6.15, 6.16) is used to calculate the amount of the mth isotope in the decay chain, providing the mth isotope is unstable.

$$N_m(t;\vec{\lambda},\vec{b},\vec{w},\vec{N_0}) = \sum_{k=1,m} r(k,m,\vec{\lambda},\vec{b}) \left[f(t;k,m,\vec{\lambda})N_{0,k} + g(t;k,m,\vec{\lambda})w_k\right] \tag{6.13}$$

$$r(k,m,\vec{\lambda},\vec{b}) = \begin{cases} \prod_{i=k,m-1}(b_{i+1}\lambda_i), & \text{if } k < m \\ 1, & \text{if } k = m \end{cases} \tag{6.14}$$

$$f(t;k,m,\vec{\lambda}) = (-1)^{m-k} \sum_{i=k,m} \left[\frac{\exp(-\lambda_i t)}{\prod_{j=k,m;j\neq i}(\lambda_i - \lambda_j)}\right] \tag{6.15}$$

$$g(t;k,m,\vec{\lambda}) = \frac{1}{\prod_{i=k,m}\lambda_i} + (-1)^{m-k+1} \sum_{i=k,m} \left[\frac{\exp(-\lambda_i t)}{\lambda_i \prod_{j=k,m;j\neq i}(\lambda_i - \lambda_j)}\right] \tag{6.16}$$

The set of equations (eqs. 6.17, 6.18, 6.19, 6.20) is used to calculate the amount of the mth isotope in the decay chain, where the mth isotope is stable.

$$N_m(t;\vec{\lambda},\vec{b},\vec{w},\vec{N_0}) = N_m + w_m t + \sum_{k=1,m-1} r(k,m,\vec{\lambda},\vec{b}) \left[f(t;k,m-1,\vec{\lambda})N_{0,k} + g(t;k,m,\vec{\lambda})w_k\right] \tag{6.17}$$

$$r(k,m,\vec{\lambda},\vec{b}) = \begin{cases} \prod_{i=k,m-1}(b_{i+1}\lambda_i), & \text{if } k < m \\ 1, & \text{if } k = m \end{cases} \tag{6.18}$$

$$f(t;k,m,\vec{\lambda}) = \frac{1}{\prod_{i=k,m-1}\lambda_i} + (-1)^{m-k+1} \sum_{i=k,m-1} \left[\frac{\exp(-\lambda_i t)}{\lambda_i \prod_{j=k,m-1;j\neq i}(\lambda_i - \lambda_j)}\right] \tag{6.19}$$

$$g(t;k,m,\vec{\lambda}) = \frac{1}{\prod_{i=k,m-1}\lambda_i} t - \frac{\sum_{i=k,m-1}\left[\prod_{j=k,m-1;j\neq i}\lambda_j\right]}{\prod_{i=k,m-1}\lambda_i^2} + (-1)^{m-k+1} \sum_{i=k,m-1} \left[\frac{\exp(-\lambda_i t)}{\lambda_i^2 \prod_{j=k,m-1;j\neq i}(\lambda_i - \lambda_j)}\right] \tag{6.20}$$

6.2.5 Preference: Analytic over Numeric

The numeric solution only requires the equation to be solved in the s-domain; the Gaver-Stehfest algorithm performs the inversion. It is worth the extra effort to derive and implement an analytic solution, as the numeric is only an approximation. Examples of the pitfalls of the numeric solution are that it can give negative amounts of an isotope and the difference between the numeric and analytic calculated amounts can become quite large