

# CSCI 5451 Homework 1 Report

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## 1 Parallelization

To determine how a parallel version of this algorithm would be implemented, we first consider how a serial implementation would work. A serial implementation of this algorithm might take the following steps:

1. Initialize  $w$  to all 0s.
2. Perform  $k$  iterations of the algorithm.
  - (a) Iterate  $i$  from 0 to  $m$ .
    - i. Compute  $X_{-i}w_{-i}$ .
    - ii. Compute the numerator,  $X_i^T(y - X_{-i}w_{-i})$
    - iii. Compute the denominator,  $X_i^T X_i$
    - iv. Set the new  $w_i$  as the numerator divided by the denominator
  - (b) Compute  $Xw$ .
  - (c) Compute and print out the loss,  $\|Xw - y\|^2$

We can implement the following optimizations:

- We save  $X^T$  in a second array, which we can read over whenever we need to iterate over  $X_i$ . This helps with caching, since data locality is increased.
- The values of  $X_i^T X_i$  never change between iterations of the algorithm. Therefore, instead of computing them each iteration, we compute them beforehand and save them to lookup later.
- Instead of computing  $X_{-i}w_{-i}$  in each loop, we compute and store  $Xw$ . Then, we can more quickly compute  $X_{-i}w_{-i}$  as  $Xw - X_iw_i$ .
- This optimization can be further improved upon. Instead of fully computing  $Xw$ , we can instead update it whenever  $w_i$  is changed, according to the equation  $Xw^{new} = Xw + X_i(w_i^{new} - w_i)$ . This also lets us skip step 2b.

After applying these optimizations, our new algorithm takes the following steps:

1. Initialize  $w$  to all 0s.
2. Initialize  $Xw$  to all 0s.
3. Compute the possible values of  $X_i^T X_i$ .
4. Compute  $X^T$ .
5. Perform  $k$  iterations of the algorithm.
  - (a) Iterate  $i$  from 0 to  $m$ .
    - i. Compute the numerator,  $X_i^T(y - X_{-i}w_{-i})$ , with  $X_{-i}w_{-i}$  computed as  $Xw - X_iw_i$ .

- ii. Compute  $w_i^{new}$  as the numerator divided by  $X_i^T X_i$ .
  - iii. Update  $Xw$  according to the equation  $Xw^{new} = Xw + X_i(w_i^{new} - w_i)$ .
  - iv. Set  $w_i$  to  $w_i^{new}$ .
- (b) Compute and print out the loss,  $\|Xw - y\|^2$

The serial version of this algorithm is implemented in `lc_serial.c`.

Steps 5 and 5a cannot be parallelized, as their iterations need to be run sequentially. Steps 5aii and 5aiv also cannot be parallelized, as they are just single operations. We can parallelize the remaining steps as follows:

1. Each thread iterates over its own range in 0 to  $m$  and sets  $w_i$  to 0 in that range.
2. Each thread iterates over its own range in 0 to  $n$  and sets the  $i$ th elements of  $Xw$  to 0 in that range.
3. Each thread iterates over its own range in 0 to  $m$  and computes  $X_i^T X_i$  on that range.
4. Each thread iterates over its own range in 0 to  $m$  and sets the  $i$ th rows of  $X^T$  on that range.
- 5ai. Each thread iterates over its own range in 0 to  $m$  and computes part of the numerator over that section of the involved matrices/vectors. The individual sums are added together to get the numerator.
- 5aiii. Each thread iterates over its own range in 0 to  $m$  and computes  $Xw^{new}$  on that range.
- 5b. Each thread iterates over its own range in 0 to  $m$  and computes part of the loss over that section over those matrices/vectors. The individual sums are then added together to get the total loss.

