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SPAKE2, a Password-Authenticated Key Exchange

Abstract

This document describes SPAKE2, which is a protocol for two parties that share a password to derive a strong shared key without disclosing the password. This method is compatible with any group, is computationally efficient, and has a security proof. This document predated the Crypto Forum Research Group (CFRG) password-authenticated key exchange (PAKE) competition, and it was not selected; however, given existing use of variants in Kerberos and other applications, it was felt that publication was beneficial. Applications that need a symmetric PAKE, but are unable to hash onto an elliptic curve at execution time, can use SPAKE2. This document is a product of the Crypto Forum Research Group in the Internet Research Task Force (IRTF).

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1. Introduction

This document describes SPAKE2, which is a means for two parties that share a password to derive a strong shared key without disclosing the password. This password-based key exchange protocol is compatible with any group (requiring only a scheme to map a random input of a fixed

length per group to a random group element), is computationally efficient, and has a security proof. Predetermined parameters for a selection of commonly used groups are also provided for use by other protocols.

SPAKE2 was not selected as the result of the CFRG PAKE selection competition. However, given existing use of variants in Kerberos and other applications, it was felt that publication was beneficial. This RFC represents the individual opinion(s) of one or more members of the Crypto Forum Research Group of the IRTF.

Many of these applications predated methods to hash to elliptic curves being available or predated the publication of the PAKEs that were chosen as an outcome of the PAKE selection competition. In cases where a symmetric PAKE is needed and hashing onto an elliptic curve at protocol execution time is not available, SPAKE2 is useful.

2. Requirements Notation

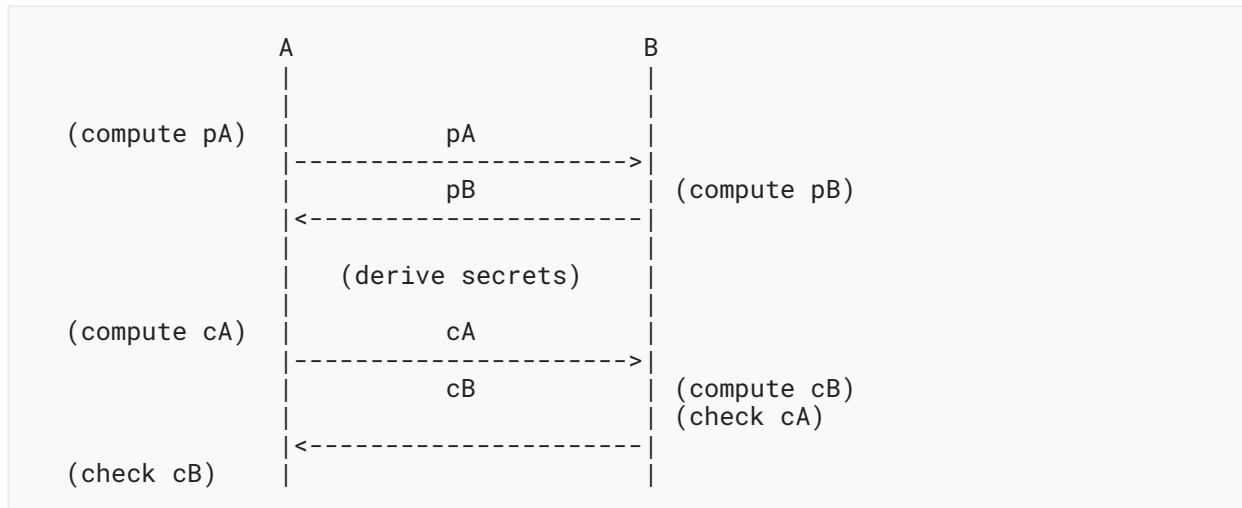
The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "NOT RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in BCP 14 [RFC2119] [RFC8174] when, and only when, they appear in all capitals, as shown here.

