# Unit 3

## Scoring
Here is how you arrive at a score for this unit:

| Lesson | Key Questions: Potential Score (PS) | Key Questions: Actual Score (AS) |
|---|---|---|
| Lesson 11 28-30 | 18 | 16 |
| Lesson 12 31-33 | 52 | 52 |
| Lesson 13 34-36 | 16 | 15 |
| Lesson 14 37-40 | 12 | 4 |
| Lesson 15 41-45 | 27 | 19 |
| Total | 125 | 106 |

**Unit Score = AS ÷ PS × 100 = __85_____%**

28. a) 1985: $97 \pm 1$
    1996: $42 \pm 1$
    b) 1986: $-5.98$
    1991: $-5.60$
    c) $\approx 1998$

29.

$$P'(t) = \frac{(1+e^{-t})(0) - (500)\left(0 + (-e^{-t})\right)}{(1+e^{-t})^2} = \frac{-500(-e^{-t})}{(1+e^{-t})^2} = \frac{500(e^{-t})}{(1+e^{-t})^2}$$

$$P'(3) = \frac{500(e^{-t})}{(1+e^{-t})^2} \approx \frac{24.894}{1.102} = 22.59$$

30. a)

$$C_A(x) = \frac{C(x)}{x}$$

$$C_A(x) = \frac{2500 + 100(200) - 0.1(200)^2}{200} = \frac{2500 + 20000 - 0.1(40000)}{200}$$

$$= \frac{22500 - 4000}{200} = \frac{18500}{200} = \frac{185}{2} = 92.5$$

$$C_M(x) = C'(x) = 100 - 0.2x$$

$$C_M(200) = 100 - 0.2(200) = 100 - 40 = 60$$

The average cost is \$92.50 and the marginal cost is \$60.00.

b) i)

$$A(t) = A_O\left(\frac{1}{2}\right)^{\frac{t}{k}}$$

$$\frac{1}{32}A_O = A_O\left(\frac{1}{2}\right)^{\frac{t}{k}}$$

$$\frac{1}{32} = \left(\frac{1}{2}\right)^{\frac{t}{k}} = \left(\frac{1}{2}\right)^{\frac{t}{(20)(24)}} = \left(\frac{1}{2}\right)^{\frac{t}{480}}$$

$t = 480\log_{0.5}\left(\frac{1}{32}\right) = 480(0.2) = 96$ 2/3 answer is $\frac{1}{32} = 1\left(\frac{1}{2}\right)^{\frac{t}{20}}$

$$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{\frac{t}{20}}$$

$$5 = \frac{t}{20}$$

$$t = 100$$

The amount of time it takes for 1/32 of this substance to be left is 96 hours or 4 days.

ii)

$$A'(t) = \left(\frac{1}{2}\right)^{\frac{t}{k}}\ln\frac{1}{2}\left(\frac{1}{k}\right) = \left(\frac{1}{2}\right)^{\frac{480h}{2400h}}\ln\frac{1}{2}\left(\frac{1}{2400h}\right) \approx 2.5\times10^{-4}$$

The rate of decay is $2.5\times10^{-4}$ kg per year.

0/1 answer is $A'(100) = \left(\frac{1}{2}\right)^{\frac{100}{20}} \cdot \ln\left(\frac{1}{2}\right)\cdot\frac{1}{20}$

$$= -0.00108304247$$

31.

$$R(x) = x\big(1200(1.50 - x)\big) = 1200x(1.50 - x) = 1800x - 1200x^2$$
$$R(x)' = 1800 - 2400x$$
$$0 = 1800 - 2400x$$
$$2400x = 1800$$
$$x = \frac{1800}{2400} = 0.75$$

The fare that will maximize the revenue is \$0.75.

32.

$$N(t) = 30t - t^2$$
$$N'(t) = 30 - 2t$$
$$0 = 30 - 2t$$
$$2t = 30$$
$$t = 15$$
$$N(15) = 30(15) - 15^2 = 450 - 225 = 225$$

The maximum number of conversations is 225 at 15 minutes.

