Midterm Report: Supply Chain

Ben McGuire

March 7, 2020

Abstract

We are given a supply

1 The Problem

Given the supply chain we are seeking to minimize the cost of shipping ducks to their destinations. The first step in approaching this problem is to translate the supply chain into a network flow diagram.

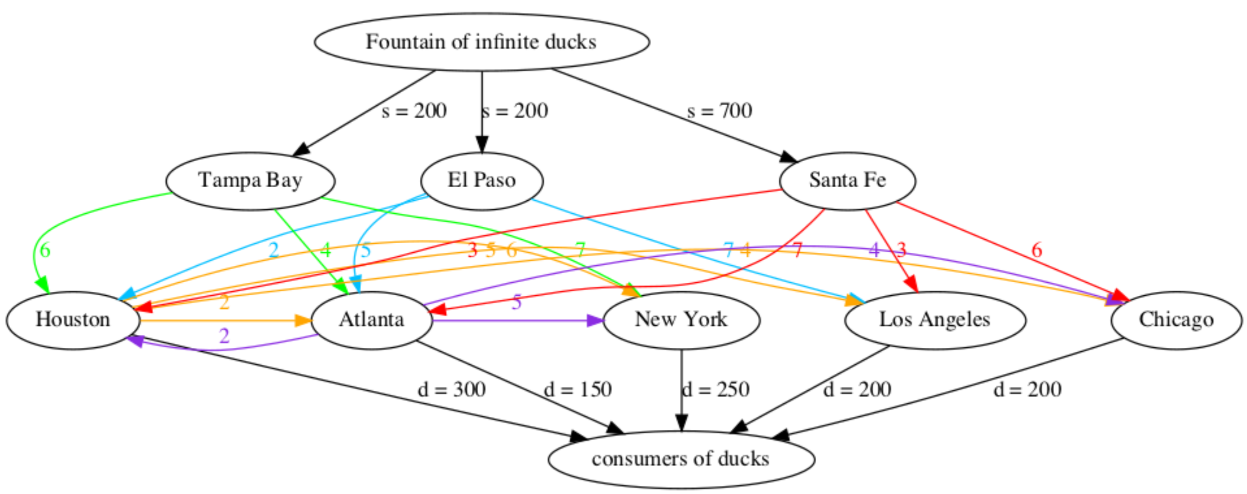

Figure 1: The duck supply chain as a network flow. Each route has it's associated shipping cost as a label

Some remarks about notation. Each city will be abbreviated by the first letter of its name regardless of whether it is a store or warehouse city. The edges from the source to the warehouse cities reflect the quantity of supplied ducks, and will be denoted s_n with n being the name of the city receiving the supply. Similarly, the edges from the store cities to consumers reflect the quantity of ducks consumed, and will be denoted d_n with n being the name of the city receiving the supply. For all other edges between cities, they represents the quantity of ducks sent along the shipping route and will be denoted r_{ij} with i being the sending city and j the receiving.

As the variation of this problem in part 6 will involve changing the quantities of ducks that are consumed and produced, the problem has been structured to include variables representing the number of ducks sent/produced at each warehouse, denoted s_n with n being the receiving warehouse. Additionally number of ducks consumed by each city is denoted d_n with n being the consuming city. In the base case, these are all just set to fixed values instead of allowing them to vary.

2 Developing a Linear Program

The next step is developing a linear program. Since there are no costs of production or varying prices for ducks, this problem is a closed system. There is a supply and demand of 1100 ducks so every duck will be moved to a destination. The only task is minimizing this cost. I set up the objective function in terms of all 25 variables. However, for the initial problem, there is no cost associated with supply and demand. Thus, in this problem are looking to minimize will be sum of all ducks per route times the unit cost of that shipping route.

$$\sum_{i,j} c_{ij} r_{ij}$$

The vectors $x, c \in \mathbb{R}^{25\times 1}$. such that we are minimizing the function $x \cdot c$. For full enumeration see figure 3 and 2 in appendix A.

