The Physics of LittleUniverse

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This is a brief description of the physics concepts and algorithms used in the code for generating a simple N-body simulation. The code is available online: ;see README.md in the github repository for instructions on compilation and use. Further document outlining some pre-determined, interesting physical situations in this N-body simulation are provided in nbody_presentation.pdf. This document is not a complete description of the physics but should be sufficient for introducing a non-expert to the relevant equations and, in conjunction with nbody_presentation.pdf, will provide a useful and simple introduction to the usage of N-body simulations in astrophysics.

1 N-Body Simulations

In general, the gravitational dynamics which dictate the behaviour of large structures in our universe are very difficult to determine analytically. As a result, a lot of progress in developing the standard model of modern cosmology has come from numerical methods, aided hugely by the development of computer codes. This combination of numerical methods and computer software has allowed for increasingly complex simulations of large systems of massive universal bodies under gravity [1]. The principle behind such simulations is simple: assuming particles such as dark matter consist of elementary particles with a high number density, it would only be possible to simulate a microscopic section of the universe. Instead we represent dark matter as macroscopic particles of mass M, and assume that they behave as dark matter particles within a volume given by[1] $V = M/\rho$.

There are a few things to note about such simulations, which apply to LittleUniverse. Obviously, given the (likely) infinite size of the universe, computer simulations are only capable of simulating behaviour in a small section. For this simulation, a 3D cube is chosen (projected into 2 dimensions in the simulation output). Given that the particles within this cube feel not only the gravitational forces from each other, but also any other bodies outside the cube, it is reasonable to impose periodic boundary conditions. In theory, what this does is assume that the cube of universe which we have chosen to simulate is surrounded on all sides by identical cubes obeying the same gravitational physics. In practice, this means that a particle which exits the simulation at the top will continue its motion by reentering at the bottom.

2 Gravitational Dynamics

Each particle in the simulation obeys Newtonian gravity, the equation for which is written in the form,

$$\mathbf{F}_i = \sum_{j \neq i} \frac{M^2(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3}. \tag{1}$$

[1]Schneider P., 2015, Extragalactic Astronomy and Cosmology: An Introduction, doi:10.1007/978-3-642-54083-7.

This describes the force on the i-th particle at position $\mathbf{r}_i$ due to all other particles at positions $\mathbf{r}_j$, where j is the total number of particles minus one.

3 Correlation Function

The correlation function is one way of extracting a more physical meaning from the distribution of the particles. Since galaxies in our universe tend to cluster and form large scale structures, the probability of finding two galaxies close together is more likely than finding two galaxies at arbitrary locations. That is, the probability of observing a particle somewhere in the box is not independent of the locations of the other particles. The correlation function is useful when written in the form of an estimator [2] (a statistical equation that can be directly applied to data sets),

$$\zeta = \left(\frac{N_{rand}}{N_{data}}\right)^2 \frac{DD(r)}{RR(r)} - 1. \tag{2}$$

Here, the functions $DD(r)$ and $RR(r)$ denote the number of pairs in the dataset within the interval $r \pm dr$ and the number of pairs of a randomly generated dataset within that same interval. The correlation function in LittleUniverse is plotted as a histogram, showing the overdensity of numbers of particles pairs found at a given separation when compared to randomly generated pairs at that same separation. It can be interpreted as the probability of finding galaxies at certain separation.

[2] Huterer D., 2020, Lecture Notes: Structure Formation in the Universe, obtained from https://sites.google.com/a/umich.edu/cosmology-summer-school-2020/schedule-of-events