33. a)

$$C(x) = (15)\sqrt{120^2 + x^2} + (10)(300 - x) = (15)(120^2 + x^2)^{\frac{1}{2}} + 3000 - 10x$$
$$C(x)' = (15)(120^2 + x^2)^{-\frac{1}{2}}\left(\frac{1}{2}\right)(2x) - 10$$
$$0 = \frac{15x}{\sqrt{120^2 + x^2}} - 10$$
$$10 = \frac{15x}{\sqrt{120^2 + x^2}}$$
$$\sqrt{120^2 + x^2} = \frac{15x}{10} = \frac{3x}{2}$$
$$120^2 + x^2 = \frac{9x^2}{4}$$
$$x^2 - \frac{9x^2}{4} = -120^2$$
$$-\frac{5x^2}{4} = -120^2$$
$$\frac{5x^2}{4} = 120^2$$
$$x^2 = \frac{4(120^2)}{5} = 11520$$
$$x = \sqrt{11520} = 16\sqrt{45}$$
$$l_{river}(x) = \sqrt{120^2 + x^2}$$
$$l_{river}(16\sqrt{45}) = \sqrt{120^2 + \left(16\sqrt{45}\right)^2} = \sqrt{14400 + 11520} = \sqrt{25920} \approx 160.996$$
$$\approx 161$$

The length across the river under the water should be 161 metres in length. You can prove that this is the minimum length by testing if it creates the minimum cost via $C(x)$.

$$C\left(16\sqrt{45} - 1\right) = (15)\sqrt{120^2 + \left(16\sqrt{45} - 1\right)^2} + (10)\left(300 - 16\sqrt{45} - 1\right) = 4361.67$$
$$C\left(16\sqrt{45}\right) = (15)\sqrt{120^2 + \left(16\sqrt{45}\right)^2} + (10)\left(300 - 16\sqrt{45}\right) = 4341.64$$
$$C\left(16\sqrt{45} + 1\right) = (15)\sqrt{120^2 + \left(16\sqrt{45} + 1\right)^2} + (10)\left(300 - 16\sqrt{45} + 1\right) = 4361.67$$

When C(x) is $16\sqrt{45}$ then the lowest cost is met.

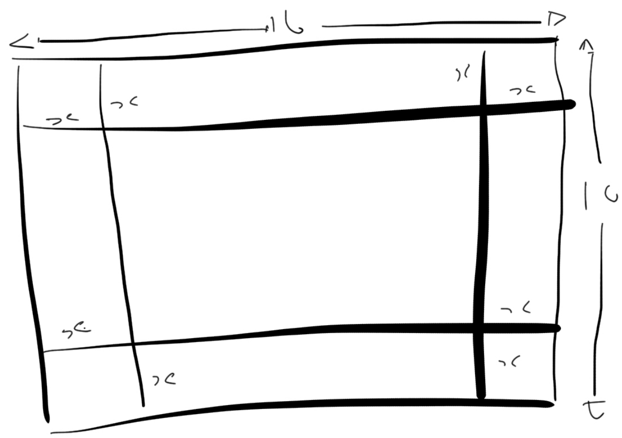
b)

$$R(n) = (40 - 2n)(2000 + 200n)$$
$$E(n) = 8(2000 + 200n)$$
$$P(n) = R(n) - E(n) = (40 - 2n)(2000 + 200n) - 8(2000 + 200n)$$
$$P'(n) = (40 - 2n)(2000 + 200n) - 8(2000 + 200n)$$

$$P'(n) = (-2)(2000 + 200n) + (40 - 2n)(200) - 8(200)$$
$$= (-4000 - 400n) + 8000 - 400n - 1600 = 2400 - 800n$$
$$0 = 2400 - 800n$$
$$800n = 2400$$
$$n = \frac{2400}{800} = 3$$
$$price(n) = 40 - 2n = 40 - 2(3) = 40 - 6 = 34$$

The price that will lead to the maximum profit is \$34.

