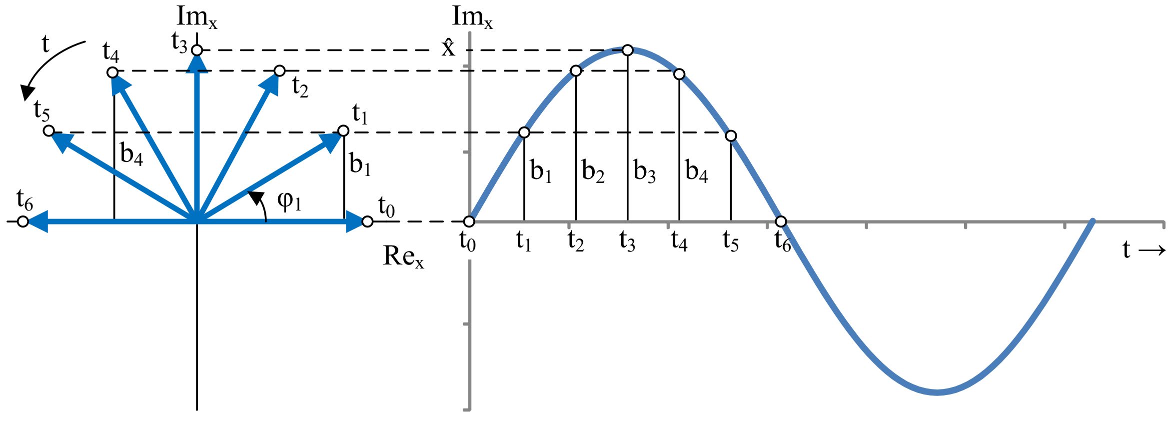
Handreichung zum Praktikumsversuch

Frequenzganganalyse



$$a_k = \hat{x} \cdot cos\omega t_k \qquad b_k = \hat{x} \cdot sin\omega t_k$$

Abbildung 1: Harmonische Schwingung in Zeigerbild und Zeitfunktion

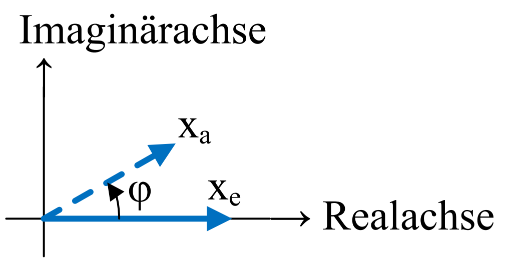


Abbildung 2: Phasenverschiebung φ von x_a, bezogen auf x_e

$e^{j\varphi} = cos\varphi + j \cdot sin\varphi$ (Eulersche Formel)

