

46. **3/3**

$$\vec{u} \cdot \vec{v} = (-1)(-2) + (0)(2) + (1)(-1) = 2 + 0 - 1 = 1$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{(-1)(-2) + (0)(2) + (1)(-1)}{\sqrt{(-1)^2 + 0^2 + 1^2}\sqrt{(-2)^2 + 2^2 + (-1)^2}} = \frac{2 + 0 - 1}{\sqrt{2}\sqrt{9}} = \frac{1}{3\sqrt{2}}$$

$$\theta \approx 76.37^\circ \quad \checkmark$$

47. **3/3**

$$(-1, 3, 4) \times (-5, 6, 0) = ((3)(0) - (6)(4), (4)(-5) - (0)(1), (-1)(6), (-5)(3)) = (0 - 24, -20 + 0, -6 + 15) = (-24, -20, 9)$$

$$(-24, -20, 9) \cdot (-1, 3, 4) = (-24)(-1) + (-20)(3) + (9)(4) = 24 - 60 + 36 = 0$$

$$(-24, -20, 9) \cdot (-5, 6, 0) = (-24)(-5) + (-20)(6) + (9)(0) = 60 - 60 = 0 \quad \checkmark$$

48. **2/2**

$$(k\vec{u}) \cdot \vec{v} = k(\vec{u} \cdot \vec{v}) = (k\vec{u}_1, k\vec{u}_2) \cdot \vec{v} = (k\vec{u}_1)(\vec{v}_1) + (k\vec{u}_2)(\vec{v}_2) = k(\vec{u}_1)(\vec{v}_1) + k(\vec{u}_2)(\vec{v}_2) = k((\vec{u}_1)(\vec{v}_1) + (\vec{u}_2)(\vec{v}_2)) = k(\vec{u} \cdot \vec{v}) \quad \checkmark$$

49. **8/8**

a)

$$\vec{u} \cdot \vec{v} = (1)(-2) + (3)(2) + (-2)(2) = -2 + 6 - 4 = 0$$

$$\vec{v} \cdot \vec{w} = (-2)(5) + (2)(1) + (2)(4) = -10 + 2 + 8 = 0$$

$$\vec{w} \cdot \vec{u} = (5)(1) + (1)(3) + (4)(-2) = 5 + 3 - 8 = 0$$

Since the dot products are 0, then  $\vec{u}$ ,  $\vec{v}$ , and  $\vec{w}$  must therefore be perpendicular to each other.

b)

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|(1) = |\vec{u}||\vec{v}| \cos 0^\circ$$

What we know is that the angle of the two vectors is zero. We do not know if they are the same magnitude, but they are in the same direction.  $\checkmark$

50. **4/4**

$$|AB \times BC| = (1+1, -1-2, 2-4) \times (2+1, 3-2, 4-4) = (2, -3, -2) \times (3, 1, 0) =$$

$$((-3)(0) - (1)(-2), (-2)(3) - (0)(2), (2)(1) - (3)(-3)) = (0+2, -6-0, 2+9) = (2, -6, 11) = \sqrt{2^2 + (-6)^2 + 11^2} = \sqrt{4+36+121} = \sqrt{161}$$

$$ABC = \frac{\sqrt{161}}{2} \approx 6.34 \text{ square units} \quad \checkmark$$

51. **4/4**

$$V = |(\vec{a} \times \vec{b}) \cdot \vec{c}| = |(2, -5, -1) \times (2, 0, -1) \cdot (-1, 0, 1)| = |((-3)(0) -$$

$$(0)(-1), (1)(2) - (-1)(2), (2)(0) - (2)(-5)) \cdot (-1, 0, 1)| = |(5-0, -2+2, 0+10) \cdot (-1, 0, 1)| = |(5)(-1) + (0)(0) + (10)(1)| = |(5)(-1) + (0)(0) + (10)(1)| =$$

$$|-5+10| = |5| = 5$$

[Make sure you are giving word answers or concluding statements: The volume is 5 cubic units.]

52. **5/5**

a)

$$|\vec{T}| = |\vec{r}||\vec{F}| \sin \theta = |0.3 \text{ m}||50 \text{ N}| \sin 30^\circ = (15 \text{ Nm})(1) = 7.5 \text{ J}$$

$$|\vec{T}| = |0.3 \text{ m}||50 \text{ N}| = (15 \text{ Nm})(1) = 15 \text{ J}$$

The torque will be 7.5 joules at an angle of 30 degrees. The maximum torque that can be applied is 15 joules when the angle is 90 degrees.

b)

$$w = |\vec{F}||\vec{d}| \cos 70^\circ = |200 \text{ N}||5 \text{ m}| \cos 70^\circ \approx (1000 \text{ J})(0.342) \approx 342 \text{ J} \quad \checkmark$$

