CS3241 Computer Graphics

Tutorial #6

Question #1

- Prove that the subdivision method draws a cubic Bezier curve with control points c_0, c_1, c_2 and c_3. Hint: derive the formula from the subdivision method (e.g. $c_{11} = (1\text{-}t)\cdot c_1 + t\cdot c_2$) and try to show the final formula is:

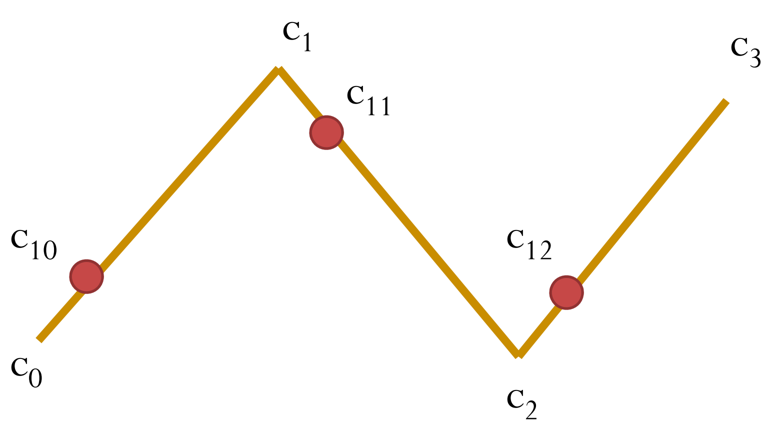
$$Q(t) = \sum_{i=0}^{3} \binom{3}{i} t^i (1-t)^{3-i} c_i$$

Question #1

- Given 4 control points c_0, c_1, c_2 and c_3

- Level 1

$$c_{10} = (1-t)c_0 + tc_1$$
$$c_{11} = (1-t)c_1 + tc_2$$
$$c_{12} = (1-t)c_2 + tc_3$$



Question #1

- Given 4 control points c_0, c_1, c_2 and c_3

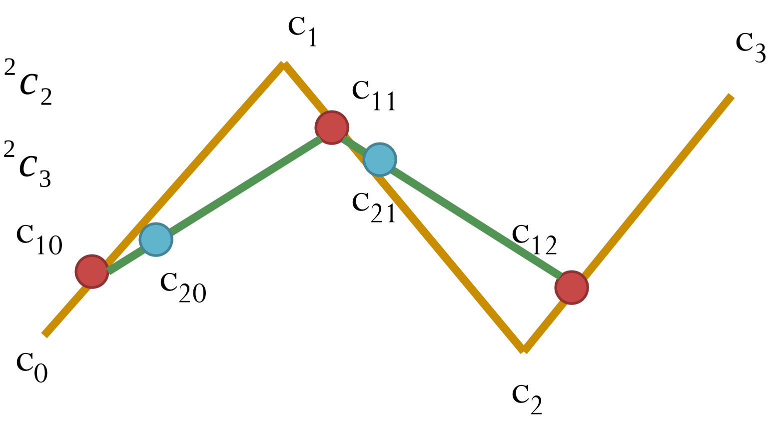
- Level 1

$$c_{10} = (1-t)c_0 + tc_1 \qquad c_{11} = (1-t)c_1 + tc_2 \qquad c_{12} = (1-t)c_2 + tc_3$$

- Level 2

$$c_{20} = (1-t)c_{10} + tc_{11} = (1-t)^2 c_0 + 2(1-t)tc_1 + t^2 c_2$$

$$c_{21} = (1-t)c_{11} + tc_{12} = (1-t)^2 c_1 + 2(1-t)tc_2 + t^2 c_3$$



Question #1

- Given 4 control points c_0, c_1, c_2 and c_3

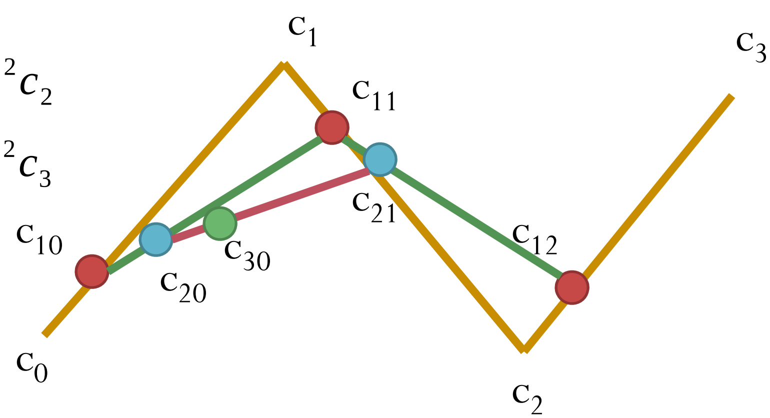
- Level 1

$$c_{10} = (1-t)c_0 + tc_1 \qquad c_{11} = (1-t)c_1 + tc_2 \qquad c_{12} = (1-t)c_2 + tc_3$$

- Level 2

$$c_{20} = (1-t)c_{10} + tc_{11} = (1-t)^2 c_0 + 2(1-t)tc_1 + t^2 c_2$$

$$c_{21} = (1-t)c_{11} + tc_{12} = (1-t)^2 c_1 + 2(1-t)tc_2 + t^2 c_3$$



- Level 3

$$c_{30} = (1-t)c_{20} + tc_{21} = (1-t)^3 c_0 + 3(1-t)^2 tc_1 + 3(1-t)t^2 c_2 + t^3 c_3 = Q(t)$$

Question #2

- Differentiate the following Bezier curve with respect to *t*:

$$Q(t) = \sum_{i=0}^{3} \binom{3}{i} t^i (1-t)^{3-i} c_i$$

$$\Downarrow$$

$$Q(t) = (1-t)c_{20} + tc_{21} = (1-t)^3 c_0 + 3(1-t)^2 t c_1 + 3(1-t)t^2 c_2 + t^3 c_3$$

