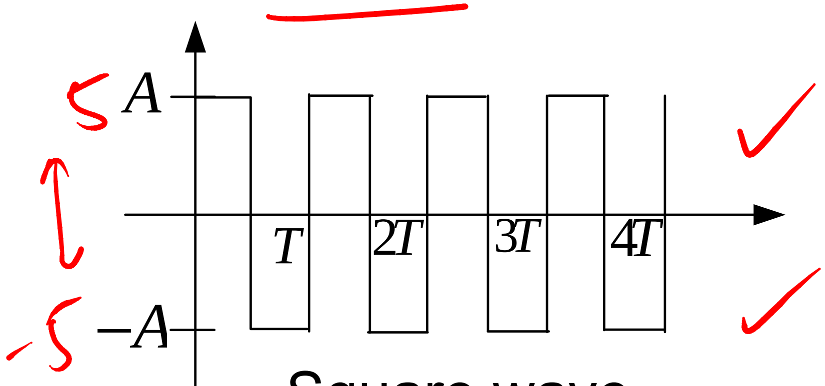
Common periodic signals



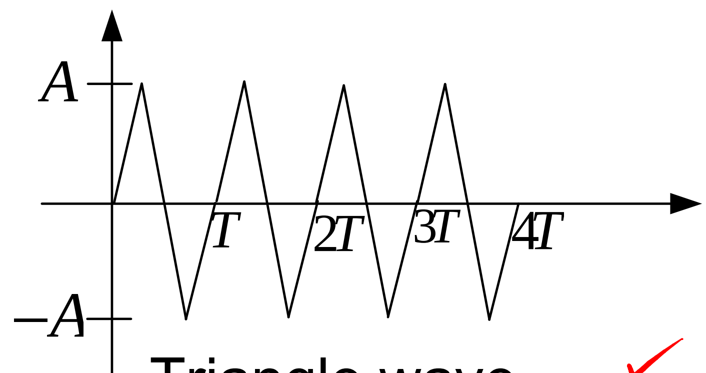
Sawtooth wave



Pulse train (TTL)



Square wave



Triangle wave

Periodic signals

T

- The **time period** of the signal is defined as the time taken to complete one cycle

$f = \frac{1}{T}$

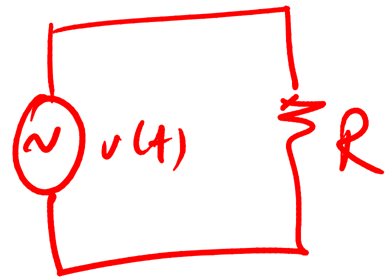
- The **frequency** of the periodic signal is the number of cycles completed in one second. The units of frequency are hertz (Hz).
- **Angular frequency** (radians per second) , as one period in time corresponds to radian.

$\omega = 2\pi \times f$

Root-mean-square (RMS) values

$p(t) = v(t) \cdot i(t)$

$$p(t) = \frac{v^2(t)}{R} \qquad E_T = \int_0^T p(t)dt = \int_0^T \frac{v^2(t)}{R}dt$$



$$P_{avg} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}dt = \frac{\left(\frac{1}{T}\int_0^T v^2(t)dt\right)}{R}$$

