Calculus and Vectors MCV4U-A
Practice Test

Time: 2 hours

Total Marks: 101   **Final Test Score** _____ **÷ 101 × 100 =** _____**%**

Instructions

- There is a label attached to this page. Compare the course code on the label with the course code printed on the Final Test to make sure that they are the same. Inform the Final Test supervisor **immediately** if they are not the same.

- The Final Test pages are numbered 1 to 13. Check to see that all thirteen pages are attached. Inform the Final Test supervisor **immediately** if there are any pages missing.

- You may use a calculator during the Final Test. You may not use any books or notes.

- You must write your answers in the space provided.

- There are two parts to the test. A breakdown of the marks and the approximate time needed for each part are given below. Look over the entire test before you begin. Manage your time carefully, and leave some time at the end to review your work.

Part	Category	Marks	Time (minutes)
	Preview		5
A	Calculus	62	70
B	Vectors	39	40
	Review		5
Total		**101**	**120**

- At the end of the test, return this test paper and all of your work (including drafts) to the Final Test supervisor. Please note that, for security reasons, marked tests are not returned to students.

Dec. 3, 2009

Part A: Calculus (62 marks) *(approximate time: 70 minutes)*

1. Find the derivative by calculating $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
   for $f(x) = \dfrac{3}{x+2}$ **(5 marks)**

2. Differentiate and simplify the following: **(6 marks: 3 marks each)**

   a) $f(x) = \sqrt{2x - 1} - \sqrt[3]{(x + 1)^2}$

   b) $y = l^{\,3x^2 - 4} \times \sin^3 x$

3. Evaluate the following limits: **(6 marks: 3 marks each)**

   a) $\displaystyle \lim_{x \to 0} \frac{x^3 - x^2 - 12x}{x^2 - 4x}$

b) $\displaystyle\lim_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$

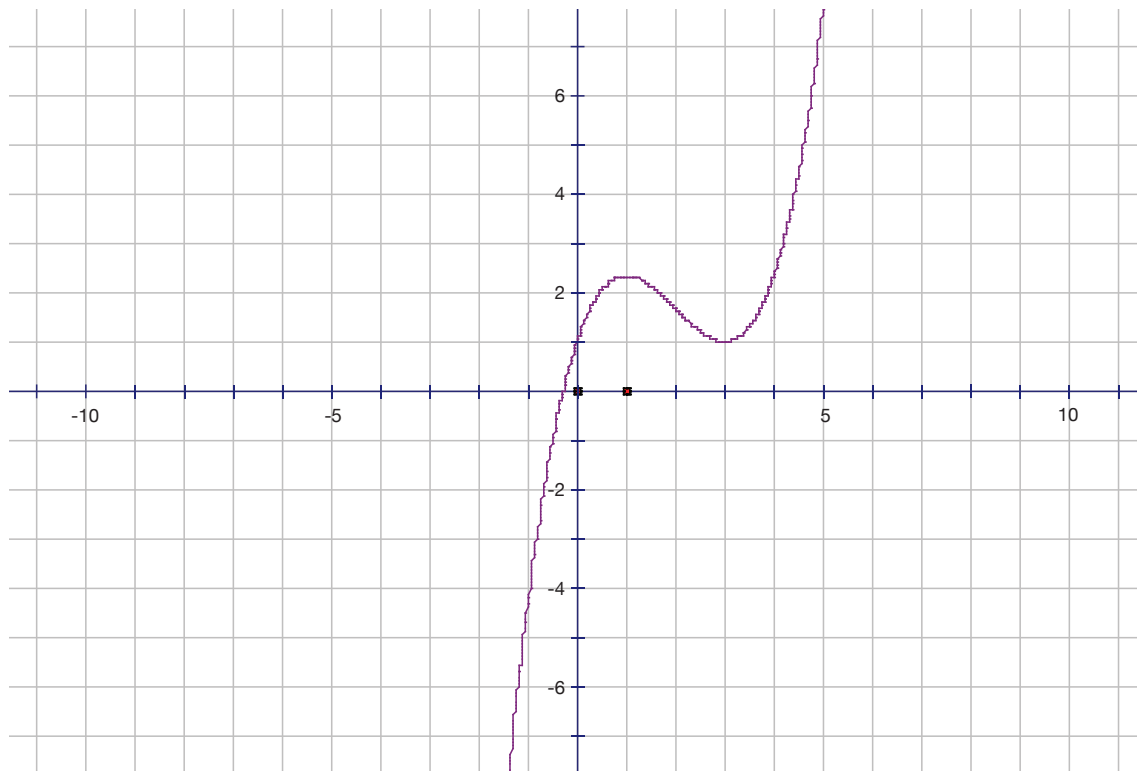4. Find the equation of the tangent line to $y = x^3 + 1$ at $x = 1$. **(4 marks)**