Question #2

- Differentiate

$$Q(t) = (1-t)^3 c_0 + 3(1-t)^2 t c_1 + 3(1-t) t^2 c_2 + t^3 c_3$$

$$-3(1-t)^2 c_0$$

$$6(1-t)tc_2 - 3t^2 c_2$$

$$3t^2 c_3$$

$$3(1-t)^2 c_1 - 6(1-t)tc_1$$

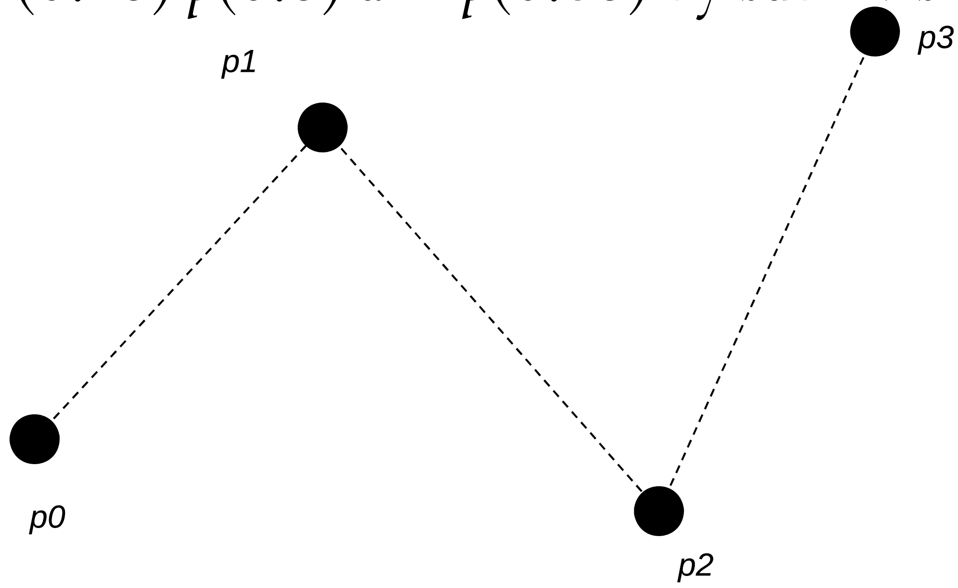
$$\frac{dQ(t)}{dt} = 3(1-t)^2 (c_1 - c_0) + 6(1-t)t(c_2 - c_1) + 3t^2 (c_3 - c_2)$$

$$\frac{dQ(t)}{dt} = (1-t)^2 3(c_1 - c_0) + 2(1-t)t3(c_2 - c_1) + t^2 3(c_3 - c_2)$$

A degree 2 Bezier curve with control points $3(c_1\text{-}c_0)$, $3(c_2\text{-}c_1)$, and $3(c_3\text{-}c_2)$,

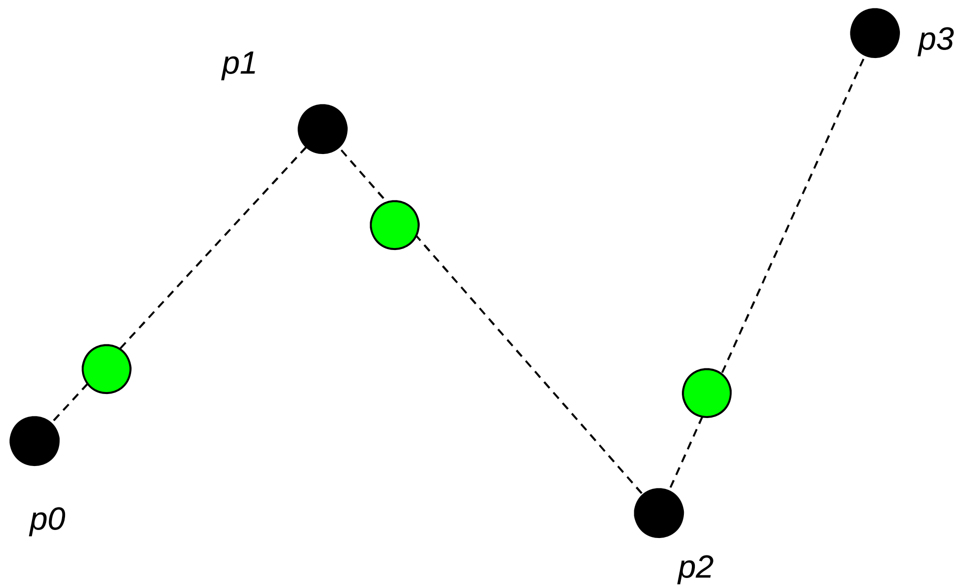
Question #3

- Given the following control points of a Bezier curve, compute $p(0.25)$ $p(0.5)$ and $p(0.75)$ by subdivision method.



