THE UNIVERSITY *of* EDINBURGH

SCHOOL OF ENGINEERING

COMPUTATIONAL METHODS AND MODELLING 3 ONLINE EXAM

MECE09033

Exam Diet: December 2020 **Duration:** 24 hours **Expected workload:** 2 hour plus upload
Exam starts: 13:00 on 18/12/2020 **Exam ends:** 13:00 on 19/12/2020 ***All times are GMT (UTC+0)***

Before commencing work, please read the academic guidance.

Examination information

- This exam has specific technical instructions, listed below. Please read these carefully before starting your exam
- This exam paper consists of FIVE questions.
- Candidates should attempt ALL questions.

Specific instructions

- Students should assume reasonable values for any data not given in a question, or not available on a datasheet, and should make any such assumption clear on their answer sheets.
- Students in any doubt as to the interpretation of the wording of a question, should make their own decision, and should state it clearly on their answer sheet.
- Write concise, complete answers. If a length limit is given, stay within it. Produce equations and diagrams to a good hand-drawn standard.
- This is an open book exam. This means you can freely access any printed or online materials to aid you in your answers. Online materials can include text, images, videos and data. You may NOT engage in interactions or discussions relating to the exam questions or examined subject matter in any form. Sharing the answers to this exam in any way, by any means and in any form is STRICTLY NOT allowed.

Technical instructions

- All Python programme files should be named with your exam number as follows, e.g., 'MECE09033-B123456-Q1a', clearly indicating the question number after the second hyphen in the name. If you choose one code to answer all questions, then please use the following format: e.g. 'MECE09033-B123456-CODES'.
- A cover report should be written giving the answers to each section with a short description of the solution strategy used with reference to the relevant codes used to solve each section. Each answer should be labelled with the relevant question number, e.g. Q1a.
- Please place your cover report (labelled with your exam number e.g. MECE09033-B123456) plus all your programme files into **one** zipped folder also labelled with your exam number (e.g. MECE09033-B123456) and upload the zip folder to the dropbox in the ONLINE EXAM area for this course on LEARN.
- If you require technical support, contact Exams.Eng@ed.ac.uk

Special Items

- None

Convenor of Board of Examiners: **Dr D Yang**
External Examiner: **Professor A Morina**

Question 1.

An engineer has been asked to evaluate the tension in a catenary cable of uniform density along its length, hanging between two points of uneven height under its own self-weight.

A differential equation that relates cable tension to the co-ordinates of the cable (x, y) along the span between the supports is given by Equation 1A.

$$\frac{d^2y}{dx^2} = \frac{w}{T_A}\sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad \text{Equation 1A}$$

Integration of Equation 1A gives an expression for the height of the cable, y, in terms of linear density, w, [kg.m^{-1}] and tension, T_A, [N], which is reproduced in Equation 1B.

$$y = \frac{T_A}{w} cosh\left(\frac{w}{T_A}x\right) + y_0 - \frac{T_A}{w} \qquad \text{Equation 1B}$$

a) Using an appropriate Python in-built function or otherwise, prove that Equation 1B is the general solution to the differential equation, Equation 1A. **(10)**

b) Calculate the tension, T_A, for the conditions: x = 50 m, y = 15 m, y_0 = 5 m, and w = 10. Demonstrate that the value calculated using this technique is correct. **(10)**

Question 2.

The simple dynamics of a pendulum can be described by the following second order differential equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \qquad \text{Equation 2A}$$

with θ being the displacement angle in radian and t being time in seconds. For this exercise, consider a gravity acceleration $g = 9.81\ m/s^2$ and a length $l = 9.81m$.

The equation above can be converted into a system of two first order differential equations:

$$\begin{cases} \dfrac{d\theta}{dt} = \omega \\ \dfrac{d\omega}{dt} = -\dfrac{g}{l}\sin\theta \end{cases} \qquad \text{Equation 2B}$$

where ω is the angular velocity. In addition, this system of equations can be linearized by assuming $\sin\theta = \theta$ (which is a good approximation for small values of θ) to obtain the following linear system:

$$\begin{cases} \dfrac{d\theta}{dt} = \omega \\ \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta \end{cases} \qquad \text{Equation 2C}$$

An engineer needs to assess the quality of this approximation by performing an analysis based on numerical simulations. To this end, a Python code which employs the Euler method for the solution of ODEs needs to be written. For the Euler method, a step $h = 0.001$ seconds should be used, and the two systems should be solved for the interval $t = [0, 30]$ seconds. In particular, due to the approximation introduced by the linearization, the oscillation period of the solution of Equation 2C will be slightly different compared to the period of the solution of Equation 2B.

