ggRandomForests: Random Forests for Survival

John Ehrlinger and Eugene H. Blackstone Cleveland Clinic

Abstract

Random Forests (Breiman 2001) (RF) are a fully non-parametric statistical method requiring no distributional assumptions on covariate relation to the response. RF are a robust, nonlinear technique that optimizes predictive accuracy by fitting an ensemble of trees to stabilize model estimates. Random Forests for survival (Ishwaran and Kogalur 2007; Ishwaran, Kogalur, Blackstone, and Lauer 2008) (RF-S) are an extension of Breiman's RF techniques to survival settings, allowing efficient non-parametric analysis of time to event data. The **randomForestSRC** package (Ishwaran and Kogalur 2014) is a unified treatment of Breiman's random forests for survival, regression and classification problems.

Predictive accuracy make RF an attractive alternative to parametric models, though complexity and interpretability of the forest hinder wider application of the method. We introduce the **ggRandomForests** package, tools for creating and plotting data structures to visually understand random forest models grown in R with the **randomForestSRC** package. The **ggRandomForests** package is structured to extract intermediate data objects from **randomForestSRC** objects and generate figures using the **ggplot2** (Wickham 2009) graphics package.

This document is formatted as a tutorial for using the randomForestSRC for building random forests for survival and ggRandomForests package for investigating how the forest is constructed. This tutorial uses the Primary Biliary Cirrhosis (PBC) Data from the Mayo Clinic (?) available in the randomForestSRC package. We use Variable Importance measure (VIMP) (Breiman 2001) as well as Minimal Depth (Ishwaran, Kogalur, Gorodeski, Minn, and Lauer 2010a), a property derived from the construction of each tree within the forest, to assess the impact of variables on forest prediction. We will also demonstrate the use of variable dependence plots (Friedman 2000a) to aid interpretation RF results in different response settings. We also will investigate interactions between covariates to demonstrate the strength of the Random Forest method in survival settings.

Keywords: random forest, survival, VIMP, minimal depth.

1. About this document

This document is an introduction to the **ggRandomForests** R package. The aim of this introduction is to provide a detailed user guide to **ggRandomForests** as well as provide a tutorial to building a Random Forest Survival model with the **randomForestSRC** package. Our attempt is to build simple, reproducible worked examples with the Primary Biliary Cirrhosis (PBC) Data from the Mayo Clinic.

This document is available as a vignette within **ggRandomForests** package, available from the Comprehensive R Archive Network via http://CRAN.R-project.org/package=ggRandomForests.

2. Introduction

Random Forests (Breiman 2001) (RF) are a robust, non-parametric statistical method that optimizes predictive accuracy by averaging an ensemble of tree models. Random Forests are not parsimonious, utilizing all provided variables in predicting the specified outcome. It does not require prior knowledge of the parametric relation of variables (linearity or non-linearity) to the response, or of interactions between variables. RF chooses the most important variables by assessing variable impact on the predictive ability of the forest of trees.

A Random Forest is built up by bagging (Breiman 1996a) a collection of classification and regression trees (Breiman, Friedman, Olshen, and Stone 1984) (CART). The method uses a set of B bootstrap (Efron and Tibshirani 1994) samples, growing a set of independent tree models on each sub-sample of the population. Trees are grown by recursively partitioning the population based on optimization of a split rule over the p dimensional covariate space. At each split, a subset of $m \leq p$ candidate variables are chosen for the splitting. Each node is split into two daughter nodes by maximizing the separation of observations according the split rule. In regression trees, node impurity is measured by mean squared error, whereas in classification problems, the Gini index is used (Friedman 2000b). Each subsequent daughter node is then split until the process reaches the stopping criteria of either node purity or node member size defining the set of terminal (unsplit) nodes for the tree. Random Forests sort each observation into one unique terminal node per tree. The Random Forest estimate for each observation is calculated by aggregation, averaging (regression) or votes (classification), the terminal node results across the collection of B trees.

