EEC 130A Introductory Electromagnetics I Midterm 1

Winter 2012

Solution

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Closed Text and Notes

1) Make sure you have 6 problems. Each problem is worth 20 points. Problem 6 is for extra credits.

2) Write only on the question sheets. Show all your work. If you need more space, please use the reverse side.

3) Write neatly. If your writing is illegible then please print.

4) A formula sheet will be provided separately from the problem sheets.

5) Smith charts are provided for Problem 1, 3, 5, and 6. You may not need any or all of them.

B

Print your name below after reading and verifying the above notes.

Name: ______________________________

1. (20 points) A lossless 100-Ω transmission line 0.3λ in length is terminated in an unknown impedance Z_L. The input impedance is measured to be Z_{in} = 40-j20 Ω,

(a) Use the Smith chart to find Z_L.
(b) Use the Smith chart to find VSWR.
(c) If a shunt resistor *R* is placed on the transmission line at a strategic distance *d* from the load, then it is possible to make Z_{in}=100 Ω. Find out *R* and *d*.

Solution:

10 (a). The normalized input impedance $z_{in} = \frac{Z_{in}}{Z_0} = \frac{40-j20}{100}$

$= 0.4 - j0.2$ 2

The location of z_{in} is shown on the Smith chart. (Point A) 2

Moving counter-clockwise from Point A by 0.3λ gives us the location of the normalized load. z_L (Point B) 2

Read z_L from the Smith chart

$z_L = 1 + j$ 2

and the load impedance $Z_L = z_L \cdot Z_0 = 100 + j100\ \Omega$. 2

5 (b). VSWR = 2.6.

5 (c). If a shunt resistor R can match the load to 100Ω, then it means that the input impedance at the location of the resistor must be purely real. From (FAEP2.30) we know that the voltage maximum point is such a location (Point C)

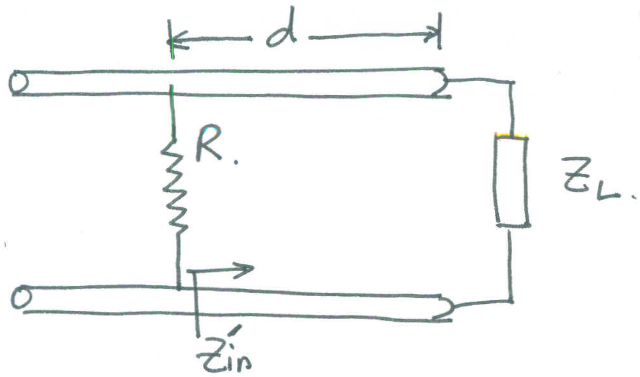

Fig. for 1-(c).

Z_{in}' must be purely real.

Then d can be read from the Smith chart

$d = 0.09\lambda$

At location d, $z_{in}' = 2.6$ (Note it has the same value as the VSWR. See practice problem P-14 if you are curious of why)

$Z_{in}' = z_{in}' \cdot Z_0 = 260\,\Omega$.

Since we are putting a resister in shunt.

$Z_{in}' \,//\, R = 100 \quad (\Omega)$.

