

$$\text{ISIS}(n, m, q, R_B) \text{ represented with } A\bar{x} = \bar{y} \quad \begin{array}{l} A \in \mathbb{Z}_q^{n \times m} \text{ (uniform)} \\ \bar{x} \in \mathbb{Z}_q^m \{ \bar{x} \mid \|\bar{x}\|_0 \leq B \} \\ \bar{y} \in \mathbb{Z}_q^n \text{ (uniform)} \end{array}$$

$$\text{ISIS}(n, m, q, R_B + \bar{\delta}) \text{ represented } A'(\bar{x}' + \bar{\delta}) = \bar{y}' \quad \text{"same as above"} \\ \forall \bar{\delta} \in \mathbb{Z}_q^m$$

Some Linear Algebra shows

$$\begin{aligned} A'(\bar{x}' + \bar{\delta}) &= \bar{y}' \\ &= A'\bar{x}' + A'\bar{\delta} = \bar{y}' \Rightarrow A'\bar{x}' = (\bar{y}' - A'\bar{\delta}) \end{aligned}$$

Therefore if $(\bar{y}' - A'\bar{\delta})$ and (\bar{y}) are PPT indistinguishable[✓], then $\text{ISIS}_{n,m,q}(R_B)$ is eqv. to $\text{ISIS}_{n,m,q}(R_B + \bar{\delta})$

for all $\bar{\delta}$ and uniform A, \bar{y}

$$\text{Notice that } \bar{y}' - A'\bar{\delta} = \bar{y}' - \underbrace{\bar{\delta}_1 a'_1 - \bar{\delta}_2 a'_2 - \dots - \bar{\delta}_m a'_m}_{\substack{\uparrow \\ \text{Is uniform}}}$$

Then for $m \gg n$ and q Prime

A uniform vector minus another uniform vector is uniform.

Therefore both ISIS instances are equivalent.