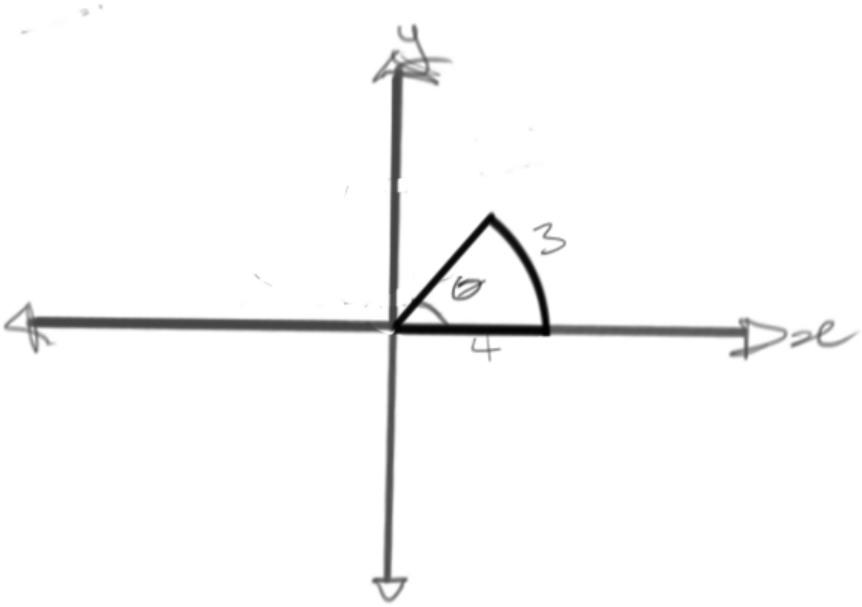
Note: * will indicate 1 mark on solutions that have not earned full marks.

2/3 marks

50. a)

$\theta = \frac{6}{4} = \frac{3}{2} = 1.5\ radians$ *

b)



$$arc\ length = 3$$

For the length of the arc:

$$\theta = \frac{3}{4} = 0.75$$

For the radius of the angle:

$$\theta = 3\left(\frac{180°}{\pi}\right) = \frac{540°}{\pi}$$

Length of arc should be approx. 3 times the length of the radius.

c)

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi$$
$$\theta = 360°$$
$$360° = 2\pi$$

$\therefore \pi = 180°$ *

4/4 marks

51. a) i)

$$10° = \frac{10\pi}{180} = \frac{\pi}{18}$$

ii)

$$225° = \frac{225\pi}{180} = \frac{45\pi}{36} = \frac{15\pi}{12} = \frac{5\pi}{4}$$

b) i)

$$\frac{\pi}{5} = \frac{\pi}{5}\left(\frac{180°}{\pi}\right) = \frac{180°}{5} = 36°$$

ii)

$$\frac{11\pi}{6} = \frac{11\pi}{6}\left(\frac{180°}{\pi}\right) = \frac{11\times180°}{6} = 11\times30° = 330°$$

0/5 marks

52. a)

$$\sin\left(\frac{2\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

b)

$$\cos\left(\frac{7\pi}{6}\right) = 7\left(\frac{\sqrt{3}}{2}\right) = \frac{7\sqrt{3}}{2}$$

c)

$$\sin\left(\frac{5\pi}{4}\right) = 5\left(\frac{1}{\sqrt{2}}\right) = \frac{5}{\sqrt{2}}$$

d)

$$\tan\left(\frac{4\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{1}\right) = 4\sqrt{3}$$

e)

$$\sec\left(\frac{7\pi}{6}\right) = 7\left(\frac{2}{\sqrt{3}}\right) = \frac{14}{\sqrt{3}}$$

a) $\sin\frac{2\pi}{3}$
$=\sin\frac{\pi}{3}$
$=\frac{\sqrt{3}}{2}$

b) $\cos\frac{7\pi}{6}$
$=-\cos\frac{\pi}{6}$
$=-\frac{\sqrt{3}}{2}$

c) $\sin\frac{5\pi}{4}$
$=-\sin\frac{\pi}{4}$
$=-\frac{1}{\sqrt{2}}$

d) $\tan\frac{4\pi}{3}$
$=\tan\frac{\pi}{3}$
$=\sqrt{3}$

e) $\sec\frac{7\pi}{6}$
$=-\sec\frac{\pi}{6}$
$=-\frac{2}{\sqrt{3}}$

5/5 marks

53. a)