$cos^2\varphi + sin^2\varphi = 1$

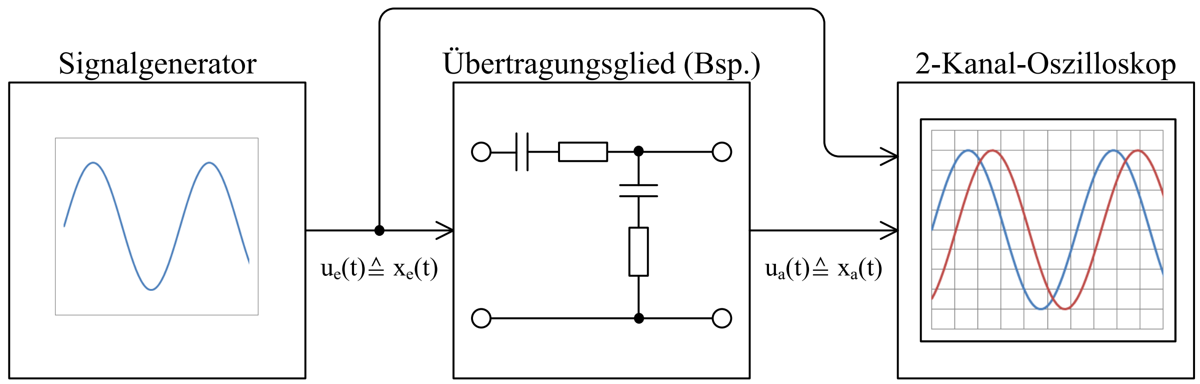


Abbildung 3: Messprinzip zur Frequenzganganalyse

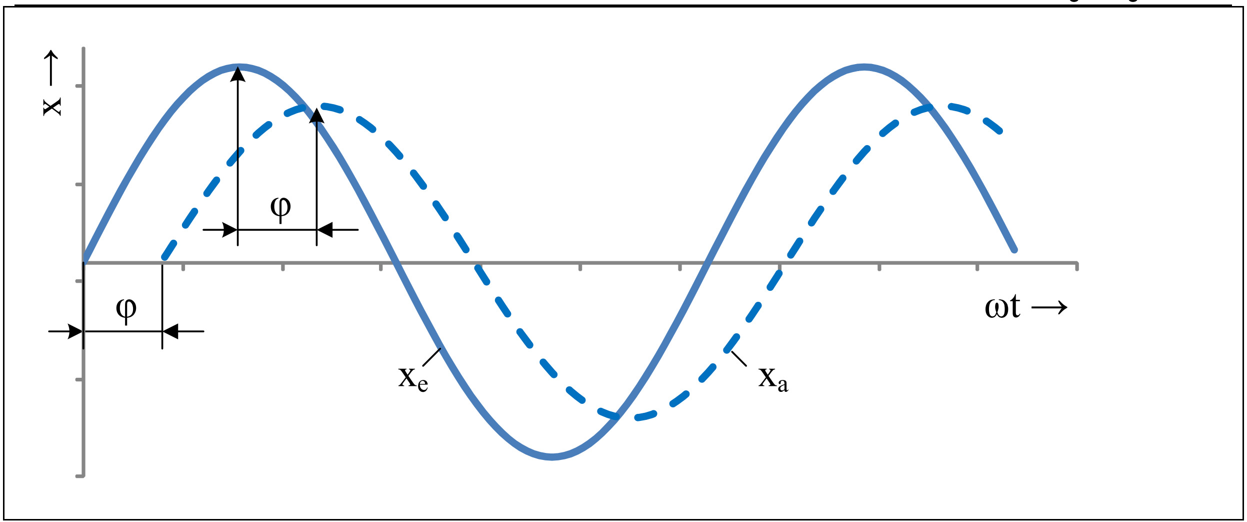


Abbildung 4: Verlauf der Signalgrößen $x_e(t)$ und $x_a(t)$

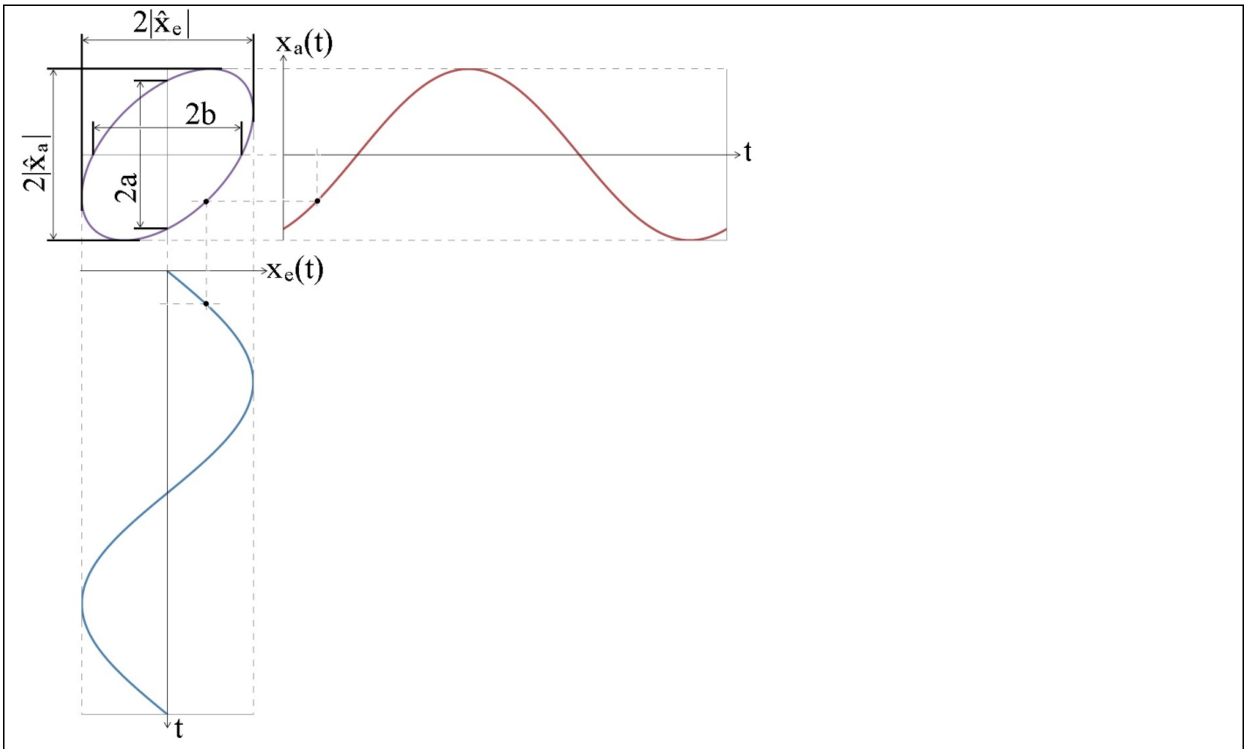


Abbildung 5: Entstehung der Lissajous-Figur (Ellipse) auf dem Oszilloskopbildschirm

Grafische Darstellung des Frequenzgangs:

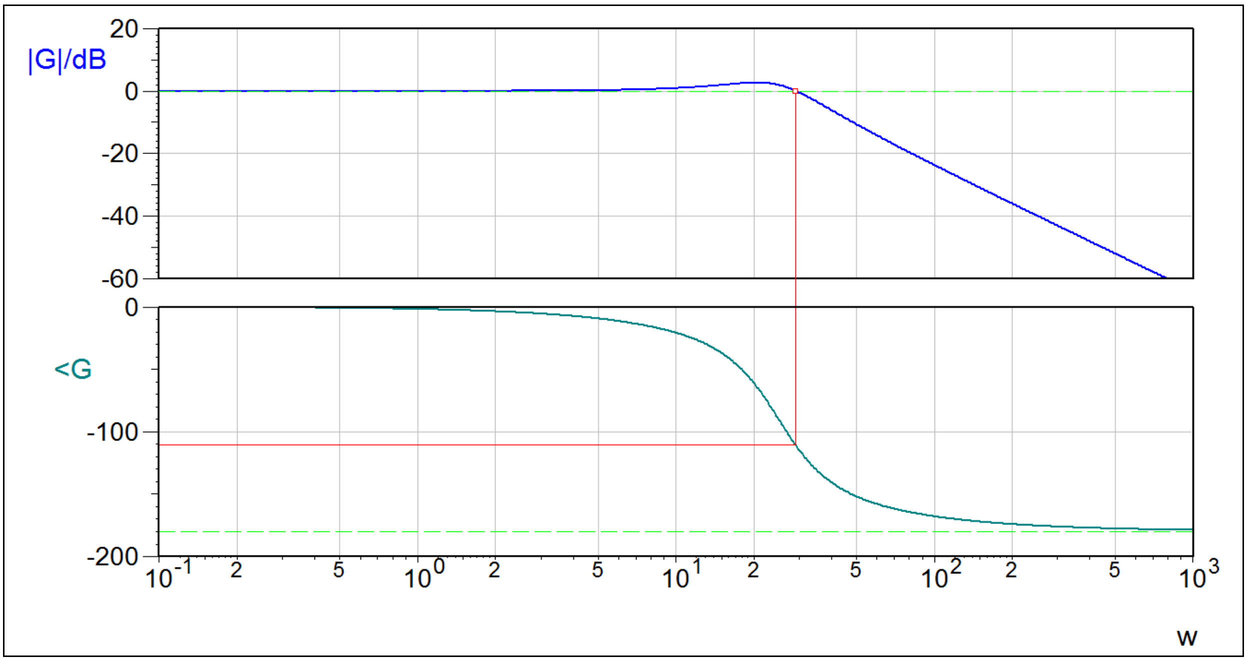


Abbildung 6: Bode-Diagramm mit Amplitudengang (oben) und Phasengang (unten)

