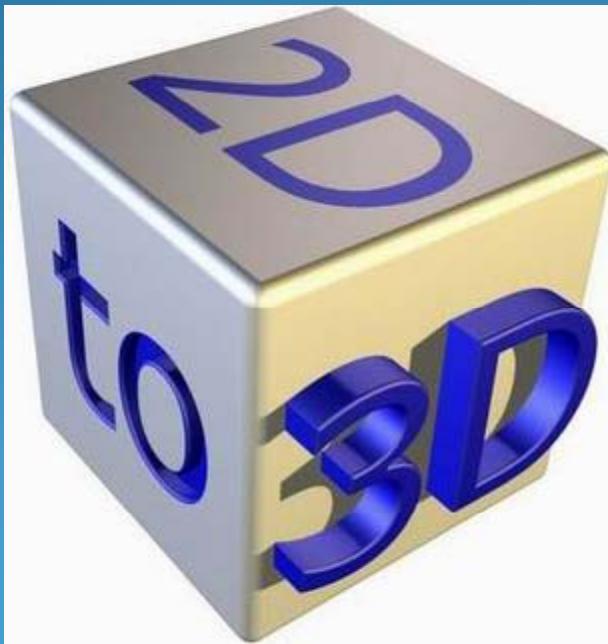


CS3241: Computer Graphics Topics Today



3D Object Representations
3D Transformation
Hierarchical Transformation

3D Object Representations

And Polygonal Meshes

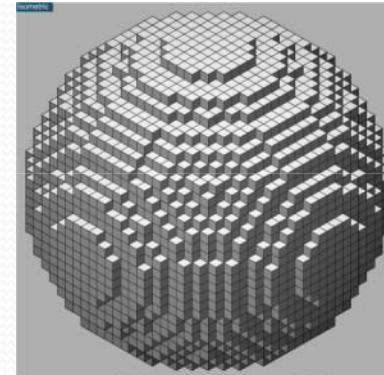
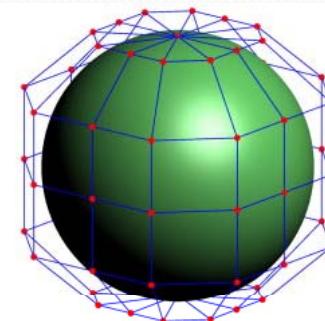
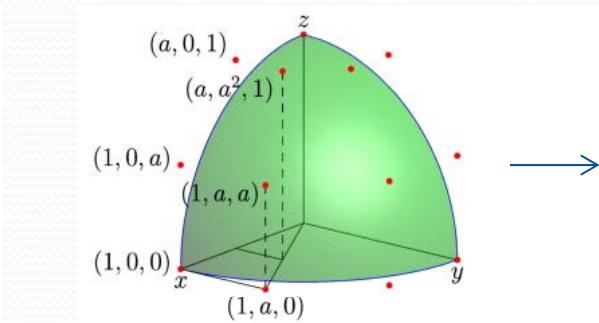
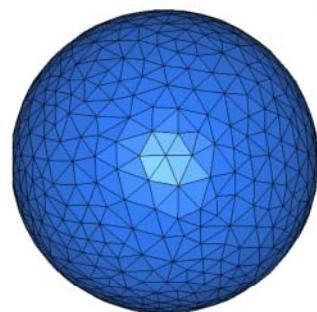


What is a “Representation”?

- For the same data/concept, we can have different representations inside the computer
 - E.g. the number 12 can be represented as an integer, or a float or a double in the memory
 - Or, a set of names, can be stored as a link list, or an array, or even a tree

Various 3D Object Representations

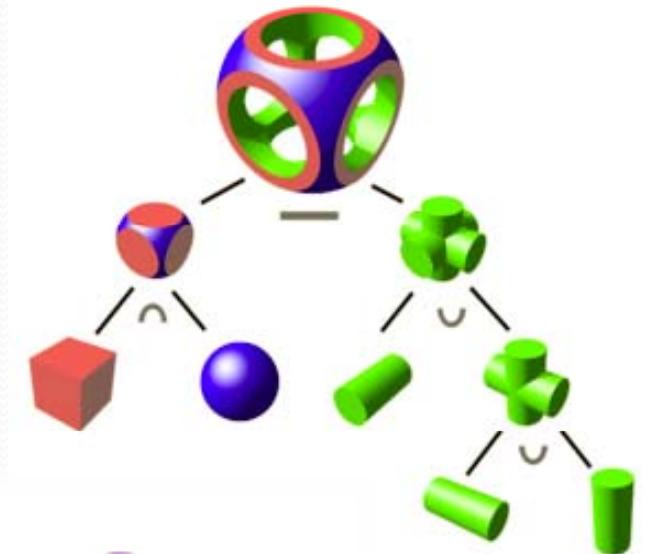
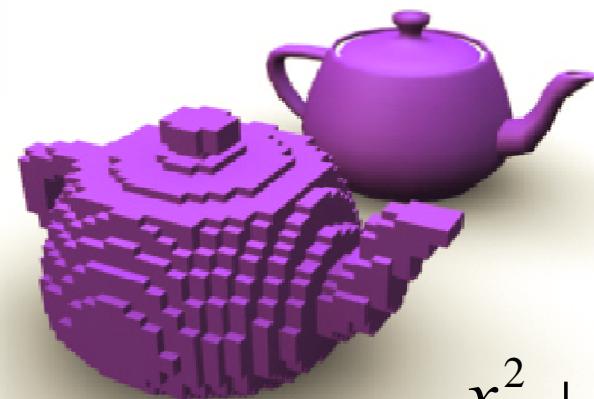
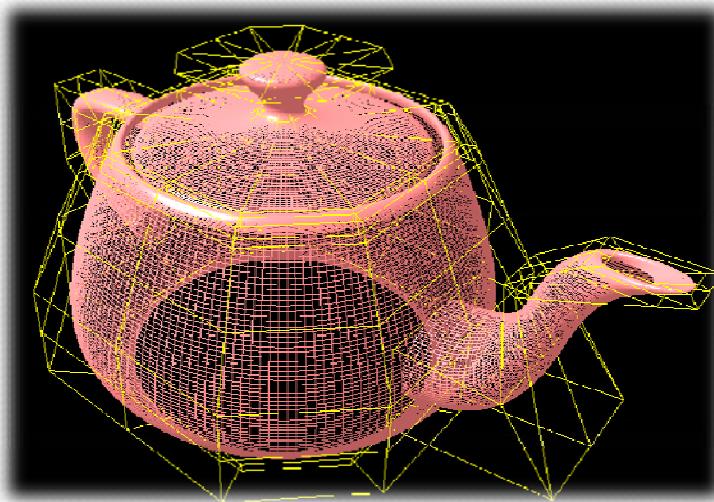
- So for the same 3D object, we can have different representations
 - E.g. a **sphere**, and its FIVE different representations



$$x^2 + y^2 + z^2 = 1$$

Various Object Representations

- Polygonal meshes
- Parametric patches (Lecture 9)
- Constructive solid geometry (CSG)
- Spatial subdivision techniques
- Implicit representation (Lecture 11)



$$x^2 + y^2 + z^2 = 1$$

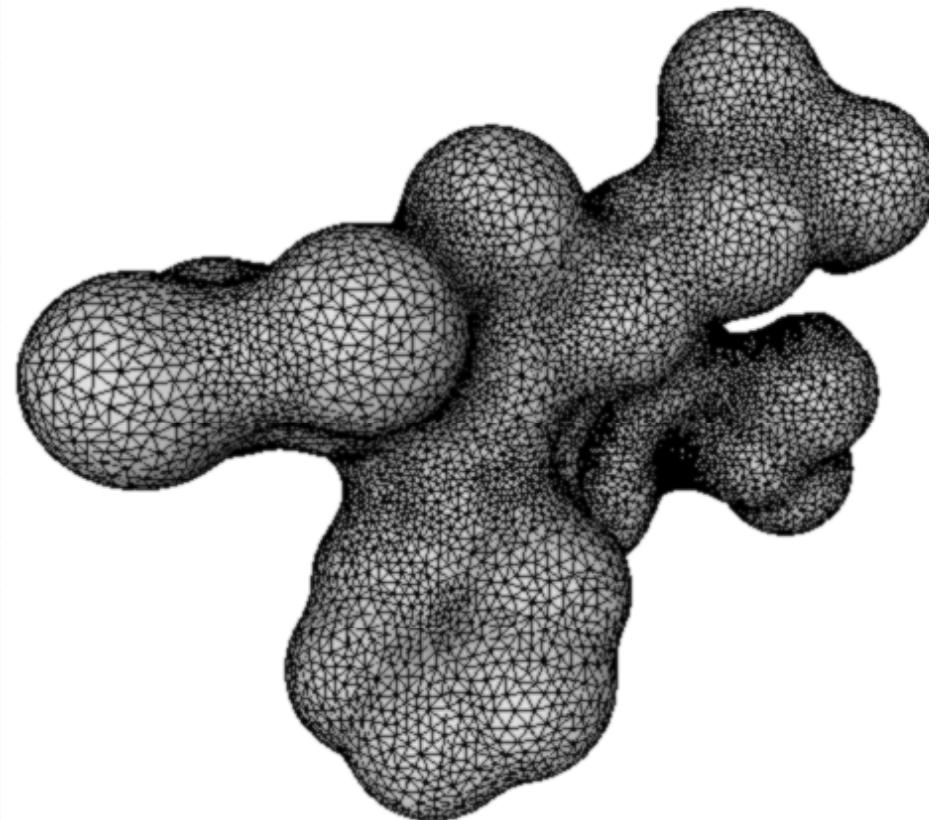
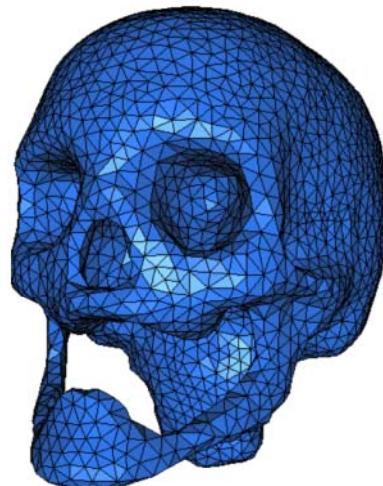
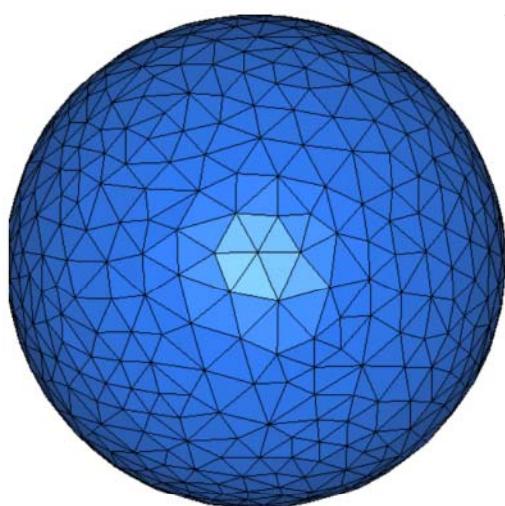