a) Write a Python code to solve the original non-linear system (Equation 2B) and the linearized system (Equation 2C) and report the value of the solutions (θ and ω) at $t = 10$, $t = 20$, and $t = 30$ seconds for both cases, starting from the following initial condition at $t = 0$: **(10)**

$$\begin{cases} \omega_0 = \omega(t = 0) = 0 \\ \theta_0 = \theta(t = 0) = \pi/4 \end{cases}$$

b) For the solutions of the non-linear and linear systems computed in Q3(a), evaluate the period of oscillation. Explain how the period has been evaluated. Evaluate the periods of oscillation also for the following initial condition: $\theta_0 = \frac{\pi}{2}$ and $\theta_0 = \frac{\pi}{8}$ **(5)**

c) Summarize your findings by discussing the difference between the oscillation periods of the non-linear and linear systems for the different values of the initial condition θ_0. **(2)**

d) Find a value of the initial condition θ_0 for which the difference between the periods of the solutions of the linear and non-linear system is (1 ± 0.05)%. *Note that (1 ± 0.05)% indicates the accepted approximation level. In other words, the task is to find a value of θ_0 such that the difference between the linear and non-linear periods is in the range between 0.95% and 1.05%. **(3)**

Question 3.

A new engineer has been asked to evaluate the interest payments on a loan needed to procure a £115,000 equipment item where no deposit is placed, and payments will be £25,500 per year for 6 years. The formula relating present worth *P*, annual payments *A*, number of years *n*, and interest rate *i* is given by Equation 3A:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} \qquad \text{Equation 3A}$$

Using a suitable numerical technique, implemented manually by calculator or Python code, calculate the interest rate being paid to service the purchase of this equipment using the equation above only. Demonstrate that the rate of interest calculated using this technique is correct. **(20)**

Question 4.

The upward velocity of a rocket can be computed by the following equation:

$$v = u\ln(\frac{m_0}{m_0 - qt}) \qquad \text{Equation 4A}$$

where v = upward velocity, u = velocity at which fuel is expelled relative to the rocket, m_0 = initial mass of the rocket at time t = 0, q = fuel consumption rate, and g = downward acceleration of gravity (assumed constant = 9.8 m/s^2).

If u = 1.8×10^3 m/s, m_0 = 160×10^3 kg, and q = 2.5×10^3 kg/s, use any appropriate technique, implemented manually by calculator or coded in Python, to determine the distance travelled by the rocket after 30 seconds of flight. **(20)**

Question 5.

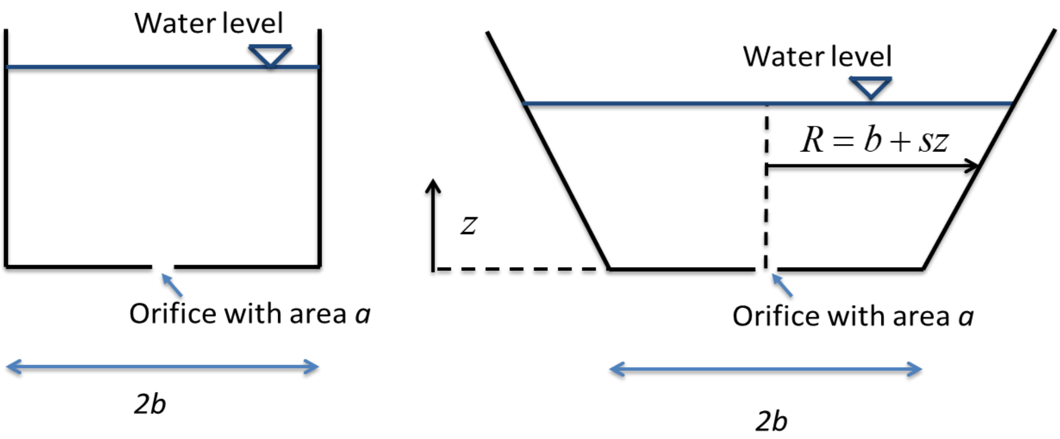

Figure A **Figure B**

A cylindrical tank with filled liquid height, h, of 1 m and width, 2b, of 2 m is designed with a slot in its base to allow drainage (Figure A). The area of the circular drainage slot is A = 0.01 m^2. An engineer realises that if she inclines the slope of the tank side to form a truncated cone shape (Figure B), she will change the flowrate and drainage time through the slot because the height of the water in the tank, h, and hence the static head, will be different for the same volume of water.

The flowrate of the water through the slot is governed by Bernoulli's Equation, while the changing radius of the tank can be expressed by the function R(z) = b + sz, where s is the slope of the tank side and z is the vertical height co-ordinate.

Find the value of s that doubles the time it takes to drain the water tank. **(20)**

END OF PAPER