EE2011 Engineering Electromagnetics - Part CXD
Tutorial 8 - Solutions

Q1

(i) $\beta_1 = k_0 = 6 \quad \text{rad/m}$

$$\omega = \beta_1 c = 1.8 \times 10^9 \quad \text{rad/s}$$

$$\varepsilon_{r2} = 2.56, \quad \Rightarrow \quad \beta_2 = k_0 \sqrt{\varepsilon_{r2}} = 9.6 \ \text{rad/m}$$

$$\eta_1 = \eta_0 = 120\pi \quad \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r2}}} = \frac{120\pi}{\sqrt{2.5}} = 238.43 \ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.225$$

$$\tau = 1 + \Gamma = 1 - 0.225 = 0.775$$

Let $\mathbf{E}_r = \hat{\mathbf{x}} E_{r0} e^{j\beta_1 z}, \quad \mathbf{H}_r = (-\hat{\mathbf{z}}) \times \dfrac{\mathbf{E}_r}{\eta_1} = -\hat{\mathbf{y}} \dfrac{E_{r0}}{120\pi} e^{j6z}.$

$$\mathbf{E}_t = \hat{\mathbf{x}} E_{t0} e^{-j\beta_2 z}, \quad \mathbf{H}_t = \hat{\mathbf{z}} \times \frac{\mathbf{E}_t}{\eta_2} = \hat{\mathbf{y}} \frac{E_{t0}}{238.43} e^{-j9.6z}$$

But $E_{r0} = \Gamma E_{i0} = -2.25$

$$E_{t0} = \tau E_{i0} = 7.75$$

$$\therefore \quad \mathbf{E}_r = -\hat{\mathbf{x}} 2.25 e^{j6z}, \quad \mathbf{H}_r = \left(-\hat{\mathbf{z}}\right) \times \frac{\mathbf{E}_r}{\eta_1} = \hat{\mathbf{y}} 0.0060 e^{j6z}$$

$$\mathbf{E}_t = \hat{\mathbf{x}} 7.75 e^{-j9.6z}, \quad \mathbf{H}_t = \hat{\mathbf{z}} \times \frac{\mathbf{E}_t}{\eta_2} = \hat{\mathbf{y}} 0.0325 e^{-j9.6z}$$

$$\mathbf{E}_r\left(z,t\right) = -\hat{\mathbf{x}} 2.25 \cos(1.8 \times 10^9 t + 6z) \quad \text{V/m}$$

$$\mathbf{H}_r\left(z,t\right) = \hat{\mathbf{y}} 0.0060 \cos(1.8 \times 10^9 t + 6z) \quad \text{A/m}$$

$$\mathbf{E}_t\left(z,t\right) = \hat{\mathbf{x}} 7.75 \cos(1.8 \times 10^9 t - 9.6z) \quad \text{V/m}$$

$$\mathbf{H}_t\left(z,t\right) = \hat{\mathbf{y}} 0.0325 \cos(1.8 \times 10^9 t - 9.6z) \quad \text{A/m}$$

(ii)

$$
\begin{aligned}
\mathbf{S}_{av_1} &= \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_1 \times \mathbf{H}_1^*\right] \\
&= \frac{1}{2}\mathrm{Re}\left[(\mathbf{E}_i + \mathbf{E}_r)\times\left(\mathbf{H}_i^* + \mathbf{H}_r^*\right)\right] \\
&= \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_i \times \mathbf{H}_i^* + \mathbf{E}_i \times \mathbf{H}_r^* + \mathbf{E}_r \times \mathbf{H}_i^* + \mathbf{E}_r \times \mathbf{H}_r^*\right] \\
&= \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_i \times \mathbf{H}_i^* + \mathbf{E}_r \times \mathbf{H}_r^*\right] \\
&= \left(\frac{|E_{i0}|^2}{2\eta_1} - \frac{|E_{r0}|^2}{2\eta_1}\right)\hat{\mathbf{z}} = \left(\frac{10^2}{2\times 377} - \frac{2.25^2}{2\times 377}\right)\hat{\mathbf{z}} = 0.126\hat{\mathbf{z}} \quad \mathrm{W/m^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{S}_{av_2} &= \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_2 \times \mathbf{H}_2^*\right] = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_t \times \mathbf{H}_t^*\right] \\
&= \frac{|E_{t0}|^2}{2\eta_2}\hat{\mathbf{z}} = \frac{7.75^2}{2\times 238.43}\hat{\mathbf{z}} = 0.126\hat{\mathbf{z}} \ \ \mathrm{W/m^2}
\end{aligned}
$$

Q2

Successive minima separated by 1.5 m $\Rightarrow \ \lambda_0 = 2\times 1.5 = 3\,\mathrm{m}$

The first minimum is thus $0.75\,\mathrm{m} = 0.25\lambda_0 = 1/4\lambda_0$ from the interface.