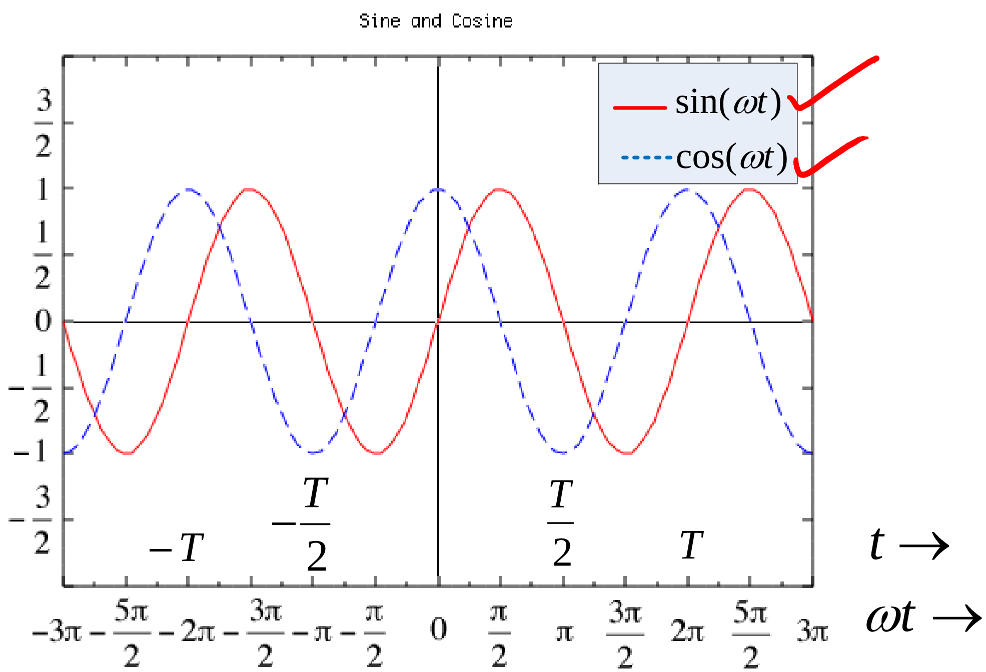
$$\frac{1}{T}\int_0^T v^2(t)dt = V_{rms}^2$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

Avg value of a sinusoid = 0

$$P_{avg} = \frac{V_{rms}^2}{R}$$

Sinusoidal signals



$$v(t) = V_m \cos(\omega t + \theta)$$

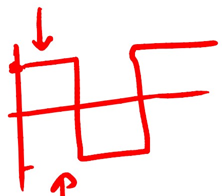
peak-to peak

angular freq.

V_m is the peak value of the voltage

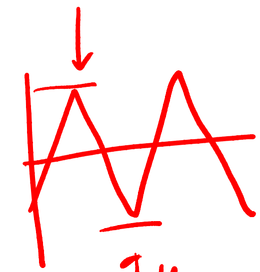
θ is the phase angle

RMS for a sinusoid



V_{pp}

$V_{Rms} = \frac{V_{pp}}{2}$



V_{pp}

$V_{Rms} = \frac{V_{pp}}{2\sqrt{3}}$

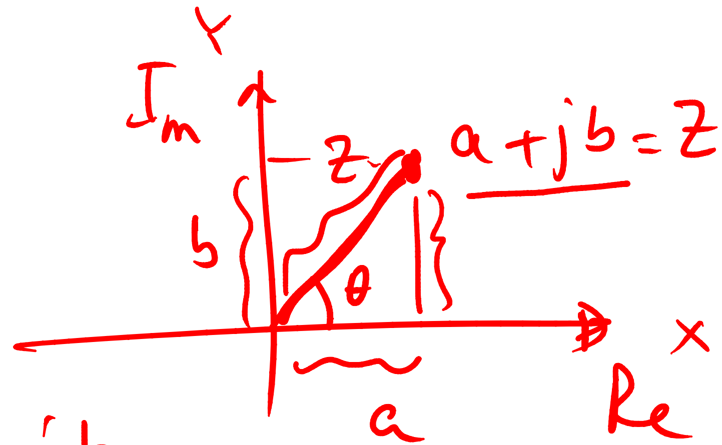
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T V_m^2\cos^2(\omega t+\theta)dt}$$

$$= \sqrt{\frac{1}{T}\frac{V_m^2}{2}\int_0^T (1+\cos 2(\omega t+\theta))dt} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

- For example when we say the PUB supply in Singapore is 230V, we mean that the RMS value of the PUB supply is 230V and its peak value would be

$$V_m = \sqrt{2}V_{rms} = 230\times\sqrt{2} = 230\times 1.414 = 325V$$

Complex number



- Complex plane
  - Real axis
  - Imaginary axis
- Rectangular form : $a + jb$
- Polar form : $|Z|\angle\theta$
  - Magnitude
  - Angle

$$|Z| = \sqrt{a^2+b^2} \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

- Complex algebra

$A = 1 + j2$ ✓

$B = 0.5 + j1$ ✓

$A - B = (1 - 0.5) + j(2 - 1) = 0.5 + j(1)$

$A * B = (1 + j2)(0.5 + j1) =$

$A / B =$

Complex algebra

- Addition/Subtraction
  - Done in rectangular form
- Multiplication/Division
  - Done in polar form

$A = a_1 + jb_1$

$= z_1 \angle \theta_1$

$B = a_2 + jb_2$

$= z_2 \angle \theta_2$

$A * B = z_1 \cdot z_2 \angle \theta_1 + \theta_2$

$A \div B = (z_1 \div z_2) \angle \theta_1 - \theta_2$

Rect $\longrightarrow$ Polar

Phasor

- Sinusoidal voltages and currents can be represented as vectors in a complex plane.

