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Bk1139

HW 3

Question 7:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.1.1, sections a-g
   1. Exercise 3.1.1, section a
      i. **True**
         1. $27 \in A$ is true because 27 is a multiple of 3 (9 X 3)
   2. Exercise 3.1.1, section b
      i. **False**
         1. $27 \in B$ is false because the square root of 27 is not an integer
   3. Exercise 3.1.1, section c
      i. **True**
         1. $100 \in B$ is true because the square root of 100 is an integer (+/-10)
   4. Exercise 3.1.1, section d
      i. **False**
         1. $E \subseteq C$ or $C \subseteq E$ is false because all elements of E are not in C and all elements of C are not in E
   5. Exercise 3.1.1, section e
      i. **True**
         1. $E \subseteq A$ is true because {3, 6, 9} are all multiples of 3
   6. Exercise 3.1.1, section f
      i. **False**
         1. $A \subset E$ is false because A is a largest set than E
   7. Exercise 3.1.1, section g
      i. **False**
         1. $E \in A$ is false because it is not an element it is a set

b) Exercise 3.1.2, sections a-e
   1. Exercise 3.1.2, section a
      i. **False**
         1. $15 \subset A$ is false because 15 is not a set and therefore cannot be a subset or a proper subset
   2. Exercise 3.1.2, section b
      i. **True**
         1. $\{15\} \subset A$ is true because 15 is an element of A and A has additional elements as well
   3. Exercise 3.1.2, section c
      i. **True**
         1. $\emptyset \subset A$ is true because the empty set is a subset of every set, and A has additional elements as well
   4. Exercise 3.1.2, section d
      i. **True**

1. $A \subseteq A$ is true because all elements of A are also in A

5. Exercise 3.1.2, section e
    i. **False**
        1. $\emptyset \in B$ is false because the empty set is not an element of B

c) Exercise 3.1.5, sections b, d
1. Exercise 3.1.5, section b
    i. $\{x \in N^{+}: x \text{ is a multiple of } 3\}$
    ii. Infinite set
2. Exercise 3.1.5, section d
    i. $\{x \in N: x \text{ is a multiple of } 10 \text{ and } x \text{ is } \leq 1000\}$
    ii. Cardinality is 101

d) Exercise 3.2.1, sections a-k
1. Exercise 3.2.1, section a
    i. True
        1. 2 is an element of X
2. Exercise 3.2.1, section b
    i. True
        1. 2 is an element of X so the set {2} is a subset of X
3. Exercise 3.2.1, section c
    i. False
        1. {2} is not an element of X
4. Exercise 3.2.1, section d
    i. False
        1. 3 is not an element of X
5. Exercise 3.2.1, section e
    i. True
        1. The set {1, 2} is an element of X
6. Exercise 3.2.1, section f
    i. True
        1. 1 and 2 are elements of X, therefore the set {1, 2} is a subset of X
7. Exercise 3.2.1, section g
    i. True
        1. 2 and 4 are elements of X, therefore the set {2, 4} is a subset of X
8. Exercise 3.2.1, section h
    i. False
        1. The set {2,4} is not an element of X
9. Exercise 3.2.1, section i
    i. False
        1. 3 is not an element of X, therefore the set {2, 3} is not a subset of X
10. Exercise 3.2.1, section j
    i. False
        1. The set {2,3} is not an element of X
11. Exercise 3.2.1, section k
    i. False
        1. The cardinality of X is 6 not 7

Question 8:

Solve Exercise 3.2.4, section b from the Discrete Math zyBook

a) Exercise 3.2.4, section b
    1. Let A = {1, 2, 3}. What is {X ∈ P(A): 2 ∈ X}
        i. P(A) = { ∅, { 1 }, { 2 }, { 3 }, { 1, 2 }, { 1, 3 }, { 2, 3 }, { 1, 2, 3 }}
        ii. {X ∈ P(A): 2 ∈ X} = {{ 2 }, { 1, 2 }, { 2, 3 }, { 1, 2, 3 }}
            1. Identify all sets where 2 is an element of that set

Question 9:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.3.1, sections c-e
    1. Exercise 3.3.1, section c
        i. A ∩ C = **{-3, 1, 17}**
    2. Exercise 3.3.1, section d
        i. A ∪ (B ∩ C) = **{-5, -3, 0, 1, 4, 17}**
            1. (B ∩ C) = {-5, 1}
    3. Exercise 3.3.1, section e
        i. A ∩ B ∩ C = **{1}**
            1. A ∩ B = {1, 4}

b) Exercise 3.3.3, sections a, b, e, f
    1. Exercise 3.3.3, section a
        i. $\bigcap_{i=2}^{5} A_i$ = **{1}**
            1. $A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$
            2. $A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$
            3. $A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$
            4. $A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$
    2. Exercise 3.3.3, section b
        i. $\bigcup_{i=2}^{5} A_i$ = **{1, 2, 3, 4, 5, 9, 16, 25}**
            1. $A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$
            2. $A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$
            3. $A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$
            4. $A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$
    3. Exercise 3.3.3, section e
        i. $\bigcap_{i=1}^{100} C_i$ = $\{x \in R : -1/100 \leq x \leq 1/100\}$ **= $\{x \in R : -0.01 \leq x \leq 0.01\}$**
            1. The intersection of all real numbers from -1 to 1 at i = 1 and -0.01 to 0.01 at i = 100 is the smaller range defined by i = 100
    4. Exercise 3.3.3, section f
        i. $\bigcup_{i=1}^{100} C_i$ = $\{x \in R : -1/1 \leq x \leq 1/1\}$ **= $\{x \in R : -1 \leq x \leq 1\}$**
            1. The union of all real numbers from -1 to 1 at i = 1 to -0.01 to 0.01 at i = 100 is the larger range defined by i = 1