3. Definition of SPAKE2

3.1. Protocol Flow

SPAKE2 is a two-round protocol, wherein the first round establishes a shared secret between A and B, and the second round serves as key confirmation. Prior to invocation, A and B are provisioned with information, such as the input password needed to run the protocol. We assume that the roles of A and B are agreed upon by both sides: A goes first and uses M, and B goes second and uses N. If this assignment of roles is not possible, a symmetric variant **MUST** be used, as described later [Section 5](#). For instance, A may be the client when using TCP or TLS as an underlying protocol, and B may be the server. Most protocols have such a distinction. During the first round, A sends a public value p_A to B, and B responds with its own public value p_B . Both A and B then derive a shared secret used to produce encryption and authentication keys. The latter are used during the second round for key confirmation. ([Section 4](#) details the key derivation and confirmation steps.) In particular, A sends a key confirmation message c_A to B, and B responds with its own key confirmation message c_B . A **MUST NOT** consider the protocol complete until it receives and verifies c_B . Likewise, B **MUST NOT** consider the protocol complete until it receives and verifies c_A .

This sample flow is shown below.



3.2. Setup

Let G be a group in which the gap Diffie-Hellman (GDH) problem is hard. Suppose G has order $p \cdot h$, where p is a large prime and h will be called the cofactor. Let I be the unit element in G , e.g., the point at infinity if G is an elliptic curve group. We denote the operations in the group additively. We assume there is a representation of elements of G as byte strings: common choices would be SEC1 [SEC1] uncompressed or compressed for elliptic curve groups or big-endian integers of a fixed (per-group) length for prime field DH. Applications **MUST** specify this encoding, typically by referring to the document defining the group. We fix two elements, M and N , in the prime-order subgroup of G , as defined in Table 1 of this document for common groups, as well as generator P of the (large) prime-order subgroup of G . In the case of a composite order group, we will work in the quotient group. For common groups used in this document, P is specified in the document defining the group, so we do not repeat it here.

For elliptic curves other than the ones in this document, the methods described in [RFC9380] **SHOULD** be used to generate M and N , e.g., via $M = \text{hash_to_curve}(\text{"M SPAKE2 seed OID x"})$ and $N = \text{hash_to_curve}(\text{"N SPAKE2 seed OID x"})$, where x is an OID for the curve. Applications **MAY** include a domain separation tag (DST) in this step, as specified in [RFC9380], though this is not required.

$||$ denotes concatenation of byte strings. We also let $\text{len}(S)$ denote the length of a string in bytes, represented as an eight-byte little-endian number. Finally, let nil represent an empty string, i.e., $\text{len}(\text{nil}) = 0$. Text strings in double quotes are treated as their ASCII encodings throughout this document.

$\text{KDF}(\text{ikm}, \text{salt}, \text{info}, L)$ is a key-derivation function that takes as input a salt, input keying material (IKM), an info string, and derived key length L to derive a cryptographic key of length L . $\text{MAC}(\text{key}, \text{message})$ is a Message Authentication Code algorithm that takes a secret key and message as input to produce an output. Let Hash be a hash function from arbitrary strings to bit strings of a fixed length that is at least 256 bits long. Common choices for Hash are SHA-256 or SHA-512 [RFC6234]. Let MHF be a memory-hard hash function designed to slow down brute-force attackers. Script [RFC7914] is a common example of this function. The output length of

MHF matches that of Hash. Parameter selection for MHF is out of scope for this document. [Section 6](#) specifies variants of KDF, MAC, and Hash that are suitable for use with the protocols contained herein.

Let A and B be two parties. A and B may also have digital representations of the parties' identities, such as Media Access Control addresses or other names (hostnames, usernames, etc.). A and B may share additional authenticated data (AAD) of a length that is at most $2^{16} - 128$ bits and separate from their identities, which they may want to include in the protocol execution. One example of AAD is a list of supported protocol versions if SPAKE2 were used in a higher-level protocol that negotiates use of a particular PAKE. Including this list would ensure that both parties agree upon the same set of supported protocols and therefore prevents downgrade attacks. We also assume A and B share integer w ; typically, $w = \text{MHF}(pw) \bmod p$ for a user-supplied password, pw . Standards, such as [\[NIST.SP.800-56Ar3\]](#), suggest taking $\bmod p$ of a hash value that is 64 bits longer than that needed to represent p to remove statistical bias introduced by the modulation. Protocols using this specification **MUST** define the method used to compute w . In some cases, it may be necessary to carry out various forms of normalization of the password before hashing [\[RFC8265\]](#). The hashing algorithm **SHOULD** be an MHF so as to slow down brute-force attackers.

