9.5	12	12.5	18	18.5	70.5	Final
10	14	23	21	35	103	68%

13.

$$f'(x) = (3(2)x + 1)e^{3x^2+x} = (6x + 1)e^{3x^2+x}$$
$$f'(2) = (6(2) + 1)e^{3(2)^2+2} = 13e^{14}$$

2/2

14.

$$f'(x) = (2x + 3)2^{x^2+3x} \ln 2$$
$$f'(2) = (2(3) + 3)2^{3^2+3(3)} \ln 2 = (9)2^{18} \ln 2$$

2/2

15. a)

$$y' = e^{2x} + 2xe^{2x}$$

5.5/6

b)

$$f'(x) = 3^x \ln 3 + 3x^2$$

c)

$$y' = -2e^{-(2x+5)}$$

d)

$$y' = (6x - 5)e^{3x^2-5x+7}$$

e) The derivative of $f(x) = e^x$ is $f'(x) = e^x$ because e is the constant to which the natural logarithm equals one. The constant is found through identifying k of the proof $f'(x) = kf(x)$ where k would be equal to $\ln x$ within an exponential function such as $f(x)$. When $\ln x$ reaches one, then you have reached e. This would therefore mean that $f'(x) = kf(x) = \ln e\, f(x) = 1 \times f(x) = f(x)$.

Lesson 7

16.

$$f(x) = \frac{\cos x}{1 - x} = (\cos x)(1 - x)^{-1}$$
$$f'(x) = (-\sin x)(1 - x)^{-1} + (\cos x)(-1)(1 - x)^{-2}(-1)$$
$$= (-\sin x)(1 - x)^{-1} + (\cos x)(1 - x)^{-2} = \frac{-\sin x}{1 - x} + \frac{\cos x}{(1 - x)^2}$$ ✓

3/4

$$f'(x) = \frac{(-\sin x)^2 + \cos x}{(1 - x)^2}$$

but $f'(0) = ?$

17.

$$y = x^2 + 4x - 3$$
$$y' = 2x + 4$$
$$-3 = 2x + 4$$
$$2x = -7$$
$$x = -\frac{7}{2}$$

3/3

$$y = \left(-\frac{7}{2}\right)^2 + 4\left(-\frac{7}{2}\right) - 3 = \frac{49}{4} - \frac{28}{2} - 3 = 12.25 + 14 - 3 = -4.75$$

Points on the curve of y where the tangent is equal to -3: $(-3, -4.75)$.

18. a)

$$f(x) = \sqrt{x}(2x + 3)^2$$
$$f'(x) = \frac{1}{\sqrt{x}}(2x + 3)^2 + 2\sqrt{x}(2x + 3)(2) = \frac{1}{\sqrt{x}}(2x + 3)^2 + 4\sqrt{x}(2x + 3)$$
$$= \frac{1}{2\sqrt{x}}(2x + 3)^2 + \sqrt{x}(8x + 12)$$

1.5/2

b)

9.5	12	12.5	18	18.5	70.5	Final
10	14	23	21	35	103	68%

$$g(x) = \frac{\sin x}{x^3 - 2x} = (\sin x)(x^3 - 2x)^{-1}$$

$\frac{1.5}{2}$

$$g'(x) = (-\cos x)(x^3 - 2x)^{-1} + (\sin x)(-1)(x^3 - 2x)^{-2}(3x^2 - 2)$$
$$= (-\cos x)(x^3 - 2x)^{-1} - (\sin x)(x^3 - 2x)^{-2}(3x^2 - 2)$$
$$= \frac{-\cos x}{x^3 - 2x} - \frac{(\sin x)(3x^2 - 2)}{(x^3 - 2x)^2} = \frac{(-\cos x)^2 - (\sin x)(3x^2 - 2)}{(x^3 - 2x)^2}$$

c)

Lesson 8

$$h(x) = 3e^{\sin(x+2)}$$
$$h'(x) = 3e^{\sin(x+2)}(-\cos(x + 2))$$

?

