Remarks of T9

Q1 By the hint given, we have
the following 6x6 transition matrix

$$
\begin{array}{cc}
\begin{array}{cccccc} \$ \; 0 & 1 & 2 & 3 & 4 & 5 \end{array} & \\
A = \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \end{pmatrix} & \begin{array}{c} \$ \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\end{array}
$$

Then use the matrix calculator at
http://wims.unice.fr/wims
to find A^5

To find A^n ,we use $A^n = PD^n P^{-1}$ $\qquad A^{\infty} = \lim_{n \to \infty} A^n$

You may use matrix calculator or MATLAB to find eigenvalues and eigenvectors of A

Q3 eigen-engine at

http://www.aw-bc.com/ide/idefiles/media/JavaTools/eignengn.html

can only be applied to matrices with entries -3, -2.5,-2,…0,0.5,1,1.5 …,3

Complex eigenvectors can’t be found there

You may use the matrix calculator at http://wims.unice.fr/wims or MATLAB to find eigenvalues and eigenvectors

Q6

(A)How to check that the vector [1,2,3] is on the plane, vector [1,2,4] is not on the plane ?

HINT:

Recall if the plane is generated by two vectors u and v ,then (uxv).w=0 iff vector w is on the plane

Cross product

Dot product

If $u=[u_1,u_2,u_3]$ $v=[v_1,v_2,v_3]$ $w=[w_1,w_2,w_3]$

then

$$(u\times v)\bullet w=\det\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$$

You may use wims website to find det

http://wims.unice.fr/wims

Q6 (cont.) (B)Why the image of the following transformation A is a plane

HINT:

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Let u, v, z be the eigenvectors of A

These three eigenvectors are linearly indep. (any pair of them are not parallel).

Hence any vector in 3-dim space can be written as $\alpha u + \beta v + \gamma z$ Then

$A(\alpha u + \beta v + \gamma z)$

$= \cdots = \alpha \lambda_1 u + \beta \lambda_2 v + \gamma \lambda_3 z$

where $\lambda_1, \lambda_2, \lambda_3$ are corresponding eigenvalues

Q6 (cont.) 2^{nd} method

First det of A is zero. So the image is either a plane or a st line. Then look at

$$Ai = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \qquad Aj = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} \qquad Aj = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

They are not parallel. Hence the image is a plane

(C) The plane is generated by

any two nonparallel vectors in the image of A

For example, we may choose any two from Ai,Aj,Ak and eigenvectors