Function	$f(x)=\sin(x)$
Amplitude	1
Period	$360°$ or 2π
¼ Wave	$90°$ or $\frac{\pi}{2}$
Starting Point	$(0,0)$

b) The starting point of $f(x)=\cos x$ is $(0,1)$

54. 3/5 marks

Amplitude	$f(x)=\sin(x)$
Period	5 8
¼ Wave	2 or $2\left(\frac{180°}{2\pi}\right)=\frac{90°}{\pi}$
Domain	$\{x\epsilon\mathbb{R}\}$
Range	$\{y\epsilon\mathbb{R}\|y\geq-2\}$ Range: $\{y\|-2\leq y\leq 8, y\in\Re\}$

4/4 marks

55. a)

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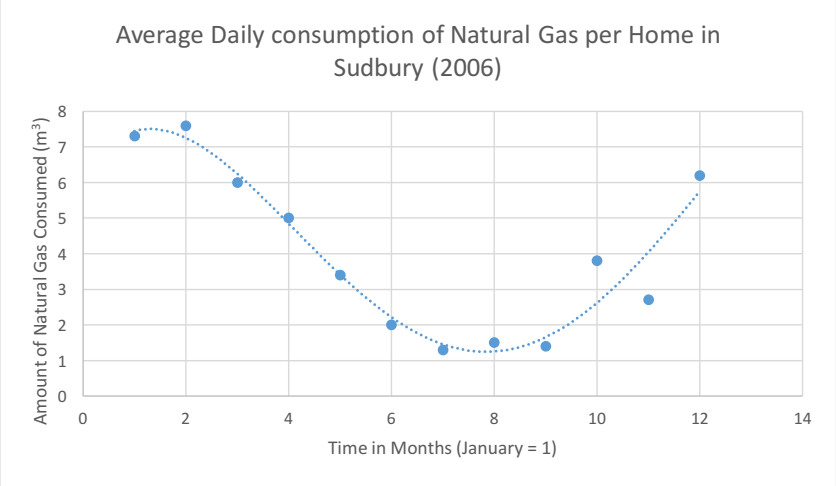


Amplitude	$\frac{7.5-1.5}{2}=\frac{6}{2}=3$
Period	$2(8-1)=2(7)=14$ or $14\left(\frac{180°}{\pi}\right)=\frac{2520°}{\pi}$
¼ Wave	$\frac{14}{4}=3.5$ or $\frac{2520°}{4\pi}=\frac{630°}{\pi}$

b) A possible reason as to why the natural gas is not zero is because some households may be using heat for cooking purposes.

56. 7/9 marks a)

Function	$f(x)=3\cos\left(2\left(x-\frac{\pi}{4}\right)\right)+1$
Period	$2\pi/2$
Amplitude	3
2* Phase Shift	$\left(\frac{\pi}{4},1\right)$

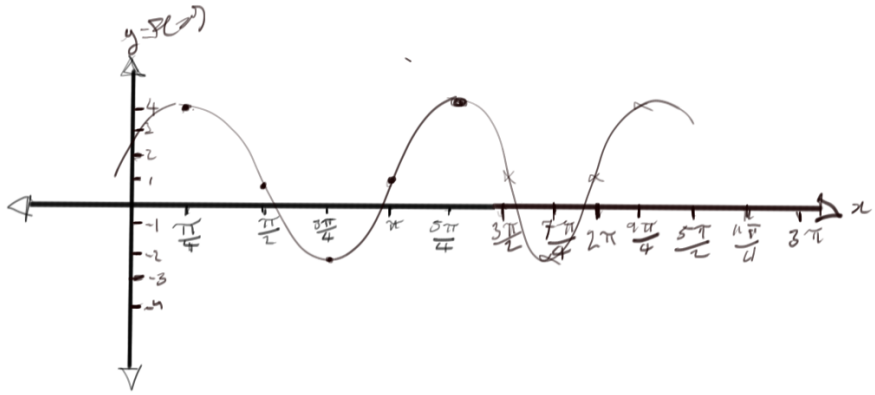
b)