Therefore $R = 162.5\ \Omega$

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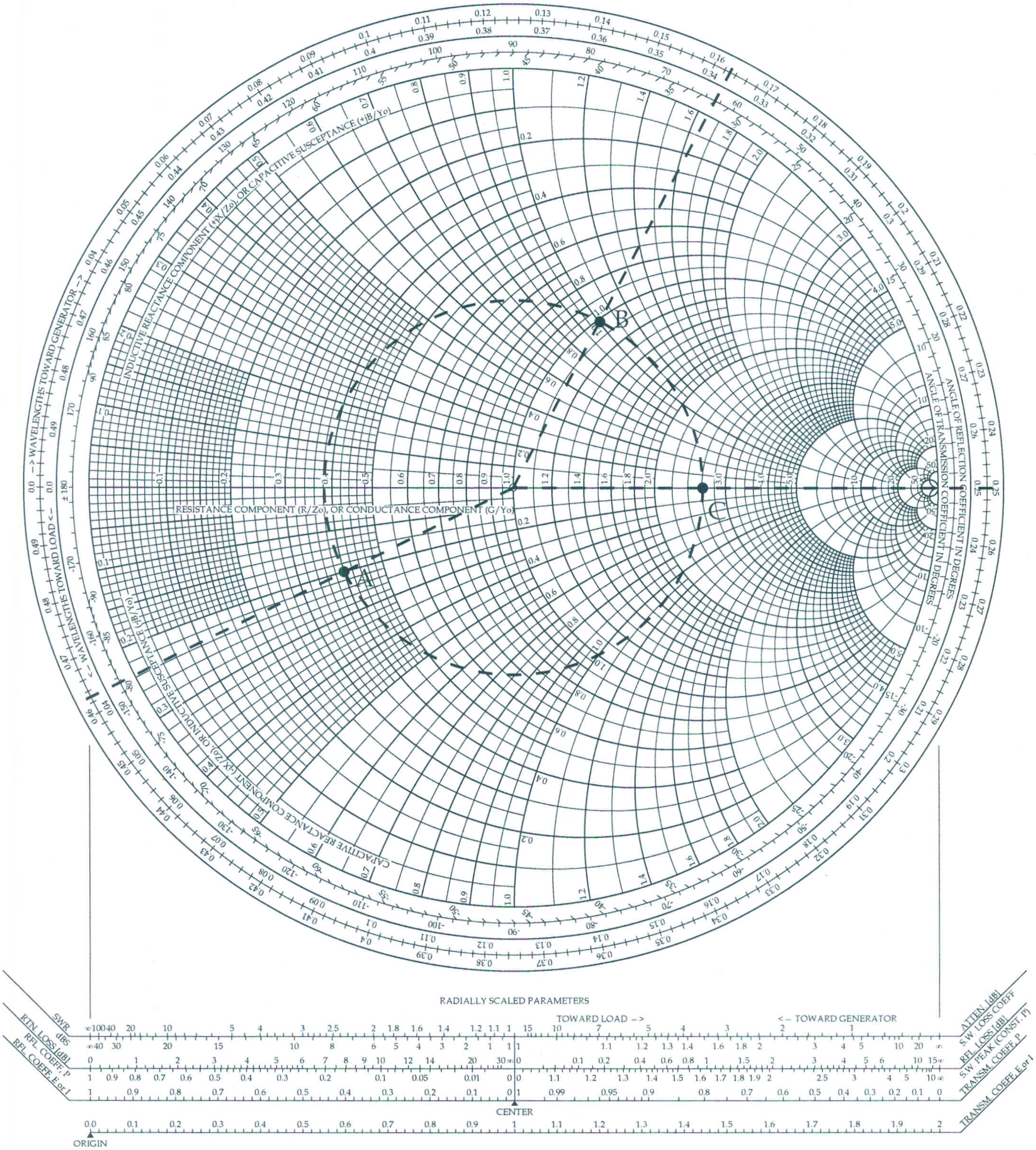
The Complete Smith Chart

Black Magic Design



2. (20 points)

(a) **Derive** an expression for the input impedance of a transmission line (with length l) terminated in an open circuit.

(b) Show that the characteristic impedance Z_0 and the propagation constant β of a transmission line (with length l) can be determined by its input impedance $Z_{in,SC}$ when the line is terminated in short circuit and its input impedance $Z_{in,OC}$ when the line is terminated in open circuit.

Solution:

12 (a). For open circuit termination, $Z_L = \infty$ 4

$$Z_{in} = Z_o \cdot \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l} = Z_o \frac{1 + j\frac{Z_o}{Z_L}\tan\beta l}{\frac{Z_o}{Z_L} + j\tan\beta l}$$ 4

Since $Z_L = \infty$, $\frac{Z_o}{Z_L} \to 0$,

therefore $Z_{in,oc} = Z_o \frac{1}{j\tan\beta l}$ 4

8 (b). Similarly, $Z_{in.sc} = jZ_o \tan\beta l$. 4

We realize that

$$Z_{in.oc} \cdot Z_{in.sc} = Z_o^2$$

and $\frac{Z_{in.sc}}{Z_{in.oc}} = -\tan^2\beta l$

Therefore,

$$Z_o = \sqrt{Z_{in.sc} \cdot Z_{in.sc}}$$ 2

$$\beta = \frac{\tan^{-1}(\sqrt{-Z_{in.sc}/Z_{in.oc}})}{l}$$ 2

3. (20 points) As shown in Fig. 1, a capacitor C=1 pF is connected to a 50-Ω air filled transmission line of length l=6 mm.