Next, is to develop the equality constraints. All supply and demand variables now have set values. Thus, the following equality constrains can be developed.

$$s_T = 200 \quad s_E = 200 \quad s_S = 700$$

$$d_H = 300 \quad d_A = 150 \quad d_N = 250 \quad d_L = 200 \quad d_C = 200$$

Additionally, since all ducks eventually reach consumers each city node is conserved. Thus the following equality constraints can be developed:

$$\begin{aligned}
\text{Tampa Bay} &: s_T - r_{TH} - r_{TA} - r_{TN} = 0 \\
\text{El Paso} &: s_E - r_{EH} - r_{EA} - r_{EL} = 0 \\
\text{Santa Fe} &: s_S - r_{SH} - r_{SA} - r_{SL} - r_{SC} \\
\text{Houston} &: r_{TH} + r_{EH} + r_{SH} + r_{AH} - r_{HA} - r_{HN} - r_{HL} - r_{HC} - d_H = 0 \\
\text{Atlanta} &: r_{HA} + r_{TA} + r_{EA} + r_{SA} - r_{AH} - r_{AN} - r_{AC} - d_A = 0 \\
\text{New York} &: r_{TN} + r_{HN} + r_{AN} - d_N = 0 \\
\text{Los Angeles} &: r_{EL} + r_{SL} + r_{HL} - d_N = 0 \\
\text{Chicago} &: r_{SC} + r_{HC} + r_{AC} - d_C = 0
\end{aligned}$$

These equations are enumerated in a matrix $A_{eq} \in \mathbb{R}^{15\times 25}$ and the zero vector $b_{eq} \in \mathbb{R}^{15\times 1}$ such that $A_{eq}x = b_{eq}$. Full enumerations can be seen in figure 4 in appendix A.

Lastly, the inequality conditions need to be set, and those are relatively simple. All routes $r_{i,j} \leq 200$. This gives a matrix $A_{ub} \in \mathbb{R}^{25\times 25}$ and $b_{ub} \in \mathbb{R}^{25\times 1}$ such that $A_{lb}x \leq b_{lb}$. Full enumerations can be found in **INSERT APPENDIX**.

3 Evaluating the Linear Program

In order to evaluate the linear program developed in section 2 we will use the linprog function in Scipy.Optimize. In order to generate the necessary matrices, they were created in Excel and read into the notebook using the code found in **Insert code appendix**. The following optimal shipping plan was found.

Shipping routes

```
r_TH ships  0.0 ducks
r_TA ships  0.0 ducks
r_TN ships  200.0 ducks
r_EH ships  200.0 ducks
r_EA ships  0.0 ducks
r_EL ships  0.0 ducks
r_SH ships  200.0 ducks
r_SA ships  100.0 ducks
r_SL ships  200.0 ducks
r_SC ships  200.0 ducks
```

Relay Routes

```
r_HA ships  50.0 ducks
r_HN ships  50.0 ducks
r_HL ships  0.0 ducks
r_HC ships  0.0 ducks
r_AH ships  0.0 ducks
r_AN ships  0.0 ducks
r_AC ships  0.0 ducks
```

These shipping quantities all fit within the given constraints and gives a minimal cost of supplying the ducks of $5300.

4 Variant: Los Angeles Workers Strike

Workers in Los Angeles are threatening to strike. They demand that the shipping costs of all incoming routes to Los Angeles are doubled or they will strike which will reduce the capacity of all routes into Los Angeles from 200 to 100. I modeled both situations to determine which outcome will incur lower costs.

Demands are met

In order to model this scenario, the initial model was slightly altered. The initial cost function was altered so that the costs of the routes r_{SL}, r_{EL} and r_{HL} were doubled. These modifications can be seen in figure 3 in appendix A. Using linprog to evaluate these new matrices did not change the shipping quantities as LA still receives 200 ducks from Santa Fe which was initially the cheapest inbound route. However because of the increase in cost, the new minimum shipping costs are now \$5900.

There is a strike

In order to model this scenario, the initial model was slightly altered. The initial upper bound(b$_u$b)$functionwasalteredsothatthemaximumcapacityoftheroutes$r$_{SL}, r_{EL}$ and r_{HL} were decreased from 200 to 100. These modifications can be seen in figure 5 in appendix A. Using linprog to evaluate these new matrices did cause shipping quantities to change. They are as follows.

```
r_TH ships  0.0 ducks
r_TA ships  0.0 ducks
r_TN ships  200.0 ducks
r_EH ships  145.89 ducks
r_EA ships  0.0 ducks
r_EL ships  54.11 ducks
r_SH ships  200.0 ducks
r_SA ships  200.0 ducks
r_SL ships  100.0 ducks
r_SC ships  200.0 ducks
r_HA ships  0.0 ducks
r_HN ships  0.0 ducks
r_HL ships  45.89 ducks
r_HC ships  0.0 ducks
r_AH ships  0.0 ducks
r_AN ships  50.0 ducks
r_AC ships  0.0 ducks
```

Because of routes into LA are limited, we can no longer send 200 ducks from Santa Fe meaning that shipping quantities must altered which caused an increase in cost for a new minimum shipping costs of $6050.

