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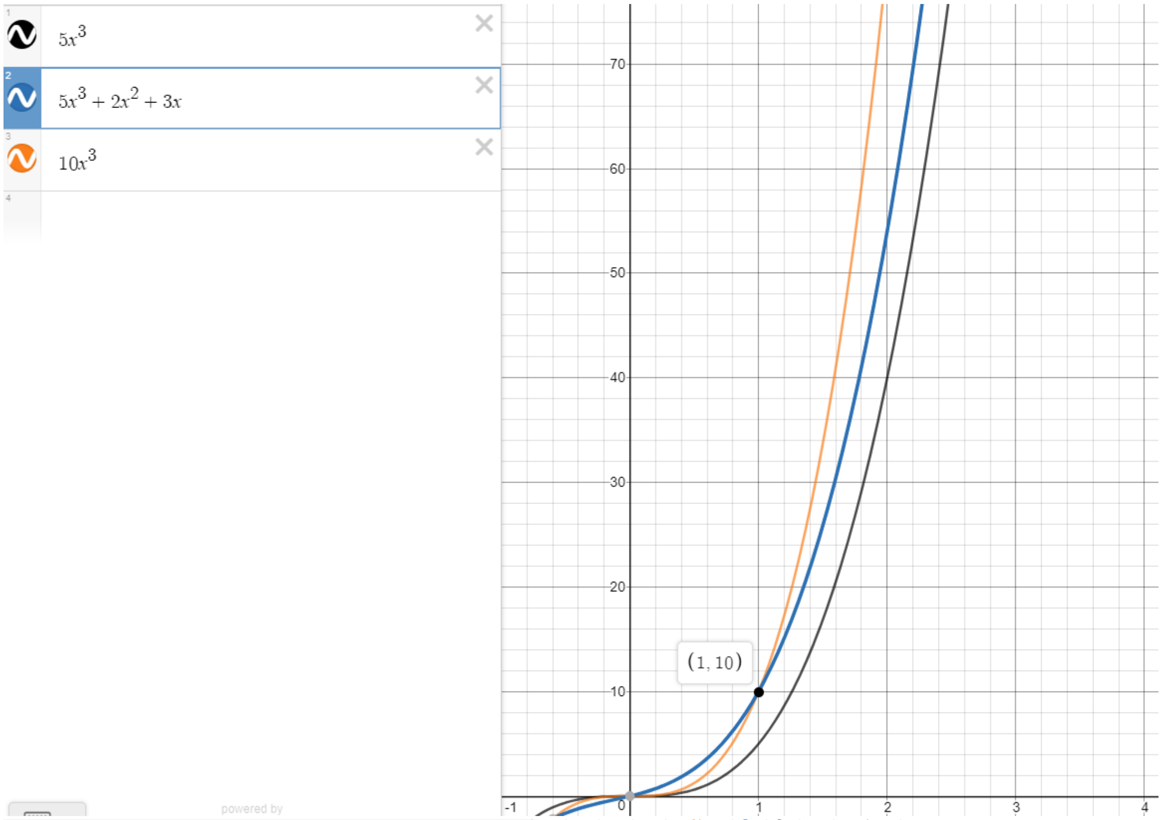
Bk1139

HW 6

<u>Question 5:</u>

a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$
   1. To prove this, we will show that $5n^3 + 2n^2 + 3n = \Theta(n^3)$, if there exist positive real constants c_1, c_2, and a positive integer constant n_0 such that $c_2*n^3 \leq 5n^3 + 2n^2 + 3n \leq c_1*n^3$ for all $n \geq n_0$
   2. Let's address c_1
      i. $5n^3 + 2n^2 + 3n \leq c_1*n^3$
      ii. $5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$
      iii. $5n^3 + 2n^2 + 3n \leq 10n^3$
      iv. This inequality will hold true for all $n \geq n_0$ where $n_0 = 1$
   3. Let's address c_2
      i. $c_2*n^3 \leq 5n^3 + 2n^2 + 3n$
      ii. $5n^3 \leq 5n^3 + 2n^2 + 3n$
      iii. This inequality will hold true for all $n \geq n_0$ where $n_0 = 1$
   4. Therefore, there exist positive real constants c_1, c_2, and a positive integer constant n_0 such that $c_2*n^3 \leq 5n^3 + 2n^2 + 3n \leq c_1*n^3$ for all $n \geq n_0$ where
      i. $c_1 = 10$
      ii. $c_2 = 5$
      iii. $n_0 = 1$
   5. Graphical support in screenshots below

6.