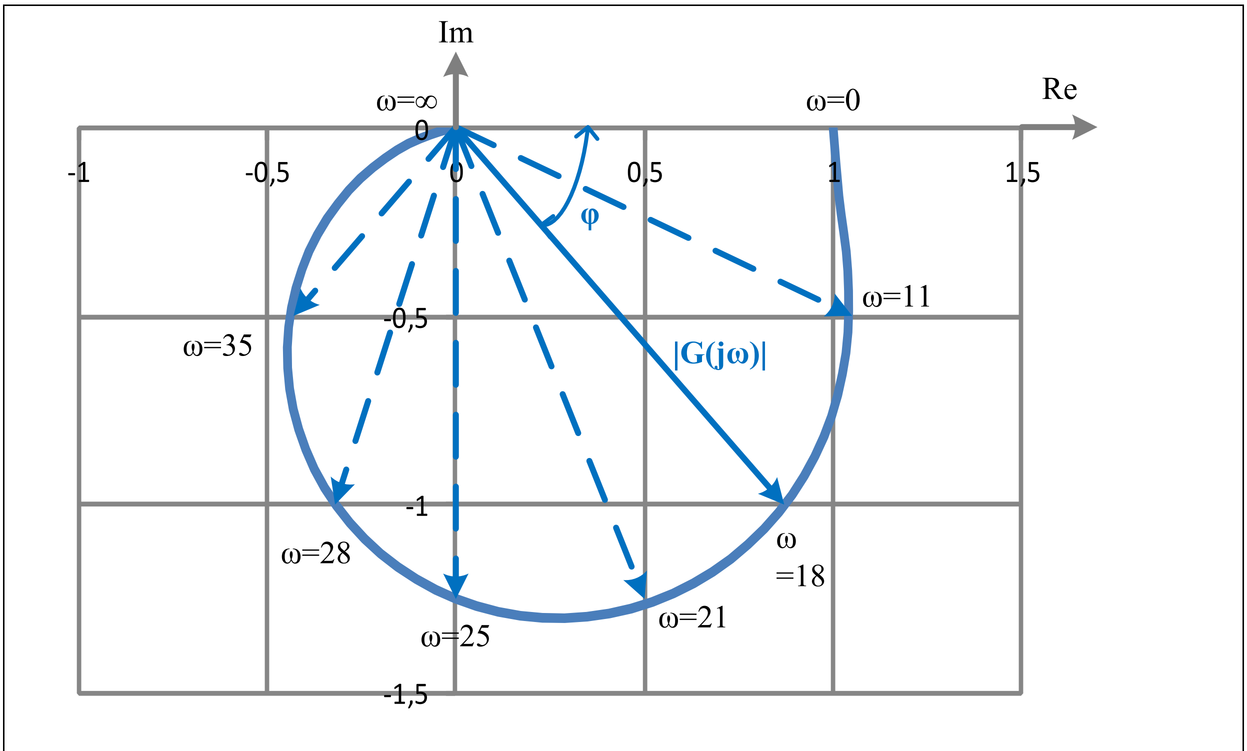


Abbildung 7: Ortskurve

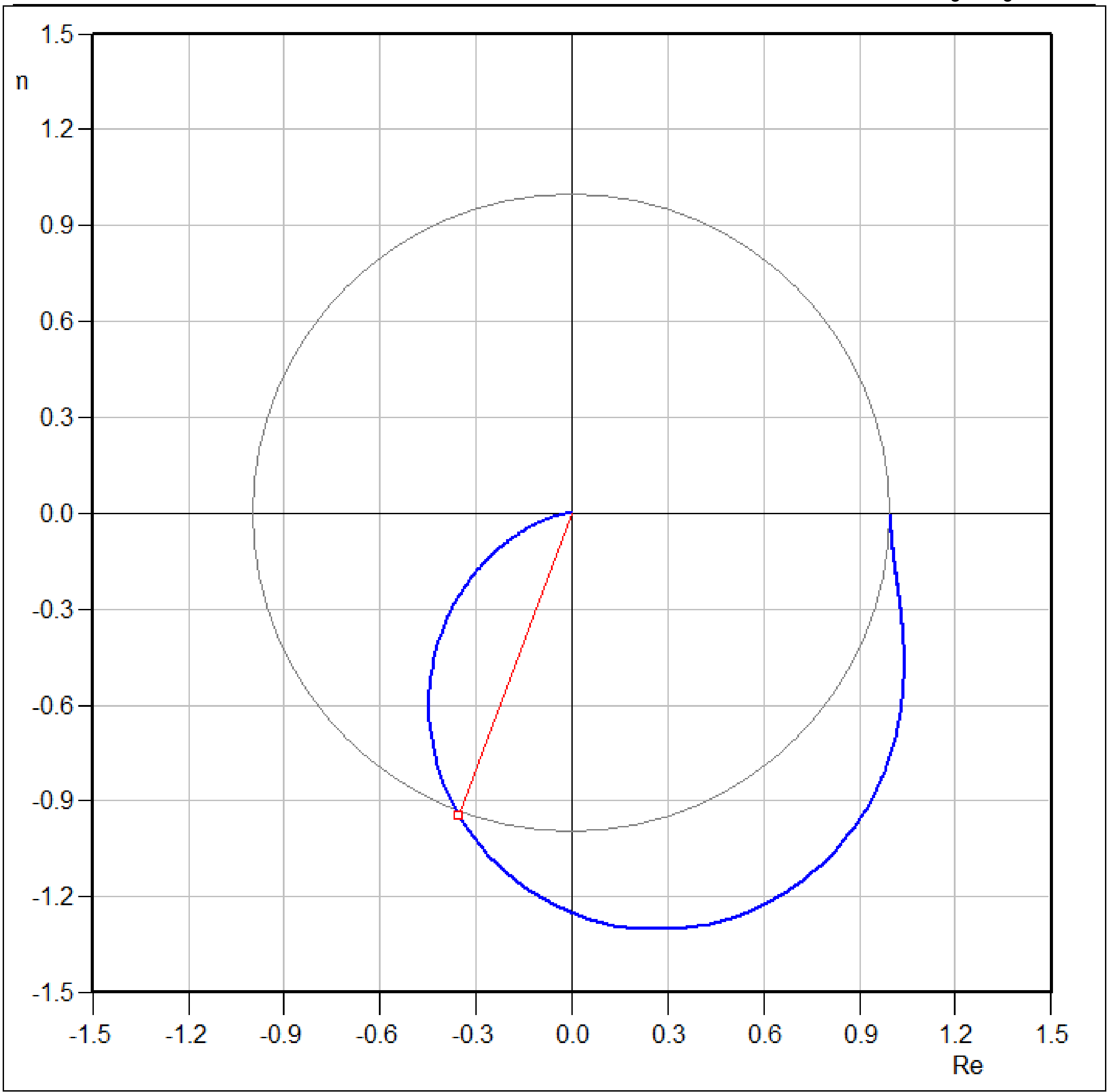
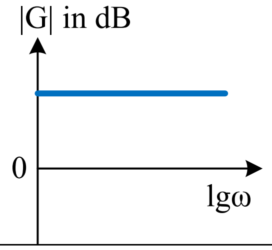
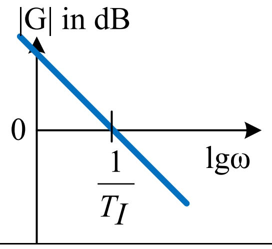
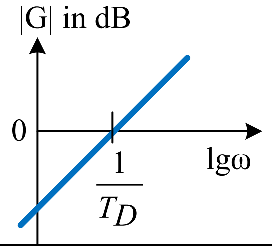
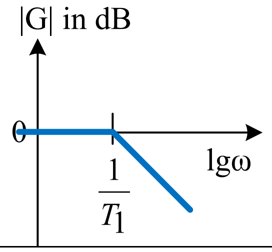
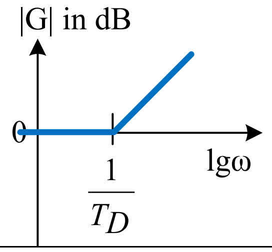


