Biophysical pasture model documentation

Model documentation for DairyMod, EcoMod and the SGS Pasture Model

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Citation: Johnson IR (2008). *Biophysical pasture model documentation: model documentation for DairyMod. EcoMod and the SGS Pasture Model.* (IMJ Consultants: Dorrigo, NSW)

This document the background to the biophysical modelling approach used in DairyMod, EcoMod and the SGS Pasture Model, collectively termed GrazeMod. It was originally published in 2005 and the latest update released in 2008.

These models have been developed as part of the WFSAT (Whole Farm Systems Analysis and Tools) project funded by Meat and Live Stock Australia (MLA), Dairy Australia (DA) and AgResearch (New Zealand).

Note:

Since writing this document, DairyMod and the SGS Pasture Model have been completely rewritten and the documentation updated. The model versions relating to this document are no longer supported. The current documentation is Johnson IR (2013). *DairyMod and the SGS Pasture Model: a mathematical description of the biophysical model structure*. IMJ Consultants, Dorrigo, NSW. This document can be obtained at www.imj.com.au, following the links to either DairyMod or the SGS Pasture Model.

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Biophysical pasture simulation modelling

1 Biophysical modelling

1.1 Introduction

DairyMod, EcoMod and the SGS Pasture Model, collectively termed GrazeMod, are a suite of biophysical simulation models of pasture systems. They include modules for pasture growth and utilization by grazing animals, animal physiology including milk production, water and nutrient dynamics, as well as a range of options for pasture management, irrigation and fertilizer application. This document provides a mathematical account of the components in the model.

The model has multiple subdivisions, or paddocks, (up to 100) that can each be defined independently to represent spatial variation within a notional 'farm'. They can have different soil types, nutrient status, pasture species, fertilizer and irrigation management. This gives considerable scope to explore the implications of management on pasture utilisation and milk production. The large number of subdivisions gives the model flexibility in selecting the appropriate area to be grazed on a particular day. These subdivisions can also be used to represent urine patches within a single paddock – this is discussed later in the *Management* chapter.

The model includes a range of continuous and rotational grazing management strategies, options for cutting, and a cutting trial that simulates the standard method of calculating seasonal pasture growth rates under cutting.

The biophysical nature of the model allows effective assessment of pasture growth and utilization in response to environmental conditions. The principal components are:

- Pasture growth. Physiologically based model with multiple species that can be C3, C4, perennial, annual, legume.
- Animal intake. Supplementary feeding of both concentrate and forage is also available.
- Animal metabolism. Energy based model that includes growth, maintenance, pregnancy and milk production.
- Water dynamics. Mechanistic model of water dynamics, including transpiration, evaporation (from canopy, litter and soil), infiltration (and therefore through drainage) and runoff.
- Nutrient dynamics. Organic matter turnover (from litter, dead roots, dung), and inorganic nutrient dynamics for N, P, K, S. The model includes plant uptake, leaching, atmospheric N losses, NH4 to NO3 transformations.
- The model also includes the ability to run up to 100 paddocks simultaneously that can each be defined with individual parameter sets, such as soil characteristics, pasture species and so on.
- There is a range of management options including set-stocked, variable stocked continuous grazing, as well as a variety of rotational grazing strategies. Cutting regimes can also be included.
- Fertilizer and irrigation options are also available.
- Climate data are read from Excel files.
- Simulations can be run for up to 150 years (this could easily be extended), and detailed simulation output can be exported to an Excel file.
- The model interface is designed for ease of use in creating and running simulations, as well as observing model behaviour as the simulation runs.

- All graphs in the model can be copied to the clipboard (right mouse) and pasted to other applications.
- The simulation results can be exported to an Excel file for further analysis. This allows the daily values of most of the variables in the model to be exported, and gives powerful access to the simulations.

This chapter gives a brief discussion of some of the underlying modelling concepts and principles that I have adopted in developing this model structure. My views on this topic have evolved over a number of years through developing models in a range of disciplines and from many discussions with modelling colleagues, particularly John Thornley and Tony Parsons.

1.2 Level of complexity

In developing the model, each module has been constructed in a relatively simple manner, with the aim of striking a balance between complexity, tractability and realism. As in the formulation of any model, this requires simplifying assumptions to be made. The modules should therefore be viewed in the context of the model as a whole and not as specific treatments of the sub-model units. For example, the treatment of leaf photosynthesis does not incorporate a mechanistic description of the biochemical processes, but uses a semi-empirical equation for the light response of photosynthesis. This is then adapted to include temperature, CO_2, water and nutrient effects. Clearly, more detailed approaches can be, and have been, adopted, but they would introduce too great a level of complexity (see, for example, Thornley and Johnson, 2000).

The modules have also been developed to be internally consistent. While it is often tempting to take sub-model units that have been built by different modelling groups and link them together, this approach has not been adopted here. By constructing each module with a view to how it fits together in the whole model, it has been possible to gain insight into the interactions between the different components of the system which, in turn, helps gain a better understanding of the structure and function of the whole system.

1.3 Physiological basis of the model

Most processes that are incorporated in the model can be described in terms of a series of fluxes, whereas the experimental data that have been collected are generally measurements of the system states. To explain what this means, consider the soil water balance. On any particular day, it is possible to measure the soil water content at a series of depths within the soil profile: the soil water content is a state variable. This state is a result of various fluxes: rainfall, evaporation (from the canopy surface, litter, and soil), transpiration, runoff, and through-drainage. Each of these fluxes has dimensions of kg water m^{-2} day^{-1}, which is equivalent to mm d^{-1}, since 1 kg water m^{-2} is equivalent to 1 mm water dept . The water balance is illustrated in Figure 1.1 (this diagram is repeated in the *Water Dynamics* chapter):

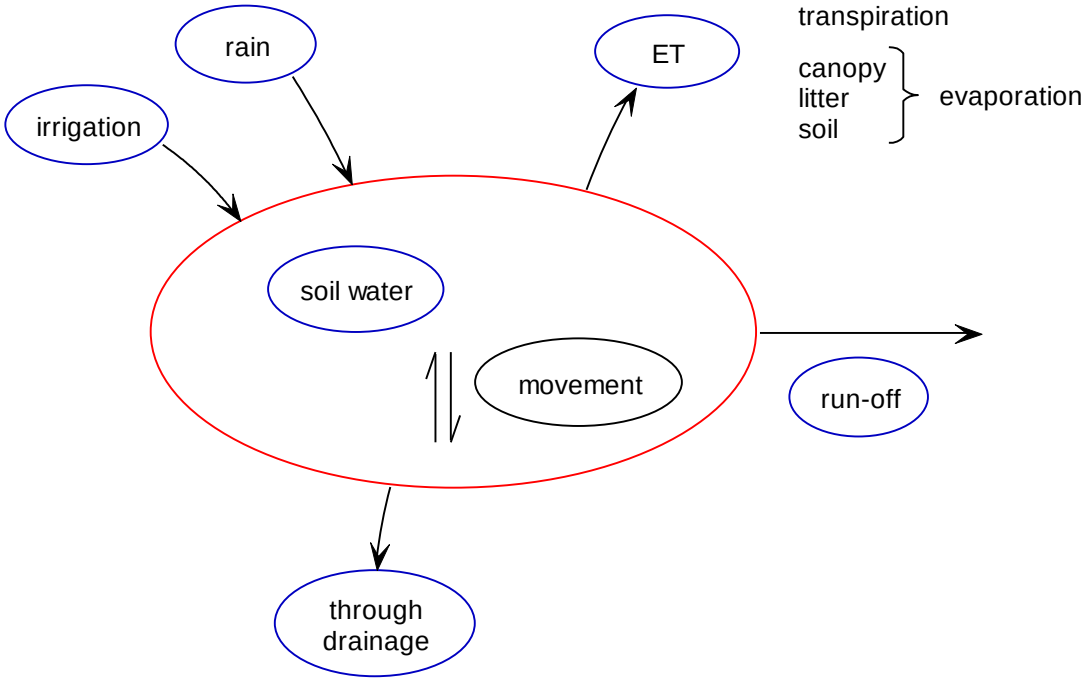
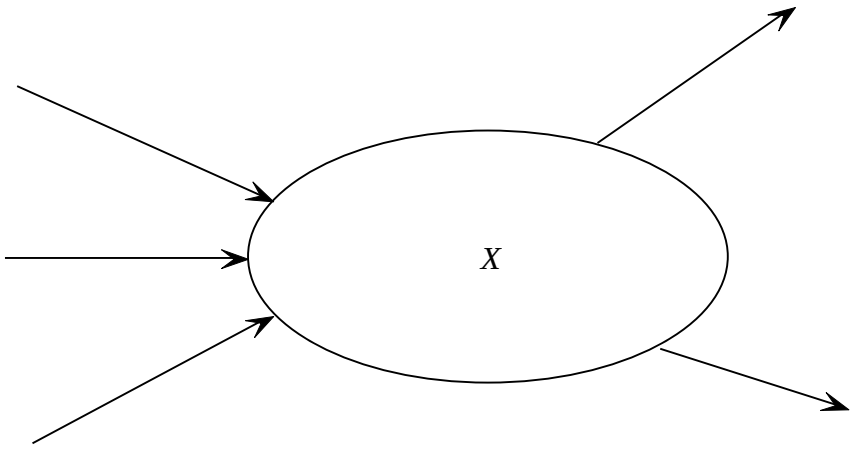


Figure 1.1: Schematic representation of the soil water balance.

For such a rate/state system, the underlying scheme is shown in Fig. 1.2, where X is the state variable and the fluxes have dimensions of X area^{-1} time^{-1}. The corresponding mathematical description is:

$$\frac{dX}{dt} = \sum \text{flux in} - \sum \text{flux out} \qquad (1.1)$$

This equation is then solved, with appropriate initial conditions, to give the state variable X.



Flux in: $(X / area) / time$ Flux out: $(X / area) / time$

Figure 1.2: Schematic representation of rate/state processes

The reason for emphasising this point is to demonstrate:

- The same solution for X may be obtained for a range of different fluxes. For example, high rates of rainfall and through drainage may give similar values for X as lower rainfall and

through drainage. Similarly, varying rates of pasture production and rates of senescence may give similar results.

- The solution for X may be compared favourably with experimental data, but be calculated from erroneous fluxes.

In other words, the solution X is not unique for all combinations of fluxes. This poses an interesting question: how do we know the model is 'right' or 'valid'. And this leads nicely to the concept of validation.

1.4 Model testing and evaluation

Testing and evaluating models is a vital step in using models for teaching, research or extension work, and for a more detailed discussion on this subject, see Thornley and Johnson (2000), Thornley and France (2008).

The traditional approach in agricultural simulation modelling for 'validation' of mechanistic models is as follows:

- Develop the model.
- Measure the system parameters (eg saturated hydraulic conductivity).
- Solve the model.
- Compare the model output with measurements (eg soil water content).
- If they agree, the model has been 'validated'; otherwise there is a problem with the model.

Similar examples could be presented for pasture production, nutrient dynamics or animal production. This approach has an immediate appeal, since agreement between the model and the data is a natural objective. However, there are associated problems. First, there is the obvious possibility that the 'right' results may be obtained for the 'wrong' reasons, as illustrated with the soil water content earlier, where errors in fluxes into the soil may be balanced by errors in fluxes out of the soil. Secondly, the inputs to the model may be in error. Finally, there may be errors in the experimental data.

The issue regarding the accuracy of model inputs is crucial in evaluating the model. Continuing with soil water as an example, it is ironic that the system state, that is the soil water content, is arguably the most accessible component of the system: certainly it is much easier to measure than either saturated or unsaturated hydraulic conductivity, provided the measurement technique has been accurately calibrated. This means that the state variable we are attempting to predict can be measured with more accuracy than the necessary parameter inputs to the model that are required to make that prediction.

Following this line of reasoning, the distinction between model inputs and model structure begins to become blurred. For example, movement of water through the profile, as described by the model, depends crucially on how the soil water characteristics of the soil are defined. So, when observations of the soil water content and data are compared, are we testing our ability to characterise the soil, or to define how much water gets into the soil (as opposed to being intercepted by the canopy or litter and evaporated directly back to the atmosphere), or the description of other processes such as runoff or water redistribution in the profile.

To summarise, testing and evaluating this, or any, model, involves examining the model structure, the parameters used in the model, and the climatic inputs that drive the processes. Time should also be spent exploring the general behaviour of the model in terms of our understanding of the system.

For example, if there is a period during the winter when it is commonly observed that there is a perched water table, we can look and see if the model displays this behaviour. If it doesn't, why doesn't it? Is this because the soil characteristics are in error, the rainfall is in error, there is too much runoff, or is there a fundamental structural problem with the model? Examining scenarios such as this will provide insight and understanding into the overall behaviour of the system and the interaction between the different components.

1.5 Use of the Model

The model can be used for general systems analysis, for example:

- Analysing experimental data;
- Identifying un-measured quantities;
- Asking *what-if* questions;
- Asking *why-if* questions.

Experimental data, particularly in the agricultural sciences, are prone to considerable variation and are often very difficult to measure. The model provides a useful structure for assessing the integrity of the data. For example, if there is disagreement between model predictions and actual measurement of soil water content is this due to model behaviour, errors in climatic data, or model parameterisation? Because the model accounts for all water in the system, it provides a useful structure for analysing the different fluxes that are involved. When there is a discrepancy between the model and the data, the process of identifying why this is often leads to an improved understanding of the behaviour of the system. Of course, it may mean that the model has to be adapted or modified due to limitations in its structure.

Using models to ask *what-if* questions is a powerful model application. For example, what would the impact on runoff and drainage be if irrigation frequency were halved but the amount applied were doubled. However, before asking *what-if* questions, it is important to have confidence in the model performance. This involves having a sound understanding of the behaviour of the system and of the observed experimental data. In other words, we need to relate our perceptions of how the system works (theory) and actual observations.

When asking *what-if* questions, the focus is generally on the answer. Using the example above, if we look at two different irrigation strategies and see variation in through drainage, runoff and crop yield, then these are the answers to our *what-if* question. However, in order to gain an understanding of the underlying behaviour of the biophysical system we can also ask *why-if* questions. Following this example, once we see what the consequences of the different irrigation strategies are, we can then try to understand why this is so from the behaviour of the different components in the model. To do so, we might start to change the soil characteristics – say from a fairly impervious B horizon to a freely draining one, and see if the general response changes. By following such lines of investigation, we increase our understanding of the behaviour of the system and how the various components interact. Each time we answer a *why-if* question we improve the accuracy of the answers to our *what-if* questions.

I would encourage all people working with the model for data analysis to regard it as a tool that allows them to define the complex fluxes that define the behaviour of the biophysical pasture system. If the model gives a good agreement with observed data, treat this with caution, and interrogate the simulation to see what the underlying implications are – is there too much drainage and not enough runoff, and so on. Conversely, if the model does not agree with the data, try to track

down why this is so – is too much water being retained in the profile, or is evaporation from the canopy surface being underestimated. By asking these questions, we can likely gain insight and understanding into the behaviour of the system and can then start to understand the implications of different management strategies or environments on the water dynamics. As we gain confidence in our ability to describe the behaviour of the system, so we can then start to ask detailed questions relating the behaviour of the system.

Model applications

There are many projects now working with the model and some of the specific questions being asked are:

- Drought analysis;
- Analysing pasture growth rate variability;
- Nitrate leaching;
- Phosphorus dynamics;
- Greenhouse gas emissions (CO_2, N_2O, CH_4);
- Climate change scenarios;
- Grazing management;
- Irrigation strategies;
- Pasture water use efficiency.

1.6 Long term simulations

After working with this model structure in a variety of projects for a number of years, I have come to the conclusion that long-term simulations are an essential part of model use and testing. They do, of course, require good quality long-term climate data, which are not always easy to obtain.

Some of the benefits of long-term simulations are:

- It is possible to let the model run for sufficient time that the choice of initial conditions has reduced effect on the simulation results. For the initial pasture dry weight, this may not be long at all, whereas for soil nutrient status it may take several years for the system to settle down.
- It allows analysis of between-season and between-year variation. It is clear from even a cursory analysis of climate data that there is rarely such a thing as a 'typical' year, which translates to considerable variation in growth potential for most regions..
- Whole farm system experiments, by their very nature, are time-consuming and expensive. By combining the findings of these with simulations, there is considerable potential to identify effective and sustainable management strategies, along with their environmental impact, under a wide variety of site and climatic conditions.
- Some aspects of the system behaviour may be subject to long-term variation that is difficult to identify through experimental programmes alone. This might include deep drainage or nitrate leaching which may be negligible in some years and yet substantial in others. As a consequence, attempts to quantify leaching over the long-term through experimental programmes alone may not be practical.

Apart from these specific examples, long-term simulations provide the opportunity to assess the general behaviour of the model and, in doing so, the underlying complex interactions between the different components of the system.

1.7 Mathematical documentation

The remainder of this document focuses on the mathematical description of the core model structure. Each component of the model has been developed at a level of complexity that is consistent with the other modules and overall model objectives. Inevitably, assumptions and approximations need to be made, and these may differ from approaches used by others. However, the overall model structure is internally consistent and provides insight into the behaviour of this complex agricultural system.

1.8 References

Thornley JHM and Johnson IR (2000). *Plant and Crop Modelling.* Blackburn Press, Caldwell, NJ (www.blackburnpress.com).

Thornley JHM and France J (2008). *Mathematical Models in Agriculture.* CABI, Oxford.

2 Management

2.1 Introduction

The description and implementation of management options in complex multi-paddock simulation models is challenging. In practice, farmers make frequent short-term tactical decisions in response to a range of information and are flexible and intuitive in doing so. And rightly so. However, our challenge has been to articulate these rules in a way that can be implemented in the model, and this has been an interesting process. The management routines should be viewed as a general treatment of some of the more complex strategies that are adopted, particularly in the dairy industry. This is an area where continued development is required.

A variety of management options are available and these need to be explored to get a feel for what they do. The number of paddocks and the size of the grazing area (farm) are defined in this module: for example, with 50 paddocks and 100 ha then each paddock is 2 ha. The actual stock density is then calculated from the number of animals as defined in the animal modules (sheep, cattle, dairy cows or deer). The rotational grazing management routines also have cutting options that are intended to simulate cutting of conserved feed.

In the management module, it is possible to select between a single or multiple paddock simulation. For the single paddock the options are continuous grazing or a cutting trial. For continuous grazing it is also possible to vary the stock number (see below). For multiple paddocks there are several options, not all of which are available in each of the SGS Pasture Model, DairyMod and EcoMod, since some of the strategies are specific to particular industries. The management strategies are considered in turn.

As well as the single and multiple paddock management options, the *single heterogeneous paddock* can also be implemented. In this case, the multiple paddocks are used to simulate a single paddock with differential dung and urine returns that can represent urine and dung patch simulations.

2.2 Single paddock

Four management options apply to a single paddock simulation:

- continuous grazing, which can include variable stock numbers;
- rotational grazing;
- daily stock numbers are read from file;
- and a cutting trial, which simulates a range of options for cutting strategy and nutrient management.

These are considered in turn.

2.2.1 Continuous grazing

Continuous grazing is the simplest grazing management option, whereby the stock remain on the paddock. There are two additional options for continuous grazing, which allow for variable stock numbers and also the addition of an exclusion cage.

Adjust stock numbers

With this option selected, stock numbers are varied at the frequency of days or months specified. Animal numbers are varied to attempt to ration the available feed above the target residual for the specified period. No allowance is made for growth, senescence, or trampling by the animals. Nevertheless, the strategy can be seen to work quite well and generally displays improved grazing management over simple set-stocked. Note that a maximum stock density can be specified.

Cage simulation

A further option is to include a *cage simulation* under continuous grazing. With this option, the model replicates the pasture (that is plant and soil status), cuts to a predefined residual, and then allows the pasture to grow under the same conditions as the grazed sward but with no animals. After a specified duration, the sward is then cut to the residual and this is stored as the yield over that growth period. The duration of the regrowth period and the dry weight (d.wt) to which the pasture is cut is defined in the *cut trial* option which is described below. Note that if the pasture is grazed below the cutting residual and, during the regrowth period, does not reach the cutting residual, then the recorded cut d.wt in the cage will be zero even though there may have been some growth during that time.

2.2.2 Rotational grazing

This option simulates a single paddock in a rotation. The animals come on to the paddock at a specified minimum target weight to graze and are taken off when it reaches the residual. Note that the minimum weight to graze and target residual can be specified as fixed throughout the year or variable. For the variable option, monthly values are specified and applied on the 15th of the month and the model extrapolates linearly between months for other dates.

2.2.3 Read from file

With the *read from file* option, the stock density, animals ha^{-1}, is read each day from the data input file. The following rules must be observed:

- The worksheet must be called 'Management';
- the dates must be in column 'A';
- and the data (animals ha^{-1}) must be in the column specified on the interface.

Note that it is also possible to include a cage simulation with this option.

2.2.4 Cut trial

The *cut trial* option allows the simulation of a standard cutting experiment, where the sward is cut at prescribed intervals and to a prescribed residual. The regrowth interval can either be a fixed number of days, use the leaf stage, be read from file or be cut on the last day of each month. The yields for each cut are accessible in the simulation summary information.

For each cutting management strategy the nutrient management is also specified. The options are:

- remove;
- return as litter;
- or return as dung and urine.

If all cuttings are removed this can represent a large nutrient export from the paddock. For example, if 12 t ha^{-1} are harvested in the year with a 4% nitrogen concentration, this corresponds to 480 kg N. Given that there may also be losses due to volatilization, denitrification and fixation, this means that at least 500 kg N may have to be applied just to offset nutrient removal. By returning the nutrients either as litter or dung and urine, this allows some treatment of nutrient cycling in the simulation. If nutrients are returned as dung and urine then the rules for partitioning between dung and urine are those specified in the animal module.

Fixed regrowth

The fixed regrowth is very straightforward and allows the sward to be cut after a specified number of days to a specified residual.

Leaf stage

With this option, cutting is done at a prescribed leaf stage and, since leaf stage depends on temperature, this will result in longer regrowth periods in colder conditions. This is analogous to the leaf stage management option for multi-paddock grazing simulations as discussed below.

Read from file

This is analogous to reading the stock numbers from file for continuous grazing where:

- the worksheet must be called 'Management';
- the dates must be in column 'A';
- and the cutting days must have a 'Y' in the column specified on the interface – it doesn't matter what is in the other cells.

Monthly

This option is similar to the fixed regrowth, but now the cutting date is always the last day of the month. This is a very convenient way of analysing monthly growth rate patterns with long-term simulations. The cutting residual can be fixed or monthly values specified. As for the rotational grazing option, these apply on the 15th of the month and linear interpolation is used for other dates.

2.3 Multiple paddocks

There are 9 management options for multiple paddock simulations but, as mentioned earlier, they are not all accessible from each industry specific model. The managements and models to which they apply are shown in Table 1. Each management also has cutting options and these are described later.

It must be emphasised here that the notion of a paddock may differ from the physical paddock with a fence around it. This is because, in practice, paddocks are often sub-divided with electric fences to reduce the area being grazed. By having a large number of notional paddocks in the model there is more flexibility in defining the grazing area.

Before looking at the individual managements, the concepts of 'chase the feed' and 'break feeding' are first described as these can be applied to several of the strategies.

Table 1: Rotational grazing management options and models to which they apply.

Management	SGS	DairyMod	EcoMod
Fixed time	✓	✓	✓
Variable time	✓	✓	✓
Feed budget	✓	✓	✓
Fixed sequence			✓
Manage by weight			✓
Animal weight	✓		
Leaf stage		✓	
Target 10		✓	
Random			✓

2.3.1 Break feeding

Break feeding only comes into effect if a paddock is to be grazed for more than one day and the aim is to restrict the animals to an even intake during the time they are on the paddock to avoid large fluctuations in intake. In practice, this will be implemented with moveable electric fences.

To implement breaks, the model calculates the amount of pasture above the desired residual and restricts the daily available pasture to this amount divided by the number of days the animals are to be on the paddock. For example, if the residual is 1 t ha^{-1}, the total d.wt is 4 t ha^{-1}, and the animals are scheduled to be on the paddock for 3 days, then the maximum they can graze on each day from this paddock is 1 t ha^{-1}.

Note that implementing breaks may give unrealistic intake levels for long rotations on a small number of paddocks. For example, if there is a 20 day 2 paddock rotation, then significant growth may occur while the animals are on the paddock and so rationing feed at the start of the grazing period may significantly underestimate the pasture available to the stock.

2.3.2 Chase-the-feed

If no paddocks are eligible for grazing then there are two options:

- either the animals are just allocated to the most abundant paddock so that they chase the feed around the farm;
- or they remain on paddock 1, with supplementary feed.

Either of these options may be applied in practice and so the model allows us to explore the impact of the strategy adopted. It should be noted that if the animals are always returned to paddock 1, and if they remain there for extended periods of time with their main feed supply from supplementary forage or concentrate, then that paddock will have a substantial influx of nutrients through the returns in dung and urine.

2.3.3 Management phases

Management strategies are generally changed throughout the year and so in the model it is possible to define up to 4 separate management phases. This allows, for example, to switch between a feed budget rotation and fixed time system. The implementation of these strategies is self-evident.

2.3.4 Fixed time rotation

This is the simplest rotational grazing option. The actual target rotation length is specified and the model then attempts to find the closest rotation length possible with the specified number of paddocks. The method for calculating either the rotation length or days on a paddock is:

$$D_p = \text{round}\left(\frac{R_T}{N}\right), \quad P_D = 1; \quad N \le R_t \tag{2.1a}$$

and

$$P_D = \text{round}\left(\frac{N}{R_T}\right), \quad D_p = 1; \quad N > R_t \tag{2.1b}$$

where D_p is the number of days on the paddock, P_D is the number of paddocks grazed per day, R_T (days) is the target rotation length, N is the total number of paddocks, and "round" is a simple numerical function that rounds a decimal number to the nearest whole number. For example, with 36 paddocks and a 30 day rotation selected, the nearest rotation length that works is 36 to 37 days (which means the rotation length will not be the same for all paddocks). Similarly, with 8 paddocks and a 30 day rotation selected, the actual rotation length is 32 or 33 days, but now the animals spend 4 days on each paddock. It is interesting to explore the behaviour of the model for different numbers of paddocks and rotation lengths.

With the fixed time rotation option, the model calculates how many paddocks are to be grazed and for how many days. It then selects the paddocks with most feed on offer to graze. As a consequence, the animals may not move round the paddock in strict sequence.

During a model run, the paddocks being grazed are coloured purple (this colour can be changed). It is interesting to see the grazed paddocks jumping around the farm. Similarly, the paddocks being cut are shown in yellow (again, this colour can be altered).

2.3.5 Variable time rotation

The variable time rotation management option is similar to the fixed time option, but now the target rotation length, R_T, is specified for each month. The model then uses this value for the 15th of each month and interpolates between these values for any day of the year. Once the target rotation length is known, the actual days per paddock and paddocks per day are calculated using the method described here.

While the rotation length should ideally be an emergent property of the model, the present method has the advantage of being easy to implement and is a considerable improvement over having one or two fixed rotation lengths throughout the year.

2.3.6 Feed budget rotation

This strategy is quite similar to the time based rotation, but now the area grazed and time spent on the paddock is calculated in response to animal requirements. The minimum d.wt at which a paddock can be grazed is defined along with the target residual. For example, if these are 2 and 1 t ha^{-1} respectively then the paddock is eligible for grazing once its d.wt exceeds 2 t ha^{-1} and the number of animals or time spent on the paddock are calculated according to the available d.wt in excess of the target residual. Again this may either involve having animals on more than one paddock per day or on a single paddock for several days. The d.wts in this option can be fixed or

monthly values specified, in which case linear interpolation is used between the values. As for the time based rotation, if more paddocks are eligible for grazing than are required, those with the greatest available d.wt (that have not been allocated for cutting) are grazed first.

If no paddocks are eligible for grazing the options are either to stay where they are, chase the feed, or use a specified holding paddock.

2.3.7 Fixed sequence

According to this option the animals move daily around the paddocks in sequence. There are no cutting options or any other elaborations for this system.

2.3.8 Manage by weight

Again, this is a very simple option and the animals graze the paddocks in sequence with no cutting options. However, the animals stay on the paddock until it has been grazed to a specified residual.

2.3.9 Animal weight

In this case the animals stay on the paddock until their weight falls below a specified value. They then move to the most abundant paddock provided the d.wt exceeds the specified minimum value.

2.3.10 Leaf stage

The leaf stage management option is generalisation of the 3-leaf approach originally developed by Bill Fulkerson and colleagues (Fulkerson and Slack, 1994; Fulkerson and Donaghy, 2001). This initial work was for perennial ryegrass, but the method can be applied generally for other species. It has been well established that there are about 3 live leaves per tiller for perennial ryegrass. In temperate summer conditions the leaf appearance interval is of the order of one leaf every 10 days, although this does depend strongly on temperature and water status. This means that there is a constant flux of material through live leaf categories and then to senescence. The appropriate time to graze a paddock is after 3 leaf appearance intervals since the last grazing, since this ensures the paddock is grazed prior to any significant senescence. Furthermore, grazing too soon after the previous grazing can result in a general depletion of plant reserves (soluble sugars). In summary, to graze too infrequently can result in excessive amounts of pasture being lost to senescence, whereas grazing too frequently can result in depletion of plant reserves which may reduce the growth rate during regrowth.

The appeal to this strategy is that it is based on the physiology of the grass plant. However, we recognise that considerable latitude is applied when using this management regime. It is therefore necessary to have a range of criteria that are applied to determine whether a paddock is eligible for grazing.

The model tracks the daily rate of leaf appearance, with units leaves day^{-1}. This is summed each day following release from grazing, and when the value reaches unity that corresponds to one leaf appearance interval. In general,

$$\text{leaf appearance interval} = \sum 1/\text{leaf appearance rate} \qquad (2.2)$$

The leaf appearance interval is then calculated for all paddocks.

The paddock is eligible for grazing if either:

- The leaf appearance interval is greater than a user-defined value (eg 3 leaves per day)

or

- The leaf appearance interval is greater than a user-defined value *and* the pasture d.wt is greater than a user-defined value (eg 2 and 2.5 t ha^{-1}). This second criterion is designed to apply during times of high growth rate.

The paddock may also be cut, and this is managed according to the principles described below.

Paddocks are ranked according to the available metabolisable energy (ME), which is calculated from pasture digestibility (see the pasture section). Paddocks eligible for grazing, but not allocated for cutting, are allocated for grazing. The number of paddocks grazed in a day or the number of days that a paddock is to be grazed are calculated from animal demand. According to this strategy, the animals will not be offered more than they require which ensures that the paddocks are not under grazed.

To illustrate this, consider the following example. Suppose there are 100 cows, each with an ME requirement of 180 MJ day^{-1}, so that the requirement of the herd is 18,000 MJ. Suppose the first 4 paddocks ranked in ME order have available ME values of 8,000, 6,000, 3,000, 2,500. The sum of the first three paddocks is 17,000 and with the next one it is 19,500. According to the management scheme, only the first 3 paddocks will be grazed since pasture in excess of ME requirement by the herd is not implemented. Thus, the stock are offered 17,000 MJ from pasture, and will have to get the extra 1,000 MJ from supplement. The exception to the rule of not offering ME that exceeds demand is when a single paddock exceeds demand.

Finally, with break feeding implemented, as described earlier, it is possible to ration the feed on a single paddock over more than one day.

2.3.11 Target 10 (T10)

The T10 management strategy has been developed by the Department of Natural Resources and the Environment, Victoria, Australia, based on Holmes and Wilson (1984). The general principal of this scheme is to determine the rotation length so that the stock return to graze each paddock when there is an appropriate level of d.wt available.

This system relies on specified values for the expected growth rate for each month. This is assumed to apply on the 15th of the month and linear interpolation is applied for other dates.

To illustrate the second of these options, suppose the September growth rate is 50 kg ha^{-1} day^{-1} and for October it is 80 kg ha^{-1} day^{-1}. The expected growth rate for 30 September is then taken to be:

$$50 + (80 - 50) \times (30 \text{ Sept} - 15 \text{ Sept}) / (15 \text{ Oct} - 15 \text{ Sept}) = 65 \text{ kg } ha^{-1} \text{ } day^{-1}$$

The other key pieces of information to be prescribed are:

- the grazing wedge available to the animals, which is the amount of pasture removed at each grazing, based on a pre-grazing target mass and a target residual;
- the target residual after grazing.

The target residual plus the wedge gives the pre-grazing target mass. So, for example, if the wedge is 1000 kg DM ha^{-1} and the target residual is 1400 kg DM ha^{-1}, then this corresponds to a pre-grazing mass of 2400 kg DM ha^{-1}. Animals will be allocated to a proportion of the grazing area once this pre-grazing mass has been reached.

Once the expected growth rate, the wedge and the target residual have been prescribed, the appropriate rotation length and the optimum area to be allocated to grazing are calculated using the simple calculations:

Rotation length = wedge / growth rate (2.3a)

Thus, for example, if the wedge = 1000 kg DM ha^{-1}, and growth rate = 50 kg DM ha^{-1} day^{-1}, then rotation length = 20 days. This is effectively the time required to regrow the wedge after grazing, at current growth rates.

Fraction of the farm to be grazed = 1 / rotation length (2.3b)

Thus for example, if rotation length = 20 days, the fraction of the farm to be grazed = 1 / 20 = 0.05 (or 5%) per day.

Substituting the terms from eqn (2.3a) in eqn (2.3b) yields eqn (2.3c), as used by the model.

Fraction of the farm to be grazed = expected growth rate / desired wedge. (2.3c)

For example, if the expected growth rate is 80 kg ha^{-1} day^{-1} and the desired wedge is 1 t ha^{-1} (which is equivalent to 1000 kg ha^{-1}), then the fraction of the farm that should be grazed is

$80 / 1000 = 0.08 \equiv 8\%$ (2.3d)

Thus, if the simulation has 100 paddocks, then 8 of them should be grazed.

One final caveat is applied. To be eligible for grazing, the available d.wt on a paddock must exceed the target residual for grazing. Those that are eligible are then ranked in order of available pasture d.wt. For example, if 12 paddocks were eligible to be grazed in the previous example then the 8 with the highest available d.wt are those that can be grazed.

Once the grazeable area is known, the actual animal requirement is calculated. If the eligible area has insufficient ME available then it will be all grazed and animal requirement will either be less than required or will need to be met by supplementary feeding. Conversely, if there is more pasture available than the animals require, then the area to be grazed is scaled back accordingly.

The rationale behind this approach is that the cycle between grazings is calculated as that time that will allow the paddock to regrow to the desired wedge. Although this is continually varying as the growth rate varies it does, nevertheless, link the regrowth interval to the expected growth rate.

Again, paddocks can also be cut, as described above. However, unlike the leaf stage approach, once the paddock is taken out of the grazing cycle for cutting it is not brought back into the calculations for grazing until after it has been cut.

2.3.12 Random

This option is self-explanatory. The number of paddocks to be grazed per day are specified by the user and these are selected randomly.

2.3.13 Cutting strategy under rotational grazing

Several of the rotational grazing systems also have the option of implementing a cutting strategy. A paddock is eligible for cutting if the d.wt exceeds a user defined value (eg 3 t ha^{-1}). It is then taken out of the grazing cycle for a further specified period (eg 21 days) after which it is cut. During this time, the paddock will only be grazed if the remaining paddocks cannot satisfy grazing demand, as defined by the specific management strategies. Note that cutting can either be applied to a range of

paddocks or to all paddocks. In addition, a date range when cutting is implemented can also be specified. When there is insufficient feed on eligible paddocks to satisfy requirements, those paddocks that have been most recently taken out of the grazing cycle are brought back in.

Finally, *forced cuts* , whereby all paddocks in a specified range can be cut regardless of their state, can be implemented. Two forced cuts per year can be implemented, which are intended to represent spraying, or slashing paddocks when a shift between species is likely to occur, such as from kikuyu to annual ryegrass.

2.4 Single heterogeneous paddock

The single heterogeneous paddock (SHP) option uses all of the 'paddocks' to represent 'cells' in a single paddock with different patterns of nutrient return. It is assumed in the SHP that there is no overlap of urine patches on any one day. This option allows the detailed exploration of the impact of urine patches on

2.4.1 Grazing options

The grazing options are:

- fixed rotation length;
- target weight;
- and read from file.

Fixed rotation length

In this case the paddock is grazed at the specified daily interval. Stock numbers are multiplied by the rotation length to represent the stock density on a paddock in a rotation (see example below in section 2.4.3). They remain on the paddock for one day.

Target weight

With a target weight specified, the animals come onto the paddock when the average weight over all the cells reaches the specified target. Again they remain on the paddock for one day.

Read from file

The animal numbers can be specified in the data file.

2.4.2 Nutrient returns options

Nutrient return options are:

- standard;
- fixed sequence;
- random;
- and uniform (all paddocks).

In addition, the dung can either be distributed with the urine or returned from the same cell where it was grazed.

Standard

In this case, dung and urine are returned in the same cell that the pasture is grazed.

Fixed sequence

With the fixed sequence, the nutrients return sequentially around the cells. The number of cells receiving nutrients per day is specified, along with the starting cell for nutrient returns.

Random

Cells are selected at random and, again, the number of cells receiving nutrients per day is specified.

Uniform (all paddocks)

For this final option, nutrient returns are distributed evenly over all cells.

2.4.3 Setting up the SHP

Consider an example with a fixed rotation length of 25 days. If the farm is 25 ha this means that each paddock in the 25 day rotation is 1 ha. Now, suppose the overall stock density is 4 animals ha^{-1}, so that the stock density when the animals are on the paddock is 25x4=100 animals ha^{-1}. If each urine patch is 0.5m^2 and there are 10 urine events per animal per day, then urine is returned to a total area of 500m^2, or 1/20 of the paddock (remembering that there is no urine patch overlap on any grazing day). Therefore, setting this system up in the SHP requires:

- Farm area (which now represents one paddock) = 1;
- No of paddocks (which now represent cells) = 20;
- No of animals = 100

Setting up the other grazing options is done in a similar way. The nutrient return options can then be specified as required.

2.5 Concluding remarks

The grazing management options implemented in the model simulate a range of strategies used in practice. These include flexible options for single- or multiple-paddock simulations with provision for cutting herbage for conservation. Coupled with supplementary feeding strategies (discussed in the *Animal* chapter), these rules provide considerable flexibility within the model for exploring the impact of grazing management on pasture productivity.

2.6 References

Fulkerson WJ and Slack K (1994). Leaf number as a criterion for determining defoliation time for Lolium perenne. 1. Effect of water soluble carbohydrates and senescence. *Grass and Forage Science* **49,** 373–377.

Fulkerson WJ and Donaghy DJ (2001). Plant soluble carbohydrate reserves and senescence – key criteria for developing an effective grazing management system for ryegrass-based pastures: a review. *Australian Journal of Experimental Agriculture* **41,** 261–275.

Holmes CW, Wilson GF (1984) *Milk production from pasture.* (Butterworths Agricultural Books, Wellington)

3 Climate

3.1 Introduction

The biophysical processes in the model are driven by climate, and in this section the climatic factors used in the model are considered. There is also a detailed discussion of the treatment of evapotranspiration in relation to climate. Since evapotranspiration includes evaporation from the soil, plant surface and litter, as well as transpiration from the plants, the description of ground cover by the plant canopy and litter is also discussed.

3.2 Precipitation and irrigation inputs

Rainfall inputs are applied from above the canopy so that they are first intercepted by the canopy and then the litter and ground, as illustrated in Fig. 3.1:

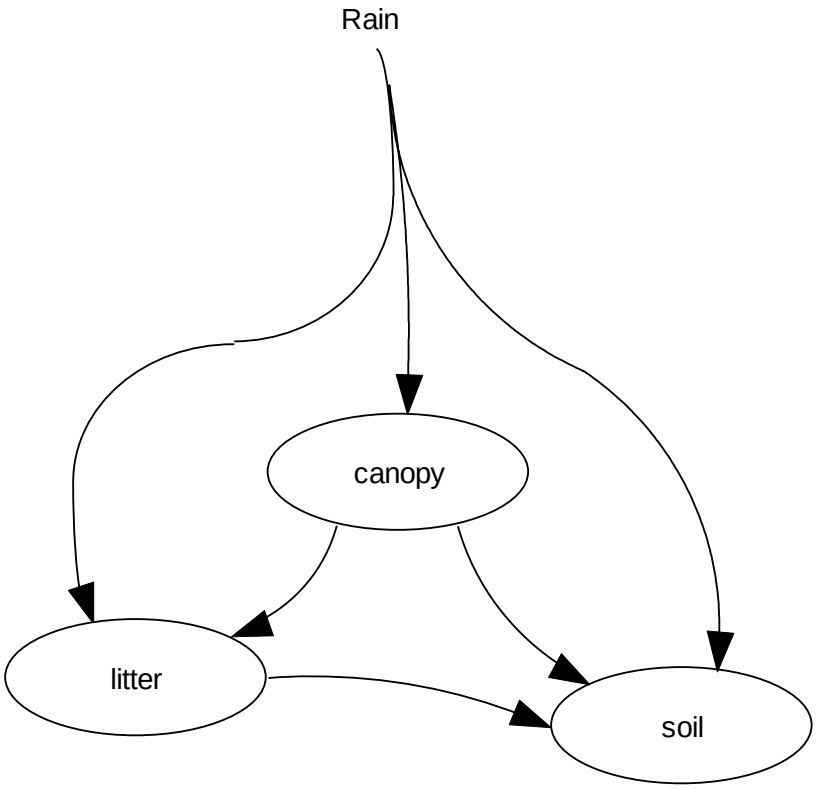


Figure 3.1: Schematic representation of the rainfall inputs

Irrigation inputs can be applied either to the canopy in the same way as rainfall, or directly to the soil and litter, depending on the irrigation application method that is selected in the model.

It is at this point that the choice of symbols becomes difficult! It would be good to use W for water and R for rainfall, but these will both be needed soon for weight and radiation. So, the compromise is to use P for rainfall and irrigation (from precipitation) and H for water stored on the canopy, litter or soil surfaces (from H_2O). The following symbols are defined:

Δt	Time step	Appropriate time unit
P	Rate of supply of rainfall and irrigation	mm water Δt^{-1}
H	Water, with various subscripts	mm water
mx, mn	maximum and minimum	

3.2.1 Rainfall inputs

Rainfall inputs are prescribed daily. Rainfall intensity is important for calculations of runoff and evapotranspiration. For example, 30 mm rainfall that falls at around noon on an otherwise hot, sunny and windy day, will be subject to considerable evaporative demand in the afternoon, as well as possibly being subject to runoff. However, if this rain falls between 8.00 pm and midnight, then the subsequent evapotranspiration and runoff may be quite different.

Daily rainfall inputs are distributed across the day in a fairly simple way. First, the user specifies the number of hours that it rains, and this number applies to all rainy days. Second, the model randomly chooses a starting time and then distributes that rainfall over those hours. For example, if it is specified that when it rains, it does so for four hours, and on a particular day the rainfall is 40 mm, then, supposing the model randomly selects the starting time for rain to be 2.00 pm, 10 mm per hour will be applied for four hours from 2.00 pm.

3.2.2 Irrigation options

Several irrigation strategies are available that should be largely self-explanatory. The options are:

- Irrigate in response to plant water status;
- Irrigate in response to soil water deficit;
- Irrigate in response to rainfall deficit;
- Irrigate at regular intervals.

There are two general options that apply to all irrigation strategies:

- Water may be applied either to the canopy or directly to the soil surface – if irrigation water is intercepted by the canopy it is then subject to direct evaporation;
- The time of irrigation application can be specified, which allows the influence of diurnal evaporative demand to be incorporated;
- A date range can be specified for irrigation;
- The minimum number of days between irrigations can be specified.

Irrigate in response to plant water status

For this option, irrigation responds to the growth limiting factor for water, GLF_{water}, that is described in *Growth* (chapter 4). In brief, if this variable has the value one then there is no water stress, and if it is zero then there is full water stress and the plants cannot grow. In practice, it is likely to lie somewhere between these two extremes. (Note that GLF_{water} is expressed as a percentage on the interface.) If GLF_{water} falls below a user specified value then irrigation water is applied.

The amount of water that is applied can either be a fixed amount, or in response to the soil water status. For the latter case, the difference between the actual soil water content over the root depth is calculated and then the irrigation water is calculated as being the amount of water required to bring this up to a user-specified percentage of field capacity – field capacity is also discussed in the

Water Balance chapter. Note that this can be greater than 100% since the soil water content can exceed field capacity.