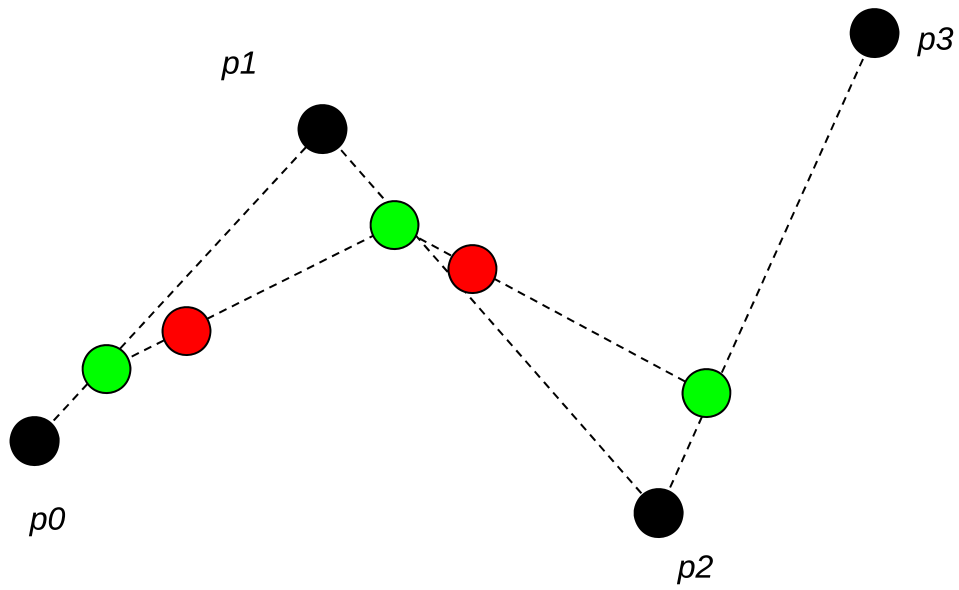
Question #3

- $p(0.25)$



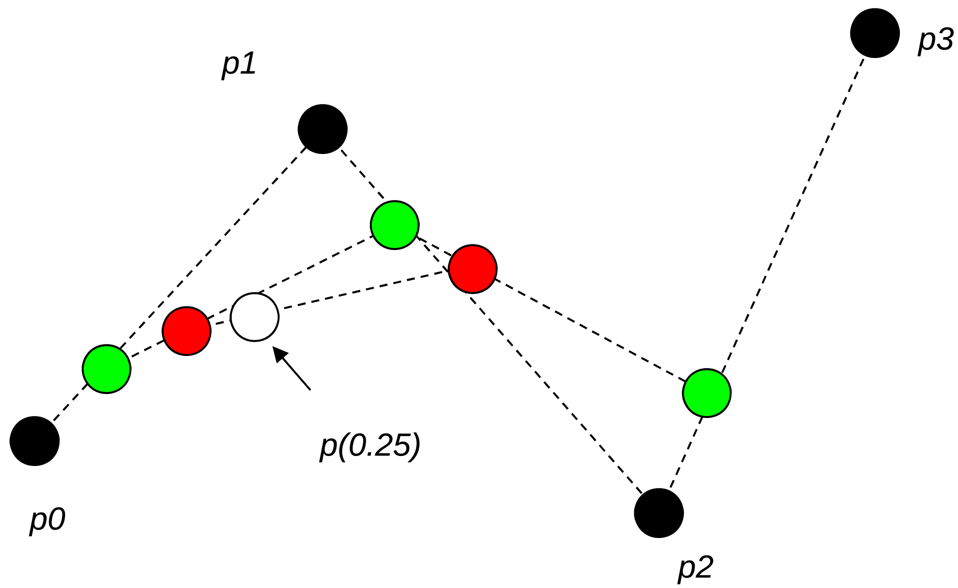
Question #3

- $p(0.25)$



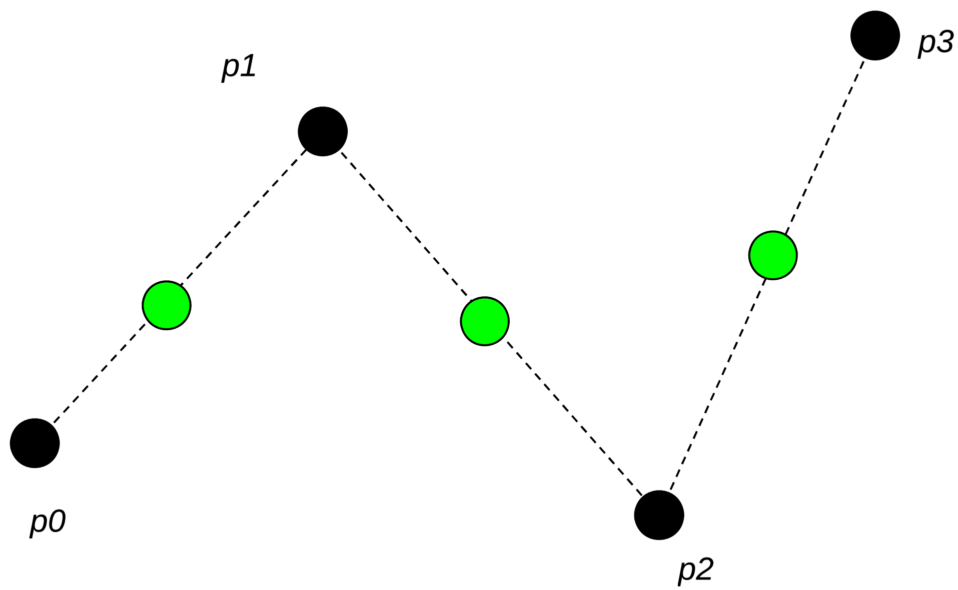
Question #3

- $p(0.25)$



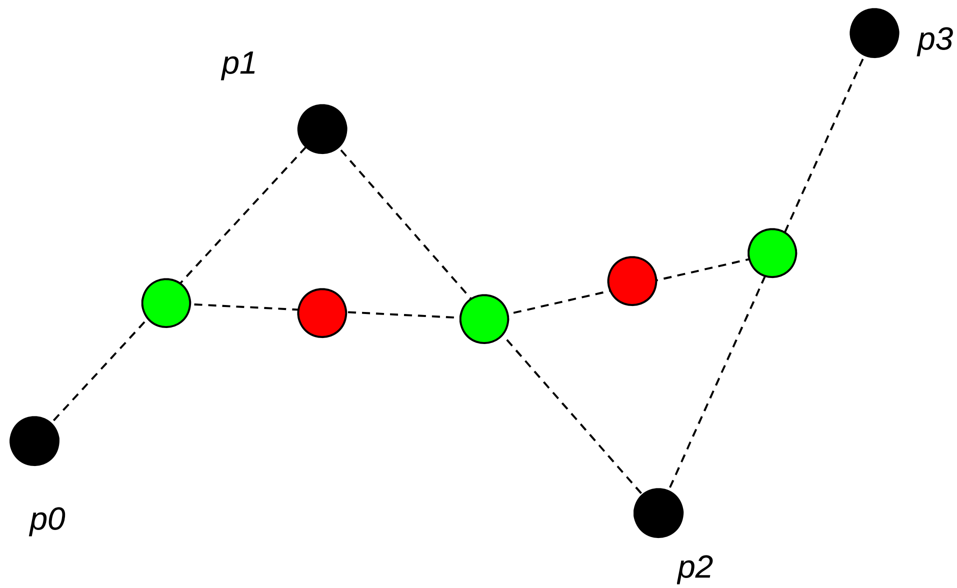
Question #3

- $p(0.5)$



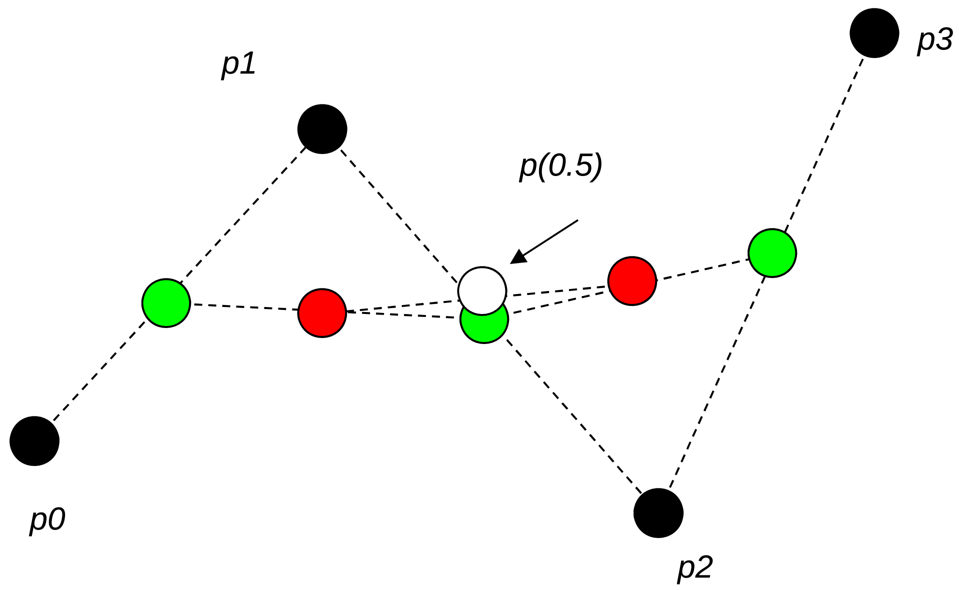
Question #3

- $p(0.5)$



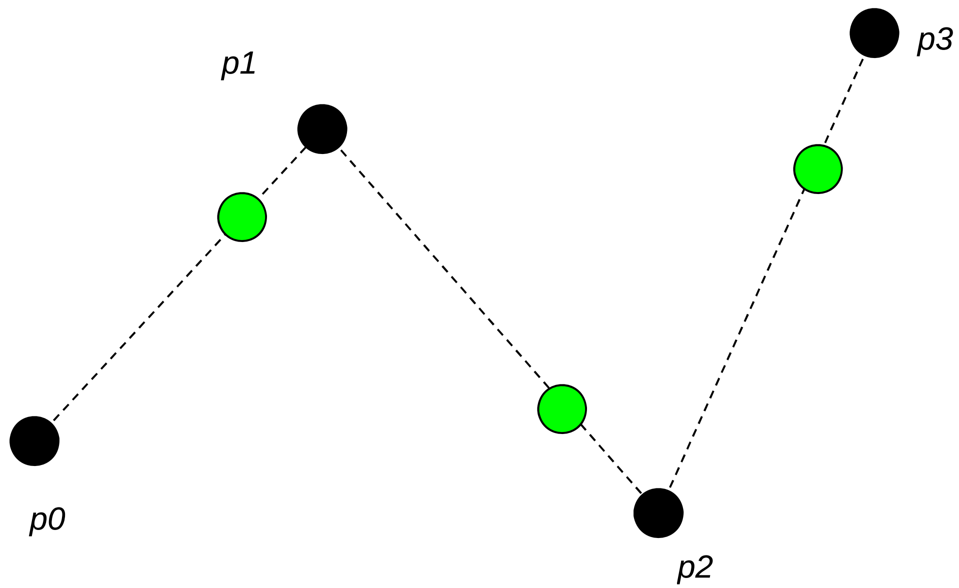
Question #3

- $p(0.5)$