$5x^3$
$5x^3 + 2x^2 + 3x$
$10x^3$
(1, 10)
70
60
50
40
30
20
10
-1
0
1
2
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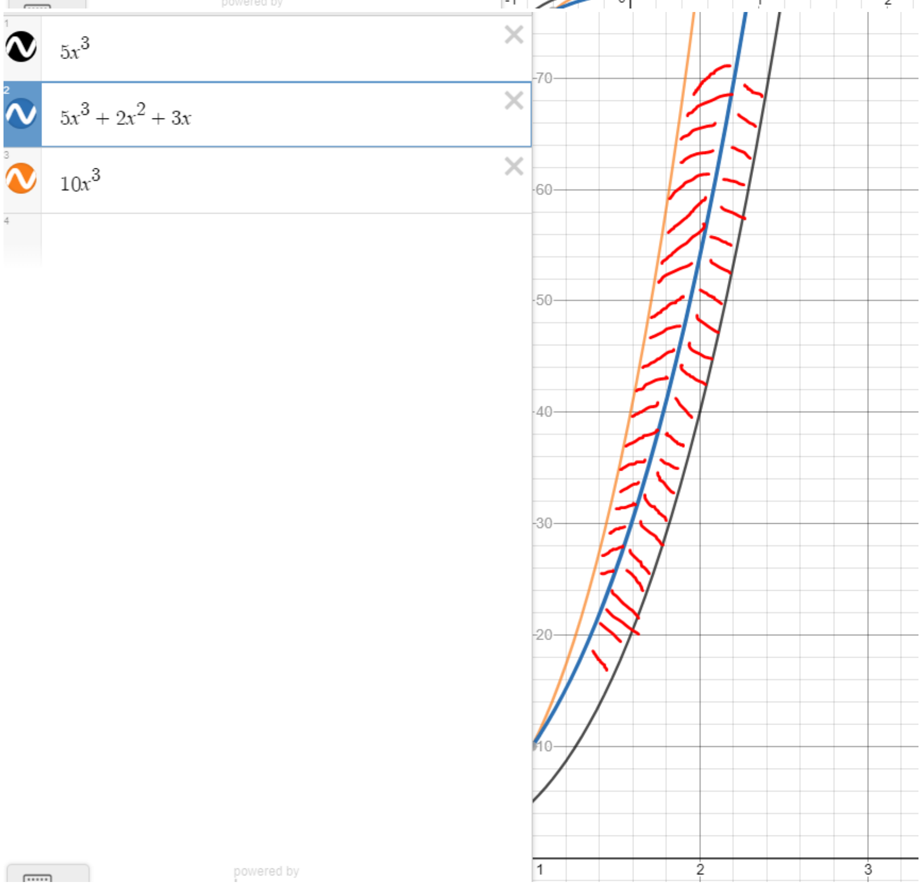
7.


