Ben Kadosh

Bk1139

HW 1

Question 1:

A. Convert the following numbers to their decimal representation:

1. 10011011_2 = **155**

$$\boxed{10011011_2} \; \boxed{=} \; \boxed{1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7}$$
$$\boxed{=} \; \boxed{1 + 2 + 0 + 8 + 16 + 0 + 0 + 128}$$
$$\boxed{=} \; \boxed{155}$$

i.

2. 456_7 = **237**

$$\boxed{456_7} \; \boxed{=} \; \boxed{6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2}$$
$$\boxed{=} \; \boxed{6 + 35 + 196}$$
$$\boxed{=} \; \boxed{237}$$

i.

3. $38A_{16}$ = **906**

$$\boxed{38A_{16}} \; \boxed{=} \; \boxed{10 \cdot 16^0 + 8 \cdot 16^1 + 3 \cdot 16^2}$$
$$\boxed{=} \; \boxed{10 + 128 + 768}$$
$$\boxed{=} \; \boxed{906}$$

i.

4. 2214_5 = **309**

$$\boxed{2214_5} \; \boxed{=} \; \boxed{4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3}$$
$$\boxed{=} \; \boxed{4 + 5 + 50 + 250}$$
$$\boxed{=} \; \boxed{309}$$

i.

B. Convert the following numbers to their binary representation:

1. 69_{10} = **1000101**

The following items are included/assumed in the approach to solving the below:
1. Work from left to right to determine which positions are populated with 1's or 0's
2. The sum of (2^0 to 2^K) = 2^(K+1) - 1 --> Determines which positions must be populated with 1's
3. Once a 1 is populated in any given position, subtract the decimal value corresponding to that index from the original/remaining amount to determine what remainder needs to be populated when working further to the right

$$= 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$69_{10} = 64 + 0 + 0 + 0 + 4 + 0 + 1$$

$$= 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= 1000101$$

i.

2. 485_{10} = **111100101**

The following items are included/assumed in the approach to solving the below:
1. Work from left to right to determine which positions are populated with 1's or 0's
2. The sum of (2^0 to 2^K) = 2^(K+1) - 1 --> Determines which positions must be populated with 1's
3. Once a 1 is populated in any given position, subtract the decimal value corresponding to that index from the original/remaining amount to determine what remainder needs to be populated when working further to the right

$$= 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$485_{10} = 256 + 128 + 64 + 32 + 0 + 0 + 4 + 0 + 1$$

$$= 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= 111100101$$

i.

3. $6D1A_{16}$ = **0110110100011010**

The following items are included/assumed in the approach to solving the below:
1. Work from left to right
2. Convert each Hexadecimal digit to it's 4 bit binary equivalent
3. Concatenate the 4 bit binary digits from left to right

$6D1A_{16}$

$6_{16} = 0110_2$

$D_{16} = 1101_2$

$1_{16} = 0001_2$

$A_{16} = 1010_2$

i. $6D1A_{16} = 0110\ 1101\ 0001\ 1010$

C. Convert the following numbers to their hexadecimal representations:

1. 1101011_2 = **$6B_{16}$**

The following items are included/assumed in the approach to solving the below:
1. Separate the binary number into sections of 4 bits from right to left to and pad the last remaining set of bits with 0's if less than 4 bits
2. Convert each 4 bit binary to it's Hexadecimal digit/letter equivalent
3. Concatenate the Hexadecimal digits/letters from left to right