3.3. SPAKE2

To begin, A picks x randomly and uniformly from the integers in $[0, p)$ and calculates $X = x * P$ and $pA = w * M + X$. Then, it transmits pA to B.

B selects y randomly and uniformly from the integers in $[0, p)$ and calculates $Y = y * P$ and $pB = w * N + Y$. Then, it transmits pB to A.

Both A and B calculate group element K . A calculates it as $h * x * (pB - w * N)$, while B calculates it as $h * y * (pA - w * M)$. A knows pB because it has received it, and likewise B knows pA . The multiplication by h prevents small subgroup confinement attacks by computing a unique value in the quotient group.

K is a shared value, though it **MUST NOT** be used or output as a shared secret from the protocol. Both A and B must derive two additional shared secrets from the protocol transcript, which includes K . This use of the transcript ensures any manipulation of the messages sent is reflected in the keys. The transcript TT is encoded as follows:

```

TT = len(A)  || A
    || len(B)  || B
    || len(pA) || pA
    || len(pB) || pB
    || len(K)  || K
    || len(w)  || w

```

Here, w is encoded as a big-endian number padded to the length of p . This representation prevents timing attacks that otherwise would reveal the length of w . $\text{len}(w)$ is thus a constant for a given group. We include it for consistency.

If an identity is absent, it is encoded as a zero-length string. This **MUST** only be done for applications in which identities are implicit. Otherwise, the protocol risks unknown key-share attacks, where both sides of a connection disagree over who is authenticated.

Upon completion of this protocol, A and B compute shared secrets K_e , K_{cA} , and K_{cB} , as specified in [Section 4](#). A **MUST** send B a key confirmation message so that both parties agree upon these shared secrets. The confirmation message cA is computed as a MAC over the protocol transcript TT , using K_{cA} as follows: $cA = \text{MAC}(K_{cA}, TT)$. Similarly, B **MUST** send A a confirmation message using a MAC that is computed equivalently, except with the use of K_{cB} . Key confirmation verification requires computing cA (or cB , respectively) and checking for equality against that which was received.

4. Key Schedule and Key Confirmation

The protocol transcript TT , as defined in [Section 3.3](#), is unique and secret to A and B (though it contains some substrings that are not secret). Both parties use TT to derive shared symmetric secrets K_e and K_a as $K_e || K_a = \text{Hash}(TT)$, with $|K_e| = |K_a|$. The length of each key is equal to half of the digest output, e.g., 128 bits for SHA-256. Keys **MUST** be at least 128 bits in length.

Both endpoints use K_a to derive subsequent MAC keys for key confirmation messages. Specifically, K_{cA} and K_{cB} are the MAC keys used by A and B, respectively. A and B compute them as $K_{cA} || K_{cB} = \text{KDF}(K_a, \text{nil}, \text{"ConfirmationKeys"} || \text{AAD}, L)$, where AAD is the associated data given to each endpoint or AAD is nil if none was provided. The length of each of K_{cA} and K_{cB} is equal to half of the underlying hash output length, e.g., $|K_{cA}| = |K_{cB}| = 128$ bits for HKDF(SHA256), with $L=256$ bits.

The resulting key schedule for this protocol, given transcript TT and AAD, is as follows.

```
TT -> Hash(TT) = Ke || Ka
AAD -> KDF(Ka, nil, "ConfirmationKeys" || AAD) = KcA || KcB
```

A and B output K_e as the shared secret from the protocol. K_a and its derived keys are not used for anything except key confirmation.