Abbildung 8: Ortskurve mit Einheitskreis und Hilfslinie

Tabelle 1: Geradenapproximation des Amplitudenganges für einfache Übertragungsglieder

<table>
<tr><th>Systemverhalten</th><th>Approximierter Amplitudengang</th></tr>
<tr><td>P-Verhalten
$G(j\omega) = K$</td><td></td></tr>
<tr><td>I-Verhalten
$G(j\omega) = \frac{1}{j\omega T_I}$</td><td></td></tr>
<tr><td>D-Verhalten
$G(j\omega) = j\omega T_D$</td><td></td></tr>
<tr><td>T$_1$-Verhalten
$G(j\omega) = \frac{1}{1 + j\omega T_1}$</td><td></td></tr>
<tr><td>PD-Verhalten
$G(j\omega) = 1 + j\omega T_D$</td><td></td></tr>
</table>

Konstruktion logarithmischer Frequenzgänge:

Beispiel: Gegeben sei folgende Frequenzganggleichung:

$$G(j\omega) = \frac{1+j\omega T_D}{(1+j\omega T_1)\cdot(1+j\omega T_2)} \quad \text{mit} \quad T_D = 0{,}5s; \quad T_1 = 2{,}5s; \quad T_2 = 0{,}025s$$

→ Zerlegung von $G(j\omega)$ in Faktoren:

$$G(j\omega) = (1 + j\omega T_D) \cdot \frac{1}{1+j\omega T_1} \cdot \frac{1}{1+j\omega T_2} = G_1(j\omega) \cdot G_2(j\omega) \cdot G_3(j\omega)$$

$$\text{mit} \quad G_1(j\omega) = 1 + j\omega T_D; \qquad G_2(j\omega) = \frac{1}{1+j\omega T_1}; \qquad G_3(j\omega) = \frac{1}{1+j\omega T_2}$$

→ Logarithmierung der zerlegten Frequenzganggleichung

$$lg[G(j\omega)] = lg[G_1(j\omega) \cdot G_2(j\omega) \cdot G_3(j\omega)] = lgG_1(j\omega) + lgG_2(j\omega) + lgG_3(j\omega)$$

$$\boxed{lg[G(j\omega)] = lg(1 + j\omega T_D) + lg\,\frac{1}{1+j\omega T_1} + lg\,\frac{1}{1+j\omega T_2}}$$

→ $\omega_{e1} = \frac{1}{T_D} = 2s^{-1}$; $\omega_{e2} = \frac{1}{T_1} = 0{,}4s^{-1}$; $\omega_{e3} = \frac{1}{T_1} = 40s^{-1}$