Irrigate in response to soil water deficit

This strategy is similar to the previous one but the trigger for irrigation is soil water deficit (mm). (Soil water deficit is also discussed in the *Water Balance* chapter.) Note that the soil water deficit must be defined to a depth defined by the user. Thus, when the soil water deficit falls below a specified value, the model applies the amount of irrigation water required to bring this up to a user-defined level. With this strategy, the user also specifies a date range for application. As for the previous option, the water applied can either be a fixed amount or a percentage of field capacity.

Irrigate in response to rainfall deficit

For this strategy, the cumulative deficit between PET and rainfall is calculated, that is,

$$\sum\left(E_{pot} - P\right) \tag{3.1}$$

where E_{pot} is the daily potential evapotranspiration rate, P, as defined above, is the daily precipitation, and the summation is calculated daily since the last irrigation. If the sum ever goes negative, so that rainfall exceeds potential evapotranspiration, then the sum is set to zero. The calculation of potential evapotranspiration is discussed in detail in the *Water Balance* chapter.

Once this sum exceeds a user-defined value irrigation is applied and, as for the previous option, the water applied can either be a fixed amount or a percentage of field capacity.

Irrigate at regular intervals

This is the simplest form of irrigation: a fixed amount of irrigation water is applied at regular intervals. As for the previous strategy, the user also specifies a date range for application.

3.2.3 Water interception

Canopy water holding capacity

The canopy can hold water, as defined by

$$H_{canopy,mx} = H_{L,mx}\, L \tag{3.2a}$$

where $H_{L,mx}$ is the water holding capacity per unit leaf area: typically $H_{L,mx}$ lies in the range 0.2 to 0.5 mm water (m^2 leaf)$^{-1}$.

Litter water holding capacity

The litter can hold water, as defined by

$$H_{litter,mx} = H_{litter,WL}\, W_{litter} \tag{3.2b}$$

where $H_{litter,WL}$ is the water holding capacity per unit mass of litter, with default 2 mm t^{-1} in the model.

Precipitation interception and storage

Equations (3.2a, b) define the maximum storage capacity of the canopy and litter, which are used to calculate the partitioning of precipitation inputs during the time period Δt. This requires the canopy and litter ground cover components to be calculated, as denoted by G_{canopy} and G_{litter} respectively, where both of these lie in the range 0 to 1. The derivation of these variables is presented in the next section. The procedure for calculating precipitation interception and storage is then quite straightforward and is as follows:

- Calculate the precipitation incident on the canopy according to PG_{canopy}. Water is retained by the canopy up to the maximum possible, as given by eqn (3.2a).
- The precipitation that is not incident on the canopy, $P\left(1 - G_{canopy}\right)$, plus the precipitation not retained by the canopy is then transferred to the litter and soil. Denote this residual by P_r.
- The precipitation incident on the litter is $P_r G_{litter}$. As for the canopy, water is retained by the litter up to the maximum possible, as given by eqn (3.2b).
- The precipitation that is not incident on the litter, $P_r(1 - G_{litter})$, plus the precipitation not retained by the litter is then transferred to the soil.
- The water that goes to the soil is then available for infiltration, runoff or evaporation as described later .

Note that the water intercepted by the canopy 'drips' to the soil and litter at a rate of 50% per hour – this parameter is fixed and is not available on the interface.

3.3 Ground cover components

3.3.1 Canopy

Canopy ground cover is defined using the standard approach to light interception, and so the attenuation of solar radiation through the canopy is described by Beer's law:

$$I\left(\ell\right) = I_0 \mathrm{e}^{-k\ell} \qquad (3.3)$$

where the symbols are:

I_0	Incident solar radiation on the canopy	W m^{-2} ≡ J m^{-2} s^{-1}
$I(\ell)$	Solar radiation within the canopy	W m^{-2} ≡ J m^{-2} s^{-1}
ℓ	Cumulative leaf area index	m^2 (leaf) (m^2 ground)$^{-1}$
L_{tot}	Total canopy leaf area index	m^2 (leaf) (m^2 ground)$^{-1}$
L_{live}	Live canopy leaf area index	m^2 (leaf) (m^2 ground)$^{-1}$
k	Light extinction coefficient	(m^2 ground) (m^2 leaf)$^{-1}$

and $0 < k < 1$.

Although this method can be extended to include direct and diffuse components of radiation, (see, for example, Johnson *et al.*, 1995), this is not included in the present analysis.

Defining the radiation that is incident on the whole canopy as the difference between that at the surface and that at the base, the canopy ground cover is therefore the proportion of light that is not intercepted:

$$G_{canopy,tot} = \left(1 - e^{-kL_{tot}}\right) \quad (3.4a)$$

In order to calculate the light intercepted by the live material it is necessary to make some assumptions about the relative distribution of live and dead material within the canopy. There are two practical options:

- Live and dead material are uniformly distributed within the canopy,
- Dead material is below the live material.

The corresponding light interception by live material is given by

$$G_{canopy,live} = \left(1 - e^{-kL_{live}}\right) \quad (3.4b)$$

and

$$G_{canopy,live} = \frac{L_{live}}{L_{tot}} G_{canopy,tot} \quad (3.4c)$$

respectively for these two conditions.

For either case, the light intercepted by the dead material is

$$G_{canopy,dead} = G_{canopy,tot} - G_{canopy,live} \quad (3.4d)$$

In the model, the assumption that the dead material is below the live is adopted, so that eqn (3.4.b) is applied.

These equations define the irradiance incident on either the live component or the total canopy. However, they do not define the absorbed irradiance, since there will be some reflection: this is considered later.

There is, in fact, a third option, whereby the live and dead are spatially separated. In the present model, spatial distribution is not specifically included, although if the light intercepting parameters of live and dead are the same, then the uniform model and the spatially separate model will be similar. Consequently, the present approach will be closely related to conditions of spatial separation between live and dead.

3.3.2 Litter

The ground cover by litter is assumed to be unrelated to the canopy cover, which simply means that the canopy is above the ground. When these equations are used to describe either rainfall inputs or radiation interception, the interaction between the two will, of course, be included.

Litter cover is defined in a similar, but not identical, way to the canopy, so that:

$$G_{litter} = \left(1 - e^{-0.69\, W_{litter} / W_{litter,h}}\right) \quad (3.5a)$$

where the symbols are:

W_{litter}	Mass of litter	t ha^{-1}
$W_{litter,h}$	Mass of litter for 50% light attenuation	t ha^{-1}

According to this description, $W_{litter,h}$ is analogous to a half-life and

$$G_{litter}\left(W_{litter}=W_{litter,h}\right)=0.5 \qquad (3.5b)$$

Note that the factor 0.69 is $\ln(2)$.

3.3.3 Bare soil

The bare soil component is the ground area that is not allocated to litter cover, and is:

$$G_{soil}=1-G_{litter}=\mathrm{e}^{-0.69\,W_{litter}/W_{litter,h}} \qquad (3.6)$$

3.4 Solar and net radiation

The radiation components that are intercepted and absorbed by the canopy, litter and soil need to be defined to describe evapotranspiration and, in the case of the green canopy component, photosynthesis. The canopy absorbs solar, or short-wave, radiation, and transmits long-wave, or terrestrial, radiation. These two components of radiation have different characteristic wavelengths by virtue of the different temperatures of the emitting bodies. For photosynthesis only the visible component of solar radiation is utilised and so the wavelength of the radiation is important. The photosynthetically active radiation (PAR–see the discussions on photosynthesis and growth in chapter 4) is approximately half the total solar radiation. However, when studying water use, it is the energy of the radiation and not its wavelength that is important.

It is essential to be familiar with the differences between short-wave, or solar radiation, which is often referred to as global radiation, and long-wave radiation, as well as the photosynthetically active component of short-wave radiation. In brief, the wavelength of radiation that is emitted by a body is a function of the temperature of that body as defined by the Stefan-Boltzmann equation, which states that the energy emitted is proportional to the absolute temperature to the fourth power. Because the external area of the sun is at around 6000K (absolute temperature), the wavelength of solar radiation is markedly different to that emitted by bodies on the earth (around 300K). Understanding this variation in the wavelength of radiation is crucial to ensuring that the correct components of the radiation balance are being measured and used appropriately in the model. For more detail, see Monteith (1973) or later editions of that text. In the present analysis, the radiation balance is defined for daily values, although the theory also applies at the instantaneous (per second) time scale.

The radiation balance at a surface is illustrated in Fig. 3.2. Note that the net outgoing long-wave radiation is itself a balance between incoming long-wave radiation and that being transmitted by the surface.

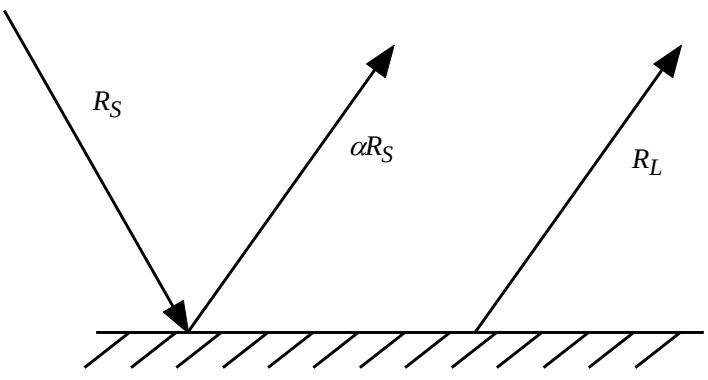


Figure 3.2: Schematic representation of the radiation balance.

The daily net radiation balance can be written, in general, as

$$\Phi_N = R_S(1-\alpha) - R_{N,L} \qquad (3.7a)$$

where the symbols are defined by

R_S	intercepted solar radiation	J m^{-2} d^{-1}
$R_{N,L}$	net out going long-wave radiation	J m^{-2} d^{-1}
α	reflection coefficient (albedo)	dimensionless
Φ_N	net radiation	J m^{-2} d^{-1}

Note that daily radiation generally uses MJ m^{-2} d^{-1}, and that J m^{-2} s^{-1} is equivalent to W m^{-2}.

Typical albedo coefficients for green material, brown material and wet soil are (Greg Lodge, *pers comm.)*

green: $\alpha_g = 0.23$

brown: $\alpha_b = 0.3$ (3.7b)

wet soil: $\alpha_s = 0.13$

The notation here uses R_S for daily solar radiation rather than I_0 as was used earlier for instantaneous solar radiation, when considering canopy light interception.

3.4.1 Light interception components

The light interception by the live and total canopy components are:

$$R_{S,canopy,live} = R_S G_{canopy,live} \qquad (3.8a)$$

and

$$R_{S,canopy,tot} = R_S G_{canopy,tot} \qquad (3.8b)$$

respectively, where I_0 was defined earlier as the incoming solar radiation and the G variables are given by eqns (3.4a, b).

The light interception by litter is:

$$R_{S,litter} = R_S\left(1 - G_{canopy,tot}\right)G_{litter} \qquad (3.8c)$$

The light interception by the soil is:

$$R_{S,soil} = R_S\left(1 - G_{canopy,tot}\right)G_{soil} \qquad (3.8d)$$

Equations (3.8a-d) can now be applied to the different components of ground cover in eqn (3.7) to define the individual net radiation balances. To do so, it now remains to derive the net outgoing longwave radiation, $R_{N,L}$.

3.4.2 Net radiation components

Daily solar radiation data can be readily measured and are available for a wide range of sites. However, net outgoing longwave radiation is generally not recorded and so has to be derived (Allen *et al.*, 1998). This can be shown to be

$$R_{N,L} = 86{,}400\sigma\left(\frac{T_{mx}^4 + T_{mn}^4}{2}\right)\left(0.34 - 0.14\sqrt{p_v}\right)\left(1.35\frac{R_S}{R_{S,0}} - 0.35\right) \qquad (3.9)$$

where T_{mx} and T_{mn} (K) are maximum and minimum daily temperatures, σ is the Stefan-Boltzmann constant with value $\sigma = 5.6704 \times 10^{-8}$ J m^{-2} s^{-1}K^{-4}, p_v (kPa) is vapour pressure, R_S (J m^2 d^{-1}) as defined above is daily solar radiation, and $R_{S,0}$ (J m^2 d^{-1}) is the clear sky solar radiation. In order to apply eqn (3.9) it is necessary to estimate the clear sky solar radiation, $R_{S,0}$.

Note that in some climate data files, including the SILO data (Jeffrey 2001), vapour pressure is expressed in hPa (hectopascal) rather than kPa. This is a return to old units as hPa is equivalent to milibar. Care must be taken to ensure the correct units are used – in the GrazeMod interface there is the option for either.

Clear sky irradiance

The theory here follows Campbell (1977), Johnson *et al* (1995), and Thornley and France (2008).

It is first necessary to calculate three variables relating to the geometry of the earth's rotation and its orbit around the sun. These are the solar declination angle, δ (rad), which is the angle between the earth's equatorial plane and the line from the earth to the sun (Thornley and France, 2008), the solar elevation angle at local noon, ϕ (rad), and the daylength, f_d, as a fraction of the 24 hour period. If t is the day of year from 1 January, λ (rad) the latitude, then

$$\delta = \frac{\pi}{180}23.45\sin\left(2\pi\frac{t-81}{365}\right) \qquad (3.10a)$$

$$\phi = \sin^{-1}\left(\sin\lambda\sin\delta + \cos\lambda\cos\delta\right) \qquad (3.10b)$$

$$f_d = \frac{1}{\pi}\cos^{-1}\left(-\tan\lambda\tan\delta\right) \qquad (3.10c)$$

Note that if λ is prescribed in degrees then, using obvious notation,

$$\lambda_{rad} = \frac{\pi}{180}\lambda_{deg} \qquad (3.10d)$$

Turning to irradiance, three sets of variables are used with appropriate subscripts (R_S has already been defined above). These are:

J, instantaneous irradiance outside the earth's atmosphere: W m^{-2} or J m^{-2} s^{-1};

I, instantaneous irradiance at the earth's surface: W m^{-2} or J m^{-2} s^{-1};

R_S, total solar radiation, J m^{-2} day.

In all cases, irradiance is measured parallel to the horizontal plane at the surface of the earth.

The irradiance outside the earth's, J_{noon}, is

$$J_{noon} = \gamma\sin\phi \qquad (3.10e)$$

where

$$\gamma = 1367 \text{ Wm}^{-2}\text{s}^{-1} \tag{3.10f}$$

is the solar constant, and is the irradiance perpendicular to the sun at the edge of the earth's atmosphere. It is now assumed that the irradiance at the earth's surface, I_{noon}, is given by

$$I_{noon} = \tau J_{noon} \tag{3.10g}$$

where τ is an atmospheric diffusivity coefficient. While more complex equations have been used this approach works well for a range of locations in Australia, as will be seen shortly, and there is no obvious reason to suggest other locations will behave much differently. Comparisons with experimental data suggest

$$\tau = 0.73 \tag{3.10h}$$

is a good default value, although it may be necessary to adjust this parameter for different sites. However, this is relatively easy to estimate, as discussed below. It should be noted that a slightly different approach than a fixed constant in eqn (3.10g) is used in FAO56 (Allen et al, 1998), although my analysis of various sites using SILO data (Jeffrey 2001) suggest that the present method is more accurate.

Assuming that the irradiance throughout the day, I_t, varies sinusoidally, it follows that

$$I_t = I_{noon} \sin(\pi t), \; t = 0\text{–}1 \text{ over the daylight period} \tag{3.10i}$$

so that the mean daily potential, or clear-sky, irradiance is

$$R_{S,0} = 86,400 \, f_d \frac{2}{\pi} I_{noon} \tag{3.10j}$$

where 86,400 is the number of seconds in a day. Thus,

$$R_{S,0} = 86,400 \, f_d \frac{2}{\pi} \tau \gamma \sin\phi \tag{3.10k}$$

This is a simple equation for the clear-sky irradiance in terms of latitude and day of year.

In order to test the approach, data from four sites in Australia from 1906 to 2007 are used from the SILO data set (Jeffrey, 2001). These sites are shown in Table 2.1. Potential irradiance for each day of the year is estimated as the maximum observed for each day in the 100 year climate file. This assumes, therefore, that for each day of the year there was at least one occasion in the 100 years where the sky was clear. The relatively smooth nature of the observed data when plotted this way suggests this to be the case. There are occasional 'blips' in the data, but these may well be due to fluctuations in the accuracy of the measurement equipment. The fact that the data and the model are virtually identical gives considerable confidence in the theoretical approach.

Table 2.1: Sites and latitudes (degrees) that are used to test eqn (3.10k) for potential, or clear-sky, daily solar radiation.

Site	Elliott, Tas	Hamilton Vic	Albany WA	Armidale, NSW
λ_{deg}	-41	-38	-35	-30

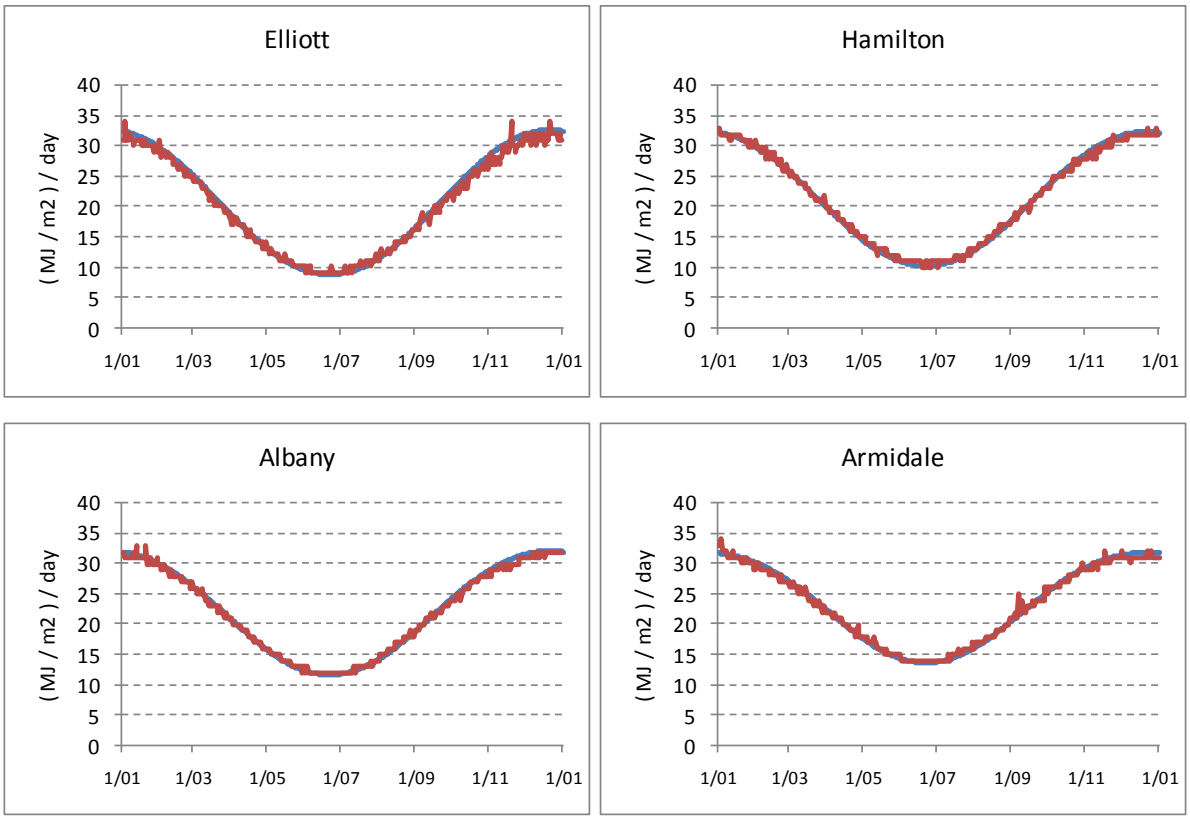


Figure 2.1: Observed maximum daily irradiance (red) and predicted values (blue) using eqn (3.10k). Note that the blue lines are obscured for much of the data.

Equation (3.10k) can now be used in (3.9a) to get the net outgoing longwave radiation in (3.7a) for the net radiation balance using appropriate albedo coefficients (3.7b) for different ground cover components.

3.4.3 Soil heat flux

The soil heat flux refers to the heating or cooling of the soil. Over the whole day this may be close to zero, although it can vary considerably from positive to negative values throughout the day. Since the above analysis has treated organic matter (live and dead) separately from the soil, and since the heat storage capacity of these components is small, there is little likelihood of any significant error in estimating the radiation components for organic matter. The main possibility of error through ignoring the soil heat flux is in the calculation of soil evaporation. In this case, the net radiation balance during the day may be over-estimated and under-estimated during the night. Since the daily evaporation from the soil is linearly related to the net radiation, any associated errors are likely to balance each other out over the full day. Soil heat flux is therefore ignored in the present analysis.

3.5 Daily values for climate parameters

The climatic parameters shown here are required for subsequent calculations of evapotranspiration (some of these parameters were defined earlier)

T	mean daily air temperature	°C
R_S	total solar, or global, radiation	(J m^{-2}) d^{-1}
H_r	relative humidity	fraction
p'_v	saturated vapour pressure	Pa or kPa
p_v	actual vapour pressure	Pa or kPa
Δp_v	vapour pressure deficit	Pa or kPa

The vapour pressure deficit is defined as

$$\Delta p_v = p'_v - p_v = p'_v\left(1 - H_r\right), \qquad (3.11)$$

The use of relative humidity is not always supported because the real driving force in the evapotranspiration calculations is vapour pressure, or density, deficit, and this depends on both relative humidity and temperature through the dependence of saturated vapour pressure on temperature. However, relative humidity data are generally available, and so is used here, recognising the interrelationship between these parameters through eqn (3.11).

3.5.1 Global radiation

This is simply the accumulated global solar radiation over the daytime period.

3.5.2 Temperature

Mean daily temperature can be either the mean of maximum and minimum temperatures or, if available, the mean of the hourly, or other time increment, temperatures.

3.5.3 Vapour pressure deficit

If only daily means of temperature and relative humidity are available, then Δp_v is calculated using eqn (3.11). However, if maximum and minimum values of temperature and relative humidity are available, it may be more appropriate to use

$$\Delta p'_v = 0.5\left[p'_v\left(T_{mx}\right)\left(1 - H_{r,mn}\right) + p'_v\left(T_{mn}\right)\left(1 - H_{r,mx}\right)\right], \qquad (3.12)$$

where subscripts *mx* and *mn* refer to maximum and minimum. The rationale behind this equation is that the daily maximum relative humidity generally occurs when the temperature is minimum and *vice versa*.

3.5.4 Windspeed

Windspeed is taken as the mean daily windspeed, which is usually calculated from accumulated daily wind run.

3.5.5 Daylength

The daylight period of the day, f_d, was given in (3.10c). Using this, the daylength in seconds is simply

$$d = 86{,}400\, f_d \qquad (3.13a)$$

3.5.6 Daily rainfall distribution

Daily rainfall distribution is calculated by specifying the rainfall duration in hours (which is fixed for all days in the simulation. A starting time for the rainfall is then randomly selected and the rain is evenly spread across that time. For example, if there is 30 mm of rain in a day and the duration selected is 3 hours, then there will be 10 mm per hour for three consecutive hours with the starting time randomly chosen.

3.6 Generic climatic patterns

If all the climate inputs required for the model are not available, generic values as displayed on the interface can be used. These are generated from fairly simple distributions and should be useful for running simulations when detailed climatic information is not accessible.

Note that it is possible to combine actual data with generic patterns, for cases where some information is missing. Generic patterns are used when the 'Use data file' box is un-checked in the data input section of the program.

3.6.1 Daily rainfall

The rainfall for each month and the number of rain days per month is prescribed. The daily rainfall is then randomly distributed across randomly selected days in the month. This is not a sophisticated rainfall generator, since rainfall data are generally accessible for most sites.

3.6.2 Temperature

Sine curves for both air and soil temperature are used, and the parameters can be adjusted. This includes specifying a diurnal temperature range (min to max) that is applied throughout the year.

3.6.3 Global radiation

The potential, or clear sky, global solar radiation was defined earlier in eqn (3.10k). The fraction of potential solar radiation to use during the simulation can be specified on the model interface.

3.6.4 Relative humidity

As for temperature, a sine curve is used for the annual pattern of relative humidity, along with a diurnal range. Note that relative humidity generally peaks in winter rather than summer.

3.6.5 Windspeed

There is no variable generic pattern for windspeed and so this can only be specified as a constant.

3.6.6 Potential evapotranspiration

Potential evapotranspiration (PET) is not prescribed by a generic pattern, but is calculated from other climatic inputs. This may be from generic climatic inputs or actual data.

3.6.7 Atmospheric CO_2 concentration

Atmospheric CO_2 concentration is also available on the interface to allow exploration of the impact of increases in CO_2 on productivity. It can be defined as fixed in the simulation or daily values can be prescribed. CO_2 concentration is given in parts per million (ppm) which, while not SI (System Internationale), is by far the most widely used unit for atmospheric CO_2 concentration. For a further discussion on this see Thornley and Johnson (2000, Chapter 2, section 2.2.4).

3.7 Concluding remarks

The climatic inputs required have been described in some detail. Since these climatic inputs drive the biophysical processes, it is important to make every effort to get accurate data. While some inputs can be directly measured, such as rainfall and temperature, other key factors in the water balance need to be calculated – this applies, in particular, to the estimates of the evapotranspiration components. It is important, therefore, to recognise that these estimates involve theory and assumptions and so should not be treated as being absolute. Even those factors, such as rainfall and temperature, that are directly measured are not without potential for error and simulation output should be examined with this possibility in mind. Generic climatic patterns have also been presented, which are useful for general exploration of the system.

3.8 References

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4 Pasture Growth

4.1 Introduction

The pasture growth module originates from the general structure of the models described by Johnson and Thornley (1983, 1985); Johnson and Parsons (1985); and Parsons, Johnson and Harvey (1988), although a number of modifications have been made. More recent discussions of physiological pasture growth models can be found in Thornley and Johnson (2000) and Thornley and France (2008). In addition, the approach has been developed in order to incorporate water and nutrient effects.

The following key points apply:

- The model is constructed for generic pasture species so that particular species are defined through the basic model parameters.
- The model includes carbon assimilation through photosynthesis and respiration followed by tissue growth, turnover and senescence.
- Pasture growth and tissue dynamics are influenced by environmental conditions (light, temperature and atmospheric CO_2 concentration) as well as soil water and nutrient status.
- N, P, K, S nutrients are incorporated.
- For annual species, vegetative (emergence to anthesis) and reproductive (anthesis to maturity) growth phases are included.
- Multiple species can be included, which may be perennial, annual, legume, C_3 or C_4.
- Pasture utilisation by grazing animals is considered in the *Animal* chapter.
- Plant digestibility is calculated in terms of the plant nutrient status, although this is described in the *Animal* chapter.
- Throughout the discussion, the area of ground that is used is m^2 although frequently in the interface the hectare, ha, is used. The choice is generally to give the user access to familiar units and should not cause any difficulty.
- Plant mass may be expressed in units of CO_2, carbon, or dry weight (d.wt). The conversion from CO_2 or carbon to dry weight, d.wt, assumes that plant d.wt is 40% carbon.

Unlike the original models referenced above, the model described here does not include specific substrate pools for labile carbon and nitrogen. Instead, in order to simplify the model, the daily carbon assimilation and respiratory costs are calculated and the net carbon balance is then directly available for growth on that day. In addition, the effect of available water and nutrients, as well as the influence of actual plant nutrient status on growth are included.

Growth is calculated as follows:

- The daily transpiration rate and the effect of water stress are calculated;
- Potential nutrient uptake is calculated in relation to root distribution and soil nutrient status;
- Daily gross photosynthesis is calculated in response to environmental conditions, available water, and plant nutrient status;
- Growth and maintenance respiration components are calculated;
- The carbon available for growth is the difference between the daily gross photosynthesis and respiratory costs, which is then partitioned between the plant shoot components as well as the root;
- Tissue turnover and senescence are then calculated.

Other processes, such as species interaction, nitrogen fixation (in legumes) and the growth of annual species are also included and a more detailed description follows.

4.2 Transpiration and the influence of water stress

The derivation of potential transpiration is discussed in the *Water* chapter, and is denoted by $E_{T,pot}$, mm water m^{-2} day^{-1}: this can be derived in a variety of ways, depending on the user selected options. The potential transpiration is derived for full ground cover, so the actual transpiration demand is given by

$$E_{T,demand} = G_{canopy,live} E_{T,pot} \quad (4.1a)$$

where $G_{canopy,live}$ is the live ground cover and is given by eqn (3.4b) in the *Climate* chapter.

Once transpiration demand is known it is then necessary to calculate the impact of soil water status and so the actual transpiration. First, the growth limiting factor, GLF_{water}, for water is defined in relation to the available soil water as shown in Fig. 4.1. If the GLF_{water} is 1 then there is no limitation to growth; if it is zero then there is total limitation. The GLF_{water} is defined in terms of the following soil water characteristics:

θ_w: Wilting point

θ_r: Recharge point

θ_{fc}: Field capacity

θ_s: Saturated water content

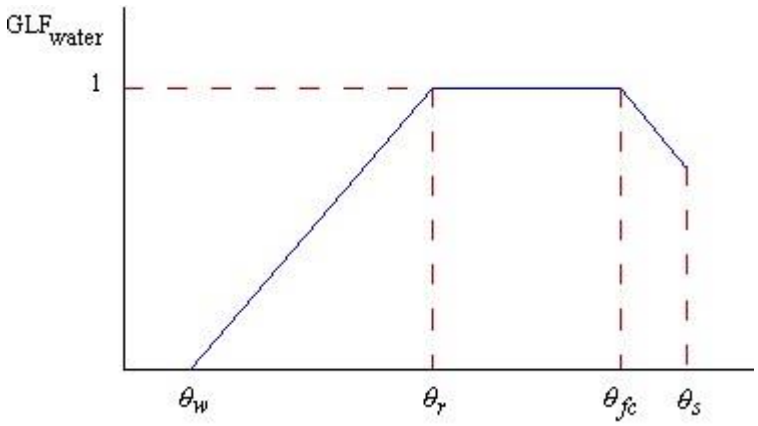

Figure 4.1: Schematic representation of the influence of limiting soil water content on transpiration.

For water contents below the wilting point, plants cannot extract water from the soil. Between the wilting point and recharge point, GLF_{water} increases from 0 to 1. Between recharge point and field capacity, GLF_{water} is 1. Between field capacity and saturation, GLF_{water} may decline, although this can be defined by the user. The reason the GLF_{water} can decline at soil water contents greater than field capacity is that plants may be susceptible to water logging. Note that wilting point, field capacity and saturation are defined in the soil water module of the interface, while the recharge point and any decline in the GLF_{water} at saturation are defined in the pasture module. The term *recharge point* is used as this is the point at which irrigation would have to be applied in order to prevent any water stress.

The strategy for calculating transpiration is to calculate the GLF_{water} for each soil layer, $GLF_{water,\ell}$ according to the scheme illustrated in Fig. 4.1. The water uptake from each layer is then given by

$$E_{T,\ell} = f_{r,\ell} GLF_{water,\ell} E_{T,demand} \quad (4.1b)$$

where $f_{r,\ell}$ is the root fraction in each layer.

If there is no limitation to water uptake from any layer due to available soil water then water uptake through the profile is taken out according to the relative root distribution. As water becomes unavailable from layers, uptake from those layers is reduced according to $GLF_{water,\ell}$.

According to eqn (4.1b) there is no compensation for water limitation in dry layers by other layers that might have abundant water. Several researchers have shown that this is likely to be the case for some plants (eg Passioura and Stirzaker 1993; Munns and Cramer, 1996). One way in which this phenomenon has been demonstrated has been to grow plants with split root systems, part of the root system in wet conditions, part in relatively dry soil. In spite of the fact that there is adequate water in the wet part of the root zone to satisfy transpiration demand, the plants still show signs of water stress. This has been interpreted as a survival adaptation: don't use all the water at once when it is clear that dry conditions are close by. In their discussion, Passioura and Stirzaker (1993) describe this as a *feed-forward* mechanism. There is a whole field of research in this area for the interested reader.

It is reasonable to suppose that not all plants have this conservative strategy. To accommodate this situation, the parameter σ in the range 0 to 1 is introduced. Once eqn (4.1b) has been applied, if actual transpiration is less than demand (which will occur if any $GLF_{water,\ell}$ is less than 1), then a second treatment of water uptake is implemented where the possible uptake from the layer is given by

$$E^*_{T,\ell} = \sigma f_{r,\ell} GLF_{water,\ell} \left(\theta_\ell - \theta_{w,\ell}\right) \Delta z_\ell \quad (4.1c)$$

where Δz_ℓ is the thickness of the layer and θ_ℓ is the soil water content in the layer after eqn (4.1b) has been applied. Equation (4.1c) is then applied sequentially through the layers from the top of the profile until it has either been applied to all layers, or the total transpiration reaches the demand. The parameter σ can be prescribed by the user.

4.2.1 Water growth limiting factor

Once the actual transpiration is known, the overall growth limiting factor is defined as

$$GLF_{water} = \frac{E_T}{E_{T,demand}} \quad (4.1d)$$

and this is used in subsequent calculations to impose the impact of water stress on plant growth.

4.3 Potential nutrient uptake

Potential nutrient uptake is calculated and then adapted to respond to plant requirements. If potential uptake exceeds plant maximum requirement then uptake is reduced. Alternatively, when uptake is less than required, the influence of nutrient limitation is evaluated.

4.3.1 Response to available soil nutrient

The potential nutrient uptake is now considered in relation to the root mass, distribution, and soil nutrient status. If this potential exceeds demand then uptake is reduced.

First consider the situation with no water stress. The basic equation describing uptake from a layer, ℓ, is

$$U_{nu,\ell} = \xi_{nu} W_{r,\ell} Nu_{\ell} \tag{4.2a}$$

where ξ_{nu} is an uptake coefficient, $W_{r,\ell}$ is the root mass in the layer, Nu_{ℓ} is the concentration of the nutrient in question. According to this equation, nutrient uptake is defined as being linearly related to both root mass and nutrient concentration in the layer.

The total uptake is given by

$$U_{nu} = \sum_{\ell} U_{nu,\ell} = \xi_{nu} \sum_{\ell} W_{r,\ell} Nu_{\ell} \,. \tag{4.2b}$$

Equation (4.2b) completely defines potential nutrient uptake in response to root mass and soil nutrient concentration. However, the challenge lies in defining the parameter ξ_{nu}. The following analysis re-formulates eqn (4.2b) without changing its underlying structure.

Consider a reference growth rate δ_{ref} and let $W_{r,ref}$ be the root mass, with normalised distribution $\zeta_{ref,\ell}$. Let $Nu_{ref,\ell}$ be a nutrient distribution that exactly satisfies demand for this growth rate with this root distribution. This combination will, of course, not be unique. It therefore follows that

$$U_{ref,nu} = f_{nu}\delta_{ref} = \sum_{\ell} U_{ref,nu,\ell} = \xi_{nu} W_{r,ref} \sum_{\ell} \zeta_{ref,\ell} Nu_{ref,\ell} \tag{4.2c}$$

where f_{nu} kg Nu (kg C)$^{-1}$ is the nutrient concentration in the new tissue. Equation (4.2c) gives

$$\xi_{nu} = \frac{f_{nu}\delta_{ref}}{W_{r,ref} \sum_{\ell} \zeta_{ref,\ell} Nu_{ref,\ell}} = \frac{f_{nu}\delta_{ref}}{W_{r,ref}\lambda_{nu}} \tag{4.2d}$$

where

$$\lambda_{nu} = \sum_{\ell} \zeta_{ref,\ell} Nu_{ref,\ell} \tag{4.2e}$$

can be interpreted as the root weighted nutrient distribution required to satisfy demand. This is analogous to the soil nutrient test value but not identical to it, although its value will be of similar magnitude to the required soil test value. It should be noted that the combination of root distribution and nutrient concentration through the profile that are required to satisfy nutrient demand is not unique.

Equation (4.2b) now becomes

$$U_{nu} = f_{nu}\delta_{ref} \frac{\sum_{\ell} W_{r,\ell} \left[Nu\right]_{\ell}}{W_{r,ref}\lambda_{nu}} \tag{4.2f}$$

This requires the parameters δ_{ref}, W_{ref} and λ_{nu} to be defined. In the model the values

$$\begin{aligned} \delta_{ref} &= 50 \text{ kg d.wt ha}^{-1} \equiv 20 \text{ kg C ha}^{-1} \\ W_{r,ref} &= 250 \text{ kg root d.wt ha}^{-1} \equiv 100 \text{ kg C ha}^{-1} \end{aligned} \tag{4.2g}$$

are used, along with the default values

$$\lambda_{NO3} = 5,\ \lambda_{NH4} = 5,\ \lambda_{P} = 30,\ \lambda_{K} = 50,\ \lambda_{S} = 2 \ \text{ mg kg}^{-1}. \tag{4.2h}$$

The theory so far allows a complete description of the potential nutrient uptake for given root and nutrient distributions through the soil.

4.3.2 Nutrient growth limiting factor

It is convenient to define the growth limiting factor for nutrients in an analogous way to that for water. This is defined in terms of the ratio of actual nutrient uptake to demand in order to satisfy optimum growth, $U_{nu,D}$ – note that uptake can occur in luxury levels and so exceed this demand.

The growth limitation factor due to each nutrient can now be defined, analogously to that for water, as

$$glf_{nu} = \min\left(1, \frac{U_{nu}}{U_{nu,D}}\right), \tag{4.2i}$$

and the overall net limitation is

$$GLF_{nu} = \min\left(glf_{nu}\right). \tag{4.2j}$$

4.3.3 Nutrient remobilisation

Although only indirectly related to nutrient uptake, it is useful to define the nutrient remobilisation at this point. Nutrients can be remobilised from senescing tissue. The maximum proportion of nutrient that can be remobilised can be specified by the user. Denoting this by $f_{nu,remob,mx}$, the nutrient available from remobilisation is defined in terms of the gross rate of senescence as:

$$Nu_{remob,mx} = f_{nu,remob,mx} \left.\frac{\mathrm{d}W_{dead}}{\mathrm{dt}}\right|_{gross} \tag{4.2k}$$

where the gross rate of senescence, that is excluding losses from standing dead to litter, is given by the derivative in this equation.

4.3.4 Nutrient composition in the shoot and root

For the calculations that follow, the nutrient composition in the shoot and root, for any nutrient Nu, is defined as f_{nu}. It is assumed that the nutrient composition in the root is less than, or equal to that in the shoot, and the scaling factor ρ is used.

4.3.5 Legumes and nitrogen fixation

Nitrogen fixation in legumes is an important source of nitrogen. Rather than being possibly limited by supply, as is nutrient uptake, fixation depends on demand for nitrogen in relation to growth potential and available nitrogen uptake. It is more appropriate, therefore, to discuss nitrogen fixation in the carbon balance section, 4.5.2.

4.4 Photosynthesis and respiration

In order to calculate the daily carbon assimilation, the instantaneous rate of canopy gross photosynthesis is first calculated using the general approach described in Thornley and Johnson (2000). The climatic inputs are the mean daily photosynthetically active solar radiation and temperature. The procedure is as follows:

- first calculate the rate of single leaf gross photosynthesis;
- integrate this through the leaves in the canopy to get the rate of canopy photosynthesis;
- sum this over the day to get the daily canopy gross photosynthesis.

4.4.1 Units

Before proceeding, note that either mass or mole units are commonly used for photosynthesis. With mass units, the rate of photosynthesis is measured in mg CO_2 m^{-2} s^{-1} and incident irradiance has units J m^{-2} s^{-1} PAR (photosynthetically active radiation), where the area may be leaf area or ground area. With mole units, the equivalent units are mole CO_2 m^{-2} s^{-1} and mole photons m^{-2} s^{-1}. While mass and moles of CO_2 can be converted directly, there is no exact conversion for the energy component as this depends on the energy of the different irradiance wavelengths. Mass units are used here for two reasons:

- the irradiance data input is the daily global, or solar, radiation, which is generally recorded at met stations in MJ m^{-2} day^{-1};
- the ultimate objective is to define pasture growth in mass units.

It must be emphasised that it is not intended to imply that mass units are always preferable: in many plant physiological studies mole units are more appropriate.

4.4.2 Photosynthetically active radiation, PAR

Only the visible component of solar radiation contributes to photosynthesis, which is the wavelength range of around 0.38–0.74 μ m. (Note that there is no precise definition of this range.) Approximately half of the solar radiation is visible, although the absolute fraction depends on environmental factors such as solar elevation and cloud cover. This visible component of solar radiation that provides the energy for photosynthesis is referred to as *photosynthetically active radiation*, or PAR. Although the PAR provides the energy for photosynthesis, not all wavelengths are equally effective in this process. Generally, red light is most effective and then blue, with green light the least effective. Note that the radiation input is total, or global, solar radiation and it is assumed that PAR is half of the total.

As mentioned earlier, the units for solar radiation (total and PAR) are J m^{-2} s^{-1}, which is equivalent to W m^{-2}. While there is no exact conversion between energy units and moles of photons for PAR due to the variation in the energy of photons depending on their wavelength and hence colour, a reasonable conversion is (Robson and Sheehy, 1981):

10^{-6} mol photons PAR $\cong$ 0.235 J PAR (4.3)

4.4.3 Leaf gross photosynthesis

Leaf photosynthesis is affected primarily by irradiance (light energy), temperature and atmospheric CO_2 concentration. These are considered in turn.

Leaf gross photosynthesis in response to irradiance

The rate of single leaf photosynthesis, P_ℓ mg CO_2 (m^2 leaf)$^{-1}$ s^{-1}, in response to irradiance is described by the non-rectangular hyperbola. This can be written as

$$\xi P_\ell^2 - (\alpha I_\ell + P_{mx}) P_\ell + \alpha I_\ell P_{mx} = 0 \qquad (4.4a)$$

where the parameters are:

P_{mx}	rate of single leaf gross photosynthesis at saturating irradiance	mg CO_2 (m^2 leaf)$^{-1}$ s^{-1}

α	photochemical, or photosynthetic, efficiency	mg CO_2 J^{-1}
ξ	curvature parameter	J kg^{-2} s^{-1}

P_ℓ is given by the lower rood of eqn (4.4a), which is

$$P_\ell = \frac{1}{2\xi}\left[\alpha I_\ell + P_{mx} - \left\{\left(\alpha I_\ell + P_{mx}\right)^2 - 4\xi\alpha I_\ell P_{mx}\right\}^{1/2}\right] \tag{4.4b}$$

Note that when $\xi = 0$ this reduces to the simpler rectangular hyperbola:

$$P_\ell = \frac{\alpha I_\ell}{\alpha I_\ell + P_{mx}} \tag{4.4c}$$

and with $\xi = 1$ it becomes the Blackman limiting response given by:

$$P_\ell = \begin{cases} \alpha I_\ell, & I_\ell \le P_{mx}/\alpha \\ P_{mx}, & I_\ell > P_{mx}/\alpha \end{cases} \tag{4.4d}$$

The influence of these parameters on the $P_\ell(I_\ell)$ response is:

- α: the initial slope;
- P_{mx}: the asymptote as the curve approaches saturating irradiance;
- ξ: the curvature of the curve.

Equation (4.4b) is shown in Fig. 4.2 for a range of ξ values as indicated.

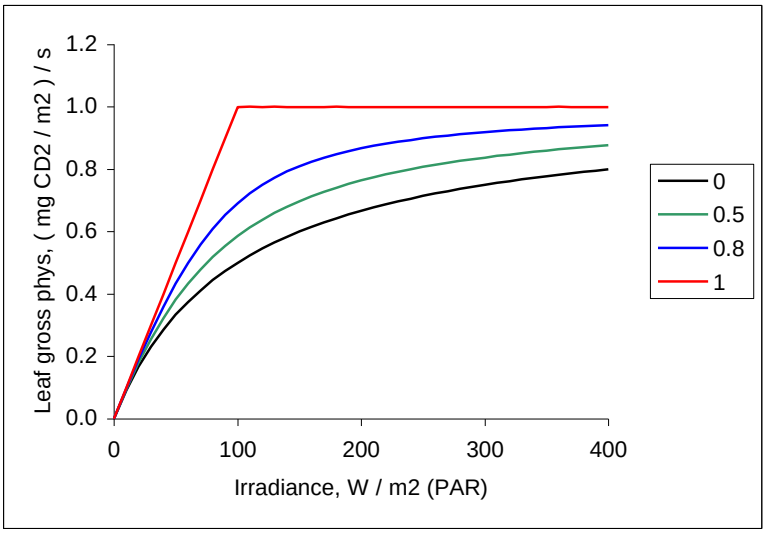

Figure 4.2: Leaf gross photosynthesis for α = 0.01 mg CO_2 J^{-1}, P_{mx} = 1 mg CO_2 (m^2 leaf)$^{-1}$ s^{-1}, and ξ as indicated.