$5x^3$
$5x^3 + 2x^2 + 3x$
$10x^3$
70
60
50
40
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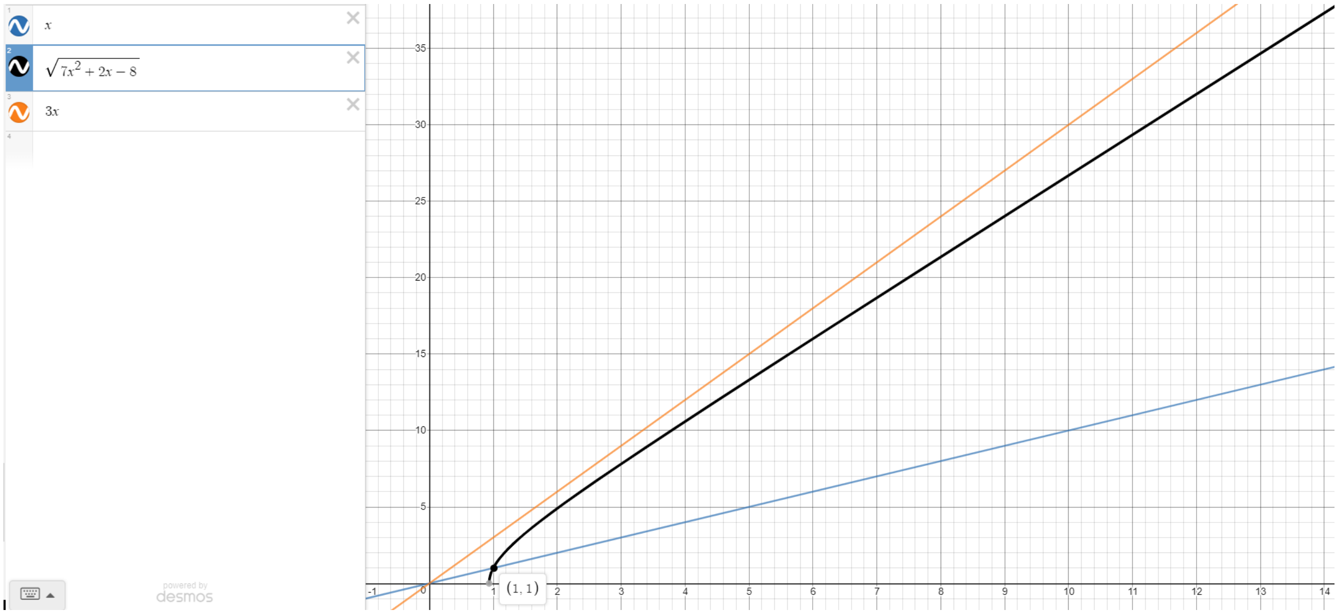
20
10
1
2
3
powered by

b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

1. To prove this, we will show that $\sqrt{7n^2 + 2n - 8} = \Theta(n)$, if there exist positive real constants c_1, c_2, and a positive integer constant n_0 such that $c_2 * n \leq \sqrt{7n^2 + 2n - 8} \leq c_1 * n$ for all $n \geq n_0$
2. In order to simplify the proof, let us square each expression in the inequality
   i. We are not concerned with accounting for the squaring of negative values as we are looking to see if there exist **positive** real constants c_1, c_2, and a **positive** integer constant n_0 which would not be negative
3. The revised inequality can be stated as follows:
   i. $\mathbf{c_2^2 * n^2 \leq 7n^2 + 2n - 8 \leq c_1^2 * n^2}$ **for all** $\mathbf{n \geq n_0}$
4. Let's address c_1
   i. $7n^2 + 2n - 8 \leq c_1^2 * n^2$
   ii. $7n^2 + 2n - 8 \leq 7n^2 + 2n^2$
   iii. $7n^2 + 2n - 8 \leq 9n^2$
   iv. $c_1^2 = 9$
   v. This inequality will hold true for all $n \geq n_0$ where $n_0 = 1$
5. Let's address c_2
   i. $c_2^2 * n^2 \leq 7n^2 + 2n - 8$
   ii. $n^2 \leq 7n^2 + 2n - 8$
   iii. $c_2^2 = 1$
   iv. This inequality will hold true for all $n \geq n_0$ where $n_0 = 1$
6. There do exist positive real constants c_1, c_2, and a positive integer constant n_0 such that $c_2^2 * n^2 \leq 7n^2 + 2n - 8 \leq c_1^2 * n^2$ for all $n \geq n_0$
   i. If we now take the square root of each expression in the inequality
      1. $\sqrt{n^2} \leq \sqrt{7n^2 + 2n - 8} \leq \sqrt{9n^2}$ for all $n \geq n_0$
      2. $c_1^2 = 9$, so $\sqrt{9n^2} = 3n$ and $\mathbf{c_1 = 3}$
         a. Take the positive root
      3. $c_2^2 = 1$, so $\sqrt{n^2} = n$ and $\mathbf{c_2 = 1}$
         a. Take the positive root

