CFD - Lecture 2

Examples :

Navier-Stokes Equation.

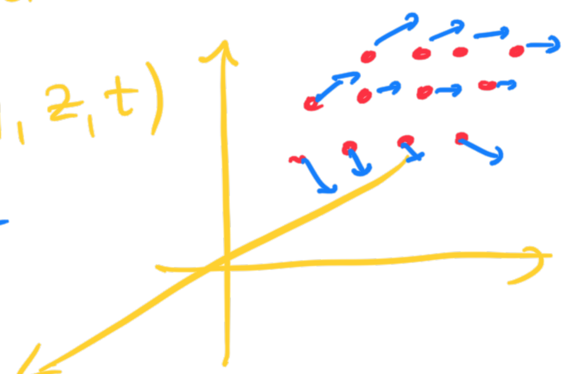
pressure

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = \vec{\nabla} p + \nu \nabla^2 \vec{v}$$

viscosity

$\vec{v}$ → velocity vector field

$\vec{v}(x, y, z, t)$



gives the velocity of the fluid at all points in space, at all times

→ if we can find $\vec{v}(x, y, z, t)$, we are done!

p → pressure scalar field

$p(x, y, z, t)$ → gives the pressure

in the fluid at all
points in space, at
all times.

→ Terms involving 2nd Order derivatives are called <u>Diffusive</u> terms.

→ Terms involving 1st Order derivative (in space) are called <u>convective</u> terms.

Most of the solutions to the N-S equations involve <u>modeling assumptions</u> (assumptions about the importance of various terms)

→ <u>All</u> of them involve a system of PDE's with
   → highest space derivative is 2nd Order
   → highest time derivative is 1st Order.

Example 1 → 1D Linear Convection

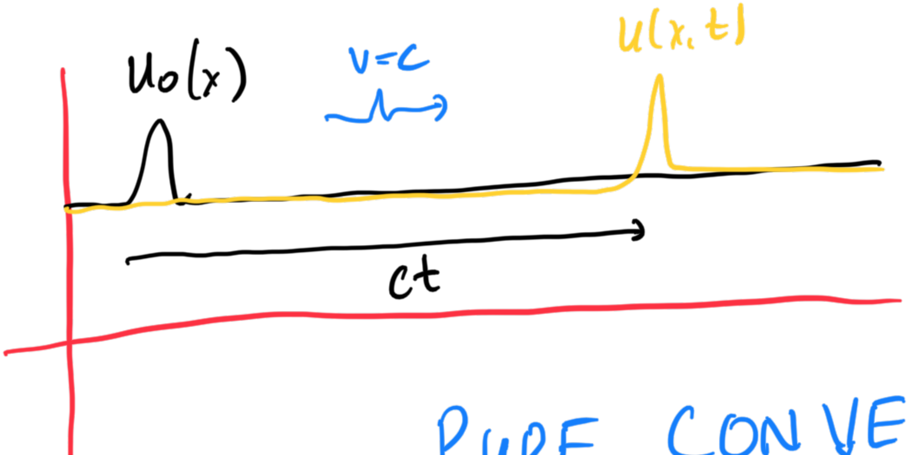
$$\frac{\partial u}{\partial t} + c\,\frac{\partial u}{\partial x} = 0$$

c ↑ transport velocity

Solution $u(x,t) = u_0(x-ct)$

↑ initial profile at $t=0$

WAVE PROPAGATION



PURE CONVECTION

DISCRETIZATION →

WHY?

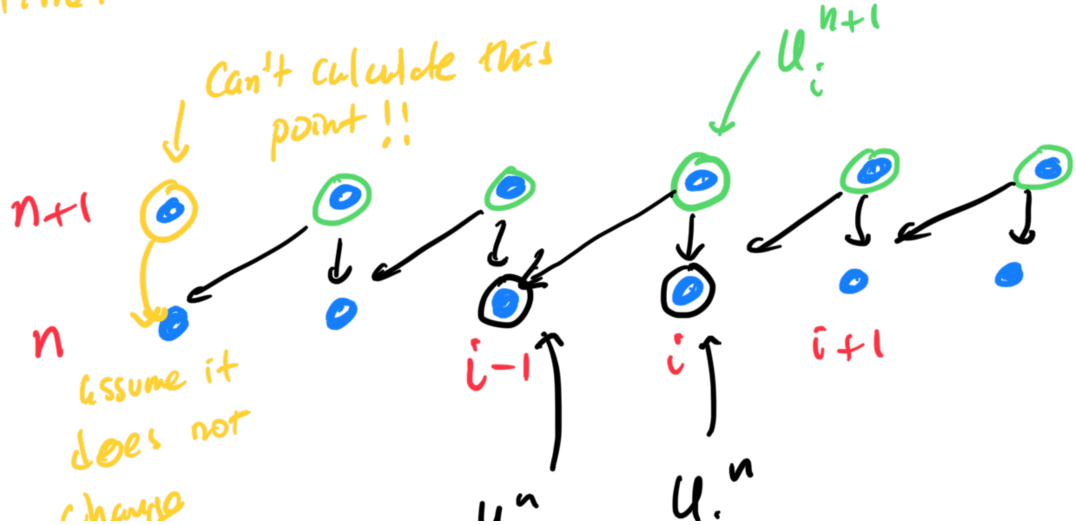
i → x axis

$n \rightarrow t$ axis

numerical scheme : forward difference in time, and backward difference in space.

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + c\,\frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

$$\underbrace{u_i^{n+1}}_{\text{next point in time.}} = \underbrace{u_i^n - \frac{c\,\Delta t}{\Delta x}\left(u_i^n - u_{i-1}^n\right.}_{\text{all at time } t_n}$$



u_{i-1}

Example 2: Inviscid Burger's Equation.

$$\boxed{\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0}$$

(interest → can generate non-linearities from smooth I.C.'s → shock waves in supersonic flows)

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \left(\frac{u_i^n - u_{i-1}^n}{\Delta x} \right) = 0$$

$$\boxed{u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x} u_i^n \left(u_i^n - u_{i-1}^n \right)}$$