

PLEASE NOTE: Each * will indicate one full mark earned on solutions where full marks are not awarded.

1/2 marks

31.

$$f(x) = 2x(x - 3)^3(x + 1)(4x - 2) = 2x(x^3 \dots)(4x^2 \dots) = 8x^6 \dots$$

The dominant term of this polynomial is $8x^6$. Degree 6

2/2 marks

32.

$$D = \{x \in \mathbb{R}\}$$
$$R = \{y \in \mathbb{R}, y \geq -5\}$$

3/3 marks

33. (a) with (ii).
(b) with (iii).
(c) with (i).

2/2 marks

34.

$$\begin{aligned}x &\rightarrow -\infty, y \rightarrow \infty \\x &\rightarrow \infty, y \rightarrow -\infty\end{aligned}$$

3/6 marks

35. a) 1.5*

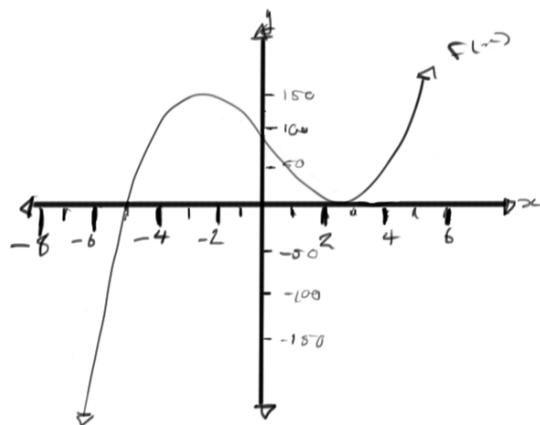
a) $f(x) = 3(x - 2)^2(x + 5)$

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x -intercept where graph cuts through the axis $x = -5$

x -intercept where graph bounces off the axis $x = 2$

$x \rightarrow -\infty \quad f(x) \rightarrow -\infty, \quad x \rightarrow \infty \quad f(x) \rightarrow \infty$

b) 1.5*

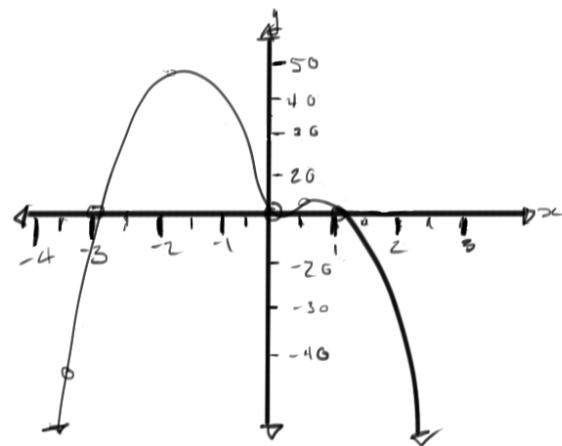
b) $g(x) = -4x^2(x+3)(x-1)$

x-intercept where graph cuts through the axis $x = -3, 1$

x-intercept where graph bounces off the axis $x = 0$

$x \rightarrow -\infty \quad f(x) \rightarrow -\infty, \quad x \rightarrow \infty \quad f(x) \rightarrow -\infty$

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36.

$$\begin{aligned}y &= ax(x + 4)^2(x - 1) \\24 &= a(-2)(4 - 2)^2(-2 - 1) \\-12 &= a(2)^2(-3) \\4 &= 4a \\a &= 1 \\\therefore y &= x(x + 4)^2(x - 1)\end{aligned}$$

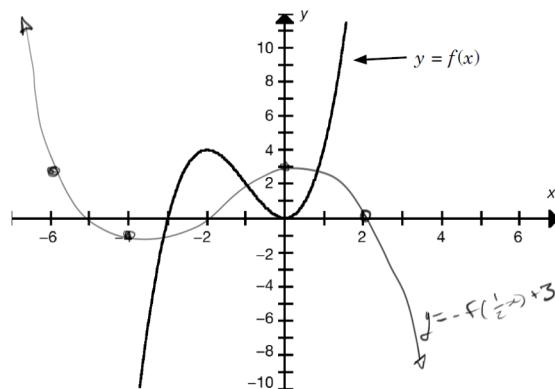
5/5 marks

37. a) Horizontal translation of 2 left
Reflection in the x -axis
Vertical translation of 5 down

b) Reflection in the y -axis
Vertical translation of 5 up

3/3 marks

38.