In practice, the parameters α and ξ are generally less variable and, within their physiological range, typical variation has a fairly small influence on the leaf photosynthetic response. P_{mx}, on the other hand, is much more variable and responds to temperature and plant nutrient status, as well as other factors such as plant morphology. P_{mx} can be varied on the interface (including its temperature and CO_2 responses which are discussed later), whereas α and ξ are kept fixed, with values:

α = 0.01 mg CO_2 J^{-1}, and ξ = 0.8. (4.4e)

Leaf gross photosynthesis in response to temperature and nutrients

While the parameters α and ξ in eqn (4.4b) are slightly temperature dependent, the temperature response of leaf photosynthesis is dominated by the parameter P_{mx}. Similarly, the leaf photosynthetic response to nutrients is also dominated by P_{mx}. While all nutrients are assumed to influence the rate of photosynthesis, nitrogen is taken as the dominant nutrient in defining the leaf photosynthetic potential.

This response is written as:

$$P_{mx} = f_p(T)\, g_p(Nu) \frac{N_{opt}}{N_{ref}} P_{mx,20,Nref} \qquad (4.5a)$$

where N, N_{opt}, N_{ref} are leaf nitrogen content, leaf nitrogen content for maximum photosynthesis, and a reference value; $f_p(T)$ is a temperature response function with $f_p(T=20)=1$; $g_p(Nu)$ is a nutrient response function; and $P_{mx,20,Nref}$ is the value of P_{mx} at 20°C and N_{ref}. According to this definition, any increase in leaf nitrogen beyond N_{opt} will not result in further increases in leaf photosynthesis, which implies that there will be some morphological restriction to continued increase in photosynthetic capacity as the level of photosynthetic enzymes increases.

The reference leaf nitrogen levels, N_{ref}, are taken to be 4% and 3% (kg N (kg d.wt)$^{-1}$) for C_3 and C_4 plants respectively, and for $P_{mx,20,Nref}$ they are 1 and 1.2 mg CO_2 (m^2 leaf) s^{-1} respectively. This means that the leaf photosynthetic potential at 20°C is controlled entirely by the actual value of N, which in turn is a reflection of the level of photosynthetic enzymes in the leaves. Note that leaf photosynthesis will be considerably higher for C_4 plants than C_3 at high temperatures (see below).

The principal differences between the temperature responses of C_3 and C_4 plants is that the latter generally require warmer temperatures, do not show a decline in photosynthesis as temperatures increase, and can achieve higher rates of photosynthesis at saturating irradiance. The physiological basis for these differences lies in the activity of PEP carboxylase in C_4 plants, which serves to eliminate photorespiration. For C_3 plants, as temperature increases there is a shift towards photorespiration, at the expense of photophosphorylation (CO_2 fixation), which is a principal cause of the decline in leaf gross photosynthesis at high temperatures. The observed shift towards photorespiration that occurs in C_3 plants as temperature increases does not occur in C_4 plants, since they do not have significant photorespiration, so that the rate of gross photosynthesis does not decline. It is beyond the scope of this document to describe C_3 and C_4 photosynthesis in detail, although treatments can be found in most standard plant physiology texts, and a detailed mathematical description is given in Thornley and Johnson (2000).

Temperature response for C_3 plants

There is considerable scope to develop empirical expressions for these types of responses, and the approach here, following Thornley (1998), is to define $f_p(T)$ for C_3 plants as

$$f_p(T) = \begin{cases} \dfrac{(T-T_{mn})^q (T_{mx}-T)}{(20-T_{mn})^q (T_{mx}-20)}, & T_{mn} \le T \le T_{mx}; \\ 0, & \text{otherwise;} \end{cases} \qquad (4.5b)$$

where

$$T_{mx} = T_{opt} + \frac{\left(T_{opt} - T_{mn}\right)}{q} \qquad (4.5c)$$

T_{mn}, T_{opt} and T_{mx} are the minimum, optimum and maximum temperatures for photosynthesis, and q is a curvature coefficient. Note that on the interface T_{mn}, T_{opt} are user-defined and T_{mx} is then calculated from eqn (4.5c). Obviously, a different reference temperature to 20°C could be used if desired.

While other approaches can and have been used, this is relatively simple and is also quite flexible. $f_p(T)$ is illustrated in Fig. 4.3 with the default parameter in the model:

T_{mn} = 5°C; T_{opt} = 20°C; q= 1.5. (4.5d)

It must be emphasised that this response is for gross photosynthesis, whereas the actual leaf net photosynthetic rate also includes respiration which is discussed later. It follows that the optimum and maximum temperatures for gross photosynthesis may not be the same as those for net photosynthesis once respiration is taken into consideration.

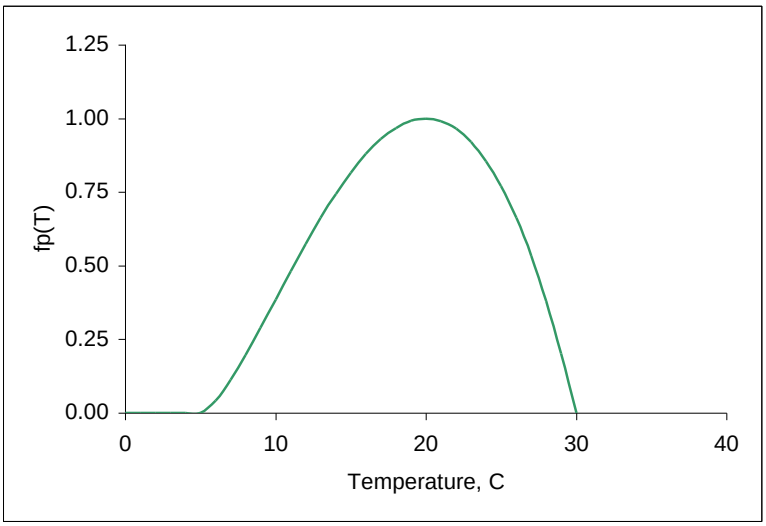

Figure 4.3: Leaf gross photosynthesis temperature response, $f_p(T)$, for C_3 plants, eqns (4.5b,c) with the parameters in eqn (4.5d).

Leaf photosynthesis can also be affected by temperature extremes so that, for example, if perennial ryegrass is exposed to sustained temperatures above around 35°C, most tillers will probably die. Similarly, exposure to cold temperatures may also have adverse medium to long-term effects. These types of responses are considered later.

Temperature response for C_4 plants

Since, as mentioned above, the rate of gross photosynthesis in C_4 plants does not tend to decline at high temperatures, eqns (4.5b,c) are adapted to use only the increasing portion of the curve, that is:

$$f_p(T)=\begin{cases} 0, & T \le T_{mn;} \\ \dfrac{(T-T_{mn})^q (T_{mx}-T)}{(20-T_{mn})^q (T_{mx}-20)}, & T_{mn} \le T \le T_{opt}; \\ \dfrac{(T_{opt}-T_{mn})^q (T_{mx}-T_{opt})}{(20-T_{mn})^q (T_{mx}-20)}, & T > T_{opt}. \end{cases} \quad (4.5e)$$

where again T_{mx} is given by eqn (4.5c). In using this formulation, T_{mx} has no physiological interpretation and is just there for mathematical convenience: it would be quite straightforward to combine eqns (4.5e) and (4.5c) to write (4.5e) independently of T_{mx}.

The response for C_4 plants is shown in Fig. 4.4 with the parameter values

T_{mx} = 10°C; T_{opt} = 35°C; q = 2. (4.5f)

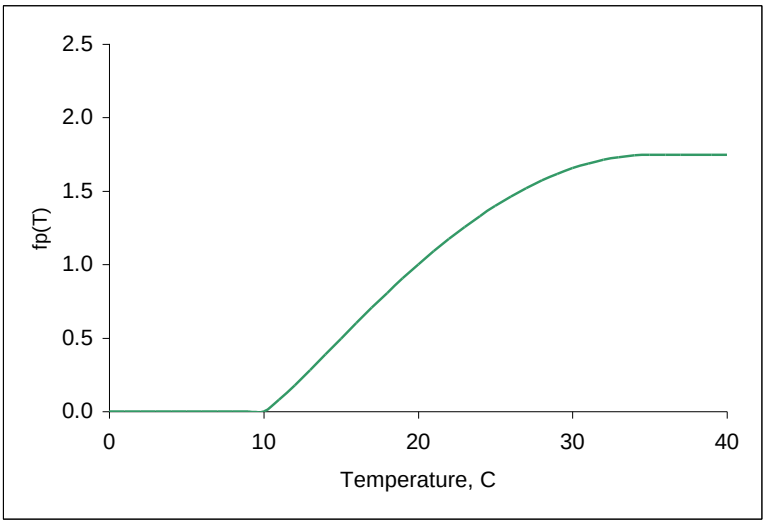

Figure 4.4: Leaf gross photosynthesis temperature response, $f_p(T)$, for C_4 plants, eqns (4.5e,c) with the parameters in eqn (4.5f).

Nutrient response

The photosynthetic potential as influenced by the optimum nitrogen level for the leaves has been described above. It now remains to incorporate the effects of nutrient stress, that is, sub-optimum nutrient levels. While only nitrogen was considered in the definition of the photosynthetic potential, all nutrients are assumed to affect the photosynthetic capacity when they are limiting.

The approach in defining the nutrient limitation function $g_p(Nu)$ in eqn (4.5a) is relatively straightforward and looks for the most limiting nutrient, so that

$$g_p(Nu)=\min\left(1,\frac{Nu}{N_{opt}}\right), \text{ where } Nu=N,P,K,S \quad (4.5g)$$

although it should be noted that only nutrients that have been activated in the model are incorporated. Eqn (4.5g) should be self-explanatory.

4.4.4 Photosynthetic response to CO_2 concentration

Elevated CO_2 concentration must be incorporate into the model for climate change studies. The three principal physiological plant responses to elevated CO_2 concentration are:

- an increase in leaf photosynthetic potential;
- a decrease in plant nitrogen content; and
- a decrease in stomatal, and therefore canopy, conductance (Long *et al.* 2004).

Although canopy conductance is not used directly in the treatment of photosynthesis here, it is convenient to consider it now for use in the treatment of transpiration which is discussed in the *Water* chapter.

While mechanistic approaches can describe these responses (eg. Johnson *et al.* 1995), simple empirical scaling functions are used in this whole-system simulation model. The simple and versatile functions presented here allow the flexibility within the model to explore the consequences of different responses to elevated CO_2.

Leaf photosynthesis response to CO_2

The response of leaf photosynthetic potential, P_{mx}, discussed in the previous section is now defined by

$$P_{mx} = P_{mx,amb} f_p(C) \qquad (4.6a)$$

with the CO_2 response defined by

$$f_p(C) = \left(\frac{C}{C + K_P}\right)\left(\frac{C_{amb} + K_P}{C_{amb}}\right) \qquad (4.6b)$$

where C, ppm, is atmospheric CO_2 concentration, C_{amb} is the current ambient level, taken to be 380 ppm, K_P, ppm, is a constant, and $P_{mx,amb}$ is the value of P_{mx} at ambient CO_2. Equation (4.6b) is a simple Michalis-Menten type response, also referred to as a rectangular hyperbola (Thornley and Johnson, 2000). The term in the second bracket is constant and is there to impose the constraint $P_{mx}(C = C_{amb}) = P_{mx,amb}$. Default values for K_P are:

$$C_3\text{: } K_p = 700; \quad C_4\text{: } K_p = 150 \qquad (4.6c)$$

Equation (4.6b), with parameters (4.6c) is illustrated in Fig. 4.5. Note that the lower response for C_4 plants is due to the lack of photorespiration – for C_3 plants elevated CO_2 not only increases the rate of carbon fixation but it also reduces photorespiration.

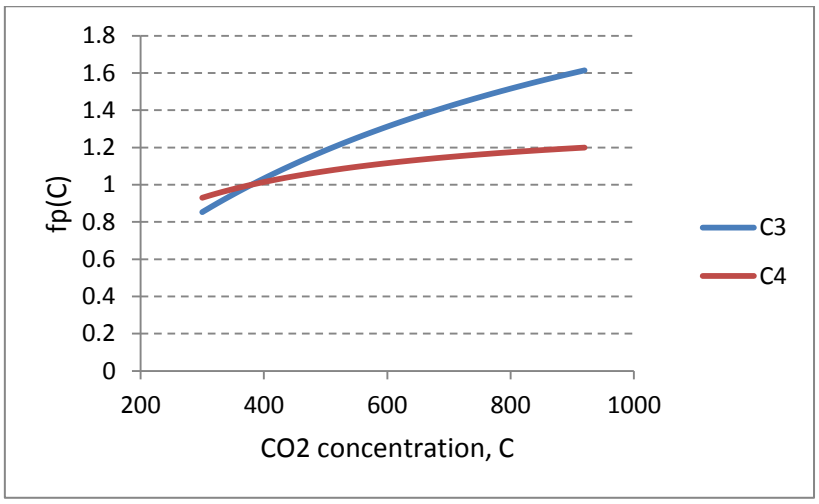

Figure 4.5: CO_2 response function, $f_p(C)$, eqn (4.6b), with C_3 and C_4 parameters given by (4.6c).

Plant nitrogen response to CO_2

The observed decline in plant nitrogen to elevated CO_2 can be explained by a balance of gross photosynthesis and plant respiration (Johnson *et al.*, 1995). In the present analysis, the response of plant nitrogen level, f_N, kg N (kg dry weight)$^{-1}$, to CO_2 concentrations is described by

$$f_N = f_{N,amb} f_N(C) \tag{4.6d}$$

where the N response function is

$$f_N(C) = \left[\lambda + (1-\lambda) \frac{(K_N - C_{amb})^q}{(K_N - C_{amb})^q + (C - C_{amb})^q} \right] \tag{4.6e}$$

where q is a curvature coefficient, and K_N, ppm, and λ are scaling parameters. According to this equation,

$$f_N(C = C_{amb}) = 1$$

$$f_N(C = K) = \frac{1+\lambda}{2} \tag{4.6f}$$

$$f_N(C \to \infty) = \lambda$$

The first of these confirms that the value of f_N at ambient CO_2 is unity; the third that this is reduced by the factor λ at saturating CO_2 while the second shows that, when $C = K$, f_N is the average of the value at ambient and saturated CO_2. The same default values are used for both C_3 and C_4 species, and are

$$\lambda = 0.7; \quad K_N = 600\,\text{ppm}; \quad q = 2 \tag{4.6g}$$

Equation (4.6e) with parameters in (4.6g) is presented in Fig. 4.6

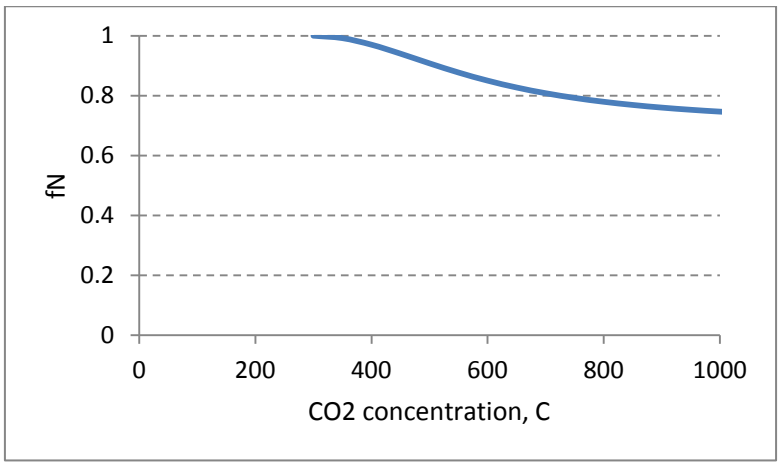

Figure 4.6: CO_2 response function, $f_N(C)$, eqn (4.6e), with parameters given by (4.6g).

Canopy conductance response to CO_2

Turning to canopy conductance, which is the sum of leaf stomatal conductances in the canopy, the decline in response to CO_2 is a result of a greater CO_2 concentration across the stomata so that the required flux of CO_2 can be maintained at lower stomatal conductance. This is described by

$$f_g(C) = g_{c,mn} + (g_{c,mx} - g_{c,mn}) \frac{(1 - g_{c,mn}) C_{amb}^{\beta}}{(g_{c,mx} - 1) C^{\beta} + (1 - g_{c,mn}) C_{amb}^{\beta}} \tag{4.6h}$$

where β is a curvature coefficient and $g_{c,mn}$ and $g_{c,mx}$ are values such that

$$f_g(C = 0) = g_{c,mx}$$
$$f_g(C \to \infty) = g_{c,mn} \tag{4.6i}$$
$$f_g(C = C_{amb}) = 1$$

The default parameters in the model are

$$g_{c,mn} = 0.2; \quad g_{c,mx} = 1.25; \quad \beta = 2.5 \tag{4.6j}$$

Equation (4.6h) with (4.6j) is illustrated in Fig. 4.7.

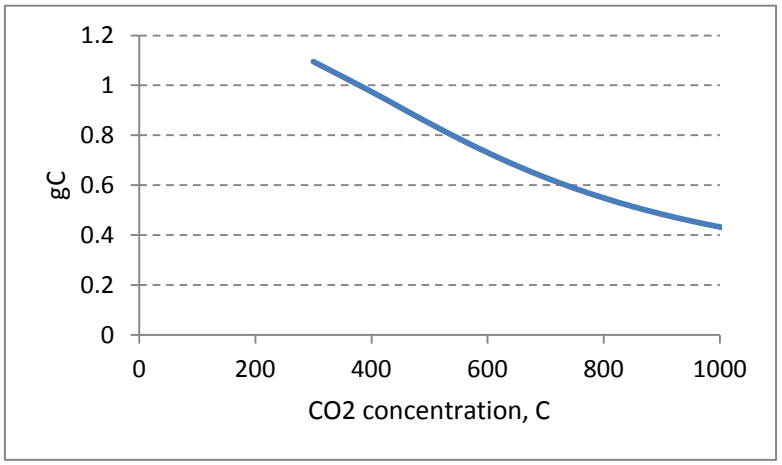

Figure 4.7: CO_2 response function, $f_g(C)$, eqn (4.6h), with parameters given by (4.6j).

4.4.5 Canopy gross photosynthesis

Once the rate of leaf gross photosynthesis, P_ℓ, is known, the rate of canopy gross photosynthesis is then calculated. To do so, it is necessary to define the rate of light attenuation through the canopy and the light intercepted by the canopy. The following treatment of light interception and canopy photosynthesis is taken from Thornley and Johnson (2000), where more detail can be found. Note that no account is made for the direct and diffuse components of radiation, which may result in a slight over-estimate of canopy gross photosynthesis.

The standard approach is to use Beer's law which describes light attenuation through the canopy as a function of the *light extinction coefficient*, k. k is dimensionless although, to be precise, it has dimensions of (m^2 ground) (m^2 leaf)$^{-1}$. The equation was presented in the *Climate* chapter [eqn (3.3)], but is repeated here for convenience:

$$I(\ell) = I_0 e^{-k\ell} \qquad (4.7a)$$

where the symbols are:

I_0	Incident solar radiation on the canopy	W m^{-2} ≡ J m^{-2} s^{-1}
$I(\ell)$	Solar radiation within the canopy	W m^{-2} ≡ J m^{-2} s^{-1}
ℓ	Cumulative leaf area index	(m^2 leaf) (m^2 ground)$^{-1}$
k	Light extinction coefficient	(m^2 ground) (m^2 leaf)$^{-1}$

and $0 < k < 1$.

The simplest physical interpretation of k is that it is the projection of the leaf area onto the ground, so that it must lie in the range $0 < k < 1$, and higher values of k correspond to more prostrate leaves. Typical values for ryegrass and white clover are 0.5 and 0.8 respectively. The rate of light attenuation as the leaf area index, ℓ, increases through the canopy is shown in Fig. 4.8 for these values of k.

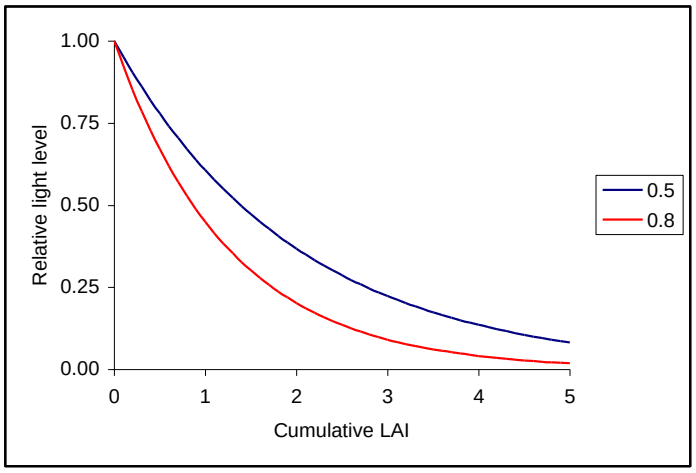

Figure 4.8: Relative light level (to that at the top of the canopy) as the LAI increases down the canopy. Curves are shown for values of the light extinction coefficient, k, of 0.5 and 0.8 as indicated.

While eqn (4.7a) defines the irradiance within the canopy on the horizontal plane, in order to calculate canopy photosynthesis it is necessary to derive the actual irradiance incident on the leaf surface. Ignoring transmission of irradiance through the leaf or reflection from the leaf surface, this is given by:

$$I_\ell(\ell) = kI(\ell) = kI_0 e^{-k\ell} \qquad (4.7b)$$

with units W $(m^2 \text{ leaf})^{-1} \equiv$ J $(m^2 \text{ leaf})^{-1}$ s^{-1}. This analysis applies to randomly distributed leaves, but can be extended to different patterns of leaf distribution (see Thornley and Johnson, 2000).

The rate of canopy gross photosynthesis is then defined by

$$P_c = \int_0^L P_\ell(I_\ell)\,d\ell \qquad (4.7c)$$

where L is the total canopy leaf area index.

Equation (4.7c) is evaluated by combining with eqns (4.4b) and (4.7b). This can be evaluated analytically although the analysis is quite complex (see Thornley and Johnson, 2000).

However, it is commonly observed that leaf photosynthetic capacity depends on the growth light environment. To understand this, consider the leaf photosynthetic response as given in eqn (4.4b) and illustrated in Fig. 4.2. The asymptote, P_{mx}, is closely related to the level of photosynthetic enzymes in the leaf, so that greater values of P_{mx} correspond to greater levels of enzymes. Now, there is a substantial maintenance respiration cost for these enzymes (see section 4.4.7). Consequently, if leaves have a high P_{mx} but are in low light then there is likely to be little benefit from these enzymes in the contribution to gross photosynthesis, while they are incurring a high respiratory cost.

A widely used approximation to define the decline in P_{mx} through the depth of the canopy as a result of the lower growth light environment is to assume that P_{mx} declines in the same way that light does (e.g. Ludlow and Charles-Edwards, 1981; Thornley and Johnson, 2000). This means that

$$P_m(\ell) = P_{m,0} e^{-k\ell} \qquad (4.7d)$$

With this assumption, the analysis for evaluating eqn (4.7c) is quite straightforward and gives

$$P_c = P_\ell\left[I_\ell(\ell=0)\right]\int_0^L e^{-k\ell}\,d\ell = P_\ell\left[I_\ell(\ell=0)\right]\frac{\left(1-e^{-kL}\right)}{k} \qquad (4.7e)$$

which is quite a simple expression to work with.

In the model there is no allowance for stored carbohydrates that allow growth to recover from severe defoliation or adverse conditions. As a simple way of addressing this, it is assumed that the ground cover by the canopy is never lower than a residual value, taken to be 5%. The consequence of this assumption is that species in the model will always persist if growth conditions recover from being severely adverse.

While there are clearly some assumptions and approximations that have been made here, eqn (4.7e) does capture the general processes that contribute to canopy gross photosynthesis, and is at an appropriate level of complexity for the present purposes.

4.4.6 Daily canopy gross photosynthesis

Once the instantaneous rate of canopy gross photosynthesis is known, this is scaled up to the day by integrating over the daylength. Before proceeding, it is necessary to change the units from mg CO_2 to kg C (carbon). This is simple, and the rate of canopy gross photosynthesis, P_c mg CO_2 m^{-2} s^{-1}, becomes $\tilde{P}_c$ kg C m^{-2} s^{-1}, which is given by

$$\tilde{P}_c = 10^{-6} \times \frac{12}{44} P_c \qquad (4.8a)$$

where the factor 12 / 44 converts from CO_2 to C.

Unit conversions in models are a potential source of programming error and are not generally recommended. However, with a model that moves from the leaf to the canopy it is desirable to use commonly used units in both areas of study. While it is, of course, easy to express leaf photosynthetic rates in kg rather than mg, this is rarely done by experimentalists. Provided care is taken, these conversions will not cause problems.

Diurnal variation in irradiance and temperature will affect the calculations and so some allowances for this variation need to be made. One option is to use actual data for these inputs throughout the day and integrate numerically, although that is not done here as it cannot be assumed that such data will always be available. Another option is to use mean daily values of irradiance and temperature. While this is quite appealing, it has been shown (eg Thornley and Johnson, 2000) that, because of the non-linear nature of the leaf photosynthetic response function to irradiance and temperature, using mean daily values for these inputs generally leads to an over-estimate of daily canopy gross photosynthesis. A more flexible approach is to use a distribution such as a sine curve to describe the diurnal distribution of irradiance and temperature. However, using the non-rectangular hyperbola analytical progress is cumbersome, although again it would be possible to integrate numerically through the day. The danger here is that the model would become unnecessarily complex when some simplification would be satisfactory. The simplification used here is to break the diurnal irradiance and temperature distributions into piece-wise linear functions.

Let the daily solar radiation be denoted by R_S (J m^{-2} day^{-1}) and the daylength by τ secs. The diurnal distribution for irradiance is illustrated in Fig. 4.9, where

$$\tilde{I} = \frac{4}{3} \frac{R_S}{\tau} \qquad (4.8b)$$

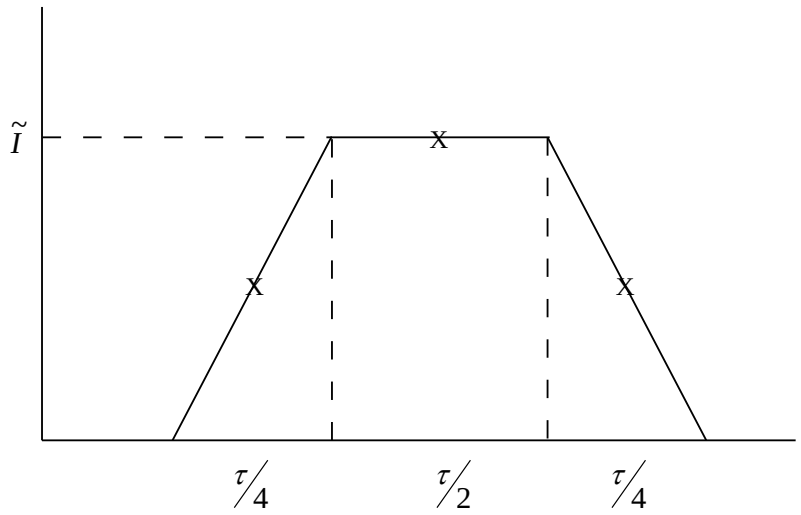

Figure 4.9: Schematic representation of the diurnal variation in irradiance.

Neglecting variation in temperature for the moment, this distribution is used by assuming that, for the three periods represented by the lines, the irradiance can be defined at the X symbol. Daily canopy gross photosynthesis is then given by:

$$\tilde{P}_g = \frac{\tau}{4}\tilde{P}_c\left(\frac{\tilde{I}}{2}\right) + \frac{\tau}{2}\tilde{P}_c\left(\tilde{I}\right) + \frac{\tau}{4}\tilde{P}_c\left(\frac{\tilde{I}}{2}\right) = \frac{\tau}{2}\left\{\tilde{P}_c\left(\frac{\tilde{I}}{2}\right) + \tilde{P}_c\left(\tilde{I}\right)\right\} \quad (4.8c)$$

with units kg C $(m^2$ ground$)^{-1}$ day^{-1}.

With temperature, a similar scheme is adopted. First note that the inputs to the model are mean, minimum and maximum daily temperature (as discussed in the *Climate* chapter). If these are not all available as inputs, the missing ones are estimated. In the calculation of daily gross photosynthesis, it is assumed that:

- during the periods of increasing and decreasing irradiance, the representative temperature is the mean daily value;
- during the period when irradiance is at the $\tilde{I}$ value, temperature is at the mean of the mean and maximum.

While refinements to these assumptions could be implemented, they do attempt to encapsulate the essence of diurnal variation, are an improvement on using mean values only, and avoid analytical or numerical complexity associated with having smoothly varying functions for diurnal patterns of irradiance and temperature which will, inevitably, be approximations only.

Thus, eqn (4.8c) can be re-written to give

$$\tilde{P}_g = \frac{\tau}{2}\left\{\tilde{P}_c\left(\frac{\tilde{I}}{2}, T_{mean}\right) + \tilde{P}_c\left(\tilde{I}, \frac{T_{mx} + T_{mean}}{2}\right)\right\} \quad (4.8d)$$

Note that if T_{mean} is the average of the maximum and minimum temperatures, which need not be the case, then

$$T = \frac{1}{4}\left(T_{mn} + 3T_{mx}\right) \quad (4.8e)$$

Influence of available water and nutrients

The expression in eqn (4.8d) includes the effects of the light and temperature environment, as well as current plant nutrient status through eqn (4.5g). It now remains to incorporate the influence of water stress as well as any further stress due to actual available nutrient uptake. In the model, the growth limiting factors due to available water or nutrients are calculated from the actual and potential transpiration rate, and actual and required nutrient uptake. These were given in sections 4.2 and 4.3, and are denoted by GLF_{water} and GLF_{nu}, which are variables between 0 and 1, with 0 representing full stress and 1 representing no stress.

The effects are incorporated by writing eqn (4.8d) as

$$P_g = GLF_{water} \times \sqrt{GLF_{nu}} \times \tilde{P}_g \quad (4.8f)$$

The square root of GLF_{nu} is used to make it a second order effect, since nutrients also influence photosynthesis through the nutrient status of the leaves.

There is a problem with circularity here. The GLF_{nu} is calculated in terms of actual nutrient uptake, but this, in turn, is related to the growth potential. To avoid this problem, the GLF_{nu} used in eqn (4.8e) is taken from the previous day.

Influence of temperature extremes

A limitation of the approach that leads to eqn (4.8d) is that, while the representative daytime temperature may be quite reasonable for the photosynthetic process, there may be day or night extremes that restrict growth. For example, in southern Queensland daily winter temperatures are adequate for the growth of C_4 pasture species, but low night temperatures can have a severe adverse effect on growth. Similarly, in Victoria the summer peak day temperatures can restrict the growth of some C_3 species, which might not be reflected by the daytime temperature used in eqn (4.8d). A simple approach to this problem is to define generic functions for initial impact and then persistence of temperature extremes.

First define the functions $\varphi_{g,mn}$ and $\varphi_{g,mx}$ as ramp functions shown in Fig. 4.10:

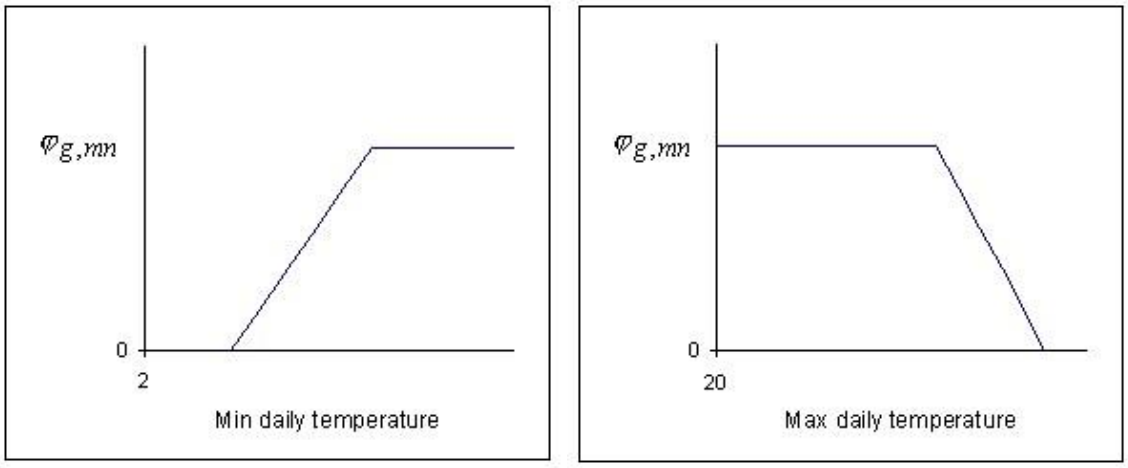

Figure 4.10: Limitation functions for daily temperature extremes..

These define the impact on any day of the temperature extremes, so that this can vary between 0 (no effect) to 1 (full effect). While this approach applied on each day can be of some use, it does not incorporate persistence.

To define the persistent temperature stress function, first consider the situation where a stress occurs for one day only. In this case, the persistent stress function is defined by:

$$\Phi_T\left(\tau_{stress}\right)=\phi_{g,0}+\left(1-\phi_{g,0}\right)\min\left(1,\frac{T_{sum}}{T_{sum\_crit}}\right) \tag{4.9a}$$

where $\varphi_{g,0}$ is the initial impact, between 0 and 1, as shown in Fig. 4.10. The temperature sum terms are

$$T_{sum}=\sum\max\left(0,T_{mean}\right) \tag{4.9b}$$

for low temperature stress and

$$T_{sum}=\sum\max\left(0,25-T_{mean}\right) \tag{4.9c}$$

for high temperature stress. The sum starts each day there is an occurrence of a temperature stress, that is $\varphi_{g,mn}$ or $\varphi_{g,mx}<1$.

As time progresses, the function approaches 1 and when this happens the stress effect has finished.

Now consider the situation where another stress event occurs while Φ_T is still active (that is, less than 1). In this case, the stress impact is calculated from $\varphi_{g,mn}$ or $\varphi_{g,mx}$ (depending on whether it is a high or low temperature stress) and the new starting value to use in eqn (4.8a) is

$$\phi_0 = \Phi_T \phi_g \qquad (4.9d)$$

so that the new effect is compounded with the old effect. The calculation in (4.9a) now starts again with the number of days since the stress being reset to zero.

This approach is completely empirical but represents a simple way of incorporating the persistence and compounding of temperature extremes.

4.4.7 Dark respiration

It is now necessary to calculate the daily respiration rate. Respiration is calculated using the McCree-Thornley (1970) approach (McCree, 1970; Thornley 1970), that has been further developed by others (eg Johnson, 1990), and is widely used. According to the McCree-Thornley approach, respiration can be regarded as having two functional components: growth and maintenance. Growth respiration is the respiration associated with the synthesis of new plant material, while maintenance is the respiration required primarily to provide energy for the re-synthesis of degraded proteins. Growth respiration is related to the growth rate of the plant, or daily carbon assimilation, while maintenance respiration is proportional to the plant d.wt. To be more precise, maintenance should be adjusted to reflect the actual protein content which may vary in response to available nutrients, particularly nitrogen. For a background on this treatment of respiration, see Johnson (1990), Thornley and Johnson (2000) or Thornley and France (2008).

In its standard form, the McCree-Thornley equation can be written:

$$R_d = \left(\frac{1-Y}{Y}\right) G + mW_{live} \qquad (4.10a)$$

where the symbols are, using carbon as the common unit:

R_d	Daily dark respiration	kg C m^{-2} day^{-1}
G	Total daily growth (shoot and root)	kg C m^{-2} day^{-1}
W_{live}	Live dry weight (shoot plus root)	kg C m^{-2}
Y	Growth efficiency	dimensionless
m	Maintenance respiration coefficient	day^{-1}

The respiration parameters are interpreted as follows:

- Y is the efficiency with which new plant material is synthesised. Typically, this is about 0.8, which means that for every 1 kg of substrate utilized for the synthesis of plant structural material, 0.2 kg is respired and 0.8 kg of structure is produced.
- m represents the relative maintenance cost for plant dry weight. For C_3 plants this is around 0.03 to 0.04 day^{-1}, which means that maintenance costs may be between 3 and 4% of the actual dry weight per day.

Growth respiration

It is readily shown that

$$R_g = \left(\frac{1 - Y_g}{Y_g} \right) G \qquad (4.10b)$$

where Y_g is the efficiency of synthesising plant material where the protein component is synthesised from amino acids. In the model, the default is

$$Y_g = 0.75 \qquad (4.10c)$$

although the user can change this value.

Maintenance respiration

The maintenance respiration for the whole plant is defined as

$$R_m = m_{20,Nref} f_m(T) \frac{f_{N,live}}{f_{N,ref}} \left(W_{sh,live} + \eta W_{r,live} \right) \qquad (4.10d)$$

where $m_{20,Nref}$, d^{-1}, is the maintenance coefficient at the reference temperature of 20°C and at the reference plant nitrogen content $f_{N,ref}$, kg N (kg C)$^{-1}$; $f_{N,live}$ is the nitrogen content of the live shoot and root plant material, $W_{sh,live}$ and $W_{r,live}$; and $f_m(T)$ is a temperature response function with $f_m(T = 20) = 1$.

The default maintenance respiration coefficient in the model is

$$m_{20,Nref} = 0.03 \ d^{-1} \qquad (4.10e)$$

Note that, for plants that cannot accumulate nitrogen to the reference level, the actual maintenance coefficient will not reach this value. This is likely to be significant for C_4 plants that generally have lower nitrogen contents than C_3.

The temperature response for maintenance respiration differs from that for photosynthesis primarily in that it does not decline at moderately high temperatures. Recall that, at least for C_3 plants, the decline in leaf photosynthesis as temperature increases is due to the activity of photorespiration. While there is opportunity to use detailed temperature responses for maintenance respiration, a simple linear approach is adopted, so that

$$f_m(T) = \frac{T}{20}. \qquad (4.10f)$$

While this may be inaccurate at very high temperatures, in such situations the response is likely to be dominated by the effects of temperature extremes described earlier.

The reference nitrogen level is taken to be 4% by dry weight which, taking the dry weight as 40% carbon, is equivalent to

$$f_{N,ref} = 0.1 \quad \text{kg N (kg C)}^{-1} \qquad (4.10g)$$

4.5 Daily pasture net carbon assimilation

The total daily carbon assimilation is the net effect of photosynthesis and respiration prior to any tissue senescence. Since the respiratory costs on nitrogen uptake and fixation are different, and since legumes will always meet their nitrogen requirements for optimum growth through nitrogen fixation, the theory for non-legumes and legumes is treated separately.

Assuming that all carbon assimilated in the day is utilised for growth and respiration, the carbon balance in the plant, that is the shoot and root, in the absence of any effect of nutrient limitation (which is considered later) can be written as

$$G^* + R_g + R_m = P_g + S_C \qquad (4.11a)$$

where the symbols are

G^*	Growth (shoot plus root)	kg C m^{-2} day^{-1}
R_g	Growth respiration	kg C m^{-2} day^{-1}
R_m	Maintenance respiration	kg C m^{-2} day^{-1}
P_g	Daily gross photosynthesis (eqn 4.7d)	kg C m^{-2} day^{-1}
S_C	Remobilised carbon from senescent tissue	kg C m^{-2} day^{-1}

Remobilisation, which was defined earlier in terms of the rate of senescence of live tissue, relates primarily to the recycling of amino acids as leaves senesce and the proportion of nutrients that are recycled can be defined by the user. It is generally not a significant component of the carbon balance, but can play an important role in the nutrient dynamics.

Combining eqns (4.10b, 11a) gives

$$\frac{G^*}{R_g} = P_g + S_C - R_m \qquad (4.11b)$$

which defines the daily growth.

4.5.1 Influence of nutrient stress

Equation (4.11b) defines the potential daily growth with no nutrient stress. If nutrients are limiting then growth will be reduced. The growth limiting factor, GLF_{nu}, was defined earlier in eqn (4.2j) and, using this, the actual growth is given by

$$G = GLF_{nu} G^* \qquad (4.11c)$$

Once the actual growth is known, nutrient uptake and remobilisation are then adjusted, if necessary, to ensure that nutrient acquisition does not exceed the maximum plant nutrient concentration.

4.5.2 Legumes

Legumes require special consideration due to nitrogen fixation. The treatment of nitrogen fixation is quite simple and yet it encapsulates the main principles that:

- fixation always occurs to some extent, even in legumes with abundant nitrogen supplied;
- fixation increases as nitrogen uptake is limited;
- the plant nitrogen level never falls below optimum.

It is assumed that $GLF_{N,leg}$ is related to the actual value GLF_N in response to available soil N by

$$GLF_{N,leg} = \eta + (1-\eta) GLF_N \qquad (4.11d)$$

which is illustrated in Fig. 4.10. According to this equation, as available soil nitrogen becomes totally limiting to growth, the legume growth limiting factor for nitrogen approaches the value η, with default value

$\eta = 0.6$. (4.11e)

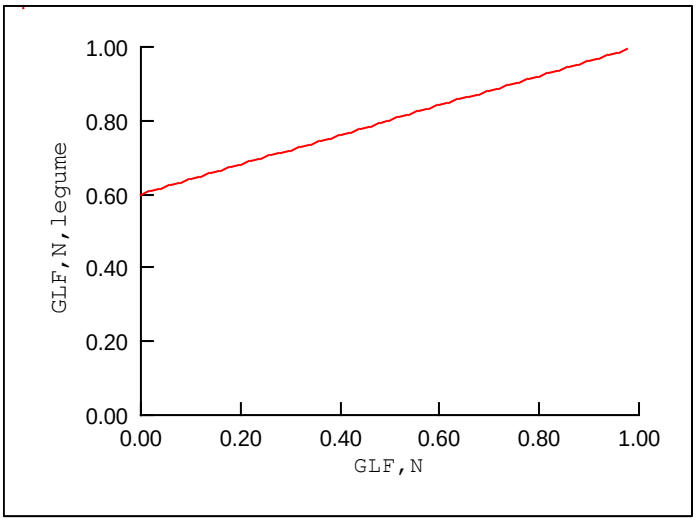

Figure 4.10: Legume N growth limiting factor in relation to the soil N growth limiting factor.

$GLF_{N,leg}$ is used to define the N limitation to growth in eqn (4.10c) to calculate the actual growth, G, in eqn (4.11c).

Nitrogen fixation by the legumes is then calculated to ensure that the plant N concentration is not less than the optimum required. In addition, a minimum fraction of total N acquisition that is through fixation occurs which, by default is 20% (the user can change this value).

4.6 Pasture growth, senescence and development

A key feature of the treatment of pasture growth here is the turnover of plant tissue, which has been shown to have a major impact on pasture growth and utilisation (eg Parsons, 1988). The flow of tissue through the system is shown schematically in Fig. 4.11:

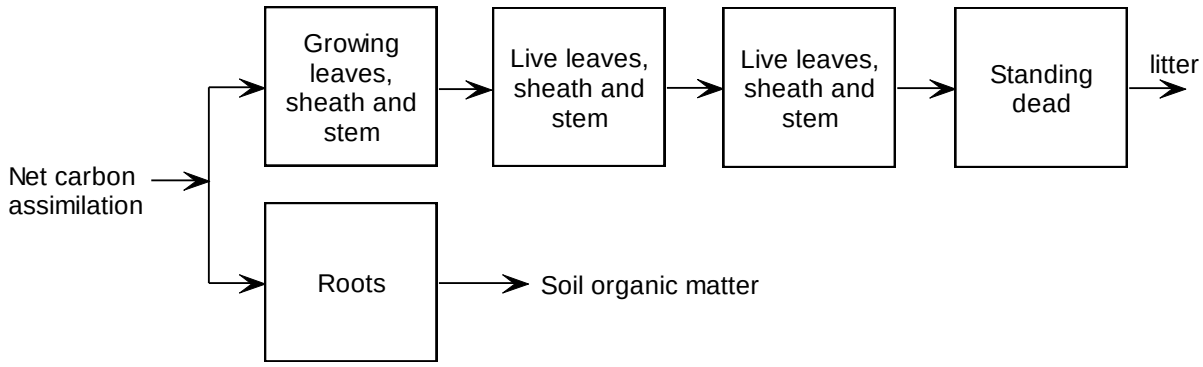

Figure 4.11: Schematic representation of growth, tissue turnover and senescence.

