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Bk1139

HW 1

**Question 5:**

a) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.12.2, sections b, e

i. Exercise 1.12.2, section b

| | | |
|---|---|---|
| **1.** | **p → (q ∧ r)** | **Hypothesis** |
| **2.** | **¬p ∨ (q ∧ r)** | **Conditional identity, 1** |
| **3.** | **(¬p ∨ q) ∧ (¬p ∨ r)** | **Distributive law, 2** |
| **4.** | **(p → q) ∧ (p → r)** | **Conditional identity, 3** |
| **5.** | **(p → q)** | **Simplification, 4** |
| **6.** | **¬q** | **Hypothesis** |
| **7.** | **¬p** | **Modus tollens, 5, 6** |

ii. Exercise 1.12.2, section e

| | | |
|---|---|---|
| **1.** | **p ∨ q** | **Hypothesis** |
| **2.** | **¬p ∨ r** | **Hypothesis** |
| **3.** | **q ∨ r** | **Resolution, 1, 2** |
| **4.** | **¬q** | **Hypothesis** |
| **5.** | **(q ∨ r) ∧ ¬q** | **Conjunction, 3, 4** |
| **6.** | **¬q ∧ (q ∨ r)** | **Commutative law, 5** |
| **7.** | **(¬q ∧ q) ∨ (¬q ∧ r)** | **Distributive law, 6** |
| **8.** | **(q ∧ ¬q) ∨ (¬q ∧ r)** | **Commutative law, 7** |
| **9.** | **F ∨ (¬q ∧ r)** | **Complement law, 8** |
| **10.** | **(¬q ∧ r) ∨ F** | **Commutative law, 9** |
| **11.** | **(¬q ∧ r)** | **Identity law, 10** |
| **12.** | **(r ∧ ¬q)** | **Commutative law, 11** |
| **13.** | **r** | **Simplification, 12** |

2. Exercise 1.12.3, section c

i. Exercise 1.12.3, section c

| | | |
|---|---|---|
| **1.** | **p ∨ q** | **Hypothesis** |
| **2.** | **¬p** | **Hypothesis** |
| **3.** | **(p ∨ q) ∧ ¬p** | **Conjunction, 1, 2** |
| **4.** | **¬p ∧ (p ∨ q)** | **Commutative law, 3** |
| **5.** | **(¬p ∧ p) ∨ (¬p ∧ q)** | **Distributive law, 4** |
| **6.** | **(p ∧ ¬p) ∨ (¬p ∧ q)** | **Commutative law, 5** |
| **7.** | **F ∨ (¬p ∧ q)** | **Complement law, 6** |
| **8.** | **(¬p ∧ q) ∨ F** | **Commutative law, 7** |
| **9.** | **¬p ∧ q** | **Identity law, 8** |
| **10.** | **q ∧ ¬p** | **Commutative law, 9** |
| **11.** | **q** | **Simplification, 10** |

3. Exercise 1.12.5, sections c, d
   i. Exercise 1.12.5, section c
      1. **j: I will get a job**
         **c: I will buy a new car**
         **h: I will buy a new house**
      2. The form of the argument is:
         a. **$(c \wedge h) \rightarrow j$**
            **$\underline{\neg j \qquad\qquad}$**
            **$\therefore \neg c$**
      3. **The argument is not valid.** When c = T, and h = j = F, the hypotheses are both true and the conclusion ¬c is false.
   ii. Exercise 1.12.5, section d
      1. **j: I will get a job**
         **c: I will buy a new car**
         **h: I will buy a new house**
      2. The form of the argument is:
         a. **$(c \wedge h) \rightarrow j$**
            **$\neg j$**
            **$\underline{h \qquad\qquad}$**
            **$\therefore \neg c$**
      3. **The argument is valid**.

| | | |
|---|---|---|
| **1.** | **$(c \wedge h) \rightarrow j$** | **Hypothesis** |
| **2.** | **$\neg j$** | **Hypothesis** |
| **3.** | **$\neg(c \wedge h)$** | **Modus tollens, 1, 2** |
| **4.** | **$\neg c \vee \neg h$** | **De Morgan's law, 3** |
| **5.** | **$\neg h \vee \neg c$** | **Commutative law, 4** |
| **6.** | **h** | **Hypothesis** |
| **7.** | **$\neg c$** | **Disjunctive Syllogism, 5, 6** |

b) Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.13.3, section b
   i. Exercise 1.13.3, section b

| | **P** | **Q** |
|---|---|---|
| **a** | **F** | **T** |
| **b** | **F** | **F** |

   ii. $(P(a) \vee Q(a)) = T$
   iii. $\neg Q(b) = T$
   **iv. However, the conclusion for both a and b is false**
      1. **P(a) = F**
      2. **P(b) = F**
2. Exercise 1.13.5, sections d, e
   i. Exercise 1.13.5, section d
      1. Defining the predicates
         a. **M(x): x missed class**
         b. **D(x): x got a detention**

