Unit 4

Lesson 16

1. a) $$\begin{aligned}\vec{u}\cdot\vec{v} &= |\vec{u}||\vec{v}|\cos\theta\\ &= (2)(3)\cos(30°)\\ &\approx 5.19\end{aligned}$$

b) $$\begin{aligned}\vec{u}\cdot\vec{v} &= |\vec{u}||\vec{v}|\cos\theta\\ &= (5)(3)\cos(135°)\\ &\approx -10.6\end{aligned}$$

2. a) $$\begin{aligned}\vec{u}\cdot\vec{v} &= (-1, 2)\cdot(4, 3)\\ &= (-1)(4)+(2)(3)\\ &= 2\end{aligned}$$

b) $$\begin{aligned}\vec{u}\cdot\vec{v} &= (4,\ -2, 3)\cdot(0,\ -1, 5)\\ &= (4)(0)+(-2)(-1)+(3)(5)\\ &= 17\end{aligned}$$

3. a) $$\begin{aligned}\vec{u}\cdot\vec{v} &= (-1, 2)\cdot(-3, -1)\\ &= (-1)(-3)+(2)(-1)\\ &= 1\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{(-1)^2+(2)^2}\\ &= \sqrt{5}\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{(-3)^2+(-1)^2}\\ &= \sqrt{10}\end{aligned}$$

Suggested Answers to Support Questions

$$\begin{aligned}\cos(\theta) &= \frac{\vec{u}\cdot\vec{v}}{\left|\vec{u}\right|\cdot\left|\vec{v}\right|}\\ &= \frac{1}{\left(\sqrt{5}\right)\left(\sqrt{10}\right)}\\ &= \frac{1}{5\sqrt{2}}\\ &\approx 0.14\end{aligned}$$

$$\begin{aligned}\theta &\approx \cos^{-1}(0.14)\\ \theta &\approx 81.95\end{aligned}$$

The angle between the two vectors is approximately 81.95°.

b) $$\begin{aligned}\vec{u}\cdot\vec{v} &= (-1,2,1)\cdot(4,0,3)\\ &= (-1)(4)+(2)(0)+(1)(3)\\ &= -1\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{(-1)^2+(2)^2+(1)^2}\\ &= \sqrt{6}\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{(4)^2+(0)^2+(3)^2}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

$$\begin{aligned}\cos(\theta) &= \frac{\vec{u}\cdot\vec{v}}{\left|\vec{u}\right|\cdot\left|\vec{v}\right|}\\ &= \frac{-1}{\left(\sqrt{6}\right)(5)}\\ &\approx -0.082\end{aligned}$$

$$\begin{aligned}\theta &\approx \cos^{-1}(-0.082)\\ \theta &\approx 94.7°\end{aligned}$$

The angle between the two vectors is approximately 94.7°.

4. $\vec{u} = (4, 6)$ and $\vec{v} = (1, 4)$

$$\begin{aligned}\vec{u}\cdot\vec{v} &= (4,6)\cdot(1,4)\\ &= (4)(1) + (6)(4)\\ &= 28\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{(4)^2 + (6)^2}\\ &= \sqrt{52}\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{(1)^2 + (4)^2}\\ &= \sqrt{17}\end{aligned}$$

$$\begin{aligned}\cos(\theta) &= \frac{\vec{u}\cdot\vec{v}}{\left|\vec{u}\right|\cdot\left|\vec{v}\right|}\\ &= \frac{28}{\left(\sqrt{52}\right)\left(\sqrt{17}\right)}\\ &\approx 0.94\\ \theta &\approx \cos^{-1}(0.94)\\ \theta &\approx 19.95^\circ\end{aligned}$$

The angle between the two vectors is approximately 19.95°.

5. a) $$\begin{aligned}\vec{u}\cdot(\vec{v}+\vec{w}) &= (1,\ -1, 2)\cdot\left[(1,\ -1, 3) + (-1, 0, 2)\right]\\ &= (1,\ -1, 2)\cdot(0,\ -1, 5)\\ &= (1)(0) + (-1)(-1) + (2)(5)\\ &= 11\end{aligned}$$

$$\begin{aligned}\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w} &= (1,\ -1, 2)\cdot(1,\ -1, 3) + (1,\ -1, 2)\cdot(-1, 0, 2)\\ &= (1)(1) + (-1)(-1) + (2)(3) + (1)(-1) + (-1)(0) + (2)(2)\\ &= 11\end{aligned}$$

Observe that $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$.

Suggested Answers to Support Questions

b) $\vec{w}\cdot\vec{w} = (-1, 0, 2)\cdot(-1, 0, 2)$
$= 5$

$|\vec{w}|^2 = (-1)^2 + (0)^2 + (2)^2$

$= 5$

Observe that $\vec{w} \cdot \vec{w} = |\vec{w}|^2$.

c) $(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = \left[(1,\ -1, 2) + (1,\ -1, 3)\right]\cdot\left[(1,\ -1, 2) + (1,\ -1, 3)\right]$
$= (2,\ -2, 5)\cdot(2,\ -2, 5)$
$= (2)(2) + (-2)(-2) + (5)(5)$
$= 33$

$|\vec{u}|^2 + |\vec{v}|^2 + 2(\vec{u}\cdot\vec{v}) = 6 + 11 + 2((1,\ -1, 2)\cdot(1,\ -1, 3))$
$= 6 + 11 + 2(8)$
$= 33$

Observe that $(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = |\vec{u}|^2 + |\vec{v}|^2 + 2(\vec{u}\cdot\vec{v})$.

