CFD - Lecture 1

→ Differential Equations involve <u>derivatives</u> with respect to various coordinates in space-time (x, y, z, t)

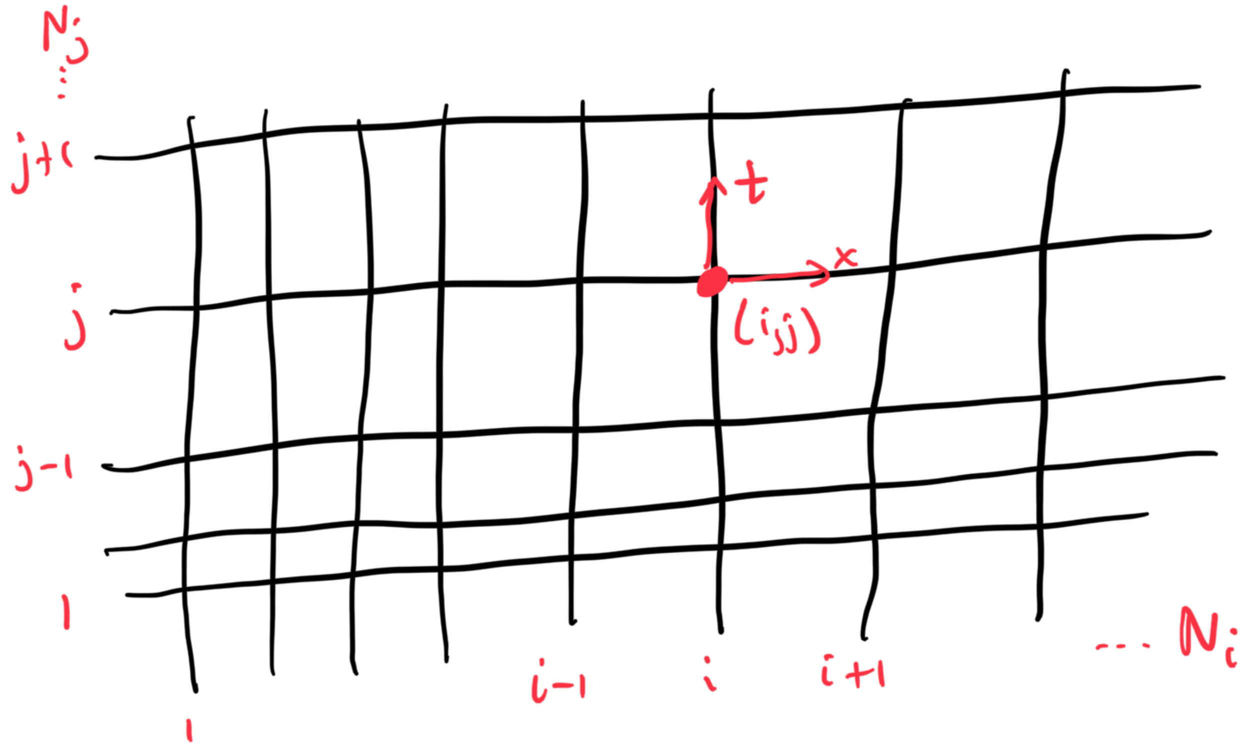
→ The first concept is that in order to solve the differential equation numerically, we will have to make the solution $\left(u(x,y,z,t) \right)$, and all of the derivatives of the solution w.r.t. all of the variables

$$\left(\frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial t^2}, \frac{\partial^4 u}{\partial z^4}, \dots \right)$$

$$\boxed{\text{DISCRETE}}$$

we imagine a

→ To do this, we ...
grid in space-time. For simplicity
we will imagine first a grid in
(x, t).



→ grid lines of the same coordinate never intersect

→ grid lines of different coordinates intersect only once.

→ grid spacing is not necessarily even.

→ grid lines are not necessarily straight
(e.g. polar, cylindrical
spherical)

Taylor series approximation of a function:

$$f(x) \approx f(x_0) + (x-x_0)\frac{\partial f}{\partial x} + \frac{(x-x_0)^2}{2!}\frac{\partial^2 f}{\partial x^2} + \dots + \frac{(x-x_0)^n}{n!}\frac{\partial^n f}{\partial x^n}$$

$$\Downarrow$$

$$\frac{\partial f}{\partial x} = \frac{f(x)-f(x_0)}{x-x_0} - \frac{x_0-x_0}{2!}\frac{\partial^2 f}{\partial x^2} - \frac{(x-x_0)^2}{3!}\frac{\partial^3 f}{\partial x^2} - \dots - \frac{(x-x_0)^{n-1}}{n!}\frac{\partial^n f}{\partial x^n}$$

If $x-x_0 \equiv \Delta x$ is small, then
higher order terms are small, and

higher

can be neglected.

DISCRETE FIRST DERIVATIVE SCHEMES :

① Forward Difference.

$$\frac{\partial f}{\partial x} \approx \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} + O(\Delta$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{f_{i+1} - f_i}{\Delta x} + O(\Delta x)}$$

truncation er

② Backward Difference.

$$\frac{\partial f}{\partial x} = \frac{f_i - f_{i-1}}{\Delta x} + O(\Delta x)$$

③ Central Difference.

$$\frac{\partial f}{} = \frac{f_{i+1} - f_{i-1}}{} + O(\Delta x^2)$$

$0x$ $2\Delta x$

$\boxed{\text{Crucial Point: Which scheme works best depends upon the differential Equation being considered.}}$