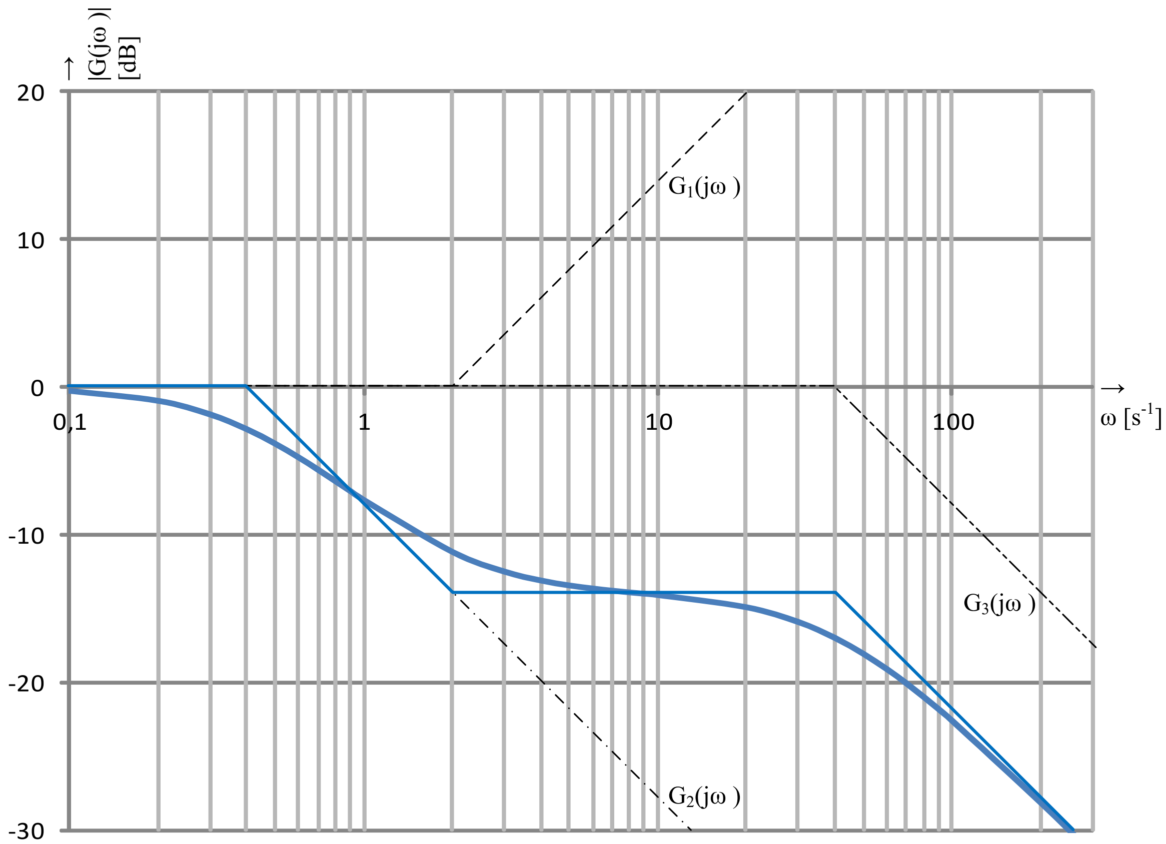
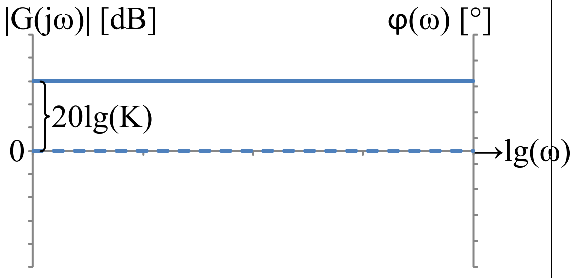
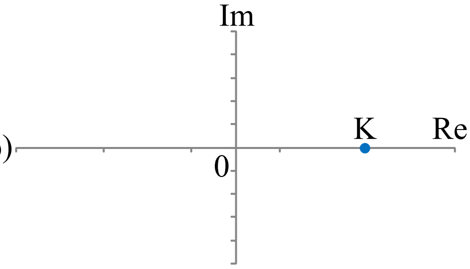
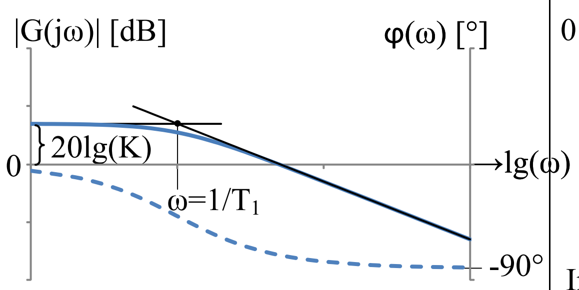
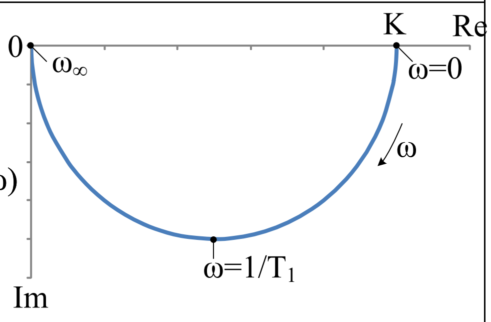
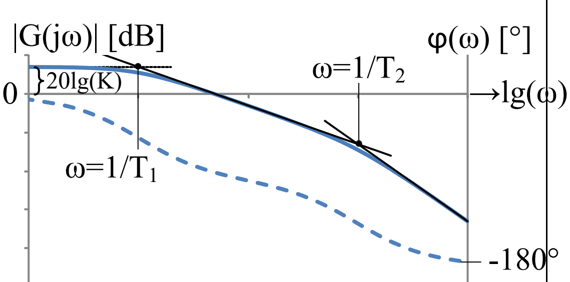
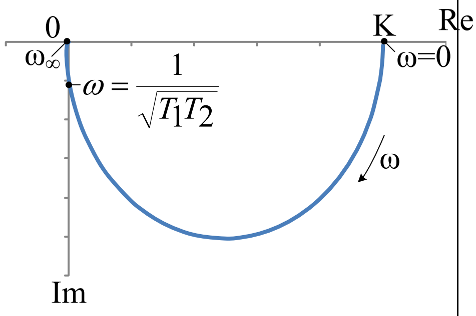
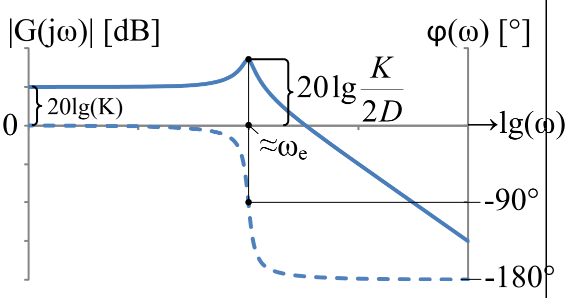
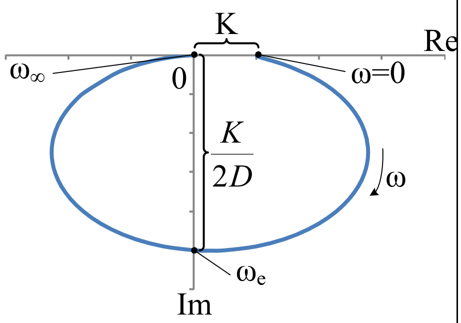


