HW 4

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I pledge my honor that I have abided by the Stevens Honor System

1 Problem

- $P(X=4) = \frac{\binom{7}{4}\binom{12-7}{6-4}}{\binom{12}{6}} = 0.378788$ $P(X \leq 4) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4) = \sum_{n=0}^{4} \frac{\binom{7}{n}\binom{12-7}{6-n}}{\binom{12}{6}} = .878788$

  $P(X=4) = \mathbf{0.378788}$

  $P(X \leq 4) = \mathbf{0.878788}$

- The mean can be represented by $E(X) = n(\frac{M}{N}) = (6)(\frac{7}{12}) = 3.5$. And the standard deviation as follows $\sigma = \sqrt{V(X)} = \sqrt{n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}} = \sqrt{6\frac{7}{12} \cdot \frac{12-7}{12} \cdot (1 - \frac{12-6}{12-1})} = 0.891883$

  Which means $\mu + \sigma = 3.5 + 0.891883 = 4.39188$ Therefore any $X > 4.39188$ will exceed the mean by more than 1 standard deviation. So $P(X > 4.39188) = P(X \geq 5) = P(X=5) + P(X=6) = \frac{\binom{7}{5}\binom{12-7}{6-5}}{\binom{12}{6}} + \frac{\binom{7}{6}\binom{12-7}{6-6}}{\binom{12}{6}} = 0.121212$ There is a **12.12%** chance that X exceeds its mean value by more than 1 standard deviation.

- Analyzing this problem we can see that the sample size of 15 is less than 5% of the total population, and because this problem can be precisely solved using a hypergeometric distrabution, we can approximate, due to the sample size, to a binomial distrabution.

  Let p be the probability of a success, then $p = \frac{M}{N} = \frac{40}{400} = 0.1$. So I can model $P(X \leq 5) = \sum_{n=0}^{5} \binom{15}{n} 0.1^n 0.9^{15-n} = 99.775$ There is approximately **99.775%** chance that, given a sample of 15 randomly selected refrigerators, five or less will have a deffective compressor.

2 Problem

p = 0.2 Geometric - purchase untill you recieve an item

- $nb(x; 2, .2) = \binom{2+x-1}{x} 0.8^2 0.2^x$

- The probability you will purchace 4 boxes means that you will purchase 2 boxes without the prize, hence $P(X=2) = \binom{2+2-1}{2} 0.2^2 0.8^2 = \mathbf{0.0768}$

- Likewise, the probability you will purchase at most 4 boxes can be restated to the probability you will purchase at most 2 boxes without the prize, and the probability is: $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = \binom{2+0-1}{0} 0.2^2 0.8^0 + \binom{2+1-1}{1} 0.2^2 0.8^1 + \binom{2+2-1}{2} 0.2^2 0.8^2 = \mathbf{0.1808}$

- The mean of a negative binomial distrabution is $\mu = \frac{pr}{1-p} = \frac{0.8 \cdot 2}{1-0.8} = 8$ You should expect to get **8** boxes without the prize and expect to get $\mu + r =$ **10** boxes total.

3 Problem

- $P(X = 3) = (1 - 0.409)^3 0.409 =$ **0.143**

  $P(X \leq 3) = \sum_{k=0}^{3} (1 - 0.409)^n 0.409 =$ **0.878003**

- This can be written as $P(X > \mu + \sigma)$ First I will find $\mu = \frac{1-p}{p} = 1.44499$. And now $\sigma = \sqrt{V} = \sqrt{\frac{1-p}{p^2}} = 1.87962$

  So our probability becomes $P(X > 1.44499 + 1.87962) = P(X > 3.32461) = P(X > 3) = 1 - P(X \leq 3)$. Now using the answer from the previous question. $P(X > 3) = 1 - 0.878003 =$ **0.121997**

The only caveat here is that I am assuming the range is {0,1,2,3...} as opposed to {1,2,3...}

4 Problem

average rate = 1 Poisson

- $P(X <= 5)$ Can be solved using Appendix Table A.2. I will choose the column with the value 1.0, and the row with the value 5 to get:

  $\sum_{y=0}^{5} \frac{e^{-1} \cdot 1^y}{y!} =$ **0.999**

- $P(X = 2)$ Can be solved using the formula $\frac{e^{-1} 1^2}{2!} = 0.18394$

  Using Apendix Table A.2 I can also solve it with: $P(X = 2) = P(X <= 2) - P(X <= 1)$ I can solve this by calculating $(1.0, 2) - (1.0, 1)$ where given, (c, r) c represents the column value, and r represents the row.

  $P(X = 2) = .920 - .736 =$ **0.184**

- $P(2 \geq X \leq 4)$ Can be calculated by subtracting $(1.0, 1)$ from $(1.0, 4)$.

  $P(2 \geq X \leq 4) = 0.996 - 0.736 =$ **0.26**

- $\sigma = \sqrt{\mu} = \sqrt{1} = 1$

  $P(X > \mu + \sigma) = P(X > 2) = \sum_{n=3}^{\infty} \frac{e^{-1} \cdot 1^n}{n!} =$ **0.080301**

5 Problem

average rate = 1 Poisson In both cases there are only two values possible for the random variable X where X denotes the number of tests. In the first case X can either be 1 (no one tested positive) or +3 (in addition to the group test 3 individual tests are preformed). Likewise when $n = 5$ the possible values for X include $1, 6$. Now all I have to do is calculate the distrabution, or probability of each value occuring. For $n = 3$ $P(X = 1) = (1 - p)^3$. Therefore $P(X = 4) = 1 - P(X = 1)$ Likewise when $n = 5$ $P(X = 1) = (1 - p)^5$, and $P(X = 6) = 1 - P(X = 1)$. Now the expected value is $1 \cdot P(X = 1) + 4 \cdot P(X = 4)$ for the case when $n = 3$ and $1 \cdot P(X = 1) + 6 \cdot P(X = 6)$ for the case when $n = 5$.

- $n = 3,\ p = .1$

  $E(X) = 1 \cdot (1 - .1)^3 + 4 \cdot (1 - (1 - .1)^3) = \mathbf{1.813}$

- $n = 5,\ p = .1$

  $E(X) = 1 \cdot (1 - .1)^5 + 6 \cdot (1 - (1 - .1)^5) = \mathbf{3.04755}$