

## Assignment 2

### Task 2)

$$\textcircled{1} \quad ax + by + c = 0$$

$$\textcircled{2} \quad dx + ey + f = 0$$

$a, b, c, d, e, f$  are real constants

Rearranging  $\textcircled{1}$ :  $by = -c - ax$   
 $\Leftrightarrow y = \frac{-c}{b} - \frac{a}{b}x$

Subbing into  $\textcircled{2}$ :  $dx - \frac{ce}{b} - \frac{ae}{b}x + f = 0$

$$\Leftrightarrow \left( \frac{bd - ae}{b} \right) x + \frac{bf - ce}{b} = 0$$

$$\Leftrightarrow \left( \frac{bd - ae}{b} \right) x = \frac{ce - bf}{b}$$

$$\Leftrightarrow x = \frac{ce - bf}{bd - ae}$$

Subbing into  $\textcircled{1}$ :  $\frac{ace - abf}{bd - ae} + by + c = 0$

$$\Leftrightarrow by = -c + \frac{abf - ace}{bd - ae}$$

$$\Leftrightarrow by = \frac{-bcd + ace + abf - ace}{bd - ae}$$

$$\Leftrightarrow by = \frac{abf - bcd}{bd - ae}$$

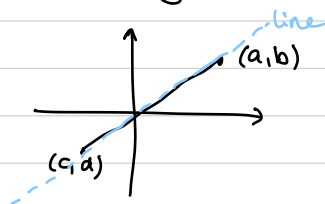
$$\Leftrightarrow y = \frac{af - cd}{bd - ae}$$

hence point of intersection at  $\left( \frac{ce - bf}{bd - ae}, \frac{af - cd}{bd - ae} \right)$

There is no point of intersection and the lines are parallel when  $bd - ae = 0$

Task 3)

a) Segment between points  $(a, b)$  and  $(c, d)$



$$\text{Eq}^n \text{ of Line: } \frac{x-a}{c-a} = \frac{y-b}{d-b}$$

$$(d-b)(x-a) = (y-b)(c-a)$$

$$(d-b)x - a(d-b) = (c-a)y - b(c-a)$$

$$(d-b)x + (a-c)y - ad + ab + bc - ab = 0$$

$$(d-b)x + (a-c)y + bc - ad = 0$$

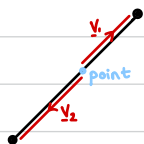
in required form  $\alpha x + \beta y + \gamma = 0$

b)

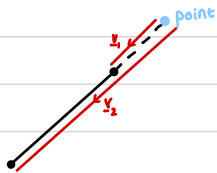
Assume point is on line from segment

$\vec{v}_1$  = vector from point to an endpoint of segment

$\vec{v}_2$  = vector from point to other endpoint of segment



if a point lies on the segment,  $\vec{v}_1$  and  $\vec{v}_2$  are in opposite directions  $\Rightarrow \vec{v}_1 \cdot \vec{v}_2 < 0$

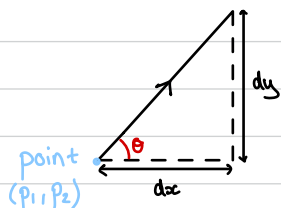


if a point does not lie on the segment,  $v_1$  and  $v_2$  are in same direction  $\Rightarrow v_1 \cdot v_2 > 0$

c) To find intersection of 2 segments:

- Find both lines from the segments
- Find intersection of these lines
- Check if this point lies on both segments

Task 4) First find eq<sup>n</sup> line which ray lies on



$$\text{gradient} = \tan \theta = \frac{dy}{dx}$$

Eq<sup>n</sup> Line from ray:  $y - y_1 = m(x - x_1)$

$$mx - y - mx_1 + y_1 = 0$$

$$\tan \theta x - y - p_1 \tan \theta + p_2 = 0$$

Check to see if a point lying on line extended from ray is also on the ray

$v_1$  = vector from ray origin point to checking point

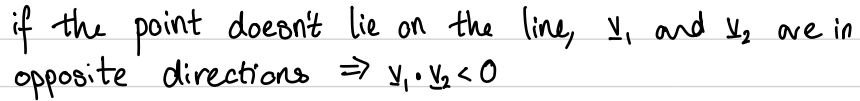
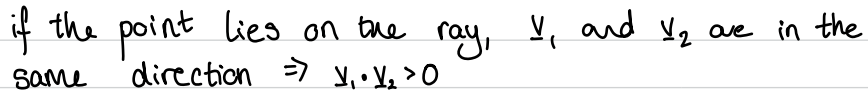
$v_2$  = direction vector of the ray

$$\text{Ray} = [a, b, \theta]$$

$$\text{Point} = [x, y]$$

$$v_1 = \begin{pmatrix} x - a \\ y - b \end{pmatrix}$$

$$v_2 = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$



- Find both lines from the segment and ray
- Find intersection of these lines

- Check if this point lies on both segment and ray

$$\alpha = \cos^{-1} \left( \frac{y_{in} \cdot \underline{s}}{|y_{in}| |\underline{s}|} \right)$$

$$\phi = \cos^{-1} \left( \frac{\underline{s} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{|\underline{s}|} \right)$$

Diagram illustrating ray reflection from a segment. A horizontal line represents the optical axis, with a point labeled  $(o')$  on the right. A ray, labeled  $v_{in} = \text{ray}$ , originates from a point  $o$  above the axis and reflects off a segment (orange line) at a point. The angle of incidence is  $\alpha$ . The reflected ray is labeled  $v_{out}$ . The angle between the incident ray and the normal is  $\frac{T-\alpha}{2}$ . The angle between the reflected ray and the normal is  $\frac{T+\alpha}{2}$ . The angle between the incident ray and the reflected ray is  $T$ . The angle between the incident ray and the optical axis is  $\phi$ . The angle between the reflected ray and the optical axis is  $\phi$ . The segment is labeled "Segment (S)". The distance from the point of reflection to the optical axis is labeled  $dy$ . The distance from the point of reflection to the optical axis is labeled  $dx$ .