One advantage of Random Forests is a built in generalization error estimate. Each bootstrap sample selects approximately 63.2% of the population on average. The remaining 36.8% of observations, the Out-of-Bag (Breiman 1996b) (OOB) sample, can be used as a hold out test set for each tree. An OOB prediction error estimate can be calculated for each observation by predicting the response over the set of trees which were NOT trained with that particular observation. Out-of-Bag prediction error estimates have been shown to be nearly identical to n-fold cross validation estimates (Hastie, Tibshirani, and Friedman 2009). This feature of Random Forests allows us to obtain both model fit and validation in one pass of the algorithm.

2.1. Random Forests for Survival

Random Forests for survival (Ishwaran 2007; Ishwaran et al. 2008) (RF-S) are an extension of Breiman (2001) Random Forests for right censored time to event data. A forest of survival trees is grown using a log-rank splitting rule to select the optimal candidate variables. Survival estimate for each observation are constructed with a Kaplan–Meier (KM) estimator within each terminal node, at each event time.

Random Forests for survival adaptively discover nonlinear effects and interactions and are fully nonparametric. Averaging over trees, with randomizing while growing a tree, enables RF-S to approximate complex survival functions, including non-proportional hazards, while maintaining low prediction error. Ishwaran and Kogalur (2010) showed that RF-S is uniformly consistent and that survival forests have a uniform approximating property in finite-sample settings, a property not possessed by individual survival trees.

2.2. ggRandomForests

3. Data Summary: Primary Biliary Cirrhosis (PBC) Data

Data from the Mayo Clinic trial in primary biliary cirrhosis (PBC) of the liver conducted between 1974 and 1984. A total of 424 PBC patients, referred to Mayo Clinic during that ten-year interval, met eligibility criteria for the randomized placebo controlled trial of the drug D-penicillamine. The first 312 cases in the data set participated in the randomized trial and contain largely complete data.

	label	type
days	survival time in days	integer
status	censoring indicator	logical
treatment	1=D-penicillamine, 2=placebo	factor
age	age in days	numeric
sex	0=female, 1=male	logical
ascites	presence of asictes	logical
hepatom	presence of hepatomegaly	logical
spiders	presence of spiders	logical
edema	presence of edema	factor
bili	serum bilirubin in mg/dl	numeric
chol	serum cholesterol in mg/dl	integer
albumin	albumin in gm/dl	numeric
copper	urine copper in ug/day	integer
alk	alkaline phosphatase in U/liter	numeric
sgot	SGOT in U/ml	numeric
trig	triglicerides in mg/dl	integer
platelet	platelets per cubic ml/1000	integer
prothrombin	prothrombin time in seconds	numeric
stage	histologic stage of disease	factor

Table 1: PBC Data field descriptions

4. Growing the Random Forest

```
Sample size: 276
Number of deaths: 111
Number of trees: 500
Minimum terminal node size: 3
Average no. of terminal nodes: 54.212
No. of variables tried at each split: 5
Total no. of variables: 17
Analysis: RSF
Family: surv
```

```
Splitting rule: logrank *random*

Number of random split points: 10

Error rate: 16.67%
```

Figure 1 shows the predicted survival from an RF-S model, where censored device prediction is colored in blue, and devices experiencing a thrombosis event are colored in red.

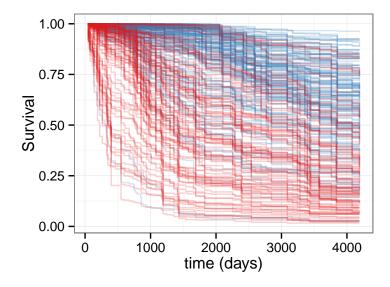


Figure 1: Freedom from pump thrombosis

5. Variable Selection

Unlike in the linear model settings, Random Forests does not require explicitly specify the functional form of the covariates to the response. Instead, we ascertain which variables contribute to the Random Forest estimates by querying the forest for variable usage.