## 2 Results

Timing results:

Threads	Small dataset		MNIST dataset	
	lc_pthreads	lc_openmp	lc_pthreads	lc_openmp
1	0.0370s	0.0542s	6.6285s	9.6588s
2	0.0279s	0.0282s	4.2439s	5.0091s
4	0.0214s	0.0157s	2.7652s	2.6089s
8	0.0252s	0.0102s	2.4685s	1.4196s
16	0.0304s	0.0096s	3.1688s	0.9735s

Weights from small dataset. Raw values can be found in the file `small_weights.txt`:

Index	Weight
0	0.085782
1	0.077807
2	0.075450
3	0.081108
4	0.082502
5	0.088705
6	0.098613
7	0.107170
8	0.106483
9	0.096340

Weights from MNIST dataset. Raw values can be found in the file `MNIST_weights.txt`:

Index	Weight	Index	Weight	Index	Weight	Index	Weight
0	0.000000	50	0.000000	100	-0.007326	150	0.021472
1	0.000000	51	0.000000	101	-0.049297	151	0.063164
2	0.000000	52	0.000000	102	0.404771	152	0.126469
3	0.000000	53	0.000000	103	-0.519793	153	0.115376
4	0.000000	54	0.000000	104	0.836800	154	0.151555
5	0.000000	55	0.000000	105	-2.676507	155	0.219725
6	0.000000	56	0.000000	106	1.214651	156	0.129269
7	0.000000	57	0.000000	107	0.000000	157	0.207881
8	0.000000	58	0.000000	108	0.000000	158	0.076247
9	0.000000	59	0.000000	109	0.000000	159	0.086470
10	0.000000	60	0.000000	110	0.000000	160	0.100062
11	0.000000	61	0.000000	111	0.000000	161	0.141866
12	0.000000	62	0.000000	112	0.000000	162	0.178016
13	0.000000	63	0.000000	113	0.000000	163	0.006952
14	0.000000	64	0.000000	114	0.000000	164	0.062815
15	0.000000	65	0.000000	115	0.000000	165	1.786331
16	0.000000	66	0.000000	116	0.239237	166	-0.822882
17	0.000000	67	12.987359	117	0.000000	167	0.000000
18	0.000000	68	0.051548	118	0.000000	168	0.000000
19	0.000000	69	0.393589	119	1.746930	169	0.000000
20	0.000000	70	-0.264375	120	-0.350130	170	-0.499954
21	0.000000	71	0.236574	121	0.376704	171	0.326779
22	0.000000	72	0.152835	122	0.271692	172	-0.031915
23	0.000000	73	-13.923561	123	0.040371	173	-0.093464
24	0.000000	74	-0.102720	124	-0.011677	174	0.000736
25	0.000000	75	0.000000	125	0.067203	175	0.129968
26	0.000000	76	0.000000	126	-0.019514	176	-0.083525
27	0.000000	77	0.000000	127	0.020456	177	-0.195940
28	0.000000	78	0.000000	128	0.053103	178	-0.045321
29	0.000000	79	0.000000	129	0.045893	179	0.006471
30	0.000000	80	0.000000	130	0.032673	180	-0.121955
31	0.000000	81	0.000000	131	-0.004911	181	-0.036847
32	0.000000	82	0.000000	132	0.007626	182	-0.025118
33	0.000000	83	0.000000	133	-0.065457	183	-0.027645
34	0.000000	84	0.000000	134	-0.045406	184	-0.040721
35	0.000000	85	0.000000	135	0.245762	185	-0.096115
36	0.000000	86	0.000000	136	-0.438799	186	0.023831
37	0.000000	87	0.000000	137	0.000000	187	-0.020653
38	0.000000	88	0.000000	138	0.000000	188	0.010907
39	0.000000	89	0.000000	139	0.000000	189	-0.138990
40	0.000000	90	0.000000	140	0.000000	190	-0.146661
41	0.000000	91	0.000000	141	0.000000	191	0.227304
42	0.403563	92	0.000000	142	1.189222	192	0.224015
43	0.000000	93	-0.147281	143	0.000000	193	-4.896863
44	0.000000	94	-0.131344	144	1.093981	194	0.000000
45	0.000000	95	-1.037606	145	-0.938003	195	0.000000
46	0.000000	96	0.683665	146	0.090064	196	-1.529101
47	0.000000	97	-0.244655	147	-0.202659	197	-0.154254
48	0.000000	98	0.136661	148	0.411374	198	0.312838
49	0.000000	99	0.000119	149	-0.507071	199	-0.184444

