MA1506 TUTORIAL 6

Question 1

In question 6 of Tutorial 5, let us assume that you are keeping the bugs not as a hobby, but because you are developing a new insecticide. Suppose that you remove 80 bugs per day from the bottle, and that all of these bugs die a painful but well-deserved death as a result of being sprayed with this insecticide. What is the limiting population in this case? What is the maximum number of bugs you can put to death per day without causing the population to die out? [Answers: 312, 141.]

Question 2

The sandhill crane is a beautiful Canadian bird with an unfortunate liking for farm crops. For many years the cranes were protected by law, and eventually they settled down to a logistic equilibrium population of 194,600 with birth rate 9.866% per year. Eventually the patience of the farmers was exhausted and they managed to have the hunting ban lifted. The farmers happily shot 10000 cranes per year, which they argued was reasonable enough since it only represents about 5% of the original population. Show that the sandhill crane is doomed. How long will it take, from the legalisation of hunting, to exterminate them? [If you don't want to do the integral yourself, you can get a computer to do it for you here: http://wims.unice.fr/wims/ Or you could use Matlab.] [Answer: about 30 years.]

Question 3

Suppose that Peruvian fishermen take a fixed number of anchovies per year from an anchovy stock which would otherwise behave logistically, apart from occasional natural disasters. According to our lecture notes, any fishing rate $\geq \mathrm{B}^2/4\mathrm{s}$ will be disastrous. Let's call this number E^*. The fishermen want to take as many anchovies as they **safely** can, meaning that they want the fish to be able to bounce back from a natural disaster that pushes their population down by 10%. Advise them. [The men, not the fish.] That is, tell them the maximum number of fish they can take, expressed as a percentage of E^*. [Hint: assume that you start with the stable equilibrium population β_2, and compute the value of E, the harvesting rate, such that β_1, the **unstable** equilibrium population, becomes 90% of β_2.] [Answer: about 99.7%.]

Question 4

In the harvesting model we considered in the lectures, the population will rebound if all harvesting is stopped. Unhappily, this is not always true: for some animals, if you drive their population down too low, they will have trouble finding mates, or they will be forced to breed with relatively close kin, which reduces genetic variability and hence their ability to resist disease. For such animals [for example, certain rare species of tigers] extinction will result if the population falls too low, even if all harvesting is forbidden. Biologists call this **depensation**. Show that this situation can be modelled by the ODE

$$\frac{dN}{dt} = -aN^3 + bN^2 - cN,$$

where N is the population and a, b, and c are positive constants such that $b^2 > 4ac$. Find the population below which extinction will occur. [Answer: $[b - \sqrt{b^2 - 4ac}]/2a$.]

Question 5

Ms Tan Ah Lian, the billionaire engineer who patented the idea of making NewImprovedWater^{TM} by mixing hydrogen and oxygen in a Plug Flow Reactor, realises that she can save money in the following way. The concentration of hydrogen in her PFR does not drop very much near its end [because the exponential function is almost flat there]. So why not make the tube narrower there, and save on construction costs? Instead of having a constant cross-sectional area, the new TALPFR has a cross-sectional area given by

$$A(x) = A_0 e^{-\gamma x},$$

where A_0 is the cross-sectional area at the top of the reactor and γ is a positive constant with units of 1/length [of course]. Show that the new TALPFR behaves like an ordinary PFR, but with a smaller rate constant [see section 3.6 of the notes]: instead of k, it is

$$k - (u\gamma/2),$$

where k is the true rate constant and u is the speed, taken to be constant, at which the mixture moves through the reactor.

Question 6

Tan Ah Lian, billionaire engineer, also has another business which was awarded a huge contract to retro-fit a large balcony onto every HDB apartment in Singapore. The contract fixes the physical properties and the total weight W of the building materials to be used in each balcony and also the length L of the balcony. Ms Tan's competitor, Ah Huat Contractor Services, proposed a cantilever design with a constant weight per unit length, $\alpha = W/L$, but the ingenious Ah Lian defeated Ah Huat by proposing a cantilever with the same total weight but with weight per unit length being given by

$$2\alpha[1 - (x/L)],$$

where x is distance from the point of attachment. [Verify that the two designs have the same total weight.] That is, the weight is concentrated near to the point where the balcony is to be attached, and it tapers off towards the end. Ah Lian claims that her balcony design is better than Ah Huat's, because the [absolute value of the] bending moment at the point of attachment is smaller so it will be cheaper to attach, since less reinforcement is needed. This is how she won the contract, even without using her considerable personal charm. How much smaller is the absolute value of the bending moment of Ah Lian's balcony compared to Ah Huat's? [Please use Euler's equation from lectures, not your knowledge of physics if any.] Which balcony dips less at the end? By what factor?

[Answer: the magnitude of AL's bending moment at $x = 0$ is 2/3 that of AH; AL's balcony dips less at the end, by a factor of 8/15.]

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