5.1. Variable Importance

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Variable importance (VIMP) was originally defined in CART using a measure involving surrogate variables (see Chapter 5 of Breiman et al. (1984)). The most popular VIMP method to date, adopts a prediction error approach involving "noising-up" a variable. VIMP for a variable x_v is the difference between prediction error when x_v is noised up by permuting its value randomly, compared to prediction error under the original predictor (Breiman 2001; Liaw and Wiener 2002; Ishwaran 2007; Ishwaran et al. 2008).

Since VIMP is the absolute difference between prediction errors before and after permutation, a large VIMP value indicates that misspecification of that variable detracts from the predictive accuracy of the forest. VIMP close to zero indicates the variable contributes nothing to

predictive accuracy, and negative values indicate the predictive accuracy improves when the variable is mispecified. In the later case, we assume noise is more informative than the variable. As such, we ignore variables with negative and near zero values of VIMP, relying on large positive values to indicate that the predictive power of the forest is dependent on those variables.

In Figure 2, we plot VIMP measures for each of the variables used to grow the forest estimates of Figure 1. Variables are shown in VIMP rank order, largest (op_yr) at the top, to smallest (iv_lospr) at the bottom. In this case, we would focus attention on the top three variables (op_yr (surgical date), ld and devno).

```
R> plot.gg_vimp(pbc_rf)+
    theme(legend.position="none")
```

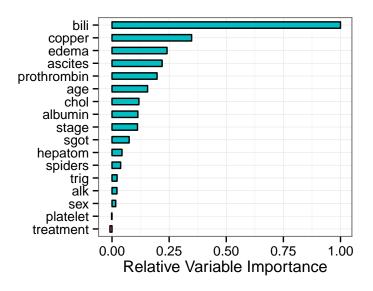


Figure 2: Variable Importance

5.2. Minimal Depth

In VIMP, prognostic risk factors are determined by inspection of the forest, ranking the most important variables according to impact on predictive ability of the forest. An alternative method recognizes that most important variables for prediction are those that most frequently split nodes nearest to the trunks of the trees (ie, at the root node) since they partition the largest portions of the population.

Node levels are numbered based on their relative distance to the trunk of the tree (ie. 0, 1, 2). A measure of important risk factors is determined by averaging the depth of first split for each variable over all trees within the forest. Lower values of this measure indicate variables that split larger groups of patients.

The maximal subtree for a variable x is the largest subtree whose root node splits on x. Thus, all parent nodes of x's maximal subtree have nodes that split on variables other than x. The largest maximal subtree possible is the root node. In general, however, there can be more

than one maximal subtree for a variable. A maximal subtree may also not exist if there are no splits on the variable. The minimal depth of a maximal subtree (the first order depth) measures predictiveness of a variable x. It equals the shortest distance (the depth) from the root node to the parent node of the maximal subtree (zero is the smallest value possible). The smaller the minimal depth, the more impact x has on prediction. The mean of the minimal depth distribution is used as the threshold value for deciding whether a variable's minimal depth value is small enough for the variable to be classified as strong.

The minimal depth plot of Figure ?? is similar to the VIMP plot in Figure 2, ranking variables from most important at the top (minimal depth measure), to least at the bottom (maximal minimal depth). Since the VIMP and Minimal Depth measures use different criteria, we expect the variable ranking to be slightly different. In this case, minimal depth indicates seven most important variables (op_yr (surgical date), age, ld, ht, wt, iv_lospr (length of stay) and inr). The vertical dashed line indicates the minimal depth threshold where smaller minimal depth values indicate higher importance and larger indicate lower importance.

```
R> pbc_vs <- var.select(pbc_rf)
R> ggMindepth <- gg_minimal_depth(pbc_vs)
R>
R> ggMindepth
```

```
gg_minimal_depth
model size
                 : 12
depth threshold : 5.4163
PE :[1] 16.671
Top variables:
           depth vimp
bili
           1.872 0.067
           2.450 0.023
copper
          2.714 0.008
albumin
chol
           2.948 0.008
prothrombin 3.132 0.013
age
           3.444 0.010
sgot
           3.820 0.005
platelet
           3.922 0.000
           3.960 0.001
trig
           3.976 0.001
alk
           4.402 0.016
edema
stage
           5.282 0.007
```

