# The Assessment of Ordinary and Universal Kriging in Estimating Rainfall distribution Computed on R Studio (A Case study of Switzerland)

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## Abstract

Kriging is a powerful geo-statistical method of spatial interpolation applied in estimation and prediction of continuous data. Kriging was pionnered by Danie Krige, a South African mining engineer from university of the Witwatersrand, South Africa in the 1950s. Initially, Kriging was developed for geological and mining applications. However, George Matheron advanced Kriging beyond mining to solve spatial phenomenas in environmental monitoring such as interpolation for temperatures, precipitation or air pollution. An accurate estimation of rainfall distribution is important for hydrological and water resources management, various spatial interpolations techniques are used to estimate the values of unknown areas using known points/ values. The objective of this project is to test the efficacy of two geo-statistical techniques (Ordinary and Universal Kriging) in estimating rainfall distribution across Switzerland. The historical rainfall records released May 1986 are used for the analysis. Cross validation is adopted to assess the performance of the two geo-statistical techniques. The results obtained from this project indicates that the universal kriging outperformed the ordinary kriging for estimating swiss rainfall. The difference is little in these two spatial interpolation technique, however it is suggested that for better accuracy estimations, digital elevation model should be added as a supplementary data.

KEYWORDS: Kriging, Cross-validation, Geostatistical, variogram models. Spherical.

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# Chapter 1

# Introduction

Climate change is imminent and the effects are already felt globally, fluctuating temperatures, intensity of extreme weather conditions and changing precipitation patterns [6]. Accurate estimation of spatial distribution of rainfall is important in climate change studies, hydrological modeling and water resource management [1]. The rainfall data which is a continuous, point data is often collected using rain gauges, however estimation of rainfall using rain gauges offers uncertainties and presenting challenges for runoff discharge prediction [7]. Indirect rainfall estimates methods such as remote sensing devices and ground-based meteorological RADARs have been employed for hydrological modeling since the late 1960s [7]. The generation of continuous precipitation predictions using indirect estimates methods needs to be fully studied, calibrated and validated using rainfall historical data [7]. The alternative to ground-based measurements, is a geo-statistical or spatial interpolation techniques. A South African mining engineer Danie Krige is a pioneer of geo-statistics, and in the 1970s French mathematician George Matheron coined the term "kriging" using the last name of Danie G. Kinge and formalised Kriging method and advanced it to be applicable beyond mining.

Geostatistics is an important part of spatial statistics, it involves the computations of mathematics and earth sciences [7]. George Matheron and team at the Fontainebleau Ecole des Mines refined the methodologies to be applicable in environmental monitoring, epidemiology and other fields. In reality, sampling the whole geographic location (domain) is often challenging, costly and time consuming. Spatial interpolation is often used to infer spatial information about locations not sampled using values from the observed or sampled locations. Spatial interpolation techniques are classified into two categories namely: deterministic (Inverse Distance Weighting "IDW", Thiessen polygon) and geostatistical (ordinary kriging, universal kriging, co-kriging and simulation). The deterministic and geo-statistical interpolation techniques are well employed in spatial interpolation of rainfall estimations, however geo-statistics techniques gained popularity over deterministic.

Kriging is a geo-spatial interpolation technique named after South African mining engineer sir Danie G. Krige in 1950s. Kriging is an important part of spatial statistics, and it has then become a powerful tool in estimating values at unsampled locations within the study region using values obtained from sampled location. Kriging takes into account the degree of spatial autocorrelation and spatial predictions uncertainties. One of the fundamental concepts driving Kriging is the degree of spatial autocorrelation, which means nearby locations will have similar values than distant locations. This is in line with Tobler's first law of Geography. Therefore autocorrelation is fundamental in modeling spatial predictions. The most important component of Kriging is semivariogram which quantifies the degree of spatial autocorrelation between measured sample points and the strength of correlation as a function of distance [5]. The variogram model is a mathematical representation of the variogram, it visualizes how the variance between observed sample points changes with distance. Types of Kriging involves simple kriging (SK), ordinary kriging (ORK) and universal kriging (UKR). Kriging is a flexible tool that can be implemented in R or various GIS software solving spatial phenomenas in mining, geology and environmental.

### 1.0.1 Problem statement

There is a vast publication in comparison of different spatial interpolation techniques in various disciplines such as mining and geology, however there is a limited research in application of spatial interpolations in meteorological studies. To estimate rainfall patterns at unsampled locations it is hindered by costs and accessibility, therefore it calls for more attention on application of spatial interpolation procedures in climate studies.To this date it is still unclear which interpolation technique is best and what are the factors affecting the performance of different spatial interpolation techniques. It is as well difficult to choose spatial interpolators for climate data, as the results do not only depend on the interpolator rather it requires knowledge on the data to be interpolated and the point sampling density and distribution [2]. Thus, it is with this project to continue reflecting from previous studies in finding the suitable spatial interpolation techniques for estimating annual rainfall pattern over a country.

### 1.0.2 Research Aim and objective

The primary aim of this project is to supplement on the existing literature on assessing the performance of geo-spatial interpolation techniques in precipitation estimations.

The objective is to present the analysis on the efficacy of ordinary (OK) and universal kringing (UK), making use of spherical semivariogram model in estimating Switzerland rainfall distribution, implemented in R studio.

### 1.0.3 Significance of the study

The preliminary results from this project will be of great importance for hydrologists and environmental decision makers. This project is very important to identify and fill in the gap of using geo-spatial interpolation techniques in climate studies, particularly identifying the best suitable geo-spatial interpolator in accurately mapping the spatial distribution of rainfall at a country level.

### 1.0.4 Scope of the project

This paper is structured into five chapters. An introduction in chapter 1, which introduces the topic, the aims and objectives of this project. Followed by chapter 2 Literature survey which reviews the published literature on the application of spatial interpolations in mapping rainfall distributions. The methodology chapter 3 provides an overview of methods and techniques followed in order to meet the objectives of this project, the data and software used. The discussion chapter 4 serves to analyse the results obtained and then chapter 5 is the conclusion summarizes the project and offer limitations and recommendations that can be implemented in future research purposes.

The R codes outputs are provided. PDF

# Chapter 2

# Literature survey

This section presents the review of previous published literature on application of spatial interpolation techniques in estimating point precipitation data.

Rainfall is an important variable for hydrological simulation models for the management of water resources at different spatial and temporal scales. The availability of uninterrupted continuous precipitation point data is crucial for the reconstruction of historical climate and hydrological conditions of a particular location, on the basis of hydrological design, modeling and water resources management [8]. Previously, Hydrologists depended on precipitation data collected from rain-gauge networks and surrounding weather stations. However, in situations where rain-gauge networks are sparse and few available weather stations often restricted by budgets and logistics increases uncertainties in rainfall distributions. This has often led to level of error and uncertainties in hydrological modeling outputs.

The fluctuations in climatic parameters can be partially measured using point data from irregular placed rain-gauge networks. Although there is increasing application of point data sources in accurately representing climatic conditions, it is often restricted by budgets [5]. This highlights the necessity of employing geo-spatial interpolation techniques which are capable of addressing problems associated with spatially restricted climatic data. The estimation of data for points in geo-locations not sampled nor recorded using known information from nearby points is termed Interpolation. Spatial interpolation is divided in two classes: deterministic and geostatistical algorithms. The commonly used deterministic spatial interpolations are Inverse Distance Weighting (IDW) and Thiessen polygons (THP).

Geostatistics a long standing discipline integrating mathematics and earth sciences, it is deep rooted in application of mining and geological purposes. It was initially developed for predicting of mineral ore distributions during mining operations (Krige 1994), and it has undergone significant methodological developments [5]. Sir George Matheron formalized Geostatistics principles and enhancing them and their applicability beyond mining to include environmental monitoring and epidemological studies. Geostatistics is more concerned with spatial-temporal datasets, but it can go beyond solving basic interpolation problems, as it takes into account the phenomena under investigation at unknown locations as a set of correlated random variables [5]. Kriging is a geo-statistical interpolation technique came into effect in the 1950s, as a work of South African mining engineer Danie Krige. The technique integrates the spatial information from the observed data to infer information or predict values at unobserved location [3]. There are different families of Kriging including but not limited to, simple kriging (SK), ordinary kriging (OK), universal kriging (UK) and Co-Kriging.

In the last decades, the employment of Kriging spatial interpolation technique beyond mining has gained popularity. It has been applied in environmental monitoring of air pollution, ocean hydrocarbons spillages and mapping rainfall distribution. A study by Bostan et al (2012) used five interpolation techniques: multiple linear regression (MLR), ordinary kriging (ORK), regression kriging (RK), universal kriging (UNK) and geographically weighted regression (GWR) were tested to map annual rainfall distribution of Turkey from 1970 to 2006 measured at 225 meteorological stations fairly distributed across the country. The results obtained showed that MLR, GWR and RK outperformed UNK and ORK [3].

Another study by Jalili et al (2020) provided an approach in selecting a semi-variogram model in interpolating rainfall using geostatistical and deterministic techniques. Two deterministic techniques (IDW) and (THI) and two geostatistical technique (ORK and UNK) were used to interpolate a 20 year daily and annual rainfall time series was generated for Zayandeh Rud basin in Iran. Seven semi-varigram models (spherical, matern, power, exponential, gaussian, circular, tetra-spherical, pentra-spherical and rational quadratic) were fitted to the experimental semi-variogram. Based on the results from root mean square error (RSME) and correlation coefficient (R) identified Gaussian model as best fit for the experimental semi-variogram. The ORK and UNK performed better than IDW and THI, which indicates that stochastic methods provides better accuracy rainfall estimates than deterministic [5].

Adhikary et al (2017) evaluated the performance of three geostatistical (ordinary kriging ORK, ordinary cokriging OCK, kriging with an external drift KED and two deterministic (inverse distance weighting IDW, radial basis function) interpolations techniques for enhanced spatial interpolation of monthly rainfall in the Middle Yarra river and the Oven river catchments in Victoria, Australia. In addition the digital elevation model of each river was used as an additional information. The assessment of each prediction model was through cross-validation. The results showed that geostatistical techniques outperformed the deterministic techniques for spatial interpolation of rainfall [1]. The OCK was found to be the best interpolator with the lowest error prediction estimates. The study by Adhikary et al (2017) encourages the addition of digital elevation as an auxiliary data in enhancing the estimation of rainfall in a catchment [1].

# Chapter 3

# Materials and Methods

The following section provides with an overview of the methodologies followed to meet the aim and objective of this project. The methodological framework adopted for this project is testing two geo-spatial interpolations techniques (Ordinary and Universal Kriging) in estimating rainfall distribution across Switzerland, computed on R studio. A brief description of these interpolation techniques, the variogram, study region and datasets is provided below.

### 3.0.1 Study area and Data

Switzerland is a mountainous landlocked country in the western and central Europe. It a home of about 20 Alps which act as climate divide and located between 45°49'05 and 47°48'30 latitudes. and the meridians 5° 57'23 and 10°29'31 longitudes. Switzerland encompasess of wide range of climate. In the west of the country the Atlantic ocean brings a lot of moisture to the country causing rainfall, and in the east the continental climate causes lower temperatures resulting in less rainfall, more of the rain occurs as snow. Altitude affects rainfall the varying altitudes of Switzerland causes imbalances of rainfall distribution. The average annual precipitation is 1229.38mm and summer receives lowest rainfall of 525mm. The are about 467 meteorological stations in the country and the coldest meteorological station is at the Jungfraujoch region.