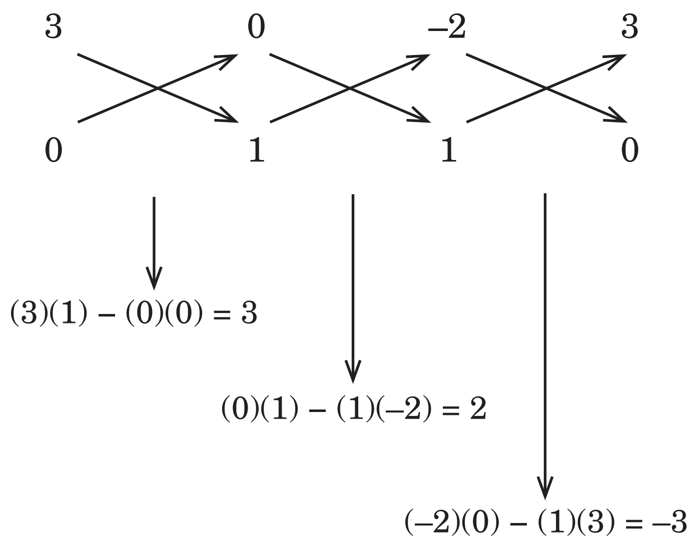
6. You need to show that the following is true for three vectors in two-space:

$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

Let $\vec{u} = (x_1, y_1)$, $\vec{v} = (x_2, y_2)$, and $\vec{w} = (x_3, y_3)$:

$$\begin{aligned}
\vec{u}\cdot(\vec{v}+\vec{w}) &= (x_1, y_1)\cdot\big((x_2, y_2) + (x_3, y_3)\big) \\
&= (x_1, y_1)\cdot(x_2 + x_3, y_2 + y_3) \\
&= x_1(x_2 + x_3) + y_1(y_2 + y_3) \\
&= x_1x_2 + x_1x_3 + y_1y_2 + y_1y_3 \\
&= x_1x_2 + y_1y_2 + x_1x_3 + y_1y_3 \\
&= \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}
\end{aligned}$$

7. $\vec{u} = (-2, 3, 0)$ and $\vec{v} = (1, 0, 1)$



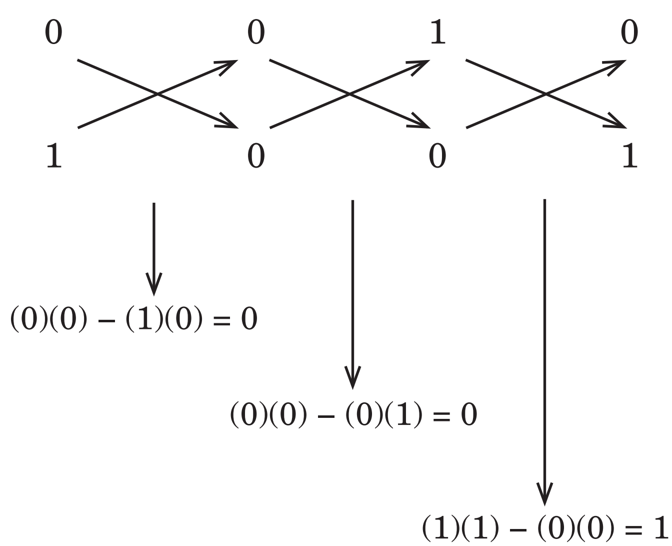
$\vec{u} \times \vec{v} = (3, 2, -3)$

You can confirm that the answer is correct by calculating the following dot products:

$(-2, 3, 0) \cdot (3, 2, -3) = (-2)(3) + (3)(2) + (0)(-3) = 0$

$(1, 0, 1) \cdot (3, 2, -3) = (1)(3) + (0)(2) + (1)(-3) = 0$

8. The cross product of two vectors is a vector that is perpendicular to both vectors:



Suggested Answers to Support Questions

(0, 0, 1) is perpendicular to both (1, 0, 0) and (0, 1, 0).

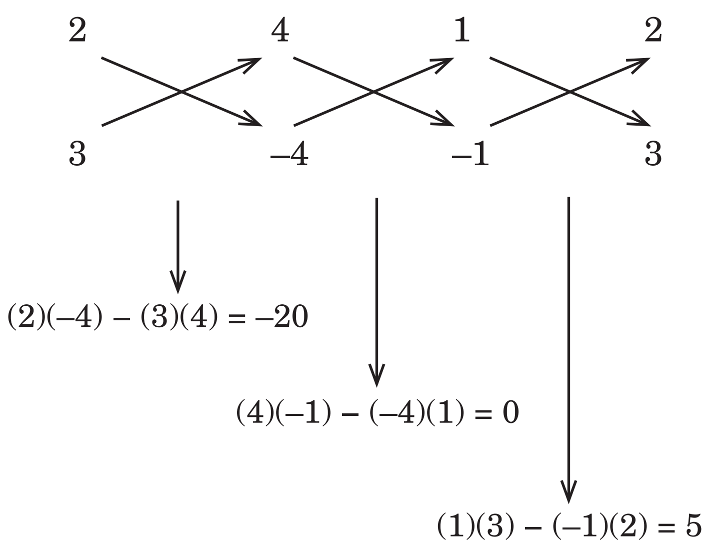
You can confirm that the answer found is correct by calculating the following dot products:

$(1, 0, 0) \cdot (0, 0, 1) = (1)(0) + (0)(0) + (0)(1) = 0$

$(0, 1, 0) \cdot (0, 0, 1) = (0)(0) + (1)(0) + (0)(1) = 0$

Lesson 17

9. The area of the parallelogram is $|\vec{u} \times \vec{v}|$ where $\vec{u} = (1, 2, 4)$ and $\vec{v} = (-1, 3, -4)$. To find the area, compute $\vec{u} \times \vec{v}$.



$\vec{u} \times \vec{v} = (-20, 0, 5)$

The area of the parallelogram:

$|\vec{u} \times \vec{v}| = |(-20, 0, 5)| = \sqrt{(-20)^2 + 0^2 + 5^2} = \sqrt{400 + 0 + 25} = \sqrt{425}$ ≈ 20.6.