   ii. $\mathbf{n_0 = 1}$
7. Therefore, $n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$ for all $n \geq 1$ and $\sqrt{7n^2 + 2n - 8} = \Theta(n)$
8. Graphical support in screenshots below

9.



x
$\sqrt{7x^2+2x-8}$
$3x$
$(1, 1)$
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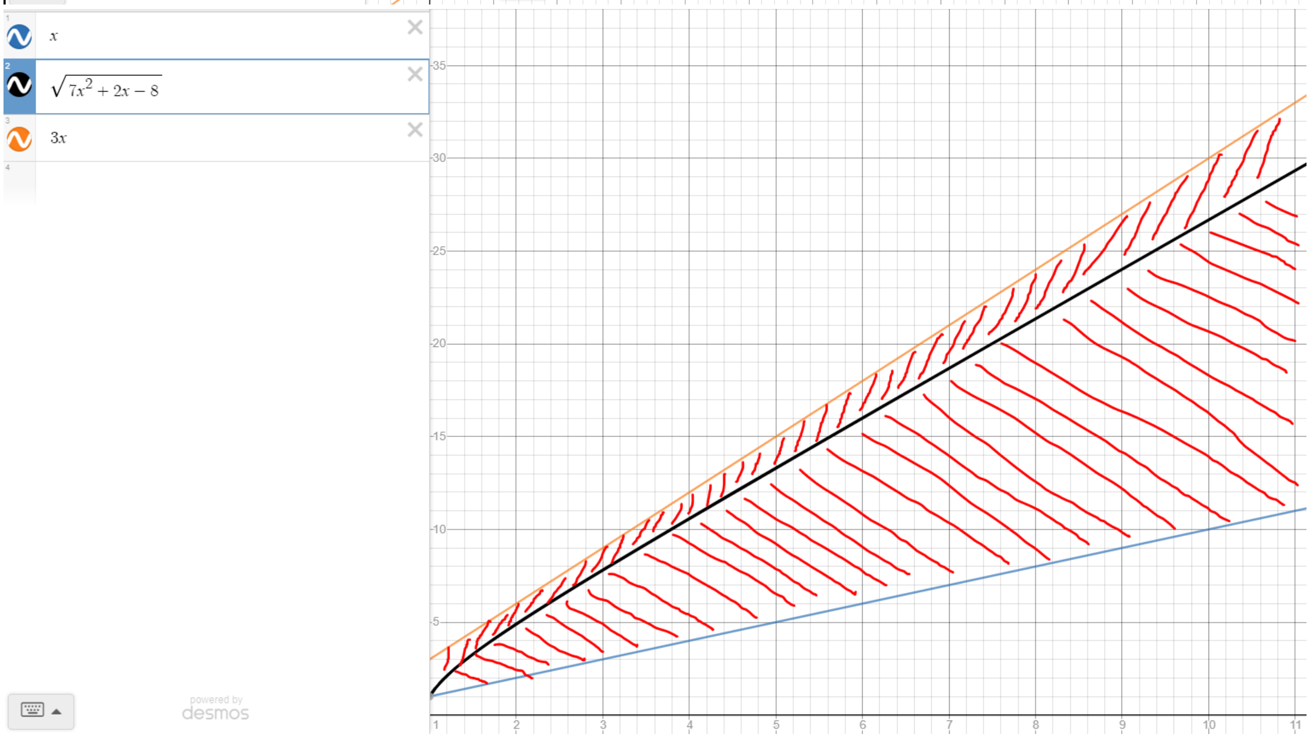
10.



x
$\sqrt{7x^2+2x-8}$
$3x$
powered by desmos