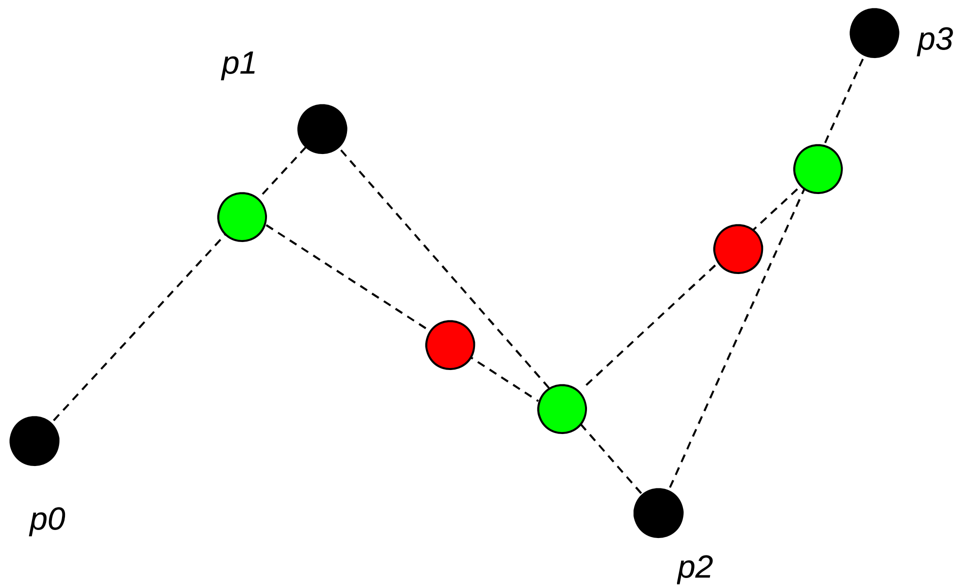
Question #3

- $p(0.75)$



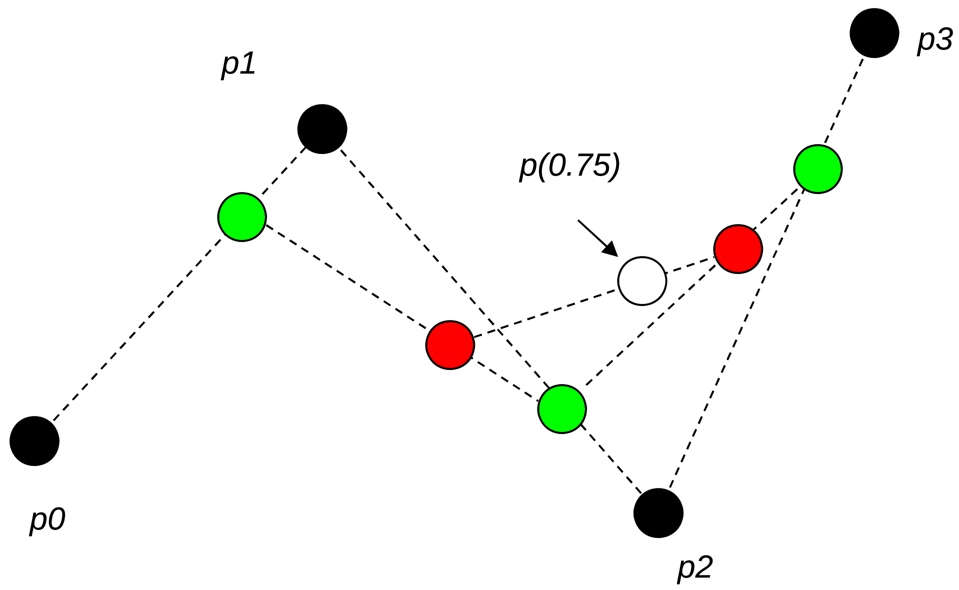
Question #3

- $p(0.75)$



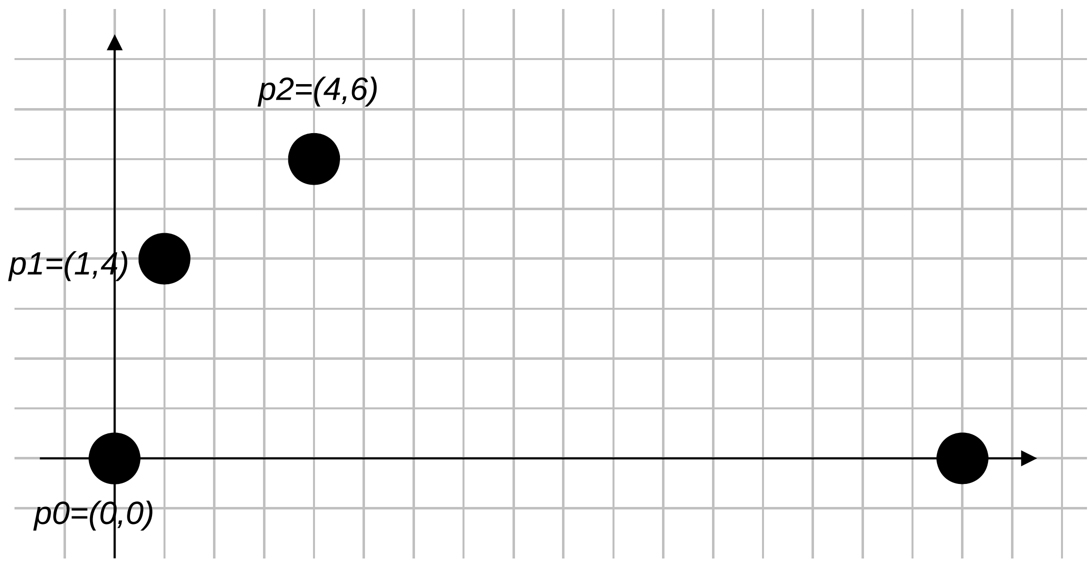
Question #3

- $p(0.75)$



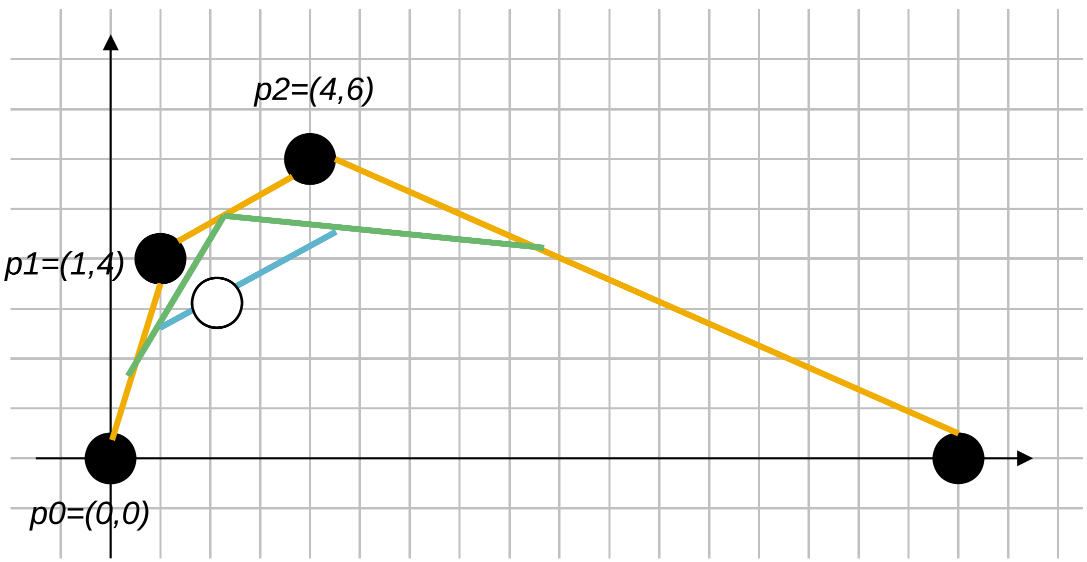
Question #4a

- Compute and connect points *p(0.0)*, *p(1/3)*, *p(2/3)*, and *p(1)* in the Bezier curve of the following diagram by calculation from the Bezier equations.