Therefore the optimal solution would be to meet the striking worker's demands and increase the cost of shipping to Los Angeles.

5 Variant: Houston Workers Strike

Workers in Houston are threatening to strike. They demand that the shipping costs of all incoming routes to Houston are doubled or they will strike which will reduce the capacity of all routes into Houston from 200 to 100. I modeled both situations to determine which outcome will incur lower costs.

Demands are met

In order to model this scenario, the initial model was slightly altered. The initial cost function was altered so that the costs of the routes r_{SH}, r_{EH}, r_{TH} and r_{AH} are doubled. These modifications can be seen in figure 3 in appendix A. Using linprog to evaluate these new matrices did cause shipping quantities to change. They are as follows.

```
r_TH ships  0.0 ducks
r_TA ships  0.0 ducks
r_TN ships  200.0 ducks
r_EH ships  129.26 ducks
r_EA ships  70.74 ducks
r_EL ships  0.0 ducks
r_SH ships  196.23 ducks
r_SA ships  103.77 ducks
r_SL ships  200.0 ducks
r_SC ships  200.0 ducks
r_HA ships  0.0 ducks
r_HN ships  25.49 ducks
r_HL ships  0.0 ducks
r_HC ships  0.0 ducks
r_AH ships  0.0 ducks
r_AN ships  24.51 ducks
r_AC ships  0.0 ducks
```

In this scenario, Houston now receives slightly fewer ducks than in the initial condition and costs are increased to $6250.

There is a strike

In order to model this scenario, the initial model was slightly altered. The initial upper bound function was altered so that the maximum capacity of the routes r_{SH}, r_{EH}, r_{TH} and r_{AH} are decreased from 200 to 100. These modifications can be seen in figure 5 in appendix A. Using linprog to evaluate these new matrices did cause shipping quantities to change. They are as follows.

```
r_TH ships  0.0 ducks
r_TA ships  0.0 ducks
r_TN ships  200.0 ducks
r_EH ships  100.0 ducks
r_EA ships  100.0 ducks
r_EL ships  0.0 ducks
r_SH ships  100.0 ducks
r_SA ships  200.0 ducks
r_SL ships  200.0 ducks
r_SC ships  200.0 ducks
r_HA ships  0.0 ducks
r_HN ships  0.0 ducks
r_HL ships  0.0 ducks
r_HC ships  0.0 ducks
r_AH ships  100.0 ducks
r_AN ships  50.0 ducks
r_AC ships  0.0 ducks
```

Because of routes into Houston are limited, we can no longer send 200 ducks from Santa Fe and El Paso meaning that shipping quantities must be altered which cause an increase in cost for a new minimum shipping costs of $6050.

Since Houston has a larger demand for ducks than Los Angeles, this outcome makes sense as an increase in shipping costs would apply to larger quantities of ducks. Therefore the cheapest option would be to allow the workers to strike.

6 Variant: Profit maximization

As the variation of this problem in part 6 will involve changing the quantities of ducks that are consumed and produced as there are now costs associated with production and consumption of ducks. The s and d variables take on a slightly different meaning. s_n now corresponds to the the number of ducks produced at warehouse n, which is bounded by the maximum capacity of the warehouse. d_n represents the number of ducks consumed by city n and is bounded above by the demand of the city. The initial model is changed to reflect these changes. First, the equality constraints on the supply from each warehouse and consumption for each city were changed to be inequalities. For example Santa Fe can store and ship up to 700 ducks but can produce less. New York will consume up to 250 ducks, but will still consume lesser quantities. This caused changes in A_{ub}, b_{ub}, A_{eq} and b_{eq} which can be seen in appendix A. Lastly the cost function needed to change. Now, there was a cost associated with all supply demand variables. The supply and shipping costs were negative and the demand costs were positive. These matrices were then evaluated using linprog and the following results were returned.

```
warehouse s_T produces  50.0 ducks
warehouse s_E produces  200.0 ducks
warehouse s_S produces  400.0 ducks
r_TH  ships  0.0 ducks
r_TA  ships  0.0 ducks
r_TN  ships  50.0 ducks
r_EH  ships  200.0 ducks
r_EA  ships  0.0 ducks
r_EL  ships  0.0 ducks
r_SH  ships  0.0 ducks
r_SA  ships  0.0 ducks
r_SL  ships  200.0 ducks
r_SC  ships  200.0 ducks
r_HA  ships  0.0 ducks
r_HN  ships  200.0 ducks
r_HL  ships  0.0 ducks
r_HC  ships  0.0 ducks
r_AH  ships  0.0 ducks
r_AN  ships  0.0 ducks
r_AC  ships  0.0 ducks
city d_H receives 0.0 ducks
city d_A receives 0.0 ducks
city d_N receives 250.0 ducks
city d_L receives 200.0 ducks
city d_C receives 200.0 ducks
```

