Assignment 3 (ML for TS) - MVA 2023/2024

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1 Introduction

Objective. The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

Warning and advice.

- Use code from the tutorials as well as from other sources. Do not code yourself well-known procedures (e.g. cross validation or k-means), use an existing implementation.
- The associated notebook contains some hints and several helper functions.
- Be concise. Answers are not expected to be longer than a few sentences (omitting calculations).

Instructions.

- Fill in your names and emails at the top of the document.
- Hand in your report (one per pair of students) by Sunday 31st December 11:59 PM.
- Rename your report and notebook as follows:
  `FirstnameLastname1_FirstnameLastname1.pdf` and
  `FirstnameLastname2_FirstnameLastname2.ipynb`.
  For instance, `LaurentOudre_CharlesTruong.pdf`.
- Upload your report (PDF file) and notebook (IPYNB file) using this link:
  docs.google.com/forms/d/e/1FAIpQLScqLsYuKeQbsDEOie5OqpOH7YwCnWmudzApMC005HvxOaOv

2 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \ldots, T-1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of (f_1, f_2) represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

Answer 1

On accède aux valeurs des 8 fréquences qui composent les symboles en regardant quelles sont les 8 fréquences pour lesquelles l'intensité moyenne sur tout le dataset d'entraînement est la plus élevée.
Maintenant qu'on a accès à ces fréquences, on trace la puissance spectrale des signaux, centrée sur les 8 fréquences (c'est à dire sur les intervalles $[f_i - 10Hz, f_i + 10Hz]$ avec f_i les 8 fréquences.)
Ainsi on se débarasse du bruit et on peut observer quand les symboles sont réellement présents.
Au lieu d'utiliser ensuite un algorithme de changepoint détection, on fixe simplement un seuil d'intensité au delà duquel on considère qu'il y a un symbole qui est présent. Ce seuil est l'un des hyperparamètres qu'on a optimisés :

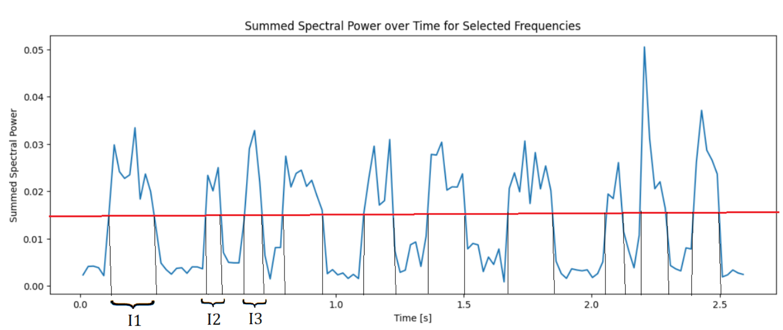


Figure 1: Schéma de la procédure d'extraction des intervalles correspondant à des symboles (un seuil de 0.014dB fonctionnait pour tous les sig,aux du dataset d'entraînement)

On extrait ainsi de tout signal les intervalles correspondant aux symboles puis on extrait de ces intervalles les deux fréquences qui ont l'intensité moyenne la plus élevée. Ainsi on arrive à une accuracy de plus de 98% sur le dataset d'entraînement

Question 2

What are the two symbolic sequences encoded in the test set?

Answer 2

- Sequence 1: 7-2-1-C-9-9
- Sequence 2: 1-#-2-#

3 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E. A specific node is denoted by v and a specific edge, by e. The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of f and $\hat{g}_m$ are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n / (M + 1 - R)$,

