Power Formula Sheet: EEE3031S & EEE4099F

Induction Machines

$a = \frac{N_1}{N_2}$
$E_1 = aE_2$
$I' = \frac{I}{a}$
$Z' = a^2 Z$
$P_{ag} = \frac{R_2'}{s} = \frac{R_2'}{s} + R_2'$
$P_{ag} = \frac{P_{mech}}{(1-s)}$
$\phi_p = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B(\theta) lr d\theta = 2B_{max}lr$
$n_s = \frac{120 f_1}{p} RPM$
$s = \frac{n_s - n}{n_s}$
$f_2 = sf_1$
$n_2 = sn_s$

IEEE Equivalent Circuit

$"V_1" = I_1(R_1 + jX_1) + E_1$
$"E_1" = aE_2 = I_\phi jX_m + I_2'(\frac{R_2'}{s} + jX_2')$

Thevenin Equivalent Circuit

$V_{th} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} V_1$
$k_{th} = \frac{X_m}{X_1 + X_m}$
$V_{th} \simeq k_{th} V_1$
$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$
$R_{th} \simeq k_{th}^2 V_1$
$X_{th} \simeq X_1$
$I_2' = \frac{V_{th}}{(R_{th} + \frac{R_2'}{s}) + j(X_{th} + X_2')}$

No-Load Formulae

$\frac{R_2'}{s} = \infty$
$X_{NL} = X_1 + X_m$

$V_p = V_1 = \frac{V_L}{\sqrt{3}}$
$I_{NL} = I_1 = I_L$
$Z_{NL} = \frac{V_1}{I_1}$

$P_{rot} = P_{NL} - 3I_1^2 R_1$
$R_{NL} = \frac{P_{NL}}{3I_1^2}$
$X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2}$

Blocked-Rotor Formulae

$s = 1$
$X_m = 0$

$V_p = V_1 = \frac{V_L}{\sqrt{3}}$
$I_{BL} = I_1 = I_L$
$Z_{BL} = \frac{V_1}{I_1}$

$R_{BL} = \frac{P_{BL}}{3I_1^2}$

$R_2' = R_{BL} - R_1$
IEEE Recommended Form
$R = R_{BL} - R_1$
$R_2' = (\frac{X_2' + X_m}{X_m})^2 R$

$X_{BL@f_{NL}} = \sqrt{Z_{BL}^2 - R_{BL}^2}$
$X_{BL@f} = X_{BL@f_{NL}} \times \frac{f_2}{f_1}$
$X_{BL@f} = X_1 + X_2'$
$X_1 = X_2' = \frac{X_{BL@f}}{2}$

Performance Formulae

$T, P = T_{ph}, P_{ph}$

$P_{mech} = T_{mech}\omega_{mech} = I_2^2 \frac{R_2}{s}(1-s)$
$\omega_{mech} = \frac{2\pi n_s}{60} = (1-s)\omega_{syn}$
$T_{mech} = \frac{1}{\omega_{syn}} \frac{V_{th}^2}{(R_{th} + \frac{R_2'}{s})^2 + (X_{th} + X_2')^2} \frac{R_2}{s}$

$s_{Tmax} = \frac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}}$
$T_{max} = \frac{1}{2\omega_{syn}} \frac{V_{th}^2}{R_t h + \sqrt{R_{th}^2 + (X_t h + X_2')^2}}$
$\frac{Tmax}{T} = \frac{s_{Tmax}^2 + s^2}{2s_{Tmax}s}$

$Z_1 = R_1 + jX_1 + \frac{jX_m(\frac{R_2'}{s} + jX_2')}{\frac{R_2'}{s} + j(X_m + X_2')} = |Z_1|\angle\theta_1$
$I_1 = \frac{V_1}{Z_1} = I_\phi + I_2'$

Efficiency Formulae

$PF = cos(\theta_1)$
$P_{in} = 3V_1 I_1 cos(\theta_1)$
$P_1 = 3I_1^2 R1$
$P_2 = 3I_2^2 R2$

$Eff_{ideal} = \frac{P_{out}}{P_{in}} = 1 - s$
$P_{ag} = P_{in}$
$P_2 = sP_{ag}$
$P_{out} = P_{mech} = P_{ag}(1-s)$

$M = +s$
$G = -s$
$P = s > 1$

Synchronous Formulae

$PF = cos(\phi)$
$S = P + jQ$

$X_s = X_{ar} + X_{al}$

$V_t^{\rightarrow} = E_f^{\rightarrow} + I_a^{\rightarrow} + jI_a^{\rightarrow} X_s$
$E_f^{\rightarrow} = |E_f^{\rightarrow}|\angle - \delta$

$P_{in} = 3V_t I_a cos(\phi)$
$E_f = V_t - jI_a X_s$
$P = \frac{3V_t E_f}{X_s} sin(\delta)$
$E_f sin(\delta) = BforP = A$
$EI_a cos(\phi) = CforP = A$
$E_A = \sqrt{(V_T cos\phi + I_a R_a)^2 + (V_T sin\phi + I_a X_s)^2}$

$E_f^{\rightarrow} = V_t^{\rightarrow} + I_a^{\rightarrow} + jI_a^{\rightarrow} X_s$
$E_f^{\rightarrow} = |E_f^{\rightarrow}|\angle\delta$

