THE UNIVERSITY *of* EDINBURGH

SCHOOL OF ENGINEERING

COMPUTATIONAL METHODS AND MODELLING 3

MECE09033

Exam Diet: December 2021
Exam starts: 13:00

Duration: 2 hours plus 1 hour upload
Exam ends: 16:00

Exam Date: 17/12/2021
All times are GMT (UTC+0)

Before commencing work, please read the academic, formatting, scanning and uploading guidance.

Examination information

- This exam has specific technical instructions, listed below. Please read these carefully before starting your exam
- This exam paper consists of FOUR questions.
- Candidates should attempt ALL FOUR questions.

Specific instructions

- Students should assume reasonable values for any data not given in a question, or not available on a datasheet, and should make any such assumption clear on their answer sheets.
- Students in any doubt as to the interpretation of the wording of a question, should make their own decision, and should state it clearly on their answer sheet.
- Write concise, complete answers. If a length limit is given, stay within it. Produce equations and diagrams to a good hand-drawn standard.
- This is an open book exam. This means you can freely access any printed or online materials to aid you in your answers. Online materials can include text, images, videos and data. You may NOT engage in interactions or discussions relating to the exam questions or examined subject matter in any form. Sharing the answers to this exam in any way, by any means and in any form is STRICTLY NOT allowed.

Technical instructions

- All Python programme files should be named with your exam number as follows, e.g., 'MECE09033-B123456-Q1a', clearly indicating the question number after the second hyphen in the name. If you choose one code to answer all questions, then please use the following format: e.g. 'MECE09033-B123456-CODES'.
- A cover report should be written giving the answers to each section with a short description of the solution strategy used. Each answer should be labelled with the relevant question number, e.g. Q1a. The cover report should be a PDF/word document.
- Please place your cover report (labelled with your exam number e.g. MECE09033-B123456.pdf) plus all your programme files into one zipped folder also labelled with your exam number (e.g. MECE09033-B123456) and upload the zip folder to the dropbox in the ONLINE EXAM area for this course on LEARN.
- If you require technical support, contact Exams.Eng@ed.ac.uk

Special Items

- None

Convenor of Board of Examiners: **Professor Dongmin Yang**
External Examiner: **Professor Ardian Morina**

Computational Methods and Modelling 3 – Open Book Exam.

Question 1.

Two blocks, each of mass, m, are connected by springs and a dashpot according to the diagram in Figure Q1.

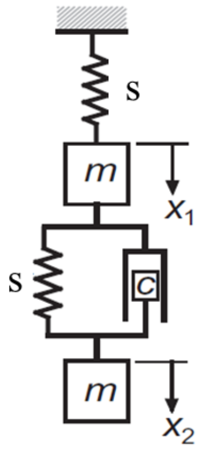

Figure Q1. Two-Mass Spring-Dashpot System

The stiffness of both springs is s, and c is the coefficient of damping of the dashpot. When the system is displaced and released, the displacement of each block (j = 1, 2) during the ensuing motion has the form given by Equation 1:

$$x_j(t) = A_j e^{\omega_r t} cos\left(\omega_i t + \varphi_j\right), \text{ j=1,2} \qquad \textbf{Equation 1}$$

where A_j = 0.1 m and $\varphi_j = \pi/8$ are constants for both springs, and $\omega = \omega_r \pm i\omega_i$ are the roots of Equation 2.

$$\omega^4 + 2\frac{c}{m}\omega^3 + 3\frac{s}{m}\omega^2 + \frac{c}{m}\frac{s}{m}\omega + \left(\frac{s}{m}\right)^2 = 0 \qquad \textbf{Equation 2}$$

a) Determine the two possible combinations of ω_r and ω_i if the ratio c/m = 12 s^{-1} and s/m = 1500 s^{-2}. Use a suitable numerical technique, performed manually, or by computer code (e.g., python or another code) to achieve this. Explain the operating principle of the code in detail. **(10)**

b) Calculate the smallest possible amount of work done on the second, damped spring in its first 10 seconds of operation using the smallest absolute values of ω_r and ω_i if the mass on the damped spring is subject to an initial tensile force of 100 N before being released. **(10)**