Polygonal Mesh

(Aka surface triangulation)

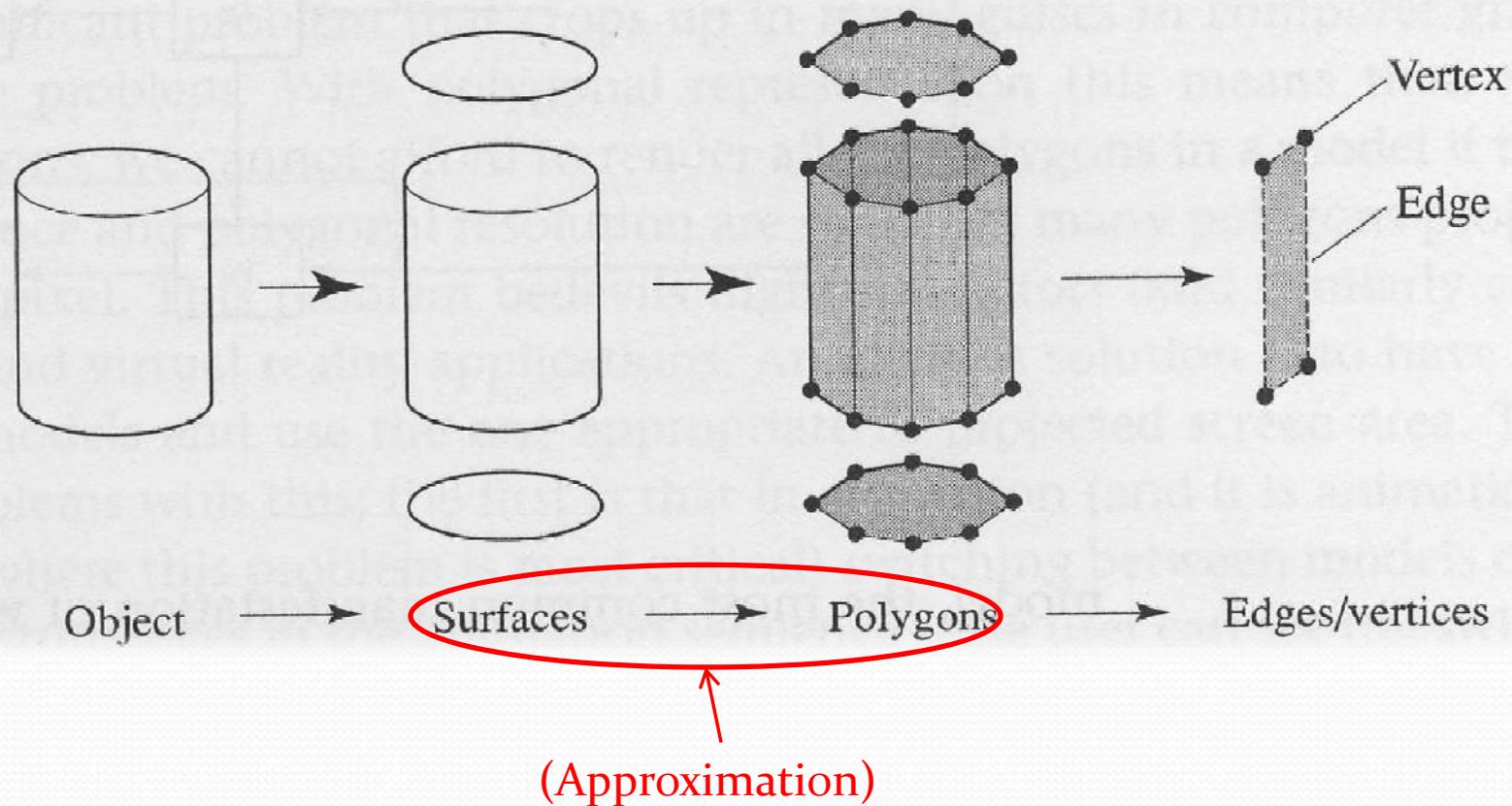
Polygonal Meshes

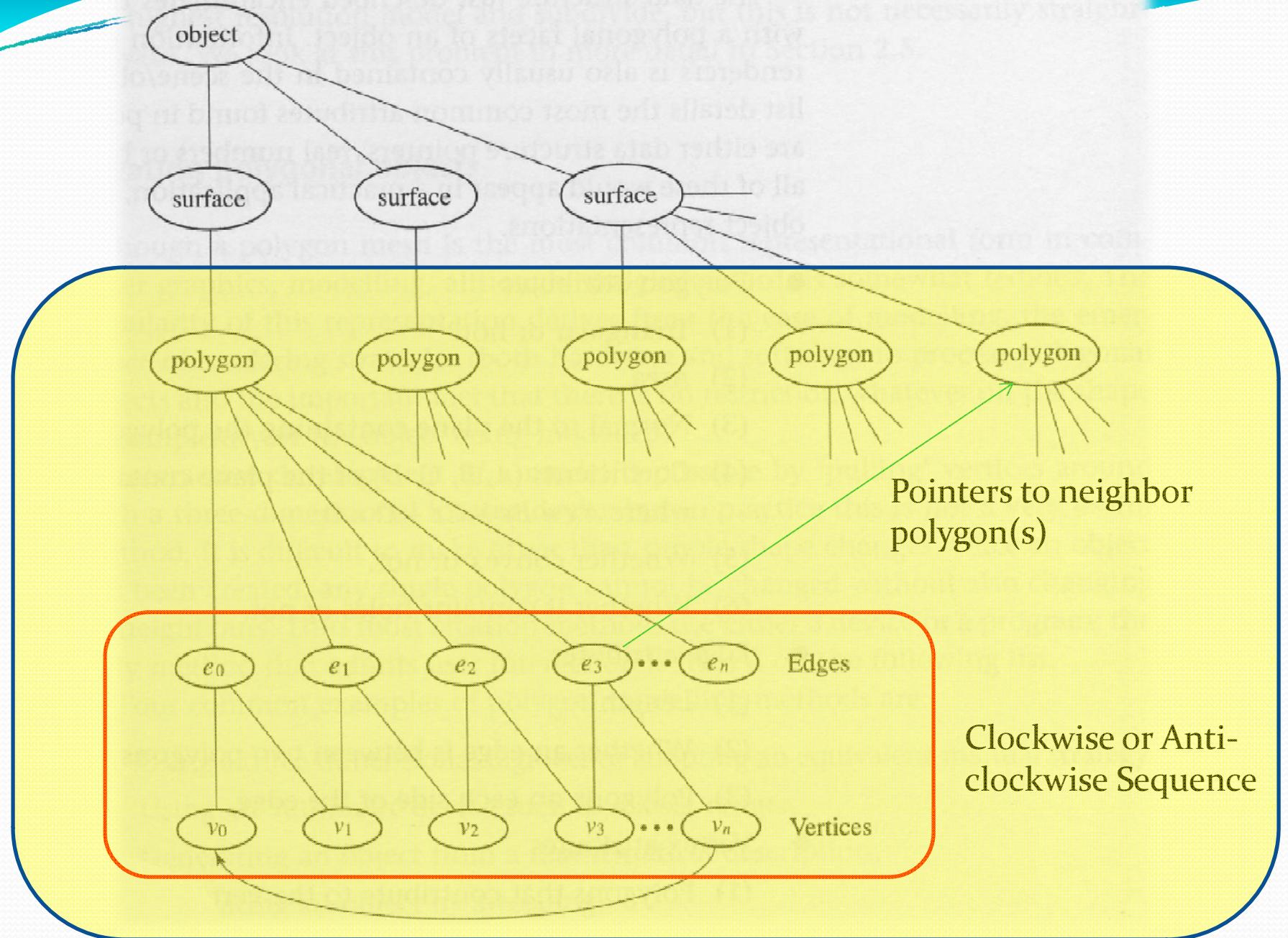
- We can use “soups” of polygons (preferably triangles) to model (actually “approximate”) objects
 - It’s basically very similar to a “graph”