10. The area of the triangle is half the area of the parallelogram determined by the vectors $\vec{AB}$ and $\vec{AC}$.

$$\begin{aligned}\vec{AB} &= \vec{OB} - \vec{OA} \\ &= (-3, 1, 0) - (-1, 0, 1) \\ &= (-2, 1, -1)\end{aligned}$$

$$\begin{aligned}\vec{AC} &= \vec{OC} - \vec{OA}\\ &= (-2, -1, 2) - (-1, 0, 1)\\ &= (-1, -1, 1)\end{aligned}$$

Compute the cross product $\vec{AB} \times \vec{AC} = (-2, 1, -1) \times (-1, -1, 1)$

$$\begin{array}{cccc} 1 & -1 & -2 & 1 \\ -1 & 1 & -1 & -1 \end{array}$$

$(1)(1) - (-1)(-1) = 0$

$(-1)(-1) - (1)(-2) = 3$

$(-2)(-1) - (-1)(1) = 3$

$\vec{AB} \times \vec{AC} = (0, 3, 3)$

The area of the parallelogram determined by $\vec{AB}$ and $\vec{AC}$ is $\sqrt{0^2 + 3^2 + 3^2} = \sqrt{0 + 9 + 9} = \sqrt{18} \approx 4.24$, and the area of the triangle is $\sqrt{18} \div 2 \approx 2.12$.

11. $$\begin{aligned}V &= \left|(\vec{u} \times \vec{v}) \cdot \vec{w}\right|\\ &= \left|\big((1, -1, 0) \times (1, 0, 3)\big) \cdot (0, -1, 1)\right|\\ &= \left|(-3, -3, 1) \cdot (0, -1, 1)\right|\\ &= \left|(-3)(0) + (-3)(-1) + (1)(1)\right|\\ &= |4|\\ &= 4\end{aligned}$$

The volume of the parallelepiped is 4 cubic units.

Suggested Answers to Support Questions

12. $$proj_{\vec{b}}\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\right)\vec{b}$$

$$= \frac{(4,\ -1,4)\cdot(-2,0,4)}{(-2,0,4)\cdot(-2,0,4)}(-2,0,4)$$

$$= \frac{-8+0+16}{4+0+16}(-2,0,4)$$

$$= \frac{8}{20}(-2,0,4)$$

$$= \left(\frac{-16}{20},0,\frac{32}{20}\right) = \left(\frac{-4}{5},0,\frac{8}{5}\right)$$

13. $$W = \vec{F}\cdot\vec{d}$$

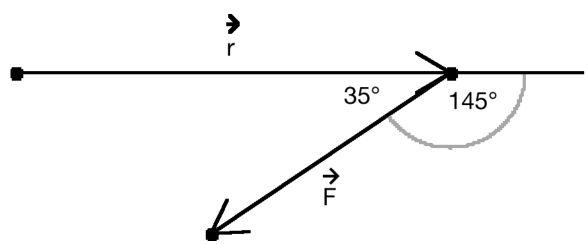
$$= \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta$$

$$= (40)(60)\cos(30)$$

$$\approx 2078.46$$

The work done is approximately 2078.46 J.

14. a)



The torque vector, $\vec{\tau}$, is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$$

The wrench is 40 cm long, so $|\vec{r}| = 0.4$.

$$\theta = 180° - 35°$$

$$\theta = 145°$$

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$$

$$= (0.4)(70)\sin(145)$$

$$\approx 16.06$$

The torque is approximately 16.06 Nm.

b) The largest value of the sine function is 1, which happens when $\theta = 90°$.

Calculate maximum torque:

$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$

$= (0.4)(70)\sin(90)$

$= 28$

The maximum torque is 28 Nm. This is achieved when the force is applied at a 90° angle.

15. $|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$

$= (0.18)(100)\sin(75)$

≈ 17.39

The torque is approximately 17.39 Nm.

Lesson 18

16. The first line can be rewritten in slope-intercept form:

$4x + 2y - 8 = 0$

$2y = -4x + 8$

$y = -2x + 4$

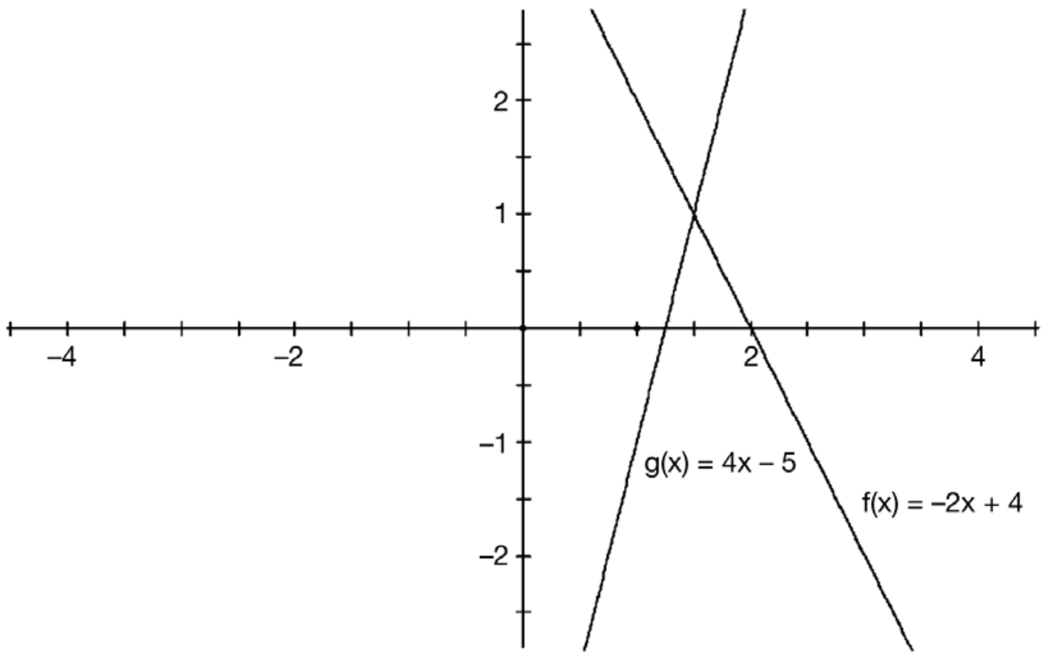
The second equation can be rewritten in slope-intercept form:

$-4x + y + 5 = 0$

$y = 4x - 5$

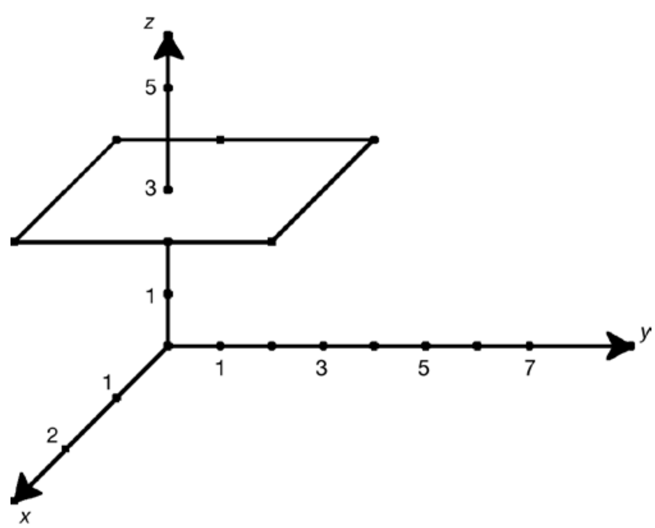
The two equations can be graphed as follows:

Suggested Answers to Support Questions

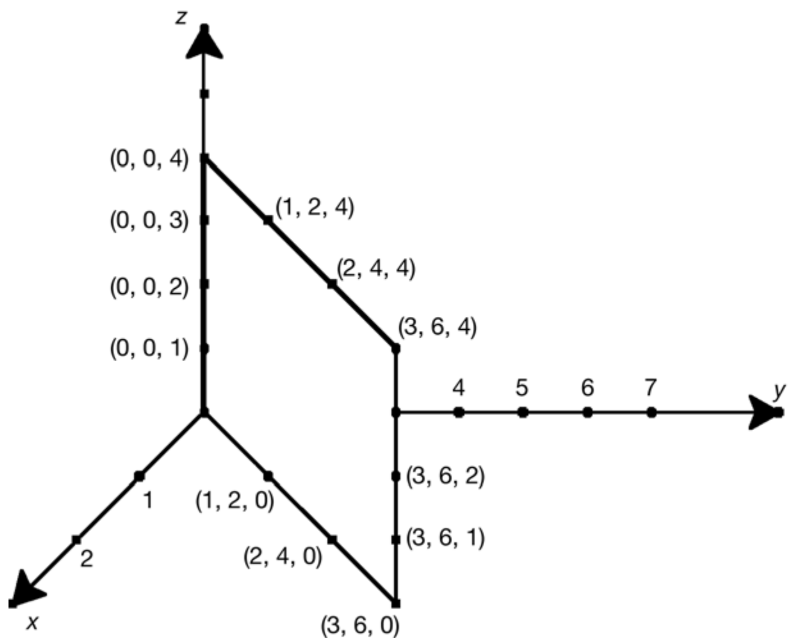


The intersection point appears to be approximately (1.5, 1). Substituting these values of x and y into the original equations satisfies both equations.

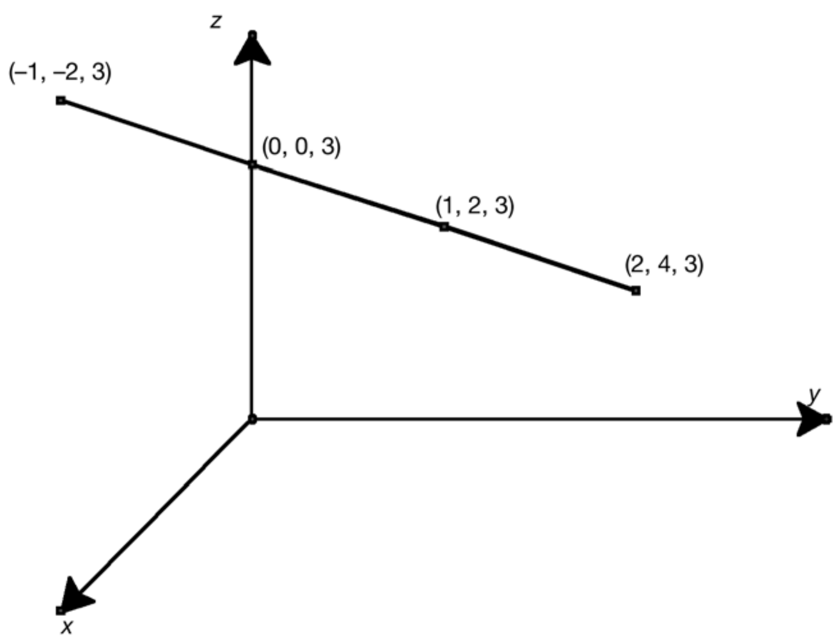
17. The plane $z = 3$ is parallel to the x-y plane, located 3 units along the z-axis in the positive direction.



The plane $y = 2x$ consists of all points where the y-coordinate is two times the x-coordinate, and the z-coordinate can have any value.



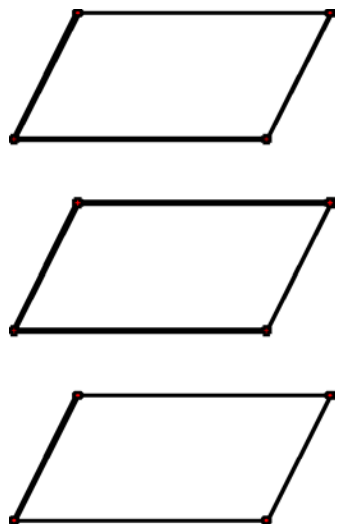
The intersection of these two planes is the line consisting of all points satisfying the relation $y = 2x$ that lie in the plane $z = 3$.



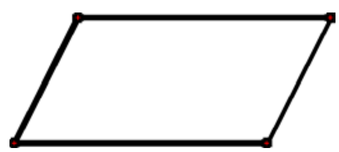
Suggested Answers to Support Questions

18. There are many possibilities:

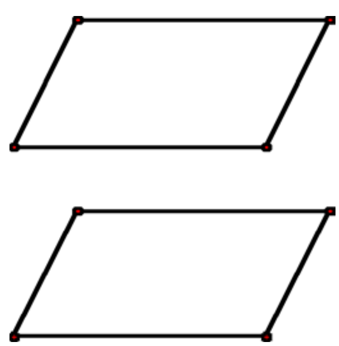
- All three planes are parallel. No intersection.