5. Per-User M and N and M=N

To avoid concerns that an attacker needs to solve a single Elliptic Curve Diffie-Hellman (ECDH) instance to break the authentication of SPAKE2, it is possible to vary M and N using [\[RFC9380\]](#) as follows:

```
M = hash_to_curve(Hash("M SPAKE2" || len(A) || A || len(B) || B))
N = hash_to_curve(Hash("N SPAKE2" || len(A) || A || len(B) || B))
```

There is also a symmetric variant where $M=N$. For this variant, we set:

```
M = hash_to_curve(Hash("M AND N SPAKE2"))
```

This variant **MUST** be used when it is not possible to determine whether A or B should use M (or N), due to asymmetries in the protocol flows or the desire to use only a single shared secret with nil identities for authentication. The security of these variants is examined in [MNVAR]. The variant with per-user M and N may not be suitable for protocols that require the initial messages to be generated by each party at the same time and that do not know the exact identity of the parties before the flow begins.

6. Ciphersuites

This section documents SPAKE2 ciphersuite configurations. A ciphersuite indicates a group, cryptographic hash function, and pair of KDF and MAC functions, e.g., SPAKE2-P256-SHA256-HKDF-HMAC. This ciphersuite indicates a SPAKE2 protocol instance over P-256 that uses SHA-256, along with HMAC-based Key Derivation Function (HKDF) [RFC5869] and Hashed Message Authentication Code (HMAC) [RFC2104] for G, Hash, KDF, and MAC functions, respectively. For Ed25519, the compressed encoding is used [RFC8032]; all others use the uncompressed SEC1 encoding.

G	Hash	KDF	MAC
P-256	SHA256 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-256	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-384	SHA256 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-384	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-521	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
edwards25519 [RFC8032]	SHA256 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
edwards448 [RFC8032]	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-256	SHA256 [RFC6234]	HKDF [RFC5869]	CMAC-AES-128 [RFC4493]

G	Hash	KDF	MAC
P-256	SHA512 [RFC6234]	HKDF [RFC5869]	CMAC-AES-128 [RFC4493]

Table 1: SPAKE2 Ciphersuites

The following points represent permissible point generation seeds for the groups listed in [Table 1](#), using the algorithm presented in [Appendix A](#). These byte strings are compressed points, as in [\[SEC1\]](#), for curves from [\[SEC1\]](#).

For P-256:

```
M =
02886e2f97ace46e55ba9dd7242579f2993b64e16ef3dcab95afd497333d8fa12f
seed: 1.2.840.10045.3.1.7 point generation seed (M)

N =
03d8bbd6c639c62937b04d997f38c3770719c629d7014d49a24b4f98baa1292b49
seed: 1.2.840.10045.3.1.7 point generation seed (N)
```

For P-384:

```
M =
030ff0895ae5ebf6187080a82d82b42e2765e3b2f8749c7e05eba366434b363d3dc
36f15314739074d2eb8613fceec2853
seed: 1.3.132.0.34 point generation seed (M)

N =
02c72cf2e390853a1c1c4ad816a62fd15824f56078918f43f922ca21518f9c543bb
252c5490214cf9aa3f0baab4b665c10
seed: 1.3.132.0.34 point generation seed (N)
```

For P-521:

```
M =
02003f06f38131b2ba2600791e82488e8d20ab889af753a41806c5db18d37d85608
cfae06b82e4a72cd744c719193562a653ea1f119eef9356907edc9b56979962d7aa
seed: 1.3.132.0.35 point generation seed (M)

N =
0200c7924b9ec017f3094562894336a53c50167ba8c5963876880542bc669e494b25
32d76c5b53dfb349fdf69154b9e0048c58a42e8ed04cef052a3bc349d95575cd25
seed: 1.3.132.0.35 point generation seed (N)
```

For edwards25519:


```
M =
d048032c6ea0b6d697ddc2e86bda85a33adac920f1bf18e1b0c6d166a5cecdaf
seed: edwards25519 point generation seed (M)

N =
d3bfb518f44f3430f29d0c92af503865a1ed3281dc69b35dd868ba85f886c4ab
seed: edwards25519 point generation seed (N)
```

