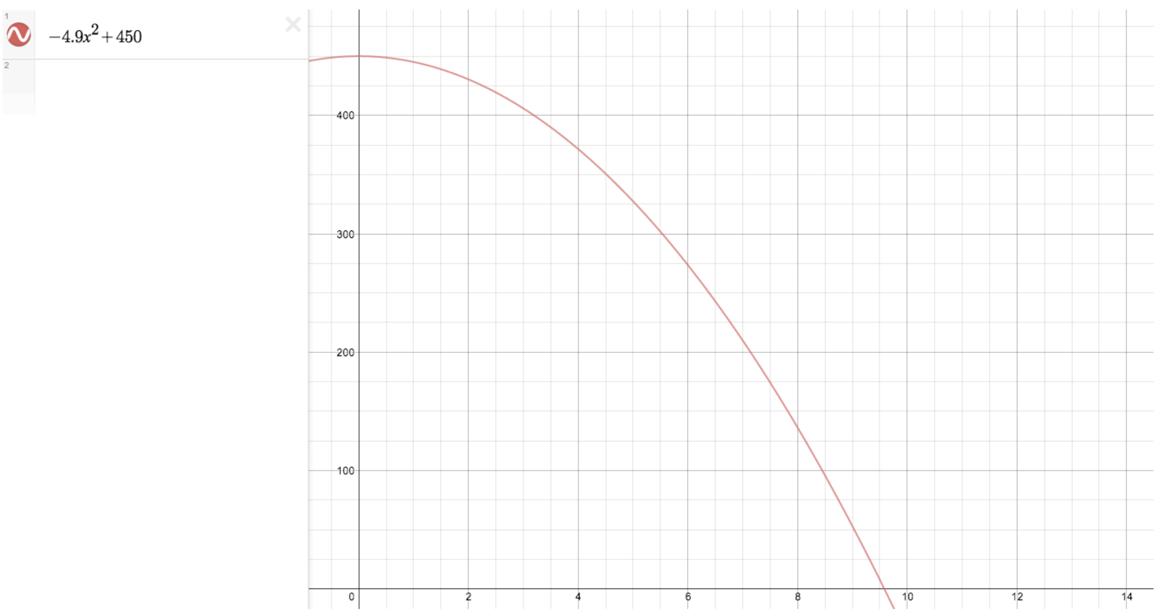
Lessons	Potential Score	Actual Score
1	20	19
2	13	10
3	14	9
4	16	14
5	26	26

Unit Total: 78/89 or 88%

1. a)



3/3

b)

$$v_a = \frac{f(b)-f(a)}{b-a} = \frac{f(2)-f(0)}{2-0} = \frac{-4.9(2)^2+4.9(0)^2}{2} = -\frac{19.6}{2} = -9.8$$

1/1

c) i)

$$v_a = \frac{f(4)-f(1)}{4-1} = \frac{-4.9\left(4\frac{\mathrm{m}}{\mathrm{s}}\right)^2+4.9\left(1\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{3} = \frac{-78.4\frac{\mathrm{m}}{\mathrm{s}}+4.9\frac{\mathrm{m}}{\mathrm{s}}}{3} = -\frac{73.5\frac{\mathrm{m}}{\mathrm{s}}}{3} = -24.5\,\frac{\mathrm{m}}{\mathrm{s}}$$

1/1

ii)

$$v_a = \frac{f(2) - f(1)}{2 - 1} = -4.9\left(2\frac{\text{m}}{\text{s}}\right)^2 + 4.9\frac{\text{m}}{\text{s}} = -19.6\frac{\text{m}}{\text{s}} + 4.9\frac{\text{m}}{\text{s}} = -14.7\frac{\text{m}}{\text{s}}$$

1/1

iii)

$$v_a = \frac{f(1.5) - f(1)}{1.5 - 1} = 2\left(-4.9\left(1.4\frac{\text{m}}{\text{s}}\right)^2 + 4.9\frac{\text{m}}{\text{s}}\right) = 2\left(-11.025\frac{\text{m}}{\text{s}} + 4.9\frac{\text{m}}{\text{s}}\right) = 2\left(-6.125\frac{\text{m}}{\text{s}}\right) = -12.25\frac{\text{m}}{\text{s}}$$

