THE UNIVERSITY of EDINBURGH

SCHOOL OF ENGINEERING

COMPUTATIONAL METHODS AND MODELLING 3

MECE09033

Exam Date: **19/12/2018** From and To: **14:30-16:30** Exam Diet: **Dec 2018**

Please read full instructions before commencing writing

Exam paper information

- This exam paper consists of THREE questions.
- Candidates should attempt TWO out of the THREE questions.

Special instructions

- This is an open book exam.
- Students should assume reasonable values for any data not given in a question nor available on a datasheet, and should make any such assumptions clear on their script.
- Students in any doubt as to the interpretation of the wording of a question, should make their own decision, and should state it clearly on their script.
- Please write your name in the space indicated at the right hand side on the front cover of the answer book. Also enter your examination number in the appropriate space on the front cover.
- Write **ONLY** your examination number on any extra sheets or worksheets used and firmly attach these to the answer book(s).
- This examination will be marked anonymously.

Special items

- This is an open book exam. There are no restrictions on the use, or introduction of, subject material to this exam. Computer networks and internet may also be accessed during the exam

Convenor of Board of Examiners: **Dr I M Viola**
External Examiner: **Professor M Cartmell**

Please submit a cover report (Word file with all answers) plus separate codes for each question, labelled with the number and part of the question, e.g. Q1b.m, to the Exam Dropbox on Learn by the Dropbox deadline indicated. Submissions after this time will not be accepted.

Question 1.

The cost of a vertical steel pipe that bears a compressive end load, P = 20 kN, is a linear function of its weight, W, and its diameter, d. The cost coefficients for the pipe's weight and diameter are £0.7/kg and £0.9/m, respectively.

The pipe has a modulus of elasticity, E, of 200 GPa, a height, H, of 2.75 m, and a density of 7800 kg.m^{-3}. Pipes are available in any combination of diameters between d_1 = 1 cm and d_2 = 10 cm, and in thicknesses between t_1 = 0.1 cm and t_2 = 1 cm.

The maximum buckling stress of the pipe at which failure occurs is given by Equation A:

$$\sigma_b = \frac{\pi E I}{H^2 d t} \qquad \text{Equation A}$$

where I is the second moment of area for a hollow cylinder, and π = 3.14159

a) For these conditions, determine the optimum values of d and t for which design buckling stress is no higher than 80% of the maximum buckling stress of the pipe material, but for which material cost is still as low as possible. Use an appropriate code or function implemented via Matlab or another technique to determine the optimum values of d and t, and report the optimum cost of the pipe. **(15)**

An engineer wishes to be able to calculate the value of the function $ln(1+x)$ to good accuracy using an appropriate computer algorithm for any value of x > 0.

b) Write the Taylor series expansion of this function with three terms. **(2)**

c) Using a suitable code in Matlab, calculate the value of the function to 7 significant decimals at x = 0.1. **(3)**

Question 2.

An engineer has been asked to evaluate the tension in a catenary cable of uniform density along its length, hanging between two points of uneven height under its own self-weight.

A differential equation that relates cable tension to the co-ordinates of the cable (x, y) along the span between the supports is given by Equation B.

$$\frac{d^2y}{dx^2} = \frac{w}{T_A}\sqrt{1+\left(\frac{dy}{dx}\right)^2} \qquad \text{Equation B}$$

Integration of Equation B gives an expression for the height of the cable, y, in terms of linear density, w, [kg.m^{-1}] and tension, T_A, [N], which is reproduced in Equation C.

$$y = \frac{T_A}{w}cosh\left(\frac{w}{T_A}x\right) + y_0 - \frac{T_A}{w} \qquad \text{Equation C}$$

a) Using an appropriate Matlab in-built function or otherwise, prove that Equation C is the general solution to the differential equation, Equation B. **(10)**

b) Using a suitable numerical root finding technique, implemented manually by calculator or Matlab code, calculate the tension, T_A, for the conditions: x = 50 m, y = 15 m, y_0 = 5 m, and w = 10. Demonstrate that the value calculated using this technique is correct. **(5)**

In a separate problem, the flow of a fluid through a packed bed can be computed by Equation D:

$$\frac{\Delta P\rho}{G_0^2}\frac{D_p}{L}\frac{\varepsilon^3}{1-\varepsilon} = 150\frac{1-\varepsilon}{D_pG_0/\mu} + 1.75 \qquad \text{Equation D}$$

where D_p is the mean diameter of the bed particles [m], ΔP is the pressure drop in the bed [Pa], ρ is the density of the fluid [kg.m^{-3}], μ is the viscosity of the fluid [N.s.m^{-2}], L is the length of the bed [m], G_0 is mass velocity of the fluid in the bed [kg.m^{-2}.s^{-1}], and ε is the porosity of the bed.

c) If the term D_pG_0/μ = 500, and the term $\frac{\Delta P\rho}{G_0^2}\frac{D_p}{L} = 20$ use fixed point iteration, implemented manually by calculator or coded in MATLAB, to determine the value of the porosity, ε. **(5)**

Question 3.

An engineer needs to solve the characteristic (quadratic) equation given by Equation E in order to solve an ordinary differential equation.

$$x^2 + 10x + 25 \qquad \text{Equation E}$$

a) Develop an appropriate Matlab code to factorise Equation E above and thereby determine its two roots. Do not use the standard Matlab functions roots, solve or fzero. **(5)**

b) Use any appropriate polynomial deflation technique to solve for all roots of the 4th order polynomial, $x^4 + 5x^3 + 15x^2 + 3x - 10$. **(15)**

END OF PAPER