According to this scheme there are leaf categories, including the associated sheath and stem, corresponding to growing leaves, two categories of live leaves, and standing dead. New growth goes to the growing leaf box. There is a flow of tissue through the categories until it is transferred to the litter. Carbon is also partitioned to the roots, although separate root categories are not included. Root senescence passes straight to the soil organic matter pool.

For legumes there is also a stolon component, which is not shown in the figure. The stolon dynamics are similar to those for leaves, and sheath plus stem, although there is no stolon contribution to the

standing dead, which means that the flux out of the third box for the stolons goes directly to the litter pool.

The net carbon assimilation was described in detail in the previous section and the dynamics of growth, partitioning, tissue turnover and senescence are now considered.

First, the basic state variables in the model are defined, including stolons if present.

Live leaf:

$$W_{live,\ell,i},\ i = 1\text{to}3 \qquad (4.12a)$$

Total live leaf:

$$W_{live,\ell} = \sum_{i=1}^{3} W_{live,\ell,i} \qquad (4.12b)$$

Live sheath + stem

$$W_{live,s,i},\ i = 1\text{to}3 \qquad (4.12c)$$

Total live sheath+stem:

$$W_{live,s} = \sum_{i=1}^{3} W_{live,s,i} \qquad (4.12d)$$

Live stolon

$$W_{live,st,i},\ i = 1\text{to}3 \qquad (4.12e)$$

Total live stolon:

$$W_{live,st} = \sum_{i=1}^{3} W_{live,st,i} \qquad (4.12f)$$

Total live shoot:

$$W_{live,shoot} = W_{live,\ell} + W_{live,s} + W_{live,st} \qquad (4.12g)$$

Dead leaf $W_{dead,\ell}$

(4.12h)

Dead sheath + stem $W_{dead,s}$

(4.12i)

Root live W_r

(4.12j)

4.6.1 Shoot:root partitioning

Shoot:root partitioning is important in terms of the plant's ability to access water and nutrients, its impact on shoot, and therefore harvestable, growth, and also on the supply of organic matter to the soil for soil organic matter and inorganic nutrient dynamics. Various models of shoot:root partitioning are discussed in Thornley and Johnson (2000). The models partition growth in a way that attempts to balance the requirements between resources acquired by the shoot and root respectively. For example, if water is limiting then plants will partition a greater proportion of growth to the roots to attempt to increase water uptake.

Define the proportion of newly assimilated carbon that is partitioned to the shoots by λ, with the remaining $(1-\lambda)$ being partitioned to the roots. Let the live shoot:root ratio be

$$\rho = \frac{W_{live,shoot}}{W_r} \tag{4.13a}$$

and let it take the value ρ_{mx} when root conditions are non-limiting. The ratio of carbon partitioned to the shoots to that partitioned to the roots is now defined by

$$\frac{\lambda}{1-\lambda} = GLF\,\rho_{mx}\frac{\rho_{mx}}{\rho} \tag{4.13b}$$

According to this equation, if the overall growth limiting factor, GLF, is 1, which means there is no water or nutrient stress, and if the shoot:root ratio is ρ_{mx}, then the partitioning ratio is ρ_{mx}. As the GLF declines partitioning shifts in favour of the roots, and as ρ declines, so that there is relatively greater root material, partitioning shifts in favour of the shoots. Equation (4.13b) is rearranged to give

$$\lambda = \frac{GLF\,\rho_{mx}^2/\rho}{1+GLF\,\rho_{mx}^2/\rho}. \tag{4.13c}$$

4.6.2 Leaf, sheath plus stem, and stolon partitioning

The above ground material now has to be partitioned into the leaf, sheath/stem and stolon components, where relevant. The sheath and stem are included in a single component. These partitioning coefficients are taken to be fixed.

The new growth to the shoot is λG, where the derivation of the total growth G is derived in sections 4.5.1 and 4.5.2, and λ is given by (4.13c). This growth is partitioned as follows:

Stolon: $f_{stolon}\lambda G$ (4.14a)

Leaf: $(1-f_{stolon})f_{leaf}\lambda G$ (4.14b)

Sheath+stem: $(1-f_{stolon})(1-f_{leaf})\lambda G$ (4.14c)

According to these equations, f_{stolon} is the proportion of above ground d.wt that is partitioned to the stolon. The remainder, $(1-f_{stolon})$, is partitioned to the leaf and sheath plus stem and, of this f_{leaf} goes to the leaf and $(1-f_{leaf})$ to the sheath plus stem. For many plants, f_{stolon} is zero so that f_{leaf} is just the new growth that is partitioned to the leaf.

4.6.3 Leaf area index

The leaf area index is required to calculate light interception, which in turn is used for the calculation of photosynthesis and also for the ground cover components which influence evaporation. This is evaluated from the leaf component of dry weight using the specific leaf area, σ m^2 leaf (kg C)$^{-1}$, so that the live and dead leaf area index components, m^2 leaf (m^2 ground)$^{-1}$, are

$$L_{live} = \sigma W_{live,\ell} \tag{4.15a}$$

$$L_{dead} = \sigma W_{dead,\ell} \tag{4.15b}$$

where

σ= 15 (m^2 leaf) (kg d.wt)$^{-1}$ ≡ 37.5 m^2 leaf (kg C)$^{-1}$ (4.15c)

being used here.

Note that although the incremental specific leaf area is constant, the leaf fraction of dry weight can vary since leaf and stem can be grazed with different relative intakes, as discussed in the *Animal* chapter.

4.6.4 Tissue turnover

Figure 4.11 shows the general scheme for tissue turnover and senescence. Defining the flux parameter as γ, day^{-1}, (with subscript $dead$ for the flux from standing dead to litter), the flux of material between each category can be defined as:

$$flux(1 \to 2) = 2\gamma W_{live,1} \quad (4.16a)$$

$$flux(2 \to 3) = \gamma W_{live,2} \quad (4.16b)$$

$$flux(3 \to dead) = \gamma W_{live,3} \quad (4.16c)$$

$$flux(dead \to litter) = \gamma_{dead} W_{dead} \quad (4.16d)$$

The factor 2 in eqn (4.16a) allows for the fact that the mean d.wt of leaves in the growing leaf category will be about half of the fully grown leaves in that category.

Influence of temperature and water stress

The rate of flux of material through the system is dependent on temperature and water stress, and these must now be considered. In the following analysis simple linear expressions are used.

The flux parameter γ is defined by

$$\gamma = f_{\gamma,T} f_{\gamma,W} \gamma_{ref} \quad (4.16e)$$

where $f_{\gamma,T}$ and $f_{\gamma,W}$ are response functions to temperature and water stress that lie between 0 and 1, and γ_{ref} is the flux parameter when there is no stress.

Leaf turnover increases as temperature increases but, unlike photosynthesis, it does not decline at high temperatures. The influence of temperature on leaf turnover is therefore defined by

$$f_{\gamma,T} = \begin{cases} 0, & T \le T_{mn;} \\ \dfrac{(T - T_{mn})}{(T_{opt} - T_{mn})}, & T_{mn} \le T \le T_{opt}; \\ 1, & T > T_{opt}; \end{cases} \quad (4.16f)$$

which is a simple ramp expression. The default parameters used in the model are

$$T_{mn} = 5 \text{ and } T_{opt} = 20. \quad (4.16g)$$

Equation (4.15f) with (4.15g) is illustrated in Fig. 4.12.

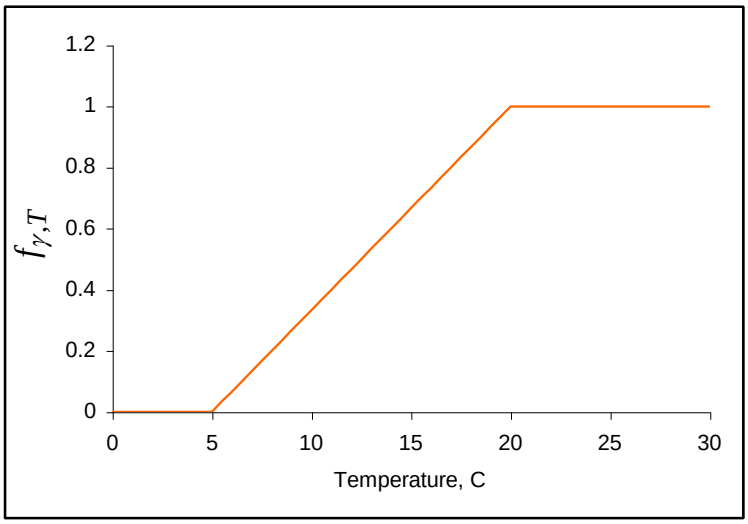

Figure 4.12: Influence of temperature on flux of leaves through the system, as given by eqn (4.17f) with (4.17g).

For water stress, it is assumed that tissue turnover increases as water stress increases, and the influence of water stress is defined through the growth limiting factor, GLF_{water}. Again, a simple expression is used, given by:

$$f_{\gamma,W} = \begin{cases} \left(f_{\gamma,W0} - 1\right)\left(\dfrac{GLF_{water,opt} - GLF_{water}}{GLF_{water,opt}}\right) + 1, & GLF_{water} \le GLF_{water,opt} \\ 1, & GLF_{water,opt} \le GLF_{water} \end{cases} \qquad (4.16h)$$

with the parameters

$$f_{\gamma,W0} = 2 \quad \text{and} \quad GLF_{water,opt} = 0.5\,. \qquad (4.16i)$$

Equation (4.16h) with (4.16i) is illustrated in Fig. 4.13.

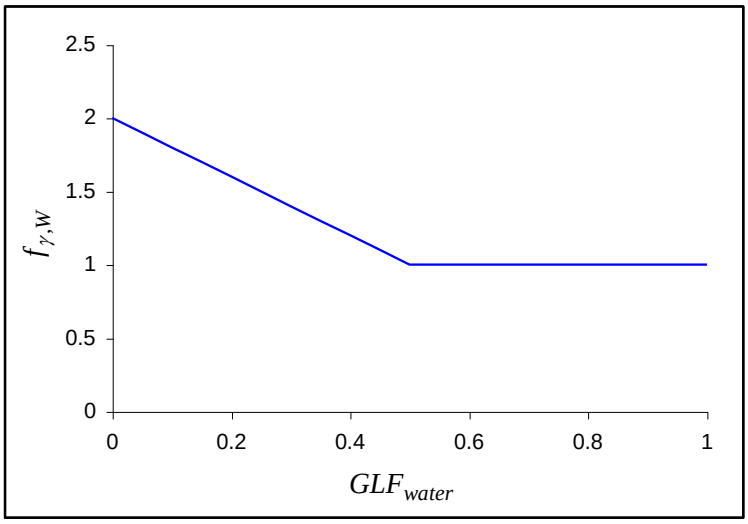

Figure 4.13: Influence of water growth limiting factor on flux of leaves through the system, as given by eqn (4.16h) with (4.16i).

Relation to leaf appearance rate

The flux parameter γ clearly will depend on the number of leaves per tiller and the number of boxes that are included for the schematic representation of leaf turnover – three live plus one dead in the present approach (see fig. 4.11). It is quite straightforward to show that:

$$\gamma_{ref} = \lambda_{ref} \frac{\text{number of boxes}}{\text{live leaves per tiller}} \qquad (4.16j)$$

where λ_{ref} is the reference rate of leaf with no temperature or water stress. The parameter λ_{ref} is has a more accessible physiological interpretation than γ_{ref}. and is available on the interface.

The leaf appearance rate is not required in the model, but is sometimes used for pasture management. This is calculated in a slightly different way to γ, even though γ is related to the leaf appearance rate with no water stress and at the reference temperature. The reasons for this difference are that, for C_3 plants, water stress or high temperatures will increase leaf senescence and therefore the flux of material through the categories, but will reduce the rate of leaf appearance. For C_4 plants it is assumed that high temperatures do not reduce the rate of leaf appearance. The actual rate of leaf appearance is therefore defined by

$$\lambda = \lambda_{ref} f(T) GLF_{water}^{1/3} \qquad (4.16k)$$

where $f(T)$ is the temperature function given by either eqn (4.5b) or (4.5e). The 1/3 power is introduced to limit the effect of water stress on the leaf appearance rate. For example with a fairly moderate water limitation of $GLF_{water} = 0.8$, the impact on the leaf appearance rate is through the value $0.8^{1/3} = 0.93$.

Flux from standing dead to litter

The treatment for the flux from standing dead to litter is similar, but not identical, to that for the flux between the live categories (including live to standing dead). In this case, water stress reduces the flux, and temperature effects are not included. However, the effect of trampling by animals is included, and the equation used is:

$$\gamma_{dead} = \gamma_{dead,ref} GLF_{water}^{3} \frac{dig_{dead}}{0.4} + \gamma_{stock} \rho_{stock} \qquad (4.16l)$$

where the parameters are:

dig_{dead} digestibility of dead material (fraction)

γ_{stock} stock influence parameter

ρ_{stock} stock density, animals ha^{-1}

According to this approach, the transfer of standing dead to litter decreases with water stress, increases with digestibility of standing dead material, and increases with stock density. The default value for γ_{stock} is 5% per 100 animals ha^{-1}, which means, for example, that if the stock density is 100 animals ha^{-1} then they will trample 5% of the standing dead in a day.

Root senescence rate

The root senescence rate is treated similarly, but with a simple linear dependence on GLF_{water}, so that

$$\gamma_r = \gamma_{r,opt} f_{\gamma,T} (2 - GLF_{water}) \qquad (4.16m)$$

4.6.5 Root distribution

The distribution of roots is important because of its influence on factors such as water and nutrient uptake as well as the input to the soil organic matter. For vegetative species, the root depth is taken to be constant, whereas for annual species it increases to its maximum value at anthesis (flowering), as described later.

The relative root distribution by weight is shown in Fig 4.14, and is defined by:

$$f_r(z) = \frac{1}{1 + \left(\frac{z}{d_{r,h}} \right)^{q_r}} \tag{4.17}$$

where $d_{r,h}$ is the depth for 50% relative root mass and q_r is a scaling parameter. This is a convenient empirical approach, whereby $f_r = 0.5$ when $z = d_{r,h}$.

Root distribution has also been described according to an exponential equation, as used by Gerwitz and Page (1974) when they analysed a large range of root distribution data. However, the data are very variable and the sigmoidal pattern is probably preferable as it allows for a concentration of roots near the surface, sometimes referred to as the plough layer.

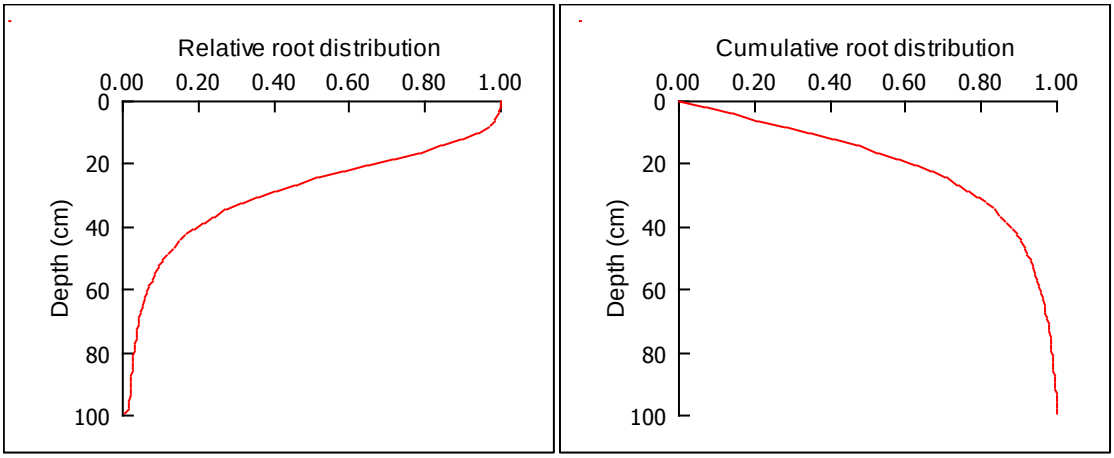

Figure 4.14: Relative root distribution, and corresponding cumulative root distribution as a function of depth using eqn (4.17), with $d_{r,h} = 25$ cm, q_r=3, and a total root depth of 100 cm.

4.6.6 Governing equations

The governing equations for tissue dynamics can now be written using the symbols introduced in eqns (4.12a-j).

Leaf

$$\frac{\mathrm{d}W_{live,\ell,1}}{\mathrm{d}t} = (1 - f_{stolon}) f_{leaf} \lambda G - 2\gamma W_{live,\ell,1} \tag{4.18a}$$

$$\frac{\mathrm{d}W_{live,\ell,2}}{\mathrm{d}t} = 2\gamma W_{live,\ell,1} - \gamma W_{live,\ell,2} \tag{4.18b}$$

$$\frac{\mathrm{d}W_{live,\ell,3}}{\mathrm{d}t} = \gamma W_{live,\ell,2} - \gamma W_{live,\ell,3} \tag{4.18c}$$

$$\frac{\mathrm{d}W_{dead,\ell}}{\mathrm{d}t} = \gamma W_{live,\ell,3} - \gamma_{dead} W_{dead,\ell} \tag{4.18d}$$

Sheath + stem

$$\frac{\mathrm{d}W_{live,s,1}}{\mathrm{d}t} = \left(1 - f_{stolon}\right)\left(1 - f_{leaf}\right)\lambda G - 2\gamma W_{live,s,1} \tag{4.18e}$$

$$\frac{\mathrm{d}W_{live,s,2}}{\mathrm{d}t} = 2\gamma W_{live,s,1} - \gamma W_{live,s,2} \tag{4.18f}$$

$$\frac{\mathrm{d}W_{live,s,3}}{\mathrm{d}t} = \gamma W_{live,s,2} - \gamma W_{live,s,3} \tag{4.18g}$$

$$\frac{\mathrm{d}W_{dead,s}}{\mathrm{d}t} = \gamma W_{live,s,3} - \gamma_{dead} W_{dead,s} \tag{4.18h}$$

Stolon

$$\frac{\mathrm{d}W_{live,st,1}}{\mathrm{d}t} = f_{stolon}\lambda G - 2\gamma W_{live,st,1} \tag{4.18i}$$

$$\frac{\mathrm{d}W_{live,st,2}}{\mathrm{d}t} = 2\gamma W_{live,st,1} - \gamma W_{live,st,2} \tag{4.18j}$$

$$\frac{\mathrm{d}W_{live,st,3}}{\mathrm{d}t} = \gamma W_{live,st,2} - \gamma W_{live,st,3} \tag{4.18k}$$

Root

$$\frac{\mathrm{d}W_{r}}{\mathrm{d}t} = \left(1 - \lambda\right)G - \gamma_r W_r \tag{4.18l}$$

Input to litter

From these equations it is immediately apparent that the input to the litter is

$$\Lambda_{litter} = \gamma_{dead}\left(W_{dead,\ell} + W_{dead,s}\right) + \gamma W_{live,st,3} \tag{4.18m}$$

Nutrient dynamics

Nutrient dynamics are implemented by applying these equations to carbon and all the nutrients that the user has implemented – available nutrients are N, P, K, S.

Input to soil organic matter from roots

Root senescence enters into the soil organic matter – soil organic matter dynamics are discussed in *Soil nutrient dynamics* (chapter 6). These inputs are distributed across soil layers according to eqns (4.17) and (4.18l) which define the input of dead roots to organic matter through the depth of the soil.

4.6.7 Annual species

Annual species are treated in a similar way to perennial species but the phase duration for vegetative and reproductive growth, and growth to maturity during vegetative growth are incorporated.

Phase duration

The only phases considered are vegetative, from germination to anthesis, and reproductive, which is anthesis to maturity. Germination is specified by date and so the model does not include the response of germination to climate (temperature and rainfall). However, growth will depend on these factors and, under poor conditions, early growth will suffer accordingly.

Models of phase duration in relation to factors such as cumulative temperature, photoperiod, irradiance, daylength, and so on, have been developed – for a discussion, see Thornley and Johnson (2000). However, since the aim here is to develop the theory for generic pasture systems, a simpler approach is taken whereby the phase duration is specified directly. While this does not encapsulate variation in phase duration in response to seasonal fluctuations in environmental conditions, it is simple to apply and provides an acceptable treatment of pasture growth and development for the present purposes.

The parameter k_{sum} is introduced, which is:

$$k_{sum} = \frac{\tau}{\tau_{phase}} \tag{4.19a}$$

where τ is the day number during the phase and τ_{phase} is the total number of days in the phase. This defines the phase development on a linear scale from the start to the end of the phase. The phase is complete when $k_{sum} = 1$.

Root distribution

It is assumed that for annuals root depth increases to the maximum during vegetative growth, so that

$$d_r = 0.05 + \left(d_{r,mx} - 0.05\right)\frac{\tau}{\tau_{veg}} \tag{4.19b}$$

where τ is the number of days since germination and τ_{veg} is the duration of the vegetative phase in days as described above. Thus, the initial root depth is 5 cm, increasing to the defined maximum at anthesis.

Equation (4.17) is now extended for annuals and written as

$$f_r\left(z\right) = \frac{1}{1 + \left(\frac{z}{d_{r,h}}\frac{d_{r,mx}}{d_r}\right)^{q_r}} \tag{4.19c}$$

so that it follows that annuals will have the same relative distribution as the root depth increases – that is, if $f_r = 0.5$ when $z = d_{r,h}\, d_r / d_{r,mx}$.

Vegetative growth

This is treated identically to that for perennials with the exception of the flux parameter. Since annuals tend to accumulate leaves before senescence starts to be significant, it is assumed that

$$\gamma_{veg} = \gamma\, k_{sum} \tag{4.19d}$$

and

$$\gamma_{r,veg} = \gamma_r\, k_{sum} \tag{4.19e}$$

Reproductive growth

The onset of reproductive growth, or anthesis, signifies the start of a reduction in leaf photosynthetic capacity, leaf growth, and an increase in senescence. Maturity is when the pasture growth is complete and growth ceases. During reproductive growth two key processes occur:

- there is a decline in the growth rate due to the decline in leaf photosynthesis;
- plant senescence occurs until maturity when all plant material is dead.

The decline in growth rate is described by introducing the scaling parameter to the leaf photosynthesis parameter

$$P_{m,opt,rep} = (1 - k_{sum}) P_{m,opt,rep} \qquad (4.19f)$$

Recall that k_{sum}, from eqn (4.19a) is zero at the start of the phase, increasing to unity at the end of the phase.

The flux parameter γ for the shoot as maturity approaches, is defined according to:

$$\gamma_{rep} = 1 - (1 - \gamma)\left(1 - k_{sum}^2\right) \qquad (4.19g)$$

so that as the end of the phase approaches there is an acceleration in movement of material through the system and all plant material is dead. The exponent 2 in this equation causes the senescence to increase towards the end of the phase.

The flux constant for dead material to litter is unaltered during this phase.

There is no root growth during the reproductive phase, and root senescence also accelerates, so that

$$\gamma_{r,rep} = 1 - (1 - \gamma_r)\left(1 - k_{sum}^2\right). \qquad (4.19h)$$

Seed bank

The model does not include an explicit treatment of seed banks. In order to accommodate the effects of a seed bank, a simple limiting function on initial growth – that is photosynthesis – has been incorporated as shown in Fig. 4.15

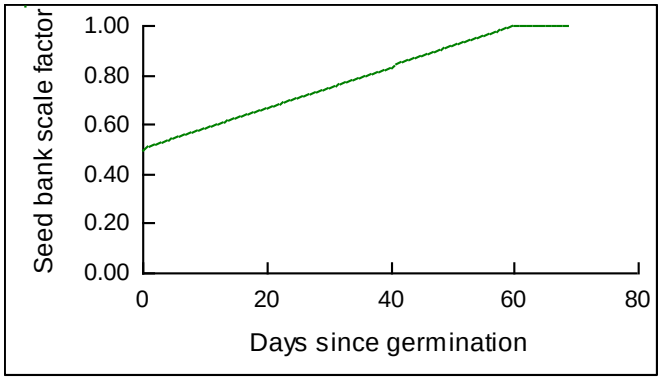

Figure 4.15 Influence of seed bank on initial growth following germination.

According to the parameters selected here, there is a 50% reduction in the initial rate of photosynthesis which gradually increases until there is no restriction after 60 days. These parameters can be adjusted on the interface. The *Implement seed bank effect* box must also be checked

4.7 Multiple species

The treatment so far has been for single species. With multiple species the main interaction is through the interception of light. For below ground processes, each species accesses water and nutrients and interaction occurs primarily because other species cause resources to be depleted more rapidly, or may access different regions of the soil profile.

4.7.1 Light interaction

Light interception by multiple species follows the analysis of Johnson, Parsons and Ludlow (1989). For a detailed discussion, see Thornley and Johnson (2000, chapter 8). In section 3.3 in the *Climate* chapter, Beer's law was introduced for light attenuation through a monoculture canopy, which is repeated here for convenience, and is

$$I(\ell) = I_0 e^{-k\ell} \tag{4.20a}$$

where I_0 and $I(\ell)$ are the irradiances [W (m^2 ground)$^{-1}$] at the top of the canopy and in the canopy at cumulative leaf area index (LAI), ℓ [(m^2 leaf) (m^2 ground)$^{-1}$], and k is the light extinction coefficient [(m^2 ground) (m^2 leaf)$^{-1}$]. The total light intercepted by the live canopy component is then

$$G_{canopy,live} = \left(1 - e^{-kL_{live}}\right) \tag{4.20b}$$

Equation (4.19b) can be extended to mixtures to give

$$G_{canopy,live,i} = \frac{k_i L_{live,i}}{k_e L_{live,tot}} \left(1 - e^{-k_e L_{live,tot}}\right) \tag{4.20c}$$

where the subscripts i and tot refer to individual species and the total canopy respectively, and the *effective* light extinction coefficient is defined by

$$k_e = \frac{\sum k_i L_i}{L_{tot}} \tag{4.20d}$$

These expressions can now be used to calculate the relative light interception by each species in the calculation of photosynthesis.

4.7.2 Water and nutrient competition

The uptake of both water and nutrients for multiple species on any particular day is calculated for each species as if the other species were not present. The rationale for this is that the relative amount of water or nutrients that is removed from the soil on any one day is generally a small fraction of the total available. However, on the next day, the pool of available soil resources will have been depleted by the amount used by all species. A minor problem may be that if a large proportion of resources is removed on any one day then the order in which species are treated may have a small effect on the simulation.

The way that plants interact below ground, or have relative advantage over other species therefore, is essentially a consequence of their parameter values, such as maximum root depth and root distribution, as well as their uptake parameters in eqn (4.2f).

4.8 Supplementary calculations

The treatment so far provides a complete description of pasture growth in response to environmental conditions. It is useful to look at some supplementary calculations relating to the animal module or the model interface.

4.8.1 Sugar levels

The model does not have an explicit sugars pool and so does not calculate the sugar content directly. The sugar concentration of leaves generally increases from a low level at the start of the day to a maximum towards the end of the day, although sugars are being utilised for growth and respiration throughout the day as well. The accumulated sugars are then generally depleted during the night, again through growth and respiration (see, for example, Gordon *et al.*, 1982 and, for a modelling treatment, Gordon and Johnson, 1984). The main purpose for using a sugar pool in the model is for the calculation of plant digestibility and, since this pool varies substantially through the day, it is sufficient here to take an indication of this component.

It is assumed that the sugar pool can be expressed as

$$f_{sugar} = \frac{P_{g,day}/2}{W_{live,shoot}} \tag{4.21a}$$

This allows for the fact that during times when photosynthesis is restricted, say due to dry conditions, sugar levels will be reduced.

4.8.2 Temperature growth limiting factor

It is useful to get an indication of the limitation to growth as a result of the temperature in an analogous way to that for water and nutrients, although this is not used directly in the model calculations. This is simply taken to be

$$GLF_T = \frac{f_p(T)}{f_p(T_{opt})} \tag{4.21b}$$

where f_p is the temperature response function in the leaf photosynthesis response, eqns (4.5a,b,e).

4.8.4 Root and stolon senescence following defoliation

Defoliation, particularly severe defoliation, generally triggers senescence of roots and, in the case of legumes, stolons. If the defoliation fraction of above ground live dry weight is denoted by f_{def}, then the senescence coefficient for the roots in eqn (4.18l) now becomes

$$\gamma_{r,def} = \gamma_r + f_{def}^2(1-\gamma_r) \tag{4.21c}$$

so that when f_{def} is small there is little change to γ_r but as f_{def} increases γ_r also increases. The second power is used to limit the effect until defoliation is quite severe.

For stolons, the analogous equation is to modify the stolon flux coefficient in eqns (4.18i-k) to

$$\gamma = \gamma + f_{def}(1-\gamma) \tag{4.21d}$$

The second power is not used here because of the so that senescence of stolons is linearly related to leaf defoliation.

4.9 Concluding remarks

The model for pasture growth described here is a physiologically based carbon assimilation model in response to environmental conditions. Daily growth rate is estimated by starting with leaf photosynthesis, summing this over the canopy and then accumulating this over the day. Dark respiration components for maintenance, nitrogen uptake and, for legumes, nitrogen fixation are incorporated. In addition, generic parameters for C_3 and C_4 pastures are included. Multiple species interactions are also incorporated. The model includes the effects of tissue turnover, phase development, annual and perennial species as well as legumes. In addition, the nutrient composition of live and dead tissue is calculated which is used both in the treatment of nutrient cycling and also animal nutrition. The water and nutrient dynamics interact with the soil water and nutrient balances in a consistent manner.

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5 Water dynamics

5.1 Introduction

The water balance in pasture and crop systems involves an intricate interaction between rainfall, evapotranspiration, runoff and infiltration. The general scheme is presented in Fig. 5.1:

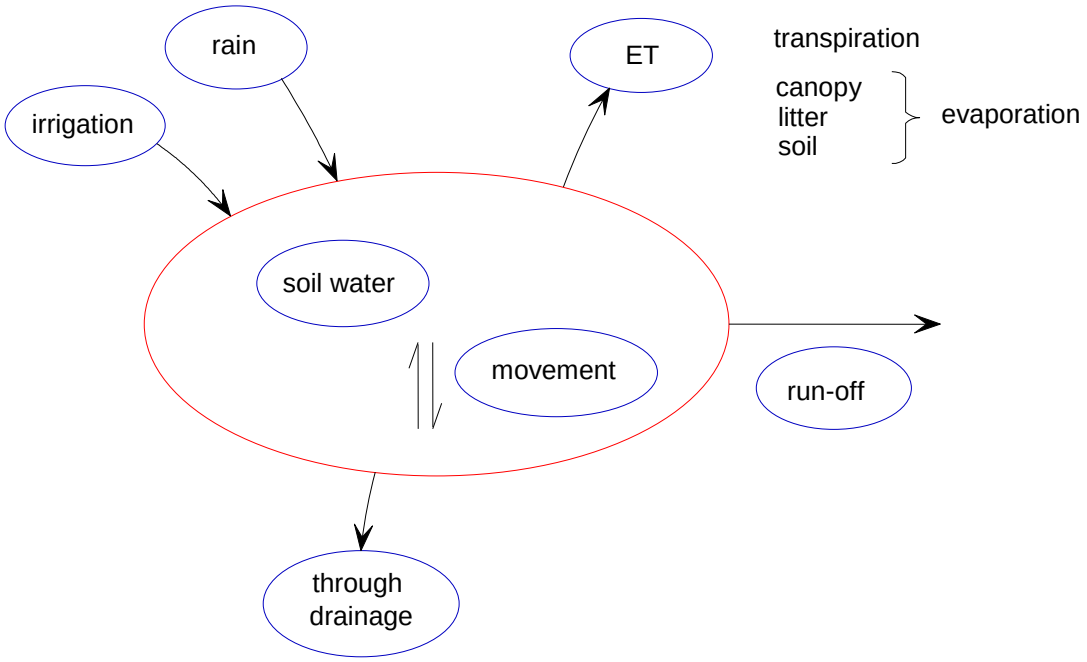

Figure 5.1: Schematic representation of the soil water balance.

In this diagram, the processes that are included in the model are shown. Some of these are quite readily measured, whereas others are more difficult. For example, while rainfall can be measured quite accurately, it is much more difficult to measure actual evapotranspiration.

The individual processes are now considered in turn.

5.2 Evapotranspiration

Evapotranspiration is the general term for the combined movement of water from the plant and soil system to the atmosphere. Transpiration is the evaporation of water from within the leaf structure. Evaporation and transpiration are influenced by the availability of water in the soil, and that held by the canopy surface and litter, as well as atmospheric conditions: net radiation, vapour pressure deficit or humidity, temperature, and windspeed.

The most widely used theory for describing evapotranspiration is the Penman-Monteith (PM) equation. The derivation of the PM equation is not presented here, but for a complete discussion see, for example, Thornley and Johnson (2000). Because, the input requirements for the PM equation are fairly extensive, simplifications are often made. The most generally accepted of these has been proposed by the FAO to calculate a reference transpiration value, as presented by Allen *et al*, (1998): throughout this document all references to FAO will be to this paper.

5.2.1 Full Penman-Monteith equation

The Penman-Monteith equation defines the canopy transpiration rate in terms of climatic conditions and canopy parameters. It can also be adapted to define evaporation from soil, litter or the canopy surface. The general approach in deriving and using the PM equation is to start with the instantaneous rate of transpiration (mm s^{-1}) and then scale that up to get daily values. There are a number of ways of expressing the PM equation, depending on the choice of pressure or density units for the water content of the air. In Thornley and Johnson (2000) the detailed derivation is given for density units and the equivalent formulations for pressure units are also presented. However, the FAO approach uses vapour pressure which is used here to avoid confusion.

The general formulation for the PM equation for the instantaneous rate of transpiration is:

$$E_T = \frac{s\varphi_N + \dfrac{c_p \rho}{r_a}\Delta p_v}{\lambda\left[s + \gamma\left(1 + r_c / r_a\right)\right]}, \qquad (5.1)$$

where the symbols (some of which have been defined earlier), with units, are defined by:

E_T	transpiration rate	mm water s^{-1}
ϕ_N	net radiation absorbed by the evaporating surface	J m^{-2} s^{-1}
c_p	specific heat capacity of air	J kg^{-1} K^{-1}
ρ	density of air	kg m^{-3}
Δp_v	vapour pressure deficit	Pa
T	air temperature	K
λ	latent heat of vaporization	J kg^{-1}
s	$\mathrm{d}p_v^{'}/\mathrm{d}T$	Pa K^{-1}
γ	psychrometric parameter	Pa K^{-1}
r_a	boundary layer resistance	s m^{-1}
r_c	canopy resistance	s m^{-1}

Note that 1 mm water is equivalent to 1 kg water m^{-2} and that in the model the actual calculations are based on the mass of water.

The canopy and boundary resistances are discussed in detail below, although a brief mention is given here. First, note that conductance, which is the reciprocal of resistance, is often used in the literature (and is used in Thornley and Johnson, 2000). The boundary layer resistance, r_a, is the resistance to water vapour movement from the surface of the leaves to the bulk air stream, for convective transport. According to the generally accepted theory, which is used here, r_a depends on the windspeed and surface roughness of the canopy. The model does not include diffuse transport of water vapour, which is generally much less significant than convective transport. The canopy resistance, r_c, is the resistance to water movement from the sub-stomatal evaporating surface to the external surface of the leaves, and is related to the leaf stomatal resistance. It is interesting to note that Penman's principal contribution to this theory was to express transpiration in terms of the

air temperature rather than leaf temperature, and Monteith's was to incorporate both the canopy and boundary layer resistances.

The PM equation is now used to estimate daily transpiration and evaporation rates, which involves making assumptions in order to scale from the second to the day. Different approaches can be used and some of these are discussed by Thornley and Johnson (2000), FAO and others. The approach used here is designed to be realistic and practical.

Boundary layer resistance

The boundary layer resistance defines how effectively water vapour is transported from the evaporating surface to the bulk air stream. As well as resistance, the term conductance is also used by many authors, where

conductance = 1 / resistance. (5.2a)

There is no obvious choice between the two and they can be regarded as inter-changeable. However, to be consistent with FAO publications, resistance is used here.

Most (but not all) analysis for the movement of water from the evaporating surface to the canopy considers turbulent transport only rather than diffusive transport. Turbulent transport occurs when the water vapour is transferred through mass transfer – that is the physical movement of the air containing the water vapour – whereas diffusive transport occurs in response to a concentration gradient. In most situations, the movement of water vapour from the evaporating surface to the bulk air stream is dominated by turbulent transport and that is the only component that is considered here. However, there may be conditions, particularly very low windspeeds and high temperatures, for which diffusive transport may be important.

A detailed coverage of the theory for the turbulent transport of water vapour is given in Thornley and Johnson (2000). The boundary layer resistance can be shown to be (cf eqn (14.9n), Thornley and Johnson, 2000):

$$r_a = \frac{\ln\left[\left(z+\varsigma-d\right)/\varsigma\right]\ln\left[\left(z+\varsigma_m-d\right)/\varsigma_m\right]}{\kappa^2 u_a}, \qquad (5.2b)$$

where the symbols are:

z	height above the ground	m
ς	roughness parameter for heat and vapour turbulent transfer	m
ς_m	roughness parameter for momentum turbulent transfer	m
d	zero-plane displacement	m
u_a	windspeed in the bulk air stream	m s^{-1}
κ	von Karman's constant	0.41

The parameter d is the apparent sink of momentum, so that the projected windspeed is zero at this height.

The derivation of eqn (5.2b) is sound, but a slightly simpler form is used in FAO, which is:

$$r_a = \frac{\ln\left[(z-d)/\varsigma\right]\ln\left[(z-d)/\varsigma_m\right]}{\kappa^2 u_a}, \qquad (5.2c)$$

and this will be used in the present analysis. In practical terms, the equations have very similar behaviour. The advantage of using (5.2c) is that the roughness coefficients and zero plane displacement parameters have been estimated (FAO) as

$$\begin{aligned} d &= 0.67h \\ \varsigma_m &= 0.123h \\ \varsigma &= 0.0123h \end{aligned} \qquad (5.2d)$$

here h (m) is the canopy height.

In order to use (5.2d) it is necessary to define the canopy height, h. The two approaches generally used are either to assume height is linearly related to leaf area index, L, or to assume an exponential relationship. The linear approach is used here, and it is assumed that (FAO)

$$h = 0.04L \qquad (5.2e)$$

The boundary layer resistance is now completely defined by eqns (5.2c, d, e) in terms of windspeed and crop height, at the height z above the ground. It is common to take z at a standard height of 2 m. However, note that if we were to apply this theory to tall canopies, such as trees, modifications would obviously have to be made.

Canopy resistance

The canopy resistance can be related to L according to

$$r_c = \frac{r_\ell}{L} \qquad (5.3a)$$

where r_ℓ is the leaf stomatal resistance, so that as L increases the canopy resistance falls. Note that r_ℓ must account for stomata on both sides of the leaf.

Potential canopy transpiration

Potential transpiration occurs when the stomata are wide open and there is no reduction in transpiration due to availability of water. This is combined with a factor that defines the availability of soil water to calculate actual transpiration. This is an alternative approach to estimating actual stomatal resistance, and a discussion of the rationale is given in the *Pasture* chapter.

In Thornley and Johnson (2000), potential transpiration is defined as occurring when the canopy resistance is zero, implying that the stomata can open wide enough to ensure there is no resistance to vapour flow. However, it is perhaps more appropriate to define a lower limit for the canopy resistance, r_c, (eg, Thornley, 1998) and this is the approach used here. The following values are reasonable defaults:

C_3: $r_c = 70\ \text{s m}^{-1}$, (5.3b)

C_4: $r_c = 140\ \text{s m}^{-1}$. (5.3c)

The C_3 value is that used in FAO, and the C_4 value has been chosen as being typical of C_4 plants. C_4 plants generally have a lower sub-stomatal CO_2 concentration due to the different leaf structure and the role of PEP carboxylase. This creates a greater concentration gradient of CO_2 between the site of

photosynthesis and the atmosphere so that the flux of CO_2 into the leaf can be maintained with smaller stomatal apertures, and hence greater stomatal resistance. The user specifies whether the species is C_3 or C_4 and the appropriate value from eqns (5.3b, c) is then used.

Once the potential transpiration is known, this is then used by the crop or pasture model to calculate actual transpiration in terms of available soil water.

Potential canopy, litter and soil evaporation

Potential evaporation of water sitting on the surface of the leaves, litter or soil is treated in a similar fashion. For freely evaporating surfaces, eqn (5.1) can be used but when canopy resistance is zero:

$$E_{pot} = \frac{s\varphi_N + \frac{c_p \rho}{r_a}\Delta p_v}{\lambda(s+\gamma)}, \qquad (5.4)$$

where the appropriate term for absorbed net radiation for green or brown vegetation, or wet soil should be used. Since the net radiation balance for the different surfaces may vary, due to their reflectivity, the potential evaporation will also be different for these surfaces.

Coefficients and temperature dependence

The temperature dependence of the PM equation parameters is now presented. The following parameters and relationships are summaries that can be found in standard texts

First note that there will be small differences in the parameters that depend on the density of air, depending on whether moist or dry air values are used. In the present analysis, dry air values are used, although differences observed in these parameters for moist and dry air parameters are small and generally have negligible effect on the results.

Saturated vapour pressure: p'_v (Pa)

$$p'_v = 610.8 \exp\left(\frac{17.27T}{T+237.3}\right) \qquad (5.5a)$$

Slope of $p'_v(T)$: s (Pa K^{-1})

$$s = \frac{4098}{(T+237.3)^2} p'_v = \frac{2503}{(T+237.3)^2} \exp\left(\frac{17.27T}{T+237.3}\right) \qquad (5.5b)$$

Density of dry air: ρ (kg m^{-3})

$$\rho = \frac{352.9}{T+273.2} \qquad (5.5c)$$

The following constant parameter values are used:

λ	latent heat of vaporization	2.45 x 10^6 J kg^{-1}
c_p	specific heat capacity of air	1013 J kg^{-1} s^{-1}
γ	psychrometric parameter	67.34 Pa K^{-1}

Although λ and γ are subject to minor temperature variation, this is ignored in the present analysis, with negligible loss of accuracy.

5.2.2 Daily Potential Evapotranspiration (PET)

The theory so far has been for steady-state instantaneous evapotranspiration, which is the underlying basis of the PM equation. However, in practice we wish to apply the equation to daily, or hourly, time-steps. The present analysis does not explicitly consider hourly time-steps, although it would be quite straightforward to adapt.

Before proceeding, some daily inputs to the model are defined.

Radiation components and daylength

The following radiation components are required. They can be derived from the total solar radiation, or global radiation, which is routinely available. This theory was discussed in the *Climate* chapter (section 3.4.2).

$\Phi_{N,g}$	net radiation absorbed by green surface	J m^{-2} d^{-1}
$\Phi_{N,b}$	net radiation absorbed by brown surface	J m^{-2} d^{-1}
	(standing dead or litter)	
$\Phi_{N,s}$	net radiation absorbed by wet soil	J m^{-2} d^{-1}

Daylength is also required, and the following notation is used:

h	daylength	hours
f_d	daytime fraction	dimensionless

This can be estimated from latitude and date, as described in *Climate,* section 3.4.2.

Daily canopy transpiration

It is assumed that canopy transpiration only occurs during the daylight period. While some evaporation may occur at night, stomata are generally closed, and so the assumption of zero transpiration is reasonable.

Equation (5.1) can now be used to give the daily transpiration rate, $E_{T,d}$ (mm d^{-1}), as

$$E_{T,d} = \frac{s\Phi_{N,g} + 86,400\dfrac{c_p\rho}{r_a}\Delta p_v}{\lambda\left[s+\gamma\left(1+r_c/r_a\right)\right]} \qquad (5.6)$$

where the factor 86,400 is the number of seconds in 24 hours. Note that the various terms on the right-hand-side that are defined on a per second basis need to be defined in terms of daily climatic inputs: this is discussed below.