1/1

d)

$$v_a = \frac{f(1.001) - f(0.999)}{1.001 - 0.999} = \frac{-4.9\left(1.001\frac{\text{m}}{\text{s}}\right)^2 + 4.9\left(0.999\frac{\text{m}}{\text{s}}\right)^2}{0.002} = \frac{-4.9\left(1.002001\frac{\text{m}}{\text{s}}\right) + 4.9(0.998001\frac{\text{m}}{\text{s}})}{0.002} = \frac{-4.9049098\frac{\text{m}}{\text{s}} + 4.8902049\frac{\text{m}}{\text{s}}}{0.002} = \frac{-0.0147049\frac{\text{m}}{\text{s}}}{0.002} = -7.35245\frac{\text{m}}{\text{s}}$$

1/1

2. a)

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0\text{s} \pm \sqrt{0\text{s} - 4(-0.4\text{s})(10.5\text{s})}}{2(0.4)} = \frac{\pm\sqrt{-4(-0.4\text{s})(10.5\text{s})}}{0.8} = \frac{\pm\sqrt{16.8\text{s}^2}}{0.8} \cong \frac{\pm 4.02\text{s}}{0.8} \cong \pm 5.03\text{s}$$

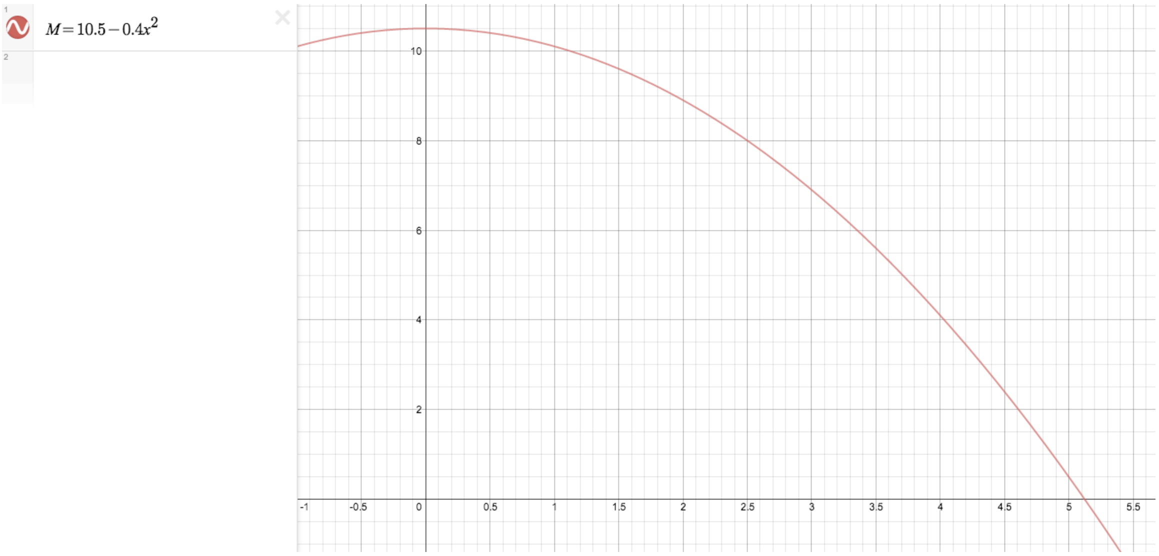
2/2

b)

$$\frac{f(1) - f(0)}{1 - 0} = f(1) - f(0) = 10.5\text{s} - 0.4\text{s}(1)^2 + 10.5\text{s} - 0.4\text{s}(0)^2 = 0.4\text{s}$$

