Pushing C-Bounds for lower approximation

Given the initial equation

$$x^T Q x + q^t x + c = 0$$

with the constraint that $Ax \le b$ we wish to discover the widest bounds on c such that we can guarantee a solution to the equation with the constraint. To begin we verify that a solution exists for a given b^*. Then we search for the largest set of bounds on c for various $b \le b^*$ using the following model which finds the largest dimensional difference in the expression $Ax - b$. First note that $Ax \le b \Rightarrow (b - Ax)(b - Ax)^T \ge 0 \Rightarrow bb^T - Axb^T - b(Ax)^T + A(xx^T)A^T \ge 0$, which can be relaxed linearly by allowing a symmetric positive semidefinite matrix $\mathcal{X}$ to replace xx^T.

$$\max z$$

Subject To:

(1) $Tr\,(Q\mathcal{X}) + q^t x + c = 0$

(2) $Ax \le b$

(3) $bb^T - Axb^T - b(Ax)^T + A\mathcal{X}A^T \ge 0$

(4) $\mathcal{X}$ is symmtric, positive semidefinite

(5) $z \le A[i,:]x - b_i + M * (1 - d_i)\ \forall i$ for some M large enough.

(6) $\sum_i d_i = 1$

(7) $d_i \in \{0, 1\}$

Notes:

Implementation of the above model was done by excluding the semidefinite constraint. Additionally an LP formulation of this model was implemented to deal with higher dimensional cases. The LP formulation eliminated the variable z, and constraints 5, 6, & 7. The objective was changed to a feasible one and an outer for loop was added to iterate over the dimensions of b so that at each dimension the constraint $(Ax)_i \le b_i$ would be changed to $(Ax)_i = b_i$ then back again after solving the model. If a solution is found then a boundary solution is identified, else if no solutions are found over all dimensions we are free to push the radius on c further. This implementation in practice was slower then its MIP counterpart for all of the smaller case files ($<$ 30). At case 30 of the matpower files the LP began to keep pace and even beat out the MIP formulation. At case 57 the LP formulation was the only model to finish in a reasonable time frame with the MIP formulation unable to complete a single model (i.e. find a single optimal z value), in the time it took the LP formulation to complete an entire model and find a max radius on c.