01101011_2

$0110\ 1011_2$

$0110_2 = 6_{16}$

$1011_2 = B_{16}$

i. $1101011_2 = 6B_{16}$

2. 895_{10} = **$37F_{16}$**

The following items are included/assumed in the approach to solving the below:
1. Divide the decimal number by the base to which you wish to convert it to (16) and keep track of the remainder at each step
2. Take the whole number that the decimal is divisible by and divide that further by the base to which you wish to convert (16)
3. Repeat this process until the whole number is 0
4. Convert any remainders greater than 9 to their hex equivalent value
5. Concatenate the hex digits from the latest remainder in the process to the earliest

$$
\begin{aligned}
\boxed{895_{10}} &= \boxed{895 \div 16} \\
&= \boxed{55\ r\ 15} & \boxed{r\ 15 = F} \\
&= \boxed{55 \div 16} & \\
&= \boxed{3\ r\ 7} & \boxed{r\ 7 = 7} \quad \boxed{\uparrow} \\
&= \boxed{3 \div 16} & \\
&= \boxed{0\ r\ 3} & \boxed{r\ 3 = 3} \\
&= \boxed{37F_{16}}
\end{aligned}
$$

i.

Question 2:

A. Solve the following, do all calculation in the given base:

1. $7566_8 + 4515_8 =$ **14303_8**

The following items are included/assumed in the approach to solving the below:
1. Any column whose sum exceeds 7 will have a carry over signified by a superscript of 1
2. As an example of the addition logic: in the one's place we have 6 + 5 which exceeds 8 by 3, which is why a 3 is placed in the one's position with a 1 carried over

$$
\begin{array}{r} 7566_8 \\ +\ 4515_8 \end{array} = \begin{array}{r} {}^{1}7\,{}^{1}5\,{}^{1}66_8 \\ +\ \ 4515_8 \\ \hline 14303_8 \end{array}
$$

i.

2. $10110011_2 + 1101_2 =$ **11000000_2**

The following items are included/assumed in the approach to solving the below:
1. Any column whose sum exceeds 1 will have a carry over signified by a superscript of 1

$$
\begin{array}{r} 10110011_2 \\ +\ \ 1101_2 \end{array} = \begin{array}{r} 1\,{}^{1}0\,{}^{1}1\,{}^{1}1\,{}^{1}0\,{}^{1}0\,{}^{1}11_2 \\ +\ \ 1101_2 \\ \hline 11000000_2 \end{array}
$$

i.

3. $7A66_{16} + 45C5_{16} =$ **$C02B_{16}$**

The following items are included/assumed in the approach to solving the below:
1. Any column whose sum exceeds 15 will have a carry over signified by a superscript of 1
2. Any column whose sum exceeds 9 will have that amount converted to the equivalent hex digit
3. As an example, in the one's place 6+5 =11 which is converted to a B in hex

$$\begin{array}{r} 7A66_{16} \\ +\ 45C5_{16} \end{array} = \begin{array}{r} {}^{1}7\,{}^{1}A66_{16} \\ +\quad 45C5_{16} \end{array}$$

$$C02B_{16}$$

i.

4. 3022_5 - 2433_5 = **34_5**

The following items are included/assumed in the approach to solving the below:
1. Any column whose top digit is less than the bottom digit will have a carryover from a digit to the left
2. When carrying over a 1 it is equivalent to carrying over 5 to the next index position
3. As an example, in the one's place 2 + 1 (carried over equivalent to 5) = 7

$$\begin{array}{r} 3022_5 \\ -\ 2433_5 \end{array} = \begin{array}{r} 302\,{}^{1}2_5 \\ +\quad 2433_5 \end{array}$$

$$= \begin{array}{r} 301\,{}^{1}2_5 \\ +\quad 2433_5 \end{array}$$

$$= \begin{array}{r} 30\,{}^{1}1\,{}^{1}2_5 \\ +\quad 2433_5 \end{array}$$

$$= \begin{array}{r} 24\,{}^{1}1\,{}^{1}2_5 \\ +\quad 2433_5 \end{array}$$

$$= 34_5$$

i.

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation:

1. 124_{10} = **01111100**

The following items are included/assumed in the approach to solving the below:
1. Work from left to right to determine which positions are populated with 1's or 0's
2. The sum of (2^0 to 2^K) = 2^(K+1) - 1 --> Determines which positions must be populated with 1's
3. Once a 1 is populated in any given position, subtract the decimal value corresponding to that index from the original/remaining amount to determine what remainder needs to be populated when working further to the right

$$= 1\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$$

$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$124_{10} = 0 + 64 + 32 + 16 + 8 + 4 + 0 + 0$$

$$= 0\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$$

$$= 01111100$$

i.