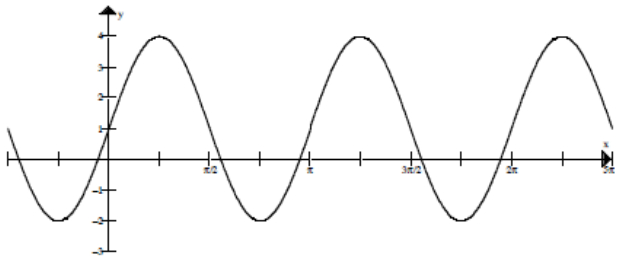
1	$\left(\frac{\pi}{4},4\right)$
2	$\left(\frac{\pi}{2},1\right)$
3	$\left(\frac{3\pi}{4},-2\right)$

4	$(\pi, 1)$
5	$\left(\frac{5\pi}{4}, 4\right)$

3*

c) 2*





¼ Wave:	$\frac{\frac{5\pi}{4} - \frac{\pi}{4}}{4} = \frac{\frac{4\pi}{4}}{4} = \frac{\pi}{4}$
Starting Point:	$\left(\frac{\pi}{4}, 4\right)$

57. 1/ 4 marks

$$y = \frac{36-4}{2}\sin\left(\frac{1}{12}\left(x - \frac{6}{2}\right)\right) + \frac{36-4}{2} + 4 = 16\sin\left(\frac{x-3}{12}\right) + 20$$

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Period = 12, therefore, $\frac{2\pi}{k} = 12 \qquad 2\pi = 12k \qquad \frac{\pi}{6} = k$

Amplitude $= \frac{36-4}{2} = 16 \qquad \therefore a = 16$

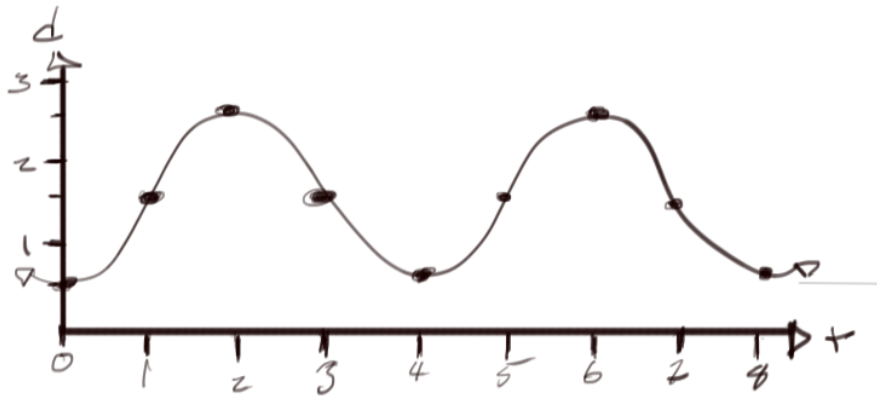
The vertical shift is $\frac{36+4}{2} = 20 : c = 20$

With a sine curve, there is a phase shift of 3 units to the right: $b = 3$

A possible equation is $d = 16\sin\frac{\pi}{6}(t-3)+20$, where d is water depth in metres and t is time in hours. **(4 marks)**

58.

a)



b)

$h = \sin\left(\frac{t\pi-\pi}{2}\right) + 1.5 = \sin\left(\frac{t}{2}(\pi-1)\right) + 1.55^{*}$

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Amplitude: $=\frac{2.5-0.5}{2}=\frac{2}{2}=1$ Period: $\frac{2\pi}{k}=4 \ \therefore 4k=2\pi \ \therefore k=\frac{\pi}{2}$