(a) Calculate the load reflection Γ_L **without** using the Smith chart.
(b) Calculate the input impedance Z_{in} at 1 GHz.
(c) Calculate the frequency at which the transmission line is a quarter wavelength.

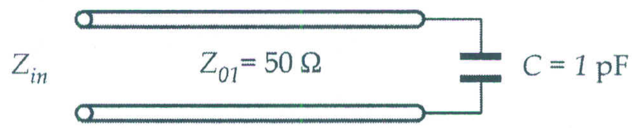


Figure 1 Circuit for Problem 1

Solution:

8 (a). $Z_L = \frac{1}{j\omega C} = \frac{1}{j2\pi\times10^9\times1\times10^{-12}} = -j160\,\Omega$ 4

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-j160-50}{-j160+50} = 0.82 - j0.57$$

$$= 1\angle -34.7^\circ$$ 4

8 (b). Since the transmission line is in air, the phase velocity is equal to the speed of light.

The wavelength $\lambda = \frac{c}{f} = \frac{3\times10^8}{10^9} = 300$ mm. 4

Therefore $Z_{in} = Z_0 \dfrac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}$

$$= 50\cdot\frac{-j160 + j50\tan(\frac{2\pi}{300}\cdot6)}{50 + j(-160j)\tan(\frac{2\pi}{300}\cdot6)}$$

$$= -j109.5\ \Omega$$ 4

(C). If the transmission line is a quarter wavelength,

4 then the wavelength needs to be $6 \times 4 = 24$ mm.

Frequency $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{24 \times 10^{-3}} = 12.5$ GHz.

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4. (20 points) In the lectures we talked about microstrip lines, which is a type of transmission line often used in high frequency circuits. Another popular type of transmission line is the coplanar waveguide (CPW), which is shown in Fig. 2. A CPW line consists of a center conductor and a pair of ground planes, all on top of a substrate. Like microstrip lines, CPW lines are quasi-TEM transmission lines. An advantage of CPW lines is that it is a lot easier to add both series and shunt lumped circuit components because the signal trace and grounds are all on the same plane.

A CPW line is measured to have characteristic impedance of Z_L=50 Ω, attenuation constant α=18.8 Np/m and β=46 rad/m at 1 GHz[1]. Find the line parameters R′, L′, G′, and C′.

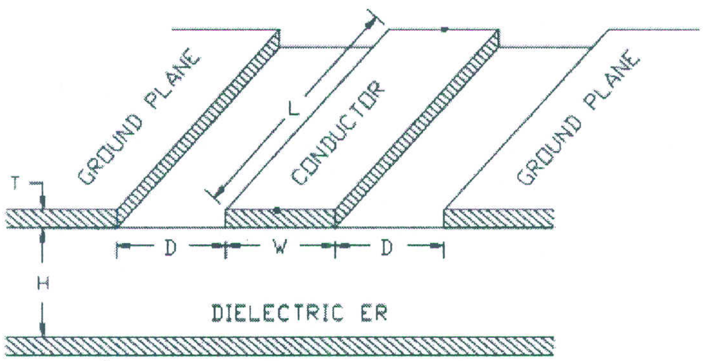

Figure 2 Coplanar waveguide (CPW)[2]

Solution

The general expression of the characteristic impedance

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

4

Z_0 is complex except when

(a) the transmission Line is lossless. i.e. $R' = G' = 0$.

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[1] Williams, D.F.; Marks, R.B., "Accurate transmission line characterization," Microwave and Guided Wave Letters, vol.3, no.8, pp.247-249, Aug 1993.