Potential daily canopy transpiration

As mentioned above, potential canopy transpiration is defined as the rate of transpiration when there is no limitation to supply of water from the soil. This is sometimes regarded as being when the

canopy resistance, r_c, is zero. However, even for well watered plants this is unlikely to be the case and so a minimum value for r_c must be defined. The default values given in eqn (5.3b, c) are used.

Potential daily canopy, litter and soil evaporation

Potential evaporation of water sitting on the surface of the leaves, litter or soil is treated in a similar fashion. For freely evaporating surfaces, eqn (5.4) can be applied, so that :

$$E_{d,pot} = \frac{s\Phi_N + 3600h\dfrac{c_p\rho}{r_a}\Delta p_v}{\lambda\left(s+\gamma\right)} \qquad (5.7)$$

where the appropriate term for absorbed net radiation for green or brown vegetation, or wet soil should be used. Since the net radiation balance for these surfaces may differ, due to varying reflectivity, the potential evaporation will also be different.

5.2.3 FAO Penman-Monteith equation

The FAO PM equation is designed, as I understand it, to provide a standard reference calculation for potential evapotranspiration. It is not intended to replace more detailed analysis if the inputs are available. Indeed, the excellent FAO documentation discusses approaches that can be used depending on the level of detail of information that is available. In this section the most widely used form of the FAO equation is considered, which is designed to give daily potential ET in terms of daily climatic parameters. The principal reference used for this section is the FAO report by Allen *et al* (1998). The approach is to define standard conditions that are representative of a wide range of conditions. These are:

- grass surface of height 0.12 cm
- meteorological measurements at 2 m
- canopy resistance of 70 s m^{-1}

Using these parameter values, the PM equation simplifies to

$$E = \frac{0.408s\Phi_N + \dfrac{60.7}{T+273}u_2\Delta p_v}{s+\gamma\left(1+0.337u_2\right)}, \qquad (5.8)$$

where u_2 denotes the windspeed at 2 m. In FAO56, γ has been introduced into the second term in the numerator: I can see no rationale for this and so eqn (5.8) is used, although the two are mathematically identical.

This is the widely used form of the FAO PM equation. By using this standardised approach, it is possible to make quantitative comparisons of different evapotranspiration demands across locations and through time.

The equation also forms the basis for predicting potential evapotranspiration in models, although it does have its limitations. Perhaps the two principal drawbacks are:

- the restriction to a grass surface of height 0.12 m
- there is no account for the fact that transpiration only occurs during the light period.

Both of these can be addressed either by modifying eqn (5.8) or applying it to different time scales, for example an hour. These issues are recognised by Allen *et al* (1998), who consider ways of extending the approach when more detailed information is available.

The FAO version of the PM equation is a useful standard indicator of potential evapotranspiration under prescribed conditions. It can also be used in models to help predict evaporation and transpiration. However, as it is defined for specific conditions, care should be taken when using this approach in models.

5.3 Soil water infiltration and redistribution

The description of soil water infiltration and redistribution is crucial in the study of water movement in soils. The aim of this section is to consider some approaches and issues relating to this topic. There is a large body of theory on modelling soil water dynamics, and so the objective here is to give a brief account of some of the principal approaches and then discuss the particular options that are available in the present model.

This section includes:

- the use of the Richards equation for infiltration;
- Campbell equations for moisture retention and unsaturated hydraulic conductivity in the Richards equation, which are adapted for the present purposes;
- a description of the van Genuchten equations for moisture retention and unsaturated hydraulic conductivity, and a brief discussion as to why these are not used in the model; and
- the simpler capacitance model taken from the WaterMod (Johnson, 2003) program developed prior to the present model.

Both methods were introduced during the early development of the model, although the capacitance approach is most widely used.

5.3.1 Infiltration with the Richards equation

The Richards equation is generally used to describe soil water infiltration. There are a number of equivalent ways to write the Richards equation: the formulation used here is:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}\left[K\left(1 - \frac{\partial \psi_m}{\partial t} \right) \right], \qquad (5.9a)$$

where the symbols are:

symbol	definition	Internal units	Interface units
θ	soil water content	fraction (vol)	% (vol)
ψ_m	matric potential	m	cm or MPa
K	hydraulic conductivity	m d^{-1}	cm d^{-1}
z	depth from soil surface	M	m or cm

The following conversions may be helpful:

1 m ≡ 100 cm ≡ 10 kPa. (5.9b)

The corresponding flux of water, q (m/day), at any depth is:

$$q = K\left(1 - \frac{\partial \psi_m}{\partial t}\right). \quad (5.9c)$$

Equation (5.9c) is Darcy's law for flow of water through a porous medium, and eqn (5.9a) is the corresponding flow equation when Darcy's law is combined with the mass balance equation.

Note that the unity terms in these equations represent the influence of gravity flow.

In order to use the Richards equation, it is necessary to relate both the matric potential and hydraulic conductivity to soil water content, which can present considerable challenges. Several approaches for describing the moisture retention curve are commonly used. These equations are empirically based, with little theoretical justification other than the fact that they give the general behaviour that is observed experimentally. During the development of WaterMod (Johnson, 2003), two approaches were explored: the Campbell equation (Campbell, 1974), which is quite simple and analytically accessible; and the van Genuchten (1980) equation which is more versatile but also a little more complex to work with. After working extensively with both of these approaches, my preference is to use the Campbell equations, with a modification to the parameterisation of the unsaturated hydraulic conductivity: the reasons are discussed below.

Campbell equations for moisture retention and hydraulic conductivity

Moisture retention

According to this equation, the matric potential is related to the soil water content by:

$$\psi_m = \psi_{ae}\left(\frac{\theta_s}{\theta}\right)^b \quad (5.10a)$$

where θ_s is the saturated water content, ψ_{ae} is known as the *air-entry potential*, or the *bubbling pressure*, and b is an empirical parameter. It is assumed that for potentials between ψ_{ae} and zero, the water content does not change and remains at saturation. Equation (5.10a) is illustrated in Fig. 5.2 on both linear and log scales.

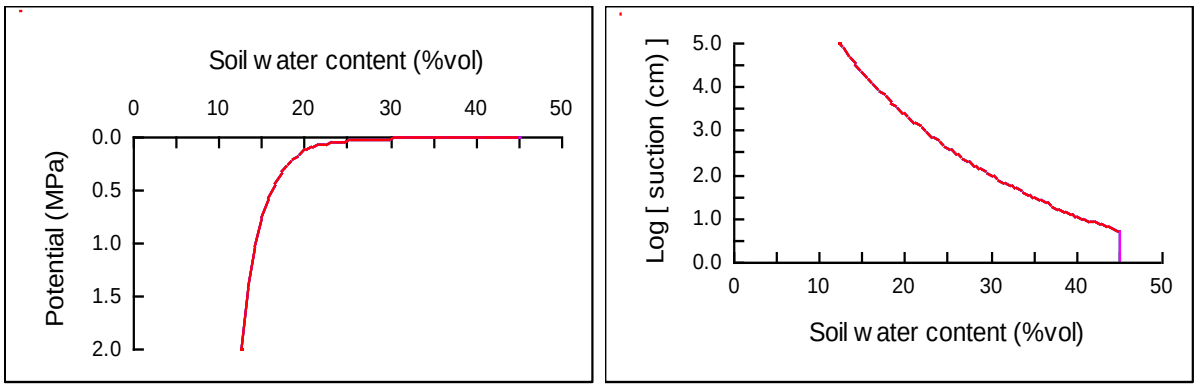

Figure 5.2: Moisture retention curve on linear and log scales using the Campbell equation. The parameters are: $\theta_s = 0.45$, $\psi_{ae} = -10$cm, and $b = 6$.

From these graphs it can be seen that:

- The linear graph shows the dry end of the response quite clearly, but not the wet end.
- The log graph gives a clearer perspective of the total response, including the behaviour of the curve around the air-entry potential.
- The choice of cm units for the log graph allows a clear impression of the discontinuity of the response curve for potentials greater (closer to zero) than the air-entry potential.

When applying eqn (5.10a), it is generally assumed that θ_s is known from measurements and so the two parameters ψ_{ae} and b need to be derived from experimental data. The Campbell equation is appealing in its simplicity, and generally gives a reasonable fit to experimental data.

Equation (5.10a) can be rearranged to give an explicit equation for the water content as a function of matric potential:

$$\theta = \begin{cases} \theta_s \left(\dfrac{\psi_{ae}}{\psi_m} \right)^{1/b}, & \psi_m < \psi_{ae}, \\ \theta_s, & \text{otherwise.} \end{cases} \qquad (5.10b)$$

This simplicity makes it possible to prescribe water contents at particular potentials in order to derive the basic parameters ψ_{ae} and b. In addition, Cresswell and Paydar (1996) have used this approach to derive these parameters according to what they term 'two-point' method. This type of approach was used in WaterMod, and I'm sure that other modelling groups will also have exploited this analytical simplicity.

To illustrate the 'two-point' method, note that for water contents at or below saturation, eqn (5.10b) can be written as

$$b \ln \theta_s - b \ln \theta = \ln \psi_m - \ln \psi_{ae} \qquad (5.10c)$$

Now suppose, for example, that the following pairs of matric potential and water content are known, corresponding to 'field capacity' and 'wilting point': (ψ_{fc}, θ_{fc}) and (ψ_w, θ_w). Typical values for the matric potential that are used for field capacity and wilting point are -100 cm (-10 kPa) and -15000 cm (-1500 kPa) respectively, but see the discussion later on these representative matric potentials. Using these values, it is easy to show that eqn (5.10b) gives

$$b = \frac{\ln\left(\psi_w / \psi_{fc}\right)}{\ln\left(\theta_{fc} / \theta_w\right)} \tag{5.10d}$$

and this can then be used to get ψ_{ae}. Several forms, all equivalent, can be derived: one is:

$$\psi_{ae} = \psi_{fc}\left(\frac{\theta_{fc}}{\theta_{sat}}\right)^b \tag{5.10e}$$

although the equivalent equation with fc replaced by w is identical.

This two-point analysis is not restricted to selecting field capacity and wilting point specifically, but any two points will do: Cresswell and Paydar (1996) discuss various options. Furthermore, the parameters ψ_{ae} and b can be derived using more general regression techniques and fitting to a range of data points.

An apparent disadvantage of the Campbell equation is that it does not include a residual water content: that is, an air-dry water content. However, the nature of the equation is such that, in practice, the water content never approaches zero, unless the choice of parameter b allows this. It should be noted that the Campbell equation is a simplification of the Brooks and Corey (1964, cited by Kutilek and Nielsen, 1994) equation that did include an air-dry water content. However, there is little advantage in including this term – for example, Cresswell and Paydar (1996) found it had negligible effect when fitting to data.

One limitation with the Campbell function is the fact that the slope of the curve is discontinuous between the air-entry potential and zero matric potential, which implies that the soil water content does not vary between these potentials. This is largely an aesthetic issue as variation in soil water content and matric potential between these points will have little effect on the solution. However, Hutson and Cass (1987) have developed a neat way to smooth the curve by using a quadratic equation close to saturation, and then adjusting the coefficients so that when this curve intercepts with the Campbell equation (5.10b), the slopes are equal. This means that eqn (5.10b) is now replaced with:

$$\psi_m = \begin{cases} \psi_{ae}\left(\dfrac{\theta_s}{\theta}\right)^b, & \psi \geq \psi_i; \\ \psi_{ae}\left(\dfrac{\theta_s}{\theta}\right)^b \left(\dfrac{\theta_s - \theta}{\theta_s - \theta_i}\right)^{1/2}, & \text{otherwise.} \end{cases} \tag{5.10f}$$

where

$$\theta_i = \frac{2b}{1+2b}\theta_s \tag{5.10g}$$

and

$$\psi_i = \left(\frac{2b}{1+2b}\right)^{-b}\psi_e \tag{5.10h}$$

While this refinement will have relatively little influence on the numerical solution of the Richards equation, it is an attractive approach as it removes the sharp discontinuity in the log(suction) graph

(see Fig. 5.2), while introducing no extra parameters. For typical b values (say in the range 6 to 30, the point (θ_i, ψ_i) will be very close to saturation.

Equation (5.10f) is illustrated in Fig. 5.3 corresponding to Fig. 5.2:

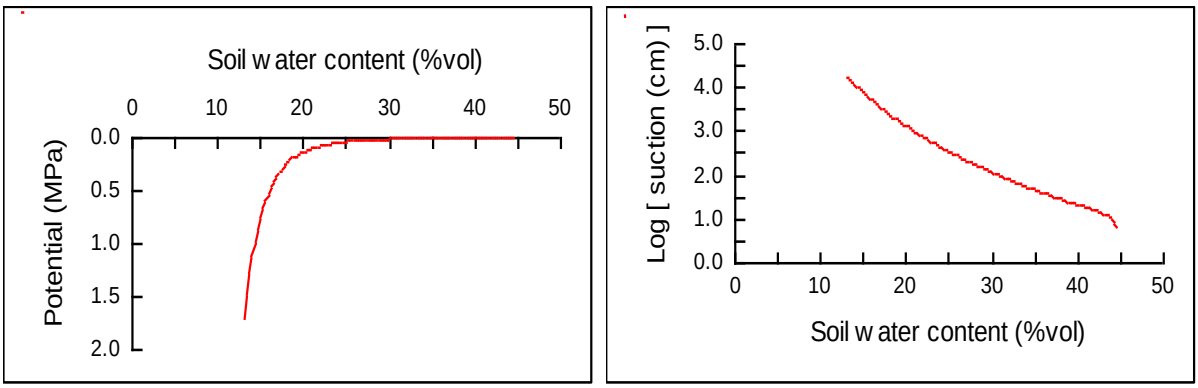

Figure 5.3: Moisture retention curve on linear and log scales using the Campbell equation with Hutson and Cass smoothing. The parameters are the same as in Fig. 5.2: $\theta_s = 0.45$, $\psi_{ae} = -10$cm, and $b = 6$.

Hydraulic conductivity

The unsaturated hydraulic conductivity corresponding to eqn (5.10a) is often taken to be:

$$K = K_s \left(\frac{\theta}{\theta_s} \right)^c \qquad (5.11a)$$

where c is an empirical coefficient. Campbell (1974) derived c as:

$$c = 2b + 3 \qquad (5.11b)$$

although slightly different relationships can be found (see, for example, Kutilek and Nielsen, 1994). A derivation of eqn (5.11b) is presented in Thornley and Johnson (2000), although it is perhaps worth mentioning one phrase in that derivation: '*…The next stages in the argument may be thought somewhat unconvincing….*'

A fundamental problem that is faced with this approach to the description of unsaturated hydraulic conductivity is that the $\psi(\theta)$ and $K(\theta)$ equations must be consistent. For example, if the former is accurate, but the latter over-estimates $K(\theta)$ as the soil dries down, then the solution to the Richards equation may predict too much through drainage. Since eqn (5.11b) is the most tenuous aspect of this approach, this is not used. Rather, it is assumed that the value of $K(\theta)$ at field capacity is fixed – the default that is used is

$$K_{fc} = 1$$

mm / day (5.11c)

Note that, as described earlier, the subscript fc refers to the field capacity which is defined as -10 kPa, or -100 cm. The strategy is therefore to use eqn (5.10a) to calculate the water content at field capacity and then substitute this into (5.11c) to derive c.

This approach is similar to that adopted by Hutson and Wagenet (1991), although in that analysis eqns (5.11b, c) were used and K_s was estimated, whereas the present method allows the user to specify their own K_s values rather than have them derived. In practice, the two methods will yield

similar results since drainage will occur until a little below field capacity, although users are probably more comfortable with the idea of specifying K_s themselves.

Equation (5.11a) with (5.11c) is illustrated in Fig. 5.4, where the main point to note is that there is a very steep decline in K as the soil dries.

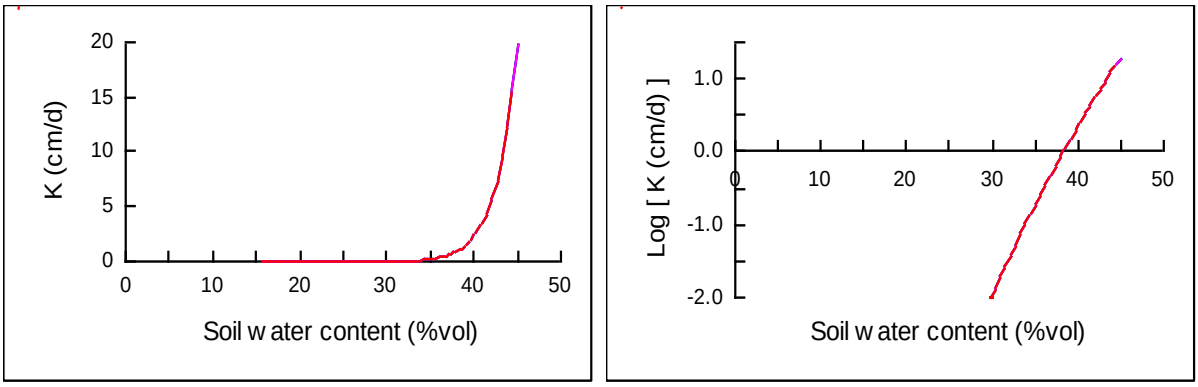

Figure 5.4: Hydraulic conductivity as defined by eqn (5.11a, c), with $K_s = 20$cm d^{-1} and the other parameters are the same as in Figs 5.2, 5.3.

Van Genuchten equations

The van Genuchten equations are sometimes used and so are discussed here. However, for various reasons that are also discussed, they are not used in the present analysis.

Moisture retention

The van Genuchten equations for moisture retention and hydraulic conductivity describe the moisture retention equation as:

$$\psi_m = \frac{1}{\alpha}\left[\frac{1}{\phi^{1/m}} - 1\right]^{1/n} \qquad (5.12a)$$

where

$$\phi = \frac{\theta - \theta_r}{\theta_s - \theta_r} \qquad (5.12b)$$

is the relative water content. θ_r is the residual (air-dry) water content, α (1/m), n, and m (both dimensionless) are empirical parameters. Note that $1/\alpha$ has a similar, but not identical, interpretation to the air-entry potential, ψ_{ae}. Equations (5.12a, b) can be rearranged to give

$$\phi = \left[1 + (\alpha\psi)^n\right]^{-m} \qquad (5.12c)$$

According to some fairly speculative theory, the number of parameters can be reduced by one by taking

$$m = 1 - \frac{1}{n} \qquad (5.12d)$$

The van Genuchten equation is illustrated in Fig. 5.5:

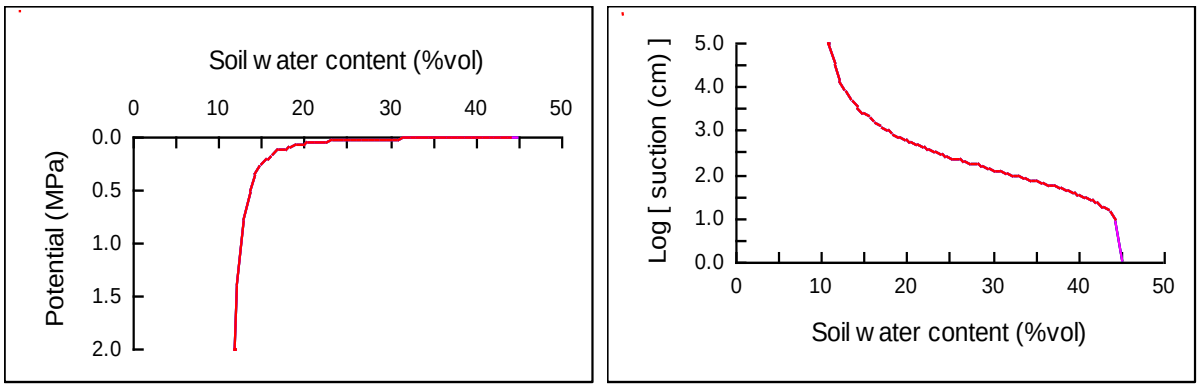

Figure 5.5: Moisture retention curves using the van Genuchten equation.
The parameters are: $\theta_s = 0.45$, $\theta_r = 0.1$, $1/\alpha = 50$cm, $n = 1.5$.

The general nature of this response is (as to be expected), similar to the Campbell function with Hutson and Cass smoothing.

The formulation of the van Genuchten equation presented here is that which is generally used in the literature but it would perhaps be better to write it with the inverse of the α parameter. Writing

$$\beta = 1/\alpha\,, \tag{5.12e}$$

eqns (5.12a, b) become

$$\psi_m = \beta\left[\frac{1}{\phi^{1/m}} - 1\right]^{1/n}, \tag{5.12f}$$

and

$$\phi = \left[1 + \left(\frac{\psi}{\beta}\right)^n\right]^{-m}, \tag{5.12g}$$

respectively, where β has the same units as ψ_m. This makes β easier to interpret as it can be related to the characteristic matric potential. It has a similar (but not identical) interpretation as the air-entry potential, or bubbling pressure, that is used in the Campbell equation.

Hydraulic conductivity

The hydraulic conductivity based on a theoretical approach developed by Maulem (1976) can be derived as:

$$K = K_s \phi^{1/2}\left[1 - \left\{1 - \phi^{1/m}\right\}^m\right]^2 \tag{5.13}$$

Note that in deriving this equation, there is a parameter that Maulem assumes to be 0.5. However, Kutilek and Nielsen mention (p 108) that Maulem's work was primarily on re-packed lab soils and for field soils this parameter can vary between -10 to more than 10. So, while eqn (5.13) may hold appeal, we should be aware that it is essentially empirical in nature. Equation (5.12) is illustrated in Fig. 5.6:

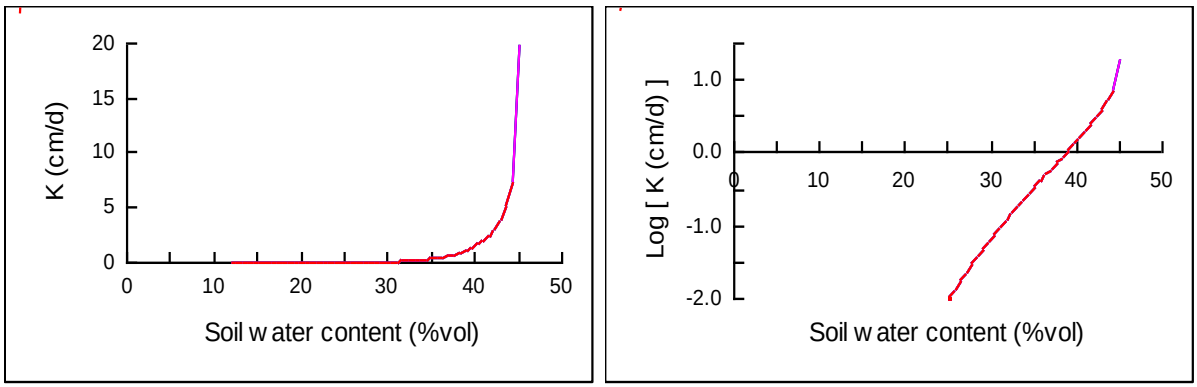

Figure 5.6. Hydraulic conductivity as defined by eqn (5.13). $K_s = 20$ cm d^{-1}; the other parameters are the same as in Figure 5.5.

Note that the general shape of the curve is similar to that for the Campbell equations (Fig. 5.4).

Which approach?

There are good reasons for using either of the approaches discussed here. The Campbell functions are easy to work with, in particular when relating specific pairs of water content and water potential. On the other hand, there is appeal to the van Genuchten equations both through the slightly greater flexibility in the shape of the curve and also because the moisture retention curve has a continuous slope everywhere. However, with Hutson and Cass smoothing, the Campbell moisture retention function also has a continuous slope.

When using either of these approaches to fit experimental data, it is important to look at the fit as well as get a statistical measure of the goodness of fit. In particular, consider the log(suction) vs water content graphs in Figs 5.3, 5.5. This is a distinct sigmoidal characteristic, and the various parameters in the equations control this curve in different regions. If the data being fitted do not display this characteristic, then the interpretation of the parameters becomes questionable. For example, if the data never approach the dry end of the curve, then there can be little confidence in the physical interpretation of the fitted value for residual water content, θ_r in the van Genuchten equation. While this may not matter too much if the sole objective is to describe the data, it could be important when extending the theory to estimate, for example, unsaturated hydraulic conductivity.

When fitting the moisture retention data to either the Campbell or van Genuchten equation, Cresswell and Paydar (1996) found little difference in the statistical fit between the two approaches, which suggests that the simpler Campbell function is no less adequate in this regard.

Given that there is little to choose between the two approaches with regard to fitting to experimental data, two principal advantages of the Campbell approach over the van Genuchten are:

- the parameters in the model have simple physical interpretations;
- the equations can be rearranged (as in eqns 5.10d, e) to derive the parameters in terms of easily understood and fairly readily estimated parameters.

Regardless of which approach is used, it is important to remember that the equation for K will have a major influence on the simulation results. The approach adopted for the Campbell equation defines K at a specific matric potential (taken to be *field capacity* here, which is defined as a suction of -100 cm). While a similar approach could be adopted for the van Genuchten equation, it is analytically more complex and is unlikely to result in a substantially different outcome.

The simple relationship between water content and water potential in the Campbell approach is attractive and allows a relatively easy analysis of the parameters in the equation across different soil types with a view to developing pedo-transfer functions (that is, generic parameter sets for different soil types). Of course, pedo-transfer functions can be developed with the more complex van Genuchten approach, and indeed such work has been done (Schaap, *et al.*, 1998).

Choice of units for water potential

The illustrations presented here use cm for soil water potential rather than kPa which is often used. cm is preferred since it gives a clearer indication of the log response curve near saturation. This is shown in Fig. 5.7 for the Campbell equation with Hutson and Cass smoothing. Note that the ranges of values on the y axis of these graphs extend to a water potential of 10^5 cm and 10^4 kPa, which are identical and equivalent to 10 Mpa. At the wet end, they start at 1 cm and 1 kPa, which are different.

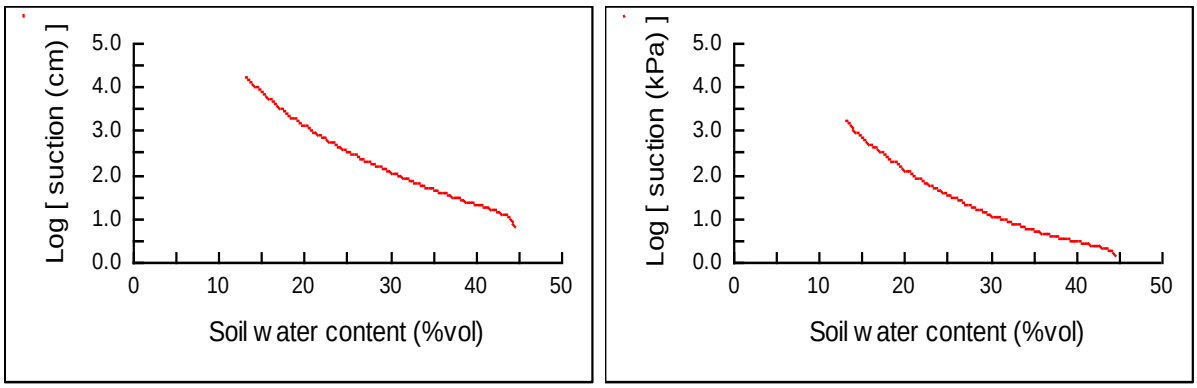

Figure 5.7: Moisture retention curves using the Campbell equation with Hutson and Cass smoothing, eqns (5.10e-h). (The parameters are the same as for Fig. 5.3.)
The graph shows the illustrative effect of using cm or kPa for water potential when using log graphs.

Using cm units gives a clearer impression of the shape of the curve near saturation and so, for the log(suction) graph, cm units are recommended.

Solving the Richards equation

Solving the Richards equation is no trivial matter! This is due to the highly non-linear nature of both the moisture retention curve and the unsaturated hydraulic conductivity, as discussed earlier. The aim here is to develop generic numerical routines that will work with any set of soil parameters – this is challenging. Occasionally, and regardless of the choice of moisture retention curve, the numerical routines break down. In these cases, if changing the time-step does not fix the problem, the soil is probably too complex for the routines and it may be necessary to revert to the simpler *capacitance model* (see below). It should be noted that when the numerical routines do fail, this is often due to sharp discontinuities in the soil profile. The common approaches that are generally used to solve the infiltration equations are now considered.

The simplest way is to use the Crank-Nicholson numerical scheme for solving a parabolic partial differential equation. There are many descriptions of this in numerical analysis texts. The approach used here derives from the discussions given by Campbell (1985) and Ross (1990). The Crank-Nicholson method results in a system of simultaneous equations that need to be solved. The most common approach, which is used here, is to solve these with the Newton-Raphson method. However, it should be noted that other methods can be used to solve the equations.

Because of the vast numerical range of water potential, and highly non-linear nature of the underlying equations, it is common to transform the variables in some way to speed up the solution. The approach used here is to take a logarithm transformation of the matric potential:

$$p = -\ln\left(1 + \frac{\psi_m}{\psi_0}\right) \tag{5.14}$$

where ψ_0 is an arbitrary scaling parameter, which can be used to help things along when the numerical routines don't work! The solution is independent of the transformation, but is generally obtained much quicker.

Other approaches are possible. One of these is (Craig Beverly, *pers com*) the Kirchoff transformation. According to this, the transformation introduces the term:

$$U = \int_{-\infty}^{\psi_m} K(\psi')d\psi' \tag{5.15a}$$

where the prime denotes the dummy variable of integration. The Richards equation then becomes:

$$\frac{\partial\theta}{\partial t} = -\frac{\partial}{\partial z}\left[K - \frac{\partial U}{\partial z}\right] \tag{5.15b}$$

I have implemented this approach and explored its behaviour. A significant advantage with the Kirchoff transformation is that the numerical analysis is simplified. However, I have found both transformations to work at about the same speed.

One of the major difficulties with either approach is the behaviour of the routines at sharp soil boundaries where the parameters are significantly different: an example is the AB horizon where the soil changes from a sandy loam to heavy clay. While there are various ways of dealing with this, the present approach is to solve the equations and then calculate the water flux terms. These are then used to update the soil water content and potential after each iteration. This approach appears to work well and is very robust. It is still required to balance mass since any errors will manifest through errors of fluxes into and out of the profile.

5.3.2 Capacitance model

While the Richards equation is well established in soil physics, it can sometimes present numerical challenges, particularly at sharp changes in soil characteristics, and can also be quite slow to solve numerically. As an alternative, the *capacitance* model that was first developed for WaterMod in 1996, can be used. This is a variation on the so-called tipping bucket model.

Model description

The parameters required for the model are:

Symbol	Definition	Internal units	Interface units
θ_s	saturated water content	fraction (vol)	% (vol)
θ_{dp}	drainage point	fraction (vol)	% (vol)
K_s	saturated hydraulic conductivity	m d^{-1}	cm d^{-1}
σ	curvature parameter	dimensionless	

As used in this model, the drainage point is alternatively known as the *drained upper limit*.

The soil is divided into layers and water moves down the layers. The flux is defined as:

$$q = \begin{cases} K_s \left(\dfrac{\theta - \theta_{dp}}{\theta_s - \theta_{dp}} \right)^{\sigma}, & \theta \geq \theta_{dp}; \\ 0, & \text{otherwise.} \end{cases} \qquad (5.16a)$$

The parameter σ controls the decline in q as the water content approaches the drainage point.

The drainage point is sometimes regarded as equivalent to the field capacity that was used earlier in discussing the Richards equation. However, in many situations soils can drain to soil water contents that are below the field capacity and so it is assumed that

$$\theta_{fc} = \theta_{dp} + \lambda\left(\theta_s - \theta_{dp}\right) \qquad (5.16b)$$

where λ is an empirical parameter taken to be

$$\lambda = 0.2 \qquad (5.16c)$$

in the model. This simply defines the field capacity as being 20% of the difference between saturation and the drained lower limit above the drained lower limit, and allows soils to drain lower than field capacity. Such drainage will be slow and is unlikely to occur except in prolonged dry periods.

Equation (5.16b) can be rearranged to give

$$\theta_{dp} = \frac{\theta_{fc} - \lambda\theta_s}{1 - \lambda} \qquad (5.16d)$$

and this is used in eqn (5.16a).

Analogous to the Campbell equation above, the parameter σ is estimated by assuming the flux of water at field capacity is fixed: the default value used is

$$q_{fc} = 0.1 \text{ mm / day.} \qquad (5.16e)$$

This value is an order of magnitude lower than that used for the Richards equation, eqn (5.11c), which has been found to give quite similar responses between the two approaches. The reason it is lower is because the Richards equation involves both gravity flow and flow due to a water potential gradient, whereas the water potential effects are not included in the capacitance model.

The parameter σ can now be evaluated as:

$$\sigma = \frac{\ln\left(q_{fc}/K_s\right)}{\ln\left\{\left(\theta_{fc} - \theta_{dp}\right)/\left(\theta_s - \theta_{dp}\right)\right\}} \qquad (5.16f)$$

According to this approach:

- only water in excess of the drainage point can move, and all movement is downwards;
- the flux decreases as the available water for movement declines, as controlled by σ, which in turn is derived from the water holding capacity of the soil.

A crucial step is to ensure that the internal time step, Δt, is sufficiently small to ensure that the distance the water moves in Δt does not exceed any of the layer thicknesses Δz (which are not constant through the profile). The constraint can be calculated from the mean pore water velocity at saturation (the speed at which the water moves through the saturated profile), and is

$$\Delta t < \left(\theta - \theta_{dp}\right)\Delta z / K_s \ . \qquad (5.16g)$$

This model is quite straightforward to implement numerically, and is formulated in terms of soil parameters that can be readily characterised.

The additional constraint is imposed that the maximum sub-daily time interval is 1 hour. This ensures that daily rainfall can be disaggregated into hourly values (see section 3.5.6 in the *Climate* chapter).

Equation (5.16a) is shown in Fig. 5.8 for a range of K_s values.

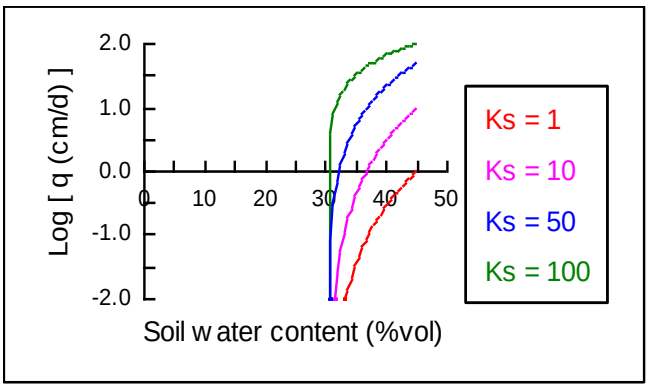

Figure 5.8: Log q as a function of soil water content, as described by eqn (5.16a) for the values of K_s as shown in cm d^{-1}

This model is readily implemented numerically and is formulated in terms of easily characterised soil parameters. The only speculative parameter is σ which controls how quickly the water is allowed to drain: the assumption used here to calculate σ (eqns 5.16b, c) seems reasonable.

On the interface where the parameters are prescribed, the wilting point and air-dry water content are also included. These are used for plant water use and soil water evaporation respectively and not directly for infiltration.

The main difference between this model and the more commonly used tipping-bucket model is the use of hydraulic conductivity and a fairly fine depth layer distribution. In the tipping-bucket model, the layers are generally coarser and water in excess of field capacity is all assumed to move from one layer to the next in a day (although variations on this are to be found).

5.3.3 Field capacity and drainage point

Field capacity is a widely used term and is generally interpreted as being the water potential to which the soil will drain in a couple of days, which is a fairly imprecise definition. However, it is a useful concept and the definition can be tightened by defining it to be the water content at a particular matric potential. Various values are used, ranging from suctions of -50 cm (equivalent to -5 kPa) to -300 cm (equivalent to -30 kPa). In the present analysis, field capacity is defined as the water content at -100 cm (equivalent to -10 kPa) suction, and it should be interpreted in this way.

The Richards equation approach does not define field capacity as the equilibrium suction: this is just a parameterisation convenience that allows us to fix a point on the moisture retention curve, and to give some feel for the effect of the choice of parameters. In the model interface it is possible to watch the soil dry down from saturation with no evapotranspiration losses. This is an interesting simulation to observe as it shows how the soil approaches an equilibrium suction. For some soils, it will approach an equilibrium suction quite quickly, whereas for others this will not be so clear.

For the capacitance model, however, the *drainage point* is defined as the point to which the soil drains, and this may well take some significant time. Both the field capacity and drainage points are defined, and these are used in the calculation of the flux parameter σ in eqns (5.16e,f).

5.3.4 Pedo-transfer functions (PTF) and generic soil types

One of the main challenges in modelling water movement through soil is in the actual parameterisation of the infiltration and re-distribution model. This has been the underlying reason for the choice of the Campbell functions and the inclusion of the capacitance model – they both give a physically realistic treatment of the movement of water in the soil and can be defined in terms of parameters that have a clear biophysical interpretation.

In the present analysis soil parameters are available for a range of generic soil types. These have been derived using the pedo-transfer functions (PDF) developed by Saxton *et al.* (1986). These parameters are approximations and if more accurate information is available then that should be used. However, they do give a good starting point and can be used to see how sensitive the model is to parameter variation around the generic values.

The Saxton *et al.* functions have been used to estimate the parameters in the Campbell equation for the moisture retention function, eqn (5.10a), and also K_s, eqn (5.11a). The procedure adopted here is as follows:

- define particle size distribution by selecting a mid-point on the texture triangle;
- use the PTF program to estimate bulk density, field capacity, wilting point, K_s ; and
- calculate the saturated water content;

where it is assumed that

$$\theta_s = 0.95\left(1 - \frac{\rho_b}{2.65}\right) \qquad (5.17)$$

where ρ_b (g cm^{-3}) is the soil bulk density. The mean density of soil particles of 2.65 g cm^{-3} is almost universally used. The value of 0.95 implies that 95% of the air spaces fill with water at saturation, although values in the range 0.9 to 1 are often used. .

These parameter values are applied to the Campbell functions to derive the basic model parameters. This means that the field capacity and wilting point are used as the two points for estimating the soil parameters. Once the generic parameters have been derived for the Campbell function, the air-dry water content is then estimated from this function and used as a prescribed parameter in the capacitance model.

The generic types are not presented here as they can be found directly on the model interface.

5.3.5 Choice of approach

The model offers both the Richards equation and capacitance model available on the interface. For the Richards equation, the Campbell equation for moisture retention is used, for the reasons discussed in detail earlier. When deciding which model to use, consideration should be given to other difficulties, such as measuring saturated water content or saturated hydraulic conductivity, that may well have greater significance than the choice of model.

Richards equation: pros and cons

As mentioned earlier, the Richards equation is the combination of Darcy's equation, which describes movement along a water potential gradient, and mass balance. Darcy's equation is widely accepted as providing a physically sound description of water movement through soil, and has general characteristics that are consistent with many experimental observations. This includes upward as well as downward movement of water in the soil profile, so that water redistribution and evaporation from the soil are basic features of the model.

One major difficulty with the Richards equation is the prescription of the moisture retention curve and unsaturated hydraulic conductivity. Deriving the moisture retention curve usually involves taking soil cores and making laboratory measurements, which means the data are derived from disturbed, and often re-packed, soil samples. Unsaturated hydraulic conductivity is extremely difficult to measure. However, the procedures described here for seeking consistency between the moisture retention curve and unsaturated hydraulic conductivity seem to work well.

Solving the Richards equation can be challenging, particularly for soils with sharp changes in the soil characteristics. However, these problems can generally be overcome, although usually at the cost of speed of simulation.

Capacitance model: pros and cons

The capacitance model is relatively quick and easy to solve. However, it is completely empirical in that it assumes only water in excess of the drainage point will move: indeed, it assumes that this point can be prescribed. A further limitation with the approach is that it does not include upward movement of water in response to a water potential gradient, although it could be adapted to include this if it was considered sufficiently important. (Surface evaporation is included, as discussed later.) This model does not require direct information about the moisture retention curve, although it does need characteristic soil water content values that are consistent with such data.

One advantage of the capacitance model is that it is easy to implement numerically and computations are not complicated by complex soil profiles. It is also considerably faster (around 5 to 10 times) than the Richards equation, which may be an issue for long-term or multiple simulations.

Which one?

The Richards equation has a sound basis in soil physics and that gives it a strong appeal, whereas the capacitance model has ease and speed. Some soils, particularly those with sharp changes in soil physical properties within the profile, can be challenging for the Richards equation. While the routines could be developed further to deal with such soils, it may be more practical to use the capacitance model in such cases.

The principal focus of much of the research on soil water dynamics is on the movement of water through the profile. While it is self-evident that this is crucial, the other key factor is actually getting

an accurate assessment of how much water gets into the profile. Large quantities of water may be evaporated from either the canopy or the litter as well as being lost to the atmosphere as soil evaporation or plant transpiration. These estimates of canopy and litter interception are likely to be crucial in any water balance study: a 50 mm error in canopy and litter evaporation may result in a 50 mm over-estimate of through drainage. Similarly, runoff is going to play an important role in soil water dynamics, although it must be noted that the significance of runoff is generally more widely accepted.

As mentioned earlier, the capacitance model is more widely used since it gives similar behaviour to the Richards equation and is fast, accurate and stable.

5.4 Influence of ground cover on K_s

Ground cover can prevent crusting of the surface of the soil. In the model there are four soil layers that can have user-defined parameters (although the model uses many more internal layers for actual numerical calculations). While these layers can be used to any depth, the top one is denoted the surface layer and does provide the opportunity to include surface effects. The minimum surface layer thickness currently available is 2 cm.

Since ground cover can influence soil properties, particularly the saturated hydraulic conductivity K_s, there is a simple option to include this in the model. With this option, K_s in the surface layer varies linearly in response to litter cover between its prescribed value and that at the top of the A horizon.

5.5 Runoff

There are several approaches in the literature for the treatment of runoff, depending on the general objectives of the model as well as the spatial- and time-scales. For the present purposes, it is necessary to be able to calculate the flux of water off the paddock on a time-scale that is consistent with the treatment of rainfall inputs, infiltration and evapotranspiration, which are calculated at sub-daily intervals.

Two widely used, and quite similar, approaches are the Manning equation and Horton's equation. Manning's equation states that the speed of water movement across the surface, v (m s^{-1})

$$v = \frac{D^{2/3} S^{1/2}}{n} \qquad (5.18a)$$

where D (m) is the depth of water on the surface, S is the profile slope (%), and n (s m$^{-1/3}$) is the 'Manning coefficient', with typical values of around 0.04 for pastures and 0.1 for bare soil. Horton's equation is similar in structure to eqn (5.18a). The main disadvantage in using eqn (5.18a) is that n depends on the ground cover characteristics which will vary throughout the simulation and so a simplified approach is used.

While runoff will clearly depend on the depth of water on the soil surface, it will also depend on the ground cover, and so it is assumed that:

$$v = \lambda \left(D - D_0 \right) S^{1/2} \qquad (5.18b)$$

where D (m) is the depth of water on the surface, D_0 is the surface detention (amount of water that the bare soil surface can hold with no runoff), and λ (s^{-1}) is related to the relative ground cover (from 0 to 1) by

$$\lambda = \lambda_0 + (\lambda_{mx} - \lambda_0) \times \text{cover} \qquad (5.18c)$$

This approach captures the essence of runoff in that it increases with profile slope and depth of water on the surface (above a threshold value), while increasing ground cover will decrease runoff. The parameters λ_0 and λ_{mx} can be prescribed on the model interface. Note that the ground cover components are derived in section 3.3 of the *Climate* chapter.

The runoff speed, as given by eqns (5.18b, c) is shown in Fig. 5.9.

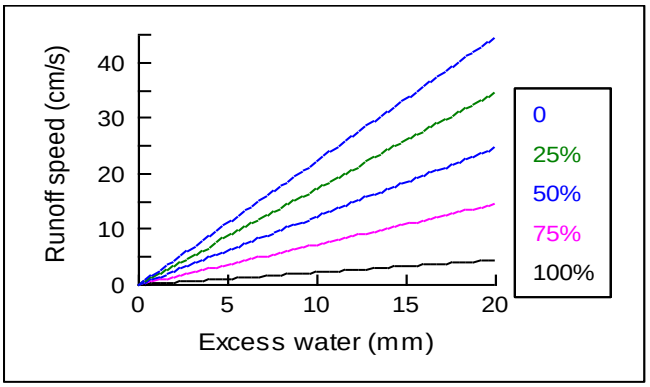

Figure 5.9: Runoff speed as a function of excess surface water for a range ground covers as shown. The parameters are: $S = 5\%$, $\lambda_0 = 100$ 100 s^{-1} and $\lambda_{mx} = 10$ 10 s^{-1}.

