CS 516: COMPILERS

Lecture 10

Topics

- Parsing

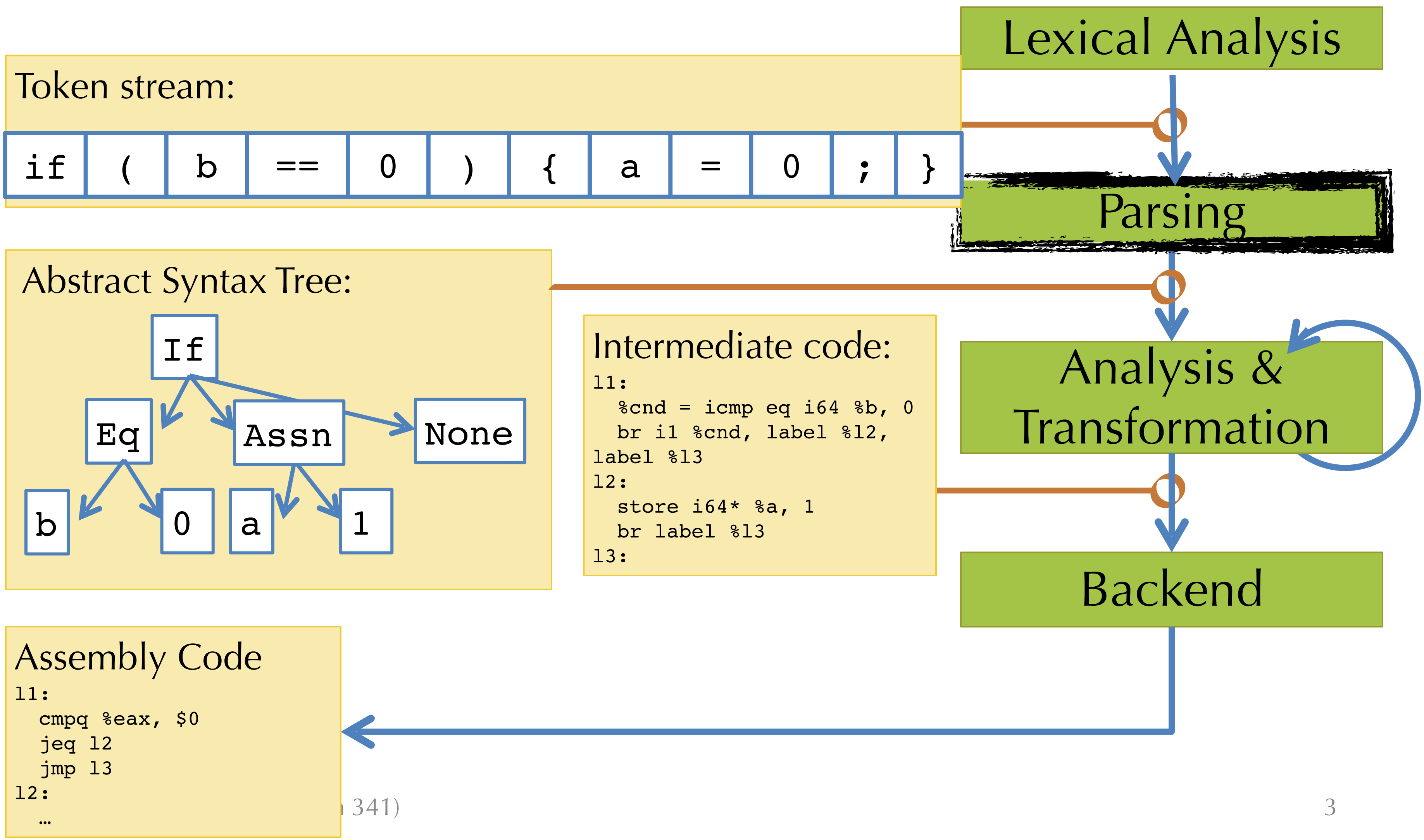
Materials

- lec10.zip

Creating an abstract representation of program syntax.