3/3 marks

39. a) Neither
b) Even
c) Odd

3/3 marks

40.

$$\begin{aligned}
 y &= a(x - 2)(x + 2)(x - 1) \\
 24 &= a(0 - 2)(0 + 2)(0 - 1) = a(-2)(2)(-1) = 4a \\
 a &= \frac{24}{4} = 6 \\
 \therefore y &= 6(x - 2)(x + 2)(x - 1)
 \end{aligned}$$

8/8 marks

41. (a) and (b):

Equation	V.A.	H.A.	x-intercepts	y-intercepts
$f(x) = \frac{4x + 8}{x - 3}$	3	4	-2	$\frac{8}{-3}$
$f(x) = \frac{1}{x^2 + 4}$	none	0	None	$\frac{1}{4}$

1.5/2 marks

42. a)

$$\begin{aligned}
 \frac{3x - 2}{x + 4} &= 0 \\
 3x - 2 &= 0; x \neq -4 \\
 3x &= 2; x \neq -4
 \end{aligned}$$

 $x = \frac{2}{3}; x \neq -4$ only the numerator

b)

$$\begin{aligned}
 \frac{2}{x - 3} &= 0 \\
 2 &= 0; x \neq 3
 \end{aligned}$$

This function at no point equals zero because the H.A. is equal to 0 and 2 is not equal to 0 by basic arithmetic. Therefore,

$$2 \neq 0; x \neq 3$$

4/4 marks

43. Finding the equation:

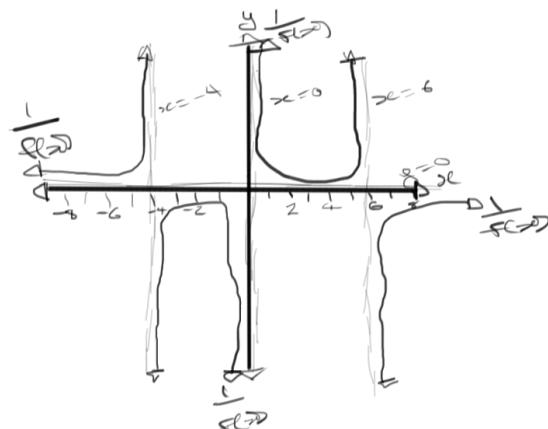
$$\begin{aligned}
 f(x) &= a(x + 4)(x - 6)x \\
 8 &= a(4 + 4)(4 - 6)(4) = -64a \\
 a &= \frac{8}{-64} = \frac{1}{-8} \\
 \therefore f(x) &= \frac{(x + 4)(x - 6)x}{-8}
 \end{aligned}$$

$$\frac{1}{f(x)} = \frac{-8}{(x+4)(x-6)x}$$

Asymptotes are:

$$x = -4, 0, 6$$

$$y = 0$$



6/6 marks

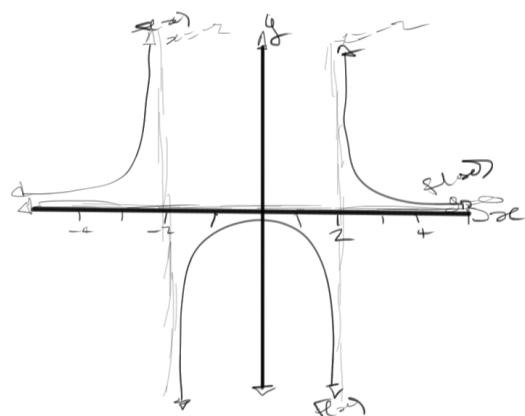
44. a) Asymptotes:

$$x = \pm 2$$

$$y = 4$$

y-intercepts: $-\frac{1}{4}$

x-intercepts: none



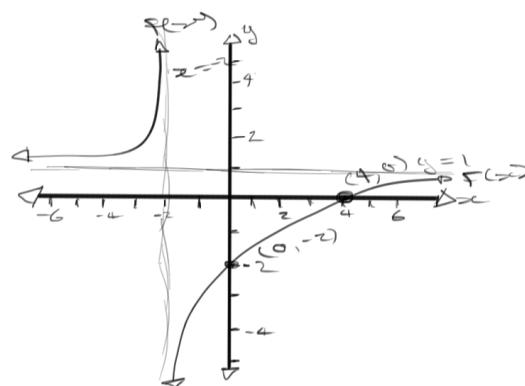
b) Asymptotes:

$$x = -2$$

$$y = 1$$

y-intercepts: -2

x-intercepts: 4



6/6 marks

45. a)