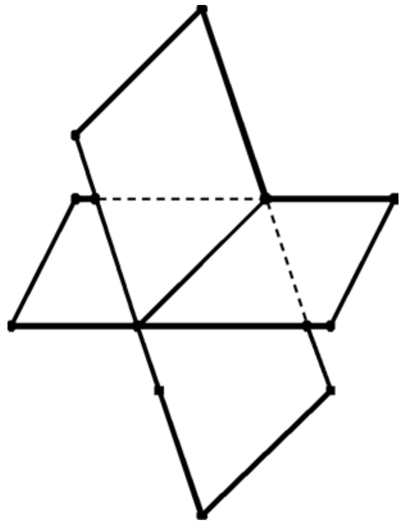
- All three planes are coincident. The intersection is a plane.



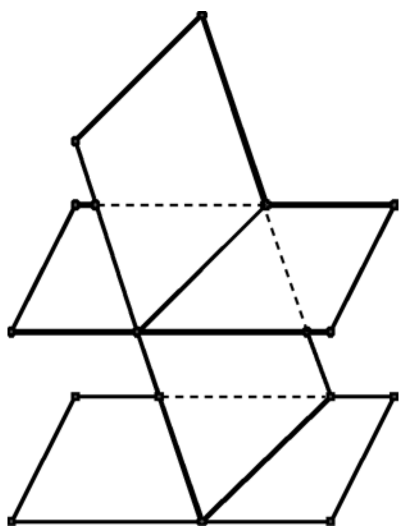
- Two planes are coincident and the other is parallel. The three planes do not intersect.



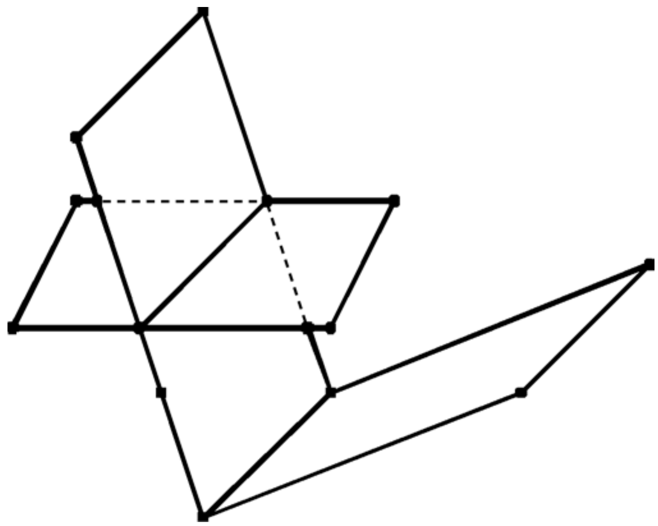
- Two planes are coincident and the other intersects the coincident planes in a line. The intersection is a line.



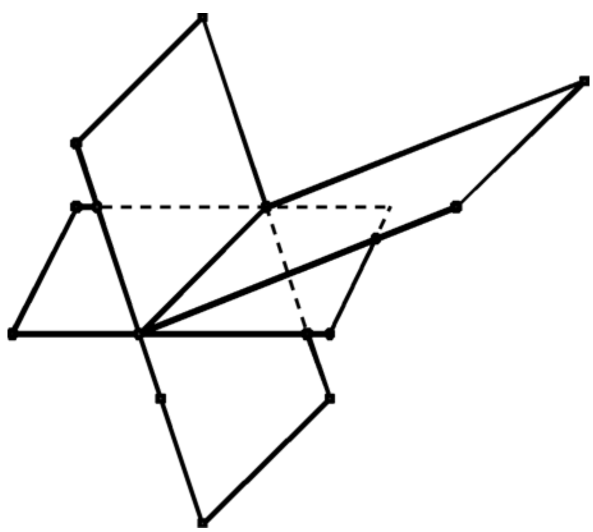
- Two planes are parallel and the other intersects both of them. The three planes do not intersect.



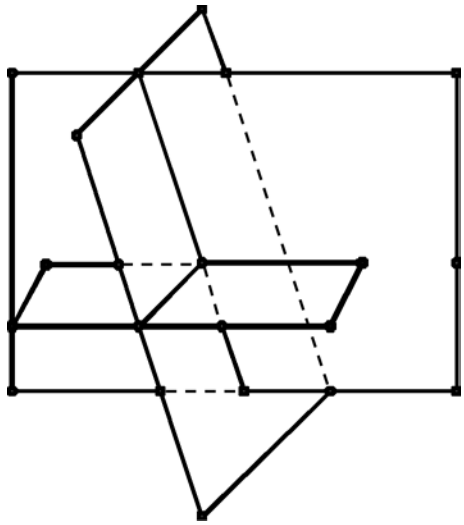
- None of the planes are parallel, and the three planes do not intersect.



- None of the planes are parallel, and the three planes intersect in a line.



- None of the planes are parallel, and the three planes intersect in a point.



Lesson 19

19. a) Find a direction vector for the line. An example of a direction vector is $\vec{AB} = \vec{OB} - \vec{OA}$ = (–3, 4) – (–1, 2) = (–2, 2). You also need one point on the line. You have been given two, so you can choose one arbitrarily. Suppose you choose A. An example of a vector equation for the line is $(x, y) = (-1, 2) + t(-2, 2)$, and an example of parametric equations for the line is

$x = -1 - 2t$

$y = 2 + 2t$

b) You need one point on the line and a direction vector. You can find a direction vector if you know two points. You can find particular points on the line by substituting a particular value for x or y.

Substituting $x = 0$ gives $3y = 10$, so $(0, \frac{10}{3})$ is a point on the line. Substituting $y = 0$ gives $2x = 10$, so (5, 0) is a point on the line.

Therefore, an example of a direction vector for the line is $(5, 0) - (0, \frac{10}{3}) = (5, -\frac{10}{3})$. (Note that $(0, \frac{10}{3}) - (5, 0)$ would be equally correct since that vector is also parallel to the line.)