PARSING

Today: Parsing



Today: Parsing

Source Code
(Character stream)

```
if (b == 0) { a = 1; }
```

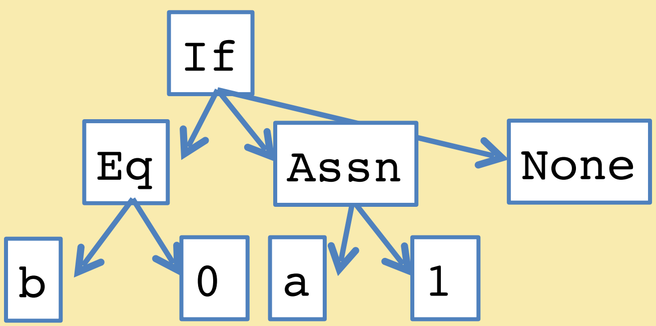
Lexical Analysis

Token stream:

if	(	b	==	0	)	{	a	=	0	;	}

Parsing

Abstract Syntax Tree:



Intermediate code:

```
l1:
  %cnd = icmp eq i64 %b, 0
  br i1 %cnd, label %l2,
label %l3
l2:
  store i64* %a, 1
  br label %l3
l3:
```

Analysis & Transformation

Backend

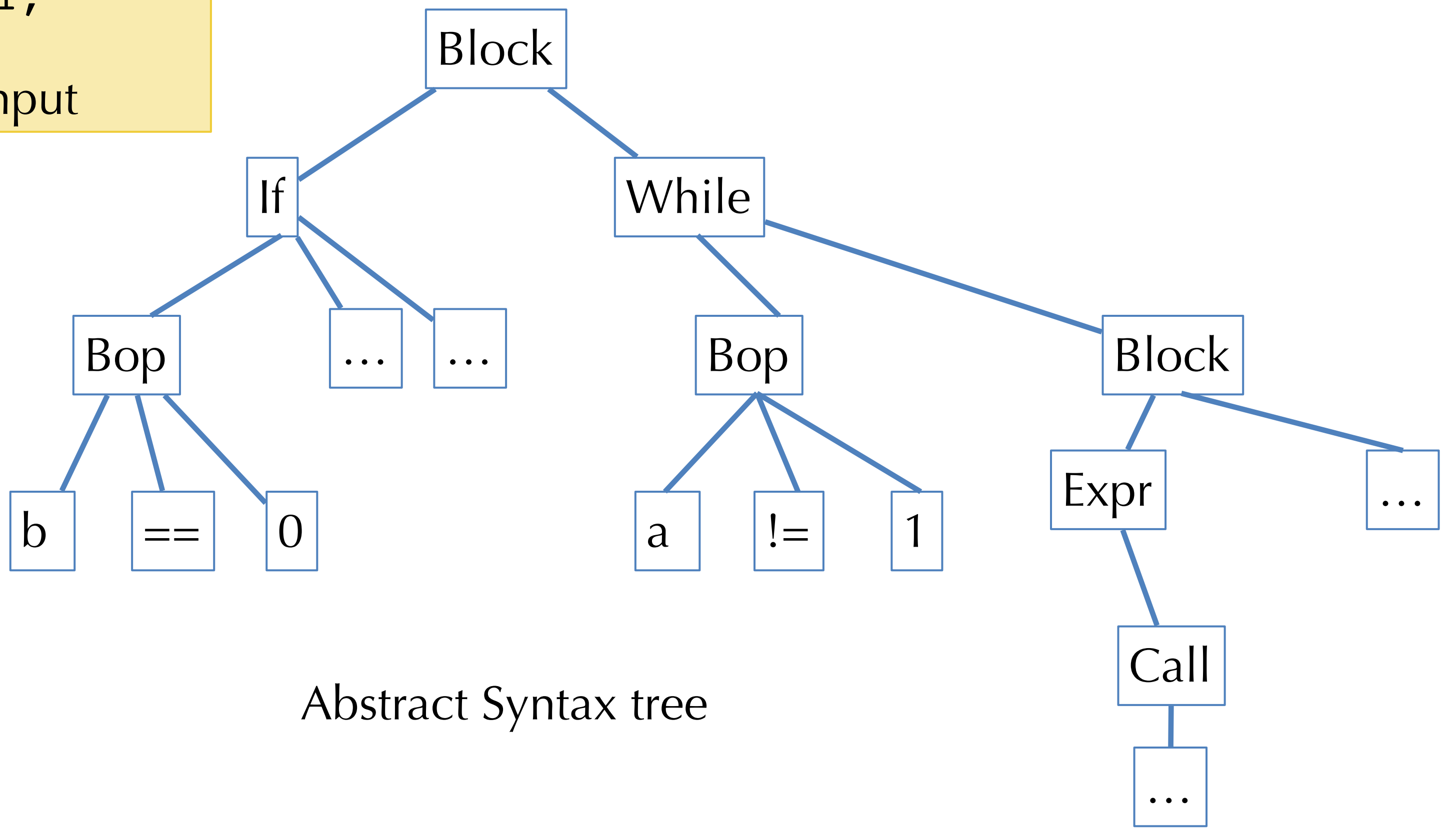
Assembly Code

```
l1:
  cmpq %eax, $0
  jeq l2
  jmp l3
l2:
  …
```