5.6 Evaporation

Water can evaporate from the canopy, litter or soil. Evaporation from the canopy occurs if there is any free standing water on the canopy. The potential evaporation is defined by eqn (5.7) or (5.8), depending on which evapotranspiration model is used. This is then used to calculate actual evaporation. The actual water holding capacity of the canopy and litter is discussed in section 3.2.3 in the *Climate* chapter.

5.6.1 Canopy

It is assumed that any water sitting on the canopy surface (leaves) is available for evaporation at the potential rate. If the amount of water on the canopy surface is H_{canopy}, then the actual evaporation of water from the canopy is simply:

$$E_{canopy} = \max\left(H_{canopy}, G_{canopy,tot} E_{d,pot}\right) \qquad (5.19a)$$

where $G_{canopy,tot}$ is the total ground cover by the canopy, eqn (3.4a) in the *Climate* chapter, and $E_{d,pot}$ is the potential daily evaporation from a free surface, as given by either eqn (5.7) or (5.8).

5.6.2 Litter

As for the canopy, it is assumed that if there is any water held in the litter then it is available for evaporation. However, the evaporative demand is attenuated in relation to canopy cover. Thus, if the amount of water held by the litter is H_{litter}, then the actual evaporation of water from the litter is

$$E_{litter} = \max\left\{H_{litter}, \left(1 - G_{canopy,tot}\right) G_{litter,tot} E_{d,pot}\right\} \qquad (5.19b)$$

where $G_{litter,tot}$ is the ground cover by the litter, eqn (3.5a) in the *Climate* chapter, and the other variables are the same as for eqn (5.19a).

5.6.3 Soil

Soil evaporation is the flux of water from the soil to the atmosphere in response to evaporative demand, ground cover and soil water content. It is treated differently for the Richards equation and capacitance model, and these are considered in turn. For either model, the potential soil evaporation is calculated from eqn (5.4) for the potential evaporation from a freely evaporating surface, E_{pot}, and the actual ground cover from both the canopy and the litter, as discussed in section 3.3 of the *Climate* chapter. Using these, the potential evaporation from the soil is

$$E_{pot,soil} = E_{pot}\left(1-G_{litter}\right)\left(1-G_{canopy,tot}\right) \qquad (5.20a)$$

where the ground cover components for litter and total canopy (live plus dead) are defined in eqns (3.4a, 3.5a).

Note that if there is any free-standing water on the soil surface this is subject to evaporation at the potential rate and the potential evaporation from the soil is adjusted accordingly.

Richards equation

With the Richards equation, water can move up the profile in response to a gradient in potential. Evaporation is therefore calculated as the flux of water out of the top layer, recognising that water will move from lower layers to this layer as it dries down. It is assumed that evaporation from this layer is not affected by the actual water content, other than to define the water that is available for evaporation. This means that as the surface layer dries down, the movement of water to the atmosphere is only restricted by the available water and not the influence that soil water has on the soil hydraulic conductivity. The evaporation from the surface layer is therefore defined as:

$$E_{soil,\Delta t} = \min\left\{E_{pot,soil,\Delta t}, \left(\theta_1 - \theta_{1,ad}\right)\mathrm{d}z_1\right\} \qquad (5.20b)$$

where Δt is the time-step, the subscript '1' refers to the top soil layer, and $\mathrm{d}z_1$ is the thickness of this layer (2 cm in the model). According to this equation, water can evaporate from this layer at the potential rate until the water content reaches the air-dry point. The most significant limitation to evaporation is probably not the flux of water from this surface layer but the flux of water into that layer from below.

The potential evaporation for the time-step Δt is calculated from the daily potential evaporation and its distribution throughout the day.

Capacitance model

Since the capacitance model does not include upward movement of water, it is not sufficient to have water being removed only from the surface layer. The following empirical approach has been developed to attenuate evaporation through the depth of the profile, recognising that evaporation can occur from deeper in the profile for soils with greater saturated hydraulic conductivity.

First, the scaling function with depth is defined as

$$\mu_L = \exp\left(-0.69\frac{z_{T,L}}{z_{e,h}}\right)\left(\frac{\min\left(\theta_L, \theta_{L,fc} - \theta_{L,ad}\right)}{\theta_{L,fc} - \theta_{L,ad}}\right) \qquad (5.20c)$$

where $z_{T,L}$ is depth at the top of layer L, $z_{e,h}$ is the depth for which the first term in this evaporation function takes the value 0.5 and the soil water content variables defined earlier are calculated for

each layer. The first term in this function scales with depth while the second one relates to the availability of soil water. The water that is available for evaporation from each layer is then defined as

$$E_{L,avail} = \mu_L \left(\theta_L - \theta_{L,ad} \right) \Delta z_L \qquad (5.20d)$$

Actual soil evaporation is then defined as

$$E_{soil} = \min \left(1, \frac{E_{d,pot}}{\sum_L E_{L,avail}} \right) \qquad (5.20e)$$

which is then removed from each layer in proportion to the water available in that layer.

Although this is completely empirical, the approach does have the general expected behaviour of evaporation. The key parameter to define on the interface is the scale parameter for 50% reduction in potential evaporation, $z_{e,h}$.

5.7 Concluding remarks

A detailed account of the water dynamics has been presented. This includes atmospheric losses through evapotranspiration, surface runoff, redistribution within the soil profile and through drainage. Various options are available for the treatment of evapotranspiration, depending on the available climatic information. Furthermore, two models for infiltration and redistribution can be used – the Richards equation and the capacitance model – which allows some flexibility in studying the water balance. As well as presenting these models, they have been put in perspective with other approaches that are used. Options for irrigation are also incorporated in the model that allow irrigation water to be applied in a variety of ways.

Water balance is an interaction between a range of complex flows. The model structure should allow the user to explore these fluxes and so gain understanding into the underlying behaviour of the system.

5.8 References

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6 Soil nutrient dynamics

6.1 Introduction

The soil nutrient dynamics component of this model includes organic matter turnover and inorganic nutrient mineralization or immobilization, inorganic nutrient movement in the soil (leaching), nutrient adsorption in the soil, and atmospheric losses of nitrogen. The model includes N, P, K, S. It is also possible to choose which, if any, of P, K, S are implemented by clicking *Options | Active nutrients…* However, because of the central role that nitrogen plays in the nutrient dynamics, nitrogen cannot be excluded. The model requires the initial organic and inorganic status to be defined in order to start the simulation – these are defined in the model either graphically or by running a simulation and using the resulting system state. For inorganic nutrients that are subject to leaching it is common to find a nutrient bulge somewhere down the profile where nutrients accumulate through leaching. The interface allows the user to construct such a bulge for the initial nutrient status or again, prescribe this from the results of a simulation that has been run. The supply of organic matter is from litter (dead plant material), dung and dead roots. There are three soil organic matter pools (in addition to surface litter, dung and live roots): fast and slow turnover, and inert. The inert material does not decay but must be accounted for as it will show up in experimental measurements. The nutrient dynamics model is illustrated schematically in Fig. 6.1 for nitrogen – other nutrients are similar, although they only exist in one inorganic form and are not subject to gaseous losses. The model described here is relatively simple in structure compared with many other soil organic models, although it does capture the general processes involved.

Since measurements of soil nutrients are made much less frequently than those for soil water, care must be taken in data analysis. For example, the nitrification of ammonium (that is, the transformation from NH_4 to NO_3) is affected by water status and temperature. Since most organic matter is near the surface, and since organic matter breakdown involves the production of NH_4 which is then transformed to NO_3, the time at which these components are measured in relation to climatic conditions will be important. This is compounded by the fact that NO_3 leaches freely with water movement, whereas there is very little movement of NH_4. Furthermore, both volatilization and denitrification (atmospheric losses of NH_4 and NO_3 respectively) are very episodic and so are extremely difficult to measure.

In the analysis, all pool dynamics are defined for the same layer distribution as is used in the soil water dynamics. Organic matter and inorganic nutrients are expressed with units kg m^{-3}.

The model structure originated from the work of McCaskill (1987) and McCaskill and Blair (1988), although it should be noted that it has evolved considerably since then.

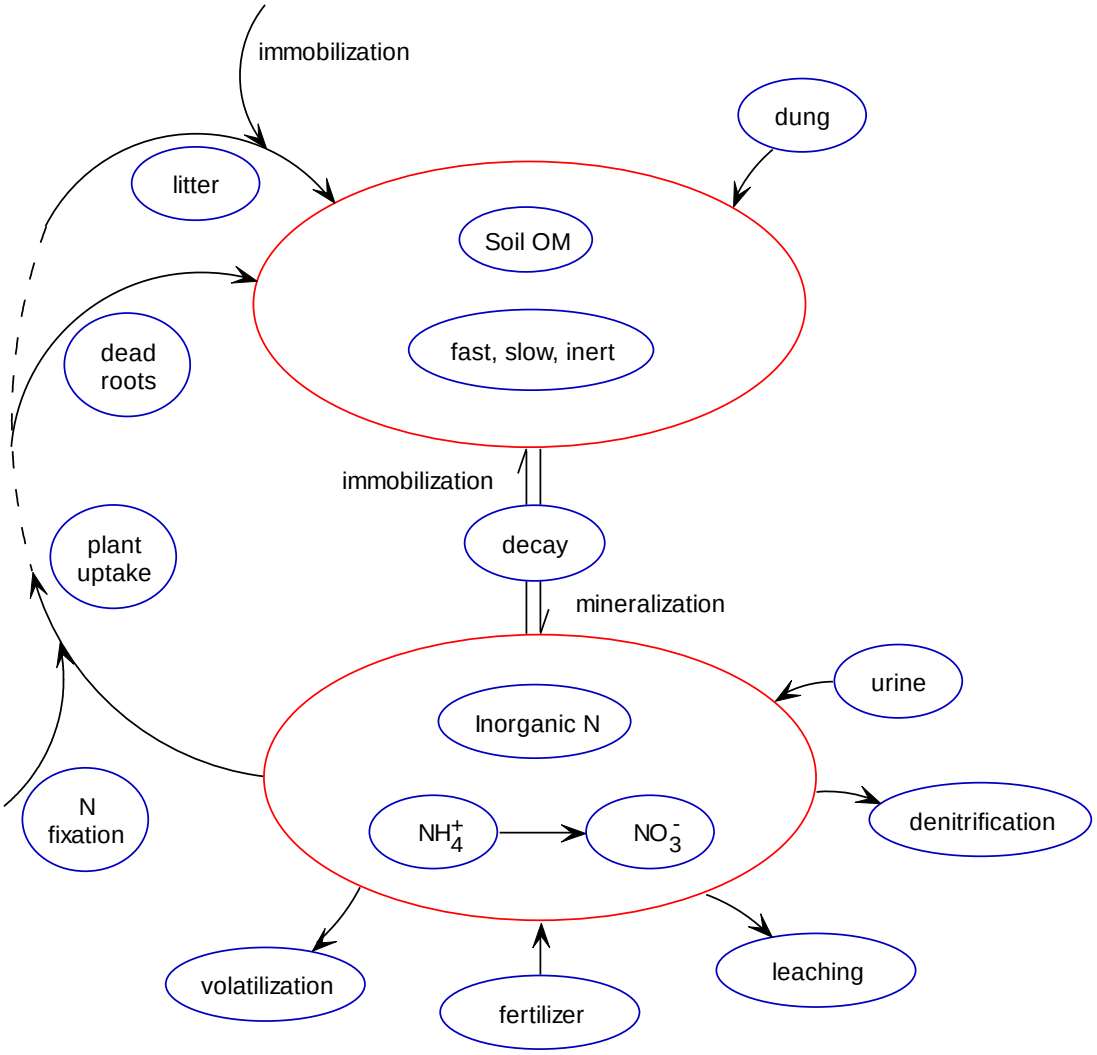

Figure 6.1: Schematic representation of the organic matter and nitrogen dynamics.

6.2 Organic matter dynamics

6.2.1 Overview

The turnover of organic matter (OM) is important both for the carbon balance of the system and also the mineralization and immobilization of nutrients. The theory is presented first for carbon and nitrogen and then extended to other nutrients, although it is assumed that all K is immediately leached out of organic matter on senescence and so it does not play a role in the OM dynamics.

There are three OM pools in the model, fast and slow turnover, as well as inert, although the inert pool does not affect the OM turnover. While there is not direct representation of the microbial biomass (BM) in the model, it is assumed that this is incorporated in the fast turnover pool. Other models (for a review see, for example, Thornley, 1998 chapter 5), often have more pools or greater complexity than in the present model. The approach here has been developed with the objective of having the features that are generally observed in organic matter dynamics while remaining as simple as possible. In other models it is common to have so-called *protected* and *unprotected* pools, with protected pools having much lower turnover rates. In the present model, the fast and slow turnover pools are intended to encapsulate these principals and, while more pools may give greater

flexibility, the aim here is to attain that flexibility through the influence of model parameters on soil conditions.

The process of organic matter decay from either the fast or slow pool is assumed to be driven by soil micro-organisms, or biomass (BM), although they are not modelled explicitly. The biomass has a prescribed C:Nu ratio, where Nu can be either N, P or S. During OM decay a proportion of the carbon is respired. The resulting C:Nu ratio (after respiration), is then compared to the C:N ratio of the BM, which determines whether there will be mineralization (release of nutrients) or immobilization (uptake from the inorganic pools). This material is now assumed to be BM. During decay from the fast pool, it is further assumed that a proportion of the BM is transferred to the slow pool, which is taken to represent the movement from protected to unprotected material. The complete OM dynamics are illustrated in Fig. 6.2 for nitrogen – P and S will be similar:

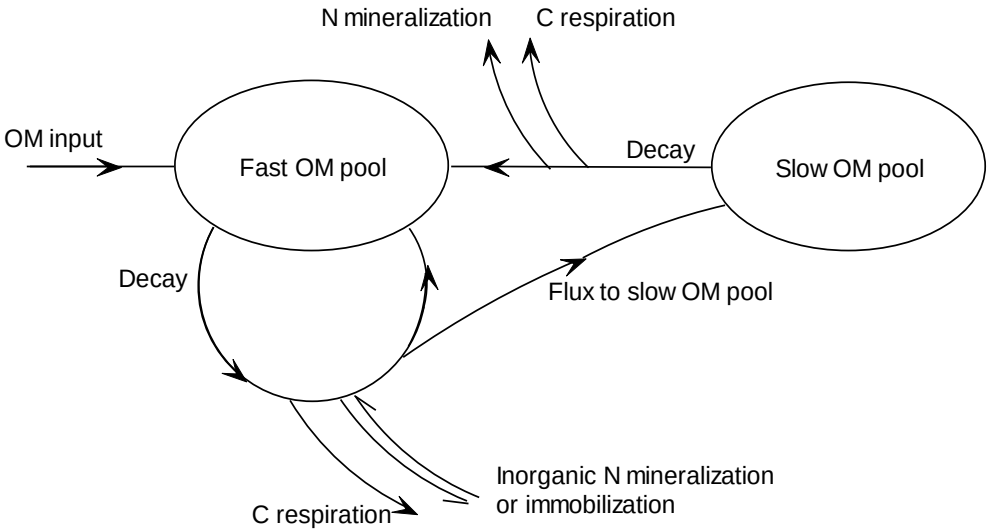

Figure 6.2: Schematic representation of the organic matter dynamics for N. P and S will be similar.

6.2.2 Organic matter turnover

The process of breakdown involves utilisation of carbon to produce microbial biomass with an associated respiratory loss. Let the rate constant for pool decay be k, and the efficiency of breakdown be Y. This means that for every kg of carbon in this pool that decays, the production of microbial biomass is Y kg with the remaining $(1-Y)$ being lost to respiration, as illustrated in Fig. 6.3:

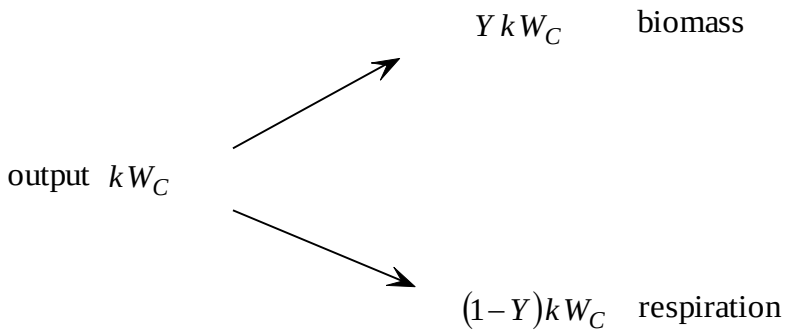

Figure 6.3: Schematic representation of OM breakdown. See text for details.

This general structure will be applied to both the slow and fast turnover pools.

The equations describing the OM dynamics as illustrated in Fig. 6.2, along with the scheme in Fig. 6.3 can be written as:

$$\begin{aligned}\frac{\mathrm{d}W_{f,C}}{\mathrm{d}t} &= I_C - k_f W_{f,C} + Y_f k_f W_{f,C}(1-\gamma) + Y_s k_s W_{s,C} \\ &= I_C - k_f W_{f,C}\left[1 - Y_f(1-\gamma)\right] + Y_s k_s W_{s,C}.\end{aligned} \qquad (6.1a)$$

and

$$\frac{\mathrm{d}W_{s,C}}{\mathrm{dt}} = \gamma Y_f\, k_f W_{f,C} - k_s W_{s,C}. \qquad (6.1b)$$

where $W_{f,C}$ and $W_{s,C}$ (kg carbon) are the carbon components of the fast and slow turnover pools; k_f and k_s are the rate constants for their decay (day^{-1}); Y_f and Y_s are the efficiencies of breakdown of the fast and slow pools (dimensionless); and γ is the proportion of BM transferred to the slow pool during the decay of the fast pool.

The decay rate constants, k_f and k_s, are assumed to be dependent on temperature and water status as discussed later. The default parameter values at 20°C and with no water stress are:

$$\begin{aligned} &k_f = 0.015 \text{ day}^{-1}, && k_f = 0.00012 \text{ day}^{-1}, \\ &Y_f = 0.6, && Y_s = 0.1, \\ &\gamma = 0.1 \end{aligned} \qquad (6.1c)$$

Equations (6.1a, b) completely define the organic carbon dynamics in the model. As a check, note that summing them gives:

$$\frac{\mathrm{d}W_C}{\mathrm{dt}} = \frac{\mathrm{d}W_{f,C}}{\mathrm{dt}} + \frac{\mathrm{d}W_{s,C}}{\mathrm{dt}} = I_C - k_f W_{f,C}(1-Y_f) - k_s W_{s,C}(1-Y_s), \qquad (6.1d)$$

as required, since the negative terms are the respiratory losses associated with organic matter breakdown.

6.2.3 Organic nutrient dynamics

The nutrient dynamics associated with organic matter breakdown are now considered. First, recall that it is assumed that there is no K in the organic matter, as it is generally observed to be immediately leached into the inorganic pool.

Let nu denote any of the other nutrients in the OM model (N, P, S), define the C:nu ratio of the biomass to be Cnu_b, and let Y and k be the efficiency and rate constant for either the fast or slow pool as appropriate. For the rate of biomass production as illustrated in Fig. 6.3, the required nitrogen is

$$Y\,k\frac{W_C}{Cnu_b}. \qquad (6.2a)$$

Now, during the breakdown process, the output of nutrient is

$$k\frac{W_C}{Cnu} \qquad (6.2b)$$

so that the corresponding flux of nutrient into the inorganic pool is

$$M_{nu} = kW_C\left(\frac{1}{Cnu} - \frac{Y}{Cnu_b}\right) \qquad (6.2c)$$

If this term is positive, then mineralization occurs; if it is negative then there is a flux of nutrient from the inorganic pool to the biomass and so the nutrient is immobilized. Mineralized nitrogen is transferred directly into the ammonium pool. The ammonium can then be transformed into nitrate through nitrification, as discussed later. However, any immobilization of nitrogen will incorporate ammonium first and if there is insufficient ammonium then nitrate will be used. This assumes that the nitrification of ammonium can occur.

This analysis is applied to both the slow and fast pools, where allowance is made for the fact that:

- for the fast pool, the biomass remains in that pool
- for the slow pool, the biomass moves to the fast pool.
- Since the biomass is the material that moves into the slow pool, the C:nu ratio of the slow pool is the same as that for the biomass. Consequently, there is only mineralization and not immobilization during decay from the slow pool.

The equations for nutrient dynamics are therefore:

$$\frac{\mathrm{d}W_{f,nu}}{\mathrm{d}t} = I_{nu} - M_{nuf} - Y_f k_f \frac{W_{f,C}}{Cnu_b} + k_s W_{f,nu} \qquad (6.2d)$$

and

$$\frac{\mathrm{d}W_{s,nu}}{\mathrm{d}t} = \lambda_f Y_f k_f \frac{W_{f,C}}{Cnu_b} - k_s W_{s,nu}\,. \qquad (6.2e)$$

In the model, the nutrient composition of the biomass is taken to be:

$$CN_b = 8\,,\ \ CP_b = 80\,,\ \ CS_b = 80\,. \qquad (6.2f)$$

(Recall that there is no K in the organic material.)

Initial conditions

The organic matter pools need to be initialised at the start of the simulation, and the theory here describes how this can be done. However, an alternative approach is to run the simulation for a long period of time and use the resulting system state as the starting point for subsequent simulations. This option is available in the model and is described in section 6.6 below.

Considering the nitrogen composition of the pools first, let the pool fractions of the total organic matter be f_f, f_s, f_i, where the subscripts denote the fast, slow and inert pools, so that

$$f_f = \frac{C_f}{C};\ f_s = \frac{C_s}{C};\ f_i = \frac{C_i}{C}\,, \qquad (6.3a)$$

where

$$C = C_f + C_s + C_i\,, \qquad (6.3b)$$

The C:N ratio of the total organic matter is

$$CN = \frac{C}{N_f + N_s + N_i} = \frac{C}{C_f/CN_f + C_s/CN_s + C_i/CNi}, \quad (6.3c)$$

and hence, using (6.3a)

$$\frac{1}{CN} = \frac{f_f}{CN_f} + \frac{f_s}{CN_s} + \frac{f_i}{CN_i} \quad (6.3d)$$

and, defining

$$f_f = \alpha(1-f_i), \quad f_s = (1-\alpha)(1-f_i), \quad (6.3e)$$

this becomes

$$\frac{1}{CN} = \frac{\alpha(1-f_i)}{CN_f} + \frac{(1-\alpha)(1-f_i)}{CN_s} + \frac{f_i}{CN_i}. \quad (6.3f)$$

The approach adopted here is to define all of the CN terms in eqn (6.3f) and the inert fraction of soil carbon, f_i, at the surface of the soil. α, which is the fast fraction of the fast plus slow pool, is then calculated from (6.3f). It is then assumed that α remains constant through the depth of the profile (although the fast plus slow component will decline). This gives sufficient information to initialise the nitrogen composition of the soil organic matter pools.

While this may seem a complex way to initialise the pools, other approaches, such as defining α, have been explored, but they can lead to problems in the equations, such as negative C:N ratios. The present approach seems quite stable and relatively straightforward to implement.

Once initial values for nitrogen have been calculated, the nutrient compositions of the P and S pools are defined by assuming that

$$\frac{CP_f}{CP_s} = \frac{CS_f}{CS_s} = \frac{CN_f}{CN_s}. \quad (6.3g)$$

Effects of water and temperature

The soil water status and temperature will influence the rate of organic matter turnover through the rate constant parameters k_f and k_s. For either of these parameters, define

$$k(T) = k_{ref} f(T) g(\theta) \quad (6.4a)$$

where the functions $f(T)$ and $g(\theta)$ incorporate the effects of temperature and water respectively (θ is the volumetric water content – see the *Water* chapter), while k_{ref} is the value of k at the reference temperature of 20°C and with no limitation due to water stress.

The temperature response is defined using a generic temperature function, adapted from Thornley (1998), and given by:

$$f(T)=\begin{cases} 0, & T<T_{mn}, \\ \dfrac{(T-T_{mn})^q\left[T_{opt}-T_{mn}+q\left(T_{opt}-T\right)\right]}{\left(T_{ref}-T_{mn}\right)^q\left[T_{opt}-T_{mn}+q\left(T_{opt}-T_{ref}\right)\right]}, & T_{mn}\le T\le T_{opt}, \\ \dfrac{(T-T_{mn})^q\left(T_{opt}-T_{mn}\right)}{\left(T_{ref}-T_{mn}\right)^q\left[T_{opt}-T_{mn}+q\left(T_{opt}-T_{ref}\right)\right]} & T>T_{opt}. \end{cases} \quad (6.4b)$$

where T_{mn} is the minimum temperature, T_{opt} is the optimum temperature, q is a curvature coefficient, and $T_{ref} = 20$°C is the reference temperature with

$$f\left(T=T_{ref}\right)=1. \quad (6.4c)$$

While this expression may seem complex, it is quite simple to work with and has all the versatility required. The response is illustrated in Fig. 6.4. Note that a similar function, that does not constrain $f(T)$ above T_{opt}, is used elsewhere in the model for temperature processes that do not have an asymptote.

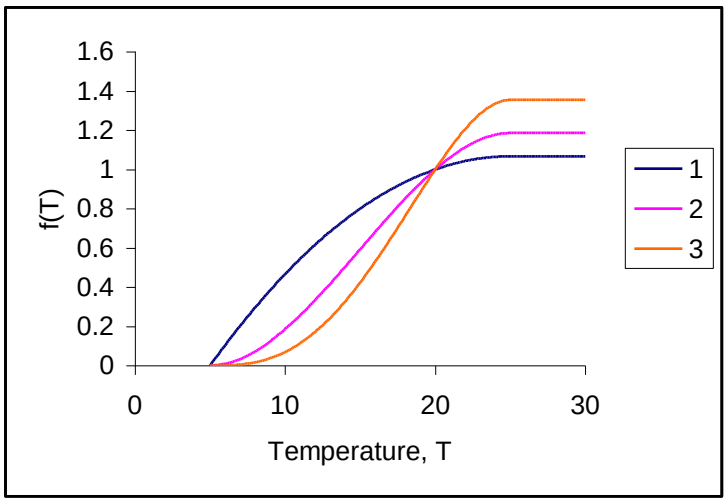

Figure 6.4: Temperature function given by eqn (6.4b). The parameter values for T_{mn}, T_{ref}, T_{opt} are 5, 20, 25 °C respectively and the curves for $q = 1,2,3$ are indicated.

Note that at present, variation in the soil temperature through the profile is not incorporated and so a single value for the soil temperature is used. However, this could be relaxed if necessary.

The water function, $g(\theta)$, is defined as a simple ramp function between wilting point and field capacity, and is expressed mathematically as:

$$g(\theta)=\begin{cases} 0, & \theta<\theta_w \\ \dfrac{\theta-\theta_w}{\theta_{fc}-\theta_w} & \theta_w\le\theta\le\theta_{fc} \\ 1 & \theta>\theta_{fc} \end{cases} \quad (6.4d)$$

where the subscripts w, fc, sat, refer to wilting point, field capacity and saturation respectively. The water content in each layer is applied to this function. This is illustrated in Fig. 6.5

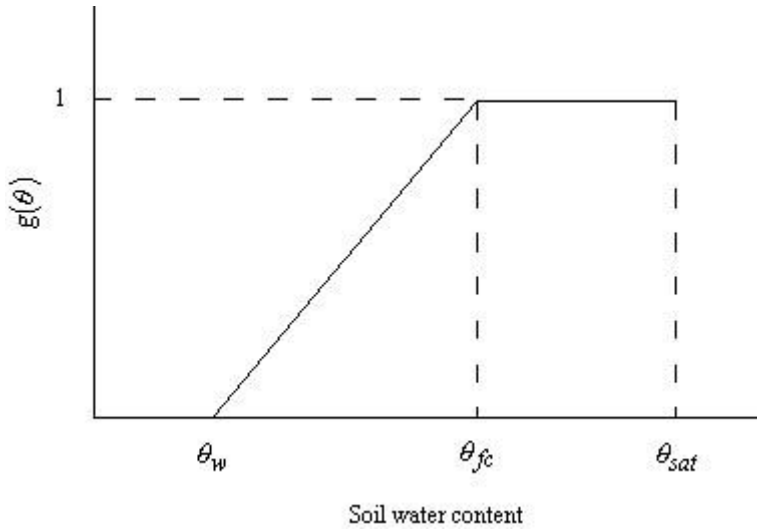

Figure 6.5: Effect of water status on the turnover of organic matter. See text for details.

6.2.4 Surface litter

Surface litter is treated exactly the same as organic matter as described above, but with the following considerations:

- It is assumed that the turnover rate for litter is half that of soil organic matter. This is because of lower microbial levels.
- The water content in the surface soil layer is used in the expression for the effect of water status on litter turnover – that is $g(\theta)$ in the above analysis.
- There is a physical transfer of litter from the soil surface to the soil.

The depth to which litter can be transported and the rate constant for this transfer (proportion per day) are prescribed. Litter is then transferred evenly to this depth at this rate.

6.2.5 Dung

Dung is also treated in the same way as organic matter, but with the following considerations:

- The nutrient content of excreta is partitioned between dung and urine, which defines the nutrient composition of dung – see section 7.6.2 in the discussion of the animal processes.
- There is a physical transfer of dung from the soil surface to the soil.

The physical transfer is described analogously to the transfer of litter.

6.3 Inorganic nutrient dynamics

Plants acquire nutrients from the inorganic pools. The mineralization, and possibly immobilization, through organic matter dynamics has been described in section 6.2.3. The other processes in the model are plant uptake, adsorption, leaching, nitrification of ammonium and gaseous nitrogen losses, as well as inputs from dung, urine and fertilizer. Nutrient uptake by the plants is described in the *Pasture* chapter (see section 4.4.2). The inorganic nutrients that are considered are NH_4, NO_3, SO_4, PO_4, and special consideration will be given for nitrogen to accommodate the differences between nitrate and ammonium.

6.3.1 Nutrient adsorption

Nutrient adsorption is a key component in the movement of nutrients through the profile: nitrate does not adsorb and so is prone to leaching, whereas for many soil types most of the phosphate is adsorbed and so is less likely to leach, although this may not be true for very sandy soils. Note that in this section, all analysis uses SI units – that is kg rather than mg – although solute concentrations are generally discussed in mg nutrient (kg soil)$^{-1}$ or mg nutrient L^{-1} for solution in water. Mixing units can create problems with the analysis and, while these are not insurmountable, it is better practice to use SI units and make conversions at the end. Two commonly applied approaches for describing nutrient leaching are to use either the Freundlich equation or the Langmuir equation.

The Freundlich equation is a power law, as given by:

$$C_a = \lambda C_s^q \qquad (6.5a)$$

where, for any nutrient nu, C_a is the nutrient concentration in the soil, kg nu (kg soil)$^{-1}$, C_s is the nutrient concentration in solution, kg nu (kg water)$^{-1}$, a and s refer to adsorbed and solution respectively, and λ and q are empirical parameters, with q usually in the range 0.4 to 0.5 for a wide range of soils. Note that the reciprocal of q is sometimes used.

The Langmuir equation is a form of the rectangular hyperbola, and can be written as:

$$C_a = \frac{\alpha C_{a,mx} C_s}{\alpha C_s + C_{a,mx}} \qquad (6.5b)$$

where α [kg P (kg soil)$^{-1}$] [kg P (kg water)$^{-1}$]$^{-1}$ is the initial slope of the curve at low values of C_s, and $C_{a,mx}$ kg nu (kg soil)$^{-1}$ is the maximum adsorption capacity of the soil at saturation. The parameters can be expressed mathematically as:

$$C_a \to \alpha C_s \text{ as } C_s \to 0, \text{ and } C_a \to C_{a,mx} \text{ when } C_s >> \alpha C_s. \qquad (6.5c)$$

With either of these equations, the analysis relates adsorbed to solution nutrient concentrations. However, since the model defines total inorganic nutrient mass, it is necessary to calculate each of these components from the total. An advantage of the rectangular hyperbola is that it allows analytical solution of the individual concentrations in terms of the total nutrient in the soil, whereas the use of the power law (eqn 6.5a) requires a numerical approach. A second advantage is that the linear characteristic of the adsorption curve at low nutrient levels is generally more realistic. . Figure (6.6) shows the rectangular hyperbola (eqn 6.5b) fitted to the data of Moody and Bolland. (1999) for phosphorous adsorption in three contrasting soils, and it can be seen that the curve gives a good fit to each data set.

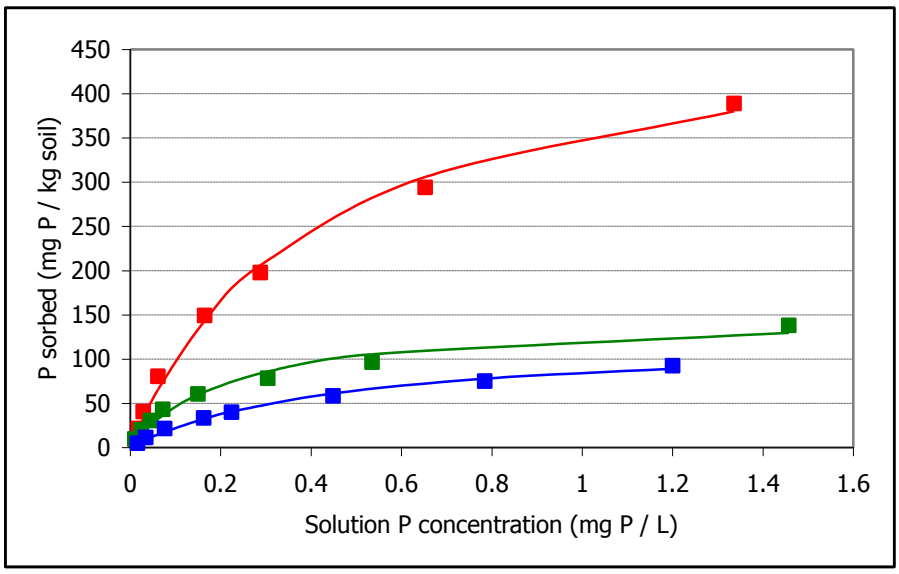

Figure 6.6: Phosphorous sorption curves, eqn (6.5b) along with data from Moody and Bolland (1999) for a Ferrosol (red), Vertosol (green), Podosol (blue).

The parameters for the soil types in Fig. 6.6 are

	β (soln / sorbed at low P)	$C_{a,mx}$ (max P sorption capacity)
Ferrosol	1225	495
Vertosol	661	150
Podosol	270	123

The default parameters used for all nutrients are:

	NH_4	PO_4	K	SO_4
β (soln / sorbed at low nu)	1000	1000	1	1
$C_{a,mx}$ (max nu sorption capacity)	500	500	100	100

The sorption curve with default parameters is shown for phosphorous in Fig. 6.7.

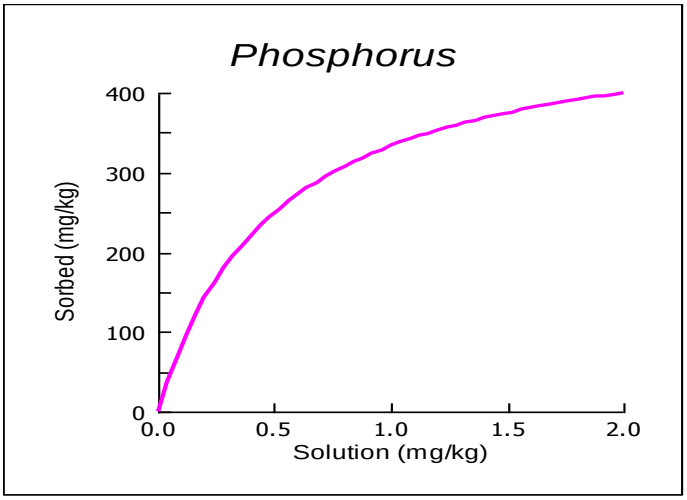

Figure 6.7: Solution phosphate as a function of sorbed phosphate using eqn (6.5b) with default parameters: see text for details.

The analysis for deriving the sorbed and solution nutrient components from the total nutrient content of any soil layer is now presented. If the mass of nutrient in any layer of depth Δz (m) is M kg nu m^{-2}, then the adsorbed and solution components are

$$M_a = C_a \Delta z \rho_b \tag{6.6a}$$

and

$$M_s = C_s \Delta z \theta \rho_w \tag{6.6b}$$

where ρ_b, kg soil m^{-3} is the soil bulk density, ρ_w, kg water m^{-3} is the density of water, and θ (m^3 water) (m^3 soil)$^{-1}$ is the volumetric soil water content, and

$$M = M_a + M_s \tag{6.6c}$$

is the total mass of nutrient in the layer.

It now remains to calculate the concentration components in terms of the total nutrient mass. Writing

$$m = M/\Delta z \tag{6.6d}$$

it is readily shown that

$$\alpha\theta\rho_w C_s^2 + \left[C_{a,mx}\left(\alpha\rho_b + \theta\rho_w\right) - \alpha m\right]C_s - mC_{a,mx} = 0 \tag{6.6e}$$

which is a quadratic equation for C_s as a function of m. While this has two solutions, one of them is always negative, and so the positive solution defines C_s: it is simple to calculate this solution.

In the model, plant uptake is described in terms of total inorganic nutrient and not the actual solution component, although for P there is the additional treatment of buffered P that is unavailable for direct uptake by the plant – see section 6.5. While nutrient in solution is the source for the plant, there is a continual flux from sorbed sites to the solution. It is likely that the roots are physically close to the soil particles so that the nutrient concentration in the region near the roots may well differ from that in the bulk soil water. Furthermore, for phosphorus (highly adsorbing), the actual nutrient in solution at any particular time is only sufficient for a few hours growth, so that there is a continual flux from sorbed to solution to the plant. The principal role of nutrient adsorption in the model is in the description of nutrient leaching, with plant uptake being described in relation to total nutrient.

6.3.2 Nutrient leaching

The treatment of nutrient leaching has been developed following discussions with John Hutson of Flinders University. It is assumed that nutrients in solution can move with the water as the water moves through the soil profile. The range of pore sizes in the soil is related to the soil water characteristics, so that the proportion of large pores that contain water increases as the soil water content increases and, conversely, as the soil dries down the water is located mainly in the smaller pores. It is therefore assumed that there is a threshold water content that defines the actual water that moves through infiltration. It is then assumed that the solute that moves is restricted to that which is in this water.

The proportion of nutrient that can move is related to the critical soil water content being defined as

$$\theta_{crit} = \theta_w + \varepsilon\left(\theta_w + \theta_{fc}\right) \tag{6.7a}$$

where θ_w and θ_{fc} are the wilting point and field capacity respectively and ε is a parameter between 0 and 1, with default 0.5. Note that ε =0.5 corresponds to a soil water content of around -100 kPa for a wide range of soils (see section 6.4.1). The proportion of nutrients that can then leach on a given day is now given by

$$\eta = \begin{cases} \dfrac{\theta - \theta_{crit}}{\theta_{sat}}, & \theta > \theta_{crit}, \\ 0, & \theta \le \theta_{crit}, \end{cases} \qquad (6.7b)$$

where θ_{sat} is the saturated soil water content and θ is the soil water content at the start of the day. This effectively means that the drier the soil, the less nutrients are available for movement. At the end of the day the remaining nutrients are mixed with any fresh water that has been moved into the profile. Thus, for example, a flush of water through the profile in one day will not be able to take all of the nitrate.

Note that since the leaching coefficient, ε, is defined in terms of soil water content parameters, it is prescribed on the soil water interface in the model.

6.3.3 Urine inputs

The partitioning of nutrients between dung and urine may play an important role in nutrient dynamics and the associated plant response, since urine returns are readily available whereas for dung the process of organic matter decay delays the release. This partitioning is discussed in the *Animal* chapter, section 7.7.2. Urine N inputs are transferred directly to the soil NH_4 pool. While nutrient dynamics in urine patches are likely to differ from the bulk soil due to the greater concentrations of nutrient in the patches, no explicit treatment for urine patch dynamics is considered here. For urine inputs, the user specifies a maximum depth and scale factor to distribute nutrient inputs.

6.3.4 Fertilizer application

Fertilizer is obviously an important source of nutrients and can be applied in the model. Up to 15 different fertilizer management options can be selected, that combine fixed dates, date ranges, rotation (cutting or grazing) and responses to soil test nutrient values. These options can be combined to provide a flexible set of conditions for fertilizer application. The structure should be self-explanatory. Some of these strategies use the nutrient growth limiting factors, or GLF_{nu} where nu represents a particular nutrient, which was defined in the *Pasture growth* chapter, section 4.3.2.

6.4 Inorganic nitrogen dynamics

Nitrogen dynamics need to be considered because inorganic nitrogen occurs as both nitrate and ammonium, ammonium is converted to nitrate through nitrification, and also gaseous nitrogen can be lost to the atmosphere through volatilisation of ammonium and denitrification of nitrate. These are considered in turn. Note that during organic matter breakdown nitrogen is added to the soil in the form of ammonium.

6.4.1 Nitrification of ammonium

Nitrification of ammonium, which is the conversion of NH_4 to NO_3 is described using a Michaelis-Menten response to available soil ammonium, so that the rate of nitrification is

$$\zeta = V_{mx,NH_4}\left(\frac{[NH_4]}{[NH_4] + K_{NH_4}}\right) f(T) f_\zeta(\theta)\gamma_C \qquad (6.8a)$$

where $[NH_4]$ is the ammonium concentration in the soil, mg N kg^{-1}, V_{mx,NH_4} is the maximum rate of nitrate production, mg N kg^{-1} day^{-1}, K_{NH_4} is the NH_4 concentration for half maximal response to ammonium concentration, the f functions are the temperature and water responses respectively, and γ_C represents the effect of soil microbial mass as described below.

According to this approach, the nitrification rate is linear in response to available soil ammonium at low concentrations and then curves to an asymptote as the concentration increases. It is apparent from (6. 8a) that at low concentrations

$$\zeta \approx \frac{V_{mx,NH_4}}{K_{NH_4}}[NH_4] f(T) f_\zeta(\theta)\gamma_C \qquad (6.8b)$$

The default parameters are

$$V_{mx,NH_4} = 20 \text{ mg N kg}^{-1} \text{ day}^{-1} \text{ and } K_{NH_4} = 90 \text{ mg N kg}^{-1} \qquad (6.8c)$$

The response with no limitations due to temperature, water or carbon is shown in Fig. 6.8.

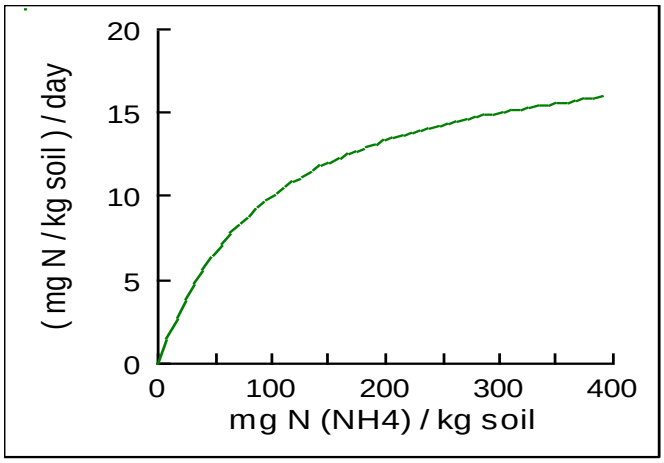

Figure 6.8: The rate of nitrification as a function of available soil ammonium for non-limiting water, temperature and carbon conditions as defined in eqn (6.8a) with parameters in eqn (6.8b) – see text for discussion.

Since there is no direct treatment of the soil microbial pool, it is assumed that the total labile soil carbon (fast plus slow turnover) reflects the level of microbes, so that

$$\gamma_{C,L} = \frac{W_{s,C}(L) + W_{f,C}(L)}{W_{s,C}(L=0) + W_{f,C}(L=0)} \qquad (6.8d)$$

where L represents the soil layer, so that γ_C is the ratio of labile soil carbon in any layer to that in the top layer.