The precipitation data used in this project is an archived Swiss rainfall data. The Swiss rainfall data is an inbuilt geodata set in the R geoR package. The geoR is a commonly used package it provides with functions for model-based geostatistics. The observations rainfall data were collected from 467 recording meteorological stations across Switzerland and were made available on 08 May 1986. The inbuilt geoR will be used for illustrative purposes.

### 3.0.2 Interpolation techniques

Spatial interpolation is an important part of spatial statistics. Kriging is a geostatistical technique developed by South African mining engineer Danie Krige in 1950s. The basic idea behind Kriging is an estimations of values at a target locations or to predict in between values of unsampled locations in a domain [6]. Kriging is widely favoured as it takes into account

Figure 3.1: Study region Switzerland, with locations of weather stations

auto-correlation of environmental variables and greater weight is assigned to the neighbouring points [4]. The following are spatial interpolation methods features that needs to be taken into consideration when choosing an interpolator: Global vs Local interpolators, Exactness vs Inexact, Gradual vs Abrupt and univariate vs multivariate. Spatial interpolation are widely applied in various disciplines and there are various factors affecting the performance of spatial interpolators that includes but not limited to sample size and density, data variation and normality, distance from prediction point, data quality and spatial autocorrelation approach.

Two geospatial interpolation techniques were selected for this project namely: Ordinary kriging (ORK) and Universal Kriging (URK) and they are briefly described below.

***ORDINARY KRIGING (ORK)***

The ordinary kriging (ORK) is a widely used univivariate geostatistical interpolator for spatial prediction. It assumes constant mean variance for the spatial process across the domain and the mean is not known, the ORK estimates values at unobserved or unsampled location by considering the weighted average of observed nearbouring/location points. The ORK is an unbiased estimator it accounts for the spatial autocorrelation in a dataset. It assumes that spatial autocorrelation of the model should depend on the distance between observed locations [3]

***UNIVERSAL KRIGING (UKR)***

The universal kriging (UKR) is an extension of ordinary kriging and it is also known as a kriging with spatial trend. URK interpolator aims to perform estimations of points using stochastic and determistic parameters. The estimations of values at unobserved location is computed by incorporating spatial autocorrelation and trend component, meaning that the assigned weights of the observed values in the kriging equation no longer depend only on the spatial distance but also on the trend model.

### 3.0.3 Methodology

For this project, R studio software was used for computation. The motivation on using R software is an open-source software, free of charge under the the terms and conditions

$$\gamma\hat{}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[ z(u_i) - z(u_i + h) \right]^2$$

Figure 3.2: semi variogram

of the Free Software Foundations GNU General public License in source code domain. R software provides with integrated functionalities capable of data processing, manipulation and visualization of outputs. The R codes and outputs can be transferred to other programs such as LaTex overleaf, ArcGIS Pro and GitHub. GeoR package with other compatible packages were installed on R. The codes (Swiss rainfall) in geoR package were run on R markdown for kriging prediction of rainfall distribution across the Switzerland. The codes were loaded on GitHub.

***SEMI-VARIANCE AND VARIONGRAM MODELING***

Before, performing Kriging the important step is to analyse the degree of spatial auto-correlation using a variogram. The variog function found in geoR library computes empirical variogram, in which the output can be returned as "binned variogram", "cloud variogram" and "smoothed variogram".

The Variogram is an important tool in geostatistics as it allows to quatify the degree of spatial correlation of spatial process. In geostatistics the semi variogram $Y(\mathbf{h})$ is used to measure dissimilarity between observed points. The semi-variogram represents a discrete function that enables the computation of spatial dependency levels between pairs of points at different distances (lags), encompassing the range of distances considered in the variogram calculation [5].

Model fitting can be done using the "fit. variogram" function.Examples of variograms models that are widely used are spherical, gaussian, exponential, power, matern and others. For this project a spherical and exponential variogram model were selected to fit the experimental variogram to Swiss precipitation data. After computation of fitting the spherical and exponential to experimental model, the exponential failed to fit the experimental variogram, see figure 4.2 therefore it was discarded. The spherical model is a mathematical model used in geostatistics to describe the variability and spatial auto-correlation of pairs of points over a distance in a dataset. The motivation to use spherical model over other varogram models is the flexibility of the model, is the best fit model in one, two and three dimensions and it has shown lowest estimation error in many publications.

After computing model fitting, the following step is Kriging prediction. Two powerful geostatistical techniques Ordinary Kriging and Universal kriging were adopted to estimate rainfall distribution across Switzerland, using spherical model. The estimate was made using 100 points See the outputs at the results and discussion chapter.

### 3.0.4 Cross-Validation

To test the performance of geostatistical interpolations techniques, it is encouraged to compute cross-validation. As it allows to infer information about the robustness and performance of the model to predict data.This approach involves a comparison of simulated and observed data, where individual observations are indivually excluded from the dataset and recalculated based on the new model using the remained observations [1]. The published literature cited in this paper mostly used cross-validation technique.The root mean square (RMSE) is cited as a critical component when assessing the efficacy of spatial interpolation of rainfall [7]. It used to indicate the extent of error and central tendencies. Therefore, the RMSE was adopted to assess the performance of ordinary and universal kriging. To minimise kriging negative estimates the transformation was performed. Transformations particularly Box-Cox assists in stabilizing variogram shape as it changes with distances, and as well improving the accuracy of kriging estimations. All the outputs are in the following chapter 4 of Results and Discussions.

# Chapter 4

# Results and Discussions

This chapter provides with the results obtained in comparison of Ordinary and Universal Kriging. The computation was done on R software and below are the R outputs together with codes.It is worth noting that an assumption was put forward, that the meteorological stations across Switzerland are fairly distributed.

In this project, two geo-spatial interpolations techniques the ordinary kriging and universal kriging are adopted to estimate the spatial distribution of rainfall at Switzerland. Additionally, two variogram models the spherical and exponential were selected before actual kriging, in order to explore between the two models which one is best fitted to the experimental variogram.

It is required that before performing kriging interpolation, there is a need for direct estimation of semi-variogram model for rainfall data. The experimental variogram see figure 4.1.

It has been suggested that using a combination of variogram models would increase the accuracy of fit models to experimental model, than using single model [4]. Hence for this project, it was opted to evaluate the performance of spherical and exponential variogram models.

The results of variogram models are as follows, the spherical model displayed a best

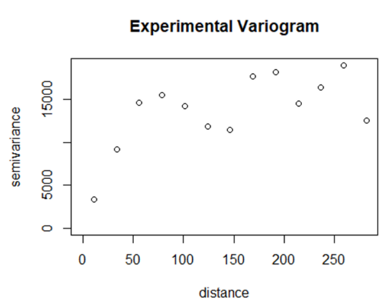


Figure 4.1: Experimental variogram

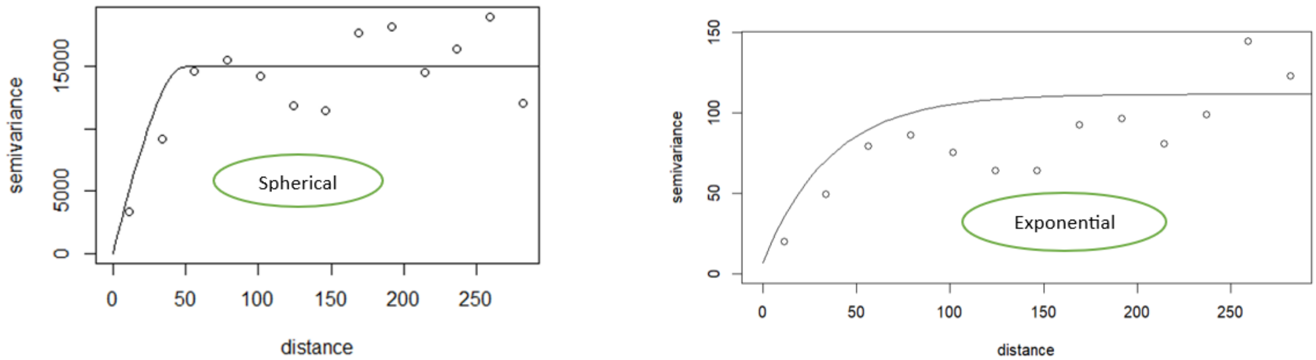


Figure 4.2: comparisons of Spherical and Exponential models

fit model over exponential model. see Figure 4.2 above. The spherical model fitted the experimental variogram very well, due to it's characteristic of linear behaviour at origin. As there is evidence of spatial correlation with the residuals, there is zero nugget, partial sill and range. The spherical model showed a lowest error. This is consistent with previous studies, where spherical model outperformed other variogram models. On the basis, the exponential variogram model showed no spatial correlation to the rainfall data.

To avoid negative kriging estimates and confusion, the exponential model was discarded and kriging estimation was performed using spherical model as it shows strong auto-correlation with the residuals.

***THE ORDINARY KRIGING INTERPOLATION***

An initial ordinary (ORK)kriging was performed using spherical model see figure 4.3. The spatial distribution of *Swiss* rainfall using this technique generally marked areas with very high and low values within the study region. The ordinary Kriging estimation shows uneven distribution of rainfall pattern in Switzerland, this estimation technique provided with unrealistic and discontinuos rainfall pattern.The Kriging variance which sought to lower the error by ordinary kriging estimator. The variance shows high rainfall distribution across Switzerland, which is true spatial variation of rainfall in Switzerland. The country is marked by heavy rainfall.

***THE UNIVERSAL KRIGING INTERPOLATION***

The estimation of Swiss rainfall distribution using universal kriging technique, Fig 4.4 showed a smooth and continuous pattern. The estimation of rainfall distribution in Switzerland using this technique was consistent with actual Swiss heavy annual rainfall.

The comparison of ordinary and universal kriging techniques was carried out on spherical variogram model. Each model performed differently, yielding different results. However,

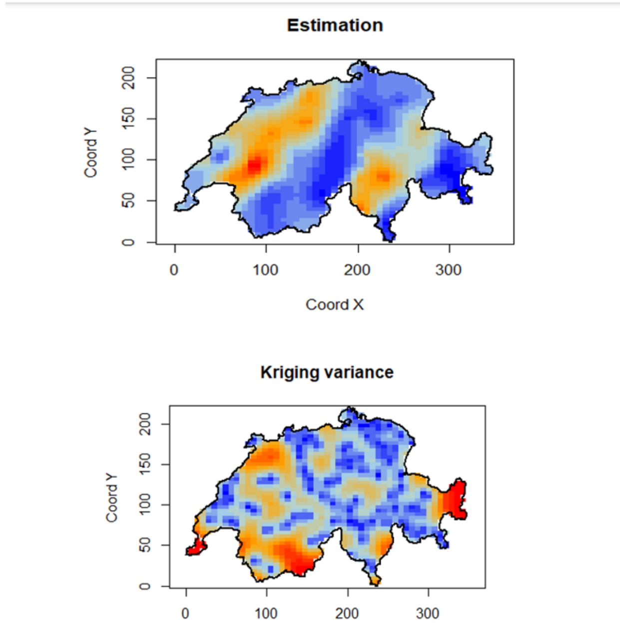


Figure 4.3: Ordinary Kriging and variance

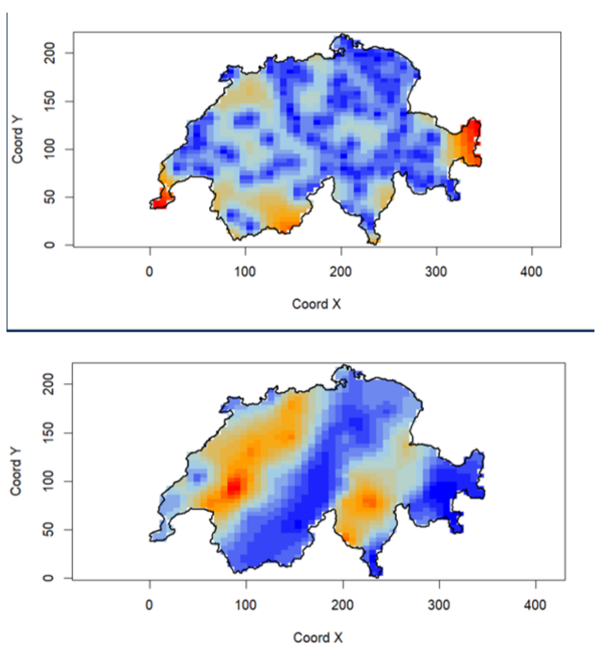


Figure 4.4: Universal kriging and variance

there are no remarkable differences between the two geo-spatial techniques when doing visual examination of the map estimates.