Parsing: Finding Syntactic Structure

```
{
  if (b == 0) a = b;
  while (a != 1) {
    print_int(a);
    a = a – 1;
  }
}
```

Source input



Abstract Syntax tree

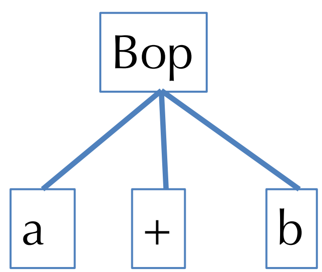
Syntactic Analysis (Parsing): Overview

- Input: stream of tokens (generated by lexer)
- Output: abstract syntax tree

- Strategy:
  - Parse the token stream to traverse the "concrete" syntax
  - During traversal, build a tree representing the "abstract" syntax

- Why abstract? Consider these three *different* concrete inputs:

```
a + b
(a + ((b)))
((a) + (b))
```



Same abstract syntax tree

- Note: parsing doesn't check many things:
  - Variable scoping, type agreement, initialization, …

Specifying Language Syntax

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- First question: **how to describe language syntax** precisely and conveniently?

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- Last time: we described tokens using regular expressions
  - Easy to implement, efficient DFA representation
  - Why not use regular expressions on tokens to specify programming language syntax?

- Limits of regular expressions:
  - DFA's have only finite # of states
  - So… DFA's can't "count"
  - For example, consider the language of all strings that contain balanced parentheses – easier than most programming languages, but not regular.

- So: we need more expressive power than DFA's

CONTEXT FREE GRAMMARS

Context-free Grammars

- Here is a specification of the language of balanced parens:

$$S \longmapsto (S)S$$
$$S \longmapsto \varepsilon$$

- The definition is *recursive* – S mentions itself.

- Idea: "derive" a string in the language by starting with S and rewriting according to the rules:

  S

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Replace

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  – Example: $S \longmapsto (S)S \longmapsto ((S)S)S \longmapsto ((\varepsilon)S)S \longmapsto ((\varepsilon)S)\varepsilon \longmapsto ((\varepsilon)\varepsilon)\varepsilon = (())$

- You can replace the "nonterminal" S by its definition anywhere
- A context-free grammar accepts a string iff there is a derivation from the start symbol

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CFGs Mathematically

- A Context-free Grammar (CFG) consists of
  - A set of *terminals* (e.g., a lexical token or ε)
  - A set of *nonterminals* (e.g., S and other syntactic variables)
  - A designated nonterminal called the *start symbol*
  - A set of productions: LHS $\longmapsto$ RHS
    - LHS is a nonterminal
    - RHS is a *string* of terminals and nonterminals

- Example: The balanced parentheses language:

$$S \longmapsto (S)S$$

$$S \longmapsto \varepsilon$$

- How many terminals? How many nonterminals? Productions?

