

46. **3/3**

$$\vec{u} \cdot \vec{v} = (-1)(-2) + (0)(2) + (1)(-1) = 2 + 0 - 1 = 1$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{(-1)(-2) + (0)(2) + (1)(-1)}{\sqrt{(-1)^2 + 0^2 + 1^2} \sqrt{(-2)^2 + 2^2 + (-1)^2}} = \frac{2 + 0 - 1}{\sqrt{2} \sqrt{9}} = \frac{1}{3\sqrt{2}}$$

$$\theta \approx 76.37^\circ \quad \checkmark$$

47. **3/3**

$$(-1, 3, 4) \times (-5, 6, 0) = ((3)(0) - (6)(4), (4)(-5) - (0)(1), (-1)(6), (-5)(3)) = \\ (0 - 24, -20 + 0, -6 + 15) = (-24, -20, 9)$$

$$(-24, -20, 9) \cdot (-1, 3, 4) = (-24)(-1) + (-20)(3) + (9)(4) = 24 - 60 + 36 = 0 \\ (-24, -20, 9) \cdot (-5, 6, 0) = (-24)(-5) + (-20)(6) + (9)(0) = 60 - 60 = 0 \quad \checkmark$$

48. **2/2**

$$(k\vec{u}) \cdot \vec{v} = k(\vec{u} \cdot \vec{v}) = (k\vec{u}_1, k\vec{u}_2) \cdot \vec{v} = (k\vec{u}_1)(\vec{v}_1) + (k\vec{u}_2)(\vec{v}_2) = k(\vec{u}_1)(\vec{v}_1) + \\ k(\vec{u}_2)(\vec{v}_2) = k((\vec{u}_1)(\vec{v}_1) + (\vec{u}_2)(\vec{v}_2)) = k(\vec{u} \cdot \vec{k}) \quad \checkmark$$

49. **8/8**

a)

$$\vec{u} \cdot \vec{v} = (1)(-2) + (3)(2) + (-2)(2) = -2 + 6 - 4 = 0$$

$$\vec{v} \cdot \vec{w} = (-2)(5) + (2)(1) + (2)(4) = -10 + 2 + 8 = 0$$

$$\vec{w} \cdot \vec{u} = (5)(1) + (1)(3) + (4)(-2) = 5 + 3 - 8 = 0$$

Since the dot products are 0, then \vec{u} , \vec{v} , and \vec{w} must therefore be perpendicular to each other.

b)

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos 0^\circ$$

What we know is that the angle of the two vectors is zero. We do not know if they are the same magnitude, but they are in the same direction. \checkmark

50. **4/4**

$$\begin{aligned}
 |AB \times BC| &= (1+1, -1-2, 2-4) \times (2+1, 3-2, 4-4) = (2, -3, -2) \times (3, 1, 0) = \\
 &((-3)(0) - (1)(-2), (-2)(3) - (0)(2), (2)(1) - (3)(-3)) = (0+2, -6-0, 2+ \\
 &9) = (2, -6, 11) = \sqrt{2^2 + (-6)^2 + 11^2} = \sqrt{4 + 36 + 121} = \sqrt{161} \\
 ABC &= \frac{\sqrt{161}}{2} \approx 6.34 \text{ square units } \checkmark
 \end{aligned}$$

51. **4/4**

$$\begin{aligned}
 V &= |(\vec{a} \times \vec{b}) \cdot \vec{c}| = |((2, -5, -1) \times (2, 0, -1)) \cdot (-1, 0, 1)| = |((-3)(0) - \\
 &(0)(-1), (1)(2) - (-1)(2), (2)(0) - (2)(-5)) \cdot (-1, 0, 1)| = |(5 - 0, -2 + 2, 0 + \\
 &10) \cdot (-1, 0, 1)| = |(5)(-1) + (0)(0) + (10)(1)| = |(5)(-1) + (0)(0) + (10)(1)| = \\
 &|-5 + 10| = |5| = 5
 \end{aligned}$$

[Make sure you are giving word answers or concluding statements: The volume is 5 cubic units.]