$$x^3 - 4x^2 - 11x + 30$$

$$f(2) = (2)^3 - 4(2)^2 - 11(2) + 30 = 8 - 16 - 22 + 30 = 38 - 38 = 0$$

$$\begin{array}{r}
 2x^2 + x - 1 \\
 \underline{-} (x^3 - 4x^2 - 11x + 30) \\
 \underline{\underline{- (x^3 - 2x^2)}} \\
 \underline{\underline{- 2x^2 - 11x}} \\
 \underline{\underline{- (-2x^2 + 4x)}} \\
 \underline{\underline{- 15x + 30}} \\
 \underline{\underline{- (-15x + 30)}} \\
 0
 \end{array}$$

$$\begin{aligned}
 &= (x - 2)(x^2 - 2x - 15) \\
 &= (x - 2)(x + 3)(x - 5)
 \end{aligned}$$

b)

$$\begin{aligned}
 2x^3 + 3x^2 - 1 \\
 f(-1) = 2(-1)^3 + 3(-1)^2 - 1 = 3 - 2 - 1 = 0
 \end{aligned}$$

$$\begin{array}{r}
 2x^2 + x - 1 \\
 \underline{- (2x^3 + 3x^2 + 0x - 1)} \\
 \underline{\underline{- (2x^3 + 2x^2)}} \\
 \underline{\underline{x^2 + 0x}} \\
 \underline{\underline{- (x^2 + x)}} \\
 \underline{\underline{- x - 1}} \\
 \underline{\underline{- (-x - 1)}} \\
 0
 \end{array}$$

$$f(x) = (x + 1)(2x^2 + x - 1) = (x + 1)(2x - 1)(x + 1) = (x + 1)^2(2x - 1)$$

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4/4 marks

46.

$$\begin{aligned}
 2x^3 + 6x^2 - x &= 3 \\
 2x^3 + 6x^2 - x - 3 &= 0 \\
 f(-3) &= 2(-3)^3 + 6(-3)^2 - (-3) - 3 = -54 + 54 + 3 - 3 = 0
 \end{aligned}$$

$$\begin{array}{r}
 \overline{2x^2 + 0 - 1} \\
 x+3 \overline{)2x^3 + 6x^2 - 2x - 3} \\
 \underline{- (2x^3 + 6x^2)} \\
 \overline{0 - 2x} \\
 \underline{-(0 - 0)} \\
 \overline{-2x - 3} \\
 \underline{-(-2x - 3)} \\
 \overline{0}
 \end{array}$$

$$(x+3)(2x^2 - 1) = 0$$

$$(x+3)\left(x - \frac{\sqrt{2}}{2}\right)\left(x + \frac{\sqrt{2}}{2}\right) = 0$$

7/7 marks

47. a)

$$\begin{aligned}
 f(2) &= 0 = 2x^3 - 3x + k = 2(2)^3 - 3(2) + k \\
 0 &= 16 - 6 + k = 10 + k \\
 \therefore k &= -10
 \end{aligned}$$

b) For $(x-1)$:

$$\begin{array}{r}
 \overline{ax^3 - 4x^2 + 5x - 3} \\
 x-1 \overline{)ax^3 - ax^2} \\
 \underline{- (ax^3 - ax^2)} \\
 \overline{(a-4)x^2 + 5x - 3} \\
 \underline{-(a-4)x^2 - (a-4)x} \\
 \overline{(a+1)x - 3} \\
 \underline{-(a+1)x - (a+1)} \\
 \overline{a-2}
 \end{array}$$

$$\therefore ax^3 - 4x^2 + 5x - 3 = \frac{ax^2 + (a-4)x + (a-1)}{x-1} + (a-2)$$

For $(x+2)$:

$$\begin{array}{r}
 \overline{ax^2 + (-2a-4)x + (4a+13)} \\
 x+2 \overline{(ax^2 - 4x^2 + 5x - 3)} \\
 \underline{- (ax^2 + 2ax^2)} \\
 \underline{(-2a-4)x^2 + 5x} \\
 \underline{- ((-2a-4)x^2 + (-4a-8)x)} \\
 \underline{(4a+13)x - 3} \\
 \underline{- ((4a+13)x + (8a+26))} \\
 \hline
 -8a - 29
 \end{array}$$

$$\therefore ax^3 - 4x^2 + 5x - 3 = \frac{ax^2 + (-2a-4)x + (4a+13)}{x+2} + (-8a-29)$$