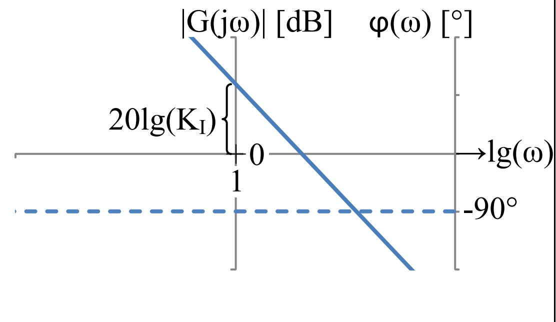
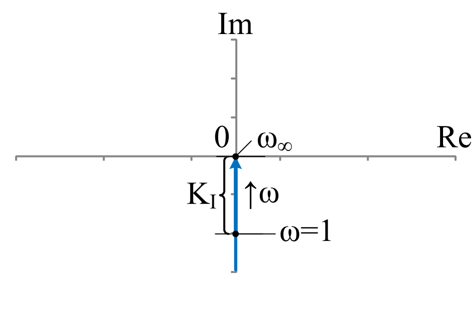
Abbildung 9: Konstruktion des logarithmischen Amplitudenganges (grafische Addition)

Beachte: Geradenanstieg a der Asymptoten im logarithmischen Amplitudengang immer:

$$a = \frac{[dB]}{[Dec]} = k \cdot \frac{20dB}{Dec} \text{ mit } k \in \mathbb{N}; (Dec \ldots \text{Dekade})$$

Tabelle 2: Übertragungsverhalten linearer Glieder (Ortskurve und Bode-Diagramm)

<table>
<tr><th>Bezeichnung
$[G(j\omega) =]$</th><th>Bode-Diagramm
Amplitudengang
Phasengang</th><th>Ortskurve
Im[G(jω)]=f{Re[G(jω)]}</th></tr>
<tr><td>P-Glied
K</td><td>
</td><td>
</td></tr>
<tr><td>P-T$_1$-Glied
$\frac{K}{1+j\omega T_1}$</td><td>
</td><td>
</td></tr>
<tr><td>P-T$_2$ (N)-Glied
$\frac{K}{(1+j\omega T_1)\cdot(1+j\omega T_2)}$</td><td>
</td><td>
</td></tr>
<tr><td>P-T$_2$ (S)-Glied
$\frac{K}{1+j\omega\frac{2D}{\omega_e}+(j\omega)^2\frac{1}{\omega_e^2}}$</td><td>
</td><td>
</td></tr>
</table>