$\frac{2.5}{3}$

19. a)

$$y = (3x + 2)^2$$
$$y' = 2(3x + 2)(3) = 6(3x + 2) = 18x + 12$$

$y'' = ?$ $\frac{1}{2}$

b)

$$f(x) = 5x^2 - 2x$$
$$f'(x) = 10x - 2$$
$$f''(x) = 10$$

$\frac{2}{2}$

c)

$$g(x) = 2(x + 4)^{-1} = (2x - 3)(x + 4)^{-1} = \frac{2}{x + 4} - \frac{\boxed{4 -} 2x - 3}{(x + 4)^2} = \frac{1 - 2x}{(x + 4)^2}$$

? $\frac{2}{3}$

20. a) Increasing: $x > -2$
Decreasing: $x < -2$

? $\frac{0}{2}$ → $x < -5$, $x > 1$ ↑
$-5 < x < 1$ ↓

b) Max: $(-5, g(-5))$
Min: $(1, g(1))$

$\frac{1}{1}$

c) Concave Up: $x > -2$
Concave Down: $x < -2$

$\frac{1}{1}$

graph $\frac{0}{4}$

21. a) Increasing: $x \neq 2$
Decreasing: None

$\frac{0}{0}$ $x \in \mathbb{R}$

b) Max: None
Min: None

✓

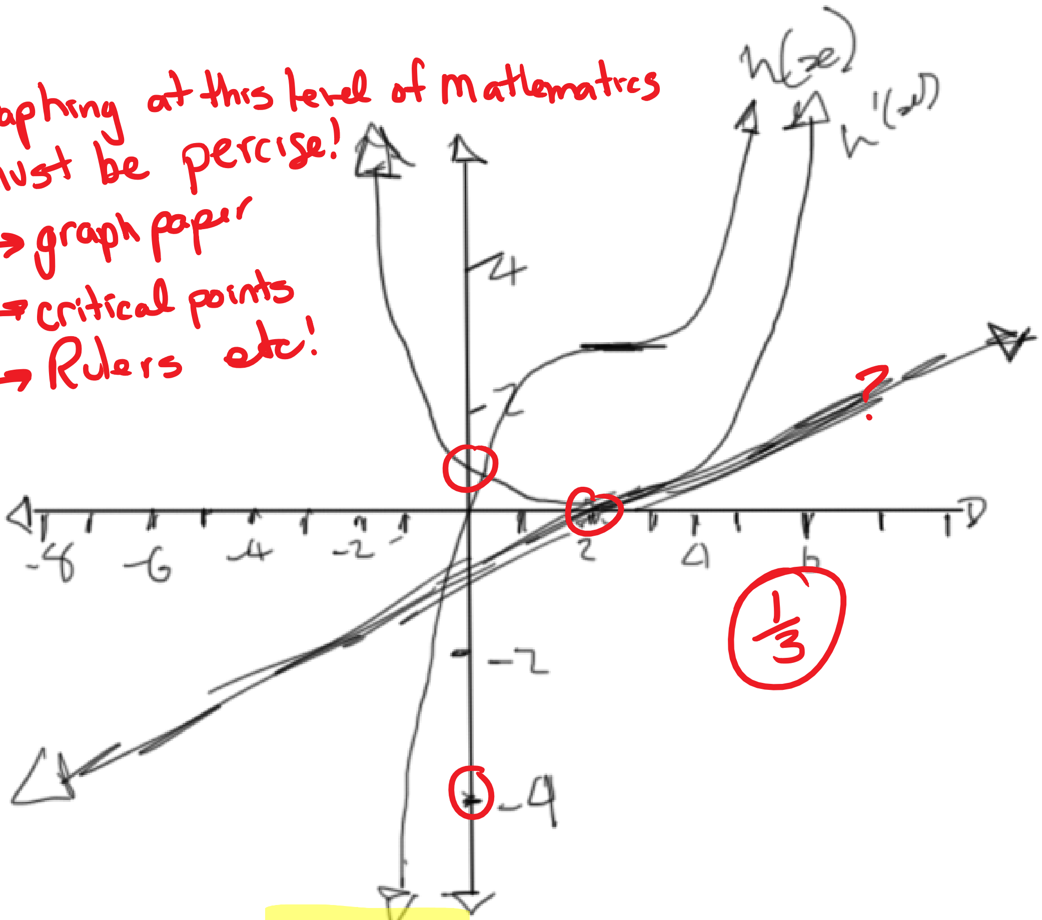
c) Concave Up: ~~None~~ $x < 2$
Concave Down: None → $x > 2$

d) Inflection Point: $(2, h(2))$ ✓

e)

9.5	12	12.5	18	18.5	70.5	Final
10	14	23	21	35	103	68%

Graphing at this level of mathematics must be percise!
↳ graph paper
↳ critical points
↳ Rulers etc!