53. **3/4**

$$3x + 4y = 20$$

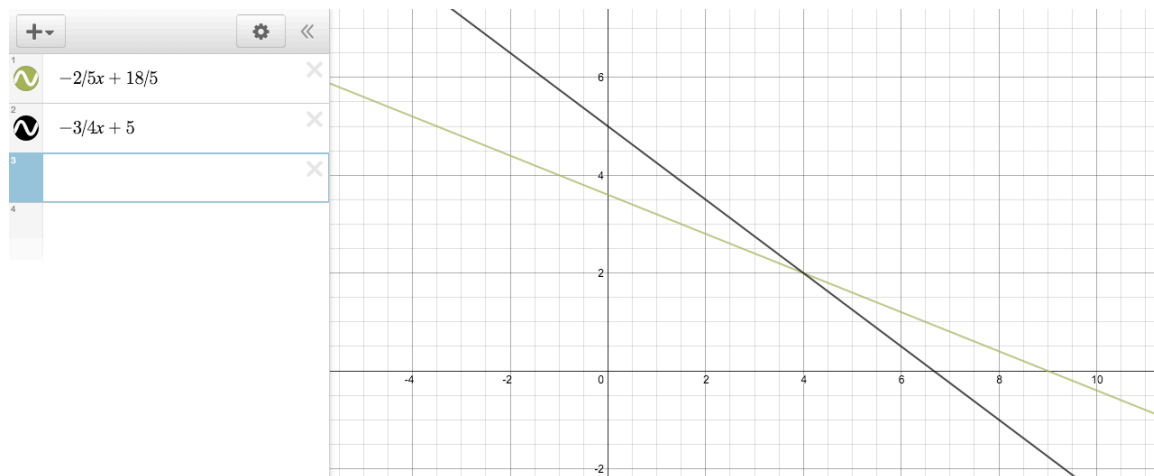
$$4y = -3x + 20$$

$$y = -\frac{3}{4}x + 5$$

$$2x + 5y = 18$$

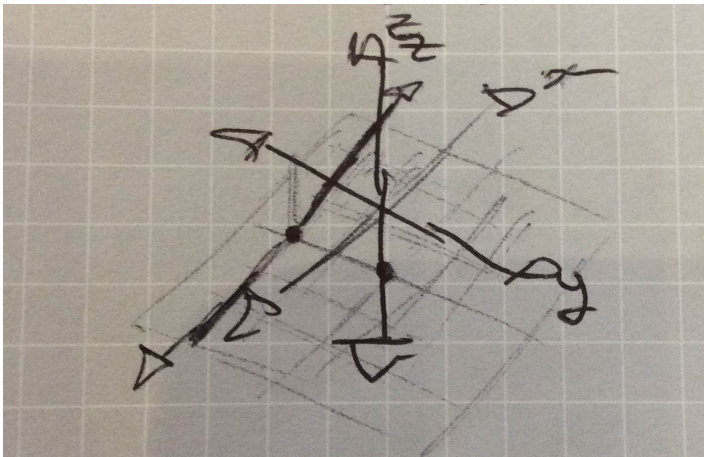
$$5y = -2x + 18$$

$$y = -\frac{2}{5}x + \frac{18}{5}$$



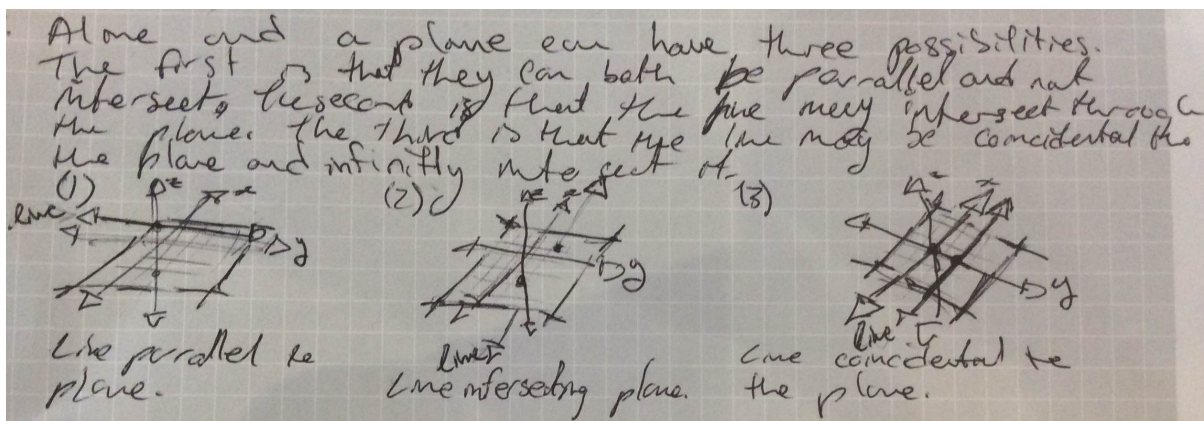
The intersection is (2, 4). [The coordinates are (4,2)]

54. 4/4



When  $z = -3$ , then you would have a horizontal plane, with a “height” of 3. When  $y$  is set equal to  $z$  ( $y = z$ ), it is also equal to -3. Therefore, you’re left with a straight line who’s only dynamic variable is  $x$ . This is the intersection: ✓