Question 2

The natural frequencies of a uniform rectangular cantilever beam are related to the roots, β_i, of the frequency equation, $y(\beta) = \cosh\beta.\cos\beta + 1 = 0$, where β is defined by Equation 3. The mass density of steel, m, is 7850 kg/m^3, the length, L, of the beam is 0.9 m, and the modulus of elasticity of the beam, E, is 200 GPa. The moment of inertia of the beam, I, is 3.255 x 10^{-11} m^4.

$$\beta_i^4 = (2\pi f_i)^2 \frac{mL^3}{EI}$$ **Equation 3**

Using an appropriate numerical technique, implemented in Python or another programming language, determine the two lowest values of the natural frequency, f_i, which satisfy Equation 4. **(20)**

Question 3.

The following code was written by a student to solve an equation using a well-known root-finding algorithm. However, there are six serious errors with the code that prevent it executing the algorithm properly.

```
MAX_ITER = 1000000
deg func( x ):
    return (x**3 - 4*(x*2) + 10)

def Code( a , c);
    if func(a) * func(b) ≤ 0:
        print("You have not assumed correct values of a and b")
        return 0
    c = a

    for i in range(MAX_ITER):

        c = (b * func(b) - a * func(a))/ (func(c) - func(a))

        if func(b) == 0:
            break

        elif func(b) * func(a) < 0:
            b = a
        else:
            a = c

    print("The value of root is : " , '%.4f' %c)
```

a) Name the algorithm being used in this code and explain, with one or more diagrams, how it works to find the root of an equation. **(5)**

b) Correct the existing errors in the code, identifying and justifying, by explanation, each correction, and add or remove any necessary commands to ensure that it delivers a valid result for the root of an appropriate equation. **(5)**

c) Use the corrected code to find the root of the equation: $x^2 + 5x + 1 = 5$. **(5)**

d) Modify the code to calculate the relative true error of this calculation, and quote this error. What alternative root finding method might result in an improved approximation to the root? **(5)**

Question 4.

The following ordinary differential equation (ODE):

$$\frac{dy}{dt} = \lambda y + (1-\lambda)cos(t) - (1+\lambda)sin(t) \qquad \textbf{Equation 4}$$

can be stiff depending on the value of the parameter λ, so care must be taken when the equation is solved with explicit approaches such as the Euler method. Here we assume that $\lambda = -10$.

An engineer needs to perform an analysis of this equation based on numerical simulations. To this end, a python code which employs the Euler method for the solution of ODEs needs to be written.

a) Write a python code to solve Equation 4 with the Euler method and report the value of the solutions y at $t = 2\pi$ and $t = 4\pi$, starting from the following initial condition at $t = 0$:

$$y(0) = 1$$

Report also the value of the step, h, that you decided to use in the numerical method implemented and explain how this was selected. **(6)**

b) The exact solution of Equation 4 is:

$$y(t) = sin(t) + cos(t)$$

which does not depend on the parameter λ.

Compute and report the maximum true error in the time range $t = (0{:}\,2\pi)$ and in the time range $t = (0{:}\,4\pi)$ for the solution computed in 4a) above and explain how the error was calculated. The maximum true error in a range is the maximum (in absolute value) of the difference between the computed and exact solution. **(4)**

c) Repeat the error assessment performed in 4b) for different values of the step, h. Calculate the maximum true error in the time ranges $t = (0{:}\,2\pi)$ and $t = (0{:}\,4\pi)$ for all the following values of the step, h:

$h = 0.025$
$h = 0.05$
$h = 0.1$
$h = 0.15$
$h = 0.2$
$h = 0.25$
$h = 0.3$

Arrange the results using the following table:

Step h	Error at $t = 2\pi$	Error at $t = 4\pi$
0.025		
0.05		
0.1		
0.15		
0.2		
0.25		
0.3		

(4)

d) Describe the trends observed in the table. In particular, consider the following aspects:
i) how does the error change for different values of the step, h? Why?
ii) Is the error trend different for different ranges of h? Why? **(6)**