A vector equation for the line is $(x, y) = (5, 0) + t(5, -\frac{10}{3})$ and parametric equations for the line are

$x = 5 + 5t$

$y = 0 + (-\frac{10}{3})t$

20. a) A vector equation for the line is simply $(x, y, z) = (1, -2, 2) + t(1, -1, 1)$ and the line can be represented with parametric equations as

$x = 1 + t$

$y = -2 - t$

$z = 2 + t$

b) An example of a vector parallel to the x-axis is $(1, 0, 0)$, so you can use this as the direction vector. A vector equation for the line is $(x, y, z) = (-1, 0, 3) + t(1, 0, 0)$ and parametric equations are

$x = -1 + t$

$y = 0$

$z = 3$

c) A direction vector is $(-1, 2, 3) - (1, 3, 4) = (-2, -1, -1)$. Thus, a vector equation for the line is $(x, y, z) = (-1, 2, 3) + t(-2, -1, -1)$ and parametric equations are

$x = -1 - 2t$

$y = 2 - t$

$z = 3 - t$

21. To get a scalar equation, you need a normal vector. You can find a normal vector by taking the cross product of two vectors in the plane. Here are two examples of vectors in the plane:

$\vec{AB} = (0, -2, 2) - (3, 2, 5) = (-3, -4, -3)$

$\vec{AC} = (1, 3, 1) - (3, 2, 5) = (-2, 1, -4)$

Compute the cross product $(-3, -4, -3) \times (-2, 1, -4)$:

$$\begin{array}{cccc} -4 & -3 & -3 & -4 \\ 1 & -4 & -2 & 1 \end{array}$$

$(-4)(-4) - (1)(-3) = 19$

$(-3)(-2) - (-4)(-3) = -6$

$(-3)(1) - (-2)(-4) = -11$

$(-3, -4, -3) \times (-2, 1, -4) = (19, -6, -11)$

A scalar equation for the plane will have the following form:

$19x - 6y - 11z + d = 0$

Substitute one of the given points to find the value of d. For example, using (1, 3, 1) gives the following:

$19(1) - 6(3) - 11(1) + d = 0$

$19 - 18 - 11 + d = 0$

$d = 10$

A scalar equation of the plane is $19x - 6y - 11z + 10 = 0$.

Suggested Answers to Support Questions

Since you have two direction vectors and at least one point on the plane, it's easy to write a vector equation for the plane:

$(x, y, z) = (3, 2, 5) + s(-3, -4, -3) + t(-2, 1, -4)$

These are the parametric equations:

$x = 3 - 3s - 2t$

$y = 2 - 4s + t$

$z = 5 - 3s - 4t$

22. If two planes are parallel, they have the same normal vector. You can find a normal vector for the given plane by taking the cross product of the two given direction vectors. Use the usual process to find the cross product:

    $(1, 2, -1) \times (0, -2, 1) = (0, -1, -2)$

    The given plane, and any plane parallel to it, will have normal vector $(0, -1, -2)$ and scalar equation of the form $0x - y - 2z + d = 0$.

    Since the plane must contain the point $(1, 0, 2)$, substitute these coordinates to find the value of d:

    $(0)(1) - (1)(0) - (2)(2) + d = 0$

    $0 - 0 - 4 + d = 0$

    $d = 4$

    A scalar equation of the plane is $-y - 2z + 4 = 0$.

Lesson 20

23. a) $\vec{l_1} = (1, 3, 2) + t(1, -2, 4)$ and $\vec{l_2} = (2, 1, 8) + s(1, -2, 6)$

    $\vec{d_1} = (1, -2, 4)$ is a direction vector of $\vec{l_1}$ and $\vec{d_2} = (1, -2, 6)$ is a direction vector of $\vec{l_2}$.

    The direction vectors of the two lines are not collinear since the ratios of the components are not equal. Therefore, the two lines intersect in a point.

Write the two lines in parametric form:

$x = 1 + t$ $\qquad$ $x = 2 + s$

$y = 3 - 2t$ $\qquad$ $y = 1 - 2s$

$z = 2 + 4t$ $\qquad$ $z = 8 + 6s$

A point on both lines must satisfy the equations of the two lines. Find values of s and t that satisfy the following linear system:

$1 + t = 2 + s$ **[1]**

$3 - 2t = 1 - 2s$ **[2]**

$2 + 4t = 8 + 6s$ **[3]**

[1] $\rightarrow t - s = 1$ **[4]**

[3] $\rightarrow 4t - 6s = 6$ **[5]**

[4] × 4 $\qquad 4t - 4s = 4$

[5] $\qquad 4t - 6s = 6$

Subtract:

$2s = -2$

$s = -1$

Substitute $s = -1$ into [4] to find t:

$t - s = 1$

$t = 1 + (-1)$

$t = 0$

Substitute $t = 0$ and $s = -1$ into both sides of [2] to check for consistency:

LS:

$3 - 2t = 3$

RS:

$1 - 2s = 1 - 2(-1)$

$= 3$

The two values are the same, so the two lines intersect.

To find the point of intersection, substitute $t = 0$ or $s = -1$ into the equations of either lines. Use $t = 0$.

$x = 1, y = 3$, and $z = 2$

The two lines intersect at (1, 3, 2).

b) $\vec{l_1} = (2, 1, 0) + t(2, 4, -10)$ and $\vec{l_2} = (11, 2, -1) + s(-8, -16, 40)$

$\vec{d_1} = (2, 4, -10)$ is a direction vector of $\vec{l_1}$ and $\vec{d_2} = (-8, -16, 40)$ is a direction vector of $\vec{l_2}$. The two vectors are collinear since $\vec{d_2} = -4\vec{d_1}$.

If the two lines have a point in common, they are coincident; otherwise, they are parallel.

The point (2, 1, 0) is on $\vec{l_1}$. Check to see if it is also on $\vec{l_2}$:

$(2, 1, 0) = (11, 2, -1) + s(-8, -16, 40)$

$2 = 11 - 8s \rightarrow 8s = 9, s = \frac{9}{8}$

$1 = 2 - 16s \rightarrow 16s = 1, s = \frac{1}{16}$

$0 = -1 + 40s$

(2, 1, 0) is not on $\vec{l_2}$. The two lines are parallel.