***CROSS-VALIDATION RESULTS***

The cross-validation was performed to evaluate the performance of the selected kriging techniques; ordinary and universal. The estimated versus observed samples are plotted to the fit model. For ordinary kriging technique an estimate was carried out with 100 point samples and the measure between the estimated and observed samples shows that ordinary kriging was unbiased. The same results were obtained for universal kriging the measure between the estimated and observed samples is unbiased. See the outputs at the appendix of this paper.

# Chapter 5

# Conclusions and Recommendations

Rainfall has been an important parameter in climate studies. Accurate estimations of rainfall distributions is imperative in hydrological modeling and water resource management systems. In this project, a geo-statistical approach is implemented in assessing its efficacy in addressing global climate issues. The objective was to evaluate the performance of ordinary and universal kriging techniques in estimating the annual rainfall distribution in Switzerland. To avoid negative estimations of the selected spatial interpolation, the present project evaluated two variogram models the spherical and exponential. The results showed that spherical model is the best fitted model to experimental variogram. Cross-validation was used to compare the performance of the selected interpolation techniques. It is difficult to recommend the best overall interpolation technique, as each technique will depend on density of sampling, and prior knowledge to the study region. Based on the results obtained, the Universal kriging outperformed ordinary kriging in accurately estimating Swiss annual rainfall pattern. Considering that universal kriging tends to overestimates rainfall values, it is still not an ideal interpolation method to use except if adding elevation as a supplementary data.

Geostatistical interpolations techniques gained popularity over deterministic techniques. For future research purposes, it is recommended to add digital elevation model (DEM) when using geostatistical interpolation techniques. The DEM might account for slope and aspect, therefore increasing accuracy of rainfall predictions.

The preliminary results of this project offered guidance in terms of strengths and weaknesses of using kriging spatial interpolations in rainfall spatial distribution.

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# Chapter 6

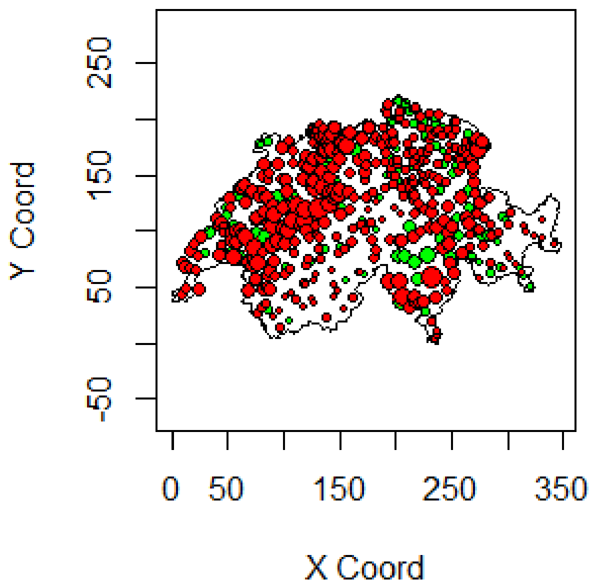
# Appendix

# Kriging Interpolation

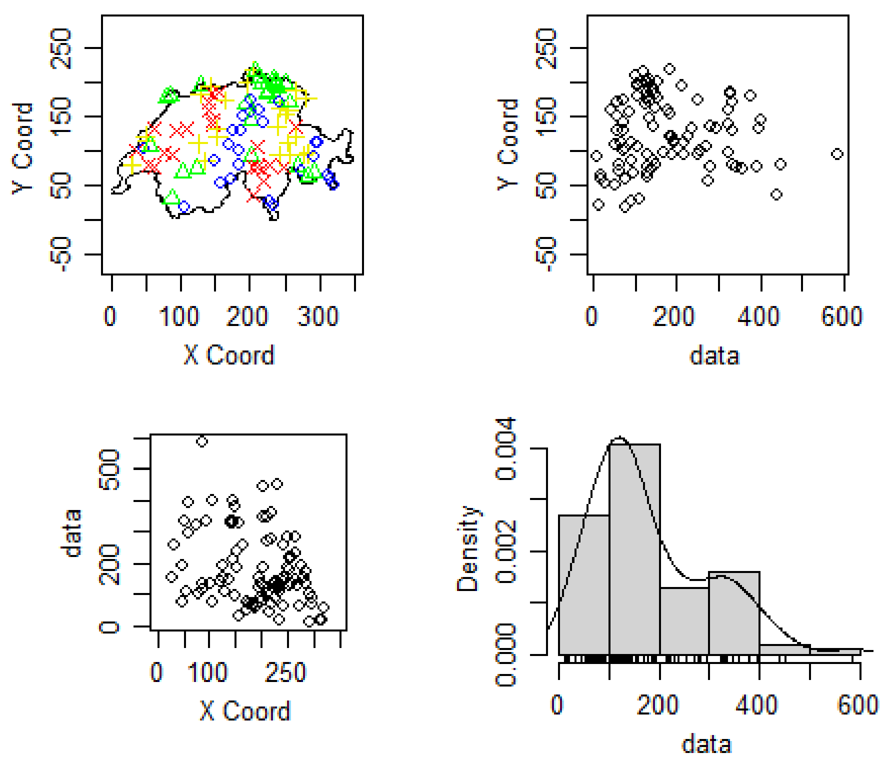
BenMakobe

2023-11-11

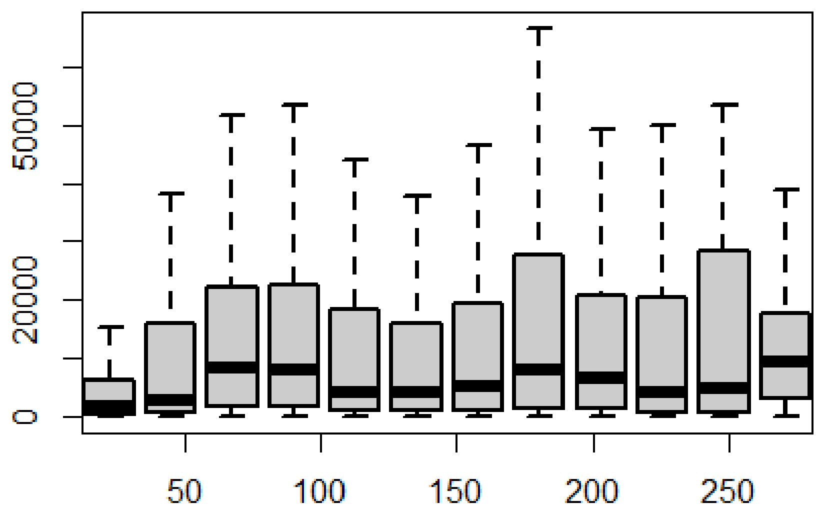
```
library(geoR)

## Warning: package 'geoR' was built under R version 4.3.2

## --------------------------------------------------------------
##  Analysis of Geostatistical Data
##  For an Introduction to geoR go to http://www.leg.ufpr.br/geoR
##  geoR version 1.9-2 (built on 2022-08-09) is now loaded
## --------------------------------------------------------------

library(fields)

## Warning: package 'fields' was built under R version 4.3.1

## Loading required package: spam

## Warning: package 'spam' was built under R version 4.3.1

## Spam version 2.9-1 (2022-08-07) is loaded.
## Type 'help( Spam)' or 'demo( spam)' for a short introduction
## and overview of this package.
## Help for individual functions is also obtained by adding the
## suffix '.spam' to the function name, e.g. 'help( chol.spam)'.

##
## Attaching package: 'spam'

## The following objects are masked from 'package:base':
##
##     backsolve, forwardsolve

## Loading required package: viridisLite

## Warning: package 'viridisLite' was built under R version 4.3.1

##
## Try help(fields) to get started.

# Loading of the data
library(geoR)
points(sic.100, borders = sic.borders, col = 'green')
points(sic.367, borders = sic.borders, col = 'red', add = TRUE)
```



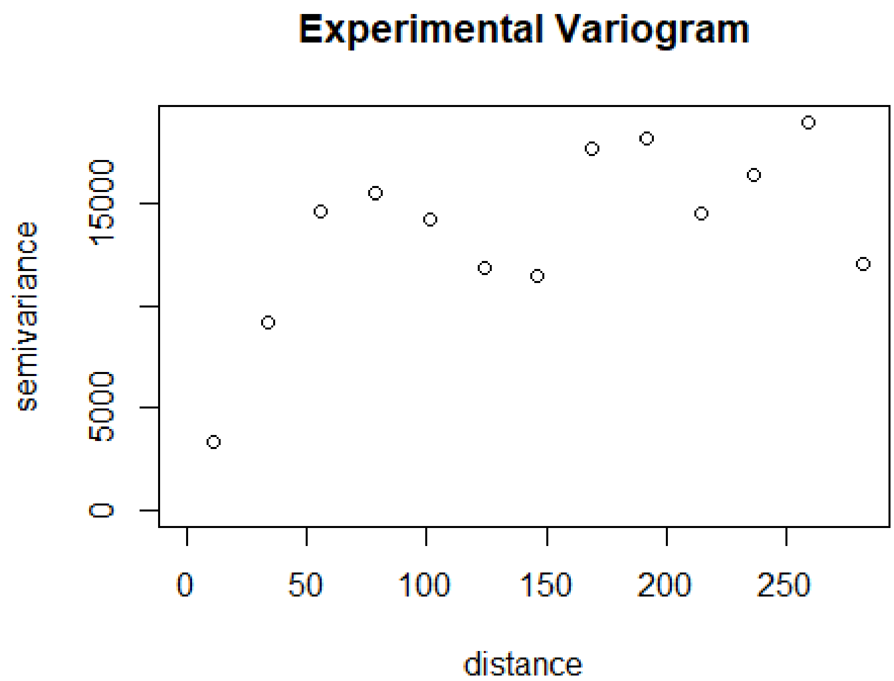

```r
# Descriptive statistics
library(geoR)
plot.geodata(sic.100, bor = sic.borders)
```



```r
# Variogram cloud and experimental variogram
library(geoR)
library(fields)
vario.b <- variog(sic.100, option = c('bin', 'cloud', 'smooth'), bin.cloud =
TRUE)

## variog: computing omnidirectional variogram

vario.c <- variog(sic.100, op = "cloud")

## variog: computing omnidirectional variogram

bplot.xy(vario.c$u, vario.c$v, breaks = vario.b$u, col = "grey80", lwd = 2, c
ex = 0.1, outline = FALSE)
```