Another Example: Sum Grammar

- A grammar that accepts parenthesized sums of numbers:

$$
\begin{array}{lllcl}
S & \longmapsto & E + S & | & E \\
E & \longmapsto & \text{number} & | & (\ S\ )
\end{array}
$$

e.g.: (1 + 2 + (3 + 4)) + 5

- Note the vertical bar '|' is shorthand for multiple productions:

$$
\left.
\begin{array}{l}
S \longmapsto E + S \\
S \longmapsto E \\
E \longmapsto \text{number} \\
E \longmapsto (S)
\end{array}
\right\}
$$

4 productions
2 nonterminals: S, E
4 terminals: (,), +, number
Start symbol: S

Derivations in CFGs

- Example: derive (1 + 2 + (3 + 4)) + 5
- $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + \mathrm{S}$

$\mathrm{S} \longmapsto \mathrm{E} + \mathrm{S} \mid \mathrm{E}$
$\mathrm{E} \longmapsto \text{number} \mid (\ \mathrm{S}\ )$

For arbitrary strings α, β, γ and production rule $A \longmapsto \beta$ a single step of the derivation is:

$$\alpha A \gamma \longmapsto \alpha \beta \gamma$$

(*substitute* β for an occurrence of A)

In general, there are many possible derivations for a given string

Note: Underline indicates symbol being expanded.

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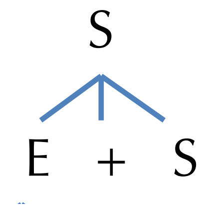

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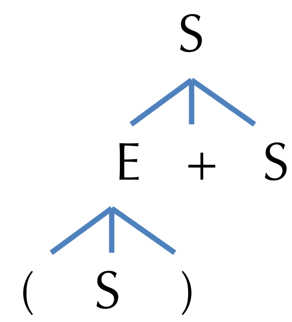

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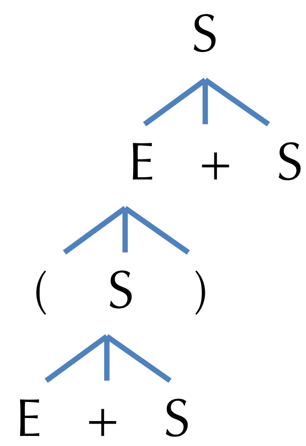

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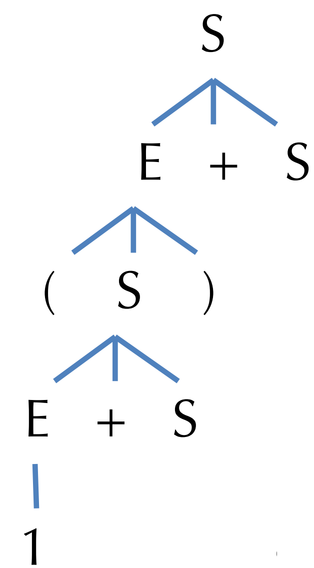

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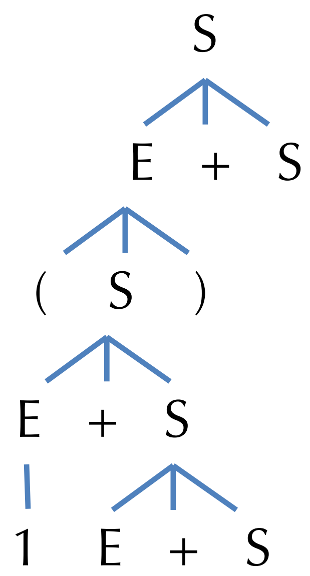

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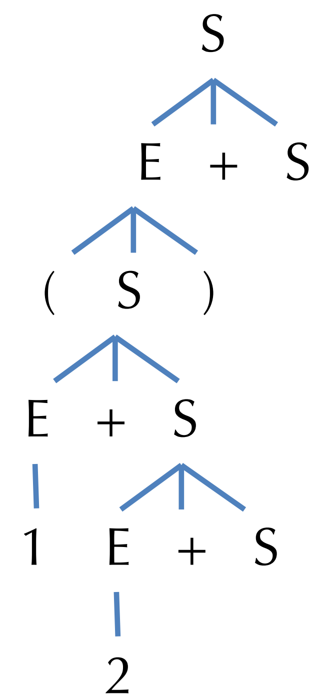

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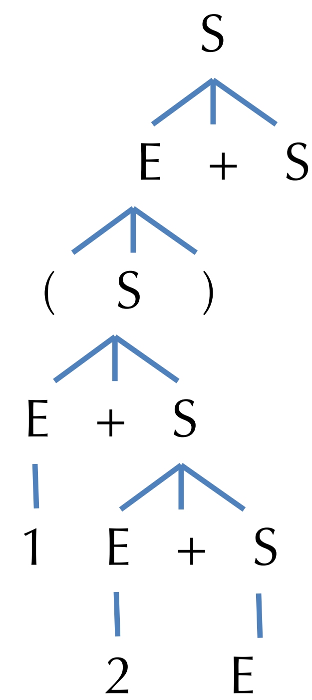

