DMAT Documentation

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1 Mathematical Background

Consider a stable relic species denoted by the index i, to which j other particles may decay. We make the assumption that annihilating particles are in thermal equilibrium with the heat bath of the universe such that their velocities follow a Maxwell-Boltzmann distribution.

$$\langle \sigma_{eff} v \rangle = \Sigma_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}} \tag{1}$$

$$= \frac{\int_0^\infty dp_{11} p_{11}^2 W_{eff} K_1(\frac{\sqrt{s}}{T})}{m_1^4 T [\Sigma_i \frac{g_i}{g_j} K_2(\frac{m_i}{T})]^2}. \tag{2}$$

Here, W_{eff} is the effective invariant annihilation rate written as,

$$W_{eff} \equiv 2\Sigma_{ij} \frac{p_{ij}}{p_{11}} \frac{g_i g_j}{g_1^2} (s - m_i^2 - m_j^2) \sigma_{ij} v_{ij,lab}, \tag{3}$$

where p_{ij} is the relative momentum between particles i and j, given by,

$$p_{ij} = \frac{1}{s\sqrt{s}} [s - (m_i + m_j)^2]^{1/2} [s - (m_i - m_j)^2]^{1/2}, \tag{4}$$

and $v_{ij,lab}$ is the lab-frame relative velocity, related to the centre-of-mass frame relative velocity by,

$$v_{ij,cms} = 2(1 - 2m^2/s) v_{ij,lab}. \tag{5}$$

We will only consider the case of a stable relic particle to which no other particle can decay, which will simplify the resulting calculation of thermal abundances. With only 1 particle (and 1 corresponding antiparticle) in being considered, we have that $i = j = 1$.

First considering the denominator, we have,

$$m_1^4 T \left[\frac{g_1}{g_1} \frac{m_1^2}{m_1^2} K_2 \left(\frac{m_1}{T} \right) \right]^2 = m_1^4 T K_2^2 \left(\frac{m_1}{T} \right). \tag{6}$$

Let $m_1 = m$ and $x = m/T$, and this term simplifies to,

$$m_1^4 T K_2 \left(\frac{m_1}{T} \right) = \frac{m^5}{x} K_2^2(x). \tag{7}$$

Now focusing on the numerator, we first simplify W_{eff},

$$\begin{aligned}
W_{eff} &\equiv 2\Sigma_{ij}\frac{p_{ij}}{p_{11}}\frac{g_i g_j}{g_1^2}(s - m_i^2 - m_j^2)\sigma_{ij}v_{ij,lab} &(8)\\
&= 2\frac{p_{11}}{p_{11}}\frac{g_1 g_1}{g_1^2}(s - m^2 - m^2)\sigma_{11}v_{11,lab} &(9)\\
&= 2(s - 2m^2)\sigma v_{11,lab} &(10)\\
&= 2s(1 - 2m^2/s)\sigma v_{cms}(2(1 - 2m^2/s))^{-1} &(11)\\
&= s\sigma v_{cms}. &(12)
\end{aligned}$$

Now, simplifying, p_{ij},

$$\begin{aligned}
p_{ij} = p_{11} &= \frac{1}{2\sqrt{s}}[s - (m_1 + m_1)^2]^{1/2}[s - (m_1 - m_1)^2]^{1/2} &(13)\\
&= \frac{1}{2\sqrt{s}}[s - 4m^2]^{1/2}\sqrt{s} &(14)\\
&= \frac{1}{2}\sqrt{s - 4m^2} &(15)
\end{aligned}$$

To simplify the integral in the numerator, we make a change of variables,

$$\begin{aligned}
s &= 4p_{11}^2 - 4m^2 &(16)\\
\rightarrow ds/dp &= 8p_{11} &(17)\\
dp &= ds/8p_{11}. &(18)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int_0^\infty dp_{11}p_{11}^2 &= \int_{4m^2}^\infty \frac{ds p_{11}}{8} &(19)\\
&= \int_{4m^2}^\infty \frac{ds}{8}\frac{1}{2}\sqrt{s - 4m^2}. &(20)
\end{aligned}$$

Combining all these terms,

$$\begin{aligned}
\langle\sigma_{eff}v\rangle(x) &= \frac{\int_{4m^2}^\infty \frac{ds}{8}\frac{1}{2}\sqrt{s - 4m^2}s\sigma v_{cms}K_1(x\sqrt{s}/m)}{\frac{m^5}{x}K_2^2(x)} &(21)\\
&= \frac{\int_{4m^2}^\infty xs\sqrt{s - 4m^2}\sigma v_{cms}(s)K_1(x\sqrt{s}/m)}{16m^5K_2^2(x)}ds &(22)
\end{aligned}$$