2. Expressing the hypotheses and conclusion using the predicates
    a. **$\forall x\ M(x) \rightarrow D(x)$**
    **Penelope is a student in the class**
    **$\underline{\neg M(\text{Penelope})}$**
    **$\therefore \neg D(\text{Penelope})$**
3. **The argument is not valid**. If M(Penelope) = F and D(Penelope) = T, then the hypotheses are all true and the conclusion is false. In other words, Penelope didn't miss class, and got a detention.

ii. Exercise 1.13.5, section e
1. Defining the predicates
    a. **M(x): x missed class**
    b. **D(x) x got a detention**
    c. **A(x): x received an A**
2. Expressing the hypotheses and conclusion using the predicates
    a. **$\forall x\ (M(x) \vee D(x)) \rightarrow \neg A(x)$**
    **Penelope is a student in the class**
    **$\underline{A(\text{Penelope})}$**
    **$\therefore \neg D(\text{Penelope})$**
3. **The argument is valid.**

| | | |
|---|---|---|
| **1.** | **$\forall x\ (M(x) \vee D(x)) \rightarrow \neg A(x)$** | **Hypothesis** |
| **2.** | **Penelope is a student in the class** | **Hypothesis** |
| **3.** | **$(M(\text{Pen}) \vee D(\text{Pen})) \rightarrow \neg A(\text{Pen})$** | **Universal instantiation, 1, 2** |
| **4.** | **$\neg(M(\text{Pen}) \vee D(\text{Pen})) \vee \neg A(\text{Pen})$** | **Conditional identity, 3** |
| **5.** | **$(\neg M(\text{Pen}) \wedge \neg D(\text{Pen})) \vee \neg A(\text{Pen})$** | **De Morgan's law, 4** |
| **6.** | **$\neg A(\text{Pen}) \vee (\neg M(\text{Pen}) \wedge \neg D(\text{Pen}))$** | **Commutative law, 5** |
| **7.** | **$\neg A(\text{Pen}) \vee (\neg D(\text{Pen}) \wedge \neg M(\text{Pen}))$** | **Commutative law, 6** |
| **8.** | **$(\neg A(\text{Pen}) \vee \neg D(\text{Pen})) \wedge (\neg A(\text{Pen}) \vee \neg M(\text{Pen}))$** | **Distributive law, 7** |
| **9.** | **$\neg A(\text{Pen}) \vee \neg D(\text{Pen})$** | **Simplification, 8** |
| **10.** | **A(Penelope)** | **Hypothesis** |
| **11.** | **$\neg D(\text{Penelope})$** | **Disjunctive syllogism, 9, 10** |

*Penelope is abbreviated to Pen where necessary**

## Question 6:

a) Solve exercise 2.2.1, sections d, c from the Discrete Math zyBook:
1. exercise 2.2.1, section d
    i. **Proof.**
    ii. Direct proof. Assume that x and y are odd integers. We will show that the product of $x \cdot y$ is an odd integer.
    iii. Since x and y are odd, $x = 2k + 1$, and $y = 2m + 1$ for some integers k and m. Plug the expression for x and y into $x \cdot y$:
    iv. $x \cdot y = (2k + 1)(2m + 1) = 4km + 2k + 2m + 1 = 2(2km + k + m) + 1$
    v. Since k and m are integers, then $(2km + k + m)$ is also an integer
    vi. Since $x \cdot y = 2c + 1$, where $c = (2km + k + m)$ is an integer, then $x \cdot y$ is odd. ■

2. exercise 2.2.1, section c
   i. **Proof.**
   ii. Direct proof. Assume that x is a real number and $x \leq 3$. We will show that $12 - 7x + x^2 \geq 0$.
   iii. Subtract 3 from both sides of the inequality for $x \leq 3$ to get $x - 3 \leq 0$
   iv. Since $x - 3 \leq 0$, then $x - 4 \leq -1$
   v. $12 - 7x + x^2$ can be simplified to be expressed as $(x - 3)(x - 4)$
   vi. This results in two potential outcomes for $(x - 3)(x - 4)$: either 0 * some negative number = 0, or some negative number multiplied by another negative number, resulting in a positive number which is greater than 0
   vii. Therefore, $12 - 7x + x^2$ equals either 0 or a positive number
   viii. $12 - 7x + x^2 \geq 0$. ■