Index	Weight	Index	Weight	Index	Weight	Index	Weight
200	-0.059179	250	0.000000	300	0.021065	350	0.317627
201	-0.027950	251	0.000000	301	0.017996	351	0.040831
202	-0.109623	252	4.465733	302	0.055413	352	-0.053667
203	0.003589	253	0.205990	303	0.171224	353	-0.047309
204	0.019878	254	0.023950	304	-0.182209	354	-0.015200
205	-0.001513	255	-0.062759	305	0.045277	355	-0.038937
206	-0.008211	256	-0.025191	306	0.597107	356	-0.038340
207	-0.060887	257	0.004727	307	-0.808531	357	0.009100
208	-0.016978	258	-0.019160	308	0.667049	358	-0.181476
209	-0.020237	259	0.074315	309	-0.002651	359	-0.034584
210	-0.028303	260	-0.130102	310	0.178182	360	0.706422
211	-0.052795	261	-0.002475	311	0.008410	361	-0.695116
212	0.000485	262	-0.050499	312	-0.119645	362	0.000000
213	-0.055951	263	-0.092816	313	0.007647	363	0.000000
214	-0.019855	264	0.008192	314	-0.025730	364	0.000000
215	0.061437	265	0.045368	315	-0.046087	365	-1.052986
216	-0.021363	266	-0.012614	316	0.029939	366	0.021074
217	0.116053	267	-0.059841	317	-0.024486	367	0.032655
218	0.082764	268	-0.011803	318	-0.019424	368	-0.274978
219	-0.065981	269	-0.008385	319	0.010958	369	0.009517
220	-0.213218	270	0.001182	320	0.002173	370	-0.079271
221	-0.095757	271	0.001842	321	-0.139689	371	0.081488
222	0.142965	272	0.004107	322	-0.032178	372	-0.062065
223	0.000000	273	-0.036257	323	0.074776	373	-0.111165
224	0.126168	274	0.027511	324	-0.059912	374	-0.013024
225	-0.285753	275	-0.085096	325	-0.068954	375	-0.054088
226	-0.122659	276	-0.004634	326	-0.051707	376	0.232984
227	0.010747	277	-0.399918	327	-0.040667	377	0.105593
228	-0.067309	278	-1.779999	328	0.014062	378	0.401891
229	0.186079	279	-0.113281	329	0.003266	379	-0.087088
230	-0.002979	280	-248.839479	330	0.016129	380	0.056730
231	-0.083971	281	-0.273016	331	0.125939	381	-0.035660
232	0.016701	282	-0.084492	332	-0.185239	382	0.016467
233	0.043679	283	0.094041	333	0.533909	383	0.014458
234	-0.040777	284	-0.007951	334	17.812770	384	-0.036232
235	-0.119654	285	0.004452	335	0.000000	385	-0.078028
236	-0.064479	286	-0.046347	336	0.000000	386	-0.103781
237	-0.017134	287	-0.036485	337	0.099877	387	0.134876
238	-0.019433	288	-0.002184	338	-0.137718	388	-0.907634
239	-0.053136	289	0.012407	339	-0.119351	389	0.192573
240	-0.030955	290	-0.073817	340	0.072362	390	-13.914623
241	0.007546	291	0.030072	341	-0.030359	391	0.000000
242	0.053089	292	0.052285	342	0.066968	392	0.000000
243	0.006358	293	-0.064317	343	0.014713	393	-8.032924
244	0.015594	294	-0.091785	344	-0.077926	394	0.578574
245	-0.031844	295	-0.035475	345	0.030972	395	0.072917
246	-0.051981	296	-0.036686	346	0.049349	396	-0.020414
247	0.039805	297	-0.049523	347	-0.104626	397	-0.010964
248	-0.040508	298	-0.044281	348	-0.062493	398	-0.083583
249	0.667843	299	-0.063768	349	0.030078	399	-0.035787