c)




$$V(x) = (10 - 2x)(16 - 2x)x = (160 - 20x - 32x + 4x^2)x = (160 - 52x + 4x^2)x$$
$$= 160x - 52x^2 + 4x^3$$
$$V'(x) = 160 - 104x + 12x^2$$
$$\frac{104 \pm \sqrt{(-104)^2 - 4(160)(12)}}{2(12)} = \frac{104 \pm \sqrt{10816 - 7680}}{24} = \frac{104 \pm \sqrt{3136}}{24}$$
$$= \frac{104 \pm 56}{24} = 2, \frac{20}{3}$$
$$V(2) = \big(10 - 2(2)\big)\big(16 - 2(2)\big)(2) = (10 - 4)(16 - 4)(2) = (6)(12)(2) = 144$$
$$V\left(\frac{20}{3}\right) = \left(10 - 2\left(\frac{20}{3}\right)\right)\left(16 - 2\left(\frac{20}{3}\right)\right)\left(\frac{20}{3}\right) = \left(10 - \frac{40}{3}\right)\left(16 - \frac{40}{3}\right)\left(\frac{20}{3}\right)$$
$$= \left(\frac{30 - 40}{3}\right)\left(\frac{48 - 40}{3}\right)\left(\frac{20}{3}\right) = \frac{(-10)(8)(20)}{3} = \frac{-1600}{3} \approx -533.3$$

The maximum volume is 144 $cm^3$.

d)

$$V = 3456cm^3$$
$$S = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$$
$$V = \pi r^2 h$$
$$h = \frac{V}{\pi r^2}$$
$$S = 2\pi r\left(r + \frac{V}{\pi r^2}\right) = 2\pi r\left(\frac{\pi r^3 + V}{\pi r^2}\right)$$

$$S(r) = \frac{2(\pi r^3 + V)}{r}$$

$$S'(r) = \frac{(r)(6\pi r^2) - \left(2(\pi r^3 + V)\right)(1)}{r^2} = \frac{6\pi r^3 - 2\pi r^3 - 2V}{r^2} = \frac{4\pi r^3 - 2V}{r^2}$$

$$0 = \frac{4\pi r^2 - 2V}{r^2}$$

$$0 = 4\pi r^3 - 2V$$

$$2V = 4\pi r^3$$

$$V = 2\pi r^3$$

$$3456\pi = 2\pi r^3$$

$$3456 = 2r^3$$

$$r^3 = 1728$$

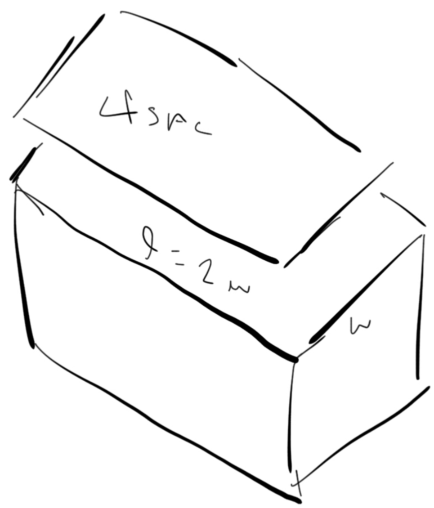
$$r = \sqrt[3]{1728}$$

$$r = 12$$

$$h = \frac{V}{\pi r^2} = \frac{3456\pi}{\pi(12)^2} = \frac{3456}{144} = 24$$

The minimum radius and height for the large soup can is 3cm in radius and 24cm in height.

e)




$$SA_c = lw + 2wh + 2lh = lw + 2h(w + l)$$

$$V = lwh$$

$$SA_l = 4lw$$

$$SA = SA_c + SA_l = lw + 2h(w + l) + 4lw = 5lw + 2h(w + l)$$

$$= 5(2w)w + 2h(w + 2w) = 10w^2 + 6hw$$

$$V = lwh$$

$$h = \frac{V}{lw}$$

$$SA = 10w^2 + 6\left(\frac{V}{lw}\right)w = 10w^2 + \frac{6W}{l} = 10w^2 + \frac{6V}{2w} = 10w^2 + \frac{3V}{w}$$
$$= 10w^2 + 3Vw^{-1}$$
$$SA' = 20w - 3Vw^{-2}$$
$$0 = 20w - 3Vw^{-2}$$
$$3Vw^{-2} = 20w$$
$$3Vw^{-3} = 20$$
$$w^{-3} = \frac{20}{3V}$$
$$w = \sqrt[3]{\frac{3V}{20}} = \sqrt[3]{\frac{3(1440)}{20}} = \sqrt[3]{3(72)} = \sqrt[3]{216} = 6$$
$$l = 2w = 2(6) = 12$$
$$h = \frac{V}{lw} = \frac{1440}{(12)(6)} = \frac{1440}{72} = 20$$