Basic Representation of 3D Objects

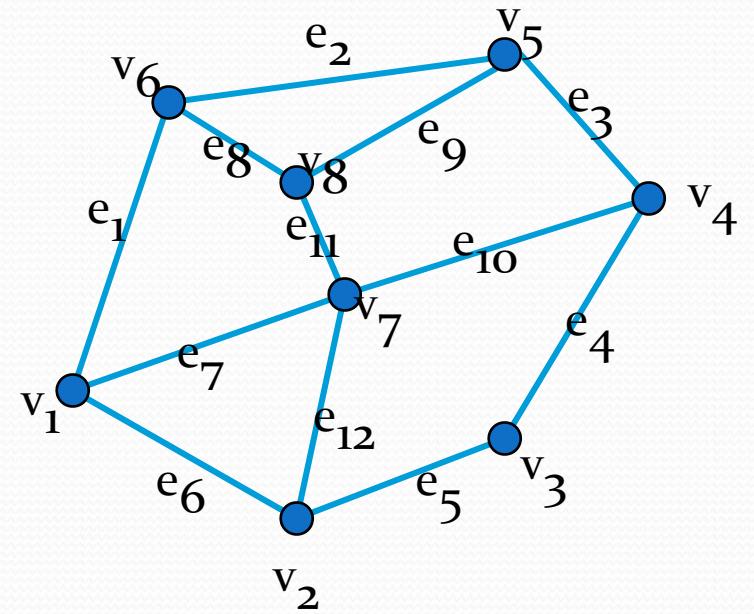
- By polygons





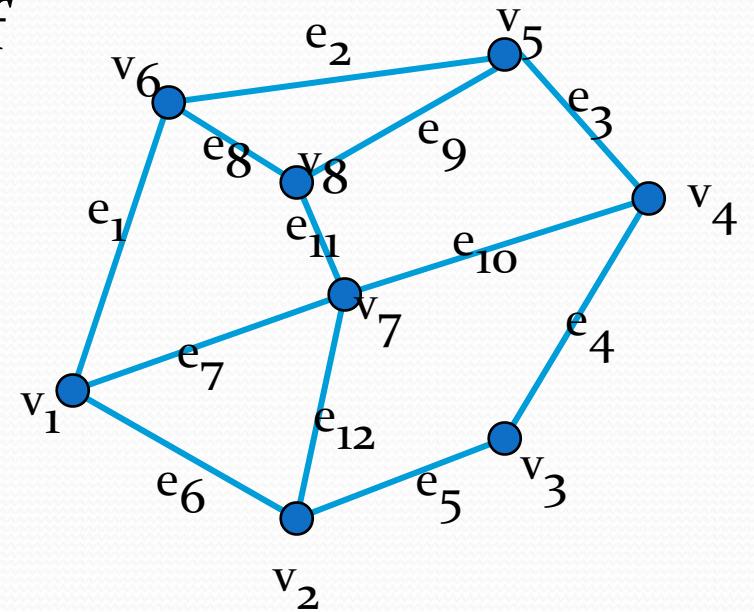
Mesh Representation

- An example of a simple mesh on the right
- There are 8 nodes and 12 edges
 - 5 interior polygons
 - 6 interior (shared) edges
- Each vertex has a location
 - $v_i = (x_i \ y_i \ z_i)$
- We can think of it as a *graph*, vertices are connected by edges



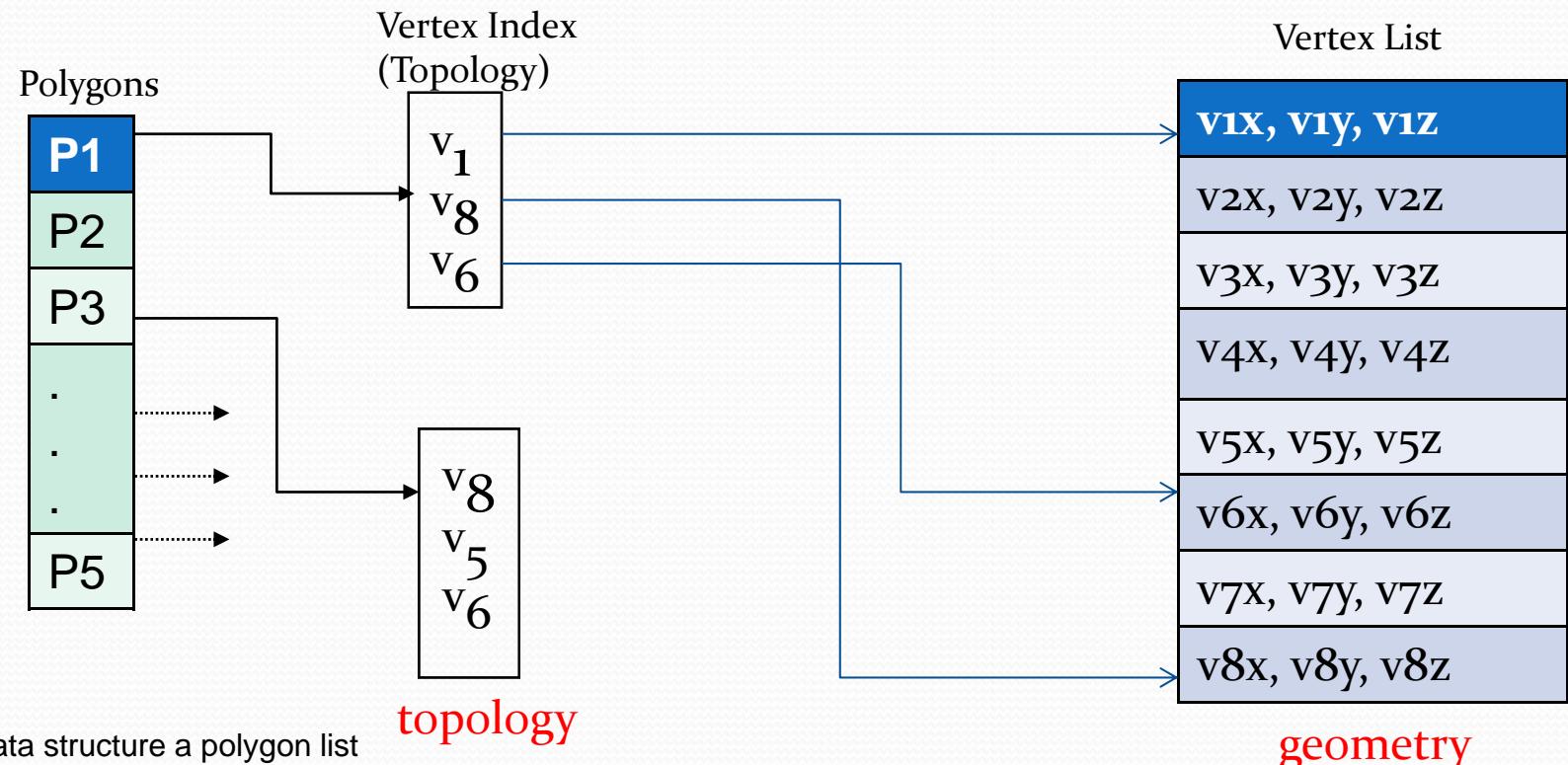
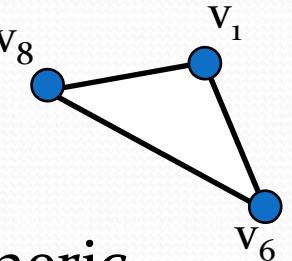
Mesh Topology

- The edges form the “Topology” of the mesh
- Note that these connections are not changed even if we modify the positions of the vertices
- We often consider the topology (connections) to be **independent** of the geometry (actual 3D point positions)



Vertex List and Polygon List

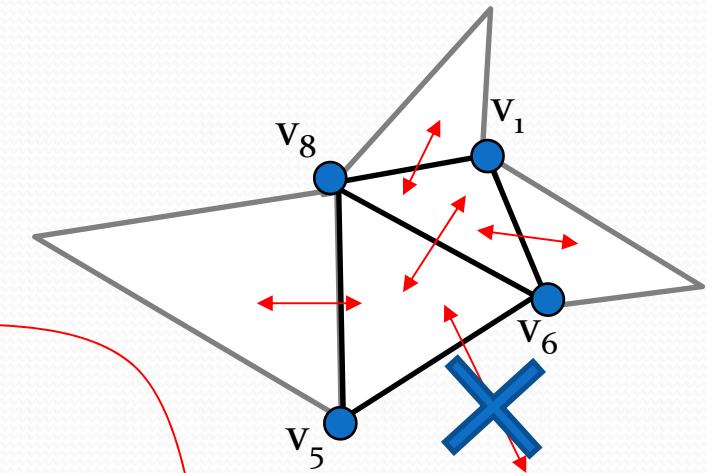
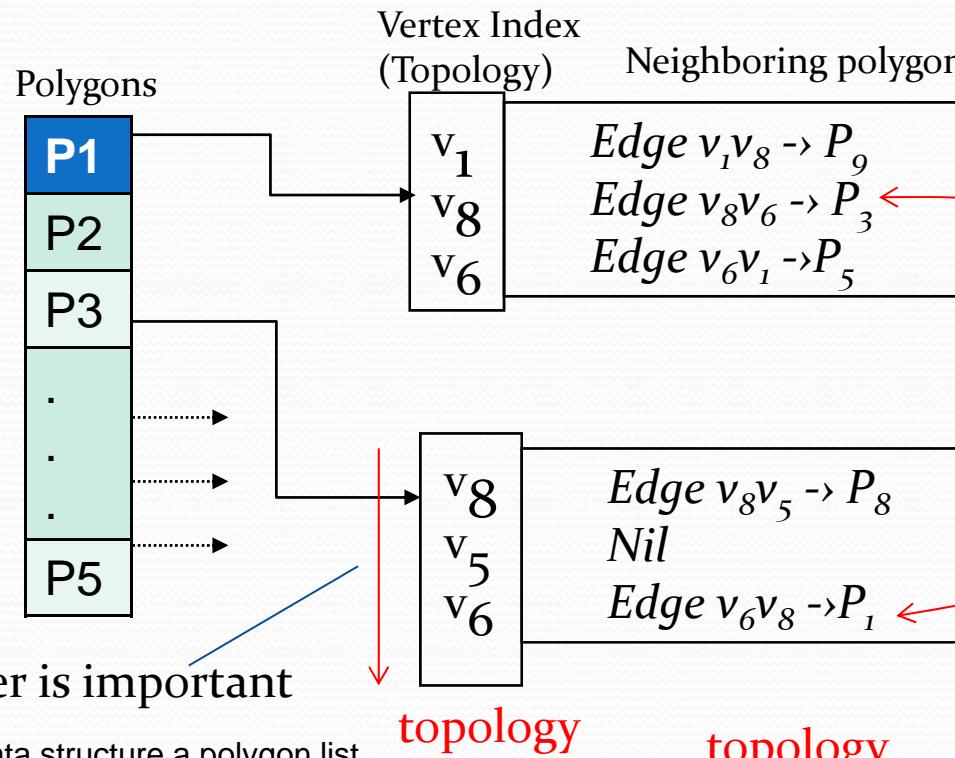
- Store the vertex coordinates in a separate list
 - **Advantages:** save memory, avoid confusion/numeric errors



* We call this data structure a polygon list or triangle list (if the polygons=triangles)
Sometimes the polygons are called “faces”