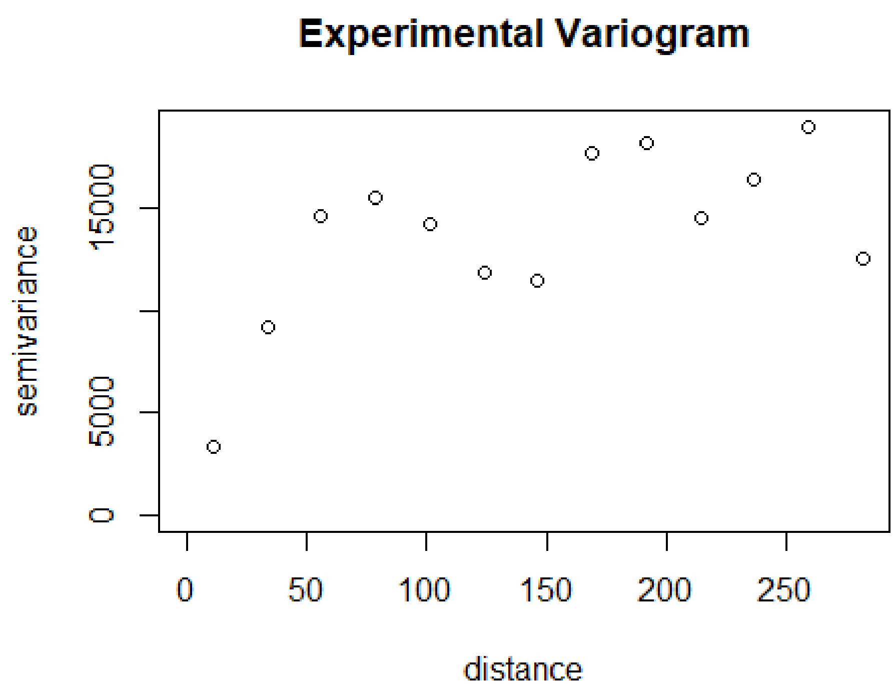

```r
library(geoR)
vario.ex <- variog(sic.100, bin.cloud = TRUE)

## variog: computing omnidirectional variogram

plot(vario.ex, main = 'Experimental Variogram')
```




```r
library(geoR)
vario4 <- variog(sic.100)
```

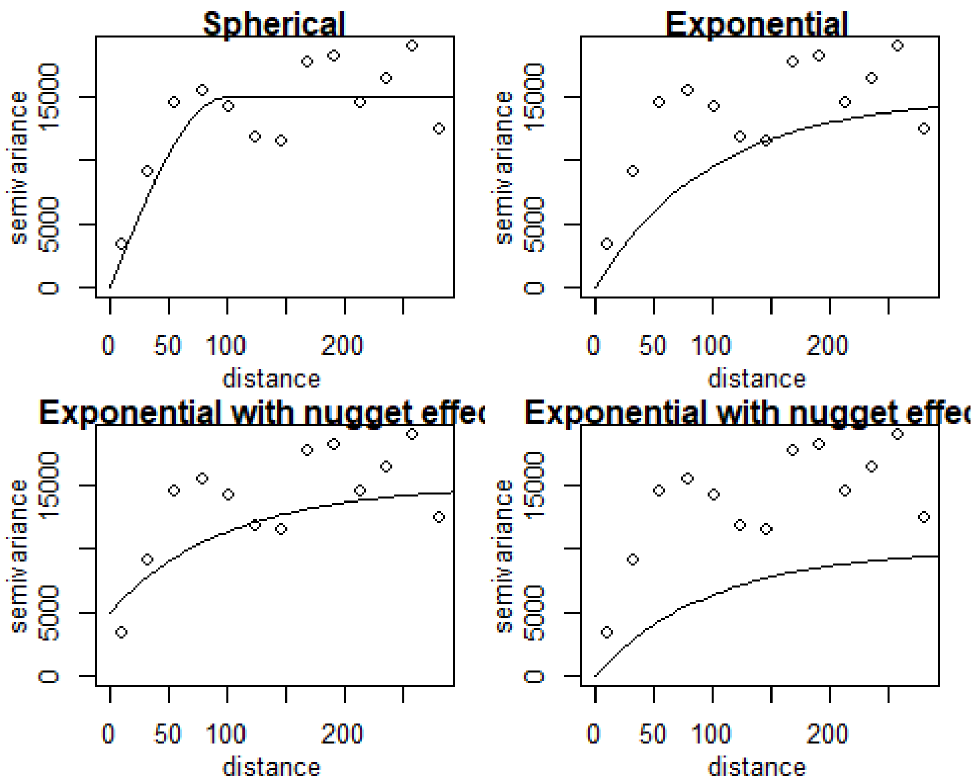
```
## variog: computing omnidirectional variogram
```

```r
plot(vario4, main = 'Experimental Variogram')
```

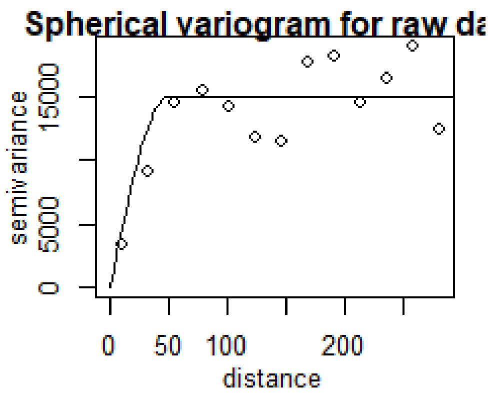



```r
# Variogram fitting
library(geoR)
vario.ex <- variog(sic.100, option = 'bin')

## variog: computing omnidirectional variogram

vario.sphe <- variofit(vario.ex, cov.model = 'spher', ini.cov.pars = c(15000,
200))

## variofit: covariance model used is spherical
## variofit: weights used: npairs
## variofit: minimisation function used: optim

par(mfrow = c(2, 2), mar = c(3, 3, 1, 1), mgp = c(2, 1, 0))

plot(vario.ex, main = 'Spherical')
lines.variomodel(cov.model = 'sphe', cov.pars = c(15000, 100), nug = 0, max.d
ist = 350)

plot(vario.ex, main = 'Exponential')
lines.variomodel(cov.model = 'exp', cov.pars = c(15000, 100), nug = 0, max.di
st = 350)

plot(vario.ex, main = 'Exponential with nugget effect')
lines.variomodel(cov.model = 'exp', cov.pars = c(10000, 100), nug = 5000, max
.dist = 350)
```

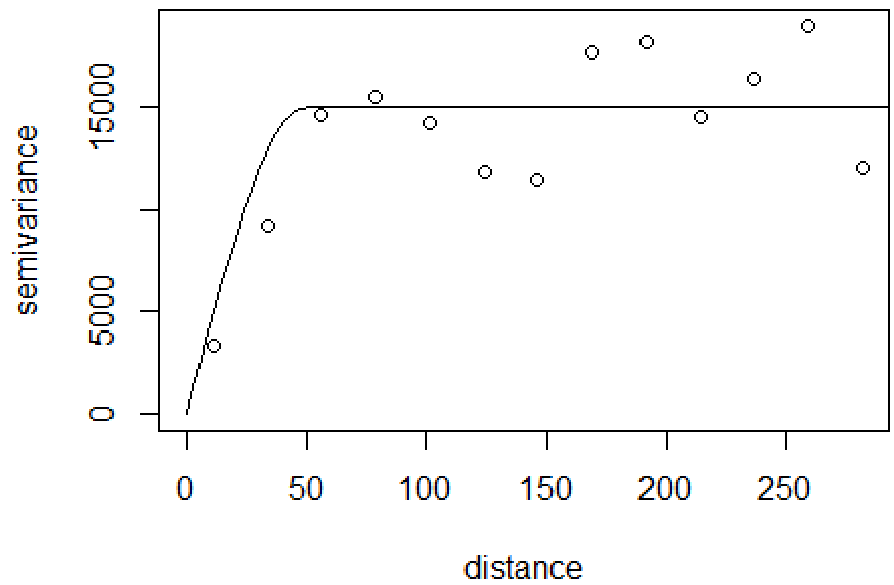
```r
plot(vario.ex, main = 'Exponential with nugget effect')
lines.variomodel(cov.model = 'matern', cov.pars = c(10000, 100), nug = 0, max
.dist = 350, kappa = 0.5)
```



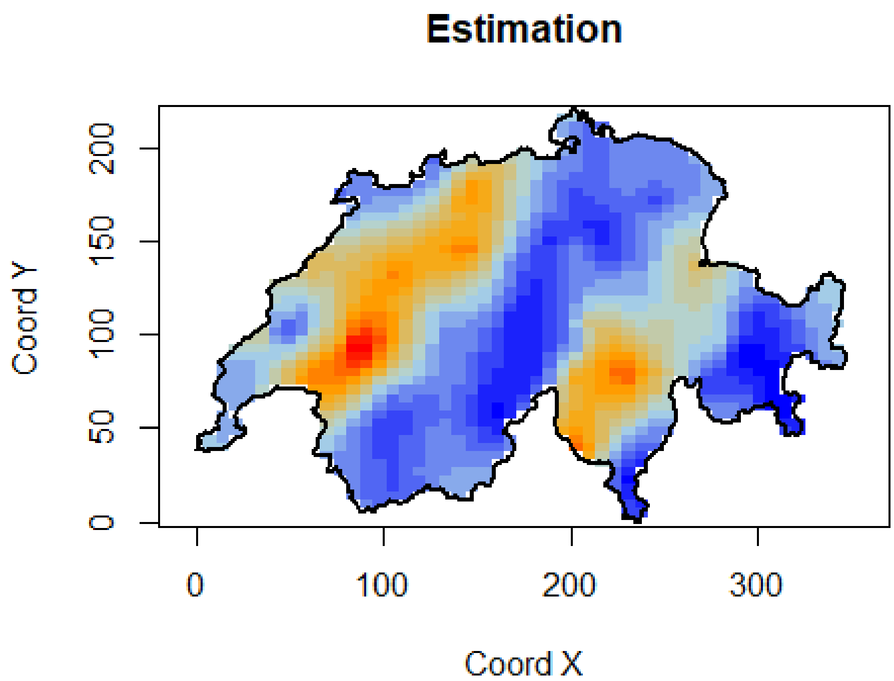

```r
plot(vario.ex, main = 'Spherical variogram for raw data')
lines.variomodel(cov.model = 'spher', cov.pars = c(15000, 50), nug = 0, max.d
ist = 300)
```



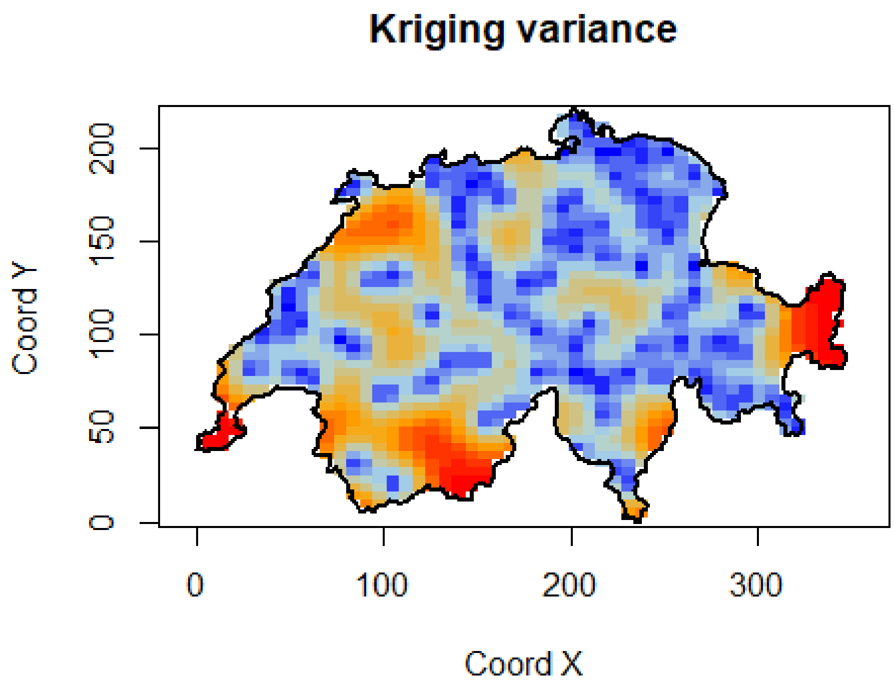

```
#Application to rainfall data
library(geoR)
vario.ex<- variog(sic.100, bin.cloud=TRUE)

## variog: computing omnidirectional variogram

plot(vario.ex,main="")
lines.variomodel(cov.model="spher",cov.pars=c(15000,50),
nug=0,max.dist=300)
```