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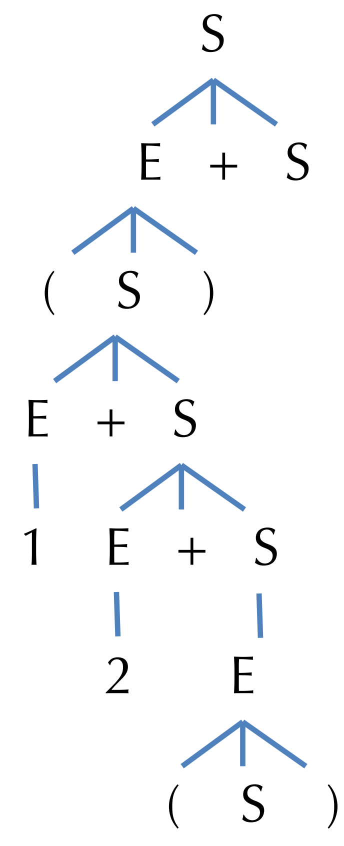

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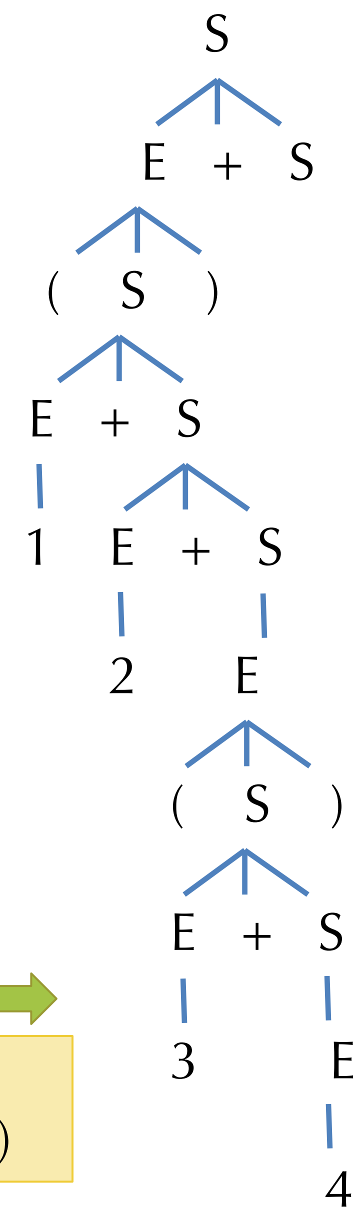

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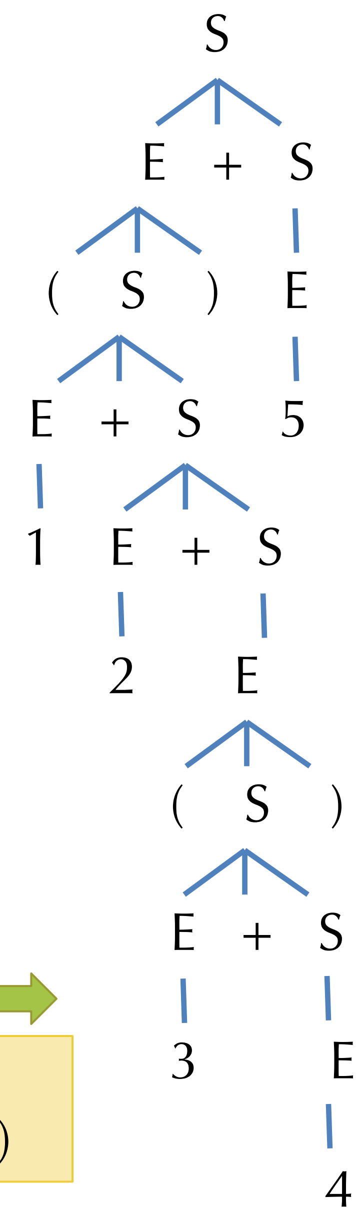
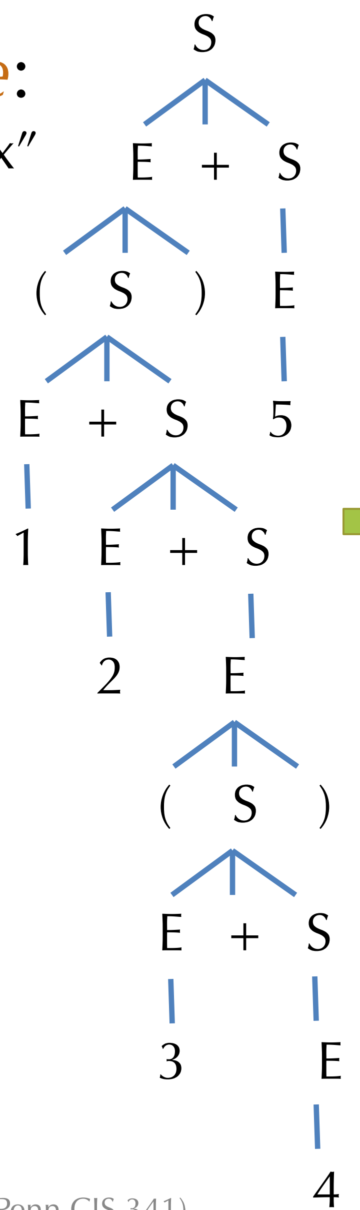