For edwards448:

```
M =
b6221038a775ecd007a4e4dde39fd76ae91d3cf0cc92be8f0c2fa6d6b66f9a12
942f5a92646109152292464f3e63d354701c7848d9fc3b8880
seed: edwards448 point generation seed (M)

N =
6034c65b66e4cd7a49b0edec3e3c9ccc4588afd8cf324e29f0a84a072531c4db
f97ff9af195ed714a689251f08f8e06e2d1f24a0ffc0146600
seed: edwards448 point generation seed (N)
```

7. Security Considerations

A security proof of SPAKE2 for prime order groups is found in [\[REF\]](#), reducing the security of SPAKE2 to the GDH assumption. Note that the choice of M and N is critical for the security proof. The generation methods specified in this document are designed to eliminate concerns related to knowing discrete logs of M and N.

Elements received from a peer **MUST** be checked for group membership. Failure to properly deserialize and validate group elements can lead to attacks. An endpoint **MUST** abort the protocol if any received public value is not a member of G.

The choices of random numbers **MUST** be uniform. Randomly generated values, e.g., x and y, **MUST NOT** be reused; such reuse violates the security assumptions of the protocol and results in significant insecurity. It is **RECOMMENDED** to generate these uniform numbers using rejection sampling.

Some implementations of elliptic curve multiplication may leak information about the length of the scalar. These **MUST NOT** be used. All operations on elliptic curve points must take time independent of the inputs. Hashing of the transcript may take time depending only on the length of the transcript but not the contents.

SPAKE2 does not support augmentation. As a result, the server has to store a password equivalent. This is considered a significant drawback in some use cases. Applications that need augmented PAKEs should use [\[CFRG-OPAQUE\]](#).

The HMAC keys in this document are shorter than recommended in [\[RFC8032\]](#). This is appropriate, as the difficulty of the discrete logarithm problem is comparable with the difficulty of brute forcing the keys.

8. IANA Considerations

This document has no IANA actions.

9. References

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- [RFC9380] Faz-Hernandez, A., Scott, S., Sullivan, N., Wahby, R. S., and C. A. Wood, "Hashing to Elliptic Curves", RFC 9380, DOI 10.17487/RFC9380, August 2023, <<https://www.rfc-editor.org/info/rfc9380>>.

9.2. Informative References

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Appendix A. Algorithm Used for Point Generation

This section describes the algorithm that was used to generate points M and N in [Table 1](#).

For each curve in [Table 1](#), we construct a string using the curve OID from [\[RFC5480\]](#) (as an ASCII string) or its name, combined with the needed constant, e.g., "1.3.132.0.35 point generation seed (M)" for P-521. This string is turned into a series of blocks by hashing with SHA-256 and hashing that output again to generate the next 32 bytes and so on. This pattern is repeated for each group and value, with the string modified appropriately.

A byte string of a length equal to that of an encoded group element is constructed by concatenating as many blocks as are required, starting from the first block and truncating to the desired length. The byte string is then formatted as required for the group. In the case of Weierstrass curves, we take the desired length as the length for representing a compressed point (Section 2.3.4 of [\[SEC1\]](#)) and use the low-order bit of the first byte as the sign bit. In order to obtain the correct format, the value of the first byte is set to 0x02 or 0x03 (clearing the first six

bits and setting the seventh bit), leaving the sign bit as it was in the byte string constructed by concatenating hash blocks. For the curves in [\[RFC8032\]](#), a different procedure is used. For `edwards448`, the 57-byte input has the least-significant 7 bits of the last byte set to zero, and for `edwards25519`, the 32-byte input is not modified. For both the curves in [\[RFC8032\]](#), the (modified) input is then interpreted as the representation of the group element. If this interpretation yields a valid group element with the correct order (p), the (modified) byte string is the output. Otherwise, the initial hash block is discarded and a new byte string is constructed from the remaining hash blocks. The procedure of constructing a byte string of the appropriate length, formatting it as required for the curve, and checking if it is a valid point of the correct order is repeated until a valid element is found.