55. 6/6

56. **6/6**

a)

$$(x, y, z) = (1, -3, 1) + t(2, -2, 1)$$

$$x = 1 + 2t$$

$$y = -3 - 2t$$

$$z = 1 + t$$

b)

$$(x, y, z) = (3, 0, 4) + t(1, 0, 0)$$

$$x = 3 + t$$

$$y = 0$$

$$z = 4$$

c)

$$\overrightarrow{AB} = (1, 2, 1) - 1, 2, 1 = (2, 0, 0)$$

$$(x, y, z) = (1, 2, 1) + t(2, 0, 0)$$

$$x = 1 + 2t$$

$$y = 2$$

$$z = 1$$

57. **5/8**

$$A(1, -2, 0), B(1, -2, 2), C(0, 3, 2)$$

$$\overrightarrow{AB} = (1, -2, 2) - (1, -2, 0) = (0, 0, 2)$$

$$\overrightarrow{AC} = (0, 3, 2) - (1, -2, 0) = (-1, 5, 2)$$

$$x = (0)(2) - (5)(2) = -10$$

$$y = (2)(-1) - (2)(0) = -2$$

$$z = (0)(5) - 1(0) = 0$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = (x, y, z) = (-10, -2, 0) \quad \checkmark$$

$$-10x - 2y + 0z + d = 0$$

$$-10x - 2y + d = 0$$

$$-10(1) - 2(-2) + d = 0$$

$$-10 + 4 + d = 0$$

$$-6 + d = 0$$

$$d = 6 \quad \checkmark$$

[Therefore, the scalar equation is ?? The vector equation is ??, and the parametric equations are ??]

58. **5/5**

$$(1)(1) - (1)(-2) = 3$$

$$(1)(4) - (1)(3) = 1$$

$$(3)(-2) - (4)(1) = -10$$

$$(3, 1, 1) \times (4, -2, 1) = (3, 1, -10)$$

$$3x + y - 10z + d = 0$$

$$3(-1) + (2) - 10(0) + d = 0$$

$$-3 + 2 + d = 0$$

$$d = 1$$

$$3x + y - 10z + 1 = 0 \quad \checkmark$$

59. **6/6**

$$\begin{array}{r} x - 2y + 3z - 1 = 0 \\ - x - 4y + 2z - 8 = 0 \\ \hline 2y + z + 7 = 0 \end{array}$$

Let  $z = t$ :

$$2y + t + 7 = 0$$

$$2y = -7 - t$$

$$y = \frac{-7-t}{2}$$

$$x - 2y + 3z - 1 = 0$$

$$x = 2y - 3z + 1 = 2\left(\frac{-7-t}{2}\right) - 3t + 1 = -7 - t - 3t + 1 = -4t - 6$$

$$x = -4t - 6$$

$$y = \frac{-7-t}{2}$$

$$z = t$$

$$(x, y, z) = (1, -4, 2) + t \left( -4, -\frac{1}{2}, 1 \right) \quad \checkmark$$

60. **3/8**

$$\begin{array}{r} 2x + y + 2z - 4 = 0 \\ + \quad x - y - z - 2 = 0 \\ \hline 3x + z - 6 = 0 \end{array}$$

$$\begin{array}{r} 4x + 2y + 4z - 8 = 0 \\ - \quad x + 2y - 6z - 12 = 0 \\ \hline 3x - 2z + 4 = 0 \\ [5x + 10z + 4 = 0, \text{ etc...}] \end{array}$$

$$\begin{array}{r} 3x + z - 6 = 0 \\ - \quad 3x - 2z + 4 = 0 \\ \hline 5z - 10 = 0 \end{array}$$

$$5z - 10 = 0$$

$$5z = 10$$

$$z = 2$$

$$\begin{array}{l} 3x - 2z + 4 = 0 \\ 3x - 2(2) + 4 = 0 \\ 3x - 4 + 4 = 0 \\ 3x = 0 \\ x = 0 \end{array}$$

$$\begin{array}{l} 2x + y + 2z - 4 = 0 \\ 2(0) + y + 2(2) - 4 = 0 \\ y + 4 - 4 = 0 \\ y = 0 \end{array}$$

Geometrically, the three planes represented by the linear equations intersect at the point  $(0, 0, 2)$ . [The three planes meet at a point:  $(64/27, 40/27, -10/9)$ ].

61. **3/4**

$$3(0) - 2(5) + 5(0) + 10 = 0$$

$$P = (0, 5, 5)$$

$$[P = (0, 5, 0)]$$

$$PA = \overrightarrow{OA} - \overrightarrow{OP} = (-2, 1, 2) - (0, 5, 0) = (-2, -4, 2) \quad \checkmark$$

$$|\overrightarrow{AH}| = \left| \frac{\overrightarrow{PA} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \right| |\vec{n}| = \left| \frac{(-2, -4, 2) \cdot (3, -2, 5)}{(3, -2, 5) \cdot (3, -2, 5)} \right| \sqrt{(3)^2 + (-2)^2 + (5)^2} =$$

$$\left| \frac{-6+8+10}{9+4+10} \right| \sqrt{9+4+10} = \frac{12\sqrt{23}}{23} \left[ = \frac{6\sqrt{38}}{19}; 5^2 = 25 \right]$$

**Final mark = 70/80 = 88%**