Assignment 2

Task 2)

① $ax + by + c = 0$

② $dx + ey + f = 0$ $\qquad$ a, b, c, d, e, f are real constants

Rearranging ①: $by = -c - ax$

$\Leftrightarrow y = \frac{-c}{b} \frac{-a}{b} x$

Subbing into ②: $dx \frac{-ce}{b} \frac{-ae}{b} x + f = 0$

$\Leftrightarrow \left(\frac{bd - ae}{b}\right) x + \frac{bf - ce}{b} = 0$

$\Leftrightarrow \left(\frac{bd - ae}{b}\right) x = \frac{ce - bf}{b}$

$\Leftrightarrow x = \frac{ce - bf}{bd - ae}$

Subbing into ①: $\frac{ace - abf}{bd - ae} + by + c = 0$

$\Leftrightarrow by = -c + \frac{abf - ace}{bd - ae}$

$\Leftrightarrow by = \frac{-bcd + ace + abf - ace}{bd - ae}$

$\Leftrightarrow by = \frac{abf - bcd}{bd - ae}$

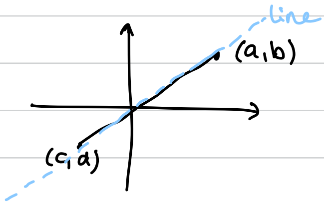
$$\Leftrightarrow \quad y = \frac{af - cd}{bd - ae}$$

Hence point of intersection at $\left(\frac{ce - bf}{bd - ae}, \frac{af - cd}{bd - ae} \right)$

There is no point of intersection and the lines are parallel when $bd - ae = 0$

Task 3)

a) Segment between points (a, b) and (c, d)



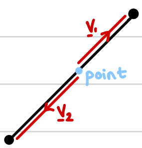
Eq^n of Line: $\frac{x - a}{c - a} = \frac{y - b}{d - b}$

$$(d-b)(x-a) = (y-b)(c-a)$$
$$(d-b)x - a(d-b) = (c-a)y - b(c-a)$$
$$(d-b)x + (a-c)y - ad + ab + bc - ab = 0$$
$$\underbrace{(d-b)x + (a-c)y + bc - ad = 0}_{\text{in required form } \alpha x + \beta y + \gamma = 0}$$

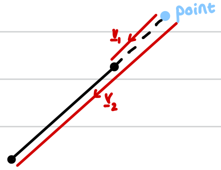
b) Assume point is on line from segment

$\underline{v}_1$ = vector from point to an endpoint of segment

$\underline{v}_2$ = vector from point to other endpoint of segment



if a point lies on the segment, $\underline{v}_1$ and $\underline{v}_2$ are in opposite directions $\Rightarrow$ $\underline{v}_1 \cdot \underline{v}_2 < 0$

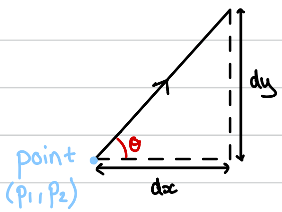


if a point does not lie on the segment, $\underline{v}_1$ and $\underline{v}_2$ are in same direction $\Rightarrow$ $\underline{v}_1 \cdot \underline{v}_2 > 0$

c) To find intersection of 2 segments:

- Find both lines from the segments
- Find intersection of these lines
- Check if this point lies on both segments

Task 4) First find eq^n line which ray lies on



$$\text{gradient} = \tan\theta = \frac{dy}{dx}$$

Eq^n Line from ray: $y - y_1 = m(x - x_1)$

$$mx - y - mx_1 + y_1 = 0$$

$$\tan\theta\, x - y - p_1 \tan\theta + p_2 = 0$$

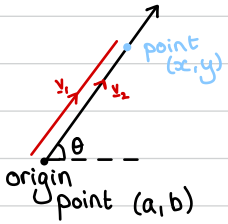
Check to see if a point lying on line extended from ray is also on the way

$\underline{v}_1$ = vector from ray origin point to checking point

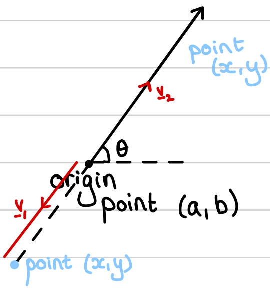
$\underline{v}_2$ = direction vector of the ray

Ray $= [[a, b], \theta]$ Point $= [x, y]$

$$\underline{v}_1 = \begin{pmatrix} x - a \\ y - b \end{pmatrix} \qquad \underline{v}_2 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$



if the point lies on the ray, $\underline{v}_1$ and $\underline{v}_2$ are in the same direction $\Rightarrow$ $\underline{v}_1 \cdot \underline{v}_2 > 0$



if the point doesn't lie on the line, $\underline{v}_1$ and $\underline{v}_2$ are in opposite directions $\Rightarrow$ $\underline{v}_1 \cdot \underline{v}_2 < 0$

To find intersection of segment and a ray

- Find both lines from the segment and ray
- Find intersection of these lines
- Check if this point lies on both segment and ray

Task 8) angle incidence, α = angle reflection

$$\alpha = \cos^{-1}\left(\frac{\underline{v}_{in} \cdot \underline{S}}{|\underline{v}_{in}||\underline{S}|}\right)$$

$$\phi = \cos^{-1}\left(\frac{\underline{S} \cdot \binom{1}{0}}{|\underline{S}|}\right)$$

Angle new ray $= \pi + \phi - \alpha$