6. Variable Dependence

Once we have an idea of which variables contribute to the predictive accuracy of the forest, it is useful to get some idea of form of this contribution. We use graphical methods to show the predicted response given dependence on covariates. We can plot the marginal effect of an covariate on the class probability (classification), response (regression), mortality (survival),

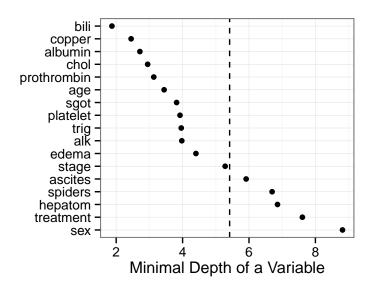


Figure 3: Minimal Depth Plot

or the expected years lost (competing risk) for a RF analysis. We plot the ensemble predicted value on the vertical axis and covariates along the horizontal axis.

6.1. Marginal Dependence

Marginal variable dependence plots the predicted response as a function of the covariate, showing each subject as a point on the plot. For classification and regression, this is straight forward predicting the response. In survival settings, we must account for the additional dimension of time. In this case, we plot the response at a specific time point of interest, for example survival at three months shown by the vertical dashed line in Figure 4. We take the predicted value of each curve at that time, and plot that against the covariate value for that observations, shown in Figure ??. Again censored cases are shown in blue circles, events are indicated by the red "x" symbols. Each predicted point is dependent on the full combination of all other covariates, not only on the covariate displayed in the dependence plot, so interpretation of these variable dependence plots can only be in general terms. The smooth loess line (Cleveland 1981; Cleveland and Devlin 1988) indicates the trend of the prediction over surgical date progression.

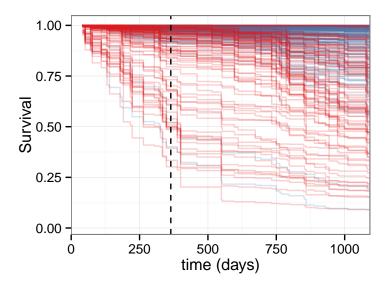


Figure 4: Freedom From Pump Thrombosis

+ scale_shape_manual(values=event.marks, labels=event.labels)

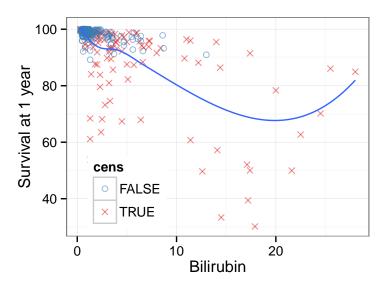


Figure 5: Freedom From Pump Thrombosis at 3 months

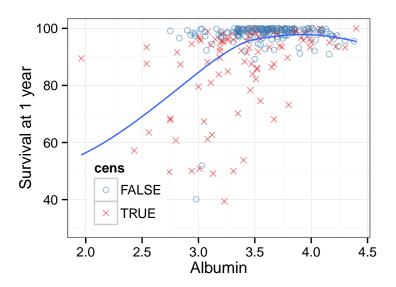


Figure 6: Survival at 1 year

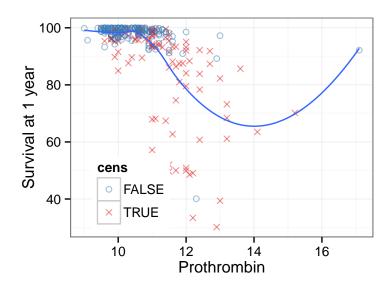


Figure 7: Survival at 1 year

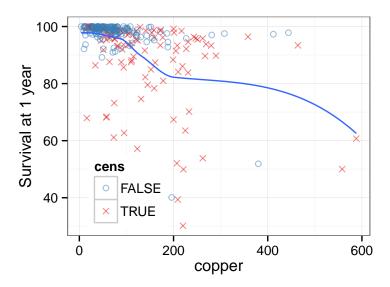


Figure 8: Survival at 1 year

6.2. Partial Dependence

Partial dependence plots are a risk adjusted alternative to marginal variable dependence. Partial plots are generated by integrating out the effects of variables beside the covariate of interest. The figures are constructed by selecting points evenly spaced along the distribution of the X variable. For each of these values (X=x), we calculate the average Random Forest prediction over all other covariates in X by (1).