Parse Tree

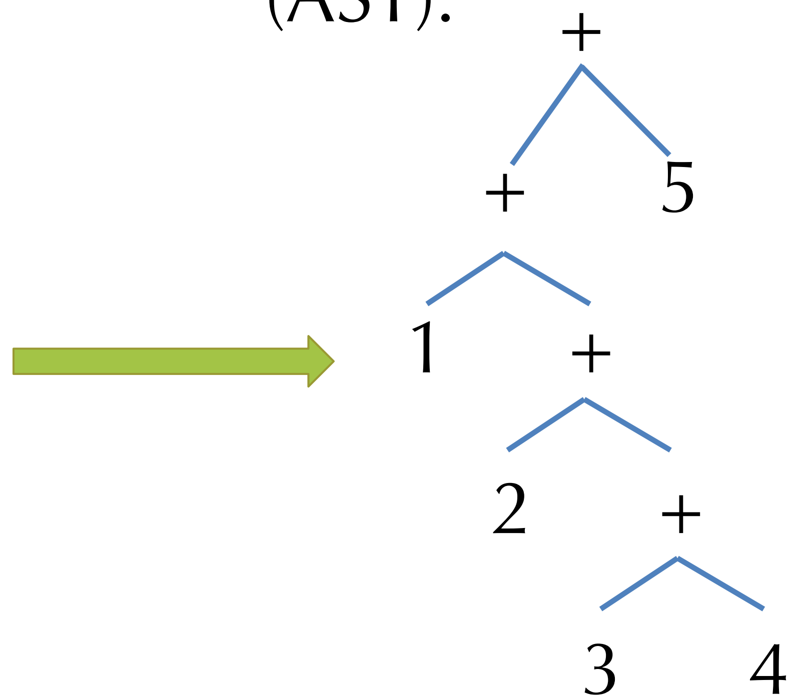
From Parse Trees to Abstract Syntax

- *Parse tree*:

"concrete syntax"



- *Abstract syntax tree* (AST):



- Hides, or *abstracts*, unneeded information.

Derivation Orders

- Productions of the grammar can be applied in any order.
- There are two standard orders:
  - *Leftmost derivation*: Find the left-most nonterminal and apply a production to it.
  - *Rightmost derivation*: Find the right-most nonterminal and apply a production there.

- Both strategies (and any other) yield the same parse tree!
  - Parse tree doesn't contain the information about what order the productions were applied.

Example: Left- and rightmost derivations

- Leftmost derivation:
- $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + S$
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- Upshot: be aware of "vacuously empty" CFG grammars.
  - Every nonterminal should eventually rewrite to an alternative that contains only terminal symbols.

Associativity, ambiguity, and precedence.