1/1

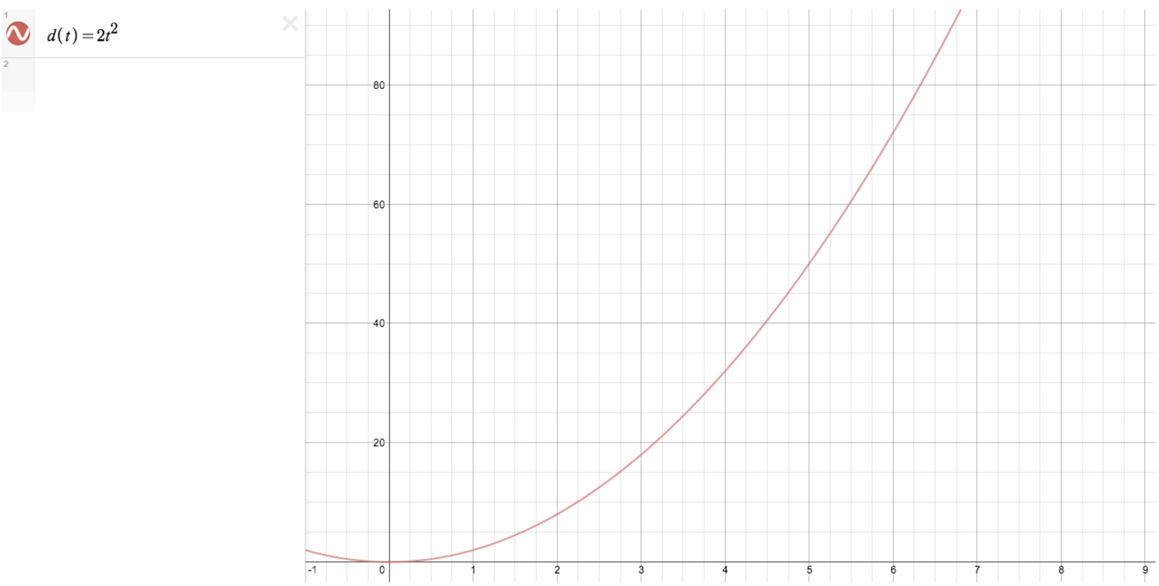
c)



3/3

What happens at time =2 seconds? 0/1

3. a)



3/3

b)

$$s_a = \frac{f(7) - f(4)}{7 - 4} = \frac{2(7)^2 - 2(4)^2}{3} = \frac{2s(49) - 2s(16)}{3} = \frac{98s - 32s}{3} = \frac{66s}{3} = 22s$$

1/1

c)

$$s = \frac{f(4.001) - f(3.999)}{4.001 - 3.999} = \frac{2s(4.001)^2 - 2s(3.999)^2}{0.002} = \frac{2s(16.008001) - 2s(15.992001)}{0.002}$$
$$= \frac{32.016002s - 31.984002s}{0.002} = \frac{0.032s}{0.002} = 16s$$

1/1

Lesson Total: 19/20

4. a) To determine the instantaneous rate of change of a function I would use function:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This is effective because it would allow me to specifically identify the rate of change at a given point instead of making approximations. 2/2 What would f(a+h) and f(a) stand for? Explain further. 0/1

b) i)

$$\lim_{x \to 0} \frac{x-3}{2x^2-5} = \frac{0-3}{2(0)^2-5} = \frac{-3}{-5} = \frac{3}{5}$$

1/1

ii)

$$\lim_{x \to 2} \frac{2x^2 - 7x + 6}{x-2} = \lim_{x \to 2} \frac{(2x-3)(x-2)}{x-2} = \lim_{x \to 2} 2x - 3 = 2(2) - 3 = 1$$

2/2

5. a)

$$= \lim_{h \to 0} -5h\frac{\text{m}}{\text{s}} + 20\frac{\text{m}}{\text{s}} = -5(0)\frac{\text{m}}{\text{s}} + 20\frac{\text{m}}{\text{s}} = 20\frac{\text{m}}{\text{s}}$$

Here there is an error in calculations so try again to ensure that there are no errors; particularly in the addition. 2/4 b)

$$\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}=\lim_{h\to 0}\frac{(4+h)^2\frac{\mathrm{m}}{\mathrm{s}}-8(4+h)\frac{\mathrm{m}}{\mathrm{s}}+15\frac{\mathrm{m}}{\mathrm{s}}-\left(4^2\frac{\mathrm{m}}{\mathrm{s}}-8(4)\frac{\mathrm{m}}{\mathrm{s}}+15\frac{\mathrm{m}}{\mathrm{s}}\right)}{h}$$
$$=\lim_{h\to 0}\frac{\left(16\frac{\mathrm{m}}{\mathrm{s}}+8h\frac{\mathrm{m}}{\mathrm{s}}+h^2\frac{\mathrm{m}}{\mathrm{s}}-32\frac{\mathrm{m}}{\mathrm{s}}-8h\frac{\mathrm{m}}{\mathrm{s}}+15\frac{\mathrm{m}}{\mathrm{s}}\right)-16\frac{\mathrm{m}}{\mathrm{s}}+32\frac{\mathrm{m}}{\mathrm{s}}-15\frac{\mathrm{m}}{\mathrm{s}}}{h}$$
$$=\lim_{h\to 0}\frac{h^2\frac{\mathrm{m}}{\mathrm{s}}+8h\frac{\mathrm{m}}{\mathrm{s}}-8h\frac{\mathrm{m}}{\mathrm{s}}-16\frac{\mathrm{m}}{\mathrm{s}}+15\frac{\mathrm{m}}{\mathrm{s}}+16\frac{\mathrm{m}}{\mathrm{s}}-15\frac{\mathrm{m}}{\mathrm{s}}}{h}=\lim_{h\to 0}\frac{h^2\frac{\mathrm{m}}{\mathrm{s}}}{h}$$
$$=\lim_{h\to 0}h\frac{\mathrm{m}}{\mathrm{s}}=0\frac{\mathrm{m}}{\mathrm{s}}$$