The dimensions are 12dm x 6dm x 20dm

f)

$$CP = \sqrt{\left(6\sqrt{2}\right)^2 + x^2}$$
$$CH = \frac{CP}{5} + \frac{4-x}{15} = \frac{\sqrt{\left(6\sqrt{2}\right)^2 + x^2}}{5} + \frac{4-x}{15} = \frac{\sqrt{72+x^2}}{5} + \frac{4-x}{15}$$
$$= \left(\frac{1}{5}\right)(72+x^2)^{\frac{1}{2}} + (4-x)\left(\frac{1}{15}\right)$$
$$CH' = \left(\frac{1}{5}\right)(72+x^2)^{-\frac{1}{2}}\left(\frac{1}{2}\right)(2x) + \left(\frac{1}{15}\right)(-1) = \frac{2x}{10\sqrt{72+x^2}} - \frac{1}{15}$$
$$\frac{1}{15} = \frac{2x}{10\sqrt{72+x^2}}$$
$$10\sqrt{72+x^2} = 30x$$
$$\sqrt{72+x^2} = 3x$$
$$72 + x^2 = 9x^2$$
$$8x^2 = 72$$
$$x^2 = 9$$
$$x = 3$$

Simon should land 3km from the campsite on the opposite bank.

34. i) $90° + 90° + 90° - 15° = 180° + 75° = 255°$

ii) $N105°W$

iii) $S75°W$

iv) $E165°W$

35. a)

$$v = \sqrt{(-8)^2 + (8)^2} = \sqrt{64 + 64} = \sqrt{128} \approx 11.31$$
$$\theta = \tan^{-1}\left(\frac{8}{-8}\right) = \tan^{-1}(-1) = -45°$$
$\vec{v} = 11.31, -45°$0/1 you must state the answers with the positive x axis measured counter clockwise

b)

$$v = \sqrt{2^2 + (-6)^2} = \sqrt{4 + 36} = \sqrt{40} \approx 6.32$$
$$\theta = \tan^{-1}\left(\frac{-6}{2}\right) = \tan^{-1}(-3) \approx -71.56°$$
$$\vec{v} = 6.32, -71.56°$$

36. a)

$$y = 6\sin\theta = 6\sin 45° \approx 4.24$$
$$x = 6\cos\theta = 6\cos 45° \approx 4.24$$
$$\vec{v} = (4.24, 4.24)$$

b)

$$y = 10\sin\theta = 10\sin 240° \approx -8.66$$
$$x = 10\cos\theta = 10\cos 240° \approx -5$$
$$\vec{v} = (-5, -8.66)$$

c)

$$y = 4\sin\theta = 4\sin 90° = 4$$
$$x = 4\cos\theta = 4\cos 90° = 0$$
$$\vec{v} = (0, 4)$$

37. a)

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} = (1,2) + (-2,3) = (-1,5)$$

$|\overrightarrow{OC}| = \sqrt{(-1)^2 + 5^2} = \sqrt{1 + 25} = \sqrt{26}$0/1 the highlight should be $= (-2-1, 3-2)$

$= (-3, 1)$

b)

$$\overrightarrow{OO} = \overrightarrow{OM} + \overrightarrow{ON} = (-3,3,4) + (-3,4,6) = (-6,7,10)$$
$$|\overrightarrow{OO}| = \sqrt{(-6)^2 + 7^2 + 10^2} = \sqrt{36 + 42 + 100} = \sqrt{178}$$

0/1 the highlight should be $= (-3-(-3), 4-3, 6-4)$

$= (0, 1, 2)$

38.