GRAMMARS FOR PROGRAMMING LANGUAGES

Associativity

$$S \longmapsto E + S \mid E$$
$$E \longmapsto \text{number} \mid (\ S\ )$$

Consider the input: $1 + 2 + 3$

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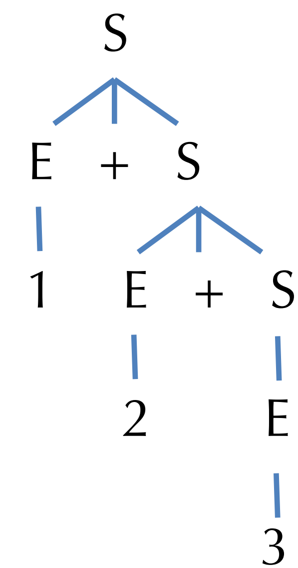

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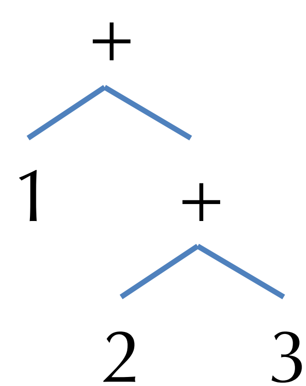

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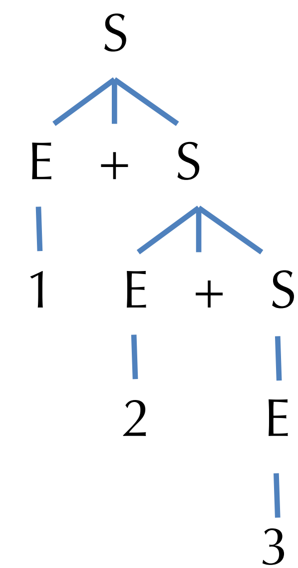

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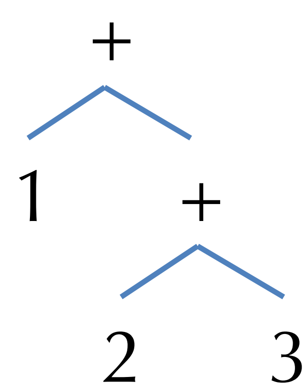

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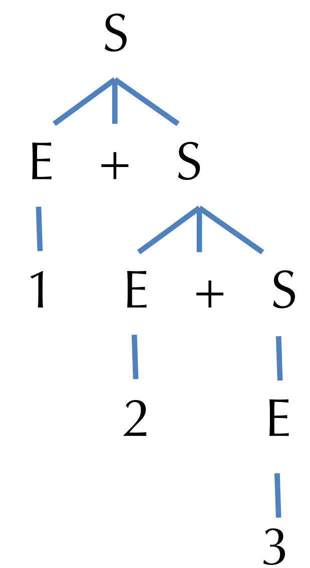

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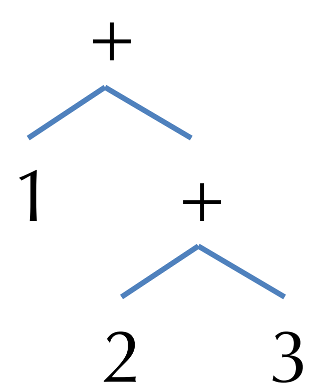

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- One derivation gives left associativity, the other gives right associativity to '+'
  - Which is which?

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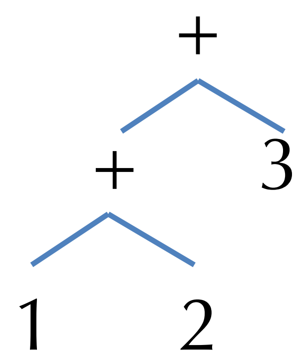
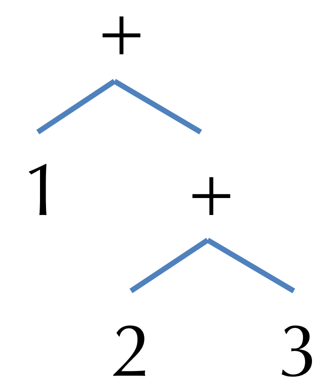
  - $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{S}} + S \longmapsto \underline{\mathbf{S}} + S + S \longmapsto 1 + \underline{\mathbf{S}} + S \longmapsto 1 + 2 + \underline{\mathbf{S}} \longmapsto 1 + 2 + 3$

- One derivation gives left associativity, the other gives right associativity to '+'
  - Which is which?