The following Python snippet generates the above points, assuming an elliptic curve implementation follows the interface of `Edwards25519Point.stdbase()` and `Edwards448Point.stdbase()` in [Appendix A](#) of [\[RFC8032\]](#):

```
def iterated_hash(seed, n):
    h = seed
    for i in range(n):
        h = hashlib.sha256(h).digest()
    return h

def bighash(seed, start, sz):
    n = -(-sz // 32)
    hashes = [iterated_hash(seed, i)
               for i in range(start, start + n)]
    return b''.join(hashes)[:sz]

def canon_pointstr(ecname, s):
    if ecname == 'edwards25519':
        return s
    elif ecname == 'edwards448':
        return s[:-1] + bytes([s[-1] & 0x80])
    else:
        return bytes([(s[0] & 1) | 2]) + s[1:]

def gen_point(seed, ecname, ec):
    for i in range(1, 1000):
        hval = bighash(seed, i, len(ec.encode()))
        pointstr = canon_pointstr(ecname, hval)
        try:
            p = ec.decode(pointstr)
            if p != ec.zero_elem() and p * p.l() == ec.zero_elem():
                return pointstr, i
        except Exception:
            pass
```

Appendix B. SPAKE2 Test Vectors

This section contains test vectors for SPAKE2, using the P256-SHA256-HKDF-HMAC ciphersuite. (The choice of MHF is omitted, and the values for *w*, *x*, and *y* are provided directly.) All points are encoded using the uncompressed format, i.e., with a 0x04 octet prefix, specified in [SEC1]. A and B identity strings are provided in the protocol invocation.