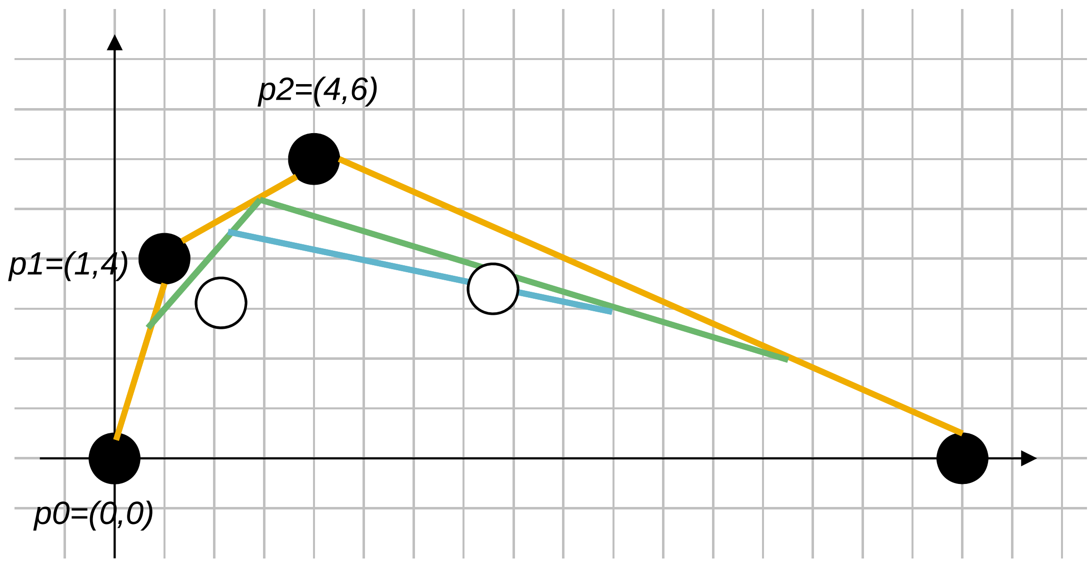
Question #4a

- Compute and connect points *p(0.0)*, *p(1/3)*, *p(2/3)*, and *p(1)* in the Bezier curve of the following diagram by calculation from the Bezier equations.



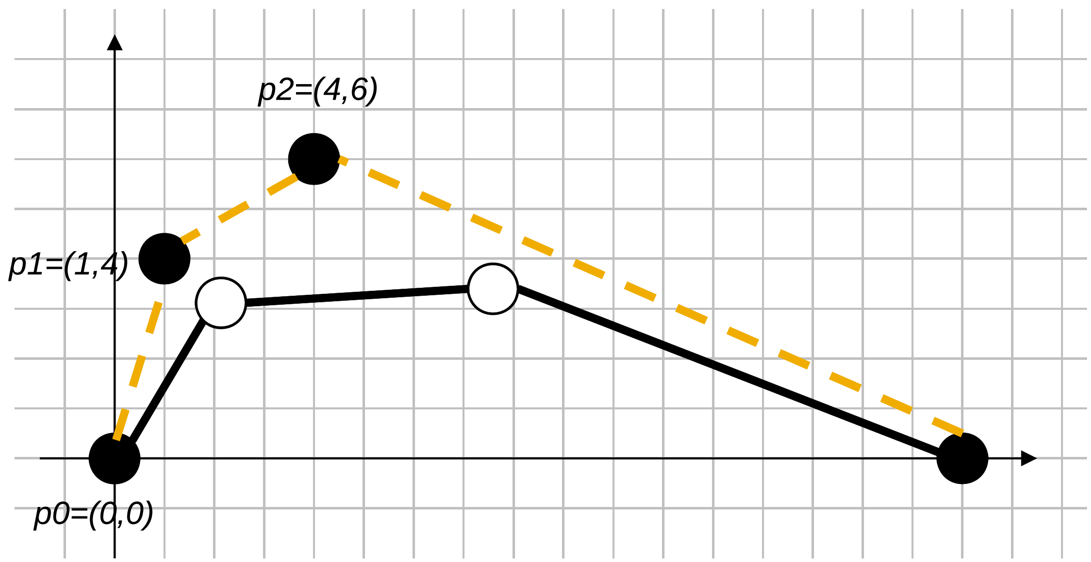
Question #4a

- Compute and connect points *p(0.0)*, *p(1/3)*, *p(2/3)*, and *p(1)* in the Bezier curve of the following diagram by calculation from the Bezier equations.