24. Two linear equations to solve:

$2x - 3y - z + 1 = 0$

$3x - 2y + 3z - 4 = 0$

Eliminate the variable z from these equations by multiplying the first equation by 3 and then adding:

$$\begin{array}{l} 6x - 9y - 3z + 3 = 0 \\ \underline{3x - 2y + 3z - 4 = 0} \\ 9x - 11y - 1 = 0 \end{array}$$

Choose one of the variables x or y to be the parameter. For example, let $y = t$:

$9x - 11t - 1 = 0$

Solve for x by rearranging:

$9x = 11t + 1$

$x = \frac{11}{9}t + \frac{1}{9}$

x and y are expressed in terms of the parameter t. Substitute $x = \frac{11}{9}t + \frac{1}{9}$ and $y = t$ into one of the equations of the given planes, such as the equation $2x - 3y - z + 1 = 0$:

$2(\frac{11}{9}t + \frac{1}{9}) - 3t - z + 1 = 0$

$\frac{22}{9}t + \frac{2}{9} - 3t - z + 1 = 0$

$\frac{22}{9}t + \frac{2}{9} - \frac{27}{9}t - z + \frac{9}{9} = 0$

$\frac{22}{9}t - \frac{27}{9}t - z = -\frac{2}{9} - \frac{9}{9}$

$\frac{-5}{9}t - z = \frac{-11}{9}$

$-z = \frac{5}{9}t - \frac{11}{9}$

$z = \frac{-5}{9}t + \frac{11}{9}$

All three variables (x, y, and z) are expressed in terms of the parameter t. Here are parametric equations of the line:

$x = \frac{11}{9}t + \frac{1}{9}$

$y = t$

$z = \frac{-5}{9}t + \frac{11}{9}$

25. Find the intersection of the two planes. Eliminate the variable y by multiplying the first equation by 2 and then adding:

$$\begin{array}{l} 2x + 2y + 6z - 20 = 0 \\ \underline{6x - 2y + z - 10 = 0} \\ 8x + 7z - 30 = 0 \end{array}$$

Let $z = t$:

$8x + 7t - 30 = 0$

$8x = -7t + 30$

$x = \frac{-7}{8}t + \frac{30}{8}$

Substitute $z = t$ and $x = \frac{-7}{8}t + \frac{30}{8}$ into the equation of one of the planes, say the first plane $x + y + 3z - 10 = 0$:

$(\frac{-7}{8}t + \frac{30}{8}) + y + 3t - 10 = 0$

$y = \frac{7}{8}t - 3t - \frac{30}{8} + 10 = \frac{7}{8}t - \frac{24}{8}t - \frac{30}{8} + \frac{80}{8}$

$y = \frac{-17}{8}t + \frac{50}{8}$

x, y, and z are expressed in terms of the parameter t, parametric equations of the intersection of the two planes:

$x = \frac{-7}{8}t + \frac{30}{8}$

$y = \frac{-17}{8}t + \frac{50}{8}$

$z = 1t + 0$

This means that a direction vector for the line is $(\frac{-7}{8}, \frac{-17}{8}, 1)$.
Here is the equation for the line:

$(x, y, z) = (3, -1, 2) + t(\frac{-7}{8}, \frac{-17}{8}, 1)$

26. Use the elimination method to solve for the intersection.
Label the equations [1], [2], and [3]:

$3x + z + 11 = 0$ **[1]**

$2x + y + z + 4 = 0$ **[2]**

$x + y + z - 3 = 0$ **[3]**

Eliminate variable z:

$3x + z + 11 = 0$ **[1]**

$2x + y + z + 4 = 0$ **[2]**

Subtract:

$x - y + 7 = 0$ **[4]**

$2x + y + z + 4 = 0$ **[2]**

$x + y + z - 3 = 0$ **[3]**

Subtract:

$x + 7 = 0$

$x = -7$

Substitute $x = -7$ into [1]:

$3x + z + 11 = 0$

$3(-7) + z + 11 = 0$

$-21 + z + 11 = 0$

$z = 10$

Substitute $x = -7$ and $z = 10$ into [2]:

$2x + y + z + 4 = 0$ **[2]**

$2(-7) + y + 10 + 4 = 0$

$-14 + y + 14 = 0$

$y = 0$

The system of linear equations has a solution $x = -7$, $y = 0$, and $z = 10$.

Geometrically, the three planes represented by the linear equations intersect at the point $(-7, 0, 10)$.

You can confirm that the three planes intersect at a point by calculating $\vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3)$ and asserting that $\vec{n}_1 \cdot (\vec{n}_2 \times \vec{n}_3) \neq 0$.

27.

$x - 2y - 2z - 6 = 0$ **[1]**

$2x - 5y + 3z + 10 = 0$ **[2]**

$3x - 7y + z + 9 = 0$ **[3]**

[1] × 2 $2x - 4y - 4z - 12 = 0$

[2] $2x - 5y + 3z + 10 = 0$

Subtract: $y - 7z - 22 = 0$ **[4]**

[1] × 3 $3x - 6y - 6z - 18 = 0$

[3] $3x - 7y + z + 9 = 0$

Subtract: $y - 7z - 27 = 0$ **[5]**

$y - 7z - 22 = 0$ **[4]**

$y - 7z - 27 = 0$ **[5]**

Subtract:

$5 = 0$

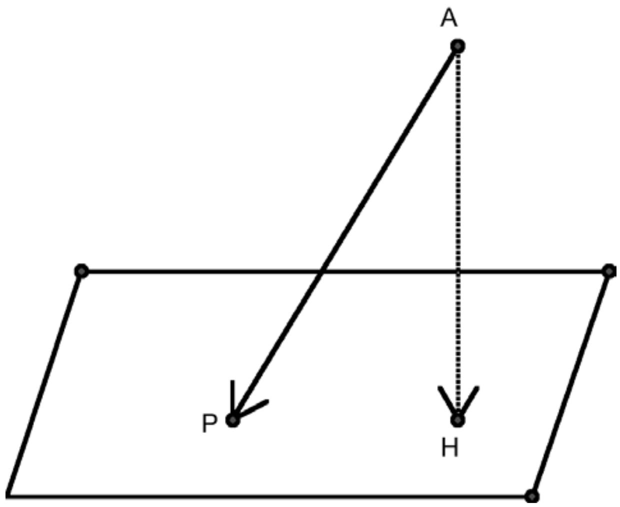
This is obviously wrong.