$\overrightarrow{uv}^2 = \vec{u}^2 + \vec{v}^2 - 2\vec{u}\vec{v}\cos(140°) = 6^2 + 2^2 - 2(6)(2)\cos(140°) \approx 36 + 4 - 24(-0.766) = 40 + 18.384 = 58.384$0/1 the angle used should be 40 not 140

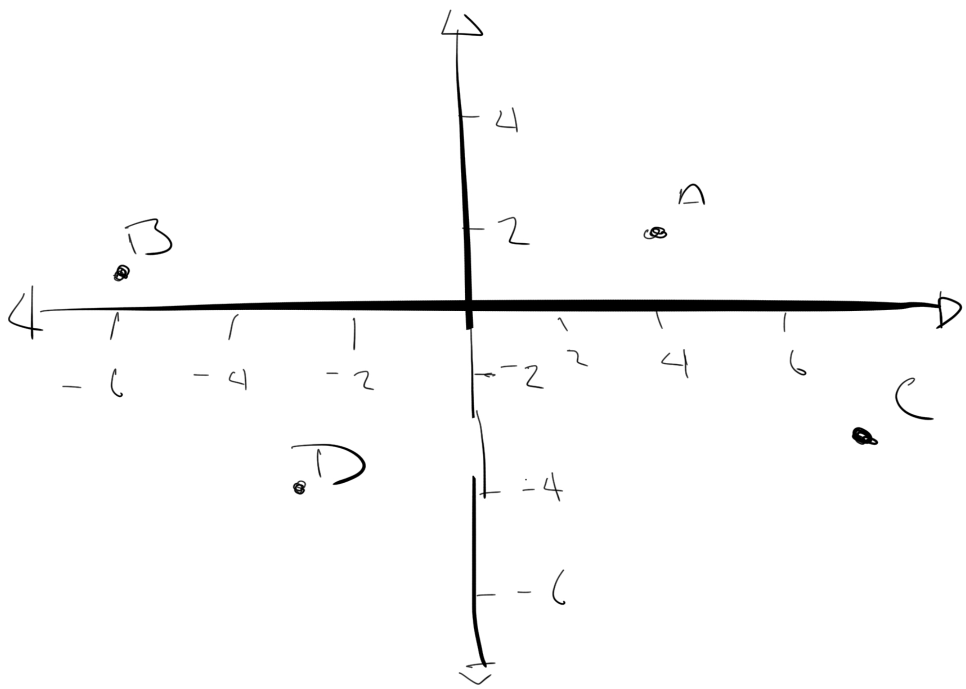
$$\overrightarrow{uv} \approx 7.64$$

$$\frac{\sin 40}{7.64} = \frac{\sin\alpha}{6}$$

$$6 \sin 40° = 7.64 \sin \alpha$$
$$7.64 \sin \alpha \approx 3.86$$
$$\sin \alpha \approx 0.5$$
$$\alpha = 30°$$

The magnitude of $\overline{uv}$ is 7.64 with an angle of 30 degrees from $\vec{v}$ to $\vec{u}$.

39.




$$\overrightarrow{AC} = \overrightarrow{BD} = \sqrt{(-6+2)^2 \ + (1+4)^2} = \sqrt{(-3)^2 + 5^2} = \sqrt{9+25} = \sqrt{36} = 6$$
$$C = (D_1 - B_1 + A_1, D_2 - B_2 + A_2) = (-3-(-6)+4, -4-1+2) = (7,-3)$$

0/1 C might be (and is located under B not under A). But what we do know is that

$\overrightarrow{AB} = \overrightarrow{CD}$. And we can solve from this which gets us (-13,-5)

40.

$$A^2 = (-8)^2 + 4^2 + (-2)^2 = 64 + 16 + 4 = 84$$
$$A \approx 9.2$$
$$B^2 = (-6)^2 + 3^2 + 5^2 = 36 + 9 + 25 = 70$$
$$B \approx 8.4$$
$$C^2 = (-10)^2 + 5^2 + (-9)^2 = 100 + 25 + 81 = 206$$
$$C \approx 14.4$$

Given the magnitudes of the sides of the figure, it is not an equilateral pyramid, but it is a pyramid where the sides are not equal.