Vertex List and Polygon List

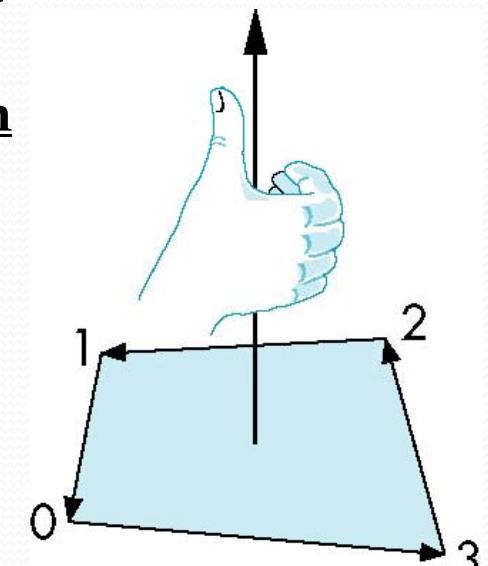
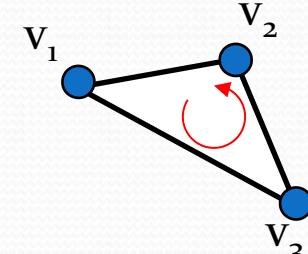
- Store the vertex coordinates in a separate list
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Sometimes the polygons are called “faces”

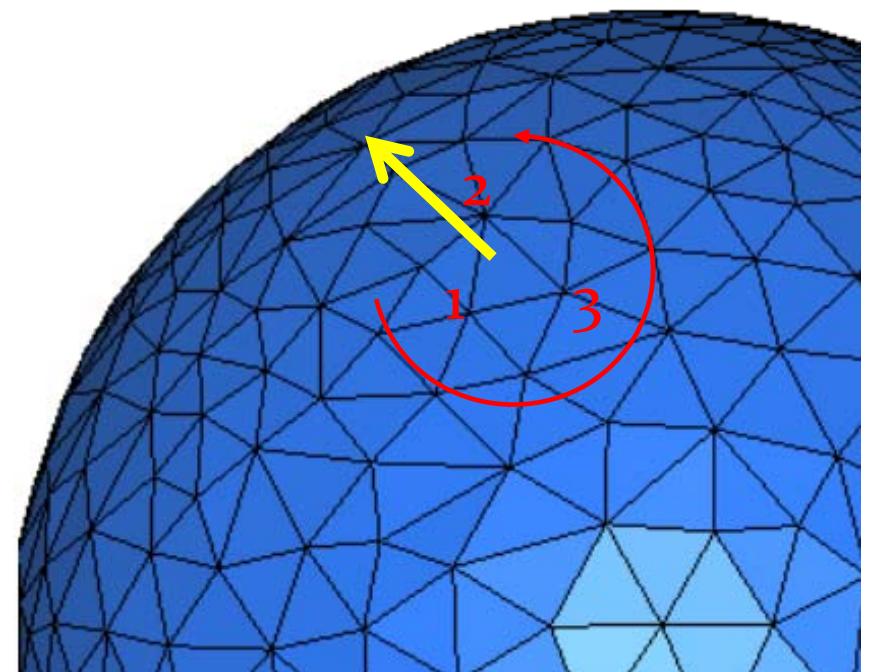
Polygon Orientations

- For a triangle with three vertices v_1 , v_2 and v_3
 - The order (v_1, v_3, v_2) and (v_3, v_2, v_1) are considered to be equivalent during drawing, e.g. `glVertex`
- But the order (v_1, v_2, v_3) is considered to be different (or “opposite”)
- (v_1, v_3, v_2) and (v_3, v_2, v_1) describe a **facing direction** of a polygon/triangle
 - Right-hand rule
 - The order (v_1, v_2, v_3) creates an opposite facing direction
- So, geometry is the same, but **normal vector** for that polygon will be different
 - Usually we follow the “natural” order in the polygon list to define the “outward” direction of an object



Vertex List and Polygon List

- For example, if it is a “solid” ball, the triangle below must be stored in the order of
 - (v_1, v_3, v_2) or (v_3, v_2, v_1) or (v_2, v_1, v_3)
 - But NOT (v_1, v_2, v_3) or some other orders



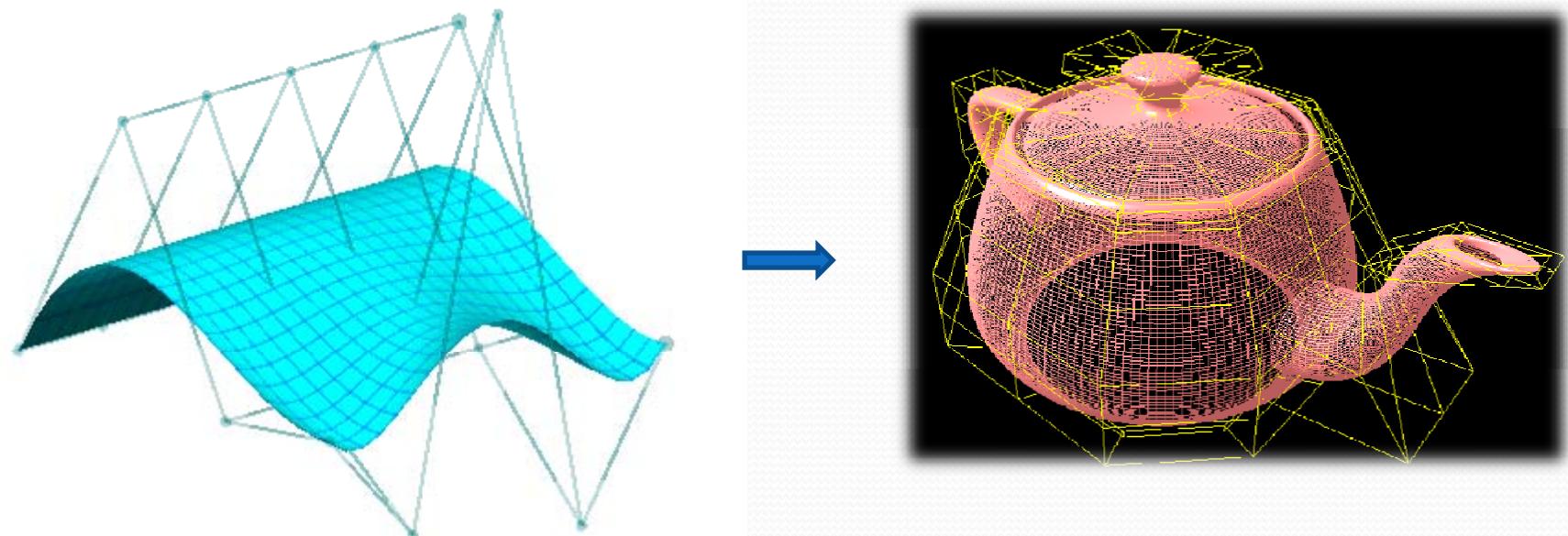
Switching the “Winding” in OpenGL

- OpenGL default is that outward facing polygons are specified in counter-clockwise fashion
 - Because, there is a speedup feature such that OpenGL will **STOP** drawing those polygons when the “outward” direction is **not** facing your eyes
- But you change this by:
 - `glFrontFace(GL_CW);` // Clockwise Winding = outward face
 - `glFrontFace(GL_CCW);` // Counter-Clockwise Winding = // outward face

Other Object Representations

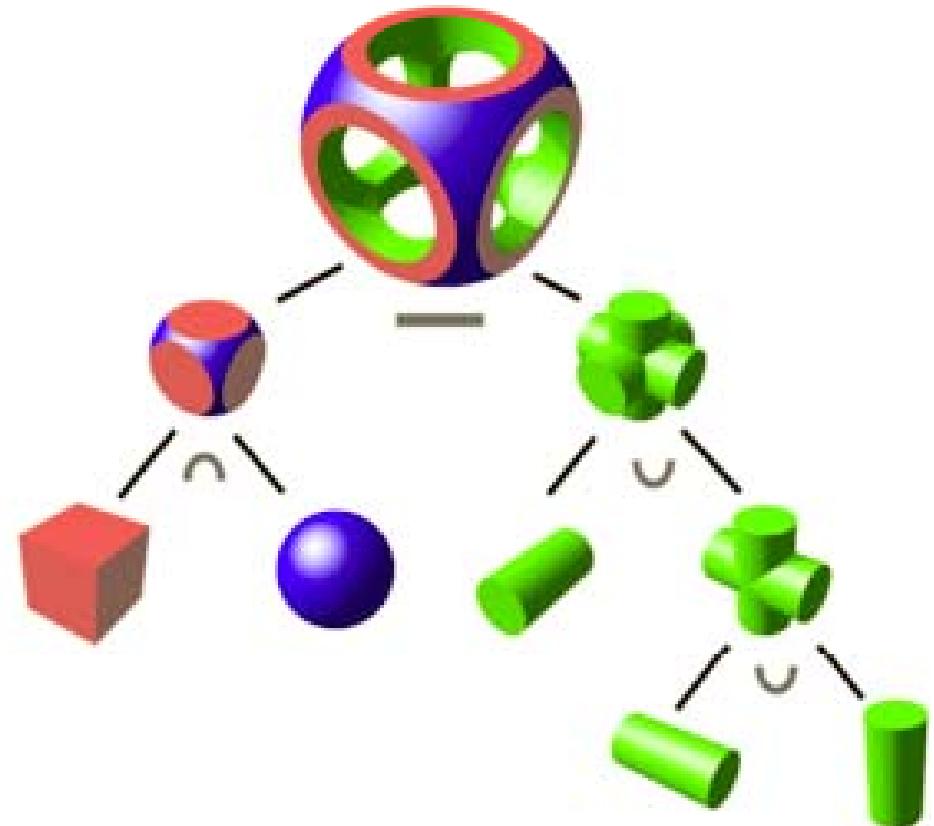
Various Object Representations

- Parametric patches (Lecture 9)



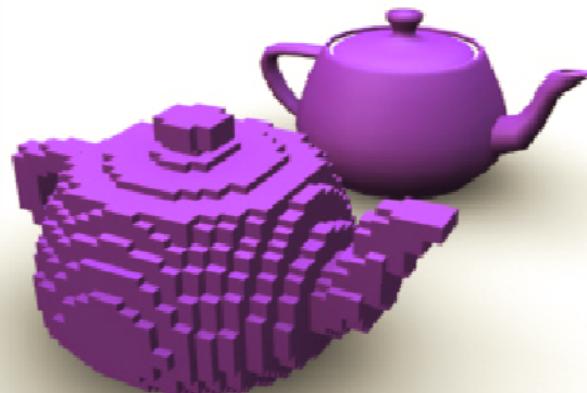
Various Object Representations

- Constructive solid geometry (CSG)
 - Constructed from a set of primitive shapes
 - E.g. spheres, cubes, cylinders
 - Then combined them with different operations:
 - E.g. intersection, union, subtractions, etc



