NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

(Semester II: 2000–2001)

ST2334 PROBABILITY AND STATISTICS

April/May 2001 — Time Allowed : 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **TEN (10)** printed pages.
2. Answer **ALL** the questions. The number in [ ] indicates the number of marks allocated for that part. The total number of marks for this paper is 60.
3. Write your answers in the spaces provided.
4. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
5. Candidates may bring in one handwritten A4-size (210 × 297 mm) help sheet.
6. Statistical tables are provided.

Matriculation No.: | | | | | | | | | |

Question No.	1	2	3	4	5	**Total**
Score						

QUESTION 1

Each front tire on an automobile is supposed to be filled to a pressure of 4 psi. Let X be the actual air pressure for the front-right tire and Y be the actual air pressure for the front-left tire. Suppose the joint density of X and Y is:

$$f(x,y) = \begin{cases} \frac{3}{392}(x^2+y^2), & \text{if } 3 < x < 5 \text{ and } 3 < y < 5, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $E[X]$ and $E[Y]$. [5 marks]

QUESTION 1 (cont'd)

(b) Find the conditional probability that Y is less than 4 given $X = 4$. [5 marks]

QUESTION 1 (cont'd)

(c) Are X and Y independent? Why? [5 marks]

QUESTION 2

A satellite system consists of 3 components and functions on any given day if at least 2 of the 3 components function on that day. On a rainy day each of the components functions independently with probability 0.90, whereas on a dry day they each functions independently with probability 0.95. If the probability of rain tomorrow is 0.60, what is the probability that the satellite system will function tomorrow? [5 marks]

QUESTION 3

Two soft-drink machines, A and B, in the Science canteen is regulated so that the amounts of drink dispensed from the machines are both approximately normally distributed. Suppose the amounts of drink dispensed from the two machines are independent. Random samples from the two machines yield the following results (unit: 10 ml):

machine	sample size	sample mean	sample variance
A	$n_A = 10$	$\bar{X}_A = 16.76$	$S_A^2 = 0.1$
B	$n_B = 10$	$\bar{X}_B = 17.92$	$S_B^2 = 0.061$

(a) Explain the meaning of a 95% confidence interval for $\frac{\sigma_A^2}{\sigma_B^2}$. [5 marks]

(b) Can we assume their variances to be equal, that is $\sigma_A^2 = \sigma_B^2$? Compute a 95% confidence interval for $\frac{\sigma_A^2}{\sigma_B^2}$ and justify your answer based on this interval. [5 marks]

QUESTION 3 (cont'd)

(c) Do the mean amounts of drink dispensed from the two machines differ? Conduct a test at significance level of 5% to support your answer. [10 marks]

QUESTION 4

Suppose that the number of customers who arrive at DBS bank, NUH branch for opening a bank account per day is a Poisson random variable with parameter λ (Its probability function is: $p(x;\lambda) = e^{-\lambda}\lambda^x/x!$, for $x = 0, 1, 2, ...$). From past experiences, each customer will open a savings account, checking account and fixed deposit account with probabilities $0.4, 0.5$ and 0.1, respectively. Assume that each customer decides independently on the type of account he or she would like to open. Let X_1 be the number of customers opening a savings account, X_2 be the number of customers opening a checking account, and X_3 be the number of customers opening a fixed deposit account tomorrow.

(a) What is the joint probability function of X_1, X_2, X_3? [5 marks]

QUESTION 4 (cont'd)

(b) What is the marginal probability function of X_1? [5 marks]

QUESTION 5

Suppose that two teams play a series of games that ends when either one of them has won 3 games. Assume that each game played is, independently, won by team A with probability p. The regular ticket price for a game is \$10 and buyers can get 20% off for a second ticket for the same game. Mary and Jon would like to watch the whole series. How much should they expect to pay for the tickets? [10 marks]

END OF PAPER