5. The height of a projectile is given by $h = -5t^2 + 40t + 10$ metres and t is measured in seconds.

    a) Calculate the average velocity for $1 \leq t \leq 3$ seconds. **(2 marks)**

    b) When is the instantaneous velocity equal to zero? **(2 marks)**

c)   Find the maximum height of the projectile. **(1 mark)**

6.   Given the following graph of $h(x)$:

a)   Identify **(4 marks: 1 mark each)**

   i)   the intervals where $h(x)$ is increasing and decreasing

   ii)   the local maximum and minimum points of $h(x)$

   iii)   the intervals where $h(x)$ is concave up and concave down

   iv)   the inflection points of $h(x)$

b)   Sketch the graphs of $h'(x)$ and $h''(x)$. **(2 marks)**

7.   Given $f(x) = \dfrac{3x + 6}{2x - 4}$:

   a)   State the domain. **(1 mark)**

   b)   Find the vertical and horizontal asymptotes. **(2 marks)**

   c)   Find the x- and y-intercepts. **(2 marks)**

8.   Given $f(x) = x^3 - 9x^2 + 15x - 7$

   a)   State whether the function is odd, even, or neither. **(1 mark)**

   b)   Find the x- and y-intercepts. (The factor theorem is needed to find
         the x-intercepts.) **(4 marks)**

c)   Use the first and second derivatives to find the coordinates of the local maximum, the local minimum, and the inflection points. **(7 marks)**

d)   Indicate the intervals where the graph is concave up and concave down. **(2 marks)**

e)   Sketch the graph of the function. **(3 marks)**

9. An open-topped cylindrical water reservoir has a volume of $320\pi\mathrm{m}^3$.
    The material used for the bottom of the reservoir costs $5/\mathrm{m}^2$ and the
    material used for the vertical portion costs $8/\mathrm{m}^2$. Determine the radius
    and the height of the cylinder so that the cost is a minimum.
    (Volume = $\pi r^2 h$ and Surface area = $\pi r^2 + 2\pi rh$) **(8 marks)**

Part B: Vectors (39 marks) *(approximate time: 40 minutes)*

10. A chandelier with a weight of 100 N is suspended at a single point by two cables that make angles of 45° and 60° with the ceiling. Calculate the tension in each cable. **(8 marks)**

11. Find the angle between the vectors $\vec{u}$ = (–5, 1, 3) and $\vec{v}$ = (–2, 1, –2). **(5 marks)**

12. Determine the scalar, vector, and parametric equations of the plane that passes through the points $A(1, -1, 2)$, $B(0, -1, 1)$, and $C(1, 2, 1)$.
(9 marks)

13. Find a vector equation of the line that is parallel to the line of intersection of $3x - y + 2z = 6$ and $2x + y - z = 4$ and passes through $P(5, -1, -3)$. **(5 marks)**

14. A normal to $3x - y + 4z = 12$ passes through $A(5, 1, 6)$. Find the point of intersection of the normal and the plane. **(5 marks)**

15. Solve the following system of linear equations and interpret your solution geometrically: **(7 marks)**

$x + y + z - 4 = 0$

$x - 2y - z - 2 = 0$

$x + y - z - 10 = 0$