Standing wave ratio S = 5. Hence the magnitude of the reflection coefficient can be found.

$$|\Gamma| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

As the medium is lossless and the first minimum is not at the interface, it is a case of $\eta_u > \eta_0$ or $\Gamma > 0$. Thus,

$$\Gamma = \frac{2}{3} = \frac{\eta_u - \eta_0}{\eta_u + \eta_0}$$

Therefore,

$$\eta_u = 5\eta_0 = 5\times 377 = 1885\ \Omega$$

Q3

$$\omega = 2\pi f = 6\pi\times 10^9 \ \text{rad/s}$$

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = 40\pi \ \text{rad/m} \qquad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = 60\pi \ \text{rad/m}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = 60\pi\,\Omega \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = 40\pi\,\Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.2 \quad , \quad \tau = 1 + \Gamma = 0.8$$

(i) $\mathbf{E}_1(z) = \hat{\mathbf{x}}\, E_{i0}\, (e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z}) = \hat{\mathbf{x}}\, E_{i0} \left[(1+\Gamma) e^{-j\beta_1 z} + \Gamma (e^{+j\beta_1 z} - e^{-j\beta_1 z}) \right]$

$$= \hat{\mathbf{x}}\, E_{i0} \left[(1+\Gamma) e^{-j\beta_1 z} + \Gamma (j2) \sin(\beta_1 z) \right]$$

$$= \hat{\mathbf{x}} \left[80 e^{-j40\pi z} - j40 \sin(40\pi z) \right]$$

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\, \frac{E_{i0}}{\eta_1} (e^{-j\beta_1 z} - \Gamma e^{+j\beta_1 z}) = \hat{\mathbf{y}}\, \frac{E_{i0}}{\eta_1} \left[(1+\Gamma) e^{-j\beta_1 z} - \Gamma (e^{+j\beta_1 z} + e^{-j\beta_1 z}) \right]$$

$$= \hat{\mathbf{y}}\, \frac{E_{i0}}{\eta_1} \left[(1+\Gamma) e^{-j\beta_1 z} - 2\Gamma \cos(\beta_1 z) \right]$$

$$= \hat{\mathbf{y}} \left[\frac{4}{3\pi} e^{-j40\pi z} + \frac{2}{3\pi} \cos(40\pi z) \right]$$

(ii) $\mathbf{E}_2(z) = \hat{\mathbf{x}}\, \tau E_{i0}\, e^{-j\beta_2 z} = \hat{\mathbf{x}}\, 80\, e^{-j60\pi z}$

$$\mathbf{H}_2(z) = \hat{\mathbf{y}}\, \frac{\tau E_{i0}}{\eta_2} e^{-j\beta_2 z} = \hat{\mathbf{x}}\, \frac{2}{\pi} e^{-j60\pi z}$$

(iii) $\Gamma = -0.2 < 0$

Electric field maxima / Magnetic field minima at

$$z'_M = \frac{\lambda_1}{4} + n\frac{\lambda_1}{2}$$
$$= 0.0125 + 0.025n \ \ \text{(m).}$$
$$= 12.5 + 25n \ \ \text{(mm)} \quad , \quad n = 0, 1, 2, .$$

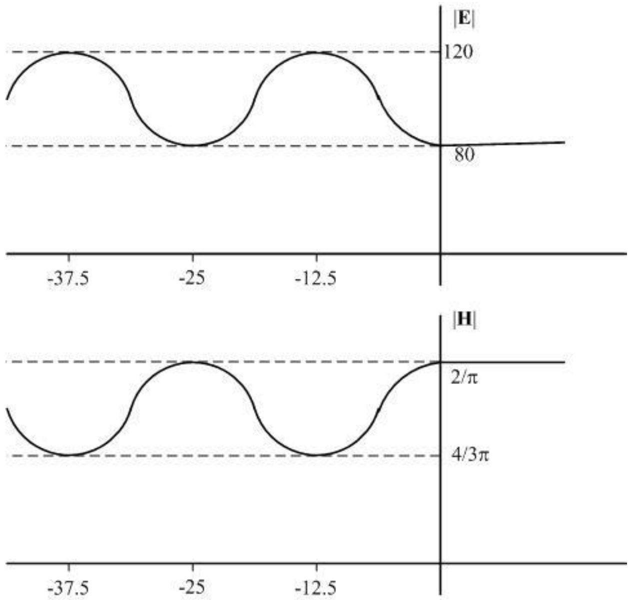
Electric field minima / Magnetic field maxima at

$$z'_m = n\frac{\lambda_1}{2}$$
$$= 0.025n \ \ \text{(m).}$$
$$= 25n \ \ \text{(mm)} \quad , \quad n = 0, 1, 2, .$$