Question #4a

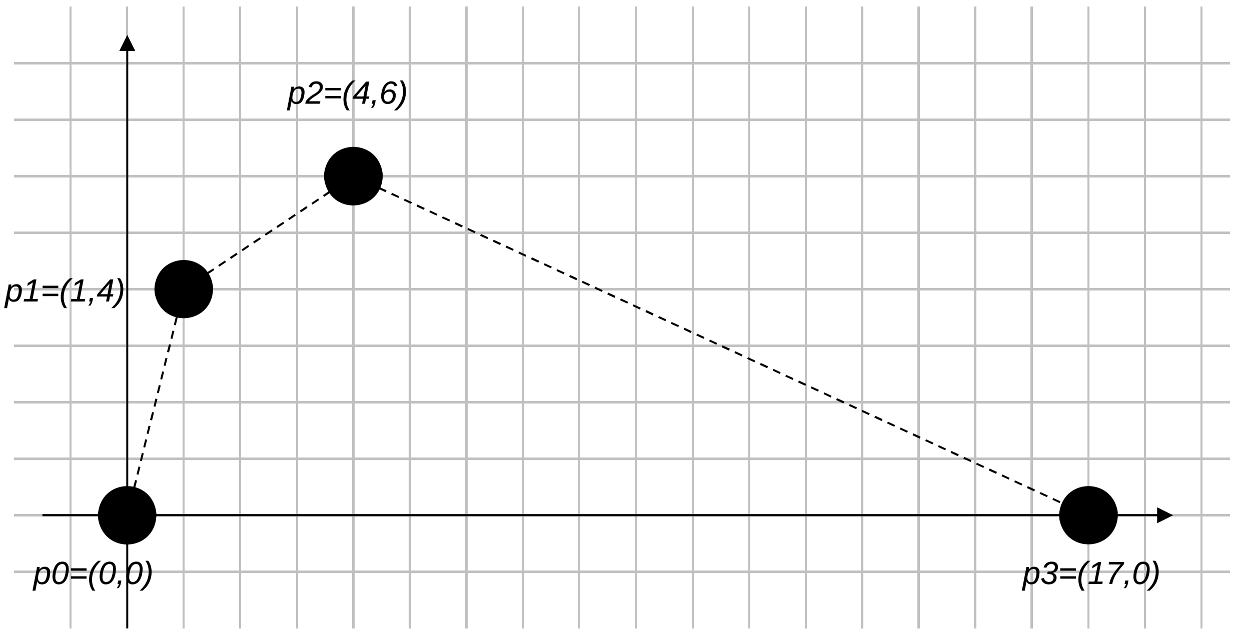
- Compute and connect points *p(0.0)*, *p(1/3)*, *p(2/3)*, and *p(1)* in the Bezier curve of the following diagram by calculation from the Bezier equations.



Question #4b

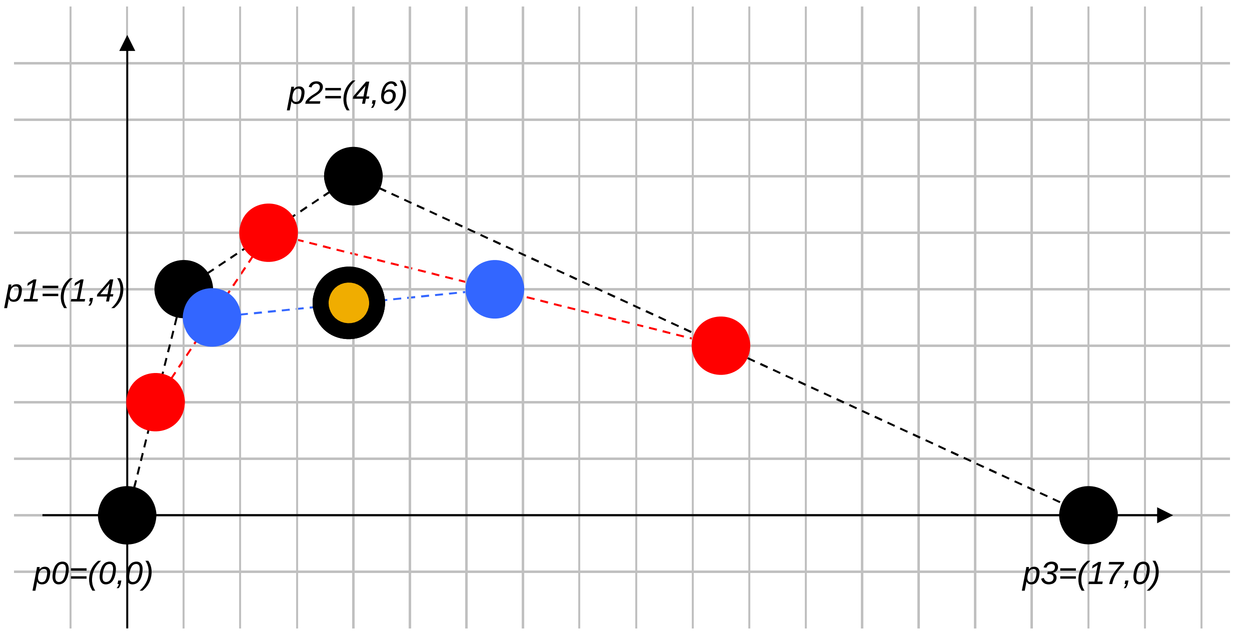
- With the same setting compute p(0.5) first by subdivision method, then the curve is divided into two smaller Bezier curves, choose the longer one and subdivide it once more. Compare it with the previous computation.

Question 4b



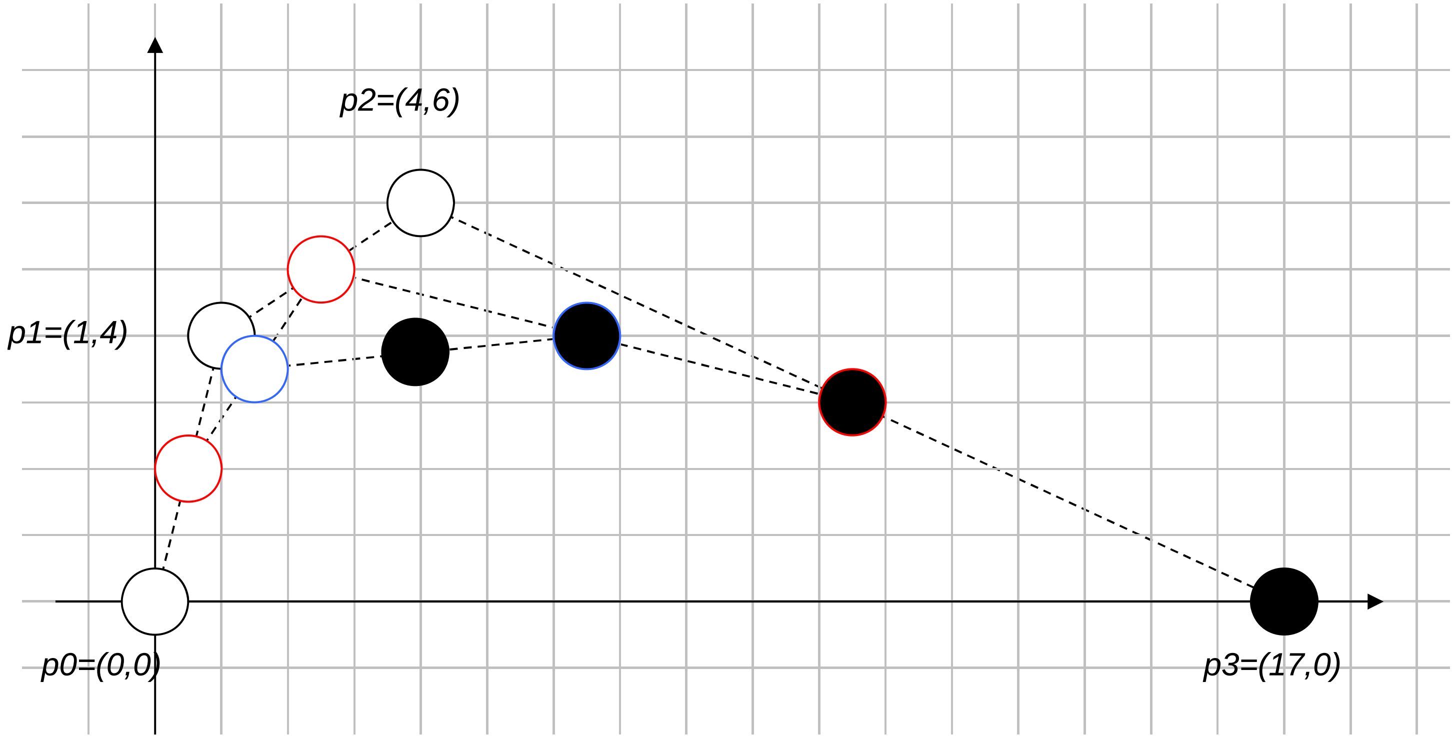
Question #4b

- p(0.5)