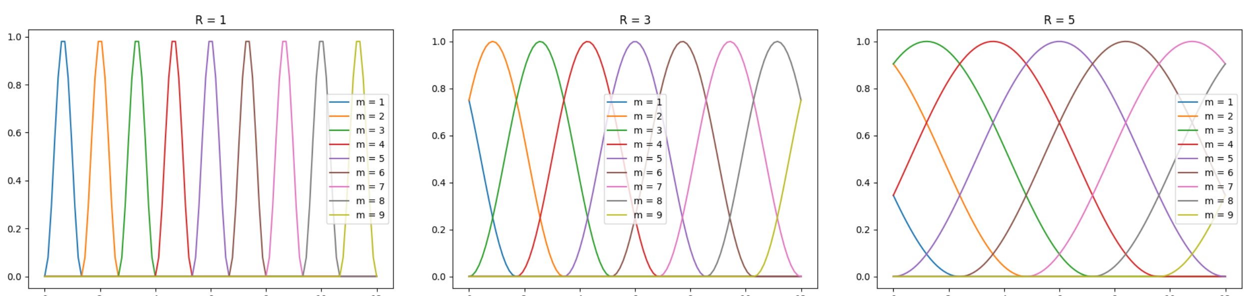
$$\hat{g}^U(\lambda) := \frac{1}{2}\left[1 + \cos\left(2\pi\left(\frac{\lambda}{aR} + \frac{1}{2}\right)\right)\right] \mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 2. What is the influence of R?

Answer 3



(a) $R = 1$ (b) $R = 3$ (c) $R = 5$

Figure 2: The SAGW kernels functions

The R parameter controls the overlap of offset cores. Filter overlap increases as R increases.

From the graphs above, we can see that for $R = 1$, there is no overlap between the shifted cores. However, for higher values, we can see that the kernels are increasingly enveloped.

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

Answer 4

The stations with missing values are:
BATZ, BEG_MEIL, CAMARET, PLOUGONVELIN, RIEC SUR BELON, ST NAZAIRE-MONTOIR, PLOUAY-SA, VANNES-MEUCON, LANNAERO, PLOUDALMEZEAU, LANDIVISIAU, SIZUN, QUIMPER, OUESSANT-STIFF, LANVEOC, ARZAL, BREST-GUIPAVAS, BRIGNOGAN.

The threshold is equal to 0.83.

The signal is the least smooth at 2014-01-21 06:00:00.

The signal is the smoothest at 2014-01-24 19:00:00.

Question 5

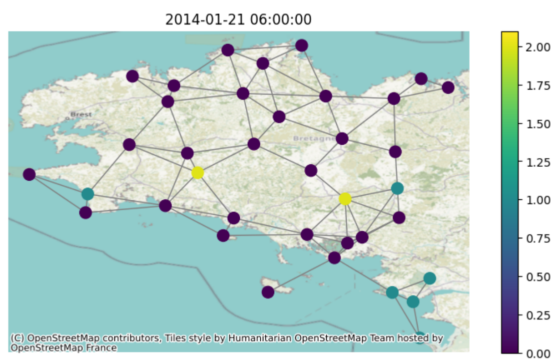
(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node v, the vector $[W_f(1, v), W_f(2, v), \ldots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low/medium/high frequency:

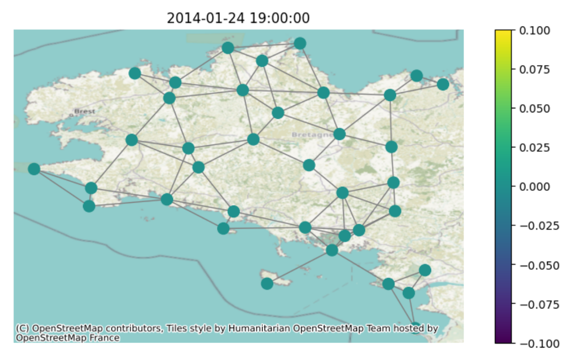
- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,
- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,
- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

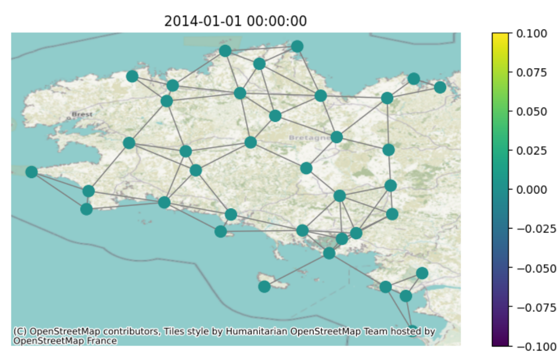
Answer 5



(a) Least smooth signal



(b) Smoothest signal



(c) First available timestamp

Figure 3: Classification of nodes into low/medium/high frequency

The figures above show that the smoothest signal and the signal with the first time stamp all have nodes with low frequencies.