[2] http://cp.literature.agilent.com/litweb/pdf/ads2008/ccdist/ads2008/CPWG_(Coplanar_Waveguide_with_Lower_Ground_Plane).html

(b). the transmission line is distortionless.

i.e. $\frac{R'}{G'} = \frac{L'}{C'} = a$, where a is a constant. 4

Because we have attenuation for this CPW line and it characteristic impedance Z_0 is real. it must be a distortionless line.

Then we have,

$$Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} = \sqrt{\frac{aG'+j\omega C'\cdot a}{G'+j\omega C'}} = \sqrt{a}. = 50.$$

So $a = 2500$.

And we know $\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$

$$= \sqrt{(aG'+j\omega aC')(G'+j\omega C')}$$

$$= \sqrt{a}\,(G'+j\omega\ C')$$

$$= \sqrt{a}\cdot G' + j\omega\sqrt{a}\,C' = \alpha + j\beta$$

Therefore.

$$G' = \frac{\alpha}{\sqrt{a}} = \frac{18.8}{50} = 0.376 \text{ S/m}$$ 3

$$C' = \frac{\beta}{\omega\sqrt{a}} = \frac{46}{2\pi\times10^9\times50} = 146 \text{ pF/m}.$$ 3

$$L' = a\cdot C' = 2500\cdot 146\times10^{-12} = 365 \text{ nH/m}$$ 3

$$R' = a\cdot G' = 0.376\times2500 = 940\ \Omega/\text{m}.$$ 3

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5. (20 points) An amplifier designer comes to you for help with matching his amplifier to an antenna, both working at 2.4 GHz. The output impedance of the amplifier and the antenna are 12.5 Ω and 75-j25 Ω. Please use the single stub matching method to do the matching. Find out *d* and *l*. You will work with 12.5-Ω microstrip lines with an effective relative permittivity of ε_{eff}=4. Assume that the via is an ideal short. You may use the Smith chart to help you.

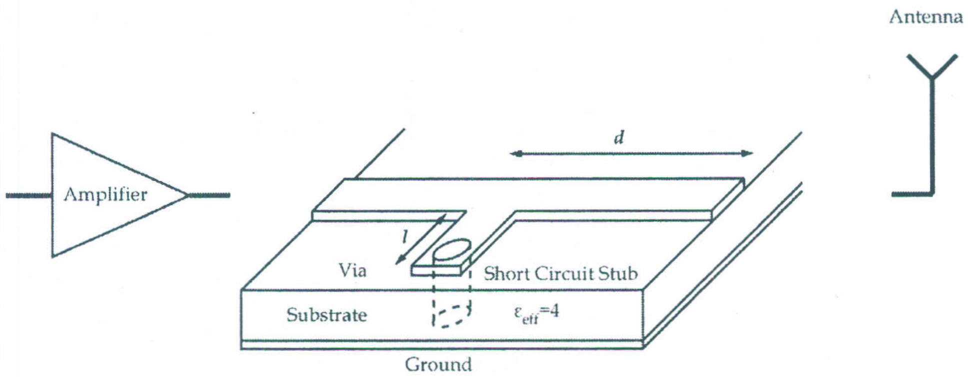
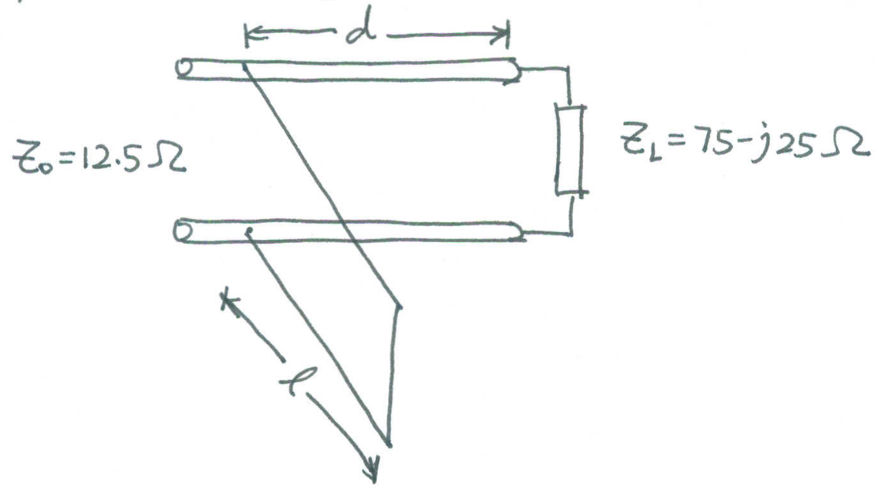