1/3

Lesson 9

22. a)

$$f(x) = x^3 - x^2 + 4x - 3$$
$$f'(x) = 3x^2 - 2x + 4$$
$$f''(x) = 6x - 2$$
$$\frac{2 \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)} = 2 \pm \sqrt{4 - 24} = 2 \pm \sqrt{-20}$$

Intervals of Increase: $n\epsilon\mathbb{R}$

Intervals of Decrease: None

b) There are no local maximum or minimums for this function.

c)

5/6

$$f''(x) = 6x - 2 = 0$$
$$6x = 2$$
$$x = \frac{2}{6} = \frac{1}{3}$$

if there is concavity then there is P.O.I.

Concave Down: $x < \frac{1}{3}$

Concave Up: $x > \frac{1}{3}$

9.5	12	12.5	18	18.5	70.5	Final
10	14	23	21	35	103	68%

? $f(\frac{1}{3}) = ?$

d) Since $P(x)$ cannot have 0 within its range, then there are no inflection points.

b/c $f''(x) > 0$? < 0 ?

23.

$$g(1) = 1$$
$$g'(1) = 3$$
$$g''(1) = 6$$

$$g(x) = ax^2 + bx + c$$
$$g'(x) = 2ax + b$$
$$g''(x) = 2a$$

$$g''(1) = g''(x)$$
$$1 = 2a$$
$$a = \frac{1}{2}$$

$\Rightarrow$ $6 = 2a$
$a = 3$

6/8

$$g'(1) = g'(x)$$
$$1 = 2ax + b = 2\left(\frac{1}{2}\right)(1) + b = 1 + b$$
$$b = 0$$

$$g(1) = g'(x)$$
$$1 = ax^2 + bx + c = \left(\frac{1}{2}\right)(1)^2 + (0)(1) + c = \frac{1}{2} + c$$
$$c = 0.5$$

$$g(x) = \left(\frac{1}{2}\right)x^2 + 0.5$$

24.

7/7

$$f(2) = -1$$
$$f'(x) = mx + b = \frac{0-8}{4-0}x + 8 = -\frac{8}{4}x + 8 = -2x + 8$$
$$f(x) = -x^2 + 8x + k$$
$$-1 = -(2)^2 + 8(2) + k = -4 + 16 + k = 12 + k$$
$$k = 13$$
$$\therefore f(x) = -x^2 + 8x + 13$$

Lesson 10

25. These characteristics state that the function is not even and not odd. The function is also quadratic and downward facing.

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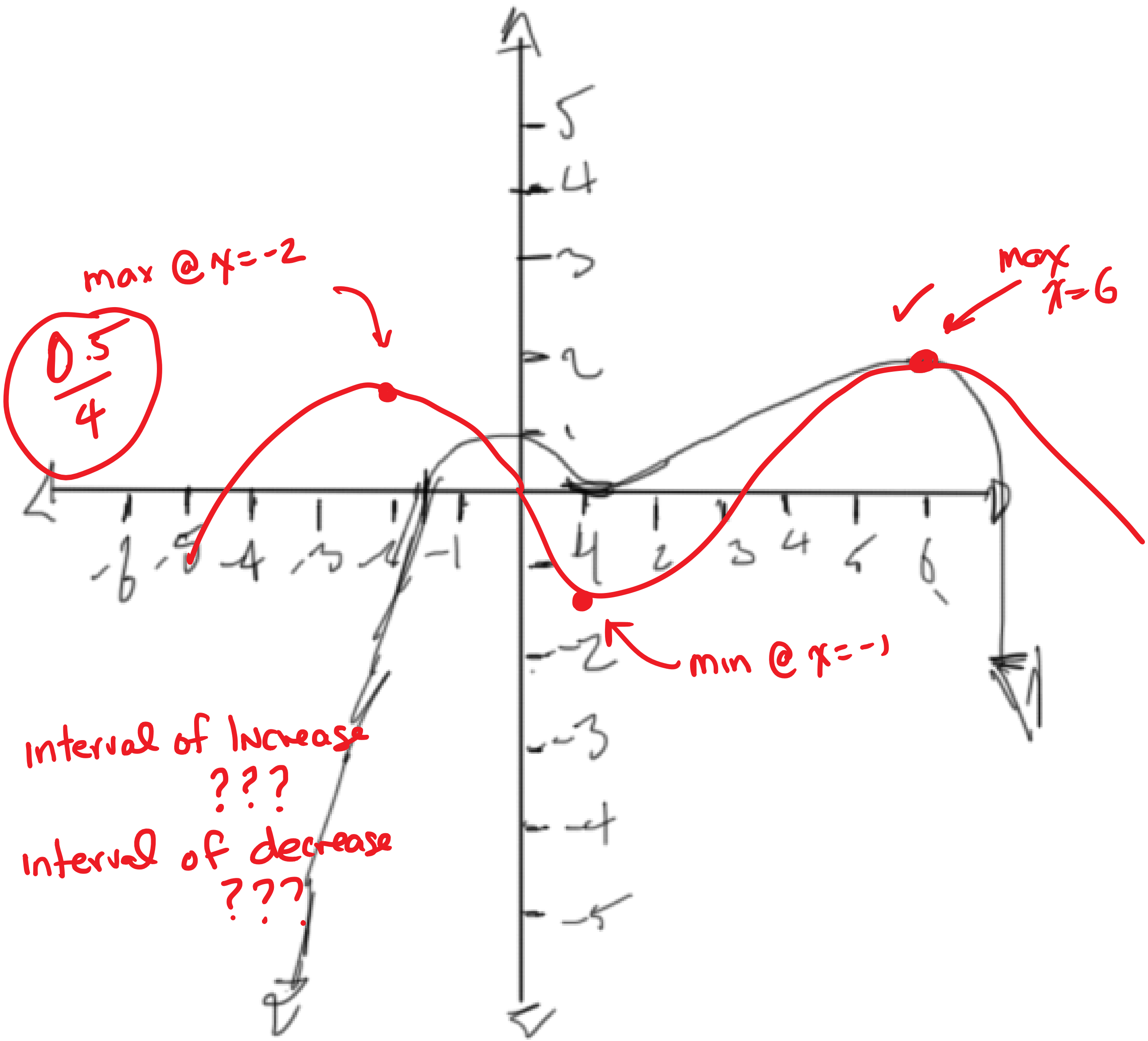