Various Object Representations

- Spatial subdivision techniques
 - 3D array of voxels
 - If the object occupies a voxel, turns the voxel on



- Implicit representation

$$x^2 + y^2 + z^2 = 1$$



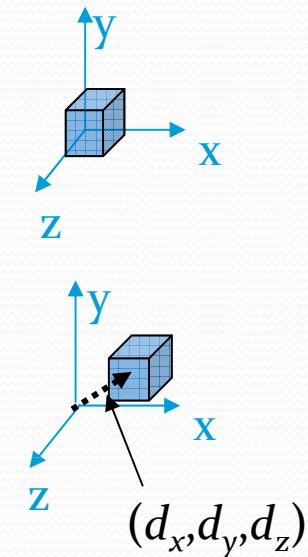
3D Transformation

3D Translation Matrix

- Now a point in 3D is
 - $p = [x \ y \ z \ 1]^T$
- We can also express translation using a 4×4 matrix T in homogeneous coordinates

$$p' = Tp$$

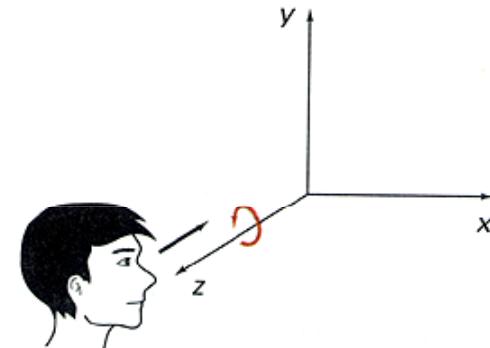
$$\text{where } T = T(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



3D Rotation Matrix

- Same argument as for rotation about z -axis

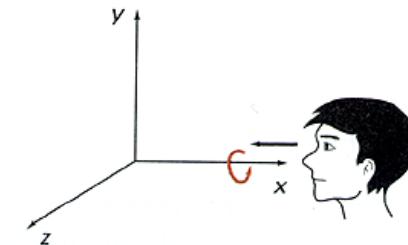
$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



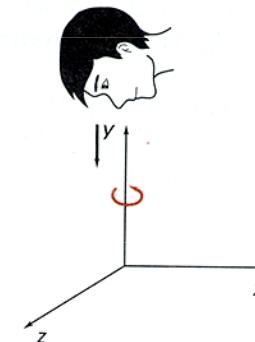
Rotation About x - and y -Axes

- x -axis and y -axis

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

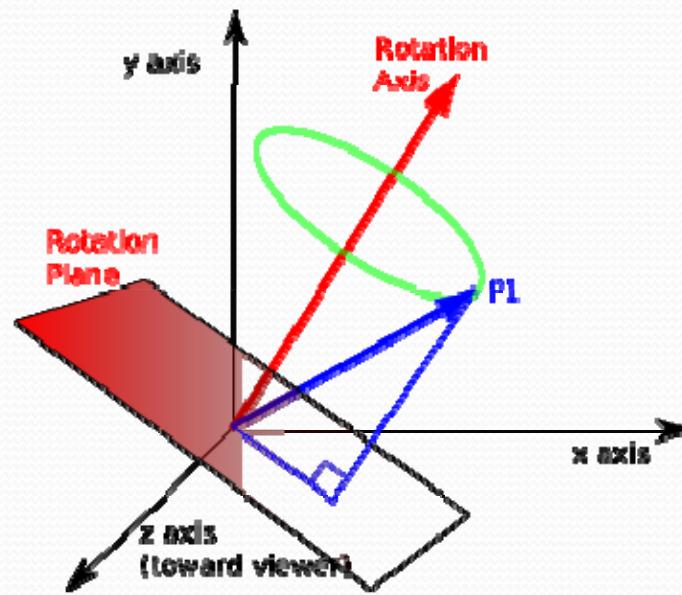


$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



3D Rotation in OpenGL

- **glRotatef(alpha, x, y, z)**
 - A point P1 will be rotated around the vector (x, y, z) (that passes through the origin) by an angle alpha



Scaling

- Expanding or contracting along any axis

$$x' = s_x x$$

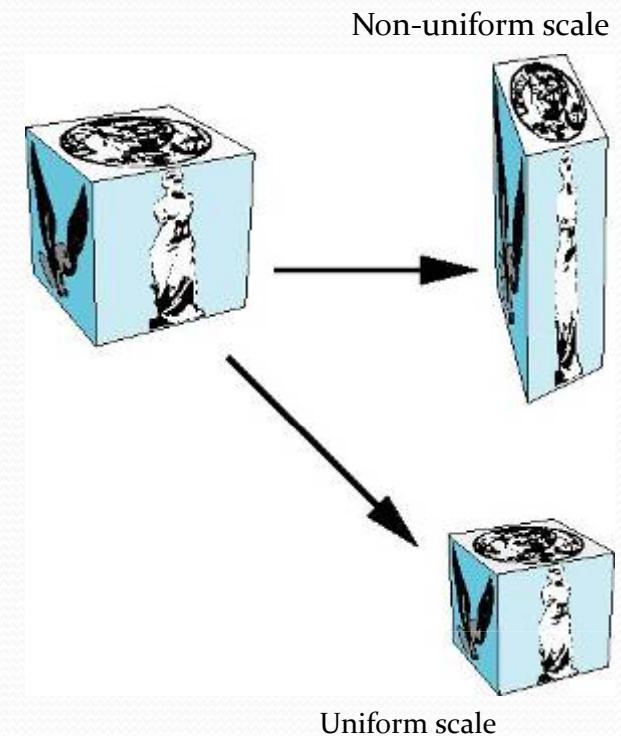
$$y' = s_y y$$

$$z' = s_z z$$

- Or in homogeneous coordinates

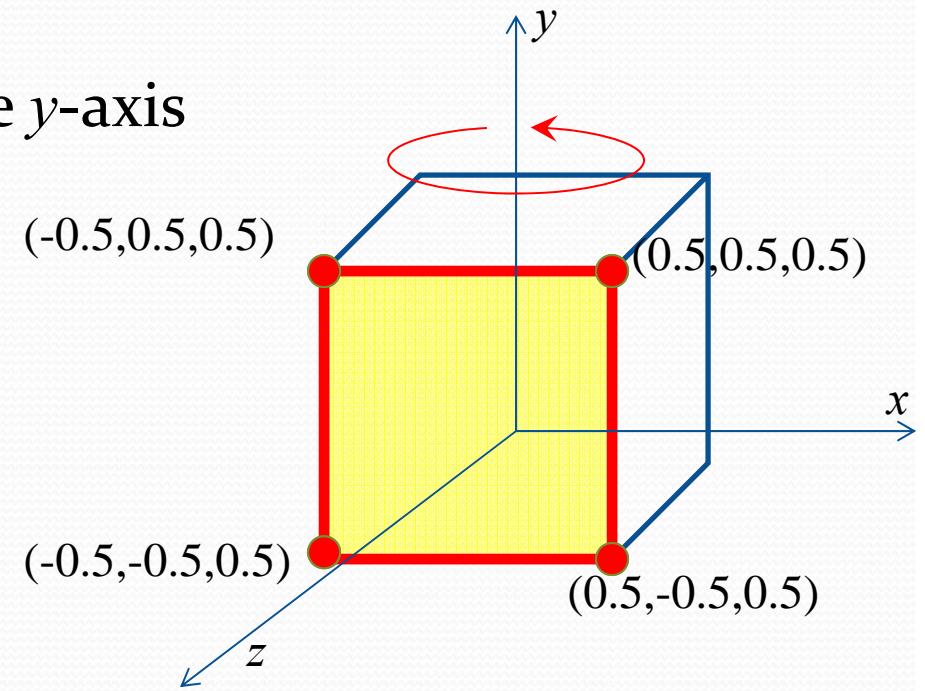
$$\mathbf{p}' = S(s_x, s_y, s_z) \mathbf{p}$$

$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



An Example: Drawing a Wireframe Cube

- All the vertices of the cubes are $(\pm 0.5, \pm 0.5, \pm 0.5)$
- Plan:
 - Draw 1 wireframe square
 - Rotate it 4 times around the y -axis

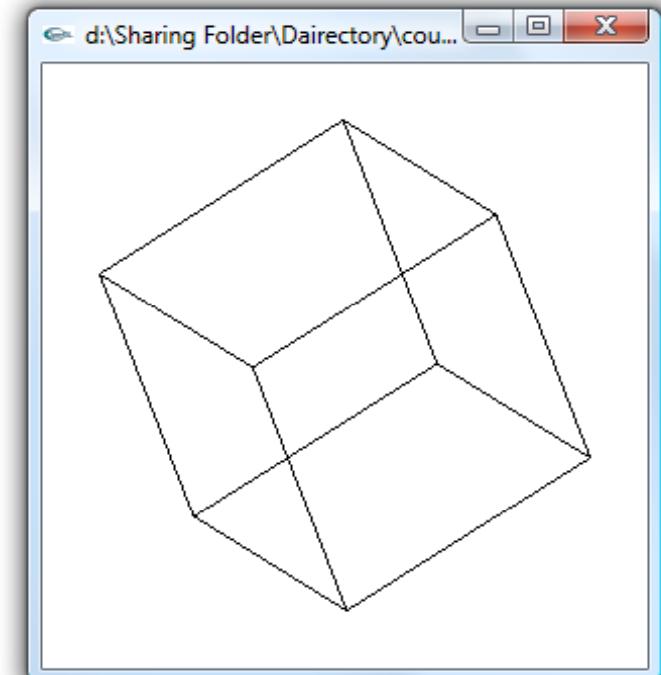
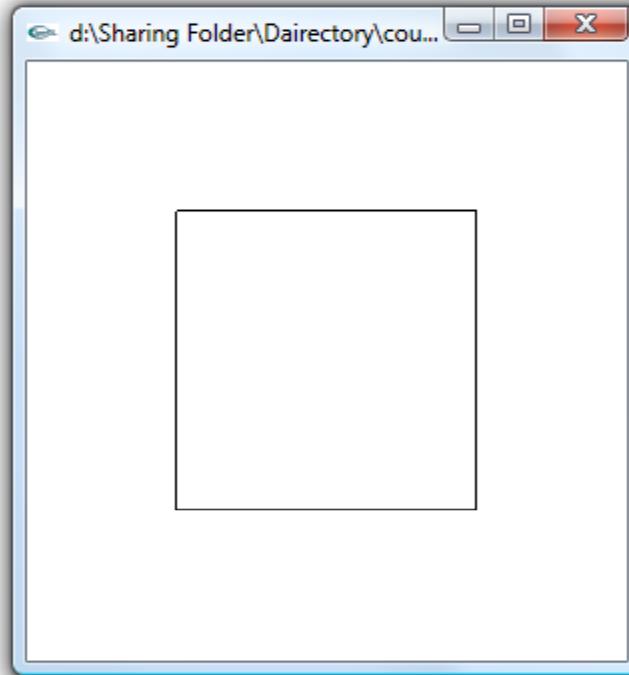