Line breaks have been added due to line-length limitations.

```
spake2: A='server', B='client'
w = 0x2ee57912099d31560b3a44b1184b9b4866e904c49d12ac5042c97dca461b1a5f
x = 0x43dd0fd7215bdcba482879fca3220c6a968e66d70b1356cac18bb26c84a78d729
pA = 0x04a56fa807caaa53a4d28dbb9853b9815c61a411118a6fe516a8798434751470
f9010153ac33d0d5f2047ffdb1a3e42c9b4e6be662766e1eeb4116988ede5f912c
y = 0xdcbb60106f276b02606d8ef0a328c02e4b629f84f89786af5befb0bc75b6e66be
pB = 0x0406557e482bd03097ad0cbaa5df82115460d951e3451962f1eaf4367a420676
d09857ccbc522686c83d1852abfa8ed6e4a1155cf8f1543ceca528afb591a1e0b7
K = 0x0412af7e89717850671913e6b469ace67bd90a4df8ce45c2af19010175e37eed
69f75897996d539356e2fa6a406d528501f907e04d97515f8e83db277b715d3325
TT = 0x0600000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000
4a56fa807caaa53a4d28dbb9853b9815c61a411118a6fe516a8798434751
470f9010153ac33d0d5f2047ffdb1a3e42c9b4e6be662766e1eeb4116988ede5f912c4
1000000000000000000000000000000000000000000000000000000000000000
67a420676d09857ccbc522686c83d1852abfa8ed6e4a1155cf8f1543ceca528afb591a
1e0b741000000000000000000000000000000000000000000000000000000000
f19010175e37eed69f75897996d539356e2fa6a406d528501f907e04d97515f8e83db2
77b715d332520000000000000000000000000000000000000000000000000000
2ac5042c97dca461b1a5f
HASH(TT) = 0x0e0672dc86f8e45565d338b0540abe6915bdf72e2b35b5c9e5663168e9
60a91b
Ke = 0x0e0672dc86f8e45565d338b0540abe69
Ka = 0x15bdf72e2b35b5c9e5663168e960a91b
KcA = 0x00c12546835755c86d8c0db7851ae86f
KcB = 0xa9fa3406c3b781b93d804485430ca27a
A conf = 0x58ad4aa88e0b60d5061eb6b5dd93e80d9c4f00d127c65b3b35b1b5281f
ee38f0
B conf = 0xd3e2e547f1ae04f2dbdbf0fc4b79f8ecff2dff314b5d32fe9fcef2fb26
dc459b
```

```
spake2: A='', B='client'
w = 0x0548d8729f730589e579b0475a582c1608138ddf7054b73b5381c7e883e2efae
x = 0x403abbe3b1b4b9ba17e3032849759d723939a27a27b9d921c500edde18ed654b
pA = 0x04a897b769e681c62ac1c2357319a3d363f610839c4477720d24cbe32f5fd8
5f44fb92ba966578c1b712be6962498834078262caa5b441ecfa9d4a9485720e918a
y = 0x903023b6598908936ea7c929bd761af60c39577a9c3f9581064187c3049d87065
pB = 0x04e0f816fd1c35e22065d5556215c097e799390d16661c386e0ecc84593974
a61b881a8c82327687d0501862970c64565560cb5671f696048050ca66ca5f8cc7fc
K = 0x048f83ec9f6e4f87cc6f9dc740bdc2769725f923364f01c84148c049a39a735e
bda82eac03e00112fd6a5710682767cfff5361f7e819e53d8d3c3a2922e0d837aa6
TT = 0x000000000000000000000000000000000000000000000000000000000000004a
897b769e681c62ac1c2357319a3d363f610839c4477720d24cbe32f5fd85f44fb92ba9
66578c1b712be6962498834078262caa5b441ecfa9d4a9485720e918a410000000000
00004e0f816fd1c35e22065d5556215c097e799390d16661c386e0ecc84593974a61b8
81a8c82327687d0501862970c64565560cb5671f696048050ca66ca5f8cc7fc4100000
000000000048f83ec9f6e4f87cc6f9dc740bdc2769725f923364f01c84148c049a39a7
35ebda82eac03e00112fd6a5710682767cfff5361f7e819e53d8d3c3a2922e0d837aa62
000000000000000000000000548d8729f730589e579b0475a582c1608138ddf7054b73b5381c7e
883e2efae
Hash(TT) = 0x642f05c473c2cd79909f9a841e2f30a70bf89b18180af97353ba198789
c2b963
Ke = 0x642f05c473c2cd79909f9a841e2f30a7
Ka = 0x0bf89b18180af97353ba198789c2b963
KcA = 0xc6be376fc7cd1301fd0a13adf3e7bfdf
KcB = 0xb7243f4ae60440a49b3f8cab3c1fba07
A conf = 0x47d29e6666af1b7dd450d571233085d7a9866e4d49d2645e2dfd9754895
21232b
B conf = 0x3313c5cefc361d27fb16847a91c2a73b766ffa90a4839122a9b70a2f6b
d1d6df
```

```