2. -124_{10} = **10000100**

The following items are included/assumed in the approach to solving the below:
1. The sum of a number and it's additive inverse is 2^k

$\boxed{124_{10}}$ $\boxed{=}$ $\boxed{{}^{1}0{}^{1}1{}^{1}1{}^{1}1{}^{1}1100}$

$\boxed{+}$

$\boxed{-124_{10}}$ $\boxed{=}$ $\boxed{10000100}$

$\boxed{100000000}$

i.

3. 109_{10} = **01101101**

The following items are included/assumed in the approach to solving the below:
1. Work from left to right to determine which positions are populated with 1's or 0's
2. The sum of (2^0 to 2^K) = 2^(K+1) - 1 --> Determines which positions must be populated with 1's
3. Once a 1 is populated in any given position, subtract the decimal value corresponding to that index from the original/remaining amount to determine what remainder needs to be populated when working further to the right

$\boxed{=}$ $\boxed{1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0}$

$\boxed{=}$ $\boxed{128 + 64 + 32 + 16 + 8 + 4 + 2 + 1}$

$\boxed{109_{10}}$ $\boxed{=}$ $\boxed{0 + 64 + 32 + 0 + 8 + 4 + 0 + 1}$

$\boxed{=}$ $\boxed{0 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0}$

$\boxed{=}$ $\boxed{01101101}$

i.

4. -79_{10} = **10110001**

The following items are included/assumed in the approach to solving the below:
1. Work from left to right to determine which positions are populated with 1's or 0's
2. The sum of (2^0 to 2^K) = 2^(K+1) - 1 --> Determines which positions must be populated with 1's
3. Once a 1 is populated in any given position, subtract the decimal value corresponding to that index from the original/remaining amount to determine what remainder needs to be populated when working further to the right

$\boxed{=}$ $\boxed{1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0}$

$\boxed{=}$ $\boxed{128 + 64 + 32 + 16 + 8 + 4 + 2 + 1}$

$\boxed{79_{10}}$ $\boxed{=}$ $\boxed{0 + 64 + 0 + 0 + 8 + 4 + 2 + 1}$

$\boxed{=}$ $\boxed{0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0}$

$\boxed{=}$ $\boxed{01001111}$

i.

The following items are included/assumed in the approach to solving the below:
1. The sum of a number and it's additive inverse is 2^k

$$79_{10} = {}^{1}0{}^{1}1{}^{1}0{}^{1}0{}^{1}1{}^{1}1{}^{1}11$$

$$+$$

$$-79_{10} = 10110001$$

$$100000000$$

ii.

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation:

1. $00011110_{8 \text{ bit 2's comp}}$ = **30**

$$= 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$00011110_{8\ bit\ 2's\ comp} = 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$$

$$= 0 + 0 + 0 + 16 + 8 + 4 + 2 + 0$$

$$= 30_{10}$$

i.

2. $11100110_{8 \text{ bit 2's comp}}$ = **-26**

$$= {}^{1}1{}^{1}1{}^{1}1{}^{1}0{}^{1}0{}^{1}110$$

$$+$$

$$= 00011010$$

$$100000000$$

i.

$$= 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$00011010_{8\ bit\ 2's\ comp} = 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$$

$$= 0 + 0 + 0 + 16 + 8 + 0 + 2 + 0$$

$$= 26_{10}$$

ii.

3. $00101101_{8\text{ bit 2's comp}}$ = **45**

$$= 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$00101101_{8\ bit\ 2's\ comp} = 0 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= 0 + 0 + 32 + 0 + 8 + 4 + 0 + 1$$

$$= 45_{10}$$

i.