An Example: Drawing a Wireframe Cube

```
void draw3DCubeWireFrame()
{
    glColor3f(0,0,0);
    glPushMatrix();
    for(int i=0;i<4;i++)
    {
        glBegin(GL_LINE_LOOP); // draw one face
        glVertex3f(0.5,0.5,0.5);
        glVertex3f(-0.5,0.5,0.5);
        glVertex3f(-0.5,-0.5,0.5);
        glVertex3f(0.5,-0.5,0.5);
        glEnd();
        glRotatef(90,0,1,0); // rotate 90 degree (x4)
    }
    glPopMatrix();
}
```

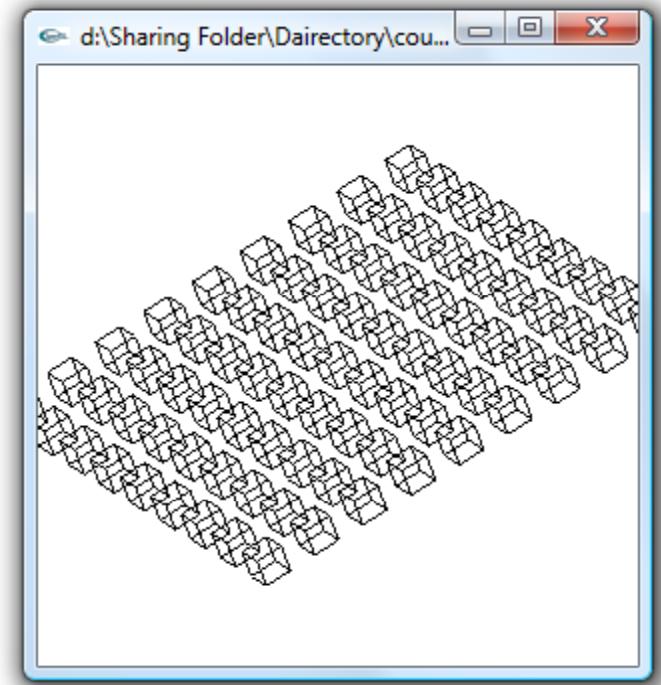
An Example: Drawing a Wireframe Cube

- Just call the routine
- With
 - glRotatef(45 , 1 , 1 , 1);
 - draw3DCubeWireFrame();



An Example: Drawing a Wireframe Cube

```
glRotatef( 45,1,1,1 );
glScalef( 0.1,0.1,0.1 );
for( int i=-8;i<=8;i+=2 )
{
    for( int j=-8;j<=8;j+=2 )
        {
            glPushMatrix();
            glTranslatef(i,0,j);
            draw3DCubeWireFrame();
            glPopMatrix();
        }
}
```



Model View Matrix in OpenGL

The Way That OpenGL Does it

- Whenever you draw a point p by `glVertex`, with a point position p , OpenGL will draw it at the position

$$p'' = Mp$$

- In which M is the *Model View Matrix*
- For example
 - After one translation, and one rotation

$$M = T(0,0.5) R(45)$$

- So when you execute `glVertex2f(1.0, 1.0)`

- Actually the “real” point will be drawn on the position

$$p'' = Mp = T(0,0.5) R(45) (1.0, 1.0)$$



And OpenGL will only save “ M ” instead of $T(0,0.5)R(45)....$ etc.

The Way That OpenGL Does it

- Whenever you draw a point p by `glVertex`, OpenGL will draw it at the position

$$p'' = Mp$$

$$p'' = T(0,0.5)R(-45^\circ)p$$

M

- In which M is the *Model View Matrix*
- When you want to change this transformation matrix M , you can:
 - `glMatrixMode(GL_MODELVIEW);`
 - Switch to this transformation matrix and stack
 - `glLoadIdentity();`
 - Set M to be the identity matrix
 - `glTranslatef()`
 - Set $M := MT$
 - `glRotatef()`
 - Set $M := MR$

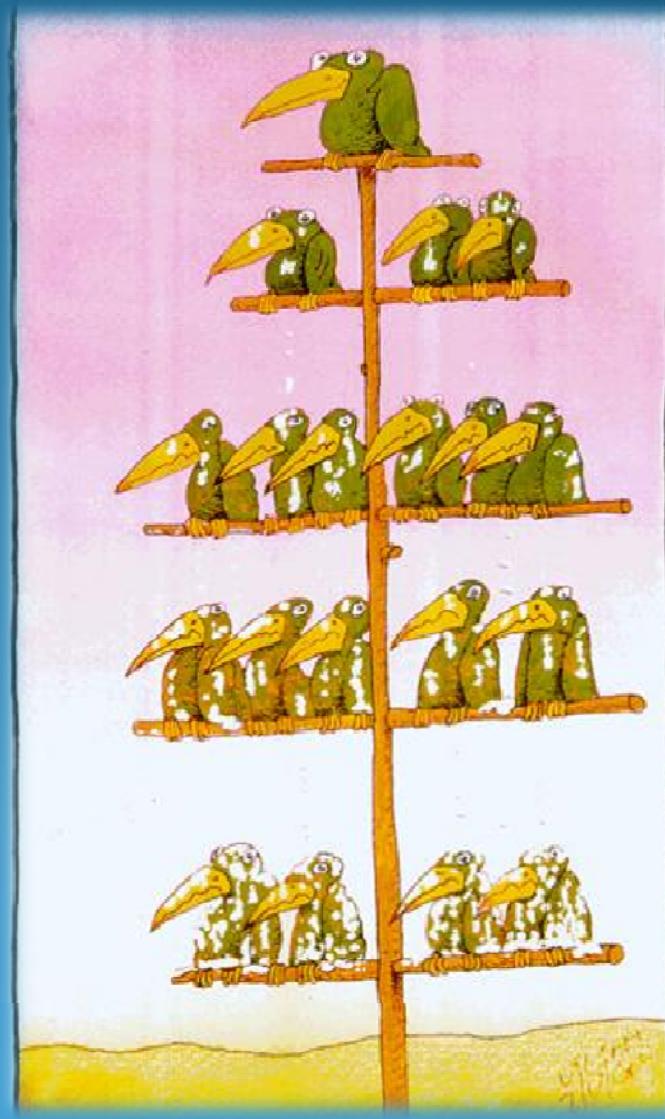
Choices:

GL_MODELVIEW
GL_PROJECTION
GL_TEXTURE
GL_COLOR

The Way OpenGL Does it

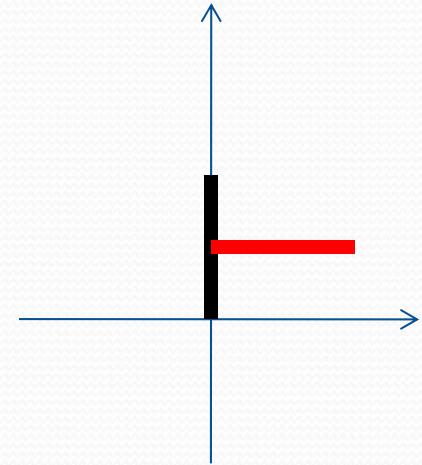
- **glScalef()**
 - Set $M := MS$
- **glPushMatrix();**
 - Push the current M onto the stack
 - Meaning “Saving the current reference frame”
- **glPopMatrix();**
 - Pop a matrix M' from the stack, and replace the current M with M'
 - Meaning “Resuming the last saved reference frame”

Hierarchical Transformation



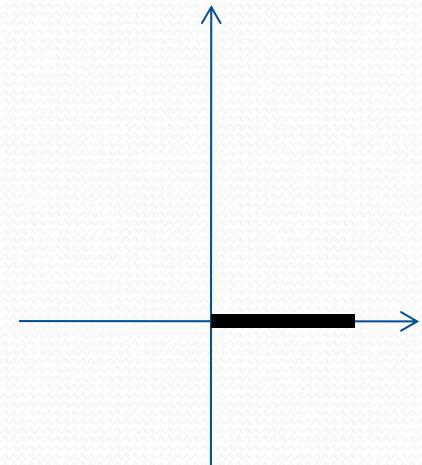
Hierarchical Transformation

- Example
 - Draw a stick figure
- A function called **drawAUnitLine()**
 - Draw a line from (0,0) to (0,1)
- Draw the “east” arm
 - Translate (0,0.5)
 - Rotate (-90)
 - drawAUnitLine()



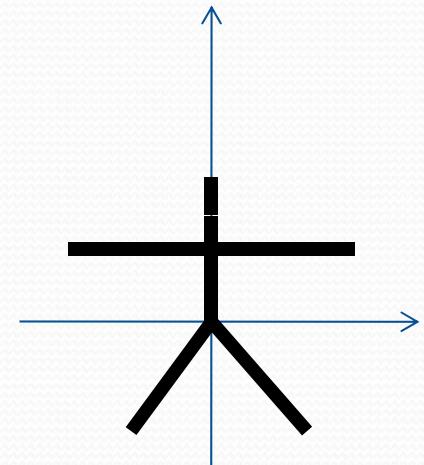
Hierarchical Transformation

```
drawLeftArm( )  
{  
    glPushMatrix( );  
    glRotatef( -90 , 0 , 0 , 1 );  
    drawAUnitLine( );  
    glPopMatrix( );  
}
```