$$\tilde{f}(x) = \frac{1}{n} \sum_{i=1}^{n} \hat{f}(x, x_{i,o}),$$
(1)

where \hat{f} is the predicted response from the random forest and $x_{i,o}$ is the value for all other covariates other than X = x for the observation i (Friedman 2000b). Partial dependence plots in time to event settings are shown at specific time points, similar to variable dependence.

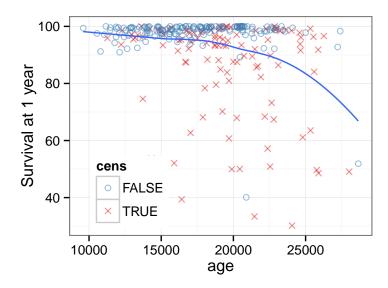


Figure 9: Survival at 1 year

Figure 10 shows the partial dependence of three month freedom from thrombosis on the surgical date covariate.

```
R> plot(ggPrtl[[1]], se=FALSE)+
+ theme(legend.position=c(.2,.2))+
+ labs(y="Freedom from Thrombosis at 3 months", x="Surgical Date")+
+ scale_color_manual(values=strCol, na.value = "black", drop=FALSE,
+ labels=event.labels)+
+ scale_shape_manual(values=event.marks, labels=event.labels)
```

7. Variable Interactions

Using the different variable dependence measures, we can calculate pairwise interactions for any pair of variables.

Using minimal depth, we calculate the maximal subtree using the normalized minimal depth of variable i relative to the root node (normalized with respect to the size of the tree) the maximal subtree interaction measure is the normalized minimal depth of a variable j wet the maximal subtree for variable i (normalized wet the size of i's maximal subtree). Smaller diagonal entries indicate predictive variables. Small interaction entries having small diagonal entries are a sign of an interaction between variable i and j (Ishwaran, Kogalur, Gorodeski, Minn, and Lauer 2010b; H., U.B., X., and A.J. 2011)

By plotting the resulting interaction measures for each variable (Figure 11), we can detect the "most interactive" pairs, and develop conditional plots Chambers (1992); Cleveland (1993).

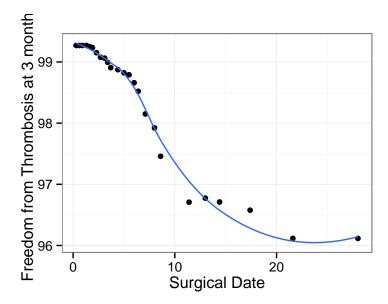


Figure 10: Risk adjusted freedom From Pump Thrombosis at 3 months

These plots are similar to stratified results, arranged in a set of panels by the interactive variable of interest.

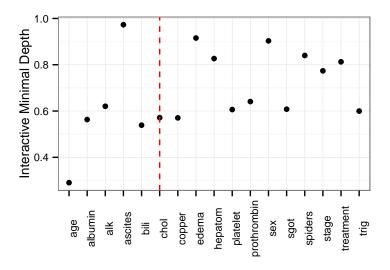


Figure 11: Minimal Depth interaction for Surgical Date

7.1. Conditional Dependence Plots

8. Conclusion

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Affiliation:

John Ehrlinger Quantitative Health Sciences Lerner Research Institute Cleveland Clinic 9500 Euclid Ave Cleveland, Ohio 44195

E-mail: john.ehrlinger@gmail.com

URL: http://www.lerner.ccf.org/qhs/people/ehrlinj/