4. $10011110_{8\text{ bit 2's comp}}$ = **-98**

The following items are included/assumed in the approach to solving the below:
1. The sum of a number and it's additive inverse is 2^k

$$= {}^{1}1{}^{1}0{}^{1}0{}^{1}1{}^{1}1{}^{1}110$$

$$+$$

$$= 01100010$$

$$100000000$$

i.

$$= 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$

$$01100010_{8\ bit\ 2's\ comp} = 0 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$$

$$= 0 + 64 + 32 + 0 + 0 + 0 + 2 + 0$$

$$= 98_{10}$$

ii.

Question 4:

1. Exercise 1.2.4, sections b, c
   a. Exercise 1.2.4, section b
      i.

p	q	(p ∨ q)	¬(p ∨ q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

   b. Exercise 1.2.4, section c

i.

p	**q**	**r**	**¬q**	**(p ∧ ¬q)**	**r ∨ (p ∧ ¬q)**
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4, sections b, d
    a. Exercise 1.3.4, section b
        i.

p	**q**	**(p → q)**	**(q → p)**	**(p → q) → (q → p)**
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

    b. Exercise 1.3.4, section d
        i.

p	**q**	**¬q**	**(p ↔ q)**	**(p ↔ ¬q)**	**(p ↔ q) ⊕ (p ↔ ¬q)**
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5:

1. Exercise 1.2.7, sections b, c
    a. Exercise 1.2.7, section b
        i. (B∧D) ∨ (B∧M) ∨ (D∧M)
            1. Determine all unique combinations of two of the three forms of identification
    b. Exercise 1.2.7, section c
        i. B ∨ (D∧M)
            1. Put parentheses around D and M to signify the AND logical operator is applied correctly
2. Exercise 1.3.7, sections b-e
    a. Exercise 1.3.7, section b
        i. (s ∨ y) → p
    b. Exercise 1.3.7, section c
        i. p → y

Table 1.3.2: English expressions of the conditional operation.

Consider the propositions:

p: You mow Mr. Smith's lawn.
q: Mr. Smith will pay you.

If p, then q.	If you mow Mr. Smith's lawn, then he will pay you.
If p, q.	If you mow Mr. Smith's lawn, he will pay you.
q if p	Mr. Smith will pay you if you mow his lawn.
p implies q.	Mowing Mr. Smith's lawn implies that he will pay you.
p only if q.	You will mow Mr. Smith's lawn only if he pays you.
p is sufficient for q.	Mowing Mr. Smith's lawn is sufficient for him to pay you.
q is necessary for p.	Mr. Smith's paying you is necessary for you to mow his lawn.

1.
2. In this example, the sentence can be presented as y is necessary for p, which is equivalent to If p, then y.

c. Exercise 1.3.7, section d
   i. **$p \leftrightarrow (s \wedge y)$**
d. Exercise 1.3.7, section e
   i. **$p \rightarrow (s \vee y)$**

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p implies q.	Mowing Mr. Smith's lawn implies that he will pay you.
p only if q.	You will mow Mr. Smith's lawn only if he pays you.
p is sufficient for q.	Mowing Mr. Smith's lawn is sufficient for him to pay you.
q is necessary for p.	Mr. Smith's paying you is necessary for you to mow his lawn.

1.

3. Exercise 1.3.9, sections c, d
   a. Exercise 1.3.9, section c
      i. **$c \rightarrow p$**
   b. Exercise 1.3.9, section d
      i. **$c \rightarrow p$**
         1. referring to the prior table – p is necessary for c can be presented as “If c, then p.”

