CSCI 5451 Homework 1 Report

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1 Parallelization

To determine how a parallel version of this algorithm would be implemented, we first consider how a serial implementation would work. A serial implementation of this algorithm might take the following steps:

1. Initialize w to all 0s.
2. Perform k iterations of the algorithm.
   (a) Iterate i from 0 to m.
      i. Compute $X_{-i}w_{-i}$.
      ii. Compute the numerator, $X_i^T(y - X_{-i}w_{-i})$
      iii. Compute the denominator, $X_i^T X_i$
      iv. Set the new w_i as the numerator divided by the denominator
   (b) Compute Xw.
   (c) Compute and print out the loss, $\|Xw - y\|^2$

We an implement the following optimizations:

- We save X^T in a second array, which we can read over whenever we need to iterate over X_i. This helps with caching, since data locality is increased.
- The values of $X_i^T X_i$ never change between iterations of the algorithm. Therefore, instead of computing them each iteration, we compute them beforehand and save them to lookup later.
- Instead of computing $X_{-i}w_{-i}$ in each loop, we compute and store Xw. Then, we can more quickly compute $X_{-i}w_{-i}$ as $Xw - X_i w_i$.
- This optimization can be further improved upon. Instead of fully computing Xw, we can instead update it whenever w_i is changed, according to the equation $Xw^{new} = Xw + X_i(w_i^{new} - w_i)$. This also lets us skip step 2b.

After applying these optimizations, our new algorithm takes the following steps:

1. Initialize w to all 0s.
2. Initialize Xw to all 0s.
3. Compute the possible values of $X_i^T X_i$.
4. Compute X^T.
5. Perform k iterations of the algorithm.
   (a) Iterate i from 0 to m.
      i. Compute the numerator, $X_i^T(y - X_{-i}w_{-i})$, with $X_{-i}w_{-i}$ computed as $Xw - X_i w_i$.

ii. Compute w_i^{new} as the numerator divided by $X_i^T X_i$.

iii. Update Xw according to the equation $Xw^{new} = Xw + X_i(w_i^{new} - w_i)$.

iv. Set w_i to w_i^{new}.

(b) Compute and print out the loss, $\|Xw - y\|^2$

Steps 5 and 5a cannot be parallelized, as their iterations need to be run sequentially. Steps 5aii and 5aiv also cannot be parallelized, as they are just single operations. We can parallelize the remaining steps as follows:

1. Each thread iterates over its own range in 0 to m and sets w_i to 0 in that range.
2. Each thread iterates over its own range in 0 to n and sets the ith elements of Xw to 0 in that range.
3. Each thread iterates over its own range in 0 to m and computes $X_i^T X_i$ on that range.
4. Each thread iterates over its own range in 0 to m and and sets the ith rows of X^T on that range.

5ai. Each thread iterates over its own range in 0 to m and computes part of the numerator over that section of the involved matrices/vectors. The individual sums are added together to get the numerator.

5aiii. Each thread iterates over its own range in 0 to m and computes Xw^{new} on that range.

5b. Each thread iterates over its own range in 0 to m and computes part of the loss over that section over those matrices/vectors. The individual sums are then added together to get the total loss.

2 Results

Timing results:

Threads	Small dataset		MNIST dataset	
	`lc_pthreads`	`lc_openmp`	`lc_pthreads`	`lc_openmp`
1	0.0370s	0.0542s	6.6285s	9.6588s
2	0.0279s	0.0282s	4.2439s	5.0091s
4	0.0214s	0.0157s	2.7652s	2.6089s
8	0.0252s	0.0102s	2.4685s	1.4196s
16	0.0304s	0.0096s	3.1688s	0.9735s

Weights from small dataset:

Index	Weight
0	0.085782
1	0.077807
2	0.075450
3	0.081108
4	0.082502
5	0.088705
6	0.098613
7	0.107170
8	0.106483
9	0.096340

Weights from MNIST dataset:

Index	Weight
0	0.000000
1	0.000000
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	0.000000
10	0.000000
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000
25	0.000000
26	0.000000
27	0.000000
28	0.000000
29	0.000000
30	0.000000
31	0.000000
32	0.000000
33	0.000000
34	0.000000
35	0.000000
36	0.000000
37	0.000000
38	0.000000
39	0.000000
40	0.000000
41	0.000000
42	0.403563
43	0.000000
44	0.000000
45	0.000000
46	0.000000
47	0.000000
48	0.000000
49	0.000000