Figure 3 Circuit for Problem 5

Solution.

This problem is a standard single stub matching problem with system impedance $Z_o = 12.5\Omega$ and $Z_L = 75 - j25\Omega$



Step 1: Find the normalized load impedance z_L and its location on the Smith chart. (Point A)

$$z_L = \frac{Z_L}{Z_0} = \frac{75-j25}{12.5} = 6-j2 \text{ .}$$

2

Step 2: Find the normalized load admittance y_L on the Smith chart. (Point B). 2

Step 3: Move y_L (Point B) along the constant-SWR circle towards the generator (clockwise) to intersect with the constant-$r=1$ circle at Point C.

The difference in wavelength between Point B & C gives you

$$d = 0.183\lambda$$

4

Step 4: Find the imaginary part of Point C from the Smith chart.

$$b = 2.1$$

4

Step 5: Find l by either

(a) Setting $jZ_0 \tan\beta l = -jbZ_0$ which gives you

$$l = \frac{\tan^{-1}(-b)}{\beta} = 0.068\lambda$$

or (b) Finding the $y_s = -jb$ point (Point D) on the Smith chart and measuring the difference in wavelength between Point D and the admittance short circuit point (Point E)

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$$l = 0.068\lambda$$

4

In order to find out the exact value of d and ℓ.
we need to know the wavelength λ.

$$\lambda = \frac{u_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{3\times10^8}{2.4\times10^9\times\sqrt{4}} = 62.5 \text{ mm.}$$ 2

Therefore $d = 11.4$ mm.

2

$\ell = 4.25$ mm.