AST 1



AST 2

Why do we care about ambiguity?

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- The '+' operation is associative, so it doesn't matter which tree we pick. Mathematically, $x + (y + z) = (x + y) + z$
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- Consider:

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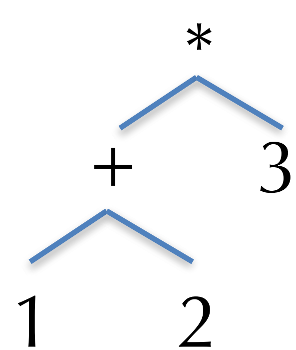
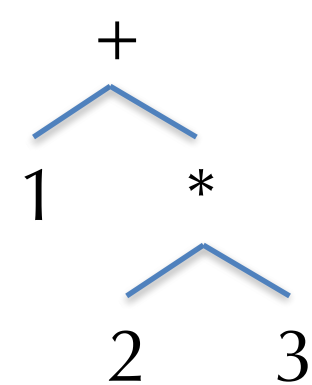
- Input: 1 + 2 * 3
  - One parse = (1 + 2) * 3 = 9
  - The other = 1 + (2 * 3) = 7



vs.



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$$\begin{aligned} S_0 &\longmapsto S_0 + S_1 \mid S_1 \\ S_1 &\longmapsto S_2 * S_1 \mid S_2 \\ S_2 &\longmapsto \text{number} \mid (S_0) \end{aligned}$$

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  - Decide (following math) to make '*' higher precedence than '+'
  - Make '+' left associative
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- Note:
  - S_2 corresponds to 'atomic' expressions

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Context Free Grammars: Summary

- Context-free grammars allow concise specifications of programming languages.
  - An unambiguous CFG specifies how to parse: convert a token stream to a (parse tree)
  - Ambiguity can (often) be removed by encoding precedence and associativity in the grammar.
- Even with an unambiguous CFG, there may be more than one derivation
  - Though all derivations correspond to the same abstract syntax tree.
- Still to come: finding a derivation
  - But first: menhir

menhir

- Consumes a context-free grammar
- Generates a parser for that grammar
- How does it cope with **ambiguity**?
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menhir

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- Example:

$$
\begin{array}{lll}
B & \longmapsto & \text{true} \\
 & | & \text{false} \\
 & | & \text{var} \\
 & | & B \;||\; B \\
 & | & B \;\&\&\; B \\
 & | & B \;\text{->}\; B \\
 & | & \sim B \\
 & | & (\; B\;)
\end{array}
$$

code demo

PARSING BOOLEAN LOGIC

1. **Unzip**

```
unzip lec10.zip ; cd lec10/
```

2. **Look at files grammar.txt, code/lexer.mll, code/range.ml, code/ast.ml, code/main.ml**
3. **Try the ambiguous parser:**

```
cp code/ambparser.mly code/parser.mly ; make
```

   A. Notice parse warnings

4. **Try the unambiguous parser:**

```
cp code/unambparser.mly code/parser.mly ; make
```

menhir

- Let's disambiguate by making
  - Ops || and &&　　left associative
  - Implication (->)　　right associative

- Giving each operator a precedence according to the usual conventions of math:

  - ~ binds "tightest" of all (i.e. with highest precedence)
  - & binds tigher than | which binds tighter than ->

- Using parentheses creates a grouping with the highest precedence

menhir

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```
B ⟼ true
  | false
  | var
  | B || B
  | B && B
  | B -> B
  | ~B
  | ( B )
```

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menhir

Right Assoc.

Left Assoc.

Left Assoc.

Precedences

menhir

$$B \longmapsto B_1$$

$$B_1 \longmapsto B_2 \;\texttt{->}\; B_1 \mid B_2$$

Right Assoc.

$$B_2 \longmapsto B_2 \;||\; B_3 \mid B_3$$

Left Assoc.

$$B_3 \longmapsto B_3 \;\&\; B_4 \mid B_4$$

Left Assoc.

$$B_4 \longmapsto \sim B_4 \mid B_5$$

$$B_5 \longmapsto \text{true} \mid \text{false} \mid \text{var} \mid (\; B_1 \;)$$

Precedences