0/4 answer is a straight line. You must find $|\overrightarrow{AB}|\ BC|and\ |\overrightarrow{AC}|$

41. a)

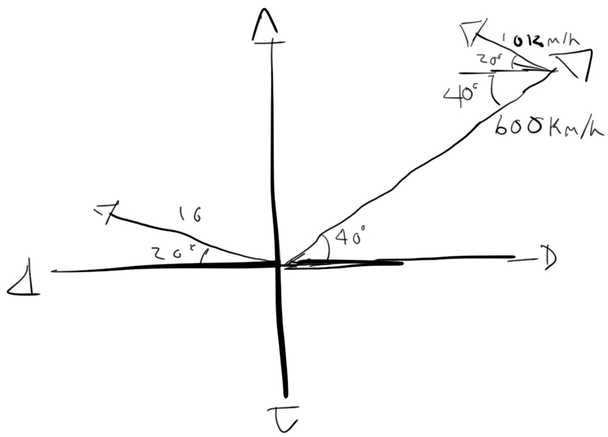
$$3(2\vec{u} - 3\vec{v}) - 2(-2\vec{v} + 3\vec{u}) = 6\vec{u} - 9\vec{v} + 4\vec{v} - 6\vec{u} = -5\vec{v}$$

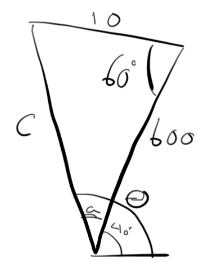
b)

$5(\vec{a}+2\vec{b}+4\vec{c})-4(2\vec{a}-3\vec{b}-5\vec{c}) = 5\vec{a}+10\vec{b}+20\vec{c}-8\vec{a}+12\vec{b}+20\vec{c} = -3\vec{a}+22\vec{b}+40\vec{c}$ 0/1 answer is

$= -3\vec{a}+2\vec{b}+40\vec{c}$

42.




$c^2 = 10^2 + 600^2 - 2(10)(600)\cos 60° = 100 + 360000 - 12000(0.5) = 360100 - 6000 = 354100$0/1 the angle is 110 not 60