For the influence of soil water status on nitrification, it is widely observed that it occurs between soil water potentials of around -100 kPa and -10 kPa (field capacity) (RE White, *personal communication*). The following analysis requires knowledge of the Campbell function for describing the moisture retention: this is discussed in section 5.3.2 and is not explained further here. Defining wilting point and field capacity as -1500 kPa and -10 kPa respectively, it is readily shown that

$$y = \frac{\theta_{100}}{\theta_w + \theta_{fc}} = \frac{\left(\frac{1}{10}\right)^{1/b}}{\left(\frac{1}{150}\right)^{1/b} + 1} \qquad (6.\ 8e)$$

The soil parameter b is typically in the range 5 to 15, and this expression is plotted for that range in Fig. 6.9. It can be seen that for a range of b values, this value is very close to 0.5. This means that for a wide range of soil types, the water content corresponding to -100 kPa is approximately the mean of wilting point and field capacity. That is:

$$\theta_{100} \approx \frac{\theta_w + \theta_{fc}}{2} \qquad (6.\ 8f)$$

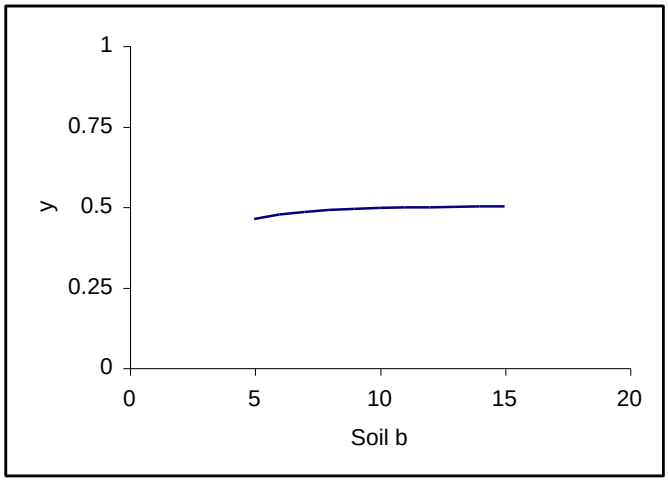

Figure 6.9: The y parameter as defined in eqn (6.8e) – see text for discussion.

Since water potential is only included in the model with the Richards equation for infiltration and redistribution of water, the effect of water on nitrification needs to be defined in terms of water content and not water potential. Using this convenient expression in (6.8f), the influence of soil water content on nitrification is defined as illustrated in Fig. 6.10. This equation is not written explicitly here, although it is easy to do so in a similar way to eqn (6.4d).

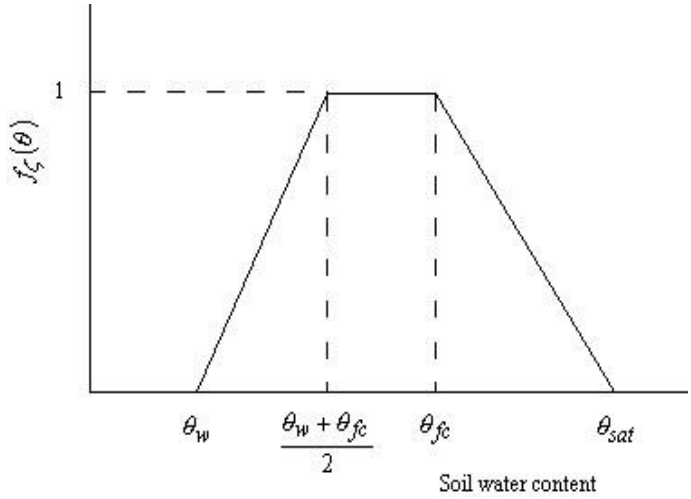

Figure 6.10: Influence of soil water content on nitrification of ammonium: see text for details.

6.4.2 Denitrification of nitrate

Denitrification is the conversion of nitrate to nitrous oxide and nitrogen gas and, while the actual denitrification losses may be relatively small in terms of the overall nitrogen dynamics in the system, the fact that nitrous oxide is such a major greenhouse gas (the CO_2 equivalent value is currently taken to be 310), care must be taken with these calculations. It is generally assumed that denitrification responds to temperature in an analogous manner to nitrification but that, since it is an anaerobic process, it only occurs in wet soils and increases towards saturation. Furthermore, as the soil gets wetter, there is a shift from losses from N_2O to N_2. In addition, as this is a microbial process, it is necessary to include the effect of soil microbial mass. The dynamics of denitrification are now considered.

Denitrification is described using Michaelis-Menten dynamics for the response to available nitrate. As for nitrification, denitrification is also related to temperature and soil water, and can be written as:

$$\rho = V_{mx,NO_3}\left(\frac{[NO_3]}{[NO_3] + K_{NO_3}}\right) f(T) f_\rho(\theta) \gamma_C \qquad (6.9a)$$

where $[NO_3]$ is the concentration of NO_3 in the soil layer, mg N kg^{-1}, V_{mx,NO_3} is the maximum rate of nitrate production, mg N kg^{-1} day^{-1}, K_{NO_3} is the NO_3 concentration for half maximal response to nitrate concentration, the f functions are the temperature and water responses respectively, and γ_C again represents the effect of soil microbial mass as described by eqn (6.8d). The effect of temperature is, once again, given by eqn (6.4b).

The default parameters are:

$$V_{mx,NO_3} = 0.25 \text{ mg N kg}^{-1} \text{ day}^{-1} \text{ and } K_{NO_3} = 50 \text{ mg N kg}^{-1} \qquad (6.9b)$$

and the rate of denitrification with these parameters is shown in Fig. 6.11.

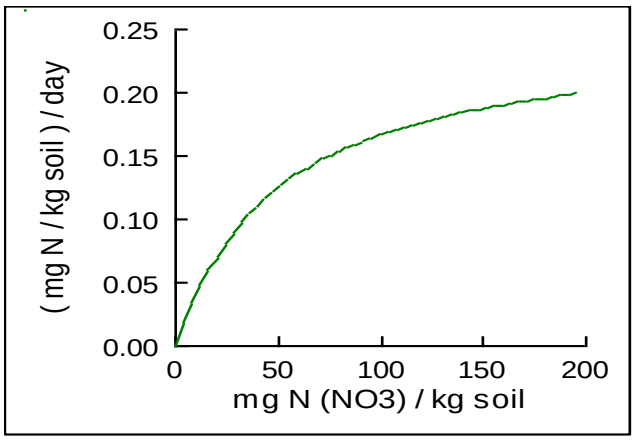

Figure 6.11: Rate of denitrification, eqn (6.9a), with the parameters in eqn (6.9b).

The effect of water is a bit more complex and is an area that can have important implications on the calculations of denitrification. The following approach is designed to allow flexibility in exploring the effects of soil water content on denitrification. Water filled pore space rather than volumetric soil water content is used, which is defined as

$$\phi = \frac{\theta}{\theta_{sat}} \tag{6.9c}$$

where (as usual) θ is the volumetric soil water content and θ_{sat} is the saturated water content. According to this definition, ϕ ranges between 0 (no water in the soil, which is generally not possible) to 1 at saturation. It is now assumed that the water function, $f_\rho(\theta)$, in eqn (6.9a) is given by

$$f_\rho(\theta) = \begin{cases} \sin\left(\frac{\pi}{2}\left(\frac{\phi-\phi_{mn,dn}}{1-\phi_{mn,dn}}\right)^{q_\rho}\right), & \phi \geq \phi_{mn,dn}; \\ 0, & \phi < \phi_{mn,dn}. \end{cases} \tag{6.9d}$$

where q_ρ is a curvature coefficient. The default parameters are

$$\phi_{mn,dn} = 0.6 \quad \text{and} \quad q_\rho = 2. \tag{6.9e}$$

Equation (6.9d) is illustrated in Fig. 6.12.

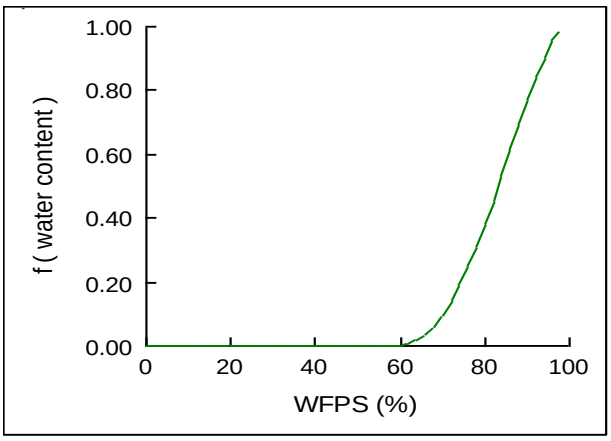

Figure 6.12: Influence of soil water content on denitrification, eqn (6.9d), with the parameters in eqn (6.9e).

The partitioning of denitrification between N_2O and N_2 is defined by assuming that:

- initially as the soil wets up all losses are to N_2O – this occurs between $\phi_{mn,dn}$ and ϕ_{mn,N_2}
- as the soil gets wetter there is a linear shift towards N_2 losses – this occurs between ϕ_{mn,N_2} and ϕ_{mn,N_2O}
- at water contents greater than ϕ_{mn,N_2O} all denitrification losses are as N_2.

Mathematically this can be expressed as

$$\begin{aligned} f_{N_2O}(\theta) &= \lambda f_v(\theta), \\ f_{N_2}(\theta) &= (1-\lambda) f_v(\theta), \end{aligned} \tag{6.9f}$$

where

$$\lambda = \begin{cases} 1, & \phi_{mn,dn} \leq \phi \leq \phi_{mn,N_2}, \\ \dfrac{\phi_{mx,N_2O} - \phi}{\phi_{mx,N_2O} - \phi_{mn,N_2}}, & \phi_{mn,N_2} \leq \phi \leq \phi_{mx,N_2O}, \\ 0, & \phi \geq \phi_{mx,N_2O}. \end{cases} \tag{6.9g}$$

The default parameters are

$$\phi_{mn,N_2} = 0.7 \quad \text{and} \quad \phi_{mx,N_2O} = 0.9, \tag{6.9h}$$

with $\phi_{mn,dn}$ prescribed in (6.9e).

The full denitrification function, with partitioning between N_2O and N_2 is illustrated in Fig. 6.13, corresponding to Fig. 6.12.

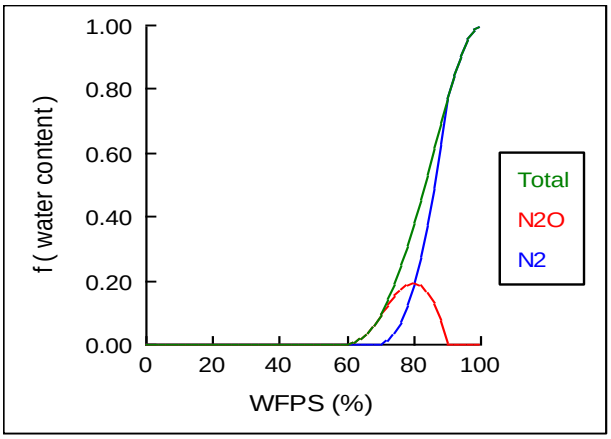

Figure 6.13: Total N denitrification function, along with the N_2O and N_2 components, as a function of water filled pore space, with the default parameters given by eqns (6.9e,h).

It is interesting to note that with this treatment of denitrification, soils with field capacity close to saturation may be susceptible to more denitrification than soils where there is quite a difference between saturation and field capacity. For example, if the saturated water content is 55% and field capacity is 45%, then field capacity occurs at a WFPS of 80% which means that denitrification occurs at field capacity and below (down to WFPS of 60% with the defaults here). Alternatively, if the field capacity is 30%, then this corresponds to a WFPS of 54%, and denitrification will not occur. This means that once the soils are wet, those soils with field capacity greater than the cut-off WFPS for denitrification may have greater rates of denitrification.

A characteristic of the mathematical treatment is that by changing the exponent q_ρ, not only does the shape of the total denitrification curve change, but so does the partitioning. This is illustrated in Fig. 6.14 which shows the responses for $q_\rho = 1$ and $q_\rho = 3$.

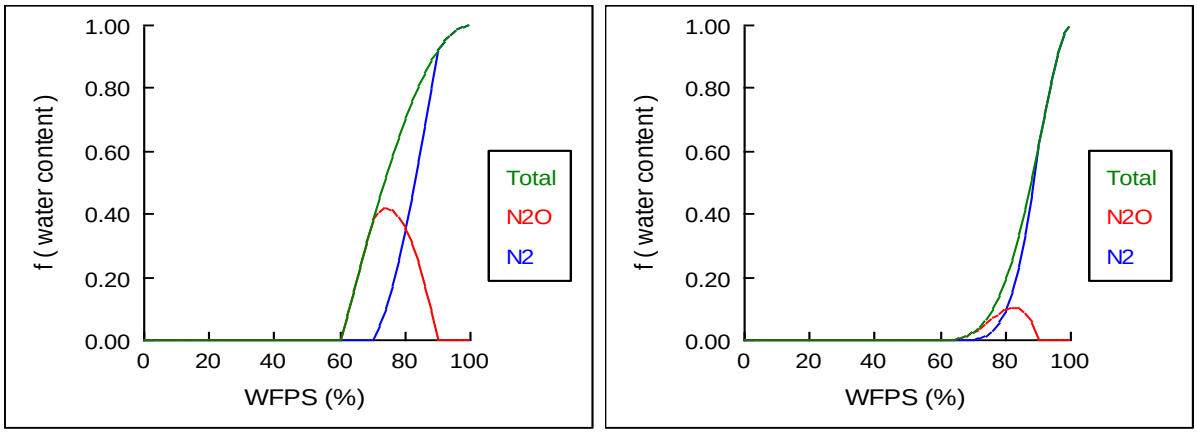

Figure 6.14: Total N denitrification function, along with the N_2O and N_2 components, corresponding to Fig. 6.13, but with $q_\rho = 1$ (left) and 3 (right). See text for details.

6.4.3 Volatilization of ammonium

Volatilization, the conversion of ammonium to ammonia gas, mainly occurs from urine patches and for urea fertilizer shortly after application. For volatilization from urine, the user can set a fixed proportion of daily urea to be lost to the atmosphere – the default is 25%. For urea fertilizer, losses occur for the two days following application: on each of these two days 50% of the urea is transferred to the surface soil ammonium pool. The rate constant for volatilization of urea is by default 5% per day.

It is also assumed that volatilization is suppressed by rainfall events. If the rainfall is greater than a user prescribed amount (default 5 mm d^{-1}) then there is no volatilization.

Note that there is no allowance for volatilization from the general soil ammonium pool as this is generally negligible (Richard Eckard, *personal communication*).

6.5 Inorganic phosphorous dynamics

An important aspect of phosphorous dynamics in the soil is the role of phosphorous buffering, whereby a pool of inorganic phosphorous is unavailable to the plant. The general scheme of inorganic P dynamics is illustrated in Fig. 6.15, where P_a and P_b are the available and buffered components of phosphorous respectively, and k_a and k_b (day^{-1}) are rate constants.

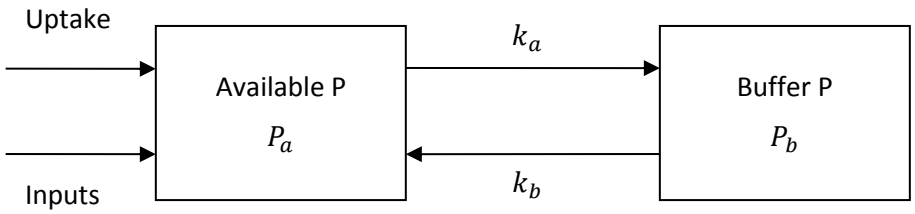

Figure 6.15: Schematic representation of the inorganic P dynamics. See text for details.

This type of structure is similar to that generally used in simulation modelling (e.g. Jones *et al.*, 1984; Karpinets *et al.*, 2004)

It is convenient to re-write the rate constants as:

$$k_a = k, \text{ and } k_b = \lambda k \qquad (6.10a)$$

where λ is a dimensionless parameter. It is mathematically equivalent to use either of the pairs (k_a, k_b) or (k, λ). Denoting the uptake and inputs by U_p and I_p respectively, and using first order kinetics, the system can be described by the equations:

$$\frac{\mathrm{d}P_a}{\mathrm{d}t} = I_p - U_p + k\left(\lambda P_b - P_a\right) \qquad (6.10b)$$

and

$$\frac{\mathrm{d}P_b}{\mathrm{d}t} = k\left(P_a - \lambda P_b\right). \qquad (6.10c)$$

These are the equations used in the model.

It is instructive to look at the special case where there is no net change in the overall P balance, so that

$$I_p = U_p \,. \tag{6.10d}$$

It is readily shown that under this condition

$$P_a = P_{a,0} - \phi\left(1 - e^{-kt}\right) \tag{6.10e}$$

and

$$P_b = P_{b,0} + \phi\left(1 - e^{-kt}\right) \tag{6.10f}$$

where $P_{a,0}$ and $P_{b,0}$ are initial values, and

$$\phi = \frac{P_{a,0} - \lambda P_{b,0}}{1 + \lambda}\,, \tag{6.10g}$$

which can be positive or negative, depending on the initial values. Figure 6.16 shows the percent P in each pool as a function of time, with each pool having the same initial size, and the parameter values given by:

$$k = 2\% \text{ per day and } \lambda = 0.1. \tag{6.10h}$$

In this case, it can be seen that there is a shift of P from the available to the buffer pool. This example can be seen as representing the situation following an initial high P application.

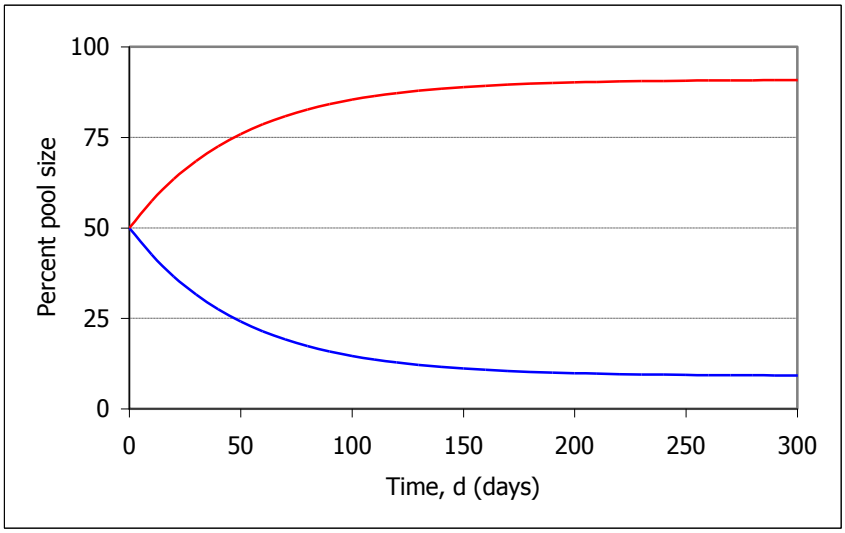

Figure 6.16: Percent pool size for P_a (blue) and P_b (red), for zero net P change conditions, starting with equal pool sizes, and parameters given by eqn (6.10h): see text for details.

A feature of this simple scenario is that, in the equilibrium state where both derivatives in eqns (6.10b, c) approach zero,

$$\frac{P_a}{P_b} = \lambda \tag{6.10i}$$

so that the parameter λ can be interpreted as the long-term equilibrium ratio of available to buffer P. Sometimes, it is convenient to think in terms of available P as a proportion of total P, in which case, eqn (6.10i) can be written as

$$\frac{P_a}{P_a + P_b} = \frac{\lambda}{1+\lambda}\,. \qquad (6.10j)$$

The program interface allows the user to set this fraction rather than the actual parameter λ.

It can also be seen that the parameter k influences how rapidly the system approaches equilibrium, so that both λ and k have readily interpretable effects on the dynamics of the system.

A further observation from this scenario is that for a soil in equilibrium, any further addition will tend, over time, to be partitioned between available and buffered P according to the ratio given by (6.10i). In practice, with fertilizer applications and product removal, P is added and removed from the soil, so that some of the added P may be utilized by the plants before being transferred to the buffered P pool.

In the model, eqns (6.10b,c) are used with the default parameter values given by eqn (6.10h). However, it should be noted that these parameters, particularly λ, are likely to vary considerably for different soil types.

6.6 Spinning-up the model

Methods for prescribing the initial soil organic and inorganic nutrient status have been described earlier, although it can be difficult to prescribe some of the pools accurately due to difficulties in measuring and their complex dynamics. In the model it is therefore possible to run a simulation for a period of time (say 100 years), and save the resulting state of the organic and inorganic pools as starting conditions for subsequent simulations. This is termed 'spinning-up', and is particularly useful since soil organic matter can take many years to stabalise.

6.7 Concluding remarks

The treatment of soil nutrients has covered organic and inorganic dynamics, leaching, nitrogen transformations, and gaseous nitrogen losses. The aim has been to avoid unnecessary complexity and yet to encapsulate the key processes.

Care must be taken in analysing soil nutrient data since, the observed data for variables such as organic carbon or inorganic nutrient concentration result from a series of fluxes. Since different combinations of these fluxes can lead to similar system state variables, attention should focus on the fluxes and not just the state variables. For example, if the rate of root growth and senescence is greater than actually occurs, but the rate of organic matter turnover is also greater than occurs, then the actual soil organic matter values from the model may agree well with observational data. Similarly, if the observations and data differ, then this could be due either to errors in estimates of inputs to the pools or to fluxes out of the pools.

The study of soil nutrient dynamics is particularly complex since data are often hard to obtain due to the slow turnover rates that are involved. The model provides a sound structure for analysing nutrient dynamics, and for interpreting observational data.

6.8 References

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7 Animal intake and metabolism

7.1 Introduction

This document describes the animal module used in DairyMod, EcoMod and the SGS Pasture Model. These models can variously implement sheep (wethers and ewes with lambs), cattle (steers and beef cows), dairy cows and deer. The approach is similar for each animal type, dealing with grazing animals that can either be growing, mature dry or mature and capable of pregnancy and lactation. Each type is therefore considered in this document.

The animal module involves calculating intake from both pasture and any supplementary feeding, and then using this intake for the metabolic processes of growth, maintenance, lactation and pregnancy, where relevant. The structure of the model is:

- Potential intake is calculated from available pasture and pasture digestibility.
- The potential intake is then converted to ME (metabolisable energy).
- The ME required is calculated in relation to the energy required, where relevant, for growth, maintenance, lactation and pregnancy.
- Potential animal growth rate is defined using a Gompertz equation – this applies to growing animals or animals that are below their mature weight.
- If the potential ME intake exceeds the total ME required then intake is scaled back to requirement.
- Supplementary feeding of both concentrate and forage is calculated in relation to the feeding rules and animal requirements.
- If the ME intake is less than the total ME required then components requiring ME mentioned earlier are scaled back. This may result in a reduction in growth and milk production, as well as weight loss.

The treatment of energy requirements is similar to the Australian Feeding Standards (AFS) (1990), but has been simplified so that the parameters have simple interpretations and can be accessed on the program interface. The actual AFS equations have been implemented in the model as an option, but are not presented here.

7.2 Digestibility and metabolisable energy

Pasture digestibility is considered first since this has an influence on pasture intake. Digestibility is calculated in terms of the plant nitrogen level. It is assumed that the plant has three main components:

- protein, which can include amino acids;
- sugars, which can include starch; and
- cell wall material.

The C:N ratio of the first and third of these is prescribed, CN_p and CN_w, respectively while the sugars have no nitrogen. The default values are:

$$CN_p = 3.5 \text{ and } CN_w = 100 \quad (7.1a)$$

where the subscripts p and w refer to protein and cell wall respectively, and it can clearly be seen that the cell wall has very low N content; indeed, the analysis is not sensitive to this number and it can be taken as zero. If the C:N ratio of the total plant material is CN, then it is readily shown that

$$\frac{1}{CN} = \frac{f_p}{CN_p} + \frac{f_w}{CN_w} \tag{7.1b}$$

where the f coefficients are the fractions of each component.

In addition,

$$f_p + f_s + f_w = 1 \tag{7.1c}$$

where the *s* subscript refers to the sugars: the sugar component is calculated in the pasture module.

Equations (7.1b,c) combine to give

$$f_{p,live} = \frac{\frac{CN_w}{CN} - (1 - f_s)}{\frac{CN_w}{CN_p} - 1} \tag{7.1d}$$

For dead plant material, it is assumed that there are no sugars, so that

$$f_s = 0 \tag{7.1e}$$

and eqns (7.1b,c) now combine to give

$$f_{p,dead} = \frac{\frac{CN_w}{CN} - 1}{\frac{CN_w}{CN_p} - 1} \tag{7.1f}$$

Equations (7.1d, f) completely define the plant composition in terms of the total CN ratio and the prescribed values for the C:N ratio for protein and cell wall material, eqn (7.1a).

It is assumed that sugars and protein are completely digestible in the rumen, while the cell wall material has a digestibility that can be prescribed by the user. While this varies between animal species, it is generally quite similar across ruminants. Thus, the digestibility of any plant material is given by

$$\delta = f_p + f_s + \delta_w f_w \tag{7.1g}$$

which is defined as the ratio of carbon utilized to that eaten. Utilized carbon is used either for growth or is respired.

The digestibilities of live and dead cell wall material are taken to be different, with the defaults being:

$$\delta_{w,live} = 0.6 \text{ and } \delta_{w,dead} = 0.2 \tag{7.1h}$$

Once the digestibility is known, the ME content per unit d.wt. eaten is given by (Spedding and Diekmahns, 1972):

$$\varphi = 16\delta \quad \text{MJ (kg d.wt)}^{-1} \tag{7.1i}$$

Note that sometimes this calculation from digestibility to ME content uses the gross energy of the material being eaten and the metabolisable quotient, which is interpreted as the energy available for metabolism once it gets to the rumen. However, this level of detail is not used in the present analysis although, should problems arise due to having a fixed relationship between digestibility and energy content, then the method could be developed as required.

7.2.1 Digestibility for combined live and dead material, and multiple species

The approach described above gives the digestibility, and ME content, for the live and dead plant material in terms of its N content and prescribed digestibility of cell wall material. In the model, it is necessary to consider combinations of species and their live and dead components.

Consider a pasture comprising multiple species that is harvested and the total digestibility calculated. Taking all components on a *pro rata* basis, the total digestibility is:

$$\delta = \frac{\sum_{sp} \delta_{\ell,sp} W_{\ell,sp} + \delta_{d,sp} W_{d,sp}}{W} \qquad (7.2a)$$

where $W_{\ell,sp}$and $W_{d,sp}$ are the live and dead dry weight components of the species denoted by sp, and W is the total dry weight. This is the digestibility term that is presented on the model interface.

In practice, when considering the influence of digestibility on animal intake, we need to account for species selection and also selection between live and dead material. To account for this, the *weighted digestibility* is defined as:

$$\delta_w = \frac{\sum_{sp} P_{sp}\left(\delta_{\ell,sp} P_\ell W_{\ell,sp} + \delta_{d,sp} W_{d,sp}\right)}{\sum_{sp} P_{sp}\left(P_\ell W_{\ell,sp} + W_{d,sp}\right)} \qquad (7.2b)$$

where $P_\ell \geq 1$ is the relative preference, or selection, for live material over dead material (default 10 in the model), and P_{sp} is the relative preference, or selection, for species sp. Note that the absolute value of P_{sp} is not important, only the relative values across the different species. Equation (7.2b) is used in the subsequent calculations for animal intake.

7.3 Pasture intake

The potential daily intake is defined in terms of available pasture d.wt, which is further influenced by pasture digestibility as calculated in the previous section.

Consider first the influence of pasture digestibility on potential intake when available pasture is non-limiting. This maximum intake, kg animal^{-1} day^{-1} is defined as:

$$I_{mx} = \begin{cases} I_{mx,80}, & \delta_w \geq 80\% \\ I_{mx,30} + \left(I_{mx,80} - I_{mx,30}\right)\phi\left(\delta_w\right), & 30\% < \delta_w < 80\% \\ I_{mx,30}, & \delta_w \leq 30\% \end{cases} \qquad (7.3a)$$

where the subscripts 30 and 80 refer to the intakes when pasture digestibility is 30% and 80% respectively, δ_w is the weighted pasture digestibility (eqn 7.2b), and $\phi(\delta_w)$ is a function that lies between 0 and 1, taking these extremes at 30% and 80% digestibility respectively. $\phi(\delta)$ is an empirical equation, that is used to give the generic expected response of intake to digestibility: it can be readily modified as required. $\phi(\delta)$ is given by:

$$\phi(\delta_w) = \frac{(\delta_w - \delta_{mn})^{q_\phi}(\delta^* - \delta_w)}{(\delta_{mx} - \delta_{mn})^{q_\phi}(\delta^* - \delta_{mx})} \quad (7.3b)$$

where δ_{mn} and δ_{mx} are minimum and maximum digestibilities used in eqn (7.3a),taken to be 30% and 80% respectively, q_ϕ is a curvature parameter, and

$$\delta^* = \frac{(1+q_\phi)\delta_{mx} - \delta_{mn}}{q_\phi} \quad (7.3c)$$

is the theoretical point where the curve would pass through the origin if δ_w were allowed to be greater than δ_{mx}, and is defined so that the slope of $\phi(\delta_w)$ is zero at δ_{mx}.

The default parameter values are given in Table 7.1:

Table 7.1 Maximum intake in response to digestibility parameters, eqns (7.3a,b,c)

	$I_{mx,80}$, kg animal^{-1} day^{-1} Max intake at 80% digestibility	$I_{mx,30}$ kg animal^{-1} day^{-1} Max intake at 30% digestibility	q_ϕ Max intake curvature parameter
Wethers	2	0.8	1.5
Ewes	3	0.8	1.5
Steers	15	6	1.5
Beef cows	20	6	1.5
Dairy cows	20	6	1.5
Deer	5	1	1.5

The maximum intake in response to digestibility is shown in Fig. 7.1 for dairy cows.

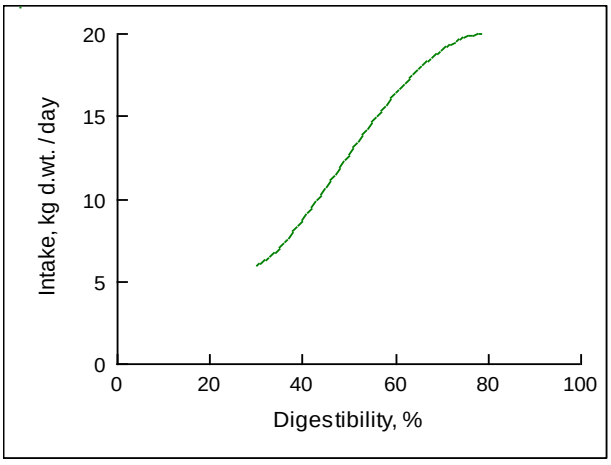

Figure 7.1: Maximum intake as a function of digestibility for the model defaults as applying to a dairy cow.

Once the intake potential in relation to pasture digestibility is known, the intake in response to available d.wt can be calculated. This uses a similar approach to that for digestibility, so that

$$I = I_{mx}\psi(W) \tag{7.3d}$$

where I_{mx} is given by (7.3a,b,c) and $\psi(W)$ is defined analogously to $\phi(\delta_w)$ above, so that

$$\psi(W)\begin{cases} 1, & W \geq W_{mx} \\ \dfrac{(W - W_{mn})^{q_\psi}\left(W^* - W\right)}{(W_{mx} - W_{mn})^{q_\psi}\left(W^* - W_{mx}\right)}, & W_{mn} < W < W_{mx} \\ 0, & W \leq W_{mn}, \end{cases} \tag{7.3e}$$

W_{mn} is the d.wt on offer for zero intake, and so is the ungrazeable residual, W_{mx} is the d.wt on offer for maximum intake (so that intake is not restricted by availability), and q_ψ is a curvature parameter. The default parameters are shown in Table 7.2.

Table 7.2 Parameters for intake in response to available pasture d.wt, eqn (7.1e)

	W_{mn} t ha^{-1}	W_{mx} t ha^{-1}	q_ψ
Sheep	0.25	1	2
Cattle	0.5	2	1
Deer	0.25	1.5	2

Equation (7.3e) applies when intake by the animal is not restricted due to high stock density, and must be modified to include stock density. For example, with intensive rotational systems if eqn (7.3d) is applied for all stock densities, intake may exceed available pasture d.wt. In this case, eqn (7.3d) now becomes:

$$\tilde{I}_{mx} = I_{mx}\frac{(W - W_{mn})}{\rho} \tag{7.3f}$$

where W is the available pasture, W_{mn} is the minimum pasture residual below which there is no intake, as defined in eqn (7.3e) and ρ is the stock density. The intake in response to available d.wt is shown in Fig. 7.2 for the model defaults for dairy cows, at stocking density 5 and 60 cows ha^{-1}. While 60 cows ha^{-1} is obviously high, it is quite feasible for strip grazing.

This approach to intake is empirical, but it does capture the general intake characteristics in response to available pasture and its quality. It is important that users explore the impact of the choice of parameter on the general model behaviour as they will affect both animal performance and pasture growth dynamics, which in turn will impact on the water and nutrient dynamics.

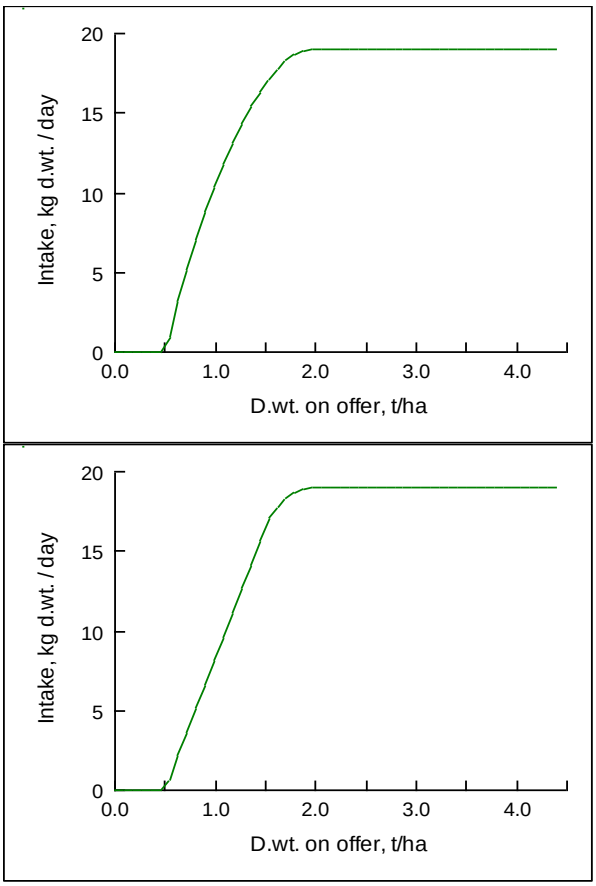

Figure 7.2: Intake as a function of d.wt. on offer, for the model defaults as applying to a dairy cow with the stocking density at 5 (left) and 60 (right) cows ha^{-1}.

7.3.1 Relative intake of species and intake of live and dead

Relative intake of species in the sward, as well as live and dead components is distributed after the total intake has been calculated according to the preceding analysis. The theory is developed here to allow for preference, or selection, between species, as well as live and dead, as used in the calculation for the weighted digestibility, δ_w, as given by eqn (7.2b). The live and dead intake components are now given by

$$I_{\ell,sp} = I \frac{P_{sp} P_\ell \delta_{\ell,sp} W_{\ell,sp}}{\sum_{sp} P_{sp} \left(P_\ell \delta_{\ell,sp} W_{\ell,sp} + \delta_{d,sp} W_{d,sp} \right)} \qquad (7.4a)$$

$$I_{d,sp} = I \frac{P_{sp} \delta_{d,sp} W_{\ell,sp}}{\sum_{sp} P_{sp} \left(P_\ell \delta_{\ell,sp} W_{\ell,sp} + \delta_{d,sp} W_{d,sp} \right)} \qquad (7.4b)$$

where the symbols are defined in section 7.2.1 above.

Consider the intake from the live component of an individual species. First, the potential intake of this species is calculated assuming the whole sward comprised this species only, using the theory described above. If this is denoted by I_s^*, where subscript s refers to the species, then the actual intake from the live component is given by

$$I_{\ell,s} = I_s^* \frac{\eta_{\ell,s} f_{\ell,s} + \eta_{d,s} f_{d,s}}{\sum_{sp} \eta_{\ell,sp} f_{\ell,sp} + \eta_{d,sp} f_{d,sp}} \qquad (7.4c)$$

where ℓ and d refer to live and dead components, the η terms are relative preference for the species components, the f terms are the live and dead fractions of the species, with

$$f_{\ell,s} + f_{d,s} = 1 \qquad (7.4d)$$

And the summation in the denominator in eqn (7.4c) is taken over all species (sp is a dummy variable).

According to this approach, intake is weighted according to the relative abundance of the live and dead components of each species, along with their preferences, or selectivity.

At present, the preference parameters are not available on the interface: it is assumed that preference between the different species is weighted according to their digestibilities, and that the weighting for live is 10 times greater than for dead.

7.4 Supplementary feeding

Supplementary feeding rules are implemented either in terms of animal body weight or ME requirements. For ME requirements, lactating and dry animals are considered separately since feed requirements can be calculated in relation to milk production. Supplement can be supplied either as concentrate or forage, each with their own energy content and nutrient composition, both of which can be prescribed on the interface. At present, animal metabolism is defined solely in terms of the animal ME dynamics, so that there is no actual distinction between the role of concentrate or forage.

The strategy for supplementation is defined in terms of the ME requirements. Before proceeding, note that the partitioning of energy between the various metabolic processes for animals that are receiving less than their required intake will impact on the supplementary feeding strategies: this partitioning is considered later.

7.4.1 Body weight requirements

Supplementary feeding can be supplied in response to animal body weight. Weights at which feeding commences and then stops are defined – for example, with wethers feeding might start when the animal weight reaches 35 kg and continues until they reach 45kg. The reason a single weight is not specified is that in the model this can result in feeding being implemented on alternate days. For example, if we attempt to maintain animal weight at 45kg then if it falls to 44.9, feed is supplied and the next day the animal weight might be 45.01kg, so feed stops and the next day it may fall to 44.95 and feeding starts, and so on.

7.4.2 Lactating animals

The strategy is to define the milk production target as a percentage of potential production, and then to aim for this through pasture intake, concentrate and forage supplements. The energy and nutrient composition of both forms of supplement are defined, along with four supplement parameters:

1. Minimum daily concentrate that will always be applied.
2. Maximum daily concentrate.

3. Minimum daily forage, and if the forage requirement is less than this then no forage is provided.
4. Maximum daily forage.

Note the difference in the minimum concentrate and forage parameters – this will become clear.

The strategy is to calculate the ME required, the ME available from pasture and the minimum daily concentrate. If this is greater than required (for maximum milk production), then pasture intake is reduced, and if it is between the target and maximum ME required then no action is taken. Otherwise, supplementary feeding is required, and is implemented as follows:

- The ME shortfall is met by increasing concentrate to its maximum daily value.
- If there is still a shortfall of ME then this is met by forage, up to its maximum amount.
- If the forage required is less than the minimum daily forage, take no action.

Some examples should help explain this process. Note that all figures are in ME units, although they are expressed in kg on the model interface.

In the examples, use the following parameters:

- Min concentrate = 20 MJ / day
- Max concentrate = 80 MJ / day
- Min forage = 10 MJ / day
- Max forage = 50 MJ / day
- ME for maximum milk production = 250 MJ
- ME for target milk production = 200 MJ

Example 1: pasture intake = 240 MJ

In this case, the minimum concentrate supplement of 20 MJ is supplied so that pasture intake is reduced to 230 MJ.

Example 2: pasture intake = 190 MJ.

Here, the pasture plus minimum concentrate supplement gives 210 MJ which is above the target, so no further action is taken.

Example 3: pasture intake = 150 MJ

The pasture plus minimum concentrate supplement is now 170 MJ and another 30 are needed. Thus, a total of 50 MJ is supplied from concentrate.

Example 4: pasture intake = 115 MJ

The pasture plus maximum concentrate is 195 MJ and this is less than the target. However, only 5 MJ are needed and the minimum from forage is 10 MJ, so no forage is supplied and production is slightly lower than target.

Example 5: pasture intake = 100 MJ

The pasture plus maximum concentrate is 180 MJ and this is less than the target. Forage (20 MJ) is now supplied to bring this up to 200 MJ.

Example 6: pasture intake = 10 MJ

The pasture plus maximum concentrate plus maximum forage is 140 MJ and this is less than the target. However, no further supplement is available and so the target is not met.

7.4.3 Dry animals

The approach is simpler for dry animals in that a target ME intake as a fraction of total ME required is prescribed. If pasture intake is less than the prescribed critical value for intake then supplementary feeding is supplied with the aim of bringing ME intake up to a target value. The same strategy of supplying concentrate and forage is implemented for dry animals.

It may be necessary to define separate concentrate and forage parameters for dry animals, although this is not currently available.

7.4.4 Substitution

Substitution is the process whereby pasture intake is reduced due to the availability of supplement, and where the d.wt reduction in pasture intake differs from the d.wt intake of supplement. This is a complex concept to quantify. For example, if the supplement is provided prior to animals being in the paddock there is likely to be a different impact than if the animals receive the supplement after grazing. There is no allowance for the timing of supplement supply in the model.

Substitution will, nevertheless, be a characteristic of the present treatment due to the different ME contents of pasture, concentrate and forage. For example, with a minimum concentrate being supplied to the animals, even if there is sufficient pasture to satisfy ME requirements, pasture intake will be reduced due to the concentrate intake. Since the ME content of the concentrate is likely to be greater than that of the pasture, the equivalent concentrate d.wt will be less than the pasture d.wt required to supply the same ME, so that less concentrate d.wt is required to provide the same ME as from pasture.

7.5 Potential animal growth and metabolism

The basis of the animal metabolism approach is that metabolisable energy, ME, is used for metabolic processes with an efficiency. This means that, for any growth process, if the energy content of the product is E, and the efficiency of utilisation of energy is Y, then the energy required to synthesise the product is

$$E_{req} = \frac{E}{Y} \qquad (7.5)$$

This approach is used for animal growth, foetus growth, and milk production. In addition, energy is required for maintenance, although this is prescribed directly and so does not have an associated efficiency. The general structure of this component of the model originated with the model described by Finlayson *et al.* (1995), although considerable simplifications and modifications have been made.

The following treatment considers growing and mature animals separately, and has been applied to dairy cows (no calves retained), ewes with lambs, lambs, wethers, beef cows with calves, calves, steers, deer and fawns.

7.5.1 Growth

The animal growth rate and its associated energy requirements are calculated by defining the potential growth curve for the animal. The animal body composition is assigned an energy content so that, with the efficiency for growth (eqn 7.4), the energy content required for growth can be calculated. Two equations that are commonly applied to animal growth are the Gompertz, which is a sigmoidal curve (e.g. France and Thornley 1984), or a negative exponential (e.g. Freer *et al.* 1997), which is sometimes referred to as monomolecular. Both curves are available in the model. Before considering the actual animal growth rate, it is first necessary to look at the birth weights.

Normal birth weight

Various approaches have been explored for relating normal birth weight to mature animal weight (eg Roy, 1980; AFRC, 1993), but these can all be approximated with the linear response:

$$W_b = c_b + m_b W_m \qquad (7.6)$$

where W_b and W_m are the birth and mature weights respectively (kg), and c_b (kg) and m_b (dimensionless) are constants. The parameters that are used in the model are given in Table 7.3. Note that these parameters are not accessible on the model interface.

Table 7.3 Parameter values for animal birth weight, eqn (7.6), with corresponding calculated birth weight.

	c_b	m_b	W_m	W_b (calculated)
Sheep	1.2	0.055	60	4.5
Cattle	-2	0.066	550	34
Deer	1.7	0.062	115	8.8

Sigmoidal growth curve: Gompertz equation

There are various ways to write the Gompertz equation (eg, Thornley and Johnson, 2000), and the choice here is to use the birth and mature weights along with a growth rate parameter. This allows animal growth rate to be expressed both in terms of age and also current body weight. The equations are presented here, but for a detailed account see Thornley and Johnson (2000, section 3.5).

$$\frac{\mathrm{d}W}{\mathrm{d}t} = DW \ln\left(\frac{W_m}{W}\right) \qquad (7.7a)$$

and

$$W = W_m \exp\left[-\frac{\mu}{D}\exp(-Dt)\right] \qquad (7.7b)$$

where

$$D = \frac{\mu}{\ln(W_b/W_m)} \qquad (7.7c)$$

and W is the animal weight (kg), t is the animal age (days), W_b and W_m (as mentioned previously) are the birth and mature weights respectively (kg), and μ (day^{-1}) is a growth parameter. The default parameters that are used for μ are shown in Table 7.4.