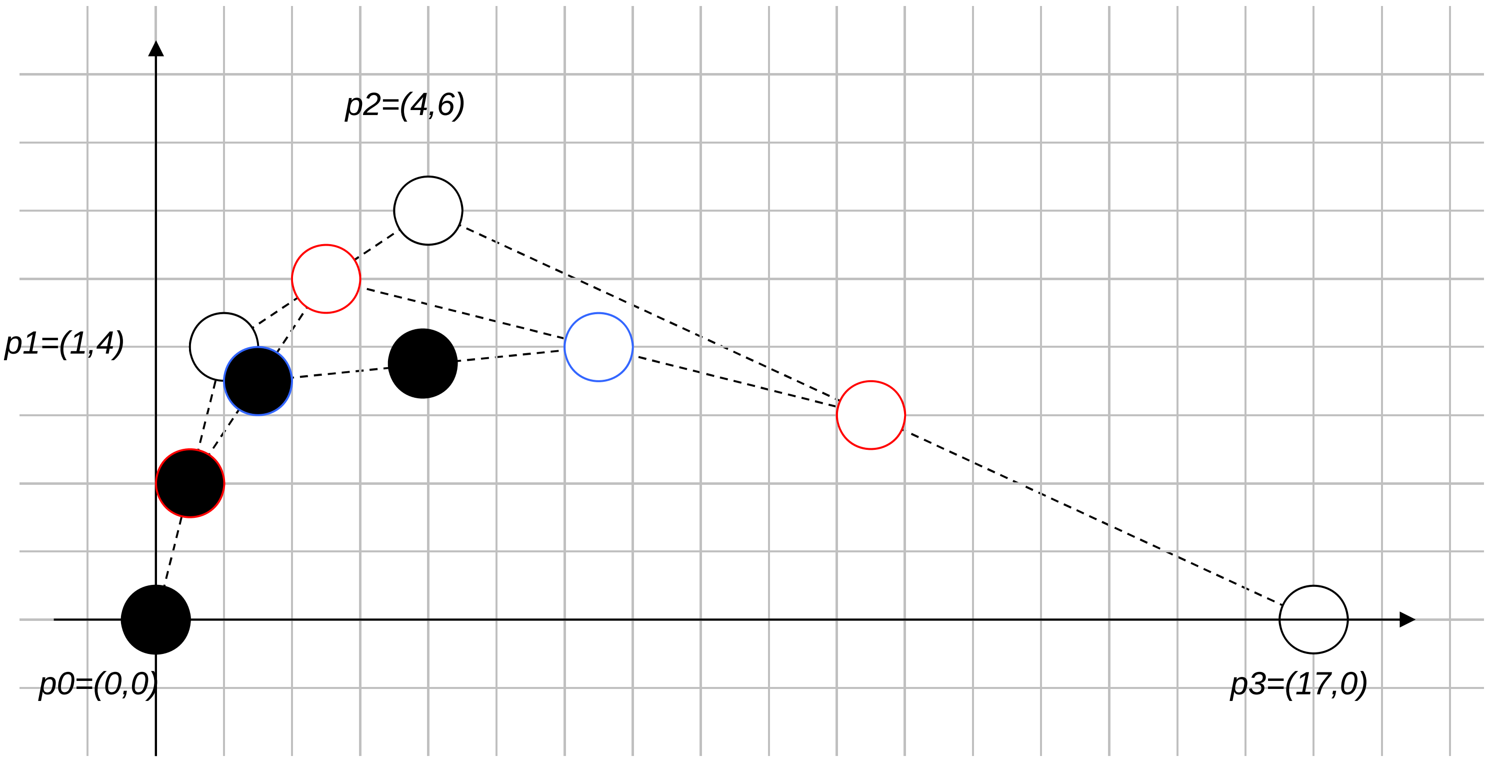
Question #4b

- 2 sides of the curve



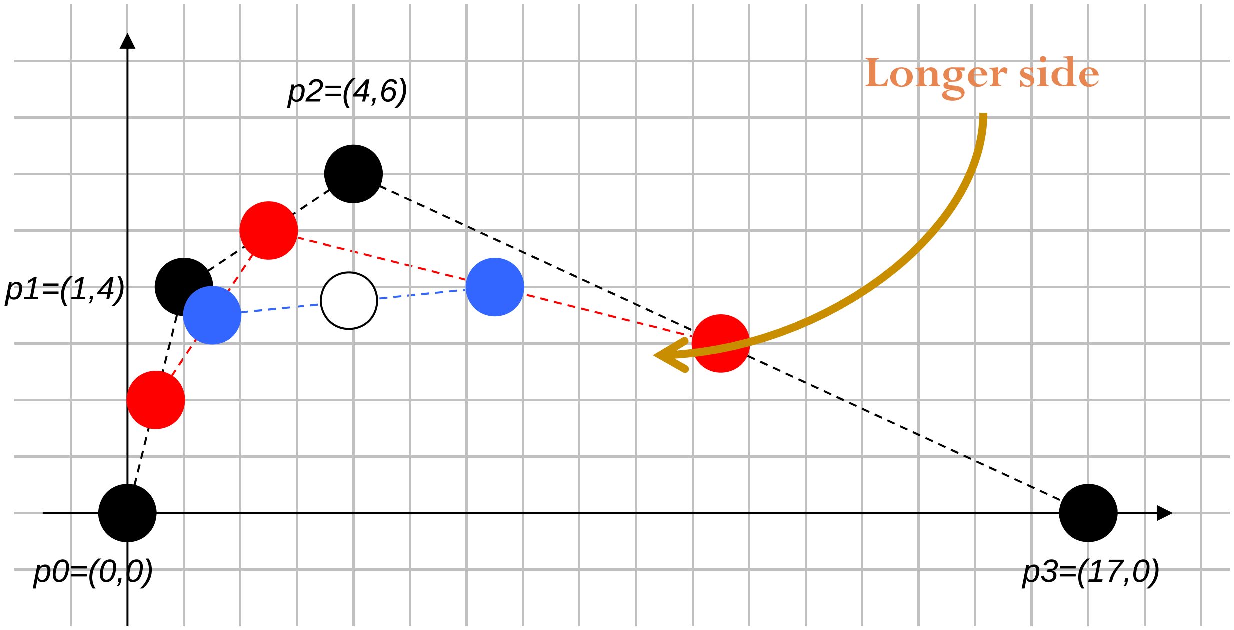
Question #4b

- 2 sides of the curve



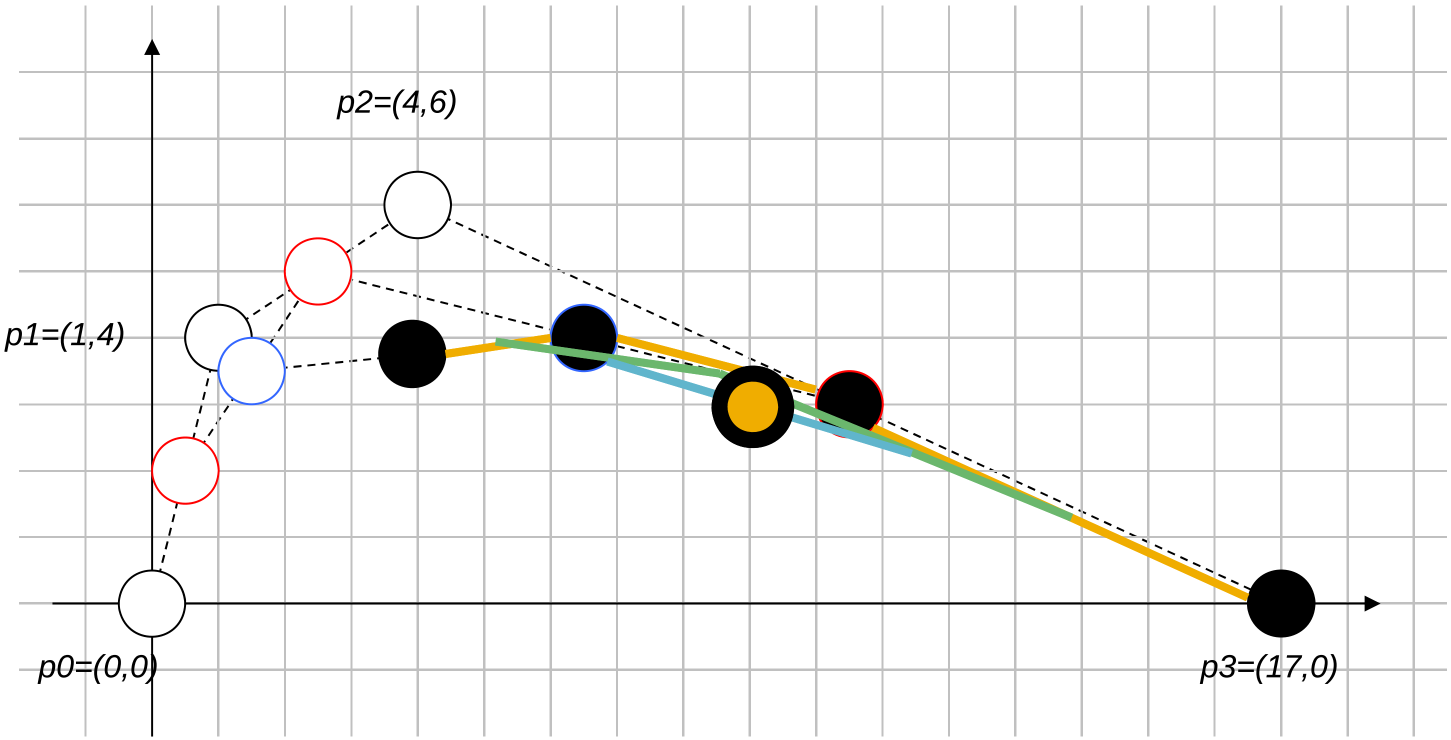
Question #4b

- Subdivide longer side



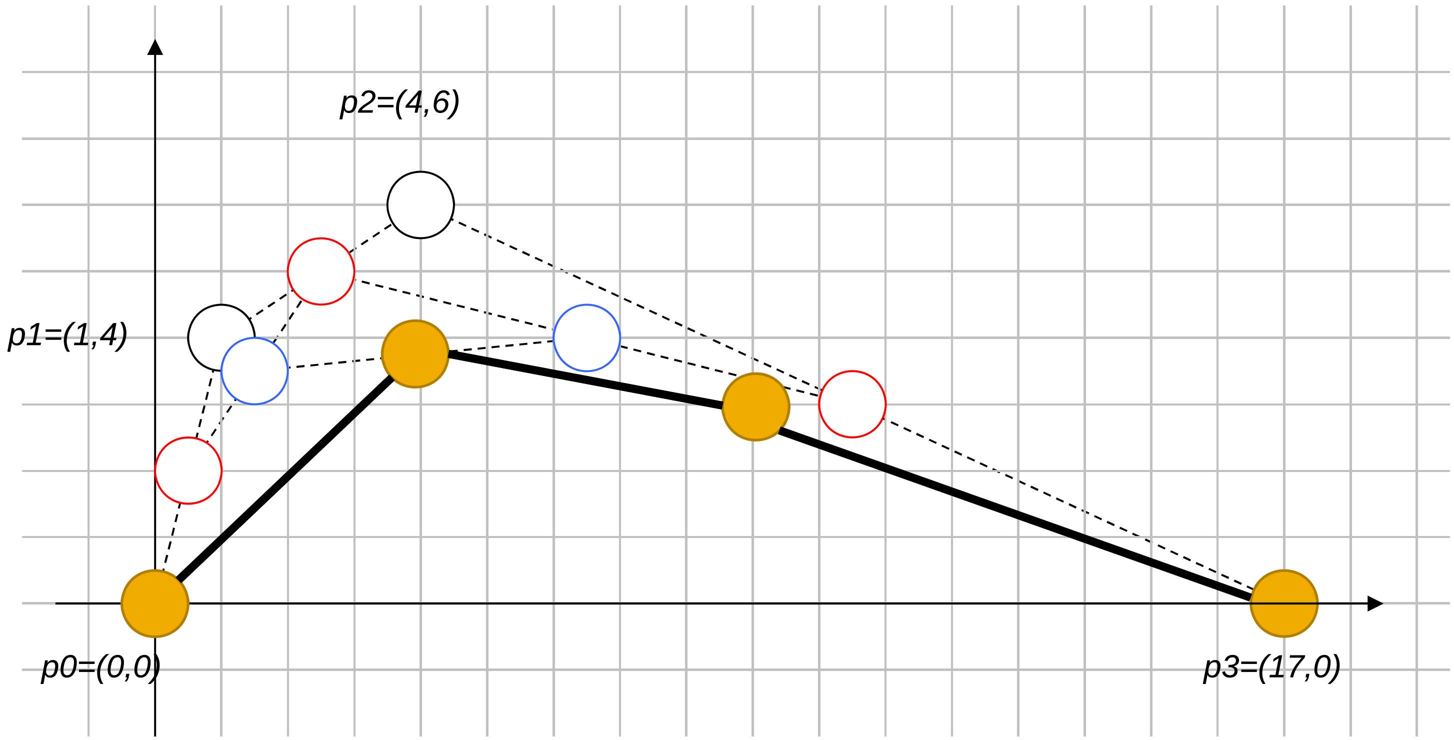
Question #4b

- 2 sides of the curve



Question #4b

- 2 sides of the curve



Question 4c

- Further improvement could be subdividing again on the curves which have:
  - Large area of the convex hull of its control points , or ,
  - Long distance of the control points.. etc.

- Discussion: What are the possible conditions/criteria to stop the recursion?