2 Case 1: Majorana Fermion

Using the code DMAT.cpp, we compute the dimensionless comoving abundance of a generic majorana fermion. This is plotted against the parameter x where $x = m/T$. Each plot contains 3 curves for different masses. These were set to be 0.8E-19GeV (less than the Plank mass), 1.2E-19GeV (equal to the plank mass), and 1.6E-19GeV (larger than the Plank mass). In total, this was done three times for different values of $\langle\sigma_{eff}v\rangle(x)$ (0.001, 1, 1000), at differing orders of magnitude. In order to make the plots more readable, we plot the log of the comoving abundance, normalised to the present day value. This makes the freeze out patterns much easier to see.

From these plots it becomes clear that the abundance of a generic majorana fermion dark matter candidate reaches a 'freeze-out' where it's abundance stabilizes as the universe cools. Although this occurs at different rates for particles of different mass, there does not appear to be a large difference for particles at varying average velocities. From figure 4, we can see that the present day abundance varies greatly for particles of higher mass, indicating a less stable abundance value for heavier particles.

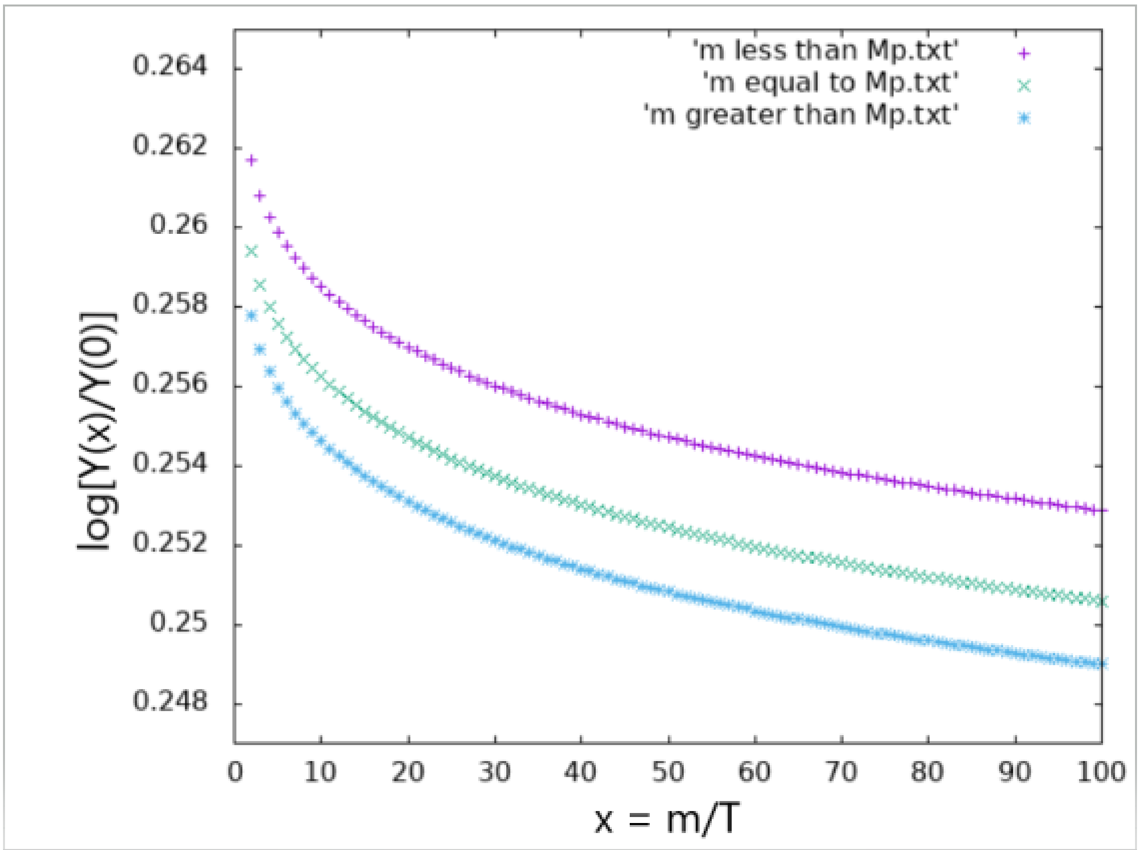

Figure 1: Normalised comoving abundance for 3 different mass majorana fermions with $\langle\sigma_{eff}v\rangle(x)$=1.

