NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2004-2005

MA2214 Combinatorial Analysis

April/May 2005 — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. Each question carries 20 marks.

3. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

4. Candidates may bring in **ONE** A4-size help sheet with handwritten notes.

Question 1 [20 marks]

(a) Sixteen people are to be seated round a circular table. Find the number of ways this can be done if three particular persons A, B and C must be seated in such a way that at least one other person is seated between A and B, at least two other persons between A and C, and at least three other persons between B and C.

(b) Thirty people of different height are to stand in three rows to take a photograph, with ten persons in each row. Find the number of ways this can be done if

(i) every person in each row is taller than the person standing on his left;

(ii) every person in the middle row is taller than the person standing right in front of him and shorter than the person standing right behind him.

Question 2 [20 marks]

(a) Find the number of ways of distributing fifteen different sweets to five children if

(i) there is no restriction;

(ii) each child must be given at least one sweet;

(iii) one particular child must be given at least three sweets and each of the remaining children must be given at least one sweet.

(b) Find the number of ways of distributing twenty identical objects into five distinct boxes if

(i) there is no restriction;

(ii) each box must contain at least one object;

(iii) one particular box must contain at least three objects and each of the remaining boxes must contain at least one object.

Question 3 [20 marks]

(a) Solve the recurrence relation

$$a_n - 3a_{n-1} + 4a_{n-3} = 2n - 7$$

with $a_1 = 0$, $a_2 = 7$ and $a_3 = 18$.

(b) Let a_n denote the number of n-digit integers formed by the digits 1, 2, 3 and 4 without containing any block of 11, 22 and 33.

(i) Find a recurrence relation of a_n with the necessary initial conditions.

(ii) Find a_n in terms of n.

Question 4 [20 marks]

(a) Find the number of 12-letter words formed by the twelve letters 4 a's, 4 b's and 4 c's such that no three consecutive letters are identical.

(b) Find the number of ways of distributing twelve distinct objects into four identical boxes such that no box is empty.

Question 5 [20 marks]

(a) Let a_n denote the number of n-digit integers formed by the digits 1, 2, 3, 4, 5, 6 and 7 such that the total number of occurrence of the six digits 1, 2, 3, 4, 5 and 6 altogether is even and the number of occurrence of the digit 7 is at least three.

(i) Find a suitable generating function for a_n.

(ii) Find a_n in terms of n.

(b) Let b_n denote the number of ways of distributing n identical objects into seven distinct boxes labelled from 1 through 7 such that the total number of objects in boxes 1, 2, 3, 4, 5 and 6 altogether is even and box 7 contains at least three objects.

(i) Find a suitable generating function for b_n.

(ii) Find b_n in terms of n.

END OF PAPER