Notation

- $y(t)$ is the output of the neuron.
- $x(t)$ is the total excitation of the neuron (that is, the weighted sum of inputs by fixed and plastic weights), before the tanh nonlinearity ($y(t) = \tanh(x(t))$).
- $w_{b,a}$ is the fixed/baseline weight of the connection from neuron a to neuron b.
- $alpha_{b,a}$ is the plastic weight (or plasticity coefficient) of the connection from neuron a to neuron b.
- $Hebb_{k,j}(t)$ is the Hebbian trace from neuron j to neuron k.

Note the time conventions: At any time step t, $Hebb_{k,j}(t)$ is an input to $y_k(t)$, so $Hebb_{k,j}(t)$ is computed before $y_k(t)$ for any given time step t. Meanwhile, $y_j(t)$ is an input to the computation of $Hebb_{k,j}(t+1)$.

Note that the following derivatives are all functions of each other, from one time step to the next.

Equations

- Derivative of $x_k(t)$ over $w_{b,a}$:

$$\begin{aligned}\frac{dx_k(t)}{dw_{b,a}} &= \sum_{j=1}^{N}\left\{w_{k,j}\frac{dy_j(t-1)}{dw_{b,a}}\right\} + \delta_{b=k}y_a(t-1)\\ &+ \sum_{j=1}^{N}\alpha_{k,j}\left\{Hebb_{k,j}(t)\frac{dy_j(t-1)}{dw_{b,a}} + y_j(t-1)\frac{dHebb_{k,j}(t)}{dw_{b,a}}\right\}\end{aligned}$$

- Derivative of $x_k(t)$ over $\alpha_{b,a}$:

$$\begin{aligned}\frac{dx_k(t)}{d\alpha_{b,a}} &= \sum_{j=1}^{N}\left\{w_{k,j}\frac{dy_j(t-1)}{d\alpha_{b,a}}\right\}\\ &+ \sum_{j=1}^{N}\alpha_{k,j}\left\{Hebb_{k,j}(t)\frac{dy_j(t-1)}{d\alpha_{b,a}} + y_j(t-1)\frac{dHebb_{k,j}(t)}{d\alpha_{b,a}}\right\}\\ &+ \delta_{b=k}Hebb_{b,a}(t)y_a(t-1))\end{aligned}$$

- Derivative of $Hebb_{k,j}(t)$ over $\alpha_{b,a}$

$$\begin{aligned}\frac{dHebb_{k,j}(t)}{d\alpha_{b,a}} &= \eta\frac{dHebb_{k,j}(t-1)}{d\alpha_{b,a}}\\ &+ (1-\eta)\left\{\frac{dy_k(t-1)}{d\alpha_{b,a}}y_j(t-2) + y_k(t-1)\frac{dy_j(t-2)}{d\alpha_{b,a}}\right\}\end{aligned}$$

- Derivative of $Hebb_{k,j}(t)$ over $w_{b,a}$

$$\begin{aligned}\frac{dHebb_{k,j}(t)}{dw_{b,a}} &= \eta \frac{dHebb_{k,j}(t-1)}{dw_{b,a}} \\ &\quad + (1-\eta)\left\{ \frac{dy_k(t-1)}{dw_{b,a}} y_j(t-2) + y_k(t-1)\frac{dy_j(t-2)}{dw_{b,a}} \right\}\end{aligned}$$

- Nonlinearity ($y = \tanh(x)$): derivative of y_k over any quantity ξ, as a function of y_k and the derivative of x_k over this same quantity ξ:

$$\frac{dy_k(t)}{d\xi} = (1 - y_k(t)^2)\frac{dx_k(t)}{d\xi}$$