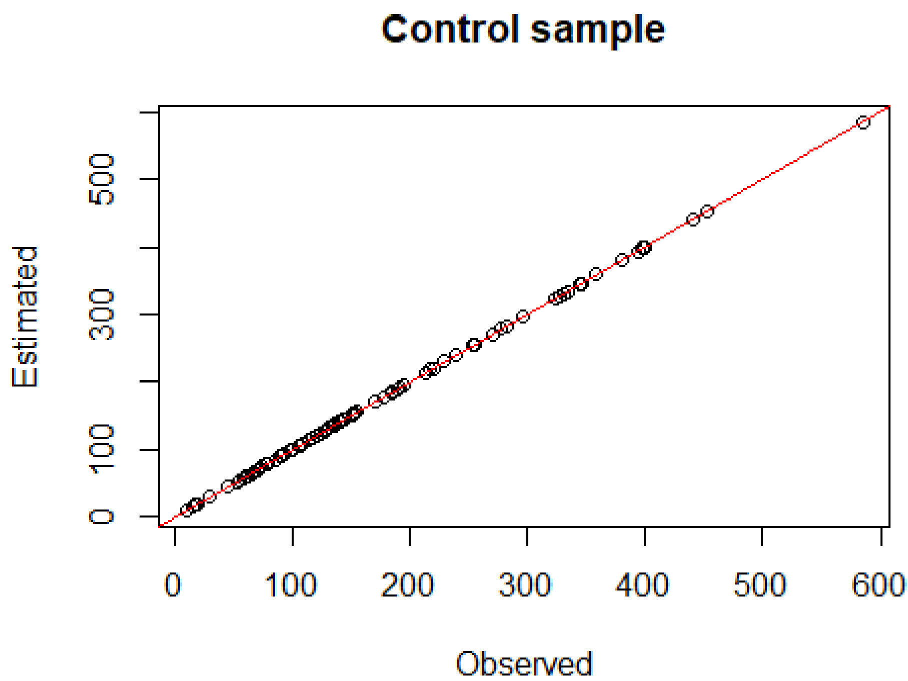

```r
# Kriging estimates and variances
library(geoR)
pred.grid <- expand.grid(seq(0, 350, length.out = 51), seq(0, 220, length.out
= 51))
rgb.palette <- colorRampPalette(c('blue', 'lightblue', 'orange', 'red'), spac
e = 'rgb')
kc <- krige.conv(sic.100, loc = pred.grid, krige = krige.control(cov.model =
'spherical', cov.pars = c(15000, 50)))

## krige.conv: model with constant mean
## krige.conv: Kriging performed using global neighbourhood

image(kc, loc = pred.grid, col = rgb.palette(20), xlab = 'Coord X', ylab = 'C
oord Y',
      borders = sic.borders, main = 'Estimation')
```



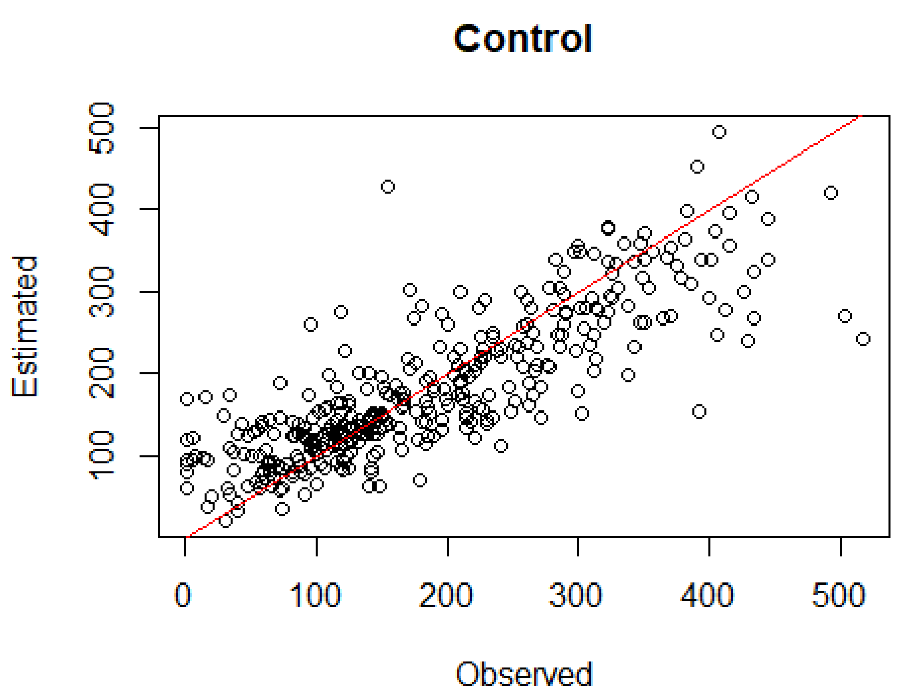

```r
# Perform kriging with the kriging variance
kc <- krige.conv(sic.100, loc = pred.grid,
                 krige = krige.control(cov.model = 'spherical', cov.pars = c(
15000, 50)))

## krige.conv: model with constant mean
## krige.conv: Kriging performed using global neighbourhood

# Compute kriging variance
krige.var <- kc$krige.var

image(kc, krige.var, loc = pred.grid, col = rgb.palette(20), xlab = 'Coord X'
, ylab = 'Coord Y',
      borders = sic.borders, main = 'Kriging variance')
```



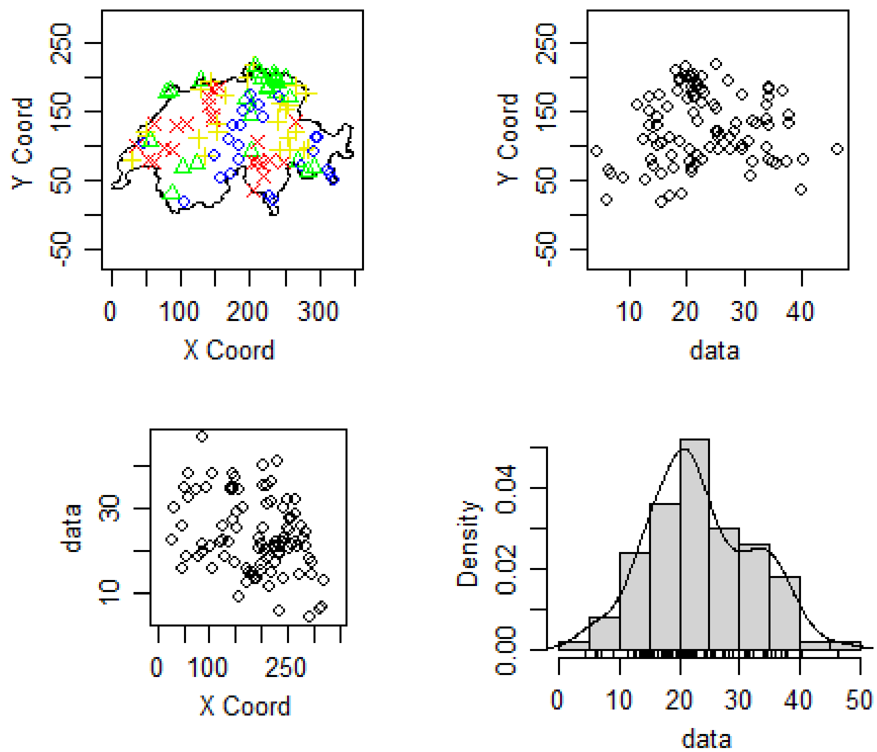

```
# Kriging for estimated or observed values
library(geoR)
kc1 <- krige.conv(sic.100, loc = sic.100$coords, krige = krige.control(cov.mo
del = 'spherical', cov.pars = c(16000, 47)))

## krige.conv: model with constant mean
## krige.conv: Kriging performed using global neighbourhood

kc2 <- krige.conv(sic.100, loc = sic.367$coords, krige = krige.control(cov.mo
del = 'spherical', cov.pars = c(16000, 47)))

## krige.conv: model with constant mean
## krige.conv: Kriging performed using global neighbourhood

plot(sic.100$data, kc1$predict, xlab = 'Observed', ylab = 'Estimated', main =
'Control sample')
abline(a = 0, b = 1, col = 'red')
```



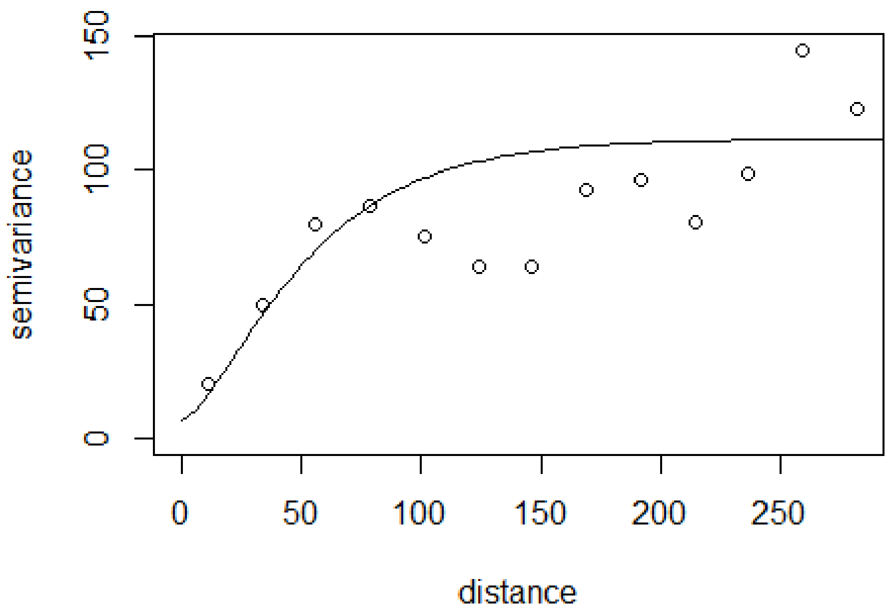

```
plot(sic.367$data, kc2$predict, xlab = 'Observed', ylab = 'Estimated', main =
'Control')
abline(a = 0, b = 1, col = 'red')
```