  Complex Number

- These are called Phasors and are very useful in steady-state analysis of sinusoidal voltages and currents.
- Phasor is just a definition. This gives rise to mathematical convenience. It has no physical significance.

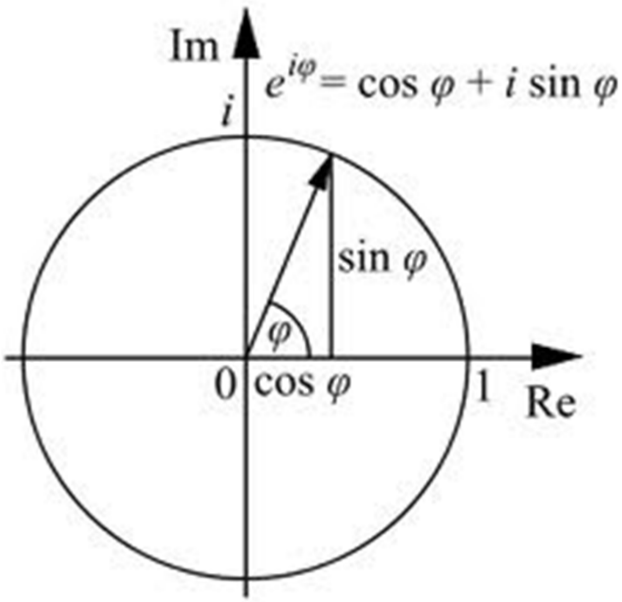
Phasor Definition

- For a sinusoidal voltage of

$$v_1(t) = V_1 \cos(\omega t + \theta)$$

- we define the phasor as: $\mathrm{V}_1 = V_1 \angle \theta_1$
- Thus, phasor of a sinusoid is a complex number having a magnitude equal to the peak value and having the same phase angle as the sinusoid.

Euler's identity



complex exponential

complex no. in rect. form

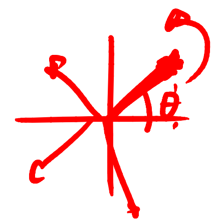
$$\boxed{e^{j\alpha} = \cos\alpha + j\sin\alpha}$$

$$\cos\alpha = \mathrm{Re}(e^{j\alpha})$$

$$\cos(\omega t + \theta) = \mathrm{Re}(e^{j(\omega t + \theta)})$$

$$V_m \cos(\omega t + \theta) = \mathrm{Re}(V_m e^{j(\omega t + \theta)})$$

$$V_m e^{j(\omega t + \theta)} = V_m e^{j\theta} \times e^{j\omega t}$$

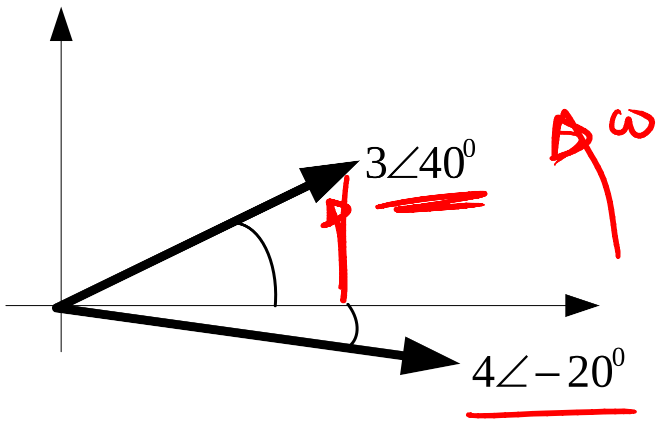


- Sinusoidal is a projection of the vector rotating in the complex plane
- The phasor can be thought of as a snap shot of the rotating vector at t=0.