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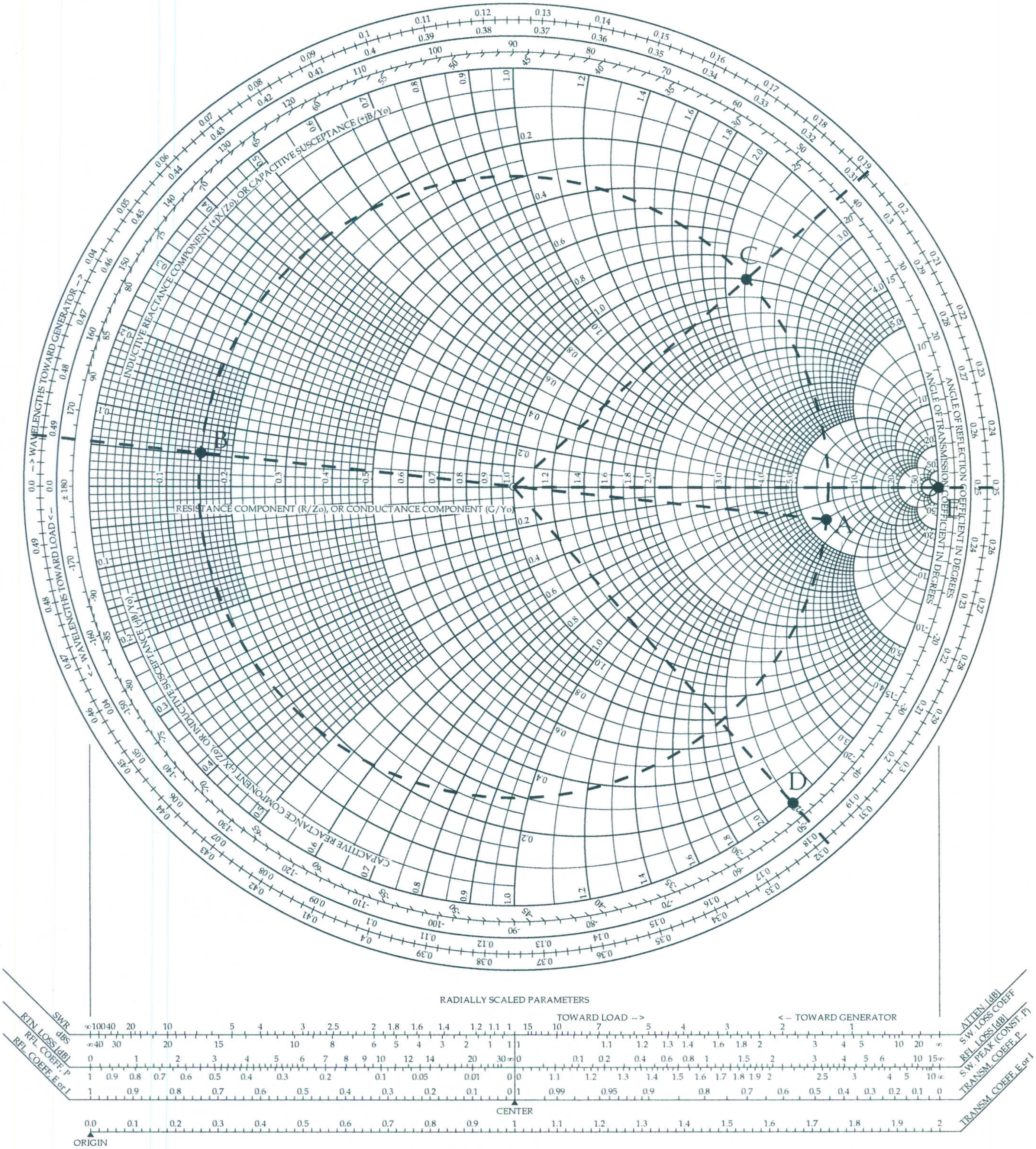
The Complete Smith Chart

Black Magic Design



6. **(Extra Credit: 20 points)** In modern high frequency integrated circuit designs, impedance matching is often done using all lumped elements to save space. Fig. 4 shows two matching circuits using series and shunt reactive elements only. Fig. 5 illustrates how the matching network in circuit #1 works with the help of a constant-g circle (dashed).

(a) Illustrate on the Smith chart how matching is achieved in circuit #2. You may arbitrarily choose your z_L (except for z_L=1 of course) to best illustrate the matching process.

(b) Given a normalized load impedance of z_L=0.2+j, find the right x and b values to achieve matching.

(c) It is known that circuit #2 can never achieve matching for some values of z_L. Either give a general expression or point out on the Smith chart what z_L values can not be matched.

