SP 4&5:

$A = 0.15$ $\quad \omega = 1 \quad \varepsilon = 0.1$

CASE 1.4:

Note: with $r(t) = \text{const}$ → $y(t)$ just has steady state solution.

0.15

$r(t)$

$$x'' + 2(0.1)(1)x' + 1^2 x = 0.15\cos(1 \cdot t)$$

$$y_p(t) = A\cos\omega t + B\sin\omega t$$

$$y_p'(t) = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$y_p''(t) = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$

$$-A\omega^2\cos\omega t - B\omega^2\sin\omega t + 0.2\omega\left[-A\omega\sin\omega t + B\omega\cos\omega t\right] + \omega^2 x = 0.15\cos(\omega t)$$

$$\left(-A\omega^2\cos\omega t + 0.2B\omega^2\cos\omega t\right) + \left(-B\omega^2 - 0.2A\omega^2\right)\sin\omega t = 0$$

$$\Leftrightarrow \begin{cases} -B\omega^2 - 0.2A\omega^2 = 0 \\ -A\omega^2 + 0.2B\omega^2 = 0.15 \end{cases} \Leftrightarrow \begin{cases} -B - 0.2A = 0 \quad (\omega \neq 0) \\ -A + 0.2B = \dfrac{0.15}{\omega^2} \end{cases}$$

$$\Leftrightarrow \begin{cases} -B - 0.2\left[0.2B - \dfrac{0.15}{\omega^2}\right] = 0 \\ A = 0.2B - \dfrac{0.15}{\omega^2} \end{cases} \Leftrightarrow \begin{cases} 1.04B = \dfrac{0.03}{\omega^2} \rightarrow B = \dfrac{3}{104\omega^2} \\ A = \dfrac{3}{520\omega^2} - \dfrac{0.15}{\omega^2} = \dfrac{-15}{104\omega^2} \end{cases}$$

$$\Rightarrow y(t) = C_1 \cdot e^{\omega t} + C_2 \cdot e^{-\omega t} - \frac{15}{104\omega^2}\cdot\cos\omega t + \frac{3}{104\omega^2}\cdot\sin\omega t$$

$$\left.\begin{matrix} y(0) = 1 \\ y'(0) = 0 \end{matrix}\right\} \rightarrow \begin{cases} C_1 + C_2 = \dfrac{15}{104\omega^2} + 1 \\ C_1\omega - C_2\omega = \dfrac{-3}{104\omega} \end{cases} \Leftrightarrow \begin{cases} C_1 + C_2 = \dfrac{15 + 104\omega^2}{104\omega^2} \\ C_1 - C_2 = \dfrac{-3}{104\omega^2} \end{cases}$$

$$\Leftrightarrow \begin{cases} C_1 = \dfrac{12 + 104\omega^2}{208\omega^2} \\ C_2 = \dfrac{18 + 104\omega^2}{208\omega^2} \end{cases} \quad \omega = 1 \rightarrow \begin{matrix} C_1 = \dfrac{29}{52} \\ C_2 = \dfrac{61}{104} \end{matrix}$$

transient

$$\Rightarrow y_p = \frac{29}{52}e^{t} + \frac{61}{104}e^{-t} - \frac{15}{104}\cdot\cos t + \frac{3}{104}\cdot\sin t$$

steady-state

t increases → y increases

SAME METHOD FOR 4,5