Index	Weight
50	0.000000
51	0.000000
52	0.000000
53	0.000000
54	0.000000
55	0.000000
56	0.000000
57	0.000000
58	0.000000
59	0.000000
60	0.000000
61	0.000000
62	0.000000
63	0.000000
64	0.000000
65	0.000000
66	0.000000
67	12.987359
68	0.051548
69	0.393589
70	-0.264375
71	0.236574
72	0.152835
73	-13.923561
74	-0.102720
75	0.000000
76	0.000000
77	0.000000
78	0.000000
79	0.000000
80	0.000000
81	0.000000
82	0.000000
83	0.000000
84	0.000000
85	0.000000
86	0.000000
87	0.000000
88	0.000000
89	0.000000
90	0.000000
91	0.000000
92	0.000000
93	-0.147281
94	-0.131344
95	-1.037606
96	0.683665
97	-0.244655
98	0.136661
99	0.000119

Index	Weight
100	-0.007326
101	-0.049297
102	0.404771
103	-0.519793
104	0.836800
105	-2.676507
106	1.214651
107	0.000000
108	0.000000
109	0.000000
110	0.000000
111	0.000000
112	0.000000
113	0.000000
114	0.000000
115	0.000000
116	0.239237
117	0.000000
118	0.000000
119	1.746930
120	-0.350130
121	0.376704
122	0.271692
123	0.040371
124	-0.011677
125	0.067203
126	-0.019514
127	0.020456
128	0.053103
129	0.045893
130	0.032673
131	-0.004911
132	0.007626
133	-0.065457
134	-0.045406
135	0.245762
136	-0.438799
137	0.000000
138	0.000000
139	0.000000
140	0.000000
141	0.000000
142	1.189222
143	0.000000
144	1.093981
145	-0.938003
146	0.090064
147	-0.202659
148	0.411374
149	-0.507071

Index	Weight
150	0.021472
151	0.063164
152	0.126469
153	0.115376
154	0.151555
155	0.219725
156	0.129269
157	0.207881
158	0.076247
159	0.086470
160	0.100062
161	0.141866
162	0.178016
163	0.006952
164	0.062815
165	1.786331
166	-0.822882
167	0.000000
168	0.000000
169	0.000000
170	-0.499954
171	0.326779
172	-0.031915
173	-0.093464
174	0.000736
175	0.129968
176	-0.083525
177	-0.195940
178	-0.045321
179	0.006471
180	-0.121955
181	-0.036847
182	-0.025118
183	-0.027645
184	-0.040721
185	-0.096115
186	0.023831
187	-0.020653
188	0.010907
189	-0.138990
190	-0.146661
191	0.227304
192	0.224015
193	-4.896863
194	0.000000
195	0.000000
196	-1.529101
197	-0.154254
198	0.312838
199	-0.184444

Index	Weight
200	-0.059179
201	-0.027950
202	-0.109623
203	0.003589
204	0.019878
205	-0.001513
206	-0.008211
207	-0.060887
208	-0.016978
209	-0.020237
210	-0.028303
211	-0.052795
212	0.000485
213	-0.055951
214	-0.019855
215	0.061437
216	-0.021363
217	0.116053
218	0.082764
219	-0.065981
220	-0.213218
221	-0.095757
222	0.142965
223	0.000000
224	0.126168
225	-0.285753
226	-0.122659
227	0.010747
228	-0.067309
229	0.186079
230	-0.002979
231	-0.083971
232	0.016701
233	0.043679
234	-0.040777
235	-0.119654
236	-0.064479
237	-0.017134
238	-0.019433
239	-0.053136
240	-0.030955
241	0.007546
242	0.053089
243	0.006358
244	0.015594
245	-0.031844
246	-0.051981
247	0.039805
248	-0.040508
249	0.667843

Index	Weight
250	0.000000
251	0.000000
252	4.465733
253	0.205990
254	0.023950
255	-0.062759
256	-0.025191
257	0.004727
258	-0.019160
259	0.074315
260	-0.130102
261	-0.002475
262	-0.050499
263	-0.092816
264	0.008192
265	0.045368
266	-0.012614
267	-0.059841
268	-0.011803
269	-0.008385
270	0.001182
271	0.001842
272	0.004107
273	-0.036257
274	0.027511
275	-0.085096
276	-0.004634
277	-0.399918
278	-1.779999
279	-0.113281
280	-248.839479
281	-0.273016
282	-0.084492
283	0.094041
284	-0.007951
285	0.004452
286	-0.046347
287	-0.036485
288	-0.002184
289	0.012407
290	-0.073817
291	0.030072
292	0.052285
293	-0.064317
294	-0.091785
295	-0.035475
296	-0.036686
297	-0.049523
298	-0.044281
299	-0.063768

