

Drag Prediction Workshop 8: Scatter Reduction RANS SA-[neg] Joukowski Airfoil

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1 Introduction

This test case is designed as a verification case of the turbulence model of the SA-[neg] RANS equations. Participants are required to use the provided grids as they are designed so the optimal convergence rate in drag coefficient error can be observed. For adjoint consistent discretizations, the theoretical optimal convergence rate in drag error is $2P$ (though it has been observed with schemes that lack adjoint consistency), where P is the polynomial degree of the discretization. Without adjoint consistency, the expected drag error convergence rate is $P + 1$. The Joukowski airfoil is used for this test as the cusped trailing edge removes the inviscid singularity at the trailing edge at zero degrees angle of attack. However, there is still a singularity in skin friction. The provided grids are design to cluster nodes at both the trailing edge singularity and the stagnation point in order to capture the expected order of accuracy. Please do not hesitate contact Marshall Galbraith as soon as possible if you are having difficulties using the provided grids.

2 Governing Equations and Models

The compressible Reynolds Averaged Navier-Stokes equations should be used with air as the working medium. The freestream Mach number is 0.15, the Reynolds number is 6×10^6 based on chord, the angle of attack is 0 degrees, a freestream static temperature of $T_\infty = T_{ref} = 520^\circ R$, and the heat capacity ratio is $\gamma = Cp/Cv = 1.4$. The Prandtl number is fixed to $Pr = 0.72$, and a turbulent Prandtl number of $Pr_t = 0.9$. The dynamic viscosity is determined by Sutherland's law:

$$\mu(T) = \mu_0 \left(\frac{T}{T_0} \right)^{3/2} \left(\frac{T_0 + S}{T + S} \right) \quad (1)$$

where $\mu_0 = 1.716 \times 10^{-5} kg/(ms)$, $T_0 = 491.6^\circ R$, $S = 198.6^\circ R$. In a non-dimensional form, normalized by μ_{ref} and $T_{ref} = 520^\circ R$, Sutherland's law is:

$$\frac{\mu(T)}{\mu_{ref}} = \left(\frac{T}{T_{ref}} \right)^{3/2} \left(\frac{1 + S/T_{ref}}{T/T_{ref} + S/T_{ref}} \right) \quad (2)$$

Participants should use a freestream value $\tilde{\nu}/\nu = 3$ (SA working variable over kinematic viscosity) for the SA-[neg] turbulence model.

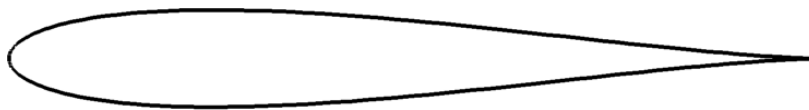


Figure 1: Joukowski airfoil geometry.

Participants must use the SA turbulence model or the equivalent “negative-SA”¹, e.g. SA-neg. Because this will have a significant effect on the “truth” drag coefficient value, the turbulence model must be carefully documented. We strongly recommend the use of test cases from the NASA Turbulence Modeling Resource (<http://turbmodels.larc.nasa.gov/>) website to verify correct implementation of the turbulence model, where details of the mathematical formulation for the SA-[neg]² model are also available.

3 Meshes

A set of Python scripts are provided that can generate both quadrilateral and triangular grids that are exported in a variety of file formats. Two families of meshes are available: The **Classic** family of meshes uses the Joukowski conformal map directly to generate the grid and produces nearly perfectly orthogonal elements and were used in the AIAA Papers 2021-1552 and 2023-1244. The **Challenge** set of meshes produces meshes with higher degrees of skew near the wall which can degrade the solution quality for some discretizations. There is also an option to generate high-order curved grids. The main script to generate the grids is `Joukowski_RANS.py`, which contains options for the grid polynomial degree, the grid family, the number of refinement levels, and the desired grid format.

4 Boundary Conditions

The far field boundary can be imposed with a Riemann invariant or characteristic boundary condition. Note that the far field boundary is not far enough away to be completely non-influential. Hence, the boundary condition used must be documented, and will likely lead to slight difference in the “truth” drag coefficient on the finest mesh. The airfoil surface is imposed as a no-slip adiabatic wall.

5 Common Inconsistencies

The following is a list of common inconsistencies that can lead to computing a different “truth” drag coefficient value.

1. Using a different Prandtl number than 0.72.
2. Using constant viscosity rather than Sutherland’s law.
3. Using isothermal wall rather than adiabatic wall.
4. Using modified SA turbulence models will have a significant effect.
5. Using a freestream value other than $\tilde{\nu}/\nu = 3$.

¹http://www.iccfd.org/iccfd7/assets/pdf/papers/ICCFD7-1902_paper.pdf

²<https://turbmodels.larc.nasa.gov/spalart.html#sa>

6 Grid Convergence Campaign

The main objective is to demonstrate grid convergence of drag coefficient error on a sequence of successively refined meshes. The provided meshes must be used for all calculations. Participants should provide a non-dimensional drag coefficient error for each of the grids. Due to variations in boundary condition implementations, participants should compute a reference drag coefficient on the finest grid with the highest order of accuracy available in their software. The drag coefficient error for each grid is computed relative to this reference drag coefficient. Participants should also verify that machine zero lift is computed on all grids.

1. Start the simulation from a uniform free stream with $M = 0.15$ everywhere, and drive the L^2 -norm of the residuals as close to machine zero as possible.
2. Perform this exercise for at least four different meshes and with different orders of accuracy to assess the performance of schemes of various accuracy.