26. i)

$$g(x) = (1-2x)^2(x-3) = (1-4x+4x^2)(x-3) = x - 3 + 4x^2 + 12x + 4x^3 - 12x^2$$
$$= 4x^3 - 8x^2 + 13x + 13$$

Since $g(x)$ has both odd and even exponents, then it is neither odd nor even.

ii) x-intercepts $(3,0), \left(\frac{1}{2}, 0\right)$

y-intercepts: $(0,-3)$

Iii) First Derivative:

$$g'(x) = 12x^2 - 16x + 13$$

10/17

Second Derivative

$$\frac{12 \pm \sqrt{12^2 - 4(12)(13)}}{2(12)}$$

max/min ?

Since the second derivative is always positive, then $g'(x)$ is always increasing.

Concavity? graph?

$g''(x) = 0$? when?

9.5	12	12.5	18	18.5	70.5	Final
10	14	23	21	35	103	68%

27. a) Domain: $x \epsilon \mathbb{R}, x \neq 2$ ✓

b) y-asymptote: $\frac{x}{2x} = \frac{1}{2} \therefore x = \frac{1}{2}$

x-asymptote: $2x - 4 = 2(x-2) \therefore y = 2$ ✓

Why? Vertical/Horizontal Asymptotes using limits??

8/14

c) x-intercept

$$f(0) = \frac{x+5}{2x-4} = \frac{0+5}{2(0)-4} = \frac{5}{-4} = -1.25$$

$(0, -1.25)$ ✓

y-intercept

$$0 = \frac{x+5}{2x-4} = x+5$$

$x = -5$

$(-5, 0)$ ✓

d)

$$f(x) = \frac{x+5}{2x-4} = (x+5)(2x-4)^2$$

$$f'(x) = (2x-4)^{-1} + (x+5)(-1)(2x-4)^{-2}(2) = (2x-4)^{-1} + \frac{-2x-10}{(2x-4)^2}$$

$$= \frac{1}{2x-4} + \frac{-2x-10}{(2x-4)^2} = \frac{2x-4-2x-10}{(2x-4)^2} = \frac{-14}{(2x-4)^2} = (-14)(2x-4)^{-2}$$

$$f''(x) = (0)(2x-4)^{-2} + (-14)(2)(-2)(2x-4)^{-3} = -\frac{56}{(2x-4)^{-3}}$$

Increasing: Never, since $f'(x)$ is never positive → ALWAYS Increase forall values of x.

Decreasing: $x \neq 2$

e) Concavity?? P.O.I? is thee one?

9.5	12	12.5	18	18.5	70.5	Final
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