spake2: A='server', B=''
w = 0x626e0cdc7b14c9db3e52a0b1b3a768c98e37852d5db30febe0497b14eae8c254
x = 0x07adb3db6bc623d3399726bfbfd3d15a58ea776ab8a308b00392621291f9633
pA = 0x04f88fb71c99bfffafa370966b7eb99cd4be0ff1a7d335caac4211c4afd855e2
e15a873b298503ad8ba1d9cbb9a392d2ba309b48bfd7879aefd0f2cea6009763b0
y = 0xb6a4fc8dbb629d4ba51d6f91ed1532cf87adec98f25dd153a75accafafedec16
pB = 0x040c269d6be017dccb15182ac6bfcd9e2a14de019dd587eaf4bdfd353f031101
e7cca177f8eb362a6e83e7d5e729c0732e1b528879c086f39ba0f31a9661bd34db
K = 0x0445ee233b8ecb51ebd6e7da3f307e88a1616bae2166121221fdc0dadb986afa
f3ec8a988dc9c626fa3b99f58a7ca7c9b844bb3e8dd9554aafc5b53813504c1cbe
TT = 0x060000000000000073657276657200000000000000041000000000000004f
88fb71c99bfffafa370966b7eb99cd4be0ff1a7d335caac4211c4afd855e2e15a873b2
98503ad8ba1d9cbb9a392d2ba309b48bfd7879aefd0f2cea6009763b0410000000000
000040c269d6be017dccb15182ac6bfcd9e2a14de019dd587eaf4bdfd353f031101e7c
ca177f8eb362a6e83e7d5e729c0732e1b528879c086f39ba0f31a9661bd34db4100000
000000000445ee233b8ecb51ebd6e7da3f307e88a1616bae2166121221fdc0dadb986
afaf3ec8a988dc9c626fa3b99f58a7ca7c9b844bb3e8dd9554aafc5b53813504c1cbe2
00000000000000626e0cdc7b14c9db3e52a0b1b3a768c98e37852d5db30febe0497b1
4eae8c254
Hash(TT) = 0x005184ff460da2ce59062c87733c299c3521297d736598fc0a1127600e
fa1afb
Ke = 0x005184ff460da2ce59062c87733c299c
Ka = 0x3521297d736598fc0a1127600efa1afb
KcA = 0xf3da53604f0aeecea5a33be7bddf6edf
KcB = 0x9e3f86848736f159bd92b6e107ec6799
A conf = 0xbc9f9bbe99f26d0b2260e6456e05a86196a3307ec6663a18bf6ac8257365
33b2
B conf = 0xc2370e1bf813b086dff0d834e74425a06e6390f48f5411900276dcccc5a2
97ec
```

```
spake2: A='', B=''  
w = 0x7bf46c454b4c1b25799527d896508afd5fc62ef4ec59db1efb49113063d70cca  
x = 0x8cef65df64bb2d0f83540c53632de911b5b24b3eab6cc74a97609fd659e95473  
pA = 0x04a65b367a3f613cf9f0654b1b28a1e3a8a40387956c8ba6063e8658563890f4  
6ca1ef6a676598889fc28de2950ab8120b79a5ef1ea4c9f44bc98f585634b46d66  
y = 0xd7a66f64074ac84652d8d623a92e20c9675c61cb5b4f6a0063e4648a2fdc02d53  
pB = 0x04589f13218822710d98d8b2123a079041052d9941b9cf88c6617ddb2fcc0494  
662eea8ba6b64692dc318250030c6af045cb738bc81ba35b043c3dcb46adf6f58d  
K = 0x041a3c03d51b452537ca2a1fea6110353c6d5ed483c4f0f86f4492ca3f378d40  
a994b4477f93c64d928edbbcd3e85a7c709b7ea73ee97986ce3d1438e135543772  
TT = 0x0000000000000000000000000000000410000000000000004a65b367a3f613  
cf9f0654b1b28a1e3a8a40387956c8ba6063e8658563890f46ca1ef6a676598889fc28  
de2950ab8120b79a5ef1ea4c9f44bc98f585634b46d664100000000000004589f132  
18822710d98d8b2123a079041052d9941b9cf88c6617ddb2fcc0494662eea8ba6b6469  
2dc318250030c6af045cb738bc81ba35b043c3dcb46adf6f58d410000000000000041  
a3c03d51b452537ca2a1fea6110353c6d5ed483c4f0f86f4492ca3f378d40a994b4477  
f93c64d928edbbcd3e85a7c709b7ea73ee97986ce3d1438e1355437722000000000000  
0007bf46c454b4c1b25799527d896508afd5fc62ef4ec59db1efb49113063d70cca  
Hash(TT) = 0xfc6374762ba5cf11f4b2caa08b2cd1b9907ae0e26e8d6234318d91583c  
d74c86  
Ke = 0xfc6374762ba5cf11f4b2caa08b2cd1b9  
Ka = 0x907ae0e26e8d6234318d91583cd74c86  
KcA = 0x5dbd2f477166b7fb6d61febbd77a5563  
KcB = 0x7689b4654407a5faeffdc8f18359d8a3  
A conf = 0xdfb4db8d48ae5a675963ea5e6c19d98d4ea028d8e898dad96ea19a80ade9  
5dca  
B conf = 0xd0f0609d1613138d354f7e95f19fb556bf52d751947241e8c7118df5ef0a  
e175
```

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