Phase shift: None Note: Reflection in the x-axis means $a=-1$

Equation: $h=-\cos\left(\frac{\pi}{2}t\right)+1.5$

59.

$$\sin(A+B)=\sin A\cos B+\sin B\cos A$$

$$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)=\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{1}{\sqrt{2}}$$

$$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{2\sqrt{3}\sqrt{2}+2\sqrt{2}}{8}=\frac{\sqrt{3}\sqrt{2}+\sqrt{2}}{4}=\frac{\sqrt{2}(\sqrt{3}+1)}{2\sqrt{2}\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

9/9 marks

60. a)

$$\cos(\pi-x)=-\cos x$$
$$R.S.=\cos(\pi-x)$$
$$=\cos\pi\cos x+\sin\pi\sin x$$
$$=-1\cos x+0\sin x$$
$$=-\cos x$$

b)

$$\cos\left(\frac{3\pi}{2}+x\right)=\sin x$$
$$R.S.=\cos\frac{3\pi}{2}\cos x-\sin\frac{3\pi}{2}\sin x$$
$$=0\cos x-\sin x$$
$$=\sin x$$

c)

$$\sin x\cos x\tan x=1-\cos^2 x$$
$$R.S.=\sin x\cos x\left(\frac{\sin x}{\cos x}\right)$$
$$=\sin^2 x$$
$$=1-\cos^2 x$$

61. 8/8 marks

a)

$$\cos\theta+\sin\theta\tan\theta=\frac{1}{\cos\theta}$$

$$R.S. = \frac{\cos\theta\left(\cos\theta + \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)\right)}{\cos\theta}$$
$$= \frac{\cos^2\theta + \frac{\cos\theta\sin^2\theta}{\cos\theta}}{\cos\theta}$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta}$$

b)

$$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$$
$$L.S. = \frac{1-\cos 2\theta}{\sin 2\theta}$$
$$= \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta$$

62. 10.5/12 marks

a)

$$2\cos^2 x + 3\cos x - 2 = 0$$
$$(2\cos x - 1)(\cos x + 2) = 0$$
$$\cos x = -2, \frac{1}{2}$$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ 4*

b)

$$\sin x = \cos 2x$$
$$\sin x = 1 - 2\sin^2 x$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$\sin x = -1, \frac{1}{2}$$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ 5*

c)

$$\sin x = \sqrt{3}\cos x$$

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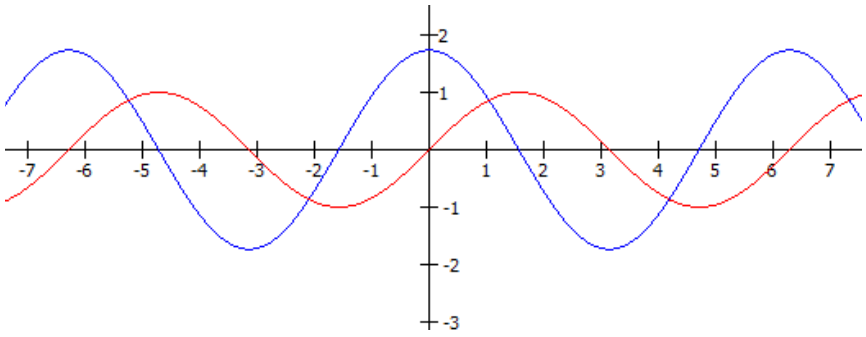
$\tan x = \sqrt{3}$ *

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{12}$ $x = \frac{\pi}{3}, \frac{4\pi}{3}$

d) The trigonometric equation in (c) is not an identity because an identity is a special equation that is true for all values of *x*. The graphs of $\sin x$ and $\sqrt{3}\cos x$ do not overlap. Red is $\sin x$ and blue is $\sqrt{3}\cos x$. 0.5*

d) The equation in part c) is not an identity since the equation is true only when $x = \frac{\pi}{3}, \frac{4\pi}{3}$, while an identity is true for all values of the variable. **(1 mark)**



63.5/80 *100 = 79%

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