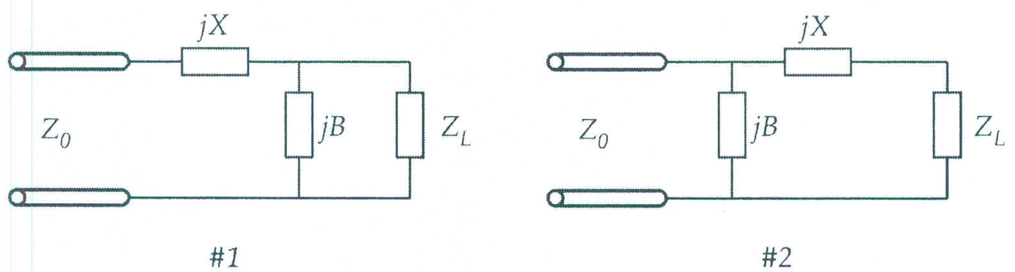


Figure 4 Lumped element matching circuits for Problem 6

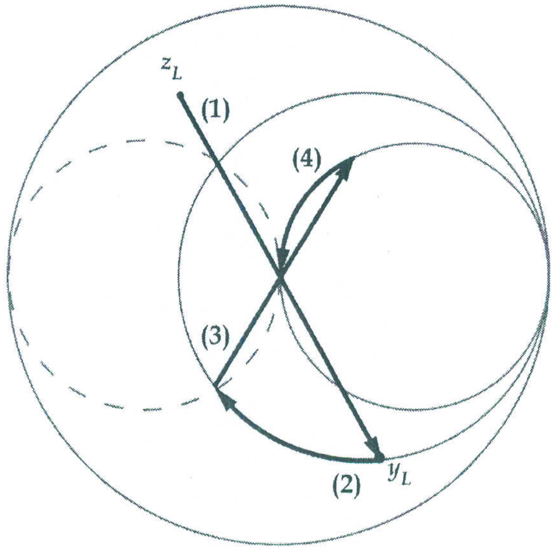
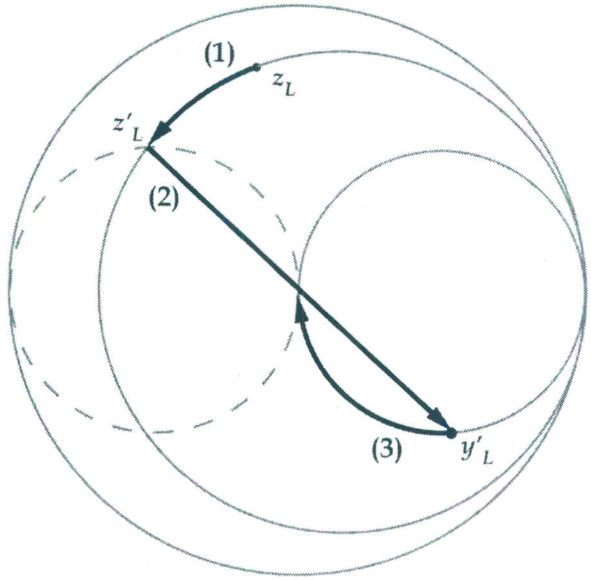


Figure 5 Smith chart illustration for impedance matching using Circuit #1

Solution:

(a) One possible solution looks like this. 10



Step (1): The series reactive element jx moves the load along a constant-r circle until it hit the constant g=1 circle, which is simply a mirror of the constant-r=1 circle.

Step (2): Find the corresponding admittance y'_L. Since the constant-g=1 circle and the constant-r=1 circle are mirror of each other, y'_L should sit on the constant-r=1 circle.

Step (3): The shunt reactive element jb moves y'_L along the constant-r=1 circle to the origin.

(b) The Smith chart solution is given on the next page. You could also solve the problem mathematically. 6

The total normalized input admittance of the circuit should be 1.