c) Exercise 3.3.4, sections b, d
    1. Exercise 3.3.4, section b
        i. P(A ∪ B) = **{∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}**
            1. (A ∪ B) = {a, b, c}
    2. Exercise 3.3.4, section d
        i. P(A) ∪ P(B) = **{∅, {a}, {b}, {c}, {a, b}, {b, c}}**
            1. P(A) = {∅, {a}, {b}, {a, b}}
            2. P(B) = {∅, {b}, {c}, {b, c}}

Question 10:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.5.1, sections b, c
    1. Exercise 3.5.1, section b
        i. (foam, tall, non-fat)
    2. Exercise 3.5.1, section c
        i. B X C = **{(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}**
b) Exercise 3.5.3, sections b, c, e
    1. Exercise 3.5.3, section b
        i. True
            1. "All integers" is a subset of "all real numbers" so the cartesian product of Z^2 is a subset of R^2
    2. Exercise 3.5.3, section c
        i. True
            1. The elements in Z^2 are pairs. The elements in Z^3 are triples. Therefore, the two sets have no elements in common and their intersection is the empty set
    3. Exercise 3.5.3, section e
        i. True
            1. If A is a subset of B, then all elements of A X C will also be in B X C
            2. B X C can be viewed as A X C and some additional "other elements" X C, and therefore A X C will be a subset of B X C
c) Exercise 3.5.6, sections d, e
    1. Exercise 3.5.6, section d
        i. {01, 011, 001, 0011}
            1. x = {0, 00}
            2. y = {1, 11}
    2. Exercise 3.5.6, section e
        i. {aaa, aaaa, aba, abaa}
            1. x = {aa, ab}
            2. y = {a, aa}

d) Exercise 3.5.7, sections c, f, g
    1. Exercise 3.5.7, section c
        i. **{aa, ab, ac, ad}**
            1. (A × B) = {ab, ac}
            2. (A × C) = {aa, ab, ad}
    2. Exercise 3.5.7, section f
        i. P(A × B) = **{ ∅, {ab}, {ac}, {ab, ac}}**
            1. (A × B) = {ab, ac}
    3. Exercise 3.5.7, section g
        i. P(A)×P(B) = **{ (∅,∅), (∅, {b}), (∅, {c}), (∅,{b, c}), ({a},∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c}) }**
            1. P(A) = {∅, {a}}
            2. P(B) = {∅, {b}, {c}, {b, c}}

Question 11:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.6.2, sections b, c
    1. Exercise 3.6.2, section b
        i. $(B \cup A) \cap (\overline{B} \cup A) = A$

1.	**$(A \cup B) \cap (A \cup \overline{B})$**	**Commutative Law**
2.	**$A \cup (B \cap \overline{B})$**	**Distributive Law**
3.	**$A \cup \emptyset$**	**Complement Law**
4.	**A**	**Identity Law**

    2. Exercise 3.6.2, section c
        i. $A \cap \overline{B} = \overline{A} \cup B$

1.	**$\overline{\overline{A} \cup \overline{B}}$**	**De Morgan's Law**
2.	**$\overline{\overline{A}} \cup B$**	**Double Complement Law**

b) Exercise 3.6.3, sections b, d
    1. Exercise 3.6.3, section b
        i. **Suppose A = {1, 2, 3} and B = {2, 3}**
        ii. **(B ∩ A) = {2, 3}**
        iii. **A - (B ∩ A) = {1}**
        iv. **{1} does not equal {1, 2, 3}**
    2. Exercise 3.6.3, section d
        i. **Suppose A = {2, 3} and B = {1, 2, 3}**
        ii. **(B - A) = {1}**
        iii. **(B - A) ∪ A = {1} ∪ {2, 3} = {1, 2, 3}**
        iv. **{1, 2, 3} does not equal {2, 3}**

c) Exercise 3.6.4, sections b, c

1. Exercise 3.6.4, section b

	$A \cap (B - A)$	
1.	$A \cap (B \cap \bar{A})$	**Set subtraction law**
2.	$A \cap (\bar{A} \cap B)$	**Commutative law**
3.	$(A \cap \bar{A}) \cap B$	**Associative law**
4.	$\emptyset \cap B$	**Complement law**
5.	$B \cap \emptyset$	**Commutative law**
6.	$\emptyset$	**Domination law**

2. Exercise 3.6.4, section c

	$A \cup (B - A)$	
1.	$A \cup (B \cap \bar{A})$	**Set subtraction law**
2.	$A \cup (\bar{A} \cap B)$	**Commutative law**
3.	$(A \cup \bar{A}) \cap (A \cup B)$	**Distributive law**
4.	$U \cap (A \cup B)$	**Complement law**
5.	$(A \cup B) \cap U$	**Commutative law**
6.	$(A \cup B)$	**Identity law**