<u>Question 6:</u>

1. Exercise 1.3.6, sections b-d
   a. Exercise 1.3.6, section b
      i. **If Joe is eligible for the honors program, then he maintains a B average.**
         1. “If p, then q” is equivalent to “q is necessary for p”
   b. Exercise 1.3.6, section c
      i. **If Rajiv can go on the roller coaster, then he is at least four feet tall.**

c. Exercise 1.3.6, section d
    i. If Rajiv is at least four feet tall, then he can go on the roller coaster.
        1. “If p, then q” is equivalent to “q if p”

2. Exercise 1.3.10, sections c-f
    a. Exercise 1.3.10, section c
        i. The truth value of the expression is: **False**
            1. $(p \vee r) \leftrightarrow (q \wedge r)$
            2. Substituting the provided truth values for the variables results in
                a. $(T \vee r) \leftrightarrow (F \wedge r)$
            3. Using the Domination Laws, this can be further simplified to
                a. $T \leftrightarrow F$
            4.
Table 1.3.4: Truth table for the biconditional operation.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

    b. Exercise 1.3.10, section d
        i. The truth value of the expression is: **Unknown**
            1. $(p \wedge r) \leftrightarrow (q \wedge r)$
            2. Substituting the provided truth values for the variables results in
                a. $(T \wedge r) \leftrightarrow (F \wedge r)$
            3. In evaluating $(T \wedge r)$, since r is unknown the left side of the biconditional operation is unknown, and therefore the truth value of the whole expression is unknown
    c. Exercise 1.3.10, section e
        i. The truth value of the expression is: **Unknown**
            1. $p \rightarrow (r \vee q)$
            2. Substituting the provided truth values for the variables results in
                a. $T \rightarrow (r \vee F)$
            3. $(r \vee F)$ is unknown, and therefore the expression is unknown
    d. Exercise 1.3.10, section f
        i. The truth value of the expression is: **True**
            1. $(p \wedge q) \rightarrow r$
            2. Substituting the provided truth values for the variables results in
                a. $(T \wedge F) \rightarrow r$
                b. $F \rightarrow r$

Table 1.3.1: Truth table for the conditional operation.

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

3.
4. Regardless of what truth value r has, because the left side of the conditional statement is false, the expression will always result to true.

Question 7:

1. Exercise 1.4.5 sections b-d
   a. Exercise 1.4.5 section b
      i. **Logically equivalent**
      ii. If Sally did not get the job, then she was late for interview or did not update her resume.
         1. $\neg j \rightarrow (l \vee \neg r)$
      iii. If Sally updated her resume and was not late for her interview, then she got the job.
         1. $(r \wedge \neg l) \rightarrow j$

j	**l**	**r**	**¬j**	**¬l**	**¬r**	**(l ∨ ¬r)**	**(r ∧ ¬l)**	**¬j → (l ∨ ¬r)**	**(r ∧ ¬l) → j**
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	T	T	F	T	T
T	F	T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	F	T	T
F	T	T	T	F	F	T	F	T	T
F	T	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	F
F	F	F	T	T	T	T	F	T	T

   b. Exercise 1.4.5 section c
      i. **Not logically equivalent**
      ii. If Sally got the job then she was not late for her interview.
         1. $j \rightarrow \neg l$
      iii. If Sally did not get the job, then she was late for her interview.
         1. $\neg j \rightarrow l$

j	**l**	**¬j**	**¬l**	**j → ¬l**	**¬j → l**
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

   c. Exercise 1.4.5 sections d
      i. **Not logically equivalent**
      ii. If Sally updated her resume or she was not late for her interview, then she got the job.
         1. $(r \vee \neg l) \rightarrow j$
      iii. If Sally got the job, then she updated her resume and was not late for her interview.
         1. $j \rightarrow (r \wedge \neg l)$

j	l	r	¬l	(r ∨ ¬l)	(r ∧ ¬l)	(r ∨ ¬l) → j	j → (r ∧ ¬l)
T	T	T	F	T	F	T	F
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	F	T	F	F	T
F	T	F	F	F	F	T	T
F	F	T	T	T	T	F	T
F	F	F	T	T	F	F	T

Question 8:

1. Exercise 1.5.2 sections c, f, i
   a. Exercise 1.5.2 sections c
      i. (p → q) ∧ (p → r) ≡ p → (q ∧ r)
         1. (¬p ∨ q) ∧ (¬p ∨ r) **Conditional Identity**
         2. ¬p ∨ (q ∧ r) **Distributive Law**
         3. p → (q ∧ r) **Conditional Identity**
   b. Exercise 1.5.2 sections f
      i. ¬(p ∨ (¬p ∧ q)) ≡ ¬p ∧ ¬q
         1. ¬((p ∨ ¬p) ∧ (p ∨ q)) **Distributive Law**
         2. ¬((T) ∧ (p ∨ q)) **Complement Law**
         3. ¬(p ∨ q) **Identity Law**
         4. ¬p ∧ ¬q **De Morgan's Law**
   c. Exercise 1.5.2 sections i
      i. (p ∧ q) → r ≡ (p ∧ ¬r) → ¬q
         1. ¬(p ∧ q) ∨ r **Conditional Identity**
         2. (¬p ∨ ¬q) ∨ r **De Morgan's Law**
         3. ¬p ∨ (¬q ∨ r) **Associative Law**
         4. ¬p ∨ (r ∨ ¬q) **Commutative Law**
         5. (¬p ∨ r) ∨ ¬q **Associative Law**
         6. (¬p ∨ ¬¬r) ∨ ¬q **Double Negation Law**
         7. ¬(p ∧ ¬r) ∨ ¬q **De Morgan's Law**
         8. (p ∧ ¬r) → ¬q **Conditional Identity**
2. Exercise 1.5.3 sections c, d
   a. Exercise 1.5.3 section c
      i. ¬r ∨ (¬r → p)
         1. ¬r ∨ (¬ (¬r) ∨ p) **Conditional Identity**
         2. ¬r ∨ (r ∨ p) **Double Negation Law**
         3. (¬r ∨ r) ∨ p **Associative Law**
         4. T ∨ p **Complement Law**
         5. T **Domination Law**
   b. Exercise 1.5.3 section d
      i. ¬(p → q) → ¬q
         1. ¬(¬(p → q)) ∨ ¬q **Conditional Identity**
         2. ¬(¬(¬p ∨ q)) ∨ ¬q **Conditional Identity**
         3. ¬(¬¬p ∧ ¬q) ∨ ¬q **De Morgan's Law**
         4. ¬(p ∧ ¬q) ∨ ¬q **Double Negation Law**

5. $(\neg p \lor \neg\neg q) \lor \neg q$ **De Morgan's Law**
6. $(\neg p \lor q) \lor \neg q$ **Double Negation Law**
7. $\neg p \lor (q \lor \neg q)$ **Associative Law**
8. $\neg p \lor T$ **Complement Law**
9. T **Domination Law**

<u>Question 9:</u>

1. Exercise 1.6.3 sections c, d
   a. Exercise 1.6.3 section c
      i. $\mathbf{\exists x\ (x = x^2)}$
   b. Exercise 1.6.3 section d
      i. $\mathbf{\forall x\ (x \leq x^2)}$
2. Exercise 1.7.4 sections b-d
   a. Exercise 1.7.4 sections b
      i. $\mathbf{\forall x\ (\neg S(x) \land W(x))}$
   b. Exercise 1.7.4 sections c
      i. $\mathbf{\forall x\ (S(x) \to \neg W(x))}$
   c. Exercise 1.7.4 sections d
      i. $\mathbf{\exists x\ (S(x) \land W(x))}$

<u>Question 10:</u>

1. Exercise 1.7.9 sections c-i
   a. Exercise 1.7.9 section c
      i. True
         1. If we test with the x value a, $a \neq c$, and therefore the hypothesis is false
         2. If the hypothesis is false, then the conditional statement is true regardless of the truth value of the conclusion
   b. Exercise 1.7.9 section d
      i. True
         1. Example: e
   c. Exercise 1.7.9 section e
      i. True
         1. $Q(a) \land P(d)$
         2. $T \land T$
   d. Exercise 1.7.9 section f
      i. True
         1. If we test with the x value {a, c, d, e} they do not equal b, so the hypothesis is true, and Q(a), Q(c), Q(d), Q(e) are all true so the conditional statement is true for the x values tested in the domain thus far
         2. When testing with x = b, then $b \neq b$ and the hypothesis is false so the conditional statement is true regardless of the truth value of the conclusion
         3. All x values result in the conditional statement being true
   e. Exercise 1.7.9 section g
      i. False
         1. Counter-example: c
   f. Exercise 1.7.9 section h
      i. True

