NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

MA1506 Laboratory 2 (scilab)
Comments and Suggested Solutions
Semester II 2010/2011

Exercise 2A

1. This is because the exponential function grows too quickly and the number e^{4t} gets too large.

$$x = \frac{2 + 2Ae^{4t}}{1 - Ae^{4t}} = \frac{\frac{2}{e^{4t}} + 2A}{\frac{1}{e^{4t}} - A} \to \frac{2A}{-A} = -2.$$

The lesson to be learnt is that while computer plots are extremely useful, we should not trust them completely. Use your theoretical knowledge to check if you are doing the right thing.

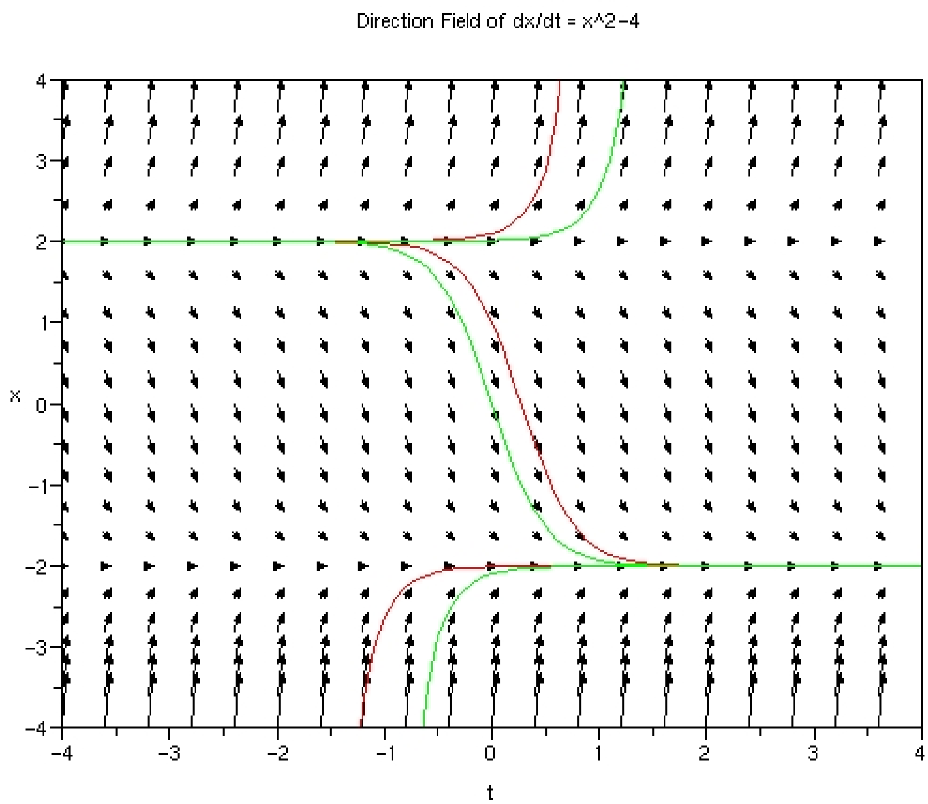
2.

$x(0)$	2.1	2.01	1	0	-2.01	-2.1
$A \approx$	0.0244	0.00244	0.333	1	401	41

Note that for $-2 < x < 2$, the curve is $x = \dfrac{2 - 2Ae^{4t}}{1 + Ae^{4t}}$.

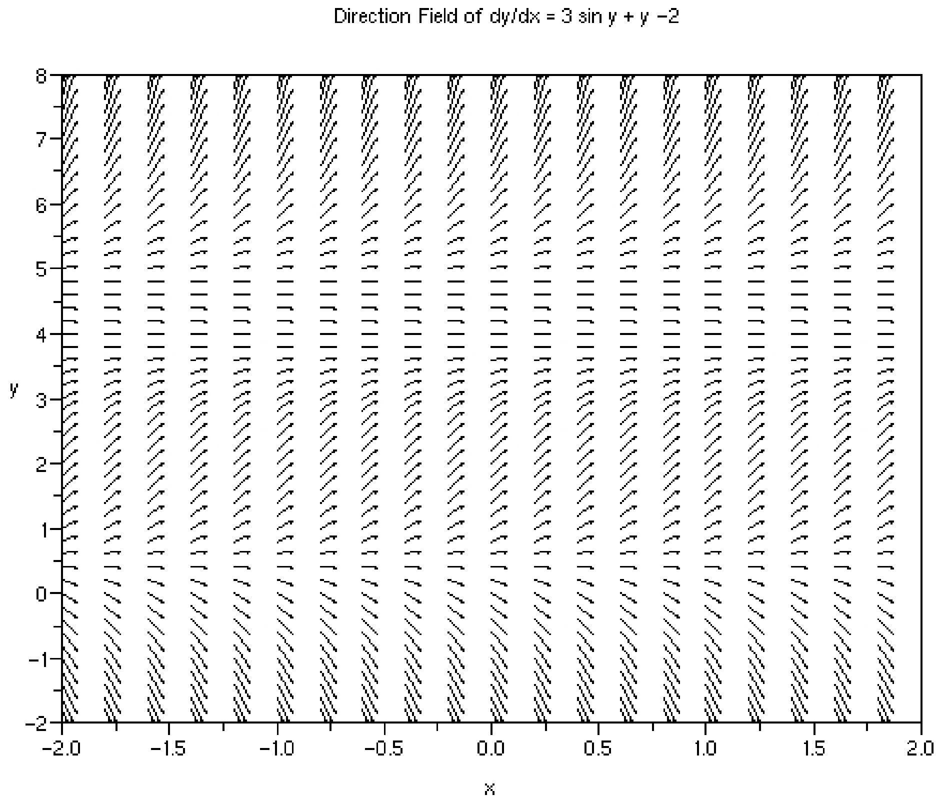
```
--> deff('[xdot]=f(a,x)','xdot=[1 ; x(2).^2-4]')
--> fchamp(f,0,-4:0.4:4,-4:0.4:4,1.5)
--> xlabel('t')
--> ylabel('x')
--> title( 'Direction Field of dx/dt = x^2 - 4')
--> t1=-4:0.1:0.7;
--> x1 = 2*(1+0.0244*exp(4*t1))./(1-0.0244*exp(4*t1));
--> plot(t1,x1,'r')
--> t2=-4:0.1:1.5;
--> x2 = 2*(1+0.00244*exp(4*t2))./(1-0.00244*exp(4*t2));
--> plot(t2,x2,'g')
--> t3=-4:0.1:4;
--> x3 = 2*(1-0.333*exp(4*t3))./(1+0.333*exp(4*t3));
--> plot(t3,x3,'r')
--> t4=-4:0.1:4;
--> x4 = 2*(1-exp(4*t4))./(1+exp(4*t4));
--> plot(t4,x4,'g')
--> t5=-1.4:0.1:4;
--> x5 = 2*(1+401*exp(4*t5))./(1-401*exp(4*t5));
```

```
--> plot(t5,x5,'r')
--> t6=-0.7:0.1:4;
--> x6 = 2*(1+41*exp(4*t6))./(1-41*exp(4*t6));
--> plot(t6,x6,'g')
--> mtlb_axis ([-4 4 -4 4]);
```