From the Remainder:

$$R = a - 2 = -8a - 29$$

$$a + 8a = 2 - 29$$

$$9a = -27$$

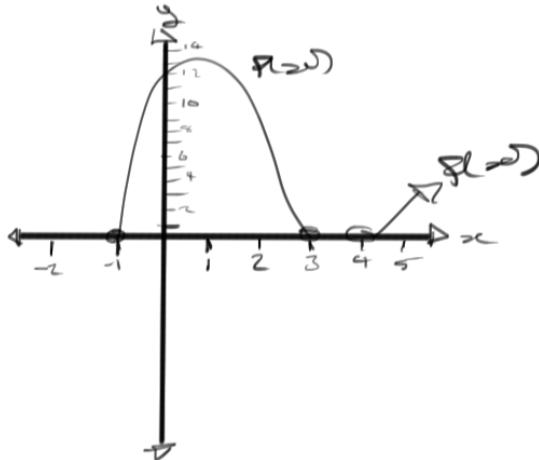
$$a = -3$$

5/10 marks

48. a) 5*

$$(x-3)(x-4)(x+1) > 0$$

$$-1 < x < 3, \quad x > 4$$



b)

$$\frac{x}{(x-4)(x+2)} \leq 0$$

$$-2 < x \leq 0, \quad x > 4$$

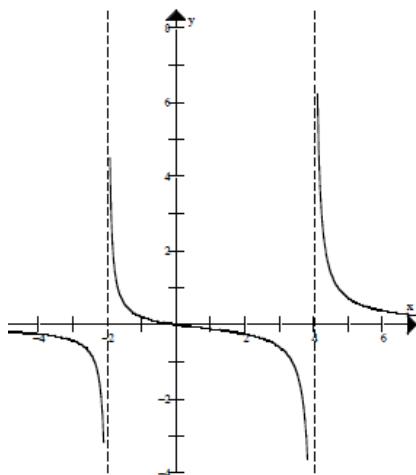
$$\frac{x}{x^2 - 2x - 8} \leq 0$$

$$\frac{x}{(x-4)(x+2)} \leq 0$$

Zero at $x = 0$

Vertical asymptotes at $x = -2, x = 4$

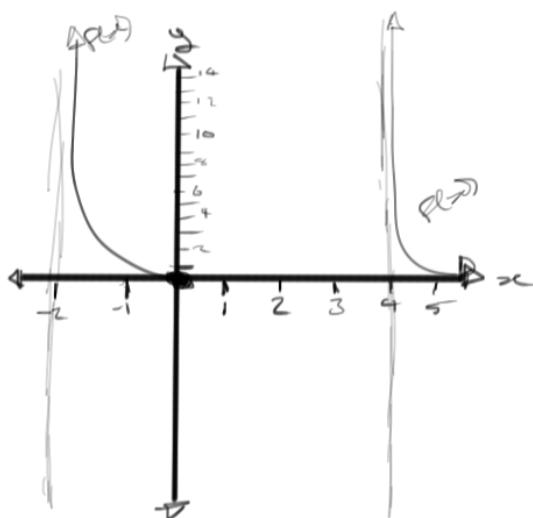
Graph $f(x) = \frac{x}{x^2 - 2x - 8}$



$$\frac{x}{x^2 - 2x - 8} \leq 0$$

$$x < -2 \text{ or } 0 \leq x < 4$$

(5 marks)

1/8 marks

49. a)

$$0 \geq x^2 + 5x + 6$$

There is no solution since at no point was the function below or equal to zero.

49. a)

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$$x^2 + 5x + 6 \leq 0$$

$$(x + 3)(x + 2) \leq 0$$

Zeros at $x = -3, -2$

Expression	$x < -3$	$x = -3$	$-3 < x < -2$	$x = -2$	$x > -2$
$(x + 3)(x + 2)$	+	0	-	0	+

$$x^2 + 5x + 6 \leq 0$$

$$-3 \leq x \leq -2$$

(4 marks)

b)

$$0 < \frac{3}{x^2 - 4} = \frac{3}{(x-2)(x+2)}$$

$x < -2, x > 2$ *

b)

$$\frac{3}{x^2 - 4} > 0$$

$$\frac{3}{(x-2)(x+2)} > 0$$

No zeros, vertical asymptotes at $x = -2, x = 2$

Expression	$x < -2$	$x = -2$	$-2 < x < 2$	$x = 2$	$x > 2$
$\frac{3}{(x-2)(x+2)}$	+	Undefined	-	Undefined	+

$$\frac{3}{x^2 - 4} > 0$$

$x < -2$ or $x > 2$

(4 marks)

70.5/87*100 = 81%

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