THE UNIVERSITY of EDINBURGH

SCHOOL OF ENGINEERING

COMPUTATIONAL METHODS AND MODELLING 3

MECE09033

Exam Date: **15/12/2017** From and To: **14:30 – 16:30** Exam Diet: **Dec 2017**

Please read full instructions before commencing writing

Exam paper information

- This exam paper consists of THREE questions.
- Candidates should attempt any TWO questions.

Special instructions

- This exam is to be performed with the aid of a computer.
- Candidates are required to save their simulations on the computer.
- Use of calculators approved by the College of Science and Engineering is permitted.
- This exam is OPEN BOOK.
- Students should assume reasonable values for any data not given in a question nor available on a datasheet, and should make any such assumptions clear, in a report document to be submitted with the simulation files.
- Students in any doubt as to the interpretation of the wording of a question, should make their own decision, and should state it clearly.
- This examination will be marked anonymously.

Special items

- None

Convenor of Board of Examiners: **Dr I M Viola**
External Examiner: **Professor M Cartmell**

<u>Computational Methods and Modelling 3 – Open Book Exam.</u>

Two hour exam

Answer TWO out of THREE questions only.

Please submit an overall Cover Report (Word file with all answers and explanations of methods used for all of your questions) plus separate codes for each question, labelled with the number and part of the question, e.g. Q1b.m, to the Exam Dropbox on Learn by the end time of the exam. Submissions after this time will not be accepted.

Question 1.

The deflection of a uniform beam (i.e. one having a constant cross-section) subject to a linearly increasing distributed load can be computed from the following equation:

$$y = \frac{\omega_0}{120EIL}\left(-x^5 + 2L^2x^3 - L^4x\right)$$

The data provided are as follows: L = 800 cm, E = 40,000 kN/cm^2, I = 40,000 cm^4, and w_0 = 3.5 kN/cm.

For these conditions, determine the point (x-value) of maximum deflection

a) graphically by plotting the function, **(3)**

b) using an appropriate optimisation technique, implemented in Matlab, until the approximate error falls below ε_s = 1% with initial guesses of lower limit, x_l = 0 and upper limit, x_u = L. **(7)**

In a separate task, an engineer needs to determine the sensitivity of the following function to a perturbation, h = 0.1, from an initial value of x = 1.

$$f(x) = exp(-x)$$

c) Write the Taylor series expansion of this function for the given perturbation value. **(3)**

d) Using a suitable code in MatLab, calculate the change in the value of *f(x)* and the corresponding sensitivity of *f(x)* for the given perturbation and write it down. The value of the function change should be accurate to six significant decimals. **(7)**

Question 2.

A new engineer has been asked to evaluate the interest payments on a loan needed to procure a £25,000 equipment item where no deposit is placed, and payments will be £5,500 per year for 6 years. The formula relating present worth *P*, annual payments *A*, number of years *n*, and interest rate *i* is given by the following equation:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

a) Using a suitable numerical technique, implemented manually by calculator or Matlab code, calculate the interest rate being paid to service the purchase of this equipment using the equation above only. Demonstrate that the rate of interest calculated using this technique is correct. **(8)**

In a separate problem, the upward velocity of a rocket can be computed by the following equation:

$$v = u\ln\left(\frac{m_0}{m_0 - qt}\right)$$

where v = upward velocity, u = velocity at which fuel is expelled relative to the rocket, m_0 = initial mass of the rocket at time t = 0, q = fuel consumption rate, and g = downward acceleration of gravity (assumed constant = 9.8 m/s^2).

b) If $u = 1.8\times10^3$ m/s, $m_0 = 160\times10^3$ kg, and $q = 2.5\times10^3$ kg/s, use any appropriate technique, implemented manually by calculator or coded in MATLAB, to determine the distance travelled by the rocket after 30 seconds of flight. **(12)**

Question 3.

A road needs to be constructed through a hill as shown in Figure 2.

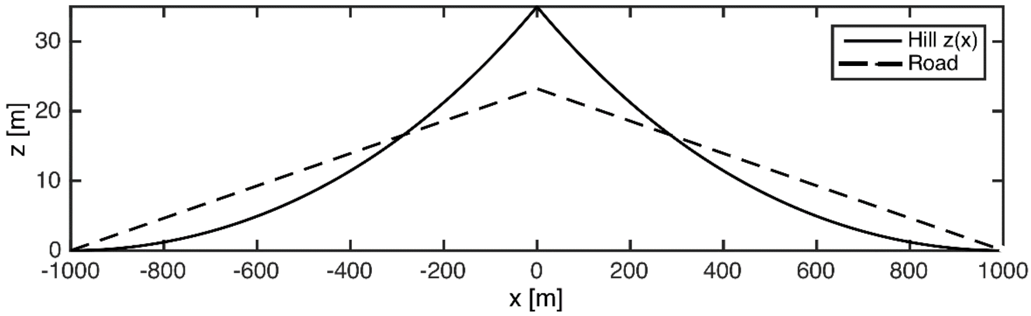

Figure 2. Road construction through a hill.

The vertical elevation of the hill is described by the equation:

$$z(x)=\begin{cases} 0 & \text{for} \quad |x|>L, \\ c_1\left(1-|x|/L\right)^2+c_2\left(1-|x|/L\right)^4 & \text{for} \quad |x|\le L, \end{cases}$$

where $c_1=30\,\text{m}$, $c_2=5\,\text{m}$ and $L=1.0\,\text{km}$. The road must connect the points $(x=-L, z=0)$ and $(x=L, z=0)$ and must be described by a symmetric piecewise function of the form:

$$z_R(x)=\begin{cases} 0 & \text{for} \quad |x|>L, \\ a+b|x| & \text{for} \quad |x|\le L. \end{cases}$$

The objective is to minimize the amount of groundwork, measured by the volume of soil that needs to be excavated or put in place.

a) Develop a simple mathematical to choose the coefficients *a* and *b* in (Eq. 3.b) to minimize the volume of earth to be moved and briefly explain your assumptions. **(6)**

b) By turning your model into a single-variable optimization problem and using a form of numerical integration to estimate the amount of soil to be moved, write an annotated MatLab algorithm to identify the optimal solution. **(12)**

c) Give the values *a* and *b* of your proposed solution, illustrate your proposed solution graphically. **(2)**

END OF PAPER