You can conclude that the system has no solution.

28. Start with any point on the plane:

$$P\left(0, 0, \frac{2}{5}\right)$$

$$\vec{AP} = \left(2,\ -1,\ -\frac{8}{5}\right)$$



$$\begin{aligned}
\left|\vec{AH}\right| &= \left|proj_{\vec{n}}(\vec{PA})\right| \\
&= \left|\frac{\vec{PA}\cdot \vec{n}}{\vec{n}\cdot\vec{n}}\vec{n}\right| \\
&= \left|\frac{\vec{PA}\cdot \vec{n}}{\vec{n}\cdot\vec{n}}\right|\left|\vec{n}\right| \\
&= \left|\frac{\left(2,-1,-\frac{8}{5}\right)\cdot(3,-2,5)}{(3,-2,5)\cdot(3,-2,5)}\right|\sqrt{(3)^2+(-2)^2+(5)^2} \\
&= \left|\frac{6+2-8}{(9+4+25)}\right|\sqrt{9+4+25} \\
&= \frac{0}{38}\sqrt{38} \\
&= 0
\end{aligned}$$

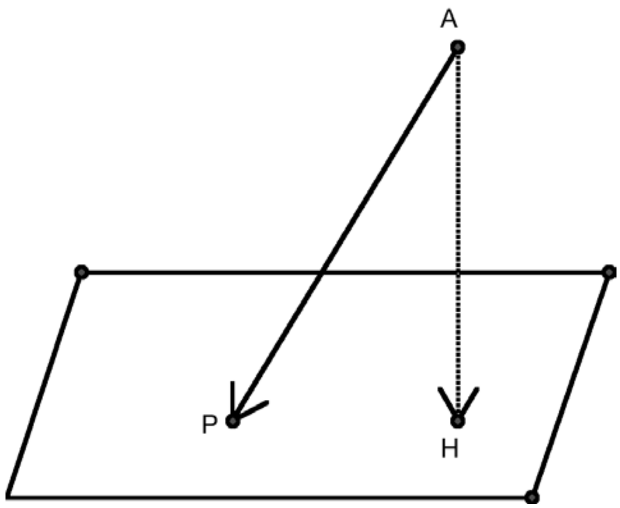
This means that point A is on the plane.

Suggested Answers to Support Questions

29. Start with any point on the plane:

$P(4, 0, 0)$

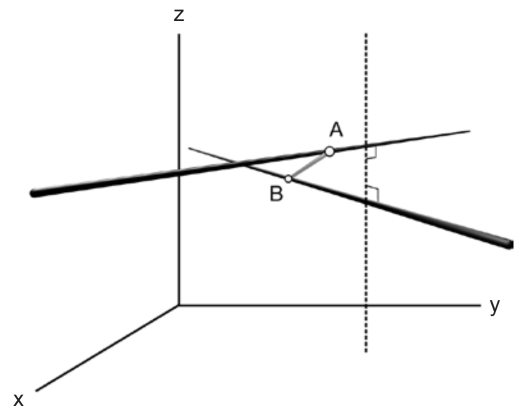
$\vec{AP} = (3, 2, -3)$



$$
\begin{aligned}
\left|\vec{AH}\right| &= \left|proj_{\vec{n}}(\vec{PA})\right| \\
&= \left|\frac{\vec{PA}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n}\right| \\
&= \left|\frac{\vec{PA}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\right|\left|\vec{n}\right| \\
&= \left|\frac{(3,2,-3)\cdot(2,-3,6)}{(2,-3,6)\cdot(2,-3,6)}\right|\sqrt{(2)^2+(-3)^2+(6)^2} \\
&= \left|\frac{6+(-6)+(-18)}{(4+9+36)}\right|\sqrt{4+9+36} \\
&= \left|\frac{-18}{49}\right|\sqrt{49} \\
&= \frac{18}{7}
\end{aligned}
$$

The distance from point A to the plane is $\dfrac{18}{7}$.

30. The two skew lines $\vec{l_1} = (6, -4, 0) + t(0, 1, -1)$ and $\vec{l_2} = (0, 5, 3) + s(2, 0, -1)$ are illustrated in the following diagram. The vertical dashed line is a common perpendicular to the two skew lines:



The projection of $\vec{AB}$ onto the normal is the distance between the two skew lines.

$A(-6, -4, 0)$ is a point on $\vec{l_1}$ and $B(0, 5, 3)$ is a point on $\vec{l_2}$.
$\vec{AB} = \vec{OB} - \vec{OA} = (6, 9, 3)$

$\vec{d_1} = (0, 1, -1)$ is a direction vector of $\vec{l_1}$ and $\vec{d_2} = (2, 0, -1)$ is a direction vector of $\vec{l_2}$.

$\vec{d_1} \times \vec{d_2}$ is perpendicular to both lines. Using the usual procedure you find:

$\vec{n} = \vec{d_1} \times \vec{d_2} = (-1, -2, -2)$

The distance between $\vec{l_1}$ and $\vec{l_2}$ is $\left|proj_{\vec{n}}\left(\vec{AB}\right)\right|$.

Suggested Answers to Support Questions

$$\left|proj_{\vec{n}}(\vec{AB})\right|$$

$$= \left|\frac{\vec{AB}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n}\right|$$

$$= \left|\frac{\vec{AB}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\right|\left|\vec{n}\right|$$

$$= \left|\frac{(6,9,3)\cdot(-1,-2,-2)}{(-1,-2,-2)\cdot(-1,-2,-2)}\right|\sqrt{(-1)^2 + (-2)^2 + (-2)^2}$$

$$= \left|\frac{(6)(-1)+(9)(-2)+(3)(-2)}{(1+4+4)}\right|\sqrt{1+4+4}$$

$$= \left|\frac{-30}{9}\right|\sqrt{9}$$

$$= 10$$

The distance between the two skew lines is 10.