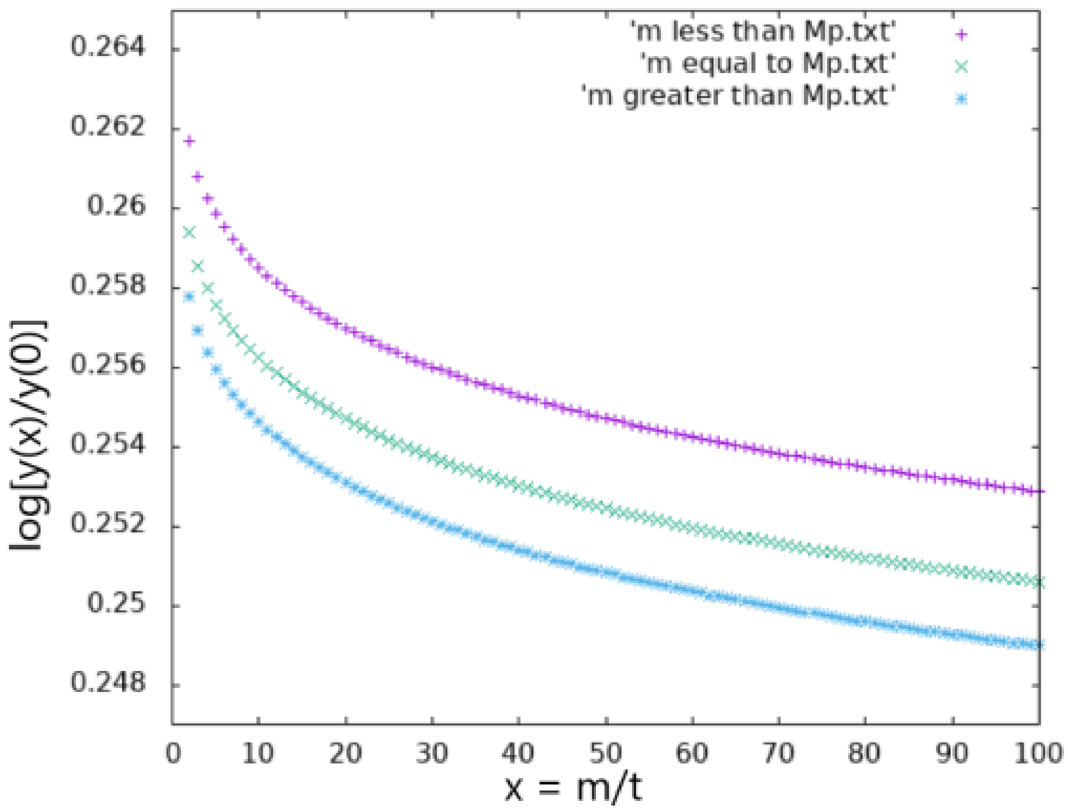

Figure 2: Normalised comoving abundance for 3 different mass majorana fermions with $\langle\sigma_{eff}v\rangle(x)$=1000

