HW 3

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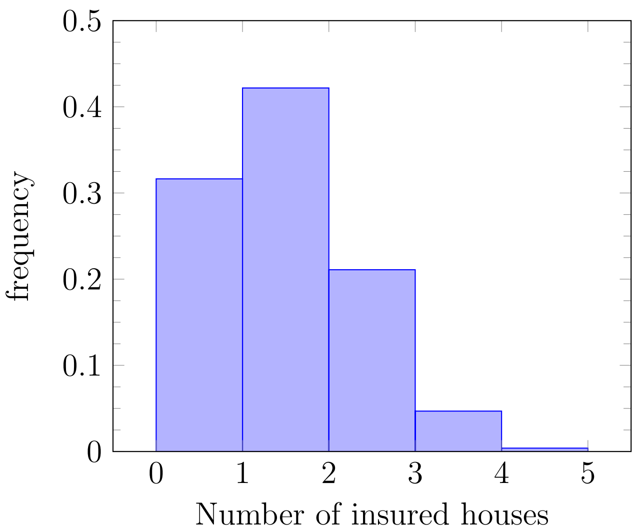
1. Flight
   (a) In order for a flight with $s = 50$ seats to accomadate k passengers who show up, $k \leq s$. Let X represent the random variable of the number of passengers who show up, then $P(X \leq s)$ is probability all k passengers will have a seat. This can be calculated as follows, $P(X \leq 50) = \rho(45) + \rho(46) + \rho(47) + \rho(48) + \rho(49) + \rho(50) = .05 + .1 + .12 + .14 + .25 + .17 = .83$. There is an **83%** chance all ticketed passengers who show up will have a seat.

   (b) The probability at least one of k passengers who show up will not be seated happens when $k > s$. Which can be shown as $P(X < 50)$ which is the complement to answer 'a'. so $P(X < 50) = 1 - P(X \leq 50) = 1 - .83 = .17$. The probability that not all of the k will recieve a seat is **17%**.

   (c) If you are the first person on standby, then if there must be one free seat on the plane for you to have a seat. This happens when $X \leq 49$. So answer derived from a $P(X < 50) = P(X \leq 50) - P(X = 50) = .83 - .17 = .66$. If you are the first person on standby there is a **66%** you will still be able to fly. Now, if you are the 3rd person on standby $X \leq 47$ must be true. $P(X \leq 47) = P(45) + P(46) + P(47) = .05 + .10 + .12 = .27$. If you are the third person on standby there is a **27%** chance you will still be able to fly.

2. Earthquake
   (a) The probability distrabution of X is

$$\rho(k) = \begin{cases} \binom{4}{k}.25^k.75^{N-k} & 0 \leq k \leq 4 \\ 0 & otherwise \end{cases}$$



   (b)

(c) Initialy I thought of calculating the expected value, $E(X) = \rho n = 1.0$, but then I interpreted the question as the mode, which value individually is most likely to be choosen, which also is **1**.

(d) The probability at least two of the four selected houses have eathquake insurance can be represented as follows, $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) = 1 - .316406 - .421875 = 0.261719$. The probability that at least two of the selected houses have earthquake insurance is **26.1719%**.

3. Allergies

(a) $P(X \leq 3) = \sum_{k=0}^{3} \binom{25}{k} .05^k .95^{25-k} = 0.965909$. There is a **96.5909%** chance 3 or less children int the sample will allergies. $P(X < 3) = P(X \leq 3) - P(X = 3) = 0.965909 - \binom{25}{3} .05^3 .95^{25-3} = 0.965909 - 0.872894 = 0.872894$. There is a **87.2894%** chance less than 3 children will have allergies.

(b) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.065909 = 0.034091$. There is a **3.4091%** chance 4 or more children in the sample will have allergies.

(c) $P(1 \leq X \leq 3) = P(X \leq 3) - P(X = 0) = .9659809 - 0.27739 = 0.68852$. There is a **68.8852%** chance between 1 and 3 children int the sample will have allergies.

(d) The given distrabution is binomial, therefore the expected value can be calculated as follows, $E(X) = \rho n = (.05)(25) = 1.25$. The expected value of the number of children in the sample size that will have allergies is **1.25**.

(e) Let $N = 50$ be the sample size maintaining $\rho = 0.05$. Let $x = 0$ be the number of children of the sample size that has allergies. Calculate the probability $P(X = x) = P(X = 0) = \binom{50}{0} .05^0 .95^{50} = 0.076945$ There is a **7.6945%** chance none of the 50 sampled children will have allergies.

4. Fine Crystal

(a) Assuming the sample population of fine crystal goblets is significantly greater than 6 I will consider this a binomial distrabution. Let X be the r.v. associated with the number of "seconds". If N is the number of goblets sampled then the distrabution of $X \sim Bin(N, \rho)$. In this case $N = 6, \rho = .10$. Therefore the probability of selecting only one "second" is $P(X = 1) = \binom{6}{1}(.1)^1(.9)^5 = \mathbf{0.354294}$.

(b) Similar to the previous problem but now there are 2 "seconds". $P(X = 2) = \binom{6}{2}(.1)^2(.9)^4 = \mathbf{0.098415}$.

(c) Selecting goblets one by one and counting the number of failures changes the distrabution from binomial to negative binomail. Classifying a failure as "not seconds" our ending condition is when we have 4 failures. Now X will be the number of successes (counter intuitive since these are bad goblets) and I am calcuating what the probability $X + 4 \leq 5$. The probability then is $P(X \leq 1) = P(X = 0) + P(X = 1) = .9^4 + \binom{4}{1}(.9)^4(.1)^1 = 0.59049 + 0.26244 = 0.85293$. There is a **85.293%** chance that at most 5 goblets must be selected to find four that are not seconds.

5. $Y = 3$ for the outcome $\{SSS\}$. $Y = 4$ for the outcomes $\{FSSS\}$. $Y = 5$ for the outcomes $\{SFSSS\}, \{FFSSS\}, \{SFSSS\}$.

6. By looking at a single round at a time, the following events are mutually exclusive: $X = 0$, a tie occurs, $X = 1$ the first player wins, $X = 2$ the second player wins. Because each round is independent, and the only possible outcomes are prefixed with ties, $(\{001\}, \{00002\}, \{002\})$,

it is valid to condition on the fact that the last event was not a tie. Now, let c_i represent the number of ways the i^{th} player can win. To calculate these, I need to know the r.v. that corraspond to the number of heads each player gets. Let X, Y represent the number of head the first and second player get respectivly (since heads and tails is equaly likely and the number of coins tossed is fixed I am counting possible outcomes as opposed to probabilites). $c_1 = X > Y = \binom{3}{3}(2^2) + \binom{3}{2}(2^2 - \binom{2}{2}) + \binom{3}{1}\binom{2}{0} = 16$. And $c_2 = Y > X = \binom{2}{2}(\binom{3}{1} + \binom{3}{0}) + \binom{2}{1}\binom{3}{0} = 6$. I will divide each of those probabilities by the sum of the total and I get: There is a **72.7273%** chance the first player will win and a **27.2727%** chance the second player will win.