$V_t = |V_t|\angle 0^0$
$Z_s = R_a + jX_s = |Z|\angle\theta_s$

$X_s = |X_s|\angle 90^0$
$E_f^* = |E_f|\angle - \delta$

$S_p = V_t I_a^*$
$I_a^* = |\frac{E_f - V_t}{Z_s}|^* = \frac{E_f^*}{Z_s^*} - \frac{V_t^*}{Z_s^*}$

$S_p = ..., P \Rightarrow cos(\gamma), Q \Rightarrow sin(\gamma)$

$R_a = 0 \Rightarrow \theta_s = 90^0 \Rightarrow P_{ag} = P_{3\phi}$
$P_{max} = \frac{2|V_t||E_f|}{|X_s|}$
$P_{3\phi} = P_{max} sin(\delta) W$
$Q_{3\phi} = P_{max} cos(\delta) - 3\frac{|V_t|^2}{|X_s|} VAR$

$\omega_{syn} = \frac{n_{syn} 2\pi}{60}$
$T = \frac{P_{3\phi}}{\omega_{syn}} = T_{max} sin(\delta) Nm$
$T_{max} = \frac{P_{max}}{\omega_{syn}}$

Machines

$B = \mu H$ $B = \mu_0 H = \frac{\mu_0 Ni}{2lg} = Bg$
$H = \frac{Ni}{l}$
$PolePitch = 180^0 ed$
$SlotPitch = \frac{720^0}{slots}$
$CoilPitch = 7/9 \times PolePitch$
$B(\theta) = B_{1max} cos(\theta) + B_{3max} cos(2\theta) + ...$

$\alpha = Angle/Pole/Phase = SlotPitch...$
$n = Slots/Pole/Phase$
$K_d = \frac{sin(n\frac{\alpha}{2})}{nsin(\frac{\alpha}{2})}$

$K_p = cos(\frac{\gamma}{2})$
$\gamma = 180^0 - CoilPitch$

$K_w = K_d \times K_p$
$K_{wh} = (\frac{sin(hn\frac{\alpha}{2})}{n \cdot sin(h\frac{\alpha}{2})}) \cdot cos(h\frac{\gamma}{2})$

$E_h = 4.44 hfN\phi_1 K_{wh}$
$N = turns/phase$ and $\phi_h = \frac{B_{max}\%h}{h}$

$E_{LN} = \sqrt{E_1^2 + E_3^2 + E_5^2 + ...}$
$E_{LL} = \sqrt{3}\sqrt{E_1^2 + E_5^2 + E_7^2 + ...}$

Machines

Important Relationships

$\omega_s = k_7 f$
$s = \frac{\omega_s - \omega_r}{\omega_s}$
$f_{sl} = sf$
$\%P_r = \frac{f_{sl}}{f - f_{sl}}$
$V_s \simeq k_3 \Phi_{ag} f$
$I_r \simeq k_5 \Phi_{ag} f_{sl}$
$T_{em} \simeq k_6 \Phi_{ag}^2 f_{sl}$
$I_m = k_8 \Phi_{ag}$
$I_s \simeq \sqrt{I_m^2 + I_r^2}$

Motor Drives

$\frac{T_m}{T_L} = \frac{\omega_L}{\omega_m} = \frac{\theta_L}{\theta_m} = \frac{n_m}{n_L} = a$
$T_{em} = \frac{\dot{\omega}}{a}[J_m + a^2 J_L] + aT_{WL} + \frac{\dot{\omega}}{a}[B_m + a^2 B_L] = J_{eq}\dot{\omega}_m + B_{eq}\omega_m + T_{Weq}$

$\frac{T_m}{F_L} = \frac{v_L}{\omega_m} = \frac{x_L}{\theta_m} = \frac{s}{2\pi} = a$
$T_{em} = \frac{\dot{v}_L}{a}[J_m + J_s + a^2(M_T + M_W)] + aF_{WL}$

$P_R = R_M I_{rms}^2$
$I_{rms}^2 = \frac{\sum_{k=1}^{m} I_k^2 t_k}{t_period}$
$T_{em,rms}^2 = k_1 I_{rms}^2$
$P_R = k_2 T_{em,rms}^2$
$P_{loss} = P_R + P_{FW} + P_{EH} + P_s + P_{stray}$
$\Delta\Theta = P_{loss} R_{TH} [C^o]$

$\frac{di}{dt} = \frac{v - e}{L}$

General

$P = T\omega$

$E_{max} = \sqrt{2}e_{rms}$

$Displacement = \frac{360^o}{M}$

$V_{D\alpha} = E_{max}\frac{sin(\pi/M)}{\pi/M}cos(\alpha)$

$F_{rms} = \left(F_0^2 + \sum_{h=1}^{\infty} F_h^2\right)^{1/2}$

$$\begin{aligned}\%THD_i &= 100\frac{I_{dis}}{I_{s1}} \\ &= 100\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} \\ &= \sum_{h=1}\left(\frac{I_{sh}}{I_{s1}}\right)^2\end{aligned}$$

Line-Frequency Diode Rectifiers (AC-DC)

$V_L = L\frac{di}{dt}$

Idealized diode bridge rectifier

$V_{do} = \frac{2}{\pi}\sqrt{2}V_s$

$I_{s1} = \frac{2}{\pi}\sqrt{2}I_d$

$I_{sh} = 0(even - h)\&I_{s1}/h(odd - h)$

$DPF = 1.0$

$PF = DPF\frac{I_{s1}}{I_s} = 0.9$