Index	Weight
300	0.021065
301	0.017996
302	0.055413
303	0.171224
304	-0.182209
305	0.045277
306	0.597107
307	-0.808531
308	0.667049
309	-0.002651
310	0.178182
311	0.008410
312	-0.119645
313	0.007647
314	-0.025730
315	-0.046087
316	0.029939
317	-0.024486
318	-0.019424
319	0.010958
320	0.002173
321	-0.139689
322	-0.032178
323	0.074776
324	-0.059912
325	-0.068954
326	-0.051707
327	-0.040667
328	0.014062
329	0.003266
330	0.016129
331	0.125939
332	-0.185239
333	0.533909
334	17.812770
335	0.000000
336	0.000000
337	0.099877
338	-0.137718
339	-0.119351
340	0.072362
341	-0.030359
342	0.066968
343	0.014713
344	-0.077926
345	0.030972
346	0.049349
347	-0.104626
348	-0.062493
349	0.030078

Index	Weight
350	0.317627
351	0.040831
352	-0.053667
353	-0.047309
354	-0.015200
355	-0.038937
356	-0.038340
357	0.009100
358	-0.181476
359	-0.034584
360	0.706422
361	-0.695116
362	0.000000
363	0.000000
364	0.000000
365	-1.052986
366	0.021074
367	0.032655
368	-0.274978
369	0.009517
370	-0.079271
371	0.081488
372	-0.062065
373	-0.111165
374	-0.013024
375	-0.054088
376	0.232984
377	0.105593
378	0.401891
379	-0.087088
380	0.056730
381	-0.035660
382	0.016467
383	0.014458
384	-0.036232
385	-0.078028
386	-0.103781
387	0.134876
388	-0.907634
389	0.192573
390	-13.914623
391	0.000000
392	0.000000
393	-8.032924
394	0.578574
395	0.072917
396	-0.020414
397	-0.010964
398	-0.083583
399	-0.035787

Index	Weight
400	-0.046153
401	0.139062
402	-0.032610
403	0.093645
404	-0.078055
405	0.093204
406	0.143825
407	-0.002579
408	-0.019800
409	-0.065768
410	-0.012284
411	0.003540
412	-0.036298
413	0.045493
414	0.017233
415	-0.262551
416	-0.458219
417	0.927049
418	0.410847
419	0.000000
420	-8.685817
421	0.000000
422	0.000000
423	0.243208
424	0.178327
425	0.024150
426	0.169748
427	-0.057198
428	0.108685
429	-0.129777
430	-0.077024
431	-0.106309
432	0.000649
433	0.081084
434	0.034155
435	0.013487
436	-0.053292
437	-0.045110
438	-0.016241
439	0.005684
440	0.125929
441	-0.166887
442	-0.039889
443	-0.120305
444	0.375053
445	-0.453969
446	1.655480
447	0.000000
448	0.302300
449	-4.435985

Index	Weight
450	-25.332182
451	-0.842183
452	-0.581136
453	0.055001
454	-0.184132
455	0.018367
456	-0.025428
457	0.001729
458	0.002517
459	-0.051129
460	-0.029314
461	-0.000440
462	0.060139
463	-0.000435
464	-0.052892
465	-0.000255
466	-0.052628
467	-0.069256
468	-0.088731
469	0.060763
470	0.006446
471	0.074751
472	0.052634
473	0.250613
474	0.000000
475	0.000000
476	0.000000
477	0.811022
478	-3.508993
479	2.424739
480	-0.421863
481	0.430718
482	-0.149879
483	0.083414
484	0.066961
485	0.074208
486	-0.044743
487	-0.054290
488	0.004557
489	-0.015733
490	0.058904
491	-0.012205
492	0.005583
493	0.029152
494	-0.009343
495	0.059740
496	-0.148277
497	0.188470
498	-0.091244
499	-0.085046

Index	Weight
500	-0.319787
501	-1.234571
502	-1.858744
503	5.688208
504	0.000000
505	14.495603
506	0.029609
507	-0.272286
508	-0.400680
509	-0.233000
510	0.102372
511	0.029991
512	-0.039485
513	-0.020060
514	-0.008819
515	0.012361
516	0.016709
517	-0.025753
518	0.001287
519	0.020156
520	0.037843
521	0.066784
522	-0.037968
523	-0.148600
524	0.130455
525	-0.256891
526	0.362308
527	0.095027
528	0.445491
529	2.191192
530	-0.340096
531	0.706388
532	0.000000
533	-0.380956
534	1.204313
535	0.105744
536	-0.508197
537	0.600230
538	-0.330184
539	0.061248
540	-0.064969
541	0.018006
542	-0.008948
543	0.080493
544	-0.041904
545	0.027249
546	-0.020755
547	0.047566
548	0.075046
549	0.017483