$$\left|\mathbf{E}_1\right|_{\max} = \left|E_{i0}\right| (1 + |\Gamma|) = 120 \text{ V/m} \qquad \left|\mathbf{E}_1\right|_{\min} = \left|E_{i0}\right| (1 - |\Gamma|) = 80 \text{ V/m}$$

$$\left|\mathbf{H}_1\right|_{\max} = \frac{\left|E_{i0}\right|}{\eta_1} (1 + |\Gamma|) = \frac{2}{\pi} \text{ A/m} \qquad \left|\mathbf{H}_1\right|_{\min} = \frac{\left|E_{i0}\right|}{\eta_1} (1 - |\Gamma|) = \frac{4}{3\pi} \text{ A/m}$$

$$\left|\mathbf{E}_2\right| = \text{constant} = 80 \text{ V/m} \qquad \left|\mathbf{H}_2\right| = \text{constant} = \frac{2}{\pi} \text{ A/m}$$



Q4

(i)

Propagation constants:

$$\beta_1 = 100 = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{16} = \frac{4\omega}{c} \;\rightarrow\; \omega = 7.5\times10^9 \text{ rad/m}$$

$$\beta_2 = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{12\times6} = \frac{\omega\sqrt{72}}{c} \;\rightarrow\; \beta_2 = 212.13 \text{ rad/m}$$

Field expressions in phasor form:

$$\mathbf{E}^i(z) = 10e^{-j\beta_1 z}\hat{\mathbf{x}} + 20e^{-j\beta_1 z}e^{j\pi/3}\hat{\mathbf{y}}$$

$$\mathbf{E}^r(z) = 10\Gamma e^{j\beta_1 z}\hat{\mathbf{x}} + 20\Gamma e^{j\beta_1 z}e^{j\pi/3}\hat{\mathbf{y}}$$

$$\mathbf{E}^t(z) = 10\tau e^{-j\beta_2 z}\hat{\mathbf{x}} + 20\tau e^{-j\beta_2 z}e^{j\pi/3}\hat{\mathbf{y}}$$

Reflection and transmission coefficients:

$$\eta_1 = \frac{120\pi}{\sqrt{16}} = 30\pi\,\Omega; \quad \eta_2 = 120\pi\sqrt{\frac{12}{6}} = 120\sqrt{2}\pi\,\Omega$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = 0.700; \quad \tau = 1+\Gamma = 1.70$$

Field expressions in instantaneous form:

$$\mathbf{E}^r(z) = 7e^{j\beta_1 z}\hat{\mathbf{x}} + 14e^{j\beta_1 z}e^{j\pi/3}\hat{\mathbf{y}}$$

$$\mathbf{E}^r(z,t) = 7\cos(\omega t + \beta_1 z)\,\hat{\mathbf{x}} + 14\cos\left(\omega t + \beta_1 z + \frac{\pi}{3}\right)\hat{\mathbf{y}}$$

$$= 7\cos\left(7.5\times10^9 t + 100z\right)\hat{\mathbf{x}} + 14\cos\left(7.5\times10^9 t + 100z + \frac{\pi}{3}\right)\hat{\mathbf{y}}\ \text{V/m}$$

$$\mathbf{E}^t(z) = 17e^{-j\beta_2 z}\hat{\mathbf{x}} + 34e^{-j\beta_2 z}e^{j\pi/3}\hat{\mathbf{y}}$$

$$\mathbf{E}^t(z,t) = 17\cos(\omega t - \beta_2 z)\,\hat{\mathbf{x}} + 34\cos\left(\omega t - \beta_2 z + \frac{\pi}{3}\right)\hat{\mathbf{y}}$$

$$= 17\cos\left(7.5\times10^9 t - 212.13z\right)\hat{\mathbf{x}} + 34\cos\left(7.5\times10^9 t - 212.13z + \frac{\pi}{3}\right)\hat{\mathbf{y}}\ \text{V/m}$$

(ii)

Poynting vector in the region $z > 0$:

$$\mathbf{S}_{\text{av},2} = \frac{1}{2}\text{Re}\left[\mathbf{E}^t \times \mathbf{H}^{t*}\right] = \frac{|E_{t0}|^2}{2|\eta_2|}\hat{\mathbf{z}} = \frac{17^2 + 34^2}{2\cdot 120\sqrt{2}\pi}\hat{\mathbf{z}} = 1.35\,\hat{\mathbf{z}} \quad \text{W/m}^2$$

Average power density in the region $z > 0$ is $\left|\mathbf{S}_{av,2}\right| = 1.35\ \text{W/m}^2$

As the region for $z > 0$ is lossless, hence the average power density is same at any distance inside the region, i.e., at $z = 4$ m the average power density is 1.35 W/m^2.