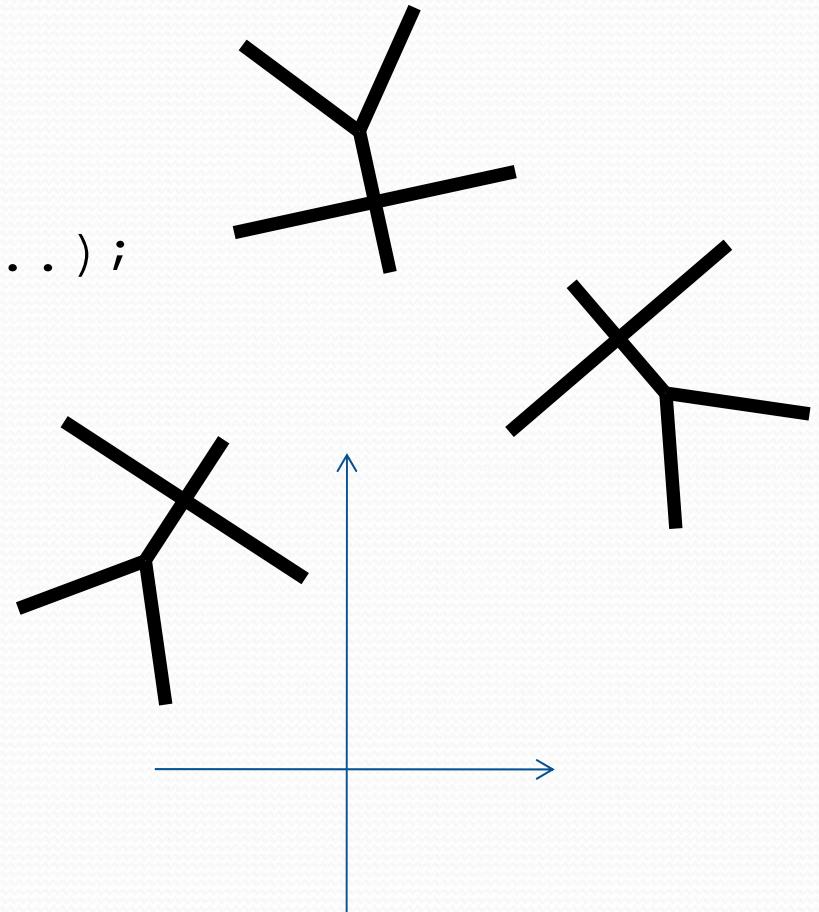
Hierarchical Transformation

```
drawAStickFigure()
{
    glPushMatrix();
        glTranslatef(0,0.5,0);
        drawLeftArm();
        drawRightArm();
    glPopMatrix();
    glPushMatrix();
        drawLeftLeg();
        drawRightLeg();
    glPopMatrix();
}
```



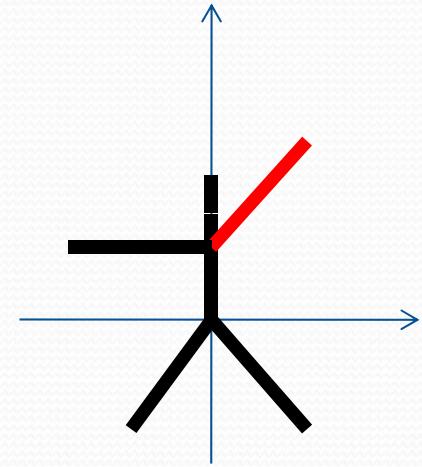
How do I Draw this?

```
myDisplay( )
{
    glPushMatrix();
        glTranslatef(some where...);
        glRotatef(some angle...);
        drawAStickFigure();
    glPopMatrix();
    .
    .
    .
    // draw many many
}
```



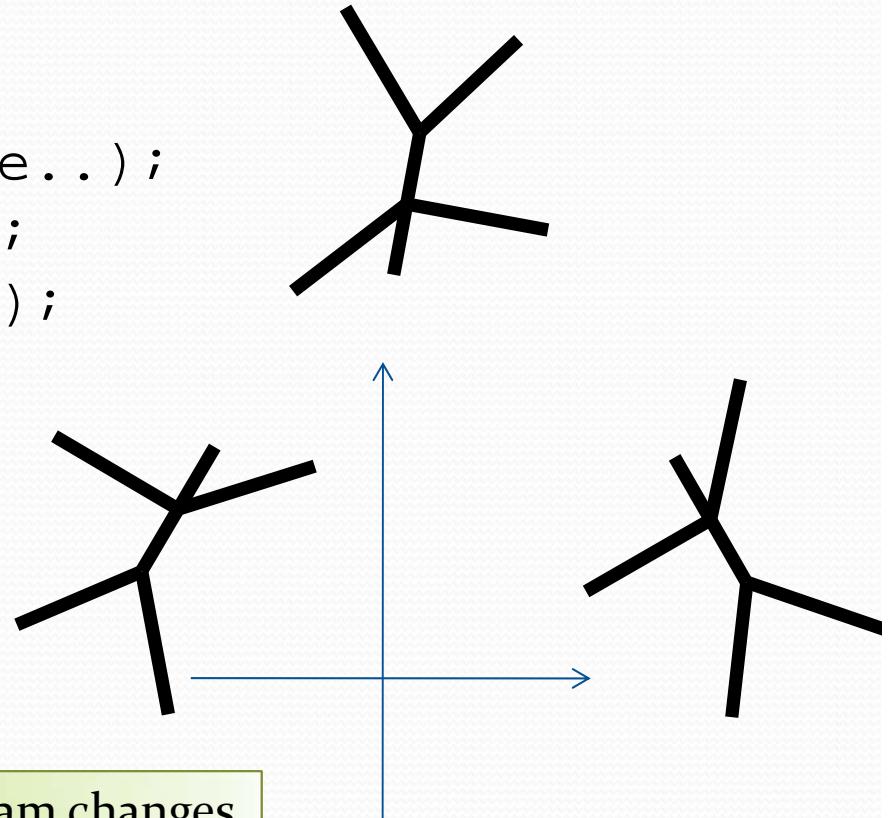
Make the Left Arm Wave

```
drawAStrickFigure(double angle)
{
    glPushMatrix();
    glTranslatef(0,0.5,0);
    glPushMatrix();
        glRotatef(angle,0,0,1);
        drawLeftArm();
    glPopMatrix();
    drawRightArm();
    glPopMatrix();
    glPushMatrix();
        drawLeftLeg();
        drawRightLeg();
    glPopMatrix();
}
```



How do I Draw this?

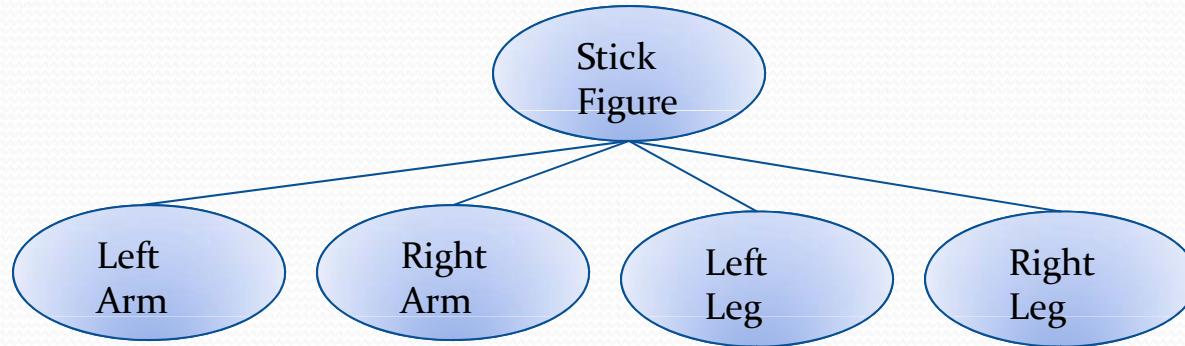
```
myDisplay( )
{
    glPushMatrix();
        glTranslatef(some where...);
        glRotatef(some angle...);
        drawAStickFigure(angle);
    glPopMatrix();
    .
    .
    .
    // draw many many
}
```