Phasor Diagram

$$v_1(t) = 3\cos(\omega t + 40^0) \qquad 3\angle 40^0$$

$$v_2(t) = 4\cos(\omega t - 20^0) \qquad 4\angle -20^0$$

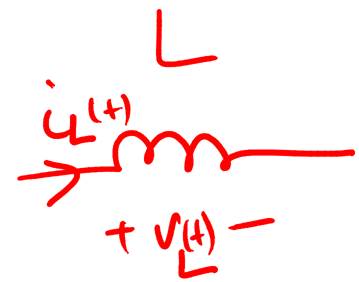


- **Leading / lagging**

Impedances

- Also known as complex resistance, frequency-dependent resistance.
- With understanding of impedances for the circuit elements like resistance, inductance and capacitance, the sinusoidal steady-state analysis will be same as analysis of purely resistive circuits under DC supply.

Inductance



$$i_L(t) = I_m \sin(\omega t + \theta) = I_m \cos(\omega t + \theta - 90^0)$$

$$v_L(t) = L\frac{di_L(t)}{dt} = L\omega I_m \cos(\omega t + \theta)$$

$$\mathrm{I_L} = I_m \angle \theta - 90^0$$

$\mathrm{I_L} = I_m \angle \theta - 90^\circ$, $\mathrm{V_L} = L\omega I_m \cdot \angle \theta$

$$\mathrm{V_L} = \omega L I_m \angle \theta = \omega L \angle 90^0 \times I_m \angle \theta - 90^0 = \mathrm{Z_L} \times \mathrm{I_L}$$

$$Z_L = \omega L \angle 90^0 = j\omega L$$

: Impedance of an inductor.

Ohm's law: $V = R \cdot I$

complex resistance

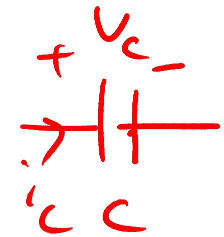
Freq. dependent resistance

Capacitance

$$V_c = Z_c I_c$$

$$\boxed{Z_c = -j\frac{1}{\omega C}} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^0$$

$$i_c = C\frac{dV_c}{dt}$$



$$V_c(t) = V_m \cos(\omega t + \theta)$$

$$i_c = C.V_m.\omega(-1).\sin(\omega t + \theta)$$

$$= C V_m \omega \cos\left(\omega t + \theta + \frac{\pi}{2}\right)$$

$$-\sin\theta = \cos\left(\theta + \frac{\pi}{2}\right)$$

$$v_c(t) \rightarrow V_C = V_m \angle\theta$$

$$i_c(t) \rightarrow I_c = C\omega V_m \angle\theta + \frac{\pi}{2} = C\omega\angle\pi/2 \,.\, V_m\angle\theta$$

$$\frac{1}{Z_c} = C\omega\angle\pi/2 \qquad = \frac{V_c}{Z_c}$$

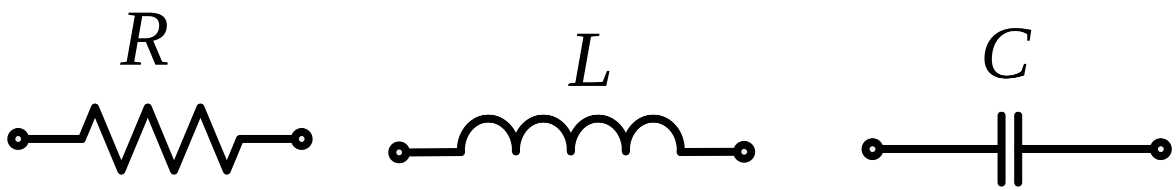
$$Z_c = \frac{1}{C\omega}\angle -\pi/2$$