Index	Weight	Index	Weight	Index	Weight	Index	Weight
400	-0.046153	450	-25.332182	500	-0.319787	550	0.129459
401	0.139062	451	-0.842183	501	-1.234571	551	-0.005925
402	-0.032610	452	-0.581136	502	-1.858744	552	0.165702
403	0.093645	453	0.055001	503	5.688208	553	-0.232492
404	-0.078055	454	-0.184132	504	0.000000	554	-0.508934
405	0.093204	455	0.018367	505	14.495603	555	0.029519
406	0.143825	456	-0.025428	506	0.029609	556	-1.111918
407	-0.002579	457	0.001729	507	-0.272286	557	0.718296
408	-0.019800	458	0.002517	508	-0.400680	558	0.000000
409	-0.065768	459	-0.051129	509	-0.233000	559	-0.304070
410	-0.012284	460	-0.029314	510	0.102372	560	0.000000
411	0.003540	461	-0.000440	511	0.029991	561	-2.462945
412	-0.036298	462	0.060139	512	-0.039485	562	-10.411344
413	0.045493	463	-0.000435	513	-0.020060	563	-0.134921
414	0.017233	464	-0.052892	514	-0.008819	564	0.586500
415	-0.262551	465	-0.000255	515	0.012361	565	-0.455416
416	-0.458219	466	-0.052628	516	0.016709	566	-0.179787
417	0.927049	467	-0.069256	517	-0.025753	567	-0.132657
418	0.410847	468	-0.088731	518	0.001287	568	-0.012116
419	0.000000	469	0.060763	519	0.020156	569	0.055675
420	-8.685817	470	0.006446	520	0.037843	570	0.013127
421	0.000000	471	0.074751	521	0.066784	571	0.011632
422	0.000000	472	0.052634	522	-0.037968	572	-0.022642
423	0.243208	473	0.250613	523	-0.148600	573	-0.026084
424	0.178327	474	0.000000	524	0.130455	574	-0.000615
425	0.024150	475	0.000000	525	-0.256891	575	-0.002987
426	0.169748	476	0.000000	526	0.362308	576	0.025979
427	-0.057198	477	0.811022	527	0.095027	577	0.063206
428	0.108685	478	-3.508993	528	0.445491	578	0.136171
429	-0.129777	479	2.424739	529	2.191192	579	-0.046480
430	-0.077024	480	-0.421863	530	-0.340096	580	-0.210957
431	-0.106309	481	0.430718	531	0.706388	581	0.150137
432	0.000649	482	-0.149879	532	0.000000	582	0.271614
433	0.081084	483	0.083414	533	-0.380956	583	0.968553
434	0.034155	484	0.066961	534	1.204313	584	-0.252393
435	0.013487	485	0.074208	535	0.105744	585	-0.243466
436	-0.053292	486	-0.044743	536	-0.508197	586	0.000000
437	-0.045110	487	-0.054290	537	0.600230	587	0.000000
438	-0.016241	488	0.004557	538	-0.330184	588	-1.113552
439	0.005684	489	-0.015733	539	0.061248	589	0.000000
440	0.125929	490	0.058904	540	-0.064969	590	0.000000
441	-0.166887	491	-0.012205	541	0.018006	591	2.261483
442	-0.039889	492	0.005583	542	-0.008948	592	-0.052684
443	-0.120305	493	0.029152	543	0.080493	593	0.374361
444	0.375053	494	-0.009343	544	-0.041904	594	0.176239
445	-0.453969	495	0.059740	545	0.027249	595	0.326205
446	1.655480	496	-0.148277	546	-0.020755	596	0.072901
447	0.000000	497	0.188470	547	0.047566	597	-0.004603
448	0.302300	498	-0.091244	548	0.075046	598	-0.022993
449	-4.435985	499	-0.085046	549	0.017483	599	0.003594