Imagine that if your program changes
the variable “angle” according to time

Hierarchical Transformation

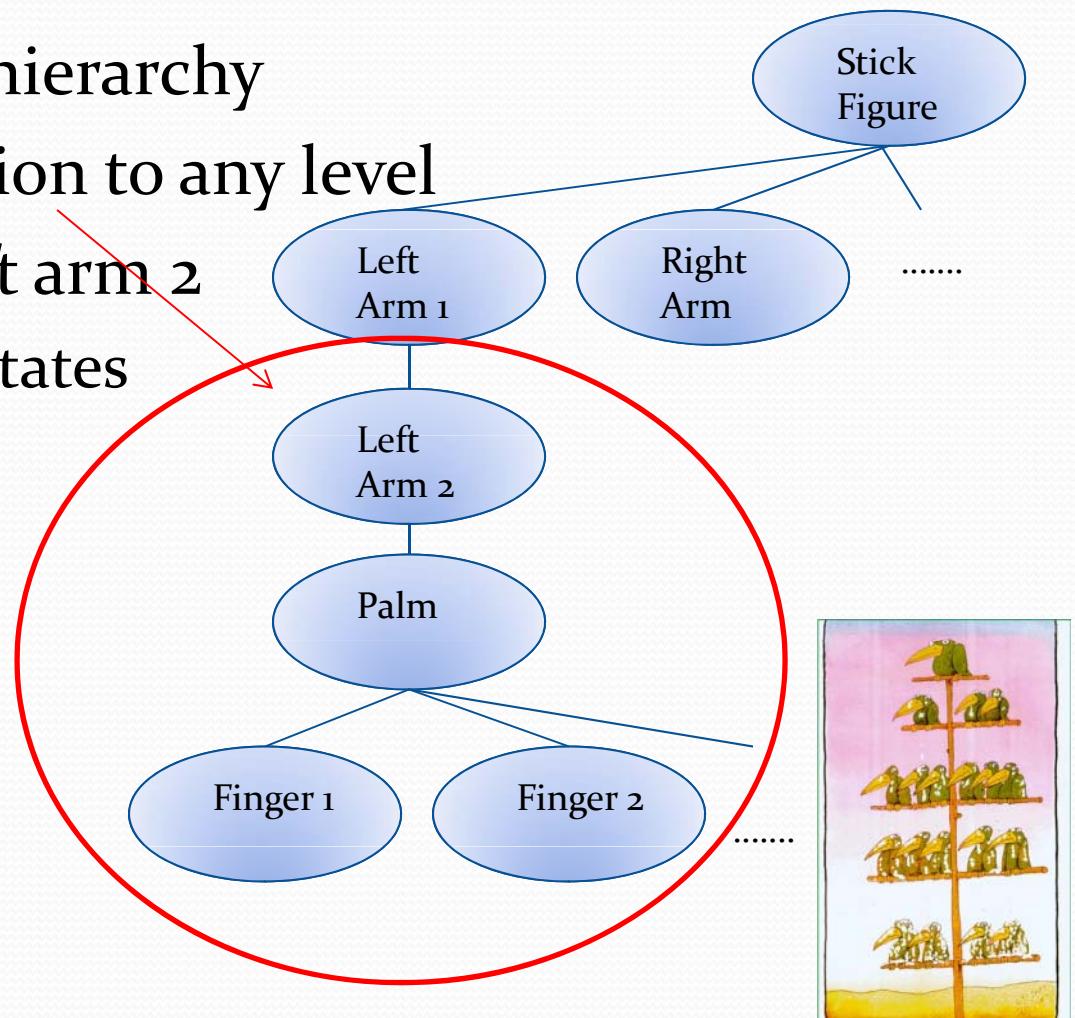
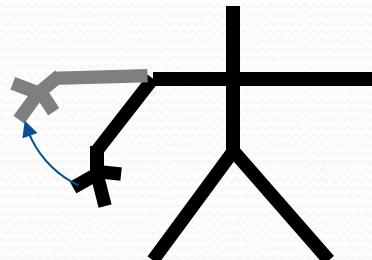
- The figure can be understood as a hierarchy as

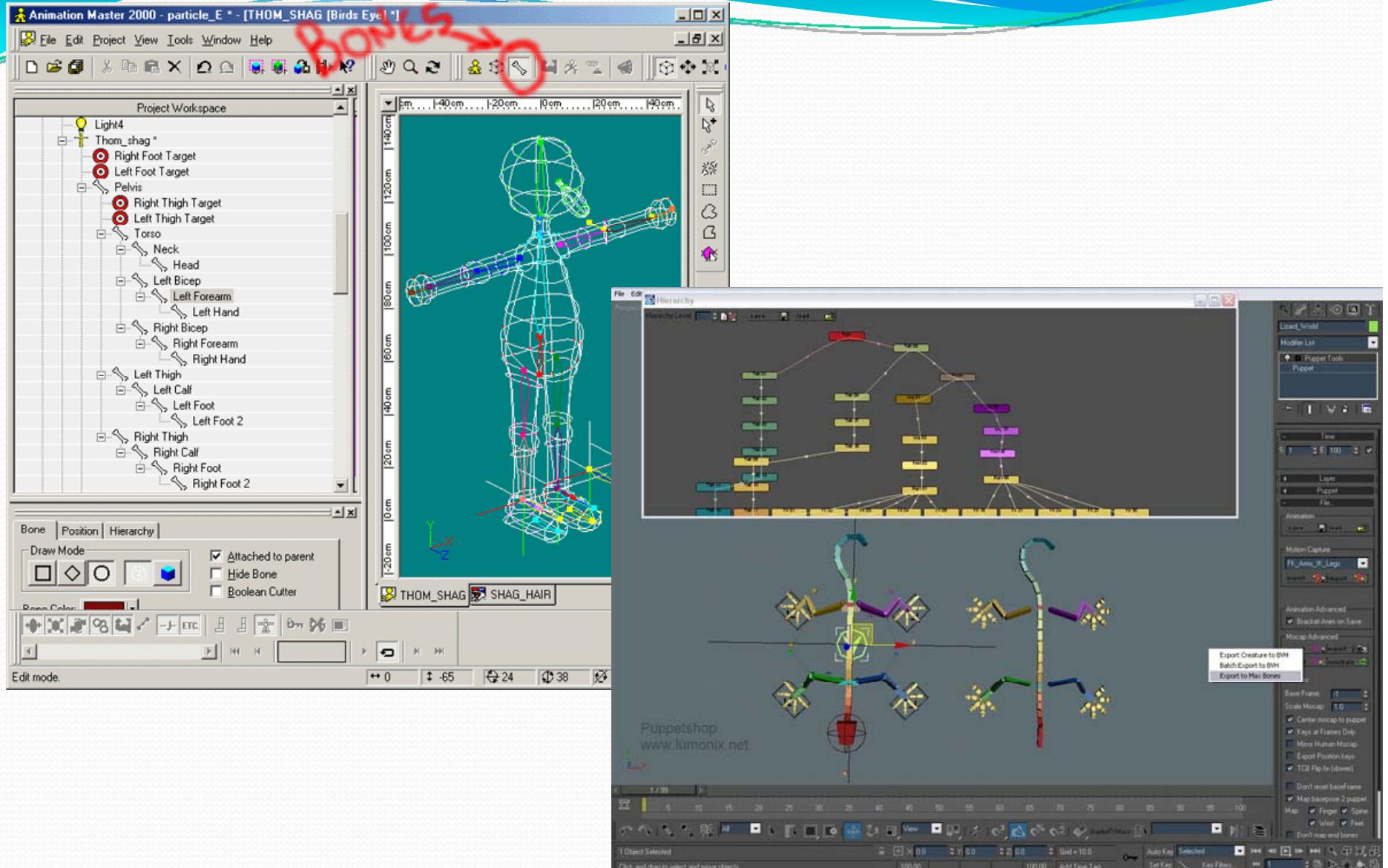


- In which each node is a new reference frame
- When we apply a transformation to a node, all its children in its sub-tree are affected
- E.g. when we translate the stick figure, all the limbs are translated

Hierarchical Transformation

- More deeper levels of hierarchy
- Apply the transformation to any level
- E.g. Rotation at the left arm 2
 - The lower sub-tree rotates





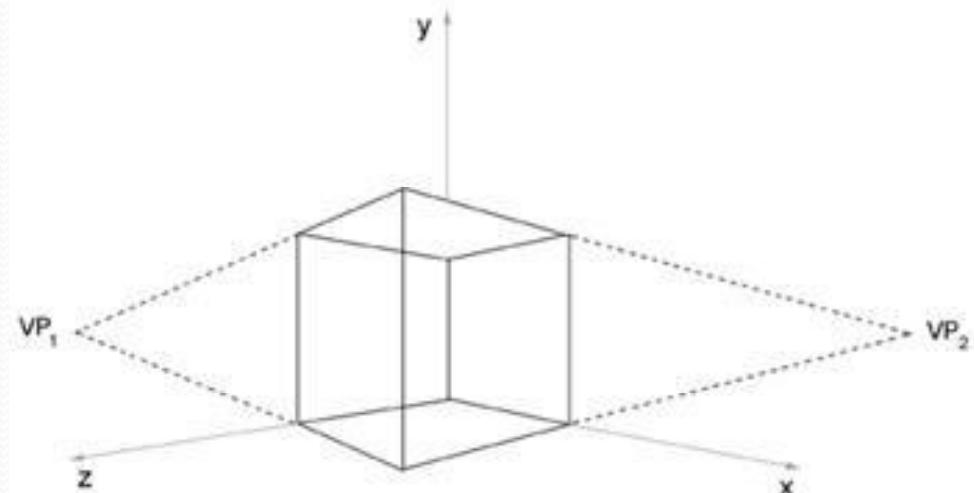
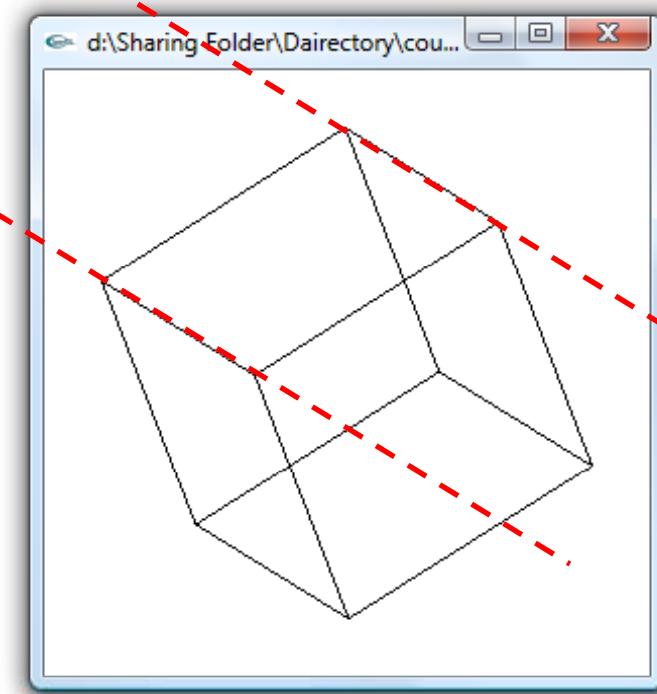
What you should be able to do now

- Romanesco broccoli



Something is not very right?!

- Parallel Projection
- Perspective Projection
- Next Lecture



Announcement

- No tutorial for this week
- Assignment #1 Deadline 8th Feb 23:59
 - Please do not start working until Friday afternoon!
 - And do NOT submit the zip file until the last minute
 - And do not spend too much time in the assignment
 - You should be able to finish the assignment within 3 hours
 - You can use either MSVS 8.0 or 10.0 or 12.0
 - Please state your version in your readme.txt
 - Or also put it in your file name like
U12345678.MSVC8.0.zip
 - In order to make your tutors happier....

Assignment 1

- Post your picture onto NUSCG facebook page
 - The first 5 have the most “like” will be voted again in the class
 - The first 3 get bonus points
- Late submission
 - 1 day late, 33% off
 - 2 day late, 66% off
 - 3 day late, you submit it for fun
 - Unless you have some serious excuses