THE UNIVERSITY of EDINBURGH

SCHOOL OF ENGINEERING

MECE09033

COMPUTATIONAL METHODS AND MODELLING 3

Exam Date: **16/12/2019** From and To: **14:30-16:30** Exam Diet: **Dec 2019**

Please read full instructions before commencing writing

Exam paper information

- This paper consists of FIVE questions.
- Candidates should attempt ANY FOUR questions.

Special instructions

- Do not remove this exam paper from the exam hall. Your exam paper must be returned with your script book.
- This is an open book exam.
- Students should assume reasonable values for any data not given in a question nor available on a datasheet, and should make any such assumptions clear on their script.
- Students in any doubt as to the interpretation of the wording of a question, should make their own decision, and should state it clearly on their script.
- Please write your name in the space indicated at the right hand side on the front cover of the answer book. Also enter your examination number in the appropriate space on the front cover.
- Write **ONLY** your examination number on any extra sheets or worksheets used and firmly attach these to the answer book(s).
- This examination will be marked anonymously.

Special items

- None

Convenor of Board of Examiners: **Dr I M Viola**
External Examiner: **Professor M Cartmell**

Computational Methods and Modelling 3 – Open Book Exam.

Two hour exam

Answer any FOUR of the five questions.

Please submit a cover report (Word file with all answers) plus separate codes for each question, labelled with the number of the question, e.g. Q1.m, to the Exam Dropbox on Learn by the end time of the exam. Submissions after this time will not be accepted.

Question 1.

Two blocks, each of mass, m, are connected by springs and a dashpot according to the diagram in Figure 1.

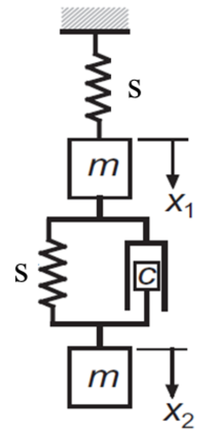

Figure 1. Two-Mass Spring-Dashpot System

The stiffness of each spring is s, and c is the coefficient of damping of the dashpot. When the system is displaced and released, the displacement of each block (j = 1, 2) during the ensuing motion has the form given by Equation 1

$$x_j(t) = A_j e^{\omega_r t} cos(\omega_i t + \varphi_j), \text{ j=1,2} \qquad \textbf{Equation 1}$$

where A_j = 0.1 m and $\varphi_j = \pi/8$ are constants for both springs, and $\omega = \omega_r \pm i\omega_i$ are the roots of Equation 2

$$\omega^4 + 2\frac{c}{m}\omega^3 + 3\frac{s}{m}\omega^2 + \frac{c}{m}\frac{s}{m}\omega + \left(\frac{s}{m}\right)^2 = 0 \qquad \textbf{Equation 2}$$

a) Determine the two possible combinations of ω_r and ω_i if the ratio c/m = 12 s^{-1} and s/m = 1500 s^{-2}. Use a suitable numerical technique, performed manually, or by computer code (e.g. MATLAB, Python or another code) to achieve this. Explain the operating principle of the code in detail. **(5)**

b) Calculate the smallest possible amount of work done on the second, damped spring in its first 10 seconds of operation using the smallest absolute values of ω_r and ω_i if the mass on the damped spring is subject to an initial tensile force of 100 N before being released. **(5)**

Question 2.

Determine the parameter p that minimizes the integral

$$\int_0^{\pi} \sin x \cos px \, dx \qquad \textbf{Equation 3}$$

a) Using any suitable numerical methods, determine the parameter p that minimizes the integral shown by Equation 3. **(5)**

b) For the value of p found in a), determine any value (or values) of x, for which the function being integrated is zero in the domain of integration. **(5)**

Question 3.

The natural frequencies of a uniform cantilever beam are related to the roots β_i of the frequency equation, $y(\beta) = \cosh \beta . \cos \beta + 1 = 0$, where β is defined by Equation 4.

$$\beta_i^4 = (2\pi f_i)^2 \frac{mL^3}{EI} \qquad \textbf{Equation 4}$$

Using an appropriate numerical technique, implemented in MATLAB or another programming language, determine the two lowest values of the natural frequency, f_i, which satisfies Equation 4. **(10)**

Question 4.

The trajectory of a satellite orbiting the earth is given by Figure 2 and described mathematically by Equation 5, where (R, θ) are the polar coordinates of the satellite, and C, e, and α are constants (e is known as the eccentricity of the orbit). The satellite is observed at three positions on its journey defined by the co-ordinates in Table 1.

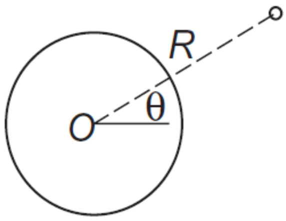

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Figure 2. Trajectory of Satellite Orbiting the Earth

$$R = \frac{C}{1 + e\sin(\theta + \alpha)}$$ **Equation 5**

θ	-30°	0°	30°
R (km)	6870	6728	6615

Table 1 Coordinates

Determine the smallest radius, R, of the satellite trajectory described by the three points in Table 1. Use any combination of appropriate numerical techniques, implemented manually or using a MATLAB or other computing code, to solve this problem. **(10)**

Question 5.

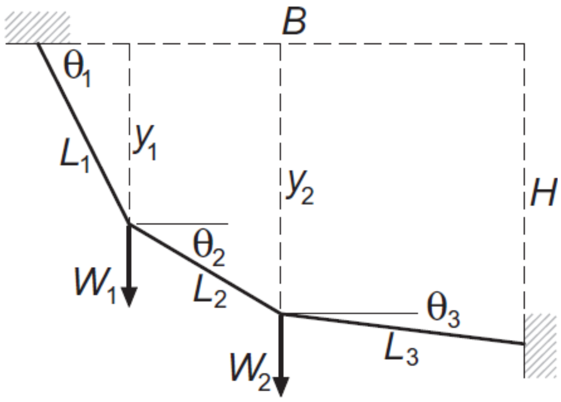

Figure 3

A cable supported at the ends carries the weights W_1 and W_2. The potential energy of the system is given by Equation 6.

$$\begin{aligned} V &= -W_1 y_1 - W_2 y_2 \\ &= -W_1 L_1 \sin\theta_1 - W_2(L_1 \sin\theta_1 + L_2 \sin\theta_2) \end{aligned}$$ **Equation 6**

The geometric constraints of the system are expressed by Equation 7 (a and b)

$$L_1 \cos\theta_1 + L_2 \cos\theta_2 + L_3 \cos\theta_3 = B$$ **Equation 7a**

$$L_1 \sin\theta_1 + L_2 \sin\theta_2 + L_3 \sin\theta_3 = H$$ **Equation 7b**

The relevant data for the system are as follows: L_1 = 1.2 m, L_2 = 1.5 m, L_3 = 1.0 m, B = 3.5 m, H = 0 m, W_1 = 20 kN, and W_2 = 30 kN.

Determine the values of the three angles θ_1, θ_2, and θ_3 that minimise the potential energy in the system while satisfying the constraints in Equation 7. **(10)**