3/3

Lesson Total: 10/13

6. a) Positive: $x<-1, x>1$ 1/1

Negative: $-1<x<1$ 1/1

b) $x=\pm 1$ 1/1

c) $max: f(-1)=2$

$min: f(1)=-2$ 1/1

7. a)

$f(x)=2x^3-7x^2+4x+1$

$f(0)=2(0)^3-7(0)^2+4(0)+1=1$

$f(1)=2(1)^3-7(1)^2+4(1)+1=2-7+4+1=0$

2/2

$f(0+h)=2(0+h)^3-7(0+h)^2+4(0+h)+1=2h^3-7h^2+4h+1$

$$f(1+h)=2(1+h)^3-7(1+h)^2+4(1+h)+1$$
$$=2(1+h+h^2+h^3)-7(1+h+h^2)+4+4h+1$$
$$=2+2h+2h^2+2h^3-7-7h-7h^2+4h+5$$
$$=2h^3-7h^2+2h^2+4h-7h+2h+5-7+2=2h^3-5h^2-h$$

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{2h^3-7h^2+4h+1-1}{h}=\lim_{h\to 0}\frac{2h^3-7h^2+4h}{h}$$
$$=\lim_{h\to 0}2h^2-7h+4=2(0)^2-7(0)+4=4$$

1/1

$$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{2h^3-5h^2-h}{h}=\lim_{h\to 0}2h^2-5h-1=-1$$

2/2

The value obtained for the limit has an error in calculation; the middle term is calculated incorrectly due to an arithmetic error in evaluating the numerator. It should compute to:

At x = 1, you should arrive at $-4h-h^2+2h^3$

b) The function is increasing at $f(0)$ because the instantaneous rate of change (4) is positive. The function is decreasing at $f(+1)$ because the instantaneous rate of change (-1) is negative. You haven't quite understood how to write the notations, f(x) and x. We input the value we are finding into f(x); so for example if we are looking for the value of x = 4, we are finding f(4) which becomes equal to some value of y; in this case you obtained the value of 0. Review these in a pre-calculus course to avoid notation errors. 0/2

c) There should be a local maximum since the slope at $x = 0$ is changing from positive to negative as it reaches $x = 1$. 0/2

Lesson Total: 9/14

8. a)

$$\begin{aligned}f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)+1-x^2+2x-1}{h}\\&=\lim_{h\to 0}\frac{x^2+2xh+h^2-2x-2xh-2h+h^2}{h}\\&=\lim_{h\to 0}\frac{x^2-x^2-2x+2x+2xh-2h+h^2}{h}=\lim_{h\to 0}\frac{2xh-2h+h^2}{h}\\&=\lim_{h\to 0}2x-2+h=2x-2\end{aligned}$$

2/2

b)

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^3-3(x+h)^2-x^3+3x^2}{h} \\ &= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-3x^2-6xh-3h^2-x^3+3x^2}{h} \\ &= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-6xh-3h^2}{h} = \lim_{h\to 0}3x^2+3xh+h^2-6x-3h \\ &= 3x^2-6x \end{aligned}$$

4/4

c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(2(x+h))^{\frac{1}{2}}-(2x)^{\frac{1}{2}}}{h} = \lim_{h\to 0}\frac{(2x+2h)^{\frac{1}{2}}-(2x)^{\frac{1}{2}}}{h} \\ &= \lim_{h\to 0}\frac{(2x+2h)^{\frac{1}{2}}-(2x)^{\frac{1}{2}}}{h}\times\frac{\sqrt{2x+2h}+\sqrt{2x}}{\sqrt{2x+2h}+\sqrt{2x}} \\ &= \lim_{h\to 0}\frac{\left(\sqrt{2x+2h}-\sqrt{2x}\right)\left(\sqrt{2x+2h}+\sqrt{2x}\right)}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)} = \lim_{h\to 0}\frac{\left(\sqrt{2x+2h}\right)^2-\left(\sqrt{2x}\right)^2}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)} \\ &= \lim_{h\to 0}\frac{2x+2h-2x}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)} = \lim_{h\to 0}\frac{2h}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)} \\ &= \lim_{h\to 0}\frac{2}{\sqrt{2x+2h}+\sqrt{2x}} = \frac{2}{\sqrt{2x}+\sqrt{2x}} = \frac{2}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}} \end{aligned}$$

