1. Literature review

1.1 Efficient Market Hypothesis

The first brick of the modern financial foundations was layout by a French financial mathematician: Louis J.B.A Bachelier (1870-1946). Previously studied in mathematical physics, he turned his interest into analyzing the prices of warrants traded in the Paris stock market when he prepared for his PhD thesis under the famous mathematician Henry Poincare. He discovered, written in his thesis “Theory of speculation” in 1914, that the prices distributed randomly, and investors could never gain profits from the past price patterns, and he concluded “the mathematical expectation of the speculator was zero”. However, his revolutionary finding was not recognized until almost half a century later by the works of two authors, who will be Nobel Prize winners, a physicist-turned-economics Paul A. Samuelson and an Italian-American financial economist Eugene F. Fama. While Samuelson (1965) originally introduced the Martingale process[1], Fama (1965) followed an established Random walk model[2] to arrive almost the same conclusion: the current traded price of an asset is already accounted all available information related to that asset. This is the heart of the Efficient Market Hypothesis (EMH) that we know today. Fama, though, coined the term *efficient market* in his writings (1965a and 1965b, respectively):

“an “efficient” market for securities, that is, a market where, given the available information, actual prices at every point in time represent very good estimates of intrinsic values”, and:

“An “efficient” market is defined as a market where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants. In an efficient market, on the average, competition will cause the full effects of new information on intrinsic values to be reflected “instantaneously” in actual prices”.

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1. A variable X follows a Martingale model if and only if: $E(X_{t+1}|X_0, \mathrm{X}_1, \dots, X_t) = X_t$
2. A random walk model assumes that the steps, in which the variable takes each time away from their current value, are to be independently and identically distributed.

Lo (2017) acknowledged two layers of EMH sophistications from Fama's works: "The Efficient Markets Hypothesis is a hypothesis about what information is available to market participants, and a second hypothesis about how prices fully reflect that information. The early tests of efficient markets focused on the what, evaluating which various types of information were or were not reflected in market prices. But the question of the how, the way markets actually incorporate information into prices is equally important— and much less obvious from the mathematics". Unlike the laws of nature which are deterministically pre-defined such as quantum mechanics, general relativity, and so on, the EMH holds because of interactions among market participants. Each one of them will try to profit from even the least possible edge in historical information, and an army of players whose movements are to exploit instantly that inefficiency, will remove the profit opportunity and bring back the balance for the asset prices.

The success of EMH was widely, empirically proven throughout the following years such that Jensen (1978) regarded "there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Market Hypothesis."

1.2 Capital Asset Pricing Model (CAPM)

The other important pillar of the financial foundations was about viewing the trade-off between risk and reward. The latter was handily derived as the expected return of the asset/portfolio over the review period. The former, however, was subtler in measuring. It could be the total volatility of the asset's returns (reflected through standard deviation) or downside risk (measured as the loss likelihood or the maximum drawdown). The relationship between risk and reward was drastically changed after William F. Sharpe published his work in 1964 (almost the same period of the EMH discovery). In his paper, he noticed that stock return variations could be divided into two components: the first element, he called "idiosyncratic", fluctuations that were generated by unique aspects of the asset, such as a change in business direction, a present of a new innovative product in the industry, to name a few, and the second component, he referred as "systematic", variations that were brought up by the general, market-wide circumstances such as unemployment rate, inflation, or oil shock. He further suggested that investors should be rewarded only for their systematic risk, not idiosyncratic one. In his reasoning, supported by mathematical formulations, since idiosyncratic risk was specifically isolated by individual assets, they could generally be eliminated by blending a substantial number of assets into a portfolio. Nonetheless, no matter how many assets an investor incorporated, the systematic risk, which was shared by all assets, would not be canceled out. This aspect was previously raised by will-be-another-Nobel-prize-winner Harry Markowitz in 1952: "This presumption, that the law of large numbers applies to a portfolio of securities, cannot be accepted. The returns from securities are too inter-correlated. Diversification cannot eliminate all variance."

Therefore, a reward should be compensated for those who accepted this risk – it is nowadays known as a *risk premium*. This analysis, combined with an independent work of Lintner (1965), helped formulate the first formal model for asset pricing, hence the name Capital Asset Pricing Model (CAPM). The model essentially illustrated the relationship between systematic risk and expected return of an asset:

$$E(r) = R_f + \beta\left(\mathrm{R_m} - R_f\right)$$

Where: R_f: risk-free rate, normally measured by the rate of 30 days to maturity Treasury Bill, $\mathrm{R_m}$ is the expected return of the market portfolio, commonly represented by S&P500 index which is a basket of 500 largest capitalization values in the US market. And β is the measure of systematic risk of the asset, the only relevant risk that investor should be rewarded for. Clearly shown from the model, an asset' expected return is linearly proportional to its β. If an asset, for instance, has beta of 3, then it would have three times the systematic risk of the market portfolio, hence the expected return of that asset should be triple the risk premium of the market portfolio.

CAPM was a revolutionary idea that transformed the entire field of investing (indeed, in 1990, Sharpe was awarded a Nobel prize for this work, among other contributions). Not only it shed the new light into asset's expected returns, but also helped provide a measure for portfolio performance. Specifically, Sharpe (1966), and Jensen (1968) leveraged the linear relationship to judge performance of a mutual fund manager by comparing directly excess return of the portfolio with the CAPM benchmark, called "alpha" (α):

$$E(r) - r_f = \alpha + \beta(R_m - r_f)$$

The manager added values to the portfolio if a positive alpha was found, meaning that the portfolio earned higher expected return then the level suggested by the CAPM, and this excess return explained the fees charged by the manger.

A clear conclusion from the EMH was that no one could consistently predict the future movements of an asset's prices, combining with the fact that only the systematic risk (as measured by β) would be rewarded, an entirely new investment vehicle was born: index funds. Based primarily upon direct results of both EMH and CAPM, an index fund would attempt to replicate the performance of an underlying market index. An investor, for instance, could trade an entire S&P500 by purchasing a S&P500 index fund with a small fee instead of individually collecting 500 different stocks into a portfolio. In other words, index funds promised to convey only beta, no alpha at all. Because index funds did not try to delivery alpha (i.e., "beat the market), they'd require less resources such as talented professionals, computing power, and so on, hence significantly lower the fees they'd charge clients. Therefore, it helped provide an accessible investment channel for the general public to approximately gain expected returns as high as the market's. And after its inception in the 1970s, index funds have blossomed into trillions of dollar industry thanks to its conveniences.

1.3 Multifactor models

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