<table>
<tr><td>I-Glied
$\frac{K_I}{j\omega}$</td><td>
</td><td>
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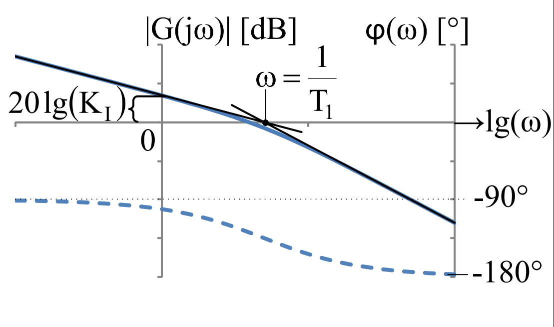
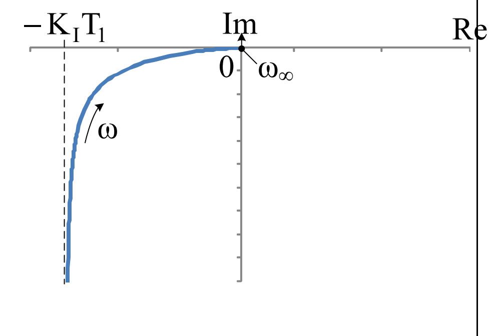
<tr><td>I-T1-Glied
$\frac{K_I}{j\omega(1+j\omega T_1)}$</td><td>
</td><td>
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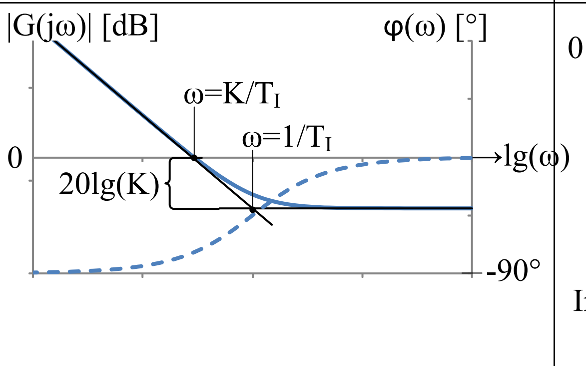
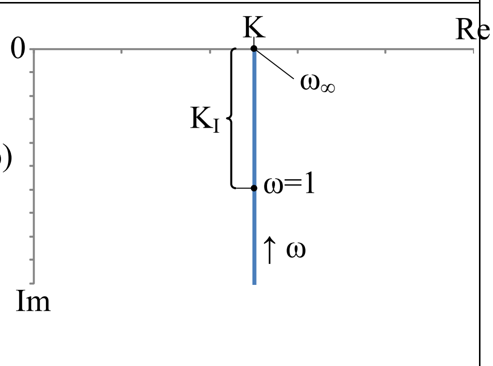
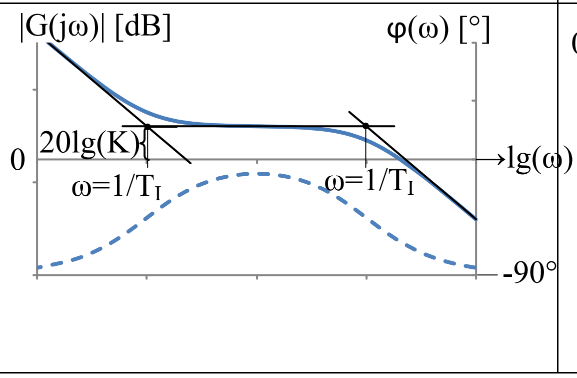
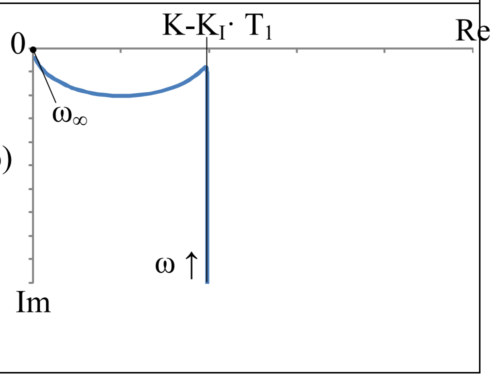
<tr><td>PI-Glied
$K\left(1+\frac{1}{j\omega T_I}\right)$</td><td>
</td><td>
</td></tr>
<tr><td>PI-T1-Glied
$\frac{K\left(1+\frac{1}{j\omega T_I}\right)}{1+j\omega T_1}$</td><td>
</td><td>
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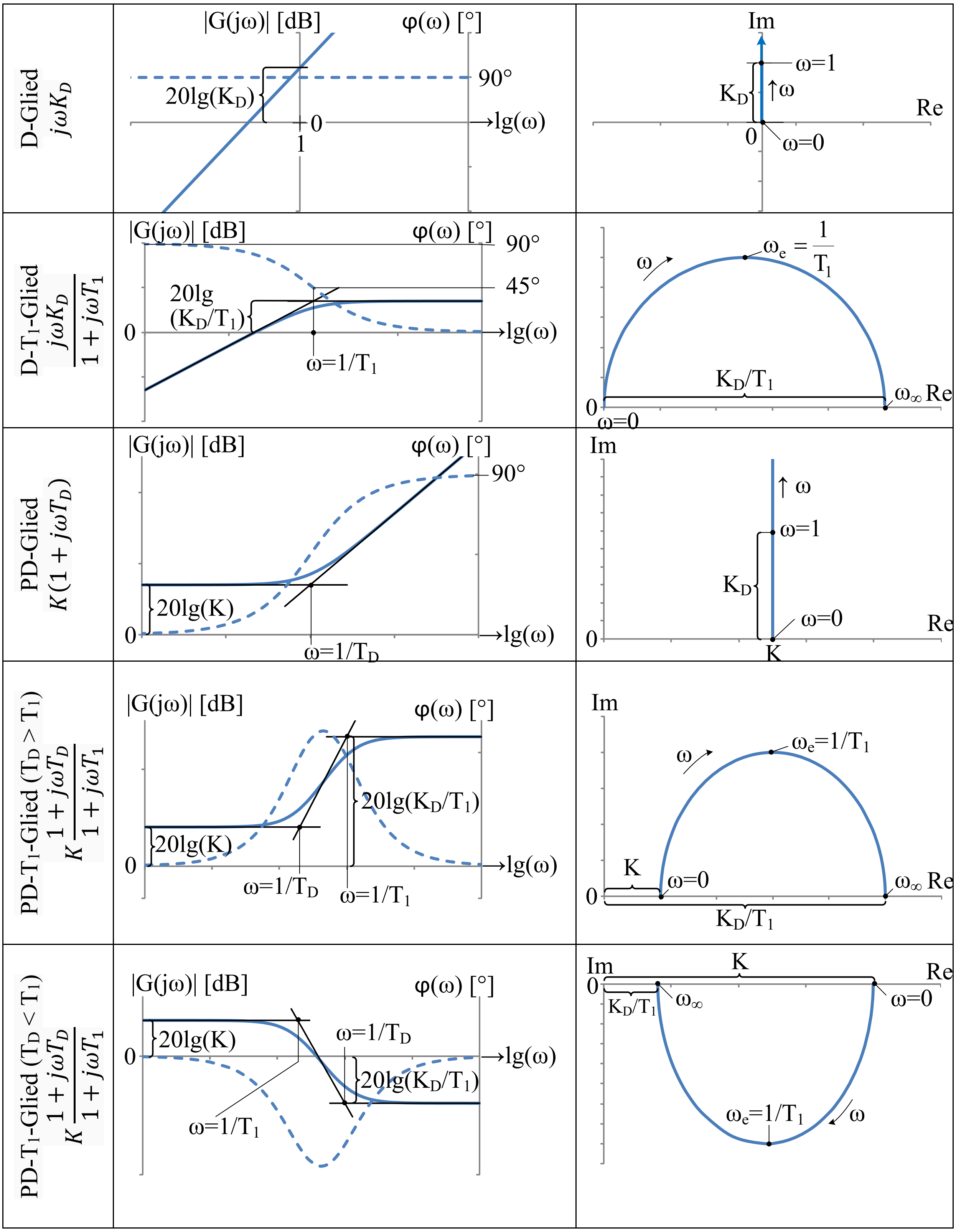
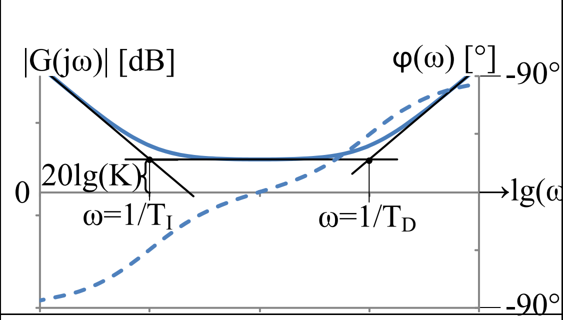
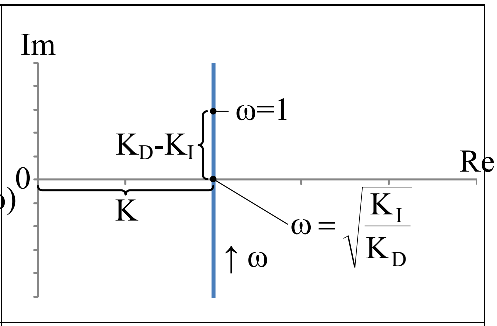
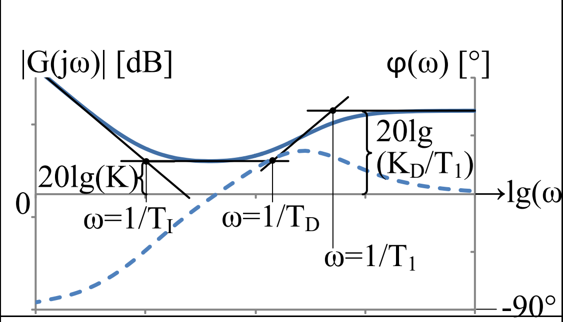
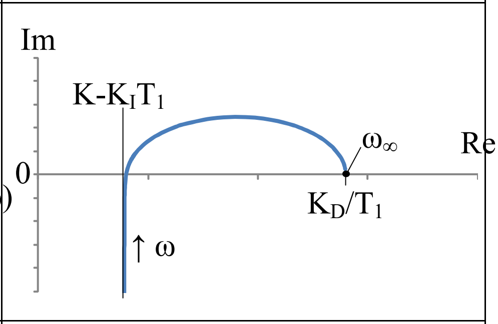