g. Exercise 1.7.9 section i
    i. **True**

2. Exercise 1.9.2 sections b-i
    a. Exercise 1.9.2 section b
        i. **True**
            1. x = 2
    b. Exercise 1.9.2 section c
        i. **True**
            1. x = 1
    c. Exercise 1.9.2 section d
        i. **False**
            1. All combinations of x and y for S(x,y) result in false
    d. Exercise 1.9.2 section e
        i. **False**
            1. There is no y such that Q(1, y), Q(2, y) and Q(3, y) are all true.
    e. Exercise 1.9.2 section f
        i. **True**
            1. y = 1
    f. Exercise 1.9.2 section g
        i. **False**
            1. Not all combinations of x and y for P(x,y) result in true
    g. Exercise 1.9.2 section h
        i. **True**
            1. x = 2, y = 1
    h. Exercise 1.9.2 section i
        i. **True**
            1. All combinations of S(x,y) result in false, meaning the negation of each results in a true value

<u>Question 11:</u>

1. Exercise 1.10.4, sections c-h
    a. Exercise 1.10.4, section c
        i. **$\exists x \exists y (x + y = x \cdot y)$**
    b. Exercise 1.10.4, section d
        i. **$\forall x \forall y ((x > 0 \land y > 0) \rightarrow x/y > 0)$**
    c. Exercise 1.10.4, section e
        i. **$\forall x ((x > 0 \land x < 1) \rightarrow 1/x > 1)$**
    d. Exercise 1.10.4, section f
        i. **$\neg\exists x \forall y (x \leq y)$**
    e. Exercise 1.10.4, section g
        i. **$\forall x \exists y (x \neq 0 \rightarrow x \cdot y = 1)$**
    f. Exercise 1.10.4, section h
        i. **$\forall x \exists y \forall z (x \neq 0 \rightarrow (x \cdot y = 1) \land (y \neq z \rightarrow (x \cdot z \neq 1)))$**
2. Exercise 1.10.7, sections c-f
    a. Exercise 1.10.7, section c
        i. **$\exists x (N(x) \land D(x))$**

    b. Exercise 1.10.7, section d
        i. $\boldsymbol{\forall x\, (D(x) \rightarrow P(Sam, x))}$
    c. Exercise 1.10.7, section e
        i. $\boldsymbol{\exists x\, \forall y\, (N(x) \land P(x, y))}$
    d. Exercise 1.10.7, section f
        i. $\boldsymbol{\exists x\, \forall y\, (N(x) \land D(x)) \land (x \neq y \rightarrow \neg(N(y) \land D(y)))}$
3. Exercise 1.10.10 sections c-f
    a. Exercise 1.10.10 section c
        i. $\boldsymbol{\forall x\, \exists y\, (T(x, y) \land y \neq \text{Math } 101)}$
    b. Exercise 1.10.10 section d
        i. $\boldsymbol{\exists x\, \forall y\, (y \neq \text{Math } 101 \rightarrow T(x, y))}$
    c. Exercise 1.10.10 section e
        i. $\boldsymbol{\forall x\, \exists y_0\, \exists y_1\, ((x \neq Sam) \land (y_0 \neq y_1) \rightarrow T(x, y_0) \land T(x, y_1))}$
    d. Exercise 1.10.10 section f
        i. $\boldsymbol{\exists y_0\, \exists y_1\, (T(Sam, y_0) \land T(Sam, y_1) \land ((y \neq y_0) \land (y \neq y_1) \rightarrow \neg T(Sam, y)))}$