Index	Weight
550	0.129459
551	-0.005925
552	0.165702
553	-0.232492
554	-0.508934
555	0.029519
556	-1.111918
557	0.718296
558	0.000000
559	-0.304070
560	0.000000
561	-2.462945
562	-10.411344
563	-0.134921
564	0.586500
565	-0.455416
566	-0.179787
567	-0.132657
568	-0.012116
569	0.055675
570	0.013127
571	0.011632
572	-0.022642
573	-0.026084
574	-0.000615
575	-0.002987
576	0.025979
577	0.063206
578	0.136171
579	-0.046480
580	-0.210957
581	0.150137
582	0.271614
583	0.968553
584	-0.252393
585	-0.243466
586	0.000000
587	0.000000
588	-1.113552
589	0.000000
590	0.000000
591	2.261483
592	-0.052684
593	0.374361
594	0.176239
595	0.326205
596	0.072901
597	-0.004603
598	-0.022993
599	0.003594

Index	Weight
600	-0.018626
601	0.001573
602	-0.011066
603	0.007779
604	-0.007261
605	-0.069711
606	-0.065467
607	0.234823
608	0.030825
609	0.227108
610	-0.021678
611	0.562302
612	-0.696347
613	1.115765
614	0.210318
615	0.000000
616	0.162797
617	0.000000
618	0.000000
619	-2.155993
620	0.322670
621	-0.166706
622	0.125919
623	-0.052014
624	0.047803
625	0.042168
626	-0.001109
627	0.005207
628	-0.016561
629	0.063505
630	-0.040736
631	0.008214
632	-0.044166
633	-0.044067
634	0.033685
635	-0.142947
636	0.086042
637	-0.012293
638	-0.496915
639	0.827429
640	0.682105
641	-1.758731
642	0.000000
643	0.000000
644	0.000000
645	0.000000
646	0.000000
647	1.272419
648	-0.512667
649	0.199520

Index	Weight
650	-0.195903
651	-0.010081
652	-0.123173
653	0.035067
654	0.009721
655	0.003292
656	-0.001983
657	0.002835
658	-0.024618
659	-0.026290
660	-0.011121
661	0.071700
662	-0.037119
663	-0.040544
664	0.132051
665	0.052738
666	0.158742
667	-0.724996
668	0.515968
669	0.000000
670	0.000000
671	0.000000
672	0.000000
673	0.000000
674	-0.343633
675	-0.170409
676	-0.334390
677	-0.047393
678	-0.025653
679	-0.049076
680	0.057906
681	0.006372
682	0.044818
683	0.042977
684	0.066207
685	0.029532
686	0.023451
687	0.082099
688	0.013682
689	0.091892
690	0.170753
691	0.136179
692	-0.397666
693	0.060628
694	0.731323
695	2.146371
696	-0.724315
697	-0.683131
698	0.000000
699	0.000000

Index	Weight
700	0.000000
701	0.000000
702	0.000000
703	0.753028
704	-0.508102
705	0.246303
706	0.077869
707	-0.150077
708	0.085474
709	-0.043162
710	-0.027328
711	-0.043222
712	0.004206
713	0.004106
714	0.047066
715	-0.060931
716	-0.044357
717	-0.107667
718	-0.258108
719	-0.001071
720	0.239737
721	0.274898
722	-0.994054
723	4.797108
724	0.126343
725	0.000000
726	0.000000
727	0.000000
728	0.000000
729	0.000000
730	0.000000
731	0.000000
732	0.325264
733	-0.403374
734	0.028788
735	0.096907
736	-0.105736
737	-0.006565
738	-0.009703
739	0.012954
740	-0.016659
741	-0.065025
742	-0.024660
743	0.042890
744	0.012772
745	0.077616
746	0.155224
747	0.110093
748	-0.075148
749	0.085505

Index	Weight
750	0.346499
751	-1.504359
752	0.000000
753	0.000000
754	0.000000
755	0.000000
756	0.000000
757	0.000000
758	0.000000
759	0.000000
760	0.079685
761	-0.535093
762	0.573362
763	-0.562727
764	0.543879
765	-0.029345
766	0.015804
767	-0.029969
768	0.001986
769	-0.115815
770	0.007334
771	-0.108838
772	0.010197
773	-0.094559
774	-0.019316
775	-0.092565
776	-0.447288
777	0.209396
778	-1.279342
779	-0.143597
780	0.000000
781	0.000000
782	0.000000
783	0.000000