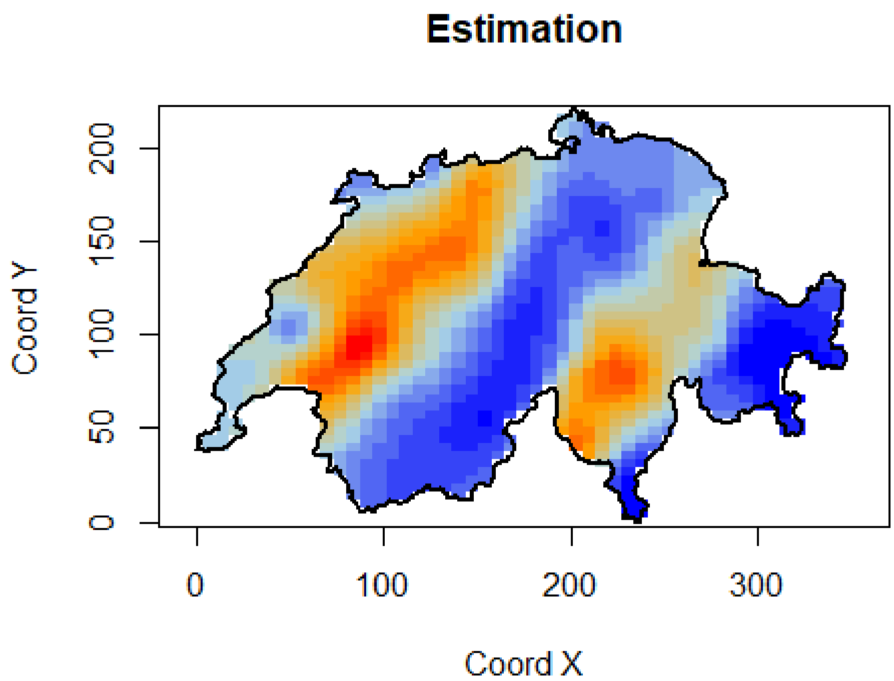
```
#Transformed data
library(geoR)
plot.geodata(sic.100,bor=sic.borders,lambda=0.5)
```



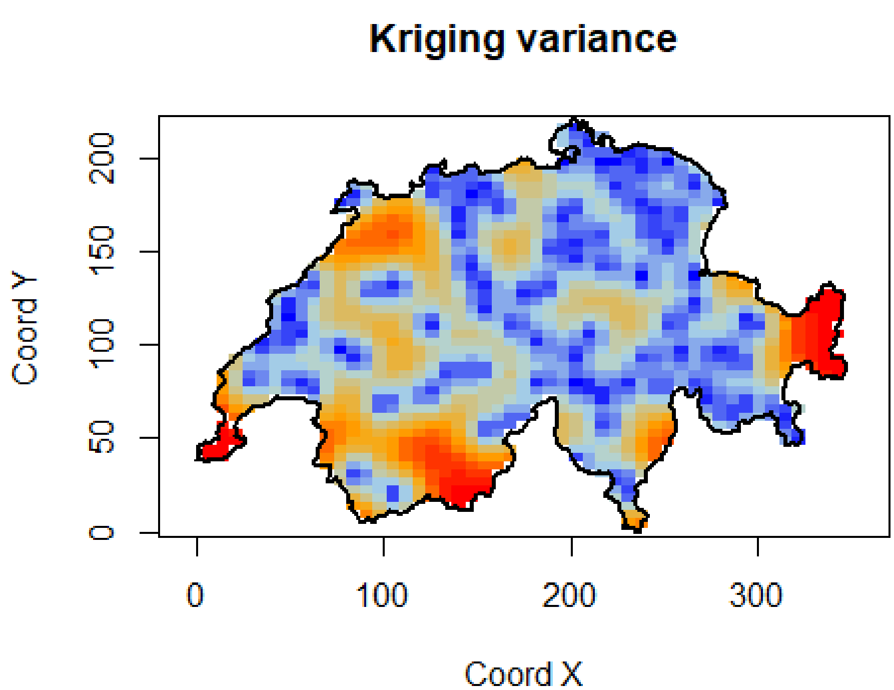

```
library(geoR)
vario.ext<- variog(sic.100,option="bin",lambda=0.5)

## variog: computing omnidirectional variogram

plot(vario.ext)
lines.variomodel(cov.m = "mat",cov.p =c (105, 36), nug = 6.9,
max.dist = 300,kappa = 1, lty = 1)
```




```
library(geoR)
kct<- krige.conv(sic.100, loc = pred.grid,
krige=krige.control(cov.model="matern",cov.pars=c(105, 36),
kappa=1,nugget=6.9,lambda=0.5))

## krige.conv: model with constant mean
## krige.conv: performing the Box-Cox data transformation
## krige.conv: back-transforming the predicted mean and variance
## krige.conv: Kriging performed using global neighbourhood

pred.grid <- expand.grid(seq(0,350, l=51),seq (0,220, l=51))
rgb.palette <- colorRampPalette(c("blue", "lightblue",
"orange", "red"),space = "rgb")
image(kct, loc = pred.grid,col =rgb.palette(20) , xlab="Coord X",
ylab="Coord Y",borders=sic.borders,main="Estimation")
```




```
image(kct, krige.var,loc = pred.grid,col =rgb.palette(20) ,
xlab="Coord X",ylab="Coord Y",borders=sic.borders,
main="Kriging variance")
```



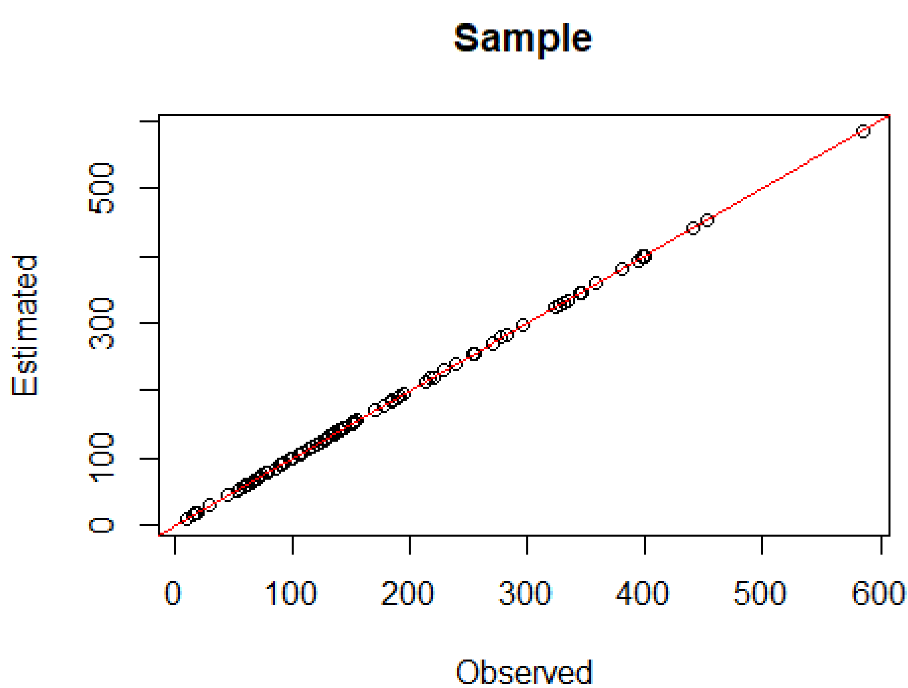
```
library(geoR)
kct1<- krige.conv(sic.100, loc = sic.100$coords,
krige=krige.control(cov.model="spherical",cov.pars=c(16000,47),
kappa=1,nugget=6.9,lambda=0.5))
```

```
## krige.conv: model with constant mean
## krige.conv: performing the Box-Cox data transformation
## krige.conv: back-transforming the predicted mean and variance
## krige.conv: Kriging performed using global neighbourhood
```

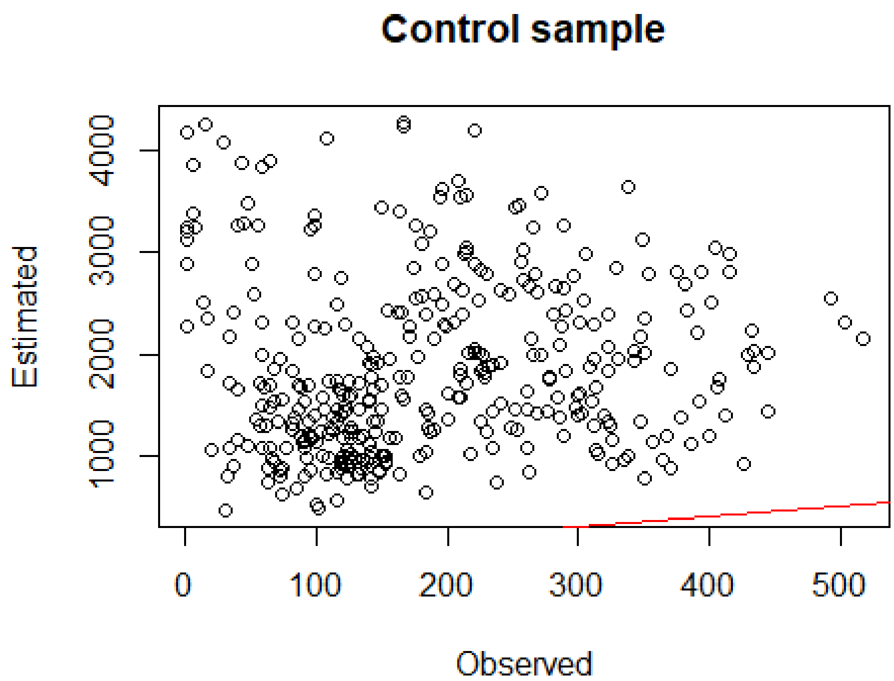
```
kct2<- krige.conv(sic.100, loc = sic.367$coords,
krige=krige.control(cov.model="spherical",cov.pars=c(16000,47),
kappa=1,nugget=6.9,lambda=0.5))
```

```
## krige.conv: model with constant mean
## krige.conv: performing the Box-Cox data transformation
## krige.conv: back-transforming the predicted mean and variance
## krige.conv: Kriging performed using global neighbourhood
```

```
plot(sic.100$data,kct1$predict,xlab="Observed",ylab="Estimated",
main="Sample")
abline(a=0,b=1,col="red")
```

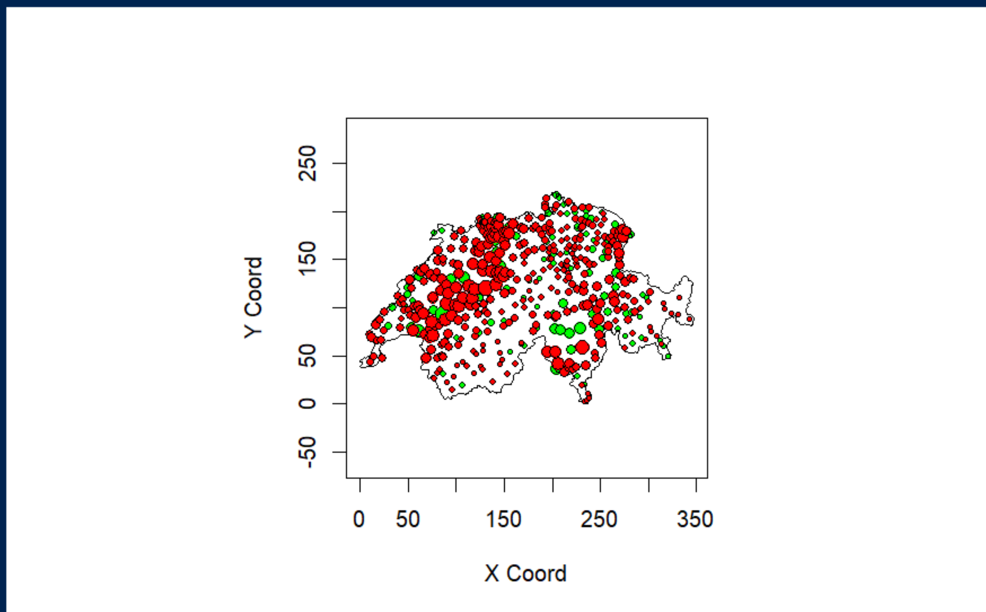



```
plot(sic.367$data,kct2$predict,,xlab="Observed",ylab="Estimated",
main="Control sample")
abline(a=0,b=1,col="red")
```