[2em]2em This gives a total profit of $4800. This shipping plan is consistent with all new constraints. No warehouse is producing more than their capacity and no city is receiving

more ducks than their demand. Additionally, the cities with the highest prices for ducks (NY, LA, CHI) receive ducks when the cities with the lowest prices (HOU, ATL) as it is more profitable for those relay cities to ship their ducks to high paying cities rather than consume them there.

A Appendix: Full matrices for base problem

s_T	s_E	s_S	r_TH	r_TA	r_TN	r_EH	r_EA	r_EL	r_SH	r_SA	r_SL	r_SC	r_HA	r_HN	r_HL	r_HC	r_AH	r_AN	r_AC	d_H	d_A	d_N	d_L	d_C

Figure 2: Vector of routes, with the order used for all calculations

	s_T	s_E	s_S	r_TH	r_TA	r_TN	r_EH	r_EA	r_EL	r_SH	r_SA	r_SL	r_SC	r_HA	r_HN	r_HL	r_HC	r_AH	r_AN	r_AC	d_H	d_A	d_N	d_L	d_C
c_ij	0	0	0	6	4	7	2	5	7	3	7	3	6	2	6	5	4	2	5	4	0	0	0	0	0
c_ny	0	0	0	6	4	7	2	5	14	3	7	6	6	2	6	10	4	2	5	4	0	0	0	0	0
c_hou	0	0	0	12	4	7	4	5	7	6	7	3	6	2	6	5	4	4	5	4	0	0	0	0	0
c_p	-10	-5	-8	-6	-4	-7	-2	-5	-7	-3	-7	-3	-6	-2	-6	-5	-4	-2	-5	-4	10	10	25	20	15

Figure 3: Horizontal vectors of costs corresponding to: the base problem, the Los Angeles workers strike, the Houston worker strike, and to the profit function

r_ij	s_T	s_E	s_S	r_TH	r_TA	r_TN	r_EH	r_EA	r_EL	r_SH	r_SA	r_SL	r_SC	r_HA	r_HN	r_HL	r_HC	r_AH	r_AN	r_AC	d_H	d_A	d_N	d_L	d_C	constraints
sup_T	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200
sup_E	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200
sup_S	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	700
Tampa Bay	1	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
El Paso	0	1	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Santa Fe	0	0	1	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
Houston	0	0	0	1	0	0	1	0	0	1	0	0	0	-1	-1	-1	-1	0	0	0	-1	0	0	0	0	0
Atlanta	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	-1	-1	-1	0	-1	0	0	0	0
New York	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	0
Los Angeles	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	-1	0	0
Chicago	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	-1	0
d_H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	300
d_A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	150
d_N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	250
d_L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	200
d_C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	200

Figure 4: $A_e q$ and $b_e q$ for the base problem

	s_T	s_E	s_S	r_TH	r_TA	r_TN	r_EH	r_EA	r_EL	r_SH	r_SA	r_SL	r_SC	r_HA	r_HN	r_HL	r_HC	r_AH	r_AN	r_AC	d_H	d_A	d_N	d_L	d_C
b_ub	0	0	0	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	0	0	0	0	0
b_ub_LA	0	0	0	200	200	200	200	200	100	200	200	100	200	200	200	100	200	200	200	200	0	0	0	0	0
b_ub_HOU	0	0	0	100	200	200	100	200	200	100	200	200	200	200	200	200	200	100	200	200	0	0	0	0	0
b_ub_p	0	0	0	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	300	150	250	200	200

Figure 5: Horizontal vectors of upper bounds on each variable corresponding to: the base problem, the Los Angeles workers strike, the Houston worker strike, and to the profit function

B Appendix: Code used in Base problem