Table 7.4 Default parameter values for μ for animal growth using the Gompertz equation, eqns (7.7a,b,c).		
Sheep	Cattle	Deer
0.02	0.015	0.015

Negative exponential growth curve

The negative exponential equation is written as

$$W = W_m - (W_m - W_b)e^{-\gamma t} \tag{7.8a}$$

where

$$\gamma = \frac{\ln(2)}{t_h} = \frac{0.69}{t_h} \tag{7.8b}$$

and t_h, days, is the time to reach the average of the birth and mature body weights. The growth rate with eqn (7.8a) is simply

$$\frac{dW}{dt} = \gamma(W_m - W) \tag{7.8c}$$

The default parameters used in the model are presented in Table 7.5.

Table 7.5 Default parameter values for t_h (days) for animal growth using the negative exponential equation, eqns (7.8a,b,c).		
Sheep	Cows	Deer
150	200	250

Growth curve illustrations

In Fig 7.3 the animal body weight and corresponding growth rate for lambs is shown using either the Gompertz or negative exponential equations (7.8a,b,c and 7.8a,b,c respectively). It can be seen that while the growth curves are quite similar, the growth rates differ substantially during early growth.

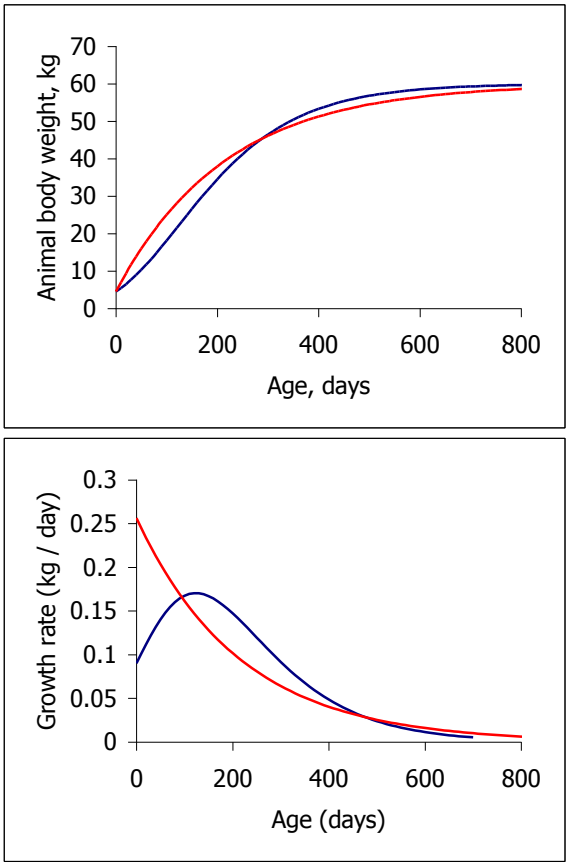

Figure 7.3 Normal animal weight (left) and growth rate (right) for lambs as a function of time using either the Gompertz (blue) or negative exponential (red) with the parameters given in Tables (7.4, 7.5).

Energy required for growth

With the animal growth rate given by either eqn (7.7a) or (7.8c), the energy required, ME_g is:

$$ME_g = \frac{E_g}{Y_g}\frac{\mathrm{d}W}{\mathrm{d}t} \qquad (7.9a)$$

The default values for all animal types are:

Growing animals: E_g = 10 MJ kg^{-1},

Mature animals: E_g = 19 MJ kg^{-1}, (7.9b)

Y_g = 0.6.

The difference in energy content for growing and mature animals is because mature animals do not grow skeletal material but mainly muscle and fat.

According to this approach, single values are used for the energy content of new tissue and the efficiency for synthesising that tissue. In practice, there will be differences between fat, protein and bone, and this could be further refined. However, the objective here is to keep things fairly simple and so the parameters in (7.9b) should be seen as being representative of typical animal composition during growth. The approach is limited for animals that are severely stressed.

There are various possibilities for describing animal weight gain in mature animals. The first is to allow mature animals to accumulate weight at a fixed potential rate until they are at their optimum mature weight. While this is simple and has appeal, it causes strange things to happen when calculating animal ME requirements. For example, a mature animal that is 1 kg below normal body weight will have an ME requirement that includes the energy for putting this kg of weight on, which may be a substantial fraction of its total ME requirement. This, in turn, can have ramifications on the calculations used for estimating overall ME requirements and supplementary feeding. In practice, it is unlikely that a mature cow that is 1 kg below its normal body weight (eg 549 kg rather than 550) would have the same priority for growth as one that is 50 kg below normal body weight. The approach adopted therefore, is to assume that in mature animals the growth requirements are identical to those for growing animals but with the exception that the energy content of this tissue is different (eqn 7.7b).

7.5.2 Maintenance

Maintenance is calculated directly in terms of animal body weight, as well as pasture digestibility which accounts for the extra energy required for foraging. Allowance is also made for extra maintenance costs associated with foraging, pregnancy and lactation as discussed below. The approach used here is a simplified treatment of the AFS (1990) system.

The commonly used approach for defining the energy required for maintenance (e.g. Finlayson *et al.* 1995; Freer *et al.* 1997) is the expression:

$$ME_m = \alpha W^{\gamma} \tag{7.10a}$$

where γ is a dimensionless parameter and α has dimensions of MJ kg^{γ} d^{-1}, which means that if the value of γ changes then the actual dimensions of α also change. In practice, γ generally takes the value 0.75 and so, to avoid any problems of this nature, eqn (7.10a) has the value

$$\gamma = 0.75 \tag{7.10b}$$

Maintenance also depends on the energy used for grazing and therefore the digestibility of the pasture since the animals will have to eat more if pasture digestibility declines. Equation (7.10a) has therefore been modified to

$$ME_m = \begin{cases} \alpha\left\{1+\lambda\left(0.8-D\right)\right\}W^{\gamma}, & D \le 0.8, \\ \alpha W^{\gamma}, & D > 0.8. \end{cases} \tag{7.10c}$$

where λ is a dimensionless parameter. The default values are presented in Table 7.6.

These parameters were derived by considering two contrasting scenarios for wethers and steers at Barraba for 100 year simulations. In the first case, the standard native C_3, C_4 pasture was used, while for the second, irrigated and fertilized perennial ryegrass with no high temperature stress limitation was implemented. The objective was to feed animals with low and high quality pasture, in order to explore the impact of digestibility on eqn (7.10c). The equivalent simulations were then run using the AFS equations. Relatively low stocking densities were used to avoid having to provide significant levels of supplement. Parameter values were chosen to give similar properties to the AFS approach for contrasting pasture qualities.

Table 7.6 Default parameter values for the maintenance parameters

α (MJ kg$^{\gamma}$ d^{-1}) and λ in eqn (7.10c).		
	α	λ
Sheep	0.35	2
Cattle	0.48	2.8
Deer	0.4	2

The illustrations in Fig. 7.4 show the annual pattern of intake for the 100 year simulations. Note that there is a dip in the native sheep simulation that is not reflected in the equivalent cattle simulation simply because animal demand was a bit higher due to the stocking density. It is clear from these simulations that the two approaches give very close animal energy requirements.

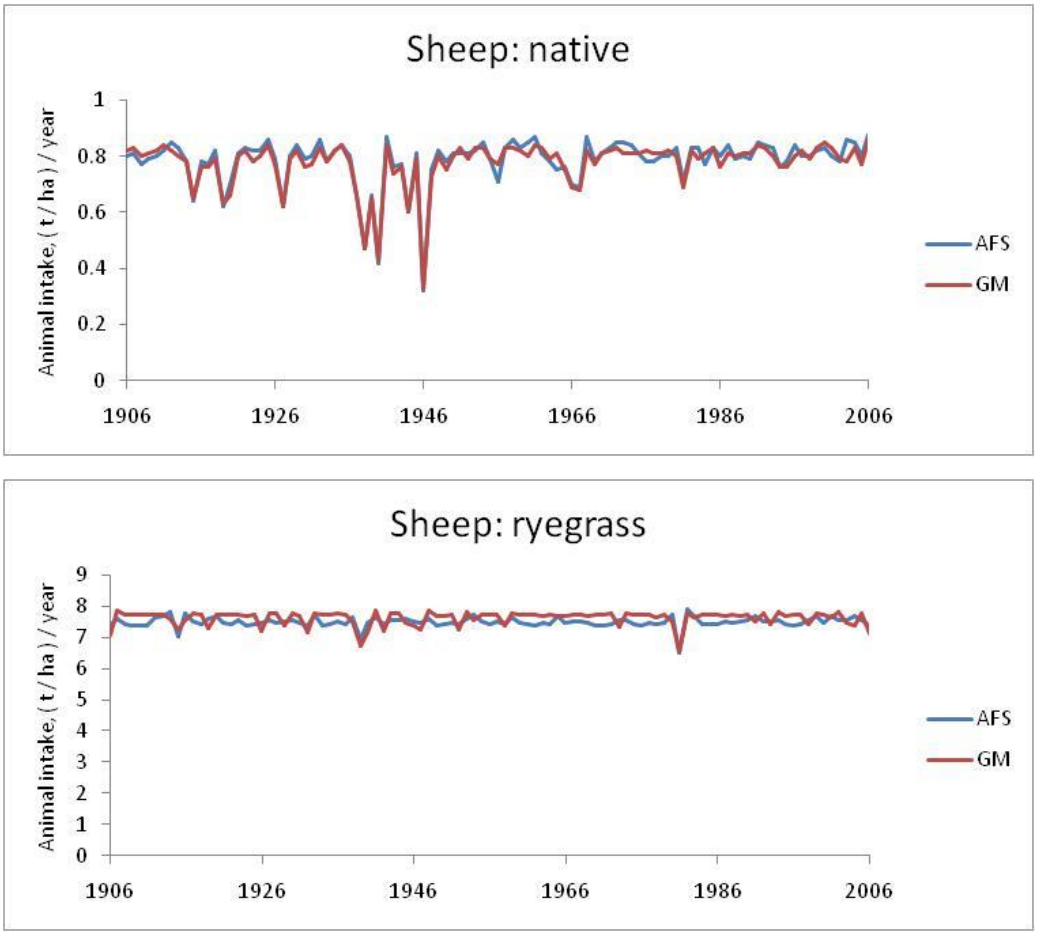

Figure 7.4: Annual intake for the sheep simulations on either native pasture or irrigated and fertilized perennial ryegrass, using either the AFS or GrazeMod equations as indicated.

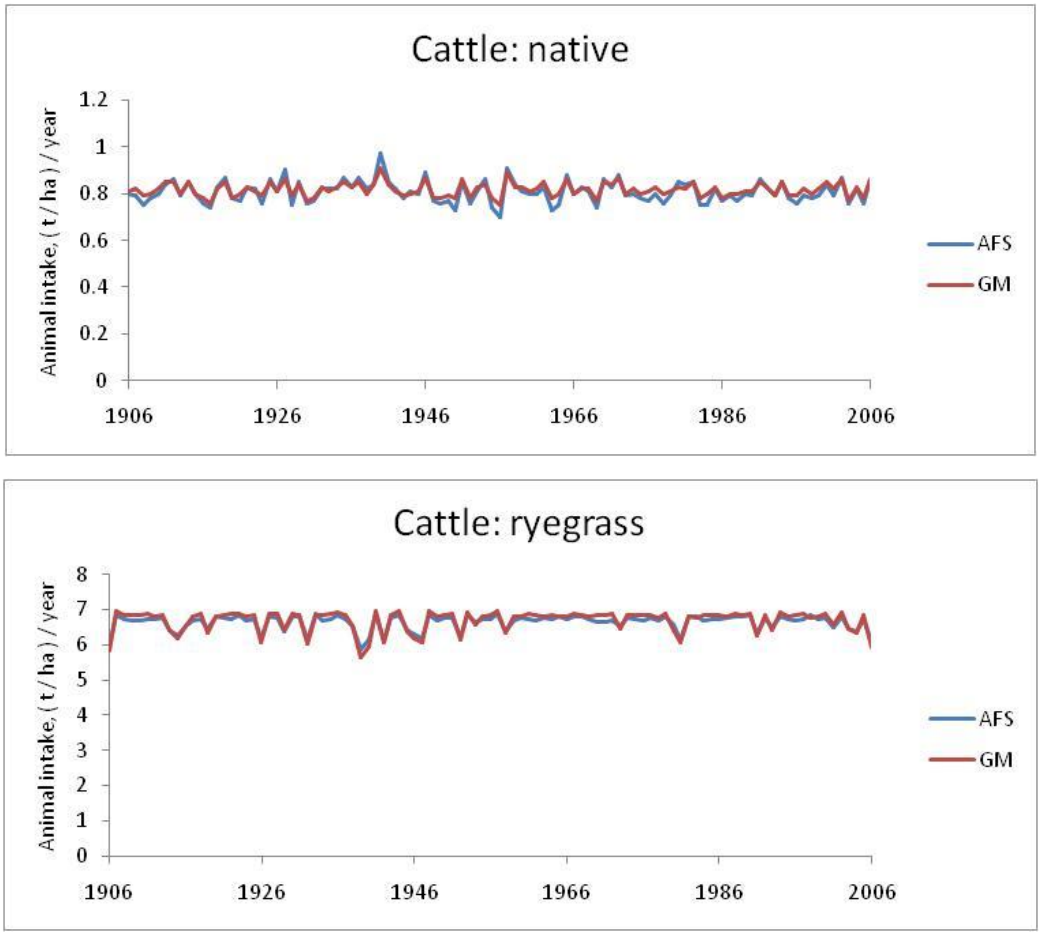

Figure 7.5: Annual intake for the cattle simulations on either native pasture or irrigated and fertilized perennial ryegrass, using either the AFS or GrazeMod equations as indicated.

The actual daily maintenance costs as a function of animal body weight for pasture of different quality is shown in Fig. 7.6 for sheep and cattle.

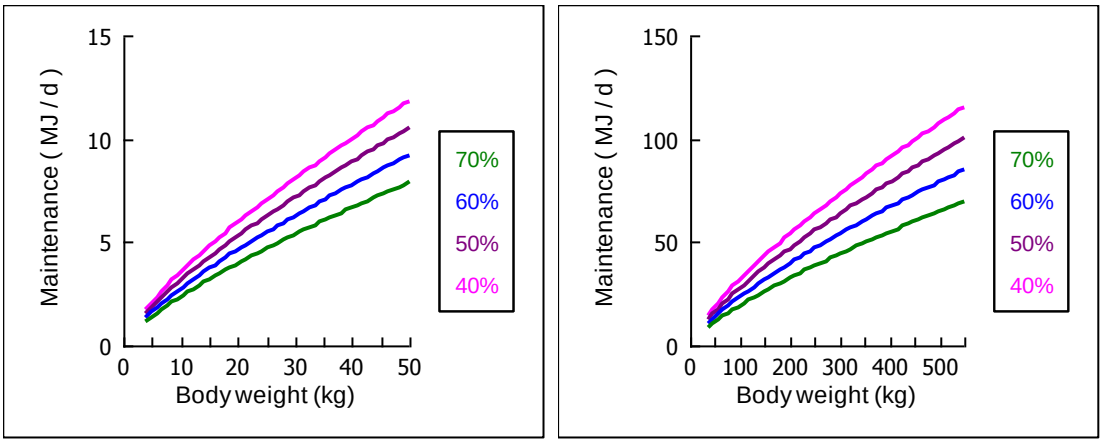

Figure 7.6: Maintenance energy costs for sheep (left) and cattle (right), from eqn (7.10a) with the parameters in Table 7.6. The pasture digestibility is indicated on the graphs.

7.5.3 Pregnancy

During pregnancy, foetal growth can be assumed to be exponential. Again, other approaches have been used, although most are generally very similar to exponential growth. The growth rate is therefore:

$$\frac{\mathrm{d}W_f}{\mathrm{dt}} = k_f W_f \qquad (7.11a)$$

where the subscript f refers to the foetus and k_f (day^{-1}) is a growth parameter.

To use this approach, it is necessary to define the weight at conception. This is derived from the normal birth weight, W_b, and the duration of the pregnancy, t_p. The weight at conception is then given by:

$$W_{f,0} = W_b \exp\left(-k_f t_p\right) \qquad (7.11b)$$

The pregnancy durations are given in Table 7.7.

Table 7.7 Pregnancy durations, days.

Ewes	Cows	Deer
150	280	200

The growth parameter, k_f, is derived from the days to 50% of normal birth weight, and is given by

$$k_f = \frac{\ln(2)}{\tau_p - \tau_{p,50}} \qquad (7.11c)$$

where it is assumed that

$$\tau_{p,50} = 0.8\tau_p \qquad (7.11d)$$

The equation for foetal weight with no limitation to growth is then

$$W_f = W_{f,0} \exp\left(k_f t\right) \qquad (7.11e)$$

The main disadvantage with this approach is the fact that the foetal weight at conception, $W_{f,0}$, has to be incorporated: in practice, this will start close to zero. The effect on the model is that immediately after conception there is a sharp increase in energy requirement to accommodate the foetal growth as defined by eqn (7.11a). This is overcome by modifying eqn (7.11e) to

$$W_f = W_b \left[\frac{\exp\left(k_f t\right) - 1}{\exp\left(k_f \tau_p\right) - 1}\right] \qquad (7.11f)$$

so that the initial weight is now zero. Equation (7.11c) is still used to calculate k_f.

Equation (7.11f) is illustrated in Fig. 7.7 for a cow – the general characteristics of the curve are similar for sheep and deer.

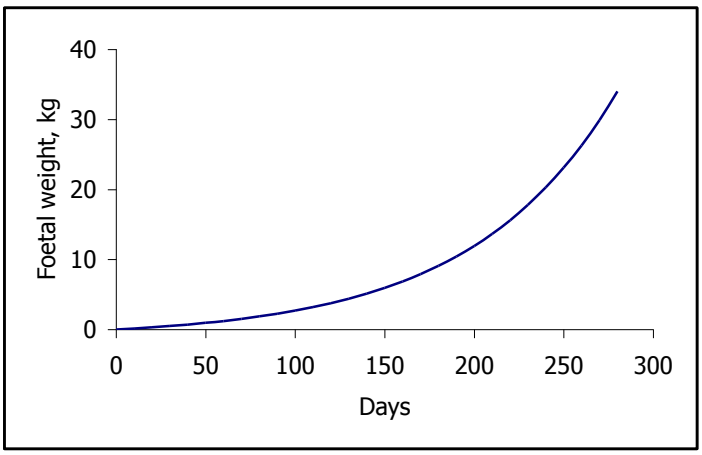

Figure 7.7 Foetal growth for cows using eqn (7.11f): see text for details.

It is still necessary to define the daily foetal growth rate in the model, which is calculated from (7.11f) as

$$\frac{\mathrm{d}W_f}{\mathrm{dt}} = \frac{k_f}{\left\{\exp\left(k_f \tau_p\right) - 1\right\}} \left[W_f \left\{\exp\left(k_f \tau_p\right) - 1\right\} + W_b \right]. \qquad (7.11g)$$

Once the growth rate is known, the energy required for pregnancy is then calculated as:

$$ME_p = \left(1 + \lambda\right) n_f \frac{E_g}{Y_g} \frac{\mathrm{d}W_f}{\mathrm{d}t} \qquad (7.11h)$$

where n_f is the number of foetuses and λ is a factor to allow for the increase in maintenance costs associated with pregnancy. The default values for λ are given in Table 7.8.

Table 7.8 Pregnancy scale factor, λ .

Ewes	Cows	Deer
0.3	0.2	0.2

7.5.4 Lactation

The energy required for lactation is defined in terms of the potential rate of milk production. This is given by the commonly applied gamma function (eg Finlayson *et al*, 1995) which can be written as:

$$L = L_{mx} \zeta \frac{t^{\alpha} \exp\left(-\alpha t / t_{mx}\right)}{t_{mx}^{\alpha} \exp\left(-\alpha\right)} \qquad (7.12a)$$

where L is the potential milk production per day, L_{mx} is the maximum potential daily milk production, t is time since parturition, t_{mx} is the time of maximum potential daily milk production, α is an empirical shape parameter, and ζ is a scale factor for multiple foetuses. The default parameters are given in Table 7.9.

Table 7.9 Lactation parameters, eqns (7.12a).			
	t_{mx}, days	α	L_{mx} litres d^{-1}
Sheep	30	0.5	1.6
Cows	65	0.5	35
Deer	50	0.2	1.8

For all animal types, the scale parameter ζ is given by

$$\zeta = 1 + 0.6\left(n_f - 1\right) \tag{7.12b}$$

(recall n_f is the number of foetuses). Equation (7.12a) is shown in Fig. 7.8 for dairy cows:

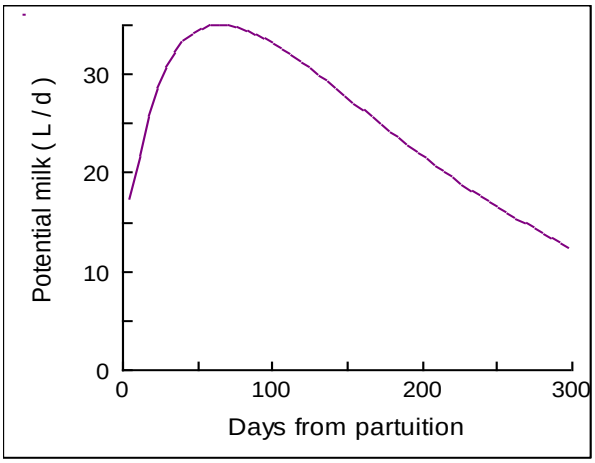

Figure 7.8: Potential milk production curve for dairy cows with the model default parameters. See text for details.

The corresponding energy required for the milk production is then:

$$ME_L = \left(1 + \mu\right) L \frac{E_L}{Y_L} \tag{7.12c}$$

where μ is a scale parameter to allow for the increase in maintenance requirements associated with lactation (analogous to λ in 7.11h). The default parameters are presented in Table 7.10. According to these parameters, the energy required to synthesise one litre of milk is 4.84 MJ for a cow, 7.42 for a ewe and 9.68 for a deer.

Table 7.10 Lactation energy parameters, eqn (7.12c).			
	E_L, MJ L^{-1}	Y_L	λ
Sheep	4.6	0.62	0.1
Ewes	3	0.62	0.35
Deer	6	0.62	0.2

In order to assess the behaviour of the model, a simulation was run for a dairy cow and, as for the maintenance illustrations in Figs 7.4 and 7.5, this was compared with the AFS equations. Although

this illustration is mainly focusing on lactation requirements, it is also influenced by pregnancy. The energy requirements for a single year are shown in Fig. 7.9 and again it can be seen that there is very good agreement between the two approaches.

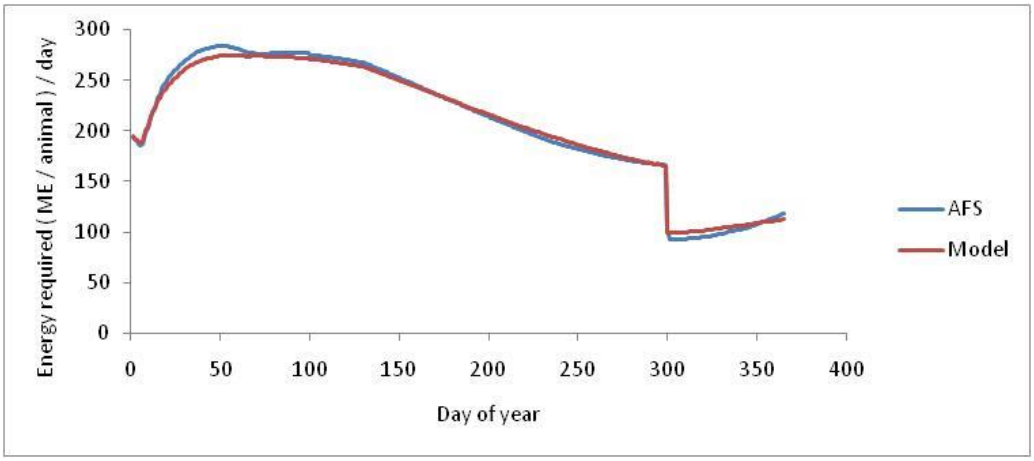

Figure 7.9: ME requirement for dairy cows on irrigated perennial ryegrass, using either the AFS or GrazeMod equations as indicated.

7.6 Actual animal growth and metabolism

The preceding section described the potential growth and metabolism – that is, the metabolism requirements for animals with no limitation due to food supply. Note that, in the present analysis, animals do not exceed their normal body weight, so that the model does not incorporate over-weight animals. It is now necessary to deal with the situation where metabolic processes are limited by food supply, or metabolic requirements limit pasture intake.

In either case, the initial calculations are:

- Potential intake is calculated as described in section 7.3 and this is converted into ME (metabolisable energy), denoted by $ME_{in,pot}$.
- Total ME required is calculated as described in section 7.5, denoted by ME_{req}.
- ME intake through supplementary feeding is calculated according to section 7.4.

7.6.1 Dry, non-pregnant animals

In this case, the only energy requirements are for growth and maintenance. The approach is as follows:

- If energy demands are not met then growth is reduced.
- If, with no growth, energy demands are still not met then weight is lost to satisfy maintenance requirements.
- If there is still not enough energy then maintenance is reduced.

Animal maximum weight loss is set at a fixed maximum amount per day with defaults 0.2 kg d^{-1} for sheep, 1 kg d^{-1} for cattle, and 0.5 kg d^{-1} for deer (these values can be altered on the interface). The energy is then passed to the ME pool and is available for metabolic processes. Thus, for example, since the energy content of mature tissue is taken to be 30 MJ kg^{-1}, eqn (7.5c), the maximum energy available per day from the remobilisation of animal tissue for sheep is 0.2 x 30 = 6 MJ d^{-1}.

Note that energy can only be lost to a prescribed minimum body weight, taken to be 30 kg, 40 kg, 400 kg for mature wethers, ewes and cows respectively.

7.6.2 Lactating and / or pregnant animals

During lactation animals will have a varying level of priority for milk production and may also lose weight specifically for the purpose of providing energy for milk production. The aim of this section is to attempt to capture the essence of the processes while avoiding too much complexity.

First define the milk priority function, λ, as

$$\lambda = \frac{1-\left(t/t^*\right)^n}{1+\left(t/t^*\right)^n} \tag{7.13a}$$

where t (days) is the time since partuition, and t^* (days) is the time when $\lambda = 0$. This function ranges from +1 to -1 and is illustrated in Fig. (7.10) with $t^* = 60$ and $n = 2,3,4$; in the model the default $n = 3$ is used.

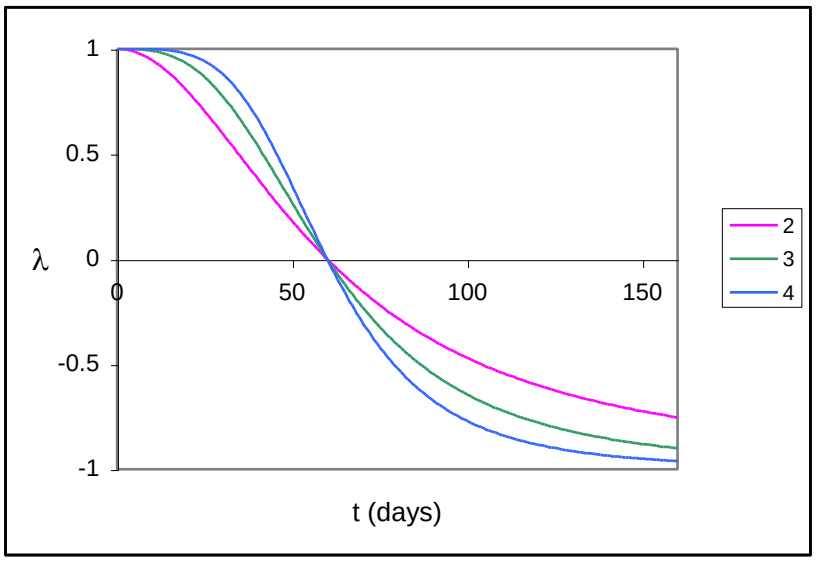

Figure 7.10: Milk priority function as given by eqn (7.13a). $t^* = 60$ and n as indicated.

When $\lambda = 1$, priority is for milk production and weight loss will be at the maximum possible rate to supply energy for milk production; when $\lambda = 0$, there will be no weight loss for milk production and no reduction in milk production to accommodate weight gain; when $\lambda = -1$, (which is only approached asymptotically with this equation), priority is for weight gain and milk production will be reduced to allow for this. The objective with this approach is to allow for a shift from the priority for milk production to weight gain.

It is also necessary to define the potential weight loss to provide energy for lactation. First define the animal body condition score, β:

$$\beta = \begin{cases} \beta_{mx} - \left(\beta_{mx} - \beta_{mn}\right)\left(\dfrac{W_{\beta,mx} - W}{W_{\beta,mx} - W_{\beta,mn}}\right), & W > W_{mn}; \\ \beta_{mn}, & W \le W_{mn}. \end{cases} \tag{7.13b}$$

where subscripts mx and mn refer to maximum and minimum respectively. This equation simply defines a straight line between the minimum and maximum body weights. For the model interface, it is assumed that the maximum body condition occurs at the normal mature body weight and that

the minimum condition occurs at a specified fraction of mature body weight, denoted by $f_{W,\beta,mn}$. The default parameters are given in Table 7.10.

Table 7.10 Body condition parameters, eqn (7.13b), see text for details.			
	β_{mn}	β_{mx}	$f_{W,\beta,mn}$
Cows	4	5.5	0.8
Ewes	4	5.5	0.8
Deer	4	5.5	0.8

The potential weight loss for metabolic processes when there is no priority for weight gain is given by:

$$\delta W_L = \delta W_{L,mx}\left[1-\left(\frac{\beta_{L,mx}-\beta}{\beta_{L,mx}-\beta_{L,mn}}\right)^q\right] \qquad (7.13c)$$

where δW_L is the actual daily weight loss, kg d^{-1}, $\delta W_{L,mx}$ is the maximum possible daily weight loss that occurs at body condition $\beta_{L,mx}$, no weight loss (by this process) is possible below a body condition of $\beta_{L,mn}$, β is the actual body condition as given by eqn (7.13b), and q is an empirical coefficient. The default parameters are given in Table 7.11.

Table 7.11 Body weight loss parameters, eqn (7.13c), see text for details.				
	$\delta W_{L,mx}$	$\beta_{L,mx}$	$\beta_{L,mn}$	q
Cows	2.5	5.5	4.5	3
Sheep	0.5	5.5	4.5	3

Equation (7.13c) is illustrated in Fig. 7.11 for a lactating cow for both $q = 3$, which is the default value used in the model, and $q = 1$: the choice of $q = 3$ is to ensure that weight loss for milk production during early lactation is fairly uniform for animals that are relatively close to their maximum body weight. Note that this response as illustrated in the figure assumes that there is no priority for weight gain: this priority varies according to eqn (7.13a), Fig 7.10.

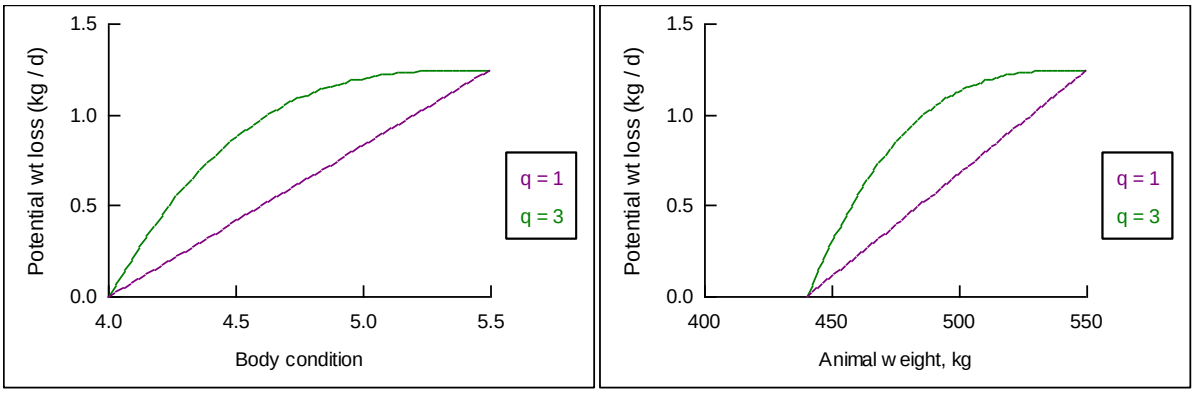

Figure 7.11: Potential weight loss for dairy cows during the first phase of lactation. The curved line is for $q = 3$ and the straight line is $q = 1$.

The procedure is to separate the energy dynamics into two lactation phases corresponding to whether λ is positive (early lactation) or negative.

Early lactation: $\lambda \geq 0$

As mentioned earlier, the energy available from pasture intake and supplement that is supplied irrespective of pasture intake is calculated, along with the energy requirements for maintenance, pregnancy and lactation. In addition, the potential weight loss to provide energy for metabolism is calculated from eqns (7.11a, b, c) as

$$\Delta W_L = \lambda \delta W_L \qquad (7.13d)$$

The total energy available to the animal is now the sum of pasture intake, supplement and energy from weight loss. Energy requirements are maintenance, lactation, and possibly pregnancy. It is assumed that maintenance and pregnancy costs must always be met before energy can be used for lactation. Supplementary feeding is then calculated as described in section 7.4.

According to this simple strategy, weight loss for the purpose of lactation will occur at a maximum rate declining to zero at time t^*. In practice, t^* is unlikely to be fixed but may depend on actual body condition. For example, animals may be able to sustain low rates of weight loss beyond t^*. However, the present approach appears to capture the essential processes involved.

Later lactation: $\lambda < 0$

When $\lambda < 0$, there is a shift from priority for milk production to body weight increase and there is no body weight decrease for the purpose of supplying energy for milk production. The energy required for growth is calculated according to section 7.5.1 and that for lactation in 7.5.4. Denoting these by $ME_{r,g}$ and $ME_{r,\ell}$ respectively, and the energy available as ME_{avail}, then the available energy is partitioned with the assumption that:

$$\frac{ME_g / ME_{r,g}}{ME_\ell / ME_{r,\ell}} = |\lambda|, \qquad (7.13e)$$

which, after a little algebra, leads to

$$ME_\ell = ME_{avail}\left(\frac{ME_{r,\ell}}{|\lambda| ME_{r,g} + ME_{r,\ell}}\right), \qquad (7.13f)$$

and

$$ME_g = ME_{avail}\left(\frac{|\lambda| ME_{r,g}}{|\lambda| ME_{r,g} + ME_{r,\ell}}\right). \qquad (7.13g)$$

According to this strategy, there is a shift towards priority for body weight gain.

Note that eqn (7.13f) can be used directly in the calculations for supplementary feeding to reach a particular milk target as described in section 7.4. If the energy required to produce the target milk is ME_ℓ^*, then the total energy required to ensure the target milk production is

$$ME_{req}^* = ME_\ell^*\left(1 + |\lambda| \frac{ME_{r,g}}{ME_{r,\ell}}\right) \qquad (7.13h)$$

which can then be used to calculate the amount of supplement needed.

7.6.3 Severe intake limitation

If intake is so low that the energy requirements for lactation and body weight gain cannot be met, then the animal will be severely stressed. In this case, body weight can be lost for the purpose of maintenance, and the maximum rate of body weight loss is the same as for dry animals down to a minimum specified body weight (see section 7.6.1) Once animals reach this weight the model will not let their weight fall although it is likely that in practice they will be severely stressed. The energy released from this body weight loss is then available for pregnancy and maintenance, both of which will be below requirement if the available energy is insufficient. It must be stressed that this is not a particularly rigorous treatment of severe stress.

7.7 Nutrient dynamics

So far, the analysis has covered the dynamics of pasture dry weight, supplementary feeding, metabolism and growth. Associated with all of these processes are the corresponding nutrient dynamics.

7.7.1 Retained nutrients

When animals grow they will retain nutrients associated with their increase in weight. This is incorporated by defining the nutrient composition of the animal, with the defaults being:

$$f_N = 0.03, \quad f_P = \frac{f_N}{10}, \quad f_K = \frac{f_N}{4}, \quad f_S = \frac{f_N}{10} \tag{7.14a}$$

with units kg nutrient (kg live weight)$^{-1}$. These parameters are not currently on the interface.

It is also necessary to define the nutrient composition of milk:

$$f_{N,milk} = 0.005, \quad f_{P,milk} = \frac{f_{N,milk}}{10}, \quad f_{K,milk} = \frac{f_{N,milk}}{3}, \quad f_{S,milk} = \frac{f_{N,milk}}{10} \tag{7.14b}$$

with units kg nutrient litre^{-1}. These parameters can be changed on the interface.

These fractions are then used to balance nutrient dynamics in the model.

7.7.2 Excreted nutrients

The balance of nutrients that are not retained are excreted. This means that for a mature animal at normal weight that is not pregnant or lactating, all nutrients that are eaten will be secreted. The partitioning of these nutrients between dung and urine can have an impact on the overall nutrient dynamics and subsequent pasture growth and so it is necessary to calculate the dung and urine components of the excreted nutrients.

The treatment for nitrogen is more developed. Working with David Pacheco, Val Snow has analysed partitioning for feeds with a range of N contents and ME values, and for animals at different metabolic stages. In this work, they developed a relationship for the proportion of N that is excreted in faeces, ϕ_F, in terms of the *N-energy density* (ρ_I) of intake (or feed), kg N MJ^{-1}, and the proportion of N intake that is excreted, λ_N. This is shown in Fig. 7.12.

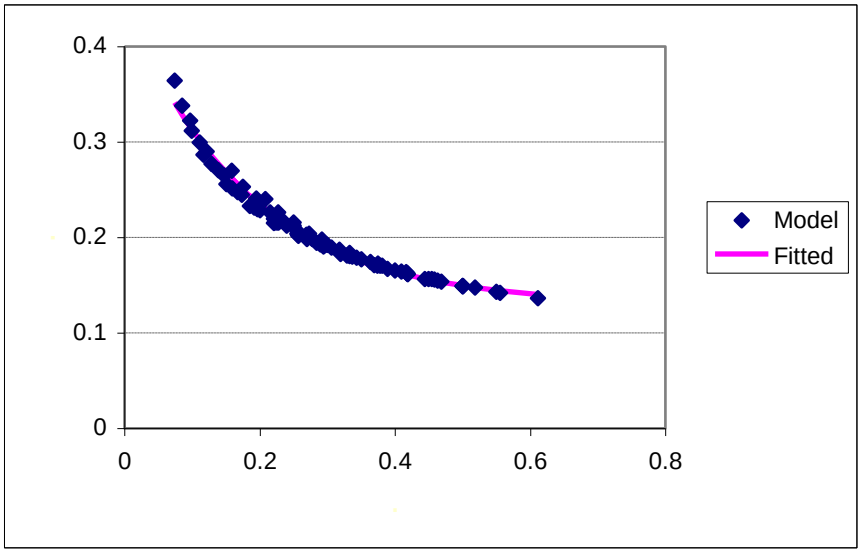

Figure 7.12: Proportion of N excreted in dung in terms of the N-energy density of intake and the proportion of N that is excreted. See text for details.

The governing equation can be written as:

$$\phi_F = \beta + (\alpha - \beta)\exp\left(-\gamma\rho_I\lambda_N^2\right) \qquad (7.15a)$$

where α and β are the values of ϕ_F for very low and very high λ_N respectively, and γ is an empirical coefficient. The default values are:

$$\alpha = 45\%,\ \ \beta = 13\%,\ \ \gamma = 560 \text{ MJ (kg N)}^{-1} \qquad (7.15b)$$

These coefficients can be altered in the Model on the 'Nutrients' tab of the 'Stock module'. The excreted proportion of N in dung, ϕ_F, is illustrated in Fig. 7.13 and uses the default parameter values with λ_N = 60%, 70%, 80%, 90% as illustrated. The ME content of the feed is taken to be 12 MJ kg^{-1} for the illustration, and it should be noted that the model generally gives an increase in ϕ_F as the ME content increases.

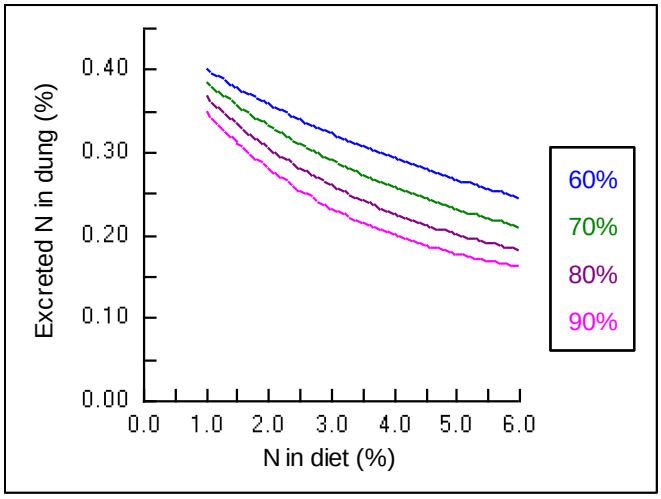

Figure 7.13: Proportion of N excreted in dung in relation to the N content of the diet. The different lines are for different proportions of N excreted. See text for details.

In the model, the proportion of N that is excreted is, of course, calculated and is not an input.

For P and S it is assumed that the partitioning between dung and urine is according to fixed fractions, with defaults 80% and 50% partitioned to dung respectively.

With K it is assumed that all excreted K is in urine. Again, while K is not generally found in dung, it may be that inorganic K is excreted in dung and then immediately leached out.

7.8 Greenhouse gas emissions

Methane (CH_4) is an important non-CO_2 greenhouse gas and is accounted for in the model. At present the treatment is quite simple and defines the energy content emitted in CH_4 as a fraction of the gross energy intake of forage or concentrate, with defaults

Energy in CH_4 per gross energy intake of: forage 6%; concentrate 4% (7.16a)

(Harry Clark, *personal communication.*)

In the model, it is necessary to convert these factors to kg C respired (kg C intake)$^{-1}$, since carbon is the base mass unit that is used. First define the energy contents of d.wt. and methane as

$$E_{CH_4} = 55.65 \text{ MJ (kg CH}_4\text{)}^{-1} \text{ and } E_W = 18.45 \text{ MJ (kg d.wt)}^{-1} \quad (7.16b)$$

which are the IPCC standards. If λ_E and λ_C are the respired fractions of intake using energy and carbon mass units respectively, then it follows that

$$\lambda_C = \lambda_E \frac{0.75/E_{CH_4}}{0.4/E_W} \quad (7.16c)$$

where the factors 0.75 and 0.4 convert from CH_4 and d.wt to carbon respectively. Combining (7.16b,c) gives

$$\lambda_C = 0.62\lambda_E . \quad (7.16d)$$

It is useful to work with units g CH_4 respired (kg d.wt. intake)$^{-1}$, so that

$$\text{g CH}_4 \text{ respired (kg d.wt intake)}^{-1} \equiv 1000\lambda_E \frac{E_W}{E_{CH_4}} . \quad (7.16e)$$

Now, the defaults for emissions from forage and concentrate are:

$$\lambda_{E,F} = 0.06 \text{ and } \lambda_{E,C} = 0.04 \quad (7.16f)$$

which correspond to 19.89 and 13.26 g CH_4 (kg d.wt intake)$^{-1}$ respectively.

This is a relatively simple treatment of methane emission, although it does incorporate the essential characteristics

7.9 Concluding remarks

Animal intake, growth and metabolism have been described. The model includes the interaction between the grazing animal and the pasture. Growth, maintenance, pregnancy and lactation have been incorporated to give a treatment of the basic animal processes. The model also accounts for nutrient dynamics and so allows for a complete description of nutrient cycling within the model. In addition, the model derives the methane emissions from ruminants supplied with either forage or concentrate.

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