D-Glied
$j\omega K_D$
|G(jω)| [dB]
φ(ω) [°]
90°
20lg(K_D)
0
1
→lg(ω)
Im
ω=1
K_D
↑ω
Re
0
ω=0
D-T_1-Glied
$\frac{j\omega K_D}{1+j\omega T_1}$
|G(jω)| [dB]
φ(ω) [°]
90°
45°
20lg
(K_D/T_1)
0
→lg(ω)
ω=1/T_1
ω
$\omega_e = \frac{1}{T_1}$
K_D/T_1
0
ω_∞ Re
ω=0
PD-Glied
$K(1+j\omega T_D)$
|G(jω)| [dB]
φ(ω) [°]
90°
20lg(K)
0
→lg(ω)
ω=1/T_D
Im
↑ ω
ω=1
K_D
ω=0
Re
0
K
PD-T_1-Glied ($T_D > T_1$)
$K\frac{1+j\omega T_D}{1+j\omega T_1}$
|G(jω)| [dB]
φ(ω) [°]
20lg(K_D/T_1)
20lg(K)
0
→lg(ω)
ω=1/T_D
ω=1/T_1
Im
ω
ω_e=1/T_1
K
ω=0
0
ω_∞ Re
K_D/T_1
PD-T_1-Glied ($T_D < T_1$)
$K\frac{1+j\omega T_D}{1+j\omega T_1}$
|G(jω)| [dB]
φ(ω) [°]
20lg(K)
ω=1/T_D
0
→lg(ω)
20lg(K_D/T_1)
ω=1/T_1
Im
K
Re
0
K_D/T_1
ω_∞
ω=0
ω_e=1/T_1
ω

<table>
<tr><td>PID-Glied
$K\left(1+\frac{1}{j\omega T_I}+j\omega T_D\right)$</td><td>
</td><td>
</td></tr>
<tr><td>PID-T₁-Glied
$K\frac{\left(1+\frac{1}{j\omega T_I}+j\omega T_D\right)}{1+j\omega T_1}$</td><td>
</td><td>
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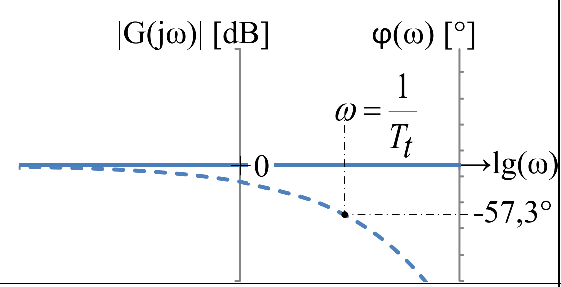
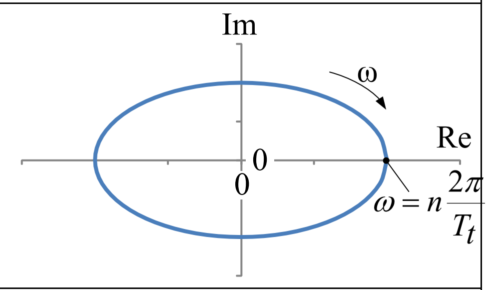
<tr><td>T_t -Glied
$e^{-j\omega T_t}$</td><td>
</td><td>
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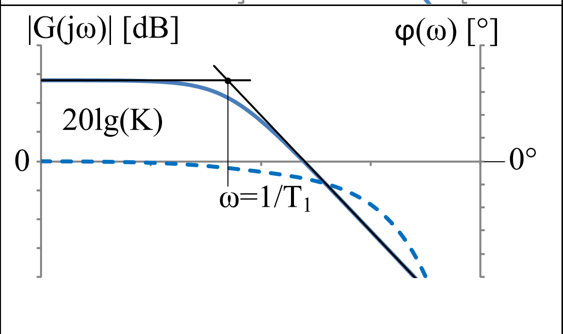
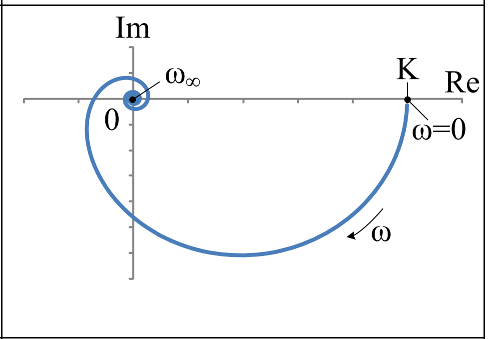
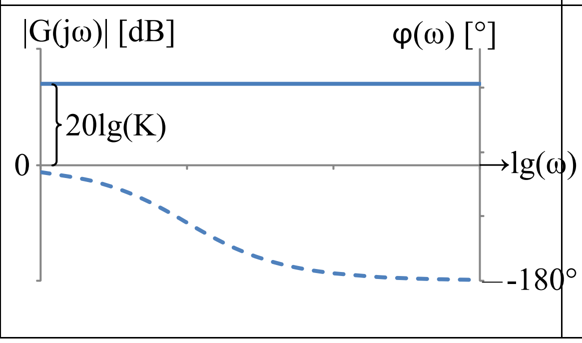
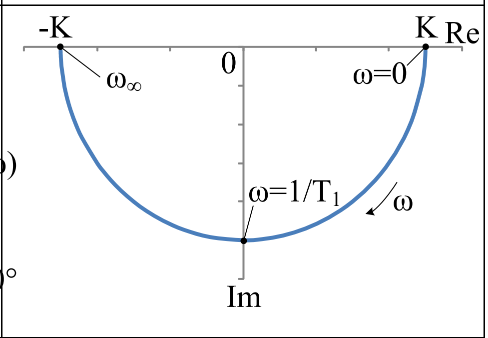
<tr><td>P-T₁-T_t-Glied
$K\cdot\frac{e^{-j\omega T_t}}{1+j\omega T_1}$</td><td>
</td><td>
</td></tr>
<tr><td>Allpass-Glied
$K\frac{1-j\omega T_1}{1+j\omega T_1}$</td><td>
</td><td>
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