Index	Weight	Index	Weight	Index	Weight	Index	Weight
600	-0.018626	650	-0.195903	700	0.000000	750	0.346499
601	0.001573	651	-0.010081	701	0.000000	751	-1.504359
602	-0.011066	652	-0.123173	702	0.000000	752	0.000000
603	0.007779	653	0.035067	703	0.753028	753	0.000000
604	-0.007261	654	0.009721	704	-0.508102	754	0.000000
605	-0.069711	655	0.003292	705	0.246303	755	0.000000
606	-0.065467	656	-0.001983	706	0.077869	756	0.000000
607	0.234823	657	0.002835	707	-0.150077	757	0.000000
608	0.030825	658	-0.024618	708	0.085474	758	0.000000
609	0.227108	659	-0.026290	709	-0.043162	759	0.000000
610	-0.021678	660	-0.011121	710	-0.027328	760	0.079685
611	0.562302	661	0.071700	711	-0.043222	761	-0.535093
612	-0.696347	662	-0.037119	712	0.004206	762	0.573362
613	1.115765	663	-0.040544	713	0.004106	763	-0.562727
614	0.210318	664	0.132051	714	0.047066	764	0.543879
615	0.000000	665	0.052738	715	-0.060931	765	-0.029345
616	0.162797	666	0.158742	716	-0.044357	766	0.015804
617	0.000000	667	-0.724996	717	-0.107667	767	-0.029969
618	0.000000	668	0.515968	718	-0.258108	768	0.001986
619	-2.155993	669	0.000000	719	-0.001071	769	-0.115815
620	0.322670	670	0.000000	720	0.239737	770	0.007334
621	-0.166706	671	0.000000	721	0.274898	771	-0.108838
622	0.125919	672	0.000000	722	-0.994054	772	0.010197
623	-0.052014	673	0.000000	723	4.797108	773	-0.094559
624	0.047803	674	-0.343633	724	0.126343	774	-0.019316
625	0.042168	675	-0.170409	725	0.000000	775	-0.092565
626	-0.001109	676	-0.334390	726	0.000000	776	-0.447288
627	0.005207	677	-0.047393	727	0.000000	777	0.209396
628	-0.016561	678	-0.025653	728	0.000000	778	-1.279342
629	0.063505	679	-0.049076	729	0.000000	779	-0.143597
630	-0.040736	680	0.057906	730	0.000000	780	0.000000
631	0.008214	681	0.006372	731	0.000000	781	0.000000
632	-0.044166	682	0.044818	732	0.325264	782	0.000000
633	-0.044067	683	0.042977	733	-0.403374	783	0.000000
634	0.033685	684	0.066207	734	0.028788		
635	-0.142947	685	0.029532	735	0.096907		
636	0.086042	686	0.023451	736	-0.105736		
637	-0.012293	687	0.082099	737	-0.006565		
638	-0.496915	688	0.013682	738	-0.009703		
639	0.827429	689	0.091892	739	0.012954		
640	0.682105	690	0.170753	740	-0.016659		
641	-1.758731	691	0.136179	741	-0.065025		
642	0.000000	692	-0.397666	742	-0.024660		
643	0.000000	693	0.060628	743	0.042890		
644	0.000000	694	0.731323	744	0.012772		
645	0.000000	695	2.146371	745	0.077616		
646	0.000000	696	-0.724315	746	0.155224		
647	1.272419	697	-0.683131	747	0.110093		
648	-0.512667	698	0.000000	748	-0.075148		
649	0.199520	699	0.000000	749	0.085505		