Resistance

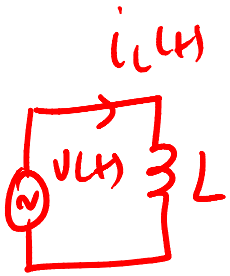
$$V_R = RI_R$$

$$Z_R = R$$

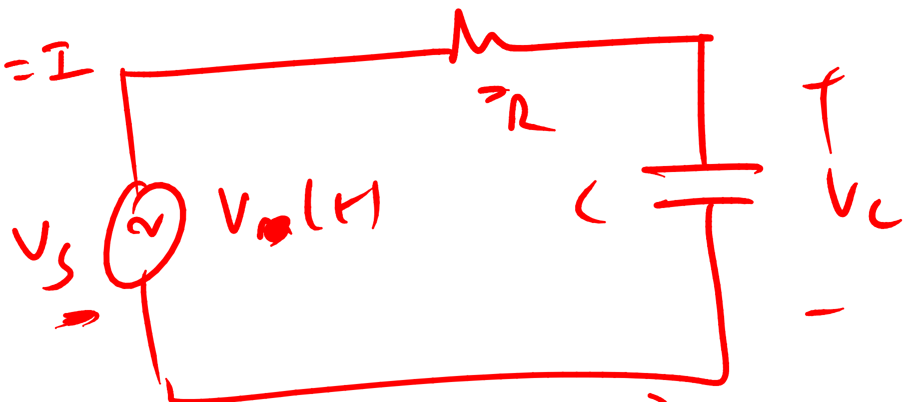
Impedances for R, L and C



$$Z_R = R \qquad Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C}$$



$\frac{V}{Z} = I$



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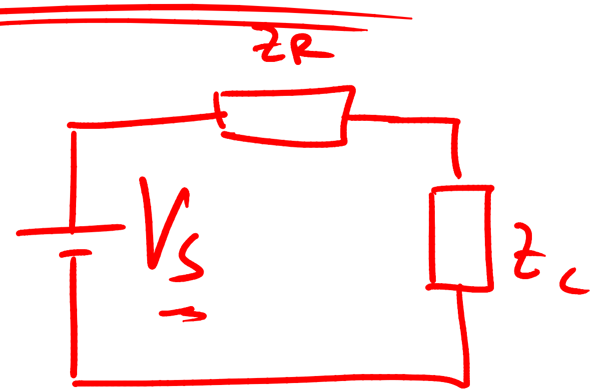


$$\tilde{V}_C = \tilde{V}_S \cdot \frac{Z_C}{Z_R + Z_C}$$

Step-by-step procedures for steady-state analysis of circuits with sinusoidal sources:

- All sources must have the same frequency. ✓
- Replace the time descriptions of the voltage and current sources with the corresponding phasors.

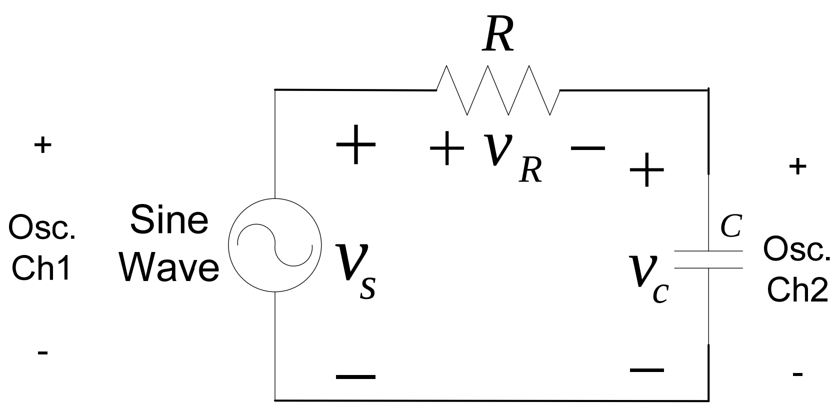
  Sinusoids → Phasors
- Replace the inductance with its impedance and capacitance with its impedance of . Resistances have the same impedance as their resistance.