$$c \approx 595$$

$$\frac{\sin a}{10} = \frac{\sin 60°}{595}$$

$$595 \sin a = 10 \sin 60° \approx 8.66$$

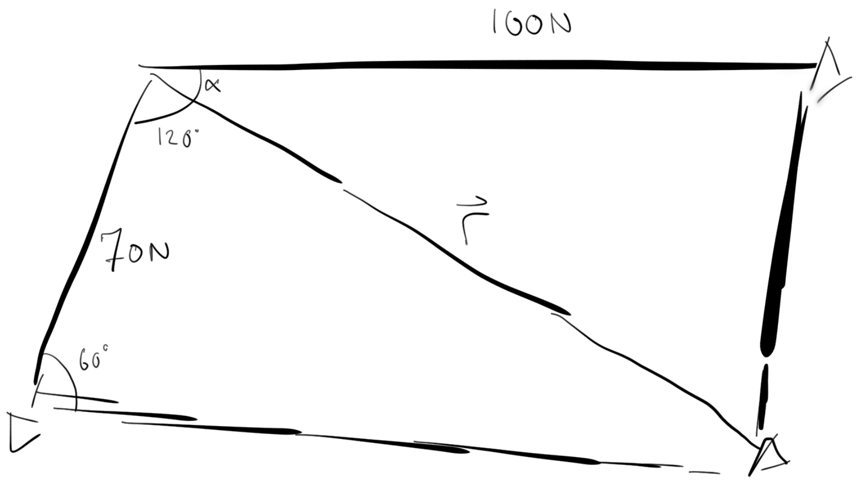
$$\sin a = \frac{8.66}{595}$$

$$a = \sin^{-1}\left(\frac{8.66}{595}\right) \approx 0.83°$$

$$\theta = 40° + a = 40° + 0.83° = 40.83$$

The plane will be flying 595 km/h at W40.83$^0$N0/1 direction will be EN

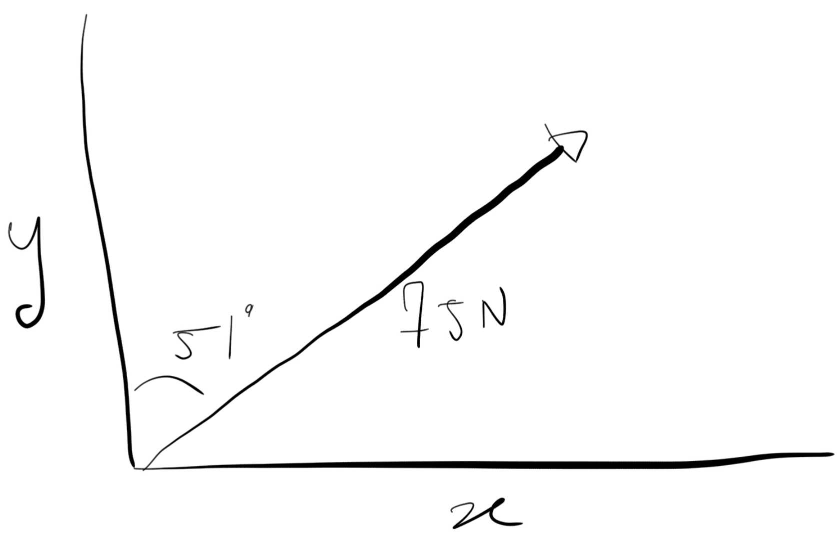
43.




$$|\vec{r}|^2 = 70^2 + 100^2 - 2(70)(100)\cos 60° = 4900 + 10000 - 14000(0.5)$$
$$= 14900 - 7000 = 7900$$
$$|\vec{r}| \approx 88.88$$
$$\frac{\sin\alpha}{70} = \frac{\sin 60°}{88.88}$$
$$88.88\sin\alpha = 70\sin 60° \approx 60.62$$
$$\sin\alpha = \frac{60.62}{88.88}$$
$$\alpha = \sin^{-1}\left(\frac{60.62}{88.88}\right) \approx 43°$$

The resultant of the two forces is 88.88N at an angle of $43^0$ from the 100N force.

44.

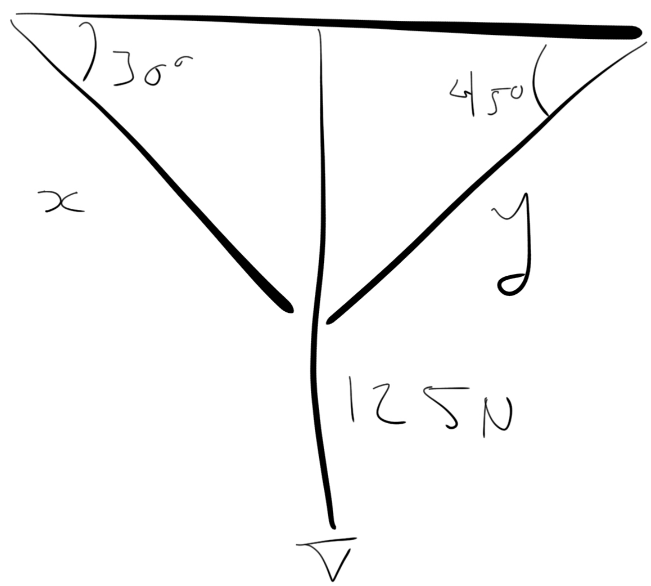



$$\sin 51° = \frac{\vec{x}}{75N}$$
$$\vec{x} = 75N\sin 51° \approx 58.29N$$
$$\cos 51° = \frac{\vec{y}}{75N}$$

$$\vec{y} = 75N\cos 51° \approx 47.20N$$

$\vec{y}$ is 47.2N at an angle of $51^0$ from the 75N force. $\vec{x}$ is 58.29N at an angle of $39^0$ from the 75N force.

45.




$$|\vec{y}|^2 = 2(125N)^2 = 2(15625N^2) = 31250N^2$$
$$|\vec{y}| = 176.78N$$
$$\sin 30° = \frac{125N}{|\vec{x}|}$$
$$|\vec{x}| = \frac{125N}{\sin 30°} = \frac{125N}{0.5} = 250N$$

3/8 start with Horizontal components:

$$T_1\cos(30) = T_2\cos(45)$$

Then find the vertical components. Then use substitution to find the two unknowns.