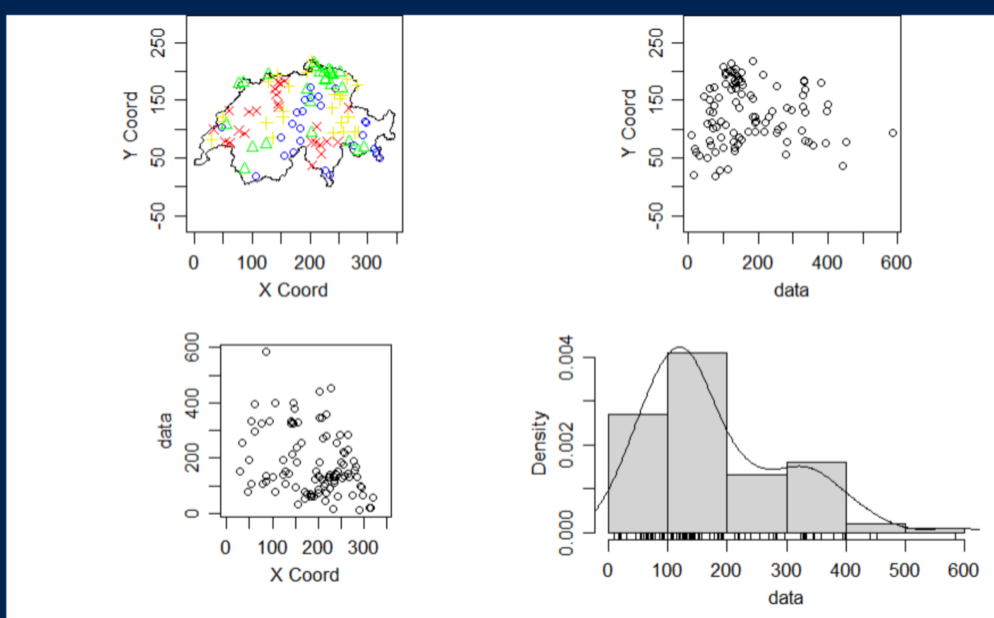



UNIVERSAL KRIGING\EXPONENTIAL

````
```{r dataload}
# Loading of the data
library(geoR)
points(sic.100, borders = sic.borders, col = 'green')
points(sic.367, borders = sic.borders, col = 'red', add = TRUE)
```
````

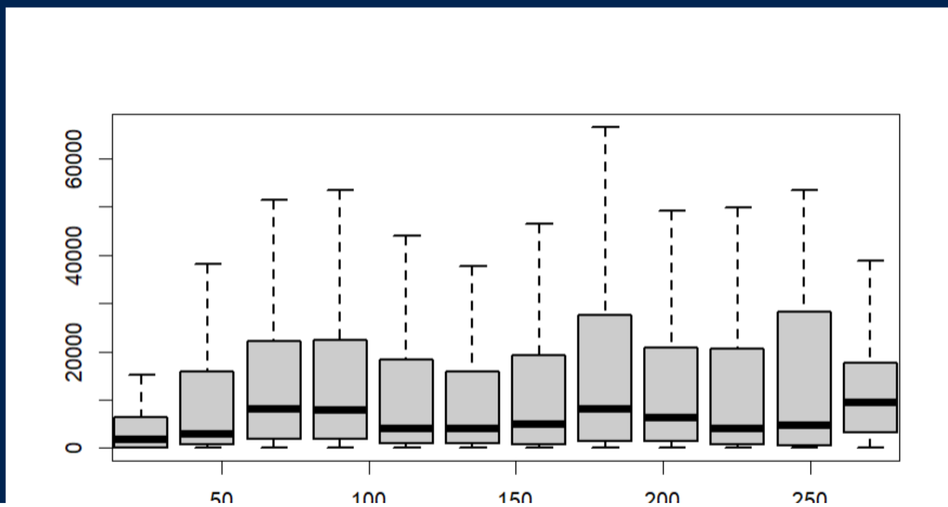



````
```{r geodataplot}
# Plot the geographical data
library(geoR)
plot.geodata(sic.100, bor = sic.borders)
```
````

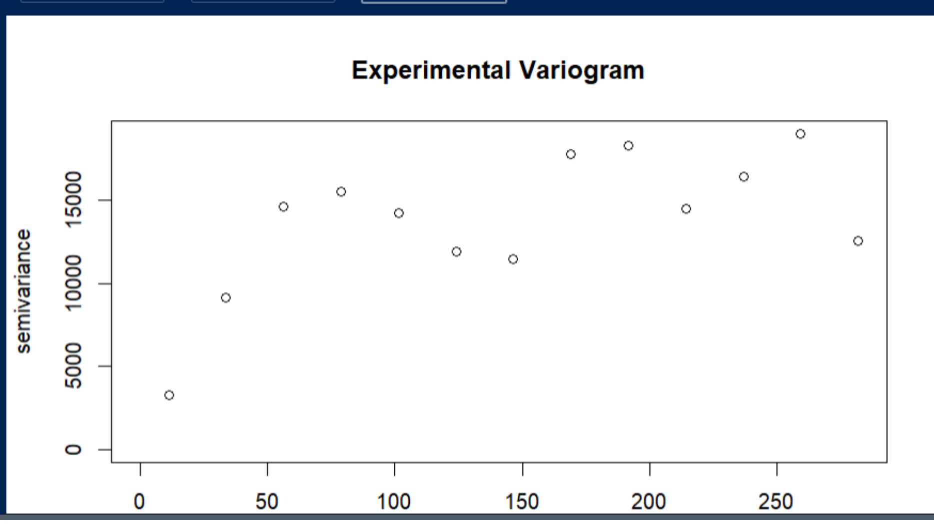


~~~~
```{r exprvariog}
# Variogram cloud and experimental variogram
library(geoR)
library(fields)
vario.b <- variog(sic.100, option = c('bin', 'cloud', 'smooth'), bin.cloud = TRUE)
vario.c <- variog(sic.100, op = "cloud")
bplot.xy(vario.c$u, vario.c$v, breaks = vario.b$u, col = "grey80", lwd = 2, cex = 0.1, outline = FALSE)
```
~~~~

R Console



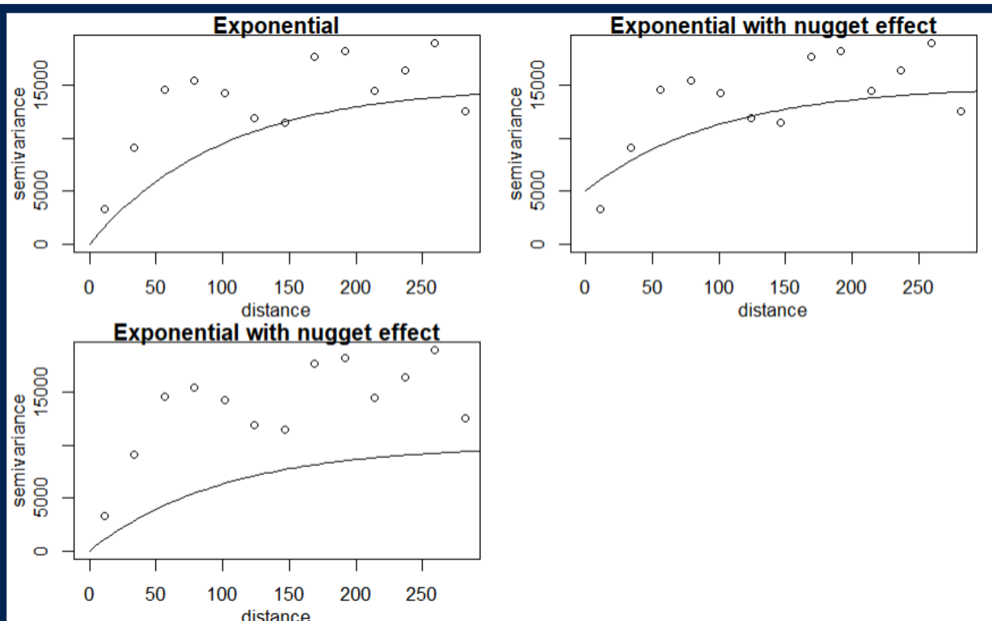

~~~~
library(geoR)
vario.ex <- variog(sic.100, bin.cloud = TRUE)
plot(vario.ex, main = 'Experimental Variogram')

# Experimental variogram (second plot)
library(geoR)
vario4 <- variog(sic.100)
plot(vario4, main = 'Experimental Variogram')
```
~~~~

R Console