  $R, L, C \rightarrow Z_R, Z_L, Z_C$

Step-by-step procedures for steady-state analysis of circuits with sinusoidal sources:

- Analyze the circuits as before with DC sources and resistances only.
- Convert the final results in phasor to time-domain form

Example – AC circuits



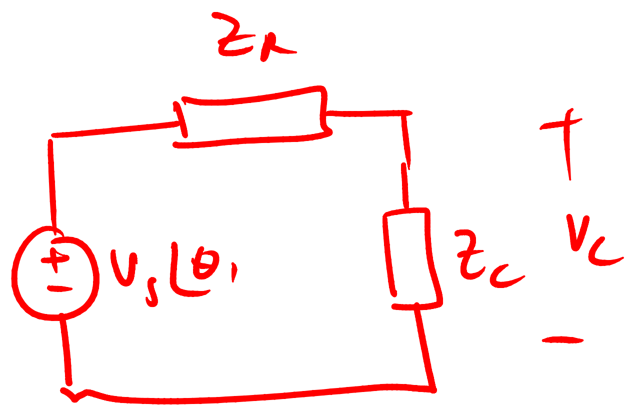
$v_c(t)$ given $v_s(t)$ as
$V_s \sin(\omega t + \theta_1)$

R, C are given

① $v_s(t) \to V_s \angle \theta_1$

$R \to R = Z_R$

$C \to \frac{1}{j\omega C} = Z_C$



$$V_C = V_s \angle \theta_1 \cdot \frac{Z_C}{Z_R + Z_C} = V_s \angle \theta_1 \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

- Find the voltage across the capacitor

$$v_c = \underline{v_s \angle\theta_1} \cdot \underbrace{\frac{1}{j\omega RC + 1}}$$

$$= v_s \angle\theta_1 \cdot |z| \angle\theta$$

$$|z| = \frac{1}{\underline{|j\omega RC + 1|}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

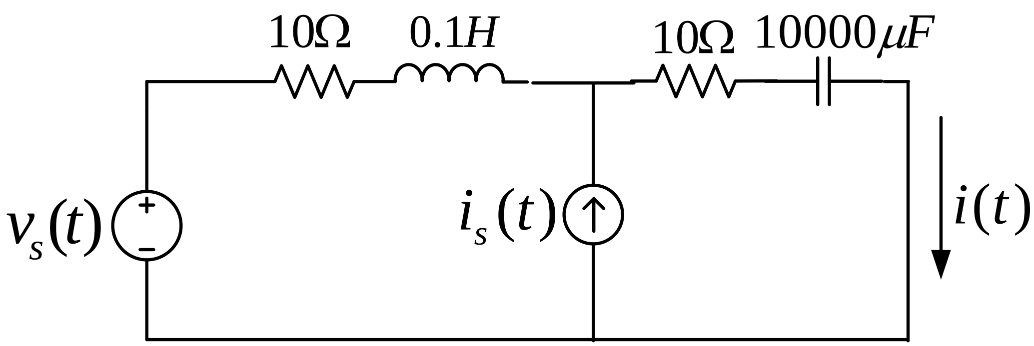
$$\theta = 0 - \angle j\omega RC + 1$$

$$= -\tan^{-1}(\omega RC)$$

$$v_c = \underline{v_s \angle\theta_1} \cdot \underline{|z| \angle\theta} = \underline{v_s |z|} \cdot \angle\theta_1 + \theta$$