52. **5/5**

a)

$$\begin{aligned}
 |\vec{T}| &= |\vec{r}| |\vec{F}| \sin \theta = |0.3 \text{ m}| |50 \text{ N}| \sin 30^\circ = (15 \text{ Nm})(1) = 7.5 \text{ J} \\
 |\vec{T}| &= |0.3 \text{ m}| |50 \text{ N}| = (15 \text{ Nm})(1) = 15 \text{ J}
 \end{aligned}$$

The torque will be 7.5 joules at an angle of 30 degrees. The maximum torque that can be applied is 15 joules when the angle is 90 degrees.

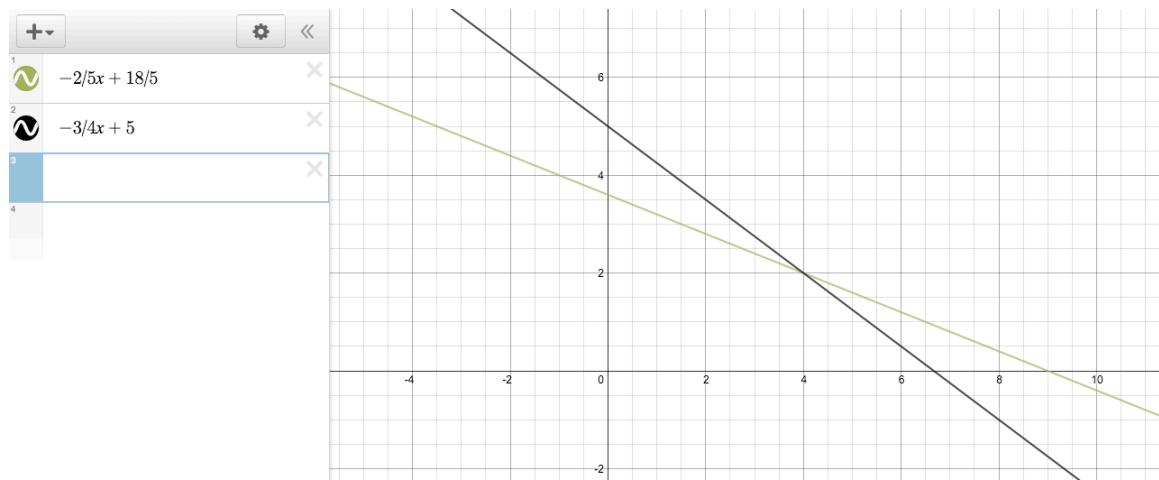
b)

$$w = |\vec{F}| |\vec{d}| \cos 70^\circ = |200 \text{ N}| |5 \text{ m}| \cos 70^\circ \approx (1000 \text{ J})(0.342) \approx 342 \text{ J } \checkmark$$

53. **3/4**

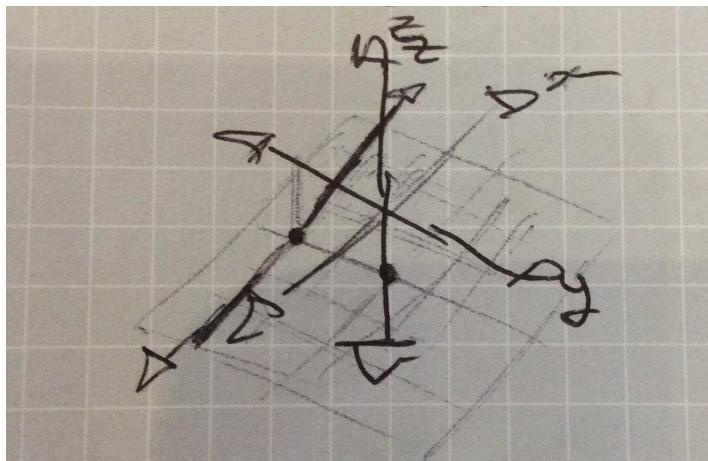
$$\begin{aligned}
 3x + 4y &= 20 \\
 4y &= -3x + 20 \\
 y &= -\frac{3}{4}x + 5
 \end{aligned}$$

$$\begin{aligned}
 2x + 5y &= 18 \\
 5y &= -2x + 18 \\
 y &= -\frac{2}{5}x + \frac{18}{5}
 \end{aligned}$$