```
```{r variogfit}
# Variogram fitting
library(geoR)
vario.ex <- variog(sic.100, option = 'bin')
vario.sphe <- variofit(vario.ex, cov.model = 'spher', ini.cov.pars = c(15000, 200))
par(mfrow = c(2, 2), mar = c(3, 3, 1, 1), mgp = c(2, 1, 0))

plot(vario.ex, main = 'Exponential')
lines.variomodel(cov.model = 'exp', cov.pars = c(15000, 100), nug = 0, max.dist = 350)

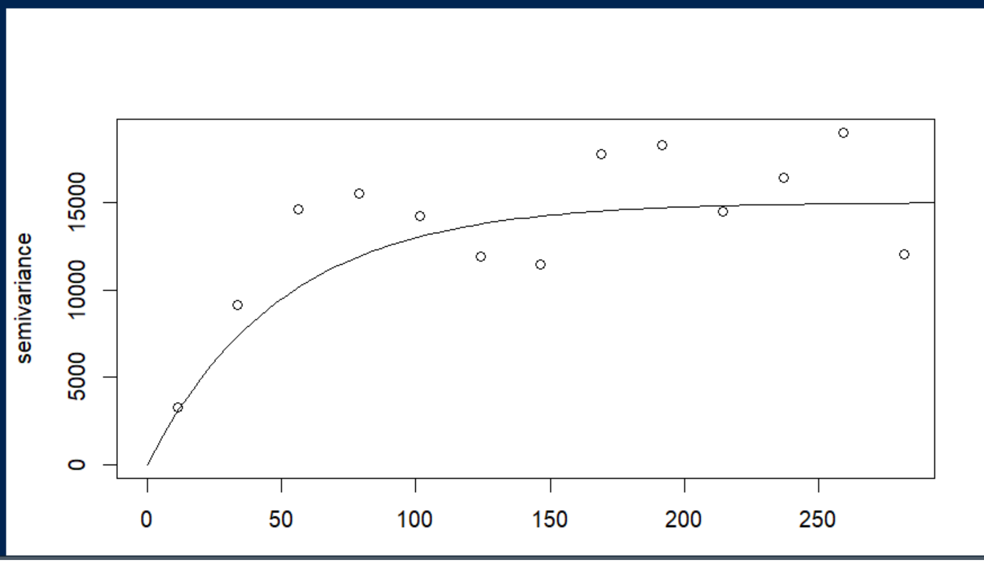
plot(vario.ex, main = 'Exponential with nugget effect')
lines.variomodel(cov.model = 'exp', cov.pars = c(10000, 100), nug = 5000, max.dist = 350)

plot(vario.ex, main = 'Exponential with nugget effect')
lines.variomodel(cov.model = 'matern', cov.pars = c(10000, 100), nug = 0, max.dist = 350, kappa = 0.5)
```
```

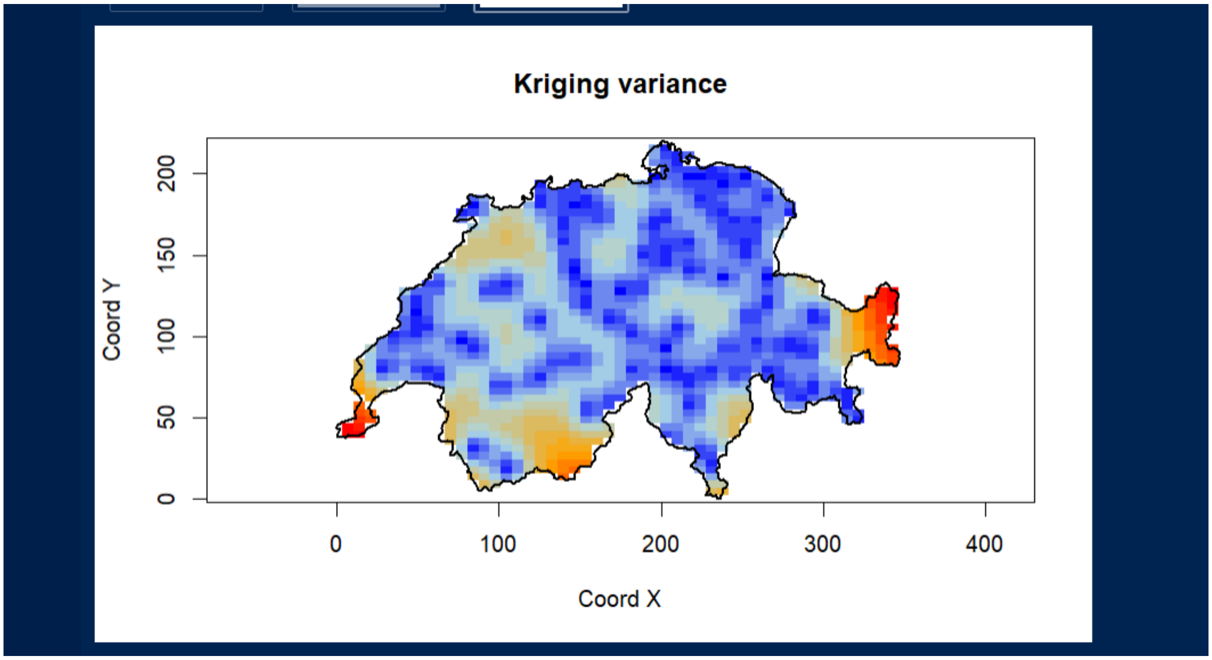


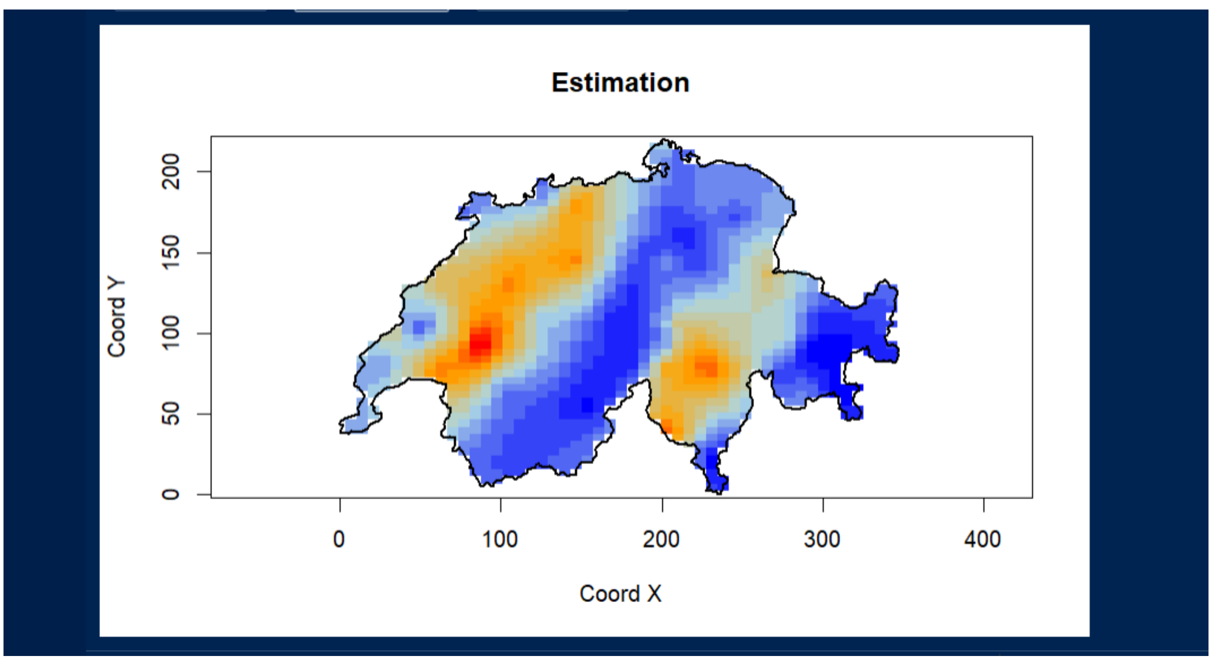
````r
```{r}
#Application to rainfall data
library(geoR)
vario.ex<- variog(sic.100, bin.cloud=TRUE)
plot(vario.ex,main="")
lines.variomodel(cov.model="exp",cov.pars=c(15000,50),
nug=0,max.dist=300)
```
````

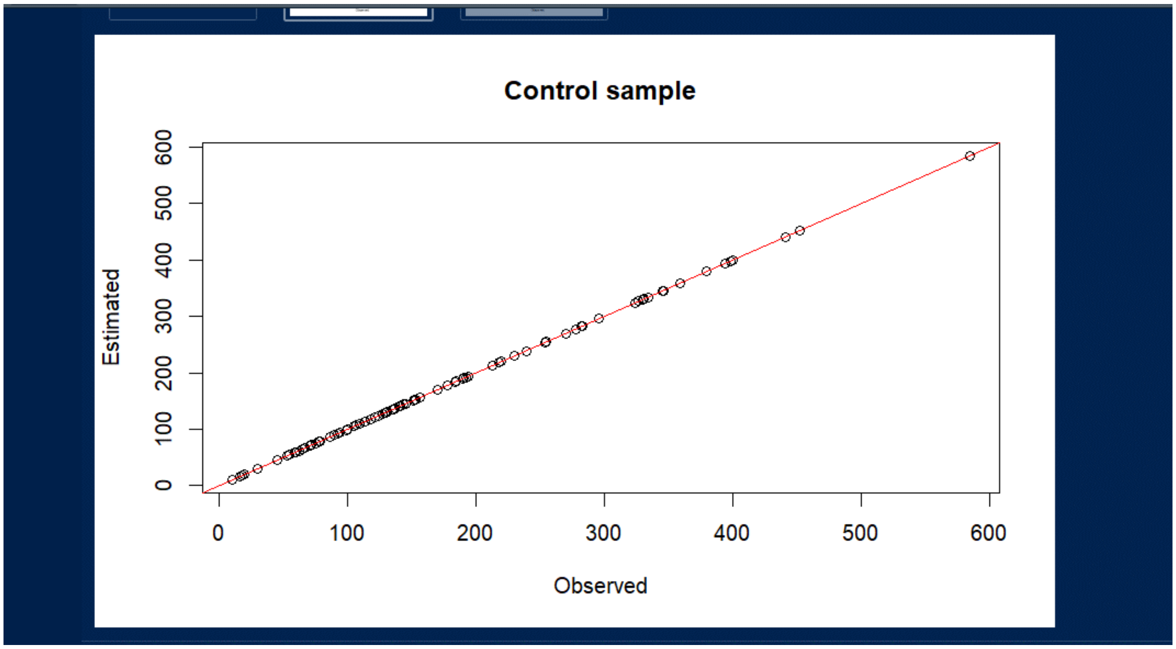
R Console

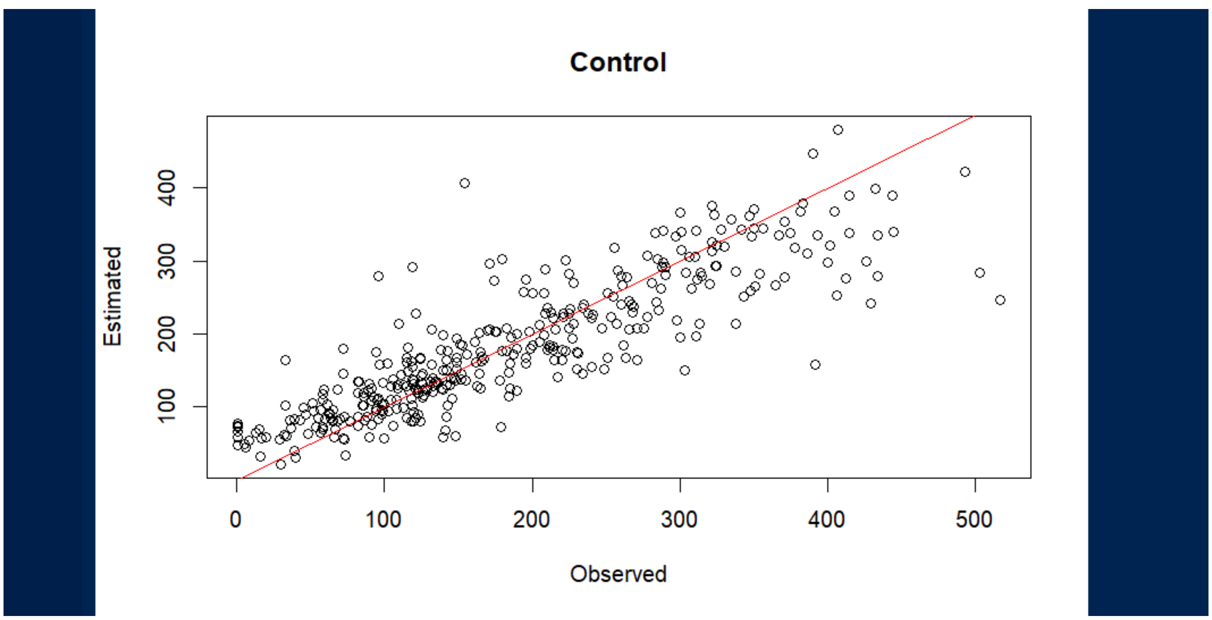



````r
```{r}
library(geoR)
pred.grid <- expand.grid(seq(0, 350, length.out = 51), seq(0, 220, length.out = 51))
rgb.palette <- colorRampPalette(c('blue', 'lightblue', 'orange', 'red'), space = 'rgb')
kc <- krige.conv(sic.100, loc = pred.grid, krige = krige.control(cov.model = 'exponential',
cov.pars = c(15000, 50)))

image(kc, loc = pred.grid, col = rgb.palette(20), xlab = 'Coord X', ylab = 'Coord Y',
      borders = sic.borders, main = 'Estimation')
image(kc, krige.var, loc = pred.grid, col = rgb.palette(20), xlab = 'Coord X', ylab = 'Coord Y',
      borders = sic.borders, main = 'Kriging variance')
```
````


Kriging variance
Coord Y
Coord X
0
50
100
150
200
0
100
200
300
400


Estimation
Coord Y
Coord X
0
50
100
150
200
0
100
200
300
400


Control sample
Estimated
Observed
0
100
200
300
400
500
600


Control
Estimated
Observed
0
100
200
300
400
500