phase diff between $v_c(t), v_s(t)$: θ

Example – AC circuits



Find $i(t)$

$v_s(t) = 100\sin(100t)\,V$ and

$i_s(t) = 100\cos(100t)\,A.$

$= 100\sin(100t + \pi/2)$

$\cos\theta = \sin(\theta + \pi/2)$

$\omega = 100$

$L = 0.1$

$\omega L = 10$

① $v_s(t) : 100\angle 0$

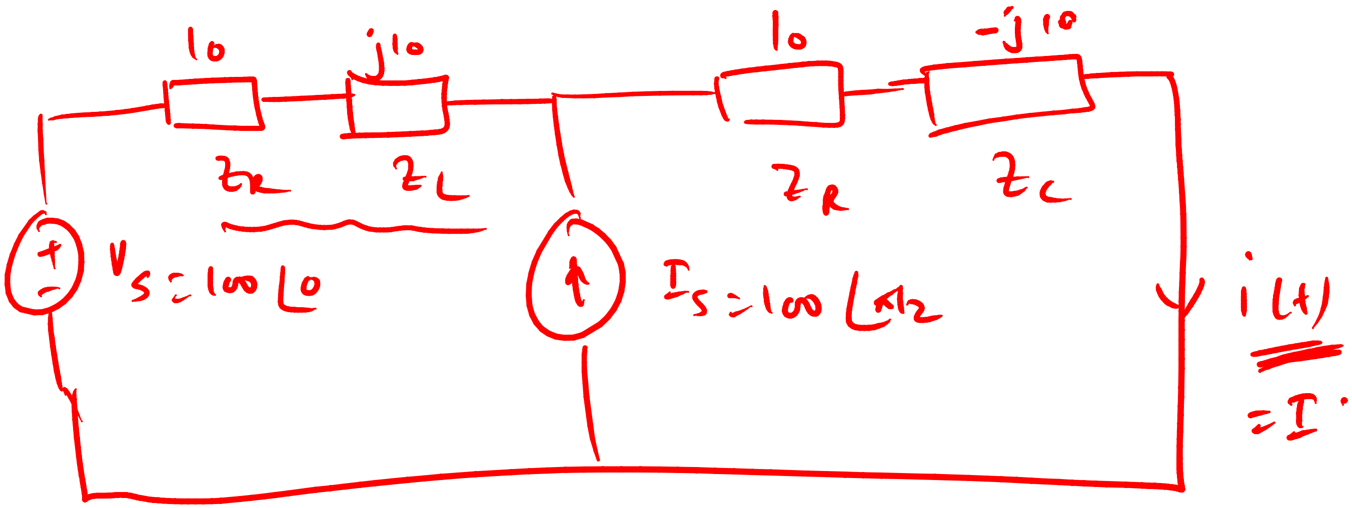
$i_s(t) = 100\angle \pi/2$

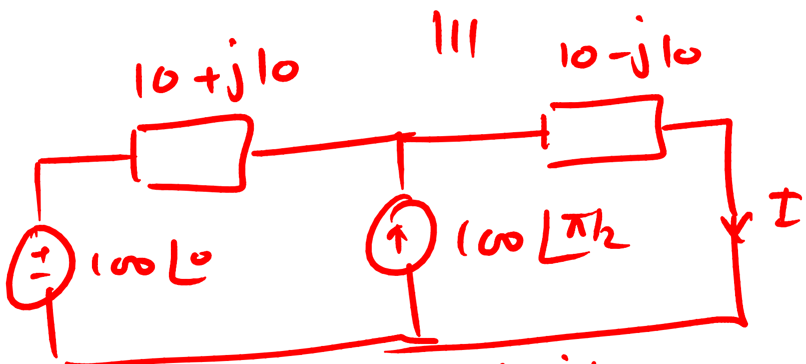
② $R \rightarrow Z_R = R$

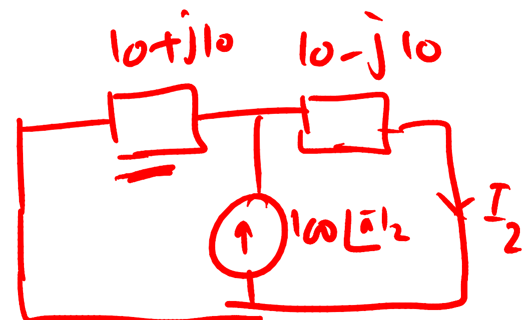
$L : Z_L = j\omega L = \omega L\angle \pi/2 = 10\angle \pi/2 = j10$

$Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{\omega C}\angle -\pi/2 = 10\angle -\pi/2$

$= -j10$







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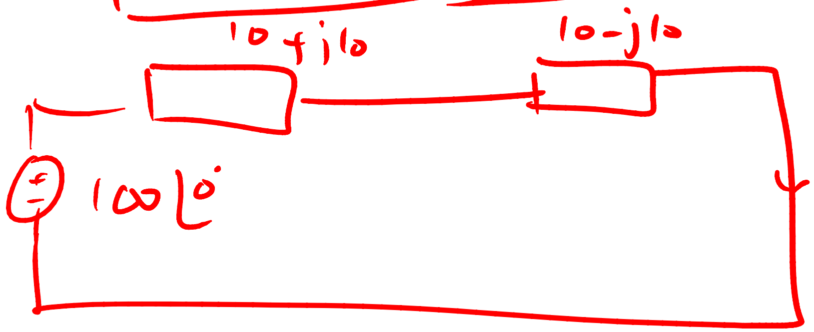


$$I_2 = 100^{5}\angle\pi/2 \cdot \frac{10+j10}{20}$$

$$I_1 = \frac{100}{20} = 5\angle 0$$

$$I_2 = 5\angle\pi/2 \cdot 10\sqrt{2}\angle 45^\circ = 50\sqrt{2}\angle 135^\circ$$