3. Some trial and error should show that the window $-2 \leq x \leq 2, -2 \leq y \leq 8$ is suitable.

```
--> deff('[xdot]=f(a,x)','xdot=[1 ; 3*sin(x(2))+x(2)-2]')
--> fchamp(f,0,-2:0.2:2,-2:0.2:8)
--> xlabel('x')
--> ylabel('y')
--> title('Direction Field of dy/dx = 3 sin y + y -2')
--> mtlb_axis([-2 2 -2 8]);
```



There are 3 equilibriums at $y \approx 5, 3.9$ and 0.5, with the equilibrium at $y \approx 3.9$ the only stable one. It is not difficult to use scilab to find the roots of $3 \sin y + y - 2$ more precisely, giving us 0.5170, 3.7745 and 4.9295.

```
--> function ftmp = f(y)
--> ftmp = 3*sin(y)+y-2
--> endfunction
--> fsolve(5,f)
--> fsolve(3.9,f)
--> fsolve(0.5,f)
```

The first three command together creates a function $f(y)$. The fsolve command will find the roots of a function, but we need to give it a starting point to search. Hence for the three roots we need to indicate the three approximate starting points.

Exercise 2B

1.
```
-->function ydot=f(x,y)
-->ydot=(x+1)/sqrt(x)
-->endfunction

-->t=1:0.1:4;
-->sol=ode(2,1,t,f);
-->sol($)
 ans  =

    8.6666661
```

   In this case since we only wanted the approximate value at $x = 4$, we could do away with the whole array t.

```
-->function ydot=f(x,y)
-->ydot=(x+1)/sqrt(x)
-->endfunction

-->ode(2,1,4,f)
 ans  =

    8.6666661
```

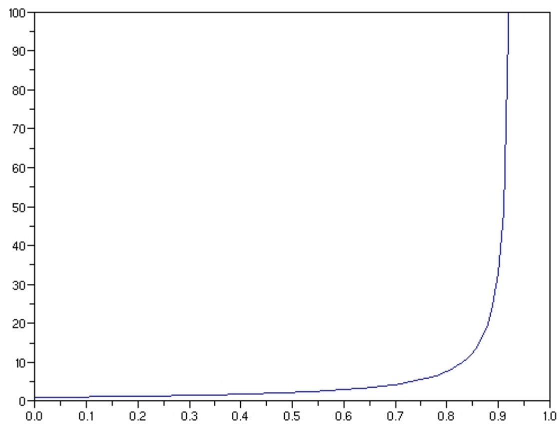
   Note that ode requires your input function to have two variables although this d.e. only has a single variable x.

2.
```
-->function xdot=f(t,x)
-->xdot= t+x^2
-->endfunction

-->t=0:0.01:1;
-->sol=ode(1,0,t,f);
-->plot(t,sol)
```

   The error message probably arose due to large numerical values. The software would also not allow you to plot the graph and would give you an error message: 'first and second arguments have incompatible dimensions'. Verify this using the browser variable function - the t variable should have 101 values but the sol variable would have less (only 94) because the ode program stops computing after the overflow. We can plot those 94 computed entries by

```
-->plot(t(1:94),sol)
-->mtlb_axis([0 1 0 100])
```



3. Using the following script file.

```
function xdot = myfunction(t,x)
xdot = [x(2) ; -2*x(2)-x(1) + 2+exp(2*t)];
endfunction
t=0:0.05:1;
x0 = [37/9; -7/9];
sol=ode(x0, 0, t, myfunction);
sol(1,$)

ans =

    3.9246649
```

—The End—