4/4

d)

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left(\frac{4}{x+h+1}\right)-\left(\frac{4}{x+1}\right)}{h} = \lim_{h\to 0}\frac{\frac{4(x+1)-4(x+h+1)}{(x+h+1)(x+1)}}{h} \\ &= \lim_{h\to 0}\frac{\frac{4x+4-4x+4h+4}{x^2+xh+x+x+xh+1}}{h} = \lim_{h\to 0}\frac{\frac{4h}{x^2+2xh+2x+1}}{h} = \lim_{h\to 0}\frac{4h}{x^2h+2xh^2+2xh+h} \\ &= \frac{4}{x^2+2x+1} \end{aligned}$$

4/4

9. The process that was used to conclude that the derivative of the cubic function is a quadratic was from identifying that leading exponent would experience a $x-1$ operationwhich resulted in the leading exponent changing from x^3 to x^2. Therefore indicating that the derivative of a cubic function was a quadratic function. You haven't quite explained why that change in exponent is made; you may want to explain your answer with reference to the first derivative

equation using a proof. However, since proofs are beyond the scope of this course, you may want to refer to technology and use the traditional plot and point equations to define your answer. 0/2

Lesson Total: 14/16

10. a)

$f'(x) = 4x^3 - 4x + 1$ 1/1

b)

$$f'(x) = 2(2x-3)(x^2-3x) = (4x-6)(x^2-3x) = 4x^3 - 12x^2 - 6x^2 + 18x$$
$$= 4x^3 - 18x^2 + 18x$$

2/2

c)

$$f'(x) = (2x)(2x^3 - 5x^2 + 4x) + (x^2+2)(6x^2 - 10x + 4)$$
$$= 4x^4 - 10^3 + 8x^2 + 6x^4 - 10x^3 + 4x^2 + 12x^2 - 20x + 8$$
$$= 8x^4 - 20x^3 + 24x^2 - 20x + 8$$

3/3

d)

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}}$$

4/4

e)

$$f'(x) = 3(x^2+4)^{-1} + (3x)(-1)(2x)(x^2+4)^{-2} = 3(x^2+4)^{-1} - (6x^2)(x^2+4)^{-2}$$
$$= \frac{3}{x^2+4} - \frac{6x^2}{(x^2+4)^2} = \frac{3(x^2+4) - 6x^2}{(x^2+4)^2}$$

4/4

11.

$$y = -\frac{3}{2}x - 1$$

$$f(x) = \frac{2}{3x-2} = 2(3x-2)^{-1}$$

$f'(x) = -2(3x-2)^{-2}(3) = -6(3x-2)^{-2} = \frac{-6}{(3x-2)^2}$

$m = -\frac{3}{2}$

1/1

$f'(x) = -\frac{3}{2}$

$\frac{-6}{(3x-2)^2} = -\frac{3}{2}$

$-12 = -3(3x-2)^2$

$0, 4 = 3x$

$0, \frac{4}{3} = x$

3/3

$f(0) = \frac{2}{3(0)-2} = \frac{2}{-2} = -1$

$f\left(\frac{4}{3}\right) = \frac{2}{3\left(\frac{4}{3}\right)-2} = \frac{2}{4-2} = \frac{2}{2} = 1$

The points are $(0,-1)$ and $\left(\frac{4}{3}, 1\right)$.

1/1

12.

$y = 7x + 3$

1/1

$f(x) = x^2 - 3x - 4$

$f'(x) = 2x - 3$

2/2

$m = 7$ from y

$f'(x) = m$

$2x - 3 = 7$

$2x = 10$

$x = 5$

1/1

$f(5) = (5)^2 - 3(5) - 4 = 25 - 15 - 4 = 1 - -4 = 6$ 1/1

$y = mx + b = 7(x - 5) + 6$

2/2

Lesson Total: 26/26