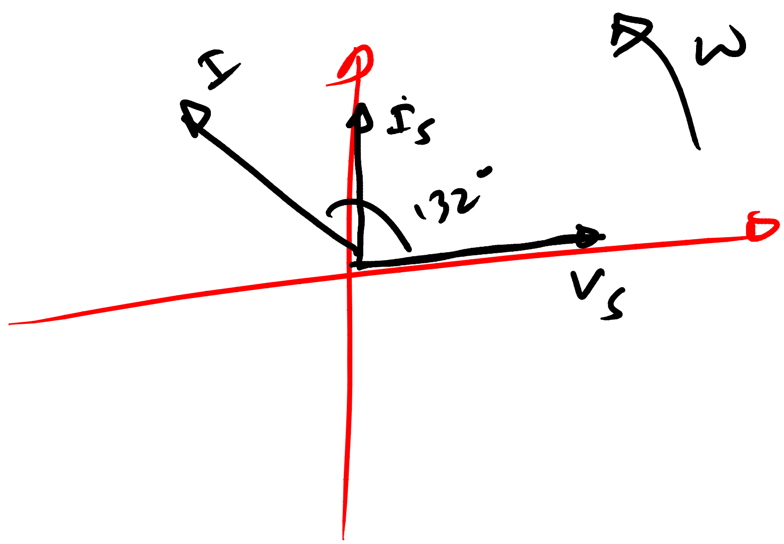
$$I = I_1 + I_2 = 5\angle 0 + 50\sqrt{2}\angle 135^\circ$$

$$I = 5\angle 0^\circ + 50\sqrt{2}\,\angle 135^\circ$$

$$= 5 - 50 + j50 = -45 + j50 = 67.27\angle 132^\circ$$

$$i(t) = 67.27\,\sin(100t + 132^\circ)$$

Phasor diagram for $v_s(t)$, $i_s(t)$, $i(t)$.



$i(t)$ is leading $i_s(t)$ by 42°.

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Autonomous Vehicle Project

Reminder on Module Assessment

- Project is worth 30% of the total marks
  - 10% Midterm
  - 40% final exam

    Theory : mathematics 50%
  - 20% (15% lab + 5% lab test)
  - 30% Project

    50%
    - No marks for individual lab
    - Requires both technical and soft skills to do well
    - Group work
    - Individual learning journal to be submitted at the end

Project work

- From week 8 to week 13 – Autonomous Vehicle project
  - Design, implementation, system integration
  - Debugging and Testing
  - Final demonstration and competition

Learning objectives of the project

- Exposure to the cycle of engineering project
  - Conceiving (C),
  - Designing (D),
  - Implementing (I)
  - Operating (O)

Soft skills

In addition to the technical skills, learn about the importance of

- Resourcefulness
- Teamwork
- Integrity
- Communications
- Timeliness
- Systematic approach

Project specification

- The autonomous vehicle can move along a curvy black line on a white surface on its on
- It should be able to turn left and right, move forward as well as backward as needed

Subsystems

A. Mechanical subsystem

1. The chassis and body of the vehicle
2. The wheels and gear box

B. Electrical subsystem

1. Power source (battery)
2. Line Sensor
3. Drive system – DC motor and driver
4. Controller – PIC Micrcontroller

How we go about it..

- We shall build the subsystems
- Integrate the subsystems together
- Debugging and Tuning the finished vehicle
- Final Demonstration
- Competition - Fastest

Timeline for the project

- Week 8 : DC Power supply and Battery
- Week 9 : DC motor drives and sensors
- Week 10 : Programming microcontroller
- Week 11 : Integration
- Week 12 : Integration
- Week 13 : Final demonstration and competition

Group and Individual grading

Sr. No.	Project Criteria	Marks from 30
Group	Project	20
Individual	a) Learning journal - 5 b) Peer feedback - 5	10

A template for the learning journal and the peer feedback is provided.