**Question 7:**

a) Solve exercise 2.3.1, sections d, f, g, l from the Discrete Math zyBook:
   1. exercise 2.3.1, section d
      i. **Proof.**
      ii. Proof by contrapositive. We assume that n is an even integer and show that $n^2 - 2n + 7$ is an odd integer.
      iii. If n is an even integer, then $n = 2k$ for some integer k. Plugging in the expression 2k for n in $n^2 - 2n + 7$ gives
      iv. $(2k)^2 - 2(2k) + 7$
      v. $4k^2 - 4k + 7$
      vi. $4k^2 - 4k + 6 + 1$
      vii. $2(2k^2 - 2k + 3) + 1$
      viii. Since k is an integer, then $(2k^2 - 2k + 3)$ is also an integer
      ix. Since $n^2 - 2n + 7 = 2c + 1$, where $c = (2k^2 - 2k + 3)$ is an integer, then $n^2 - 2n + 7$ is odd. ■
   2. exercise 2.3.1, section f
      i. **Proof.**
      ii. Proof by contrapositive. We assume for every non-zero real number x that 1/x is not irrational and prove that x must be rational.
      iii. Since x is a non-zero real number, 1/x is also a real number. Every real number is either rational or irrational. Therefore since 1/x is not irrational and x is a non-zero real number, 1/x must be rational. By the definition of a rational number, $1/x = 1/(a/b)$, where a and b are integers and $b \neq 0$.
      iv. Therefore, x is equal to the ratio of two integers, a and b, such that the denominator $b \neq 0$. Therefore, x is rational. ■

3. exercise 2.3.1, section g
    i. **Proof.**
    ii. Proof by contrapositive. We assume for every pair of real numbers x and y that $x > y$ and prove that $x^3+xy^2 > x^2y+y^3$.
    iii. Multiplying both sides of the inequality $x > y$ by $(x^2+y^2)$ results in $x^3+xy^2 > x^2y+y^3$
    iv. For the inequality to hold true, we must prove that $(x^2+y^2) \neq 0$
    v. The square of any number results in either 0 or a positive number
    vi. Since $x > y$ we know that $(x^2+y^2)$ can result in either 0 or a positive number, plus a positive number which will result in a positive number which isn't 0
    vii. Therefore, $(x^2+y^2) \neq 0$
    viii. Therefore, $x^3+xy^2 > x^2y+y^3$. ■
4. exercise 2.3.1, section l
    i. **Proof.**
    ii. Proof by contrapositive. We assume for every pair of real numbers x and y that $x \leq 10$ and $y \leq 10$ and prove that $x + y \leq 20$.
    iii. Since x and y can be no larger than 10, assume that x and y = 10, then 10 + 10 = 20
    iv. $20 \leq 20$ so the statement holds true
    v. For any combination of values for x and y such that either is less than 10, the result will be a number smaller than 20
    vi. Therefore, $x + y \leq 20$. ■

**Question 8:**

a) Solve exercise 2.4.1, sections c, e from the Discrete Math zyBook:
1. exercise 2.4.1, section c
    i. **Proof.**
    ii. Proof by contradiction. Suppose the average of three real numbers x, y, and z is less than each of the three real numbers x, y, and z.
    iii. Let the average be expressed as follows: $a = (x + y + z)/3$
    iv. We therefore argue:
        1. $a < x$
        2. $a < y$
        3. $a < z$
    v. If we add up the three inequalities, the result is $3a < x + y + z$
    vi. Since $a = (x + y + z)/3$, then $3 ((x + y + z)/3) < x + y + z$
    vii. The inequality can be simplified to $x + y + z < x + y + z$
    viii. This inequality contradicts itself as both sides of the expressions are equal and a number cannot be strictly smaller than itself
    ix. Therefore, the average of three real numbers is greater than or equal to at least one of the numbers. ■
2. exercise 2.4.1, section e
    i. **Proof.**
    ii. Proof by contradiction. Assume there is a smallest integer x.
    iii. Subtracting one from x results in $x – 1$
    iv. However, we argue that $x < x – 1$ which is equivalent to $0 < -1$
    v. This results in a contradiction, therefore there is no smallest integer. ■

**Question 9:**

a) Solve exercise 2.5.1, section c from the Discrete Math zyBook:
    1. exercise 2.5.1, section c
        i. **Proof.**
        ii. We consider two cases: x and y are both even, and x and y are both odd
        iii. **Case 1**: x and y are both even
            1. $x = 2k$ for some integer k
            2. $y = 2m$ for some integer m
            3. $x + y = 2k + 2m = 2(k + m)$
            4. $x + y = 2c$, where $c = (k + m)$ is an integer
            5. Therefore, $x + y$ is even
        iv. **Case 2**: x and y are both odd
            1. $x = 2k + 1$ for some integer k
            2. $y = 2m + 1$ for some integer m
            3. $x + y = 2k + 2m + 2 = 2(k + m + 1)$
            4. $x + y = 2c$, where $c = (k + m + 1)$ is an integer
            5. Therefore, $x + y$ is even
        v. ■