<u>Question 12:</u>

1. Exercise 1.8.2, sections b – e
    a. Exercise 1.8.2, section b
        i. $\boldsymbol{\forall x\, (D(x) \lor P(x))}$
        ii. Negation: $\boldsymbol{\neg\forall x\, (D(x) \lor P(x))}$
        iii. Applying De Morgan’s Law: $\boldsymbol{\exists x\, (\neg D(x) \land \neg P(x))}$
        iv. English: Some patient was not given both the medication and the placebo
    b. Exercise 1.8.2, section c
        i. $\boldsymbol{\exists x\, (D(x) \land M(x))}$
        ii. Negation: $\boldsymbol{\neg\exists x\, (D(x) \land M(x))}$
        iii. Applying De Morgan's law: $\boldsymbol{\forall x\, (\neg D(x) \lor \neg M(x))}$
        iv. English: Every patient was either not given the medication or did not have migraines (or both).
    c. Exercise 1.8.2, section d
        i. $\boldsymbol{\forall x\, (P(x) \rightarrow M(x))}$
        ii. Negation: $\boldsymbol{\neg\forall x\, (P(x) \rightarrow M(x))}$
        iii. Applying De Morgan's law: $\boldsymbol{\exists x\, (P(x) \land \neg M(x))}$
        iv. English: Some patient was given the placebo and did not have migraines
    d. Exercise 1.8.2, section e
        i. $\boldsymbol{\exists x\, (M(x) \land P(x))}$
        ii. Negation: $\boldsymbol{\neg\exists x\, (M(x) \land P(x))}$
        iii. Applying De Morgan's law: $\boldsymbol{\forall x\, (\neg M(x) \lor \neg P(x))}$
        iv. English: Every patient either did not have migraines or was not given the placebo (or both).
2. Exercise 1.9.4, sections c – e
    a. Exercise 1.9.4, section c
        i. $\exists x\, \forall y\, (P(x, y) \rightarrow Q(x, y))$
            1. $\forall x\, \exists y\, \neg(P(x, y) \rightarrow Q(x, y))$
            2. $\forall x\, \exists y\, \neg(\neg P(x, y) \lor Q(x, y))$ Conditional Identity
            3. $\forall x\, \exists y\, (\neg\neg P(x, y) \land \neg Q(x, y))$ De Morgan’s Law
            4. $\boldsymbol{\forall x\, \exists y\, (P(x, y) \land \neg Q(x, y))}$ Double Negation

b. Exercise 1.9.4, section d
   i. $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$
      1. $\forall x\ \exists y\ \neg(P(x, y) \leftrightarrow P(y, x))$
      2. $\forall x\ \exists y\ \neg((P(x, y) \to P(y, x)) \land (P(y, x) \to P(x, y)))$ Conditional Identity
      3. $\forall x\ \exists y\ \neg(P(x, y) \to P(y, x)) \lor \neg(P(y, x) \to P(x, y))$ De Morgan's
      4. $\forall x\ \exists y\ \neg(\neg P(x, y) \lor P(y, x)) \lor \neg(\neg P(y, x) \lor P(x, y))$ Conditional Identity
      5. $\forall x\ \exists y\ (\neg\neg P(x, y) \land \neg P(y, x)) \lor (\neg\neg P(y, x) \land \neg P(x, y))$ De Morgan's
      6. **$\forall x\ \exists y\ (P(x, y) \land \neg P(y, x)) \lor (P(y, x) \land \neg P(x, y))$** Double Negation

c. Exercise 1.9.4, section e
   i. $\exists x\ \exists y\ P(x, y) \land \forall x\ \forall y\ Q(x, y)$
      1. $\neg(\exists x\ \exists y\ P(x, y) \land \forall x\ \forall y\ Q(x, y))$
      2. $\neg(\exists x\ \exists y\ P(x, y)) \lor \neg(\forall x\ \forall y\ Q(x, y))$ De Morgan's
      3. **$(\forall x\ \forall y\ \neg P(x, y)) \lor (\exists x\ \exists y\ \neg Q(x, y))$** De Morgan's