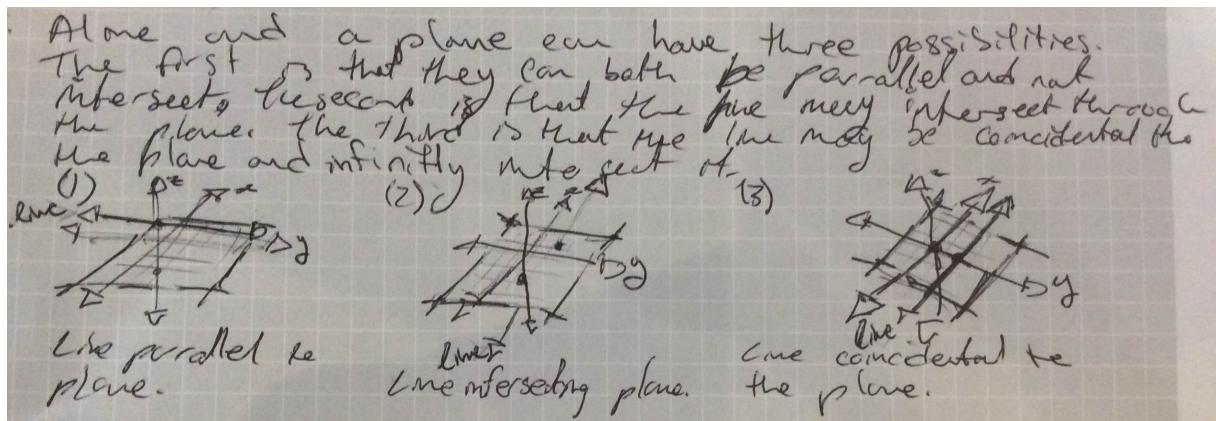
The intersection is $(2, 4)$. [The coordinates are $(4, 2)$]

54. **4/4**



When $z = -3$, then you would have a horizontal plane, with a "height" of 3. When y is set equal to z ($y = z$), it is also equal to -3. Therefore, you're left with a straight line who's only dynamic variable is x . This is the intersection: **✓**

55. **6/6**

56. **6/6**

a)

$$(x, y, z) = (1, -3, 1) + t(2, -2, 1)$$

$$x = 1 + 2t$$

$$y = -3 - 2t$$

$$z = 1 + t$$

b)

$$(x, y, z) = (3, 0, 4) + t(1, 0, 0)$$

$$x = 3 + t$$

$$y = 0$$

$$z = 4$$

c)

$$\overrightarrow{AB} = (1, 2, 1) - (1, 2, 1) = (0, 0, 0)$$

$$(x, y, z) = (1, 2, 1) + t(0, 0, 0)$$

$$x = 1 + 0t$$

$$y = 2$$

$$z = 1$$

57. **5/8**

$$A(1, -2, 0), B(1, -2, 2), C(0, 3, 2)$$

$$\overrightarrow{AB} = (1, -2, 2) - (1, -2, 0) = (0, 0, 2)$$

$$\overrightarrow{AC} = (0, 3, 2) - (1, -2, 0) = (-1, 5, 2)$$

$$x = (0)(2) - (5)(2) = -10$$

$$y = (2)(-1) - (2)(0) = -2$$

$$z = (0)(5) - 1(0) = 0$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = (x, y, z) = (-10, -2, 0) \quad \checkmark$$

$$-10x - 2y + 0x + d = 0$$

$$-10x - 2y + d = 0$$

$$-10(1) - 2(-2) + d = 0$$

$$-10 + 4 + d = 0$$

$$-6 + d = 0$$

$$d = 6 \quad \checkmark$$

[Therefore, the scalar equation is ?? The vector equation is ??, and the parametric equations are ??]