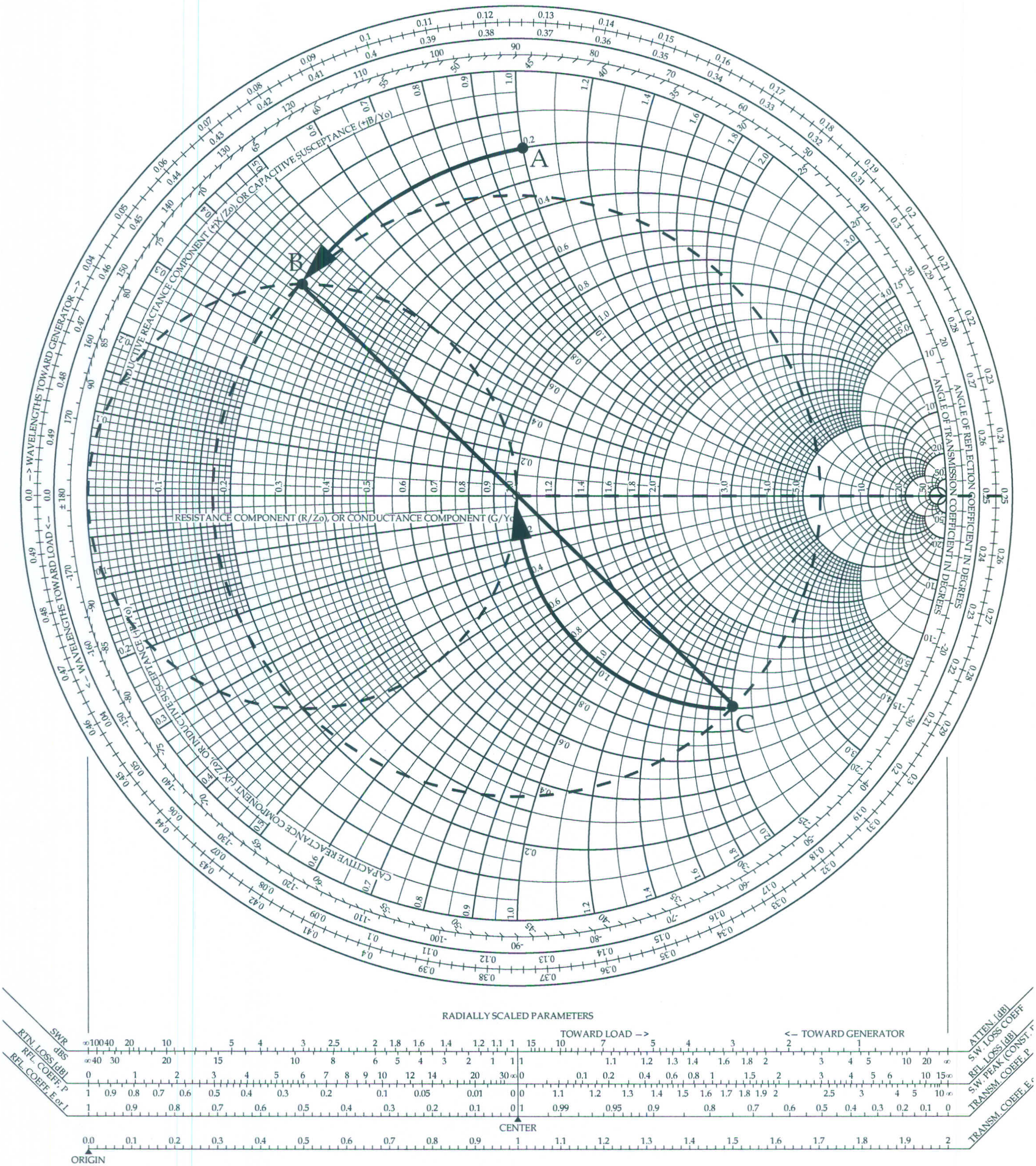
$$y_{in} = \frac{1}{0.2 + j(1+x)} + jb = 1$$

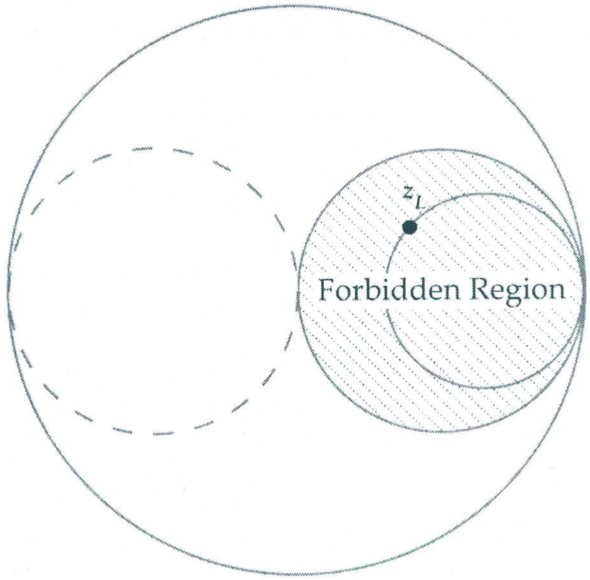
Solving the above equation gives x=0.6, b=2.

(c) Shown below. If the z_L lies inside the constant-r=1 circle, then you can never move to to the constant-g=1 circle no matter what x value you have. 4

The Complete Smith Chart

Black Magic Design

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