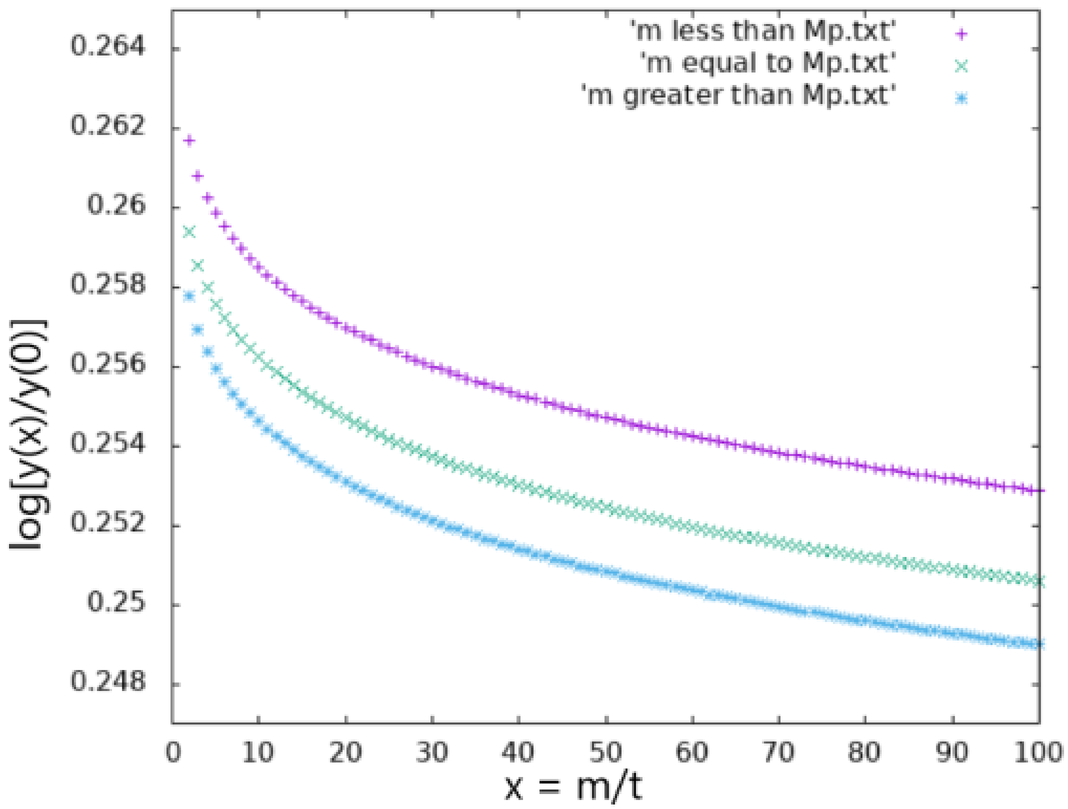

Figure 3: Normalised comoving abundance for 3 different mass majorana fermions with $\langle \sigma_{eff} v \rangle (x)$=0.001

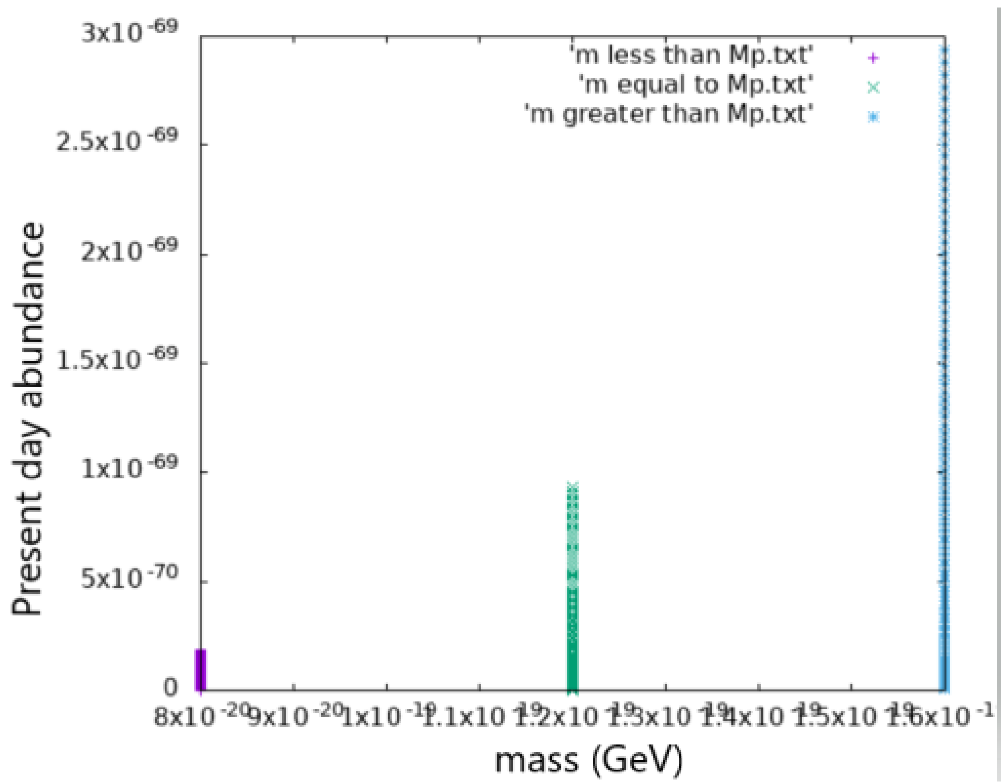

Figure 4: Present day abundance of Majorana fermions at different masses.