58. **5/5**

$$(1)(1) - (1)(-2) = 3$$

$$(1)(4) - (1)(3) = 1$$

$$(3)(-2) - (4)(1) = -10$$

$$(3, 1, 1) \times (4, -2, 1) = (3, 1, -10)$$

$$3x + y - 10z + d = 0$$

$$3(-1) + (2) - 10(0) + d = 0$$

$$-3 + 2 + d = 0$$

$$d = 1$$

$$3x + y - 10x + 1 = 0 \quad \checkmark$$

59. **6/6**

$$\begin{array}{r} x-2y+3z-1=0 \\ -x-4y+2z-8=0 \\ \hline 2y+z+7=0 \end{array}$$

Let $z = t$:

$$2y + t + 7 = 0$$

$$2y = -7 - t$$

$$y = \frac{-7-t}{2}$$

$$x - 2y + 3z - 1 = 0$$

$$x = 2y - 3z + 1 = 2\left(\frac{-7-t}{2}\right) - 3t + 1 = -7 - t - 3t + 1 = -4t - 6$$

$$x = -4t - 6$$

$$y = \frac{-7-t}{2}$$

$$z = t$$

$$(x, y, z) = (1, -4, 2) + t \left(-4, -\frac{1}{2}, 1 \right) \quad \checkmark$$

60. **3/8**

$$\begin{array}{r} 2x + y + 2z - 4 = 0 \\ + x - y - z - 2 = 0 \\ \hline 3x + z - 6 = 0 \end{array}$$

$$\begin{array}{r} 4x + 2y + 4z - 8 = 0 \\ - x + 2y - 6z - 12 = 0 \\ \hline 3x - 2z + 4 = 0 \\ [5x + 10z + 4 = 0, \text{etc...}] \end{array}$$

$$\begin{array}{r} 3x + z - 6 = 0 \\ - 3x - 2z + 4 = 0 \\ \hline 5z - 10 = 0 \end{array}$$

$$5z - 10 = 0$$

$$5z = 10$$

$$z = 2$$

$$\begin{array}{r} 3x - 2z + 4 = 0 \\ 3x - 2(2) + 4 = 0 \\ 3x - 4 + 4 = 0 \\ 3x = 0 \\ x = 0 \end{array}$$

$$\begin{array}{r} 2x + y + 2z - 4 = 0 \\ 2(0) + y + 2(2) - 4 = 0 \\ y + 4 - 4 = 0 \\ y = 0 \end{array}$$

Geometrically, the three planes represented by the linear equations intersect at the point $(0, 0, 2)$. [The three planes meet at a point: $(64/27, 40/27, -10/9)$].

61. **3/4**

$$3(0) - 2(5) + 5(0) + 10 = 0$$

$$P = (0, 5, 5)$$

$$[P = (0, 5, 0)]$$

$$PA = \overrightarrow{OA} - \overrightarrow{OP} = (-2, 1, 2) - (0, 5, 0) = (-2, -4, 2) \quad \checkmark$$

$$|\vec{AH}| = \left| \frac{\vec{PA} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \right| |\vec{n}| = \left| \frac{(-2, -4, 2) \cdot (3, -2, 5)}{(3, -2, 5) \cdot (3, -2, 5)} \right| \sqrt{(3)^2 + (-2)^2 + (5)^2} =$$

$$\left| \frac{-6+8+10}{9+4+10} \right| \sqrt{9+4+10} = \frac{12\sqrt{23}}{23} \quad \left[= \frac{6\sqrt{38}}{19}; 5^2 = 25 \right]$$

Final mark = 70/80 = 88%