The smoothest signal has all nodes with low frequencies. On the other hand, the least smooth signal has nodes with low, medium and high frequencies, which is normal. On the other hand, the least smooth signal has nodes with low, medium and high frequencies, which is normal since it is the signal with the highest smoothness value.

Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph (see notebook for more details).

Answer 6

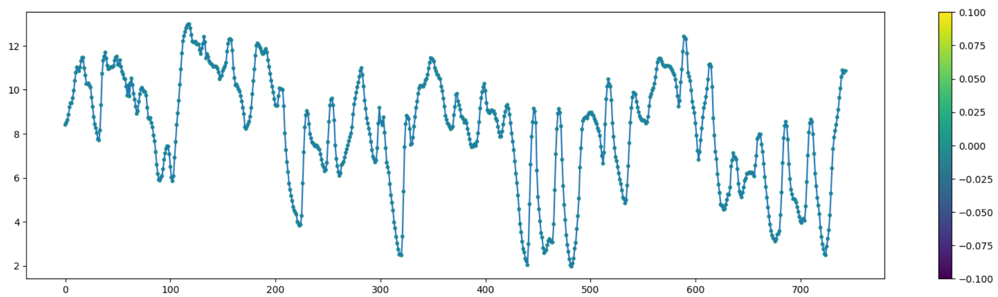


Figure 4: Average temperature. Markers' colours depend on the majority class.

We can see that the low-frequency category is the most dominant for all timestamps in the previous graph.

Question 7

The previous graph G only uses spatial information. To take into account the temporal dynamic, we construct a larger graph H as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to G) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' (which is simply a line graph, without loop).

- Express the Laplacian of H using the Laplacian of G and G' (use Kronecker products).
- Express the eigenvalues and eigenvectors of the Laplacian of H using the eigenvalues and eigenvectors of the Laplacian of G and G'.
- Compute the wavelet transform of the temperature signal.
- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

Answer 7

Let $\otimes$ be the Kronecker product and $H = G \times G'$.
And let n and m be the respective number of vertices of G and G'.
The Laplacian of H is given by:

$$\nabla^2 H = \nabla^2 G \otimes I_n + I_m \otimes \nabla^2 G'$$

Let u and v be respective eigenvectors of G and G', associated to respective eigenvalues λ and μ.
We have :

$$\begin{aligned}
\nabla^2 H(u \otimes v) &= (\nabla^2 G \otimes I_n + I_m \otimes \nabla^2 G')(u \otimes v) \\
&= (\nabla^2 G \otimes I_n)(u \otimes v) + (I_m \otimes \nabla^2 G')(u \otimes v) \\
&= (\nabla^2 G u) \otimes (I_n v) + (I_m u) \otimes (\nabla^2 G' v) \\
&= (\lambda u) \otimes v + u \otimes (\mu v) \\
&= (\lambda + \mu)(u \otimes v)
\end{aligned}$$

Thus $u \otimes v$ is an eigenvector of $\nabla^2 H$ associated to the eigenvalue of $\lambda + \mu$. Then all of the eigenvectors of $\nabla^2 H$ are the Kronecker products of the respective eigenvectors of G and G′.

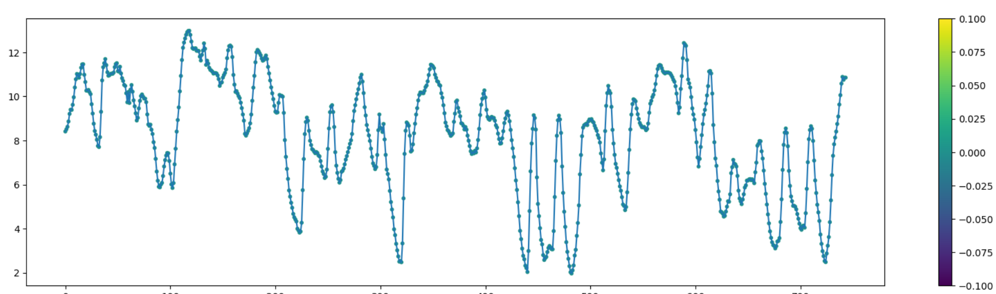


Figure 5: Average temperature. Markers' colours depend on the majority class.