

Part 2:

$$x=0; (x^2+4)y''+y=x$$

$$P(x) = \frac{0}{(x^2+4)} = 0$$

$$Q(x) = \frac{1}{(x^2+4)} = \frac{1}{4}$$

$\left. \begin{array}{l} P(x) = 0 \\ Q(x) = \frac{1}{4} \end{array} \right\} x \text{ is ordinary}$

$$y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + a_{n+1}x^{n+1} + a_{n+2}x^{n+2} \dots$$

$$y' = a_1 + 2a_2x + 3a_3x^2 + \dots + na_nx^{n-1} + (n+1)a_{n+1}x^n + (n+2)a_{n+2}x^{n+1} + \dots$$

$$y'' = 2a_2 + 6a_3x + 12a_4x^2 + \dots + (n)(n-1)a_nx^{n-2} + (n+1)(n)a_{n+1}x^{n-1} + (n+2)(n+1)a_{n+2}x^n + \dots$$

$$(x^2+4) [2a_2 + 6a_3x + 12a_4x^2 + \dots + (n)(n-1)a_nx^{n-2} + (n+1)(n)a_{n+1}x^{n-1} + (n+2)(n+1)a_{n+2}x^n + \dots] + [a_0 + a_1x + a_2x^2 + \dots + a_nx^n + a_{n+1}x^{n+1} + a_{n+2}x^{n+2} + \dots] = x$$

$$2a_2x^2 + 8a_2 + 6a_3x^3 + 24a_3x + 12a_4x^4 + 48a_4x^2 + \dots + (n-1)(n)a_nx^n + 4(n-1)(n)a_nx^{n-2} + (n)(n+1)a_{n+1}x^{n+1} + 4(n)(n+1)a_{n+1}x^{n-1} + (n+1)(n+2)a_{n+2}x^{n+2} + 4(n+1)(n+2)a_{n+2}x^n + a_0 + a_1x + a_2x^2 + \dots + a_nx^n + a_{n+1}x^{n+1} + a_{n+2}x^{n+2} + \dots - x = 0$$

We only care about the x^n terms so here is our equation:

$$(n-1)(n)a_n + 4(n+1)(n+2)a_{n+2} + a_n = 0$$

$$a_{n+2} = \frac{-(n^2 - n + 1)a_n}{4(n+1)(n+2)}$$

$$n=0, a_2 = -\frac{1}{8}a_0$$

$$n=1, a_3 = -\frac{1}{24}a_1$$

$$n=2, a_4 = -\frac{3}{48}a_2 = \frac{1}{128}a_0$$

$$n=3, a_5 = -\frac{7}{80}a_3 = \frac{7}{960}a_1$$

$$n=4, a_6 = -\frac{13}{120}a_4 = -\frac{13}{15360}a_0$$

$$n=5, a_7 = -\frac{21}{168}a_5 = -\frac{7}{15360}a_1$$

$$n=6, a_8 = -\frac{31}{224}a_6 = \frac{403}{3440640}a_0$$

$$n=7, a_9 = -\frac{43}{280}a_7 = \frac{301}{4423680}a_1$$

$$n=8, a_{10} = -\frac{57}{360}a_8 = -\frac{7657}{412876800}a_0$$

$$y = a_0 \left[1 - \frac{1}{8}x^2 + \frac{1}{128}x^4 - \frac{13}{15360}x^6 + \frac{403}{3440640}x^8 - \frac{7497}{41287680}x^{10} \right] + \dots$$

$$+ a_1 \left[x - \frac{1}{24}x^3 + \frac{7}